Governance Through Exit and Voice: A Theory of Multiple Blockholders*

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September 29, 2008

Abstract

Traditional theories argue that governance is strongest under a single large blockholder, as she has strong incentives to undertake value-enhancing interventions (engage in "voice"). However, most firms are held by multiple small blockholders. This paper shows that, while such a structure generates free-rider problems that hinder voice, the same co-ordination difficulties strengthen a second governance mechanism: disciplining the manager through trading (engaging in "exit"). Since multiple blockholders cannot co-ordinate to limit their orders and maximize combined profits, they trade competitively, impounding more information into prices. This makes the threat of disciplinary exit more credible, inducing higher managerial effort. The optimal blockholder structure depends on the relative effectiveness of manager and blockholder effort, the complementarities in their outputs, liquidity, monitoring costs, and the manager's contract.

Keywords: Multiple blockholders, corporate governance, market efficiency, exit, voice, free-rider problem, Wall Street Rule, voting with your feet

JEL Classification: D82, G14, G32

*We thank Anat Admati, Itay Goldstein, Uli Hege, Rich Mathews, Holger Mueller, Stew Myers, Tom Noe, Jun Qian, Rafael Repullo, A. Subrahmanyam, Alex Wagner, Jiang Wang and seminar participants at the 2008 European Finance Association, 2008 Texas Finance Festival, 2008 China International Conference in Finance, 2008 European Winter Finance Conference, 2007 Conference on Financial Economics and Accounting at NYU, MIT, Notre Dame, University of Illinois at Urbana-Champaign, and Wharton for helpful comments, and Qi Liu for excellent research assistance. AE gratefully acknowledges the Goldman Sachs Research Fellowship from the Rodney White Center for Financial Research. E-mails: aedmans@wharton.upenn.edu and manso@mit.edu.

1 Introduction

Corporate governance can have substantial effects on firm value. Through ensuring that managers act in shareholders' interest, it reduces the agency costs arising from the separation of ownership and control. In turn, traditional theories argue that concentrated ownership is critical for effective governance, since only large investors have incentives to monitor the manager and, if necessary, intervene to correct value-destructive actions.

However, most large firms in reality have multiple small blockholders (see, e.g., Zwiebel (1995), Barca and Becht (2001), Faccio and Lang (2002), Maury and Pajuste (2005), Laeven and Levine (2007), Holderness (2008), and Gregoric et al. (2008)). Such a structure appears to be suboptimal for governance, as splitting equity between numerous shareholders leads to a free-rider problem: each investor individually has insufficient incentives to bear the cost of monitoring, and shareholders cannot co-ordinate to share this cost.

One interpretation is that policymakers should encourage more concentrated stakes, as suggested by existing models. This paper instead demonstrates that a multiple blockholder structure may be optimal for governance. While splitting a block reduces the effectiveness of direct intervention ("voice"), we show that it increases the power of a second governance mechanism: "exit".[1] By trading on private information, blockholders move the stock price towards fundamental value, and thus cause it to more closely reflect the effort exerted by the manager to enhance firm value. Through following the "Wall Street Rule" of "voting with their feet" and selling to liquidity traders if the manager has shirked, blockholders drive down the stock price. This reduces the compensation of an equity-aligned manager, thus punishing him ex post for his inactivity. However, such a mechanism only elicits effort ex ante if it is dynamically consistent. Once effort has been exerted, blockholders cannot change the manager's action and are only concerned with maximizing their trading profits. A single blockholder will strategically limit her order to reduce the revelation of her private information. This optimizes her profit, but also lowers the extent to which prices reflect fundamental value and thus managerial effort. By contrast, multiple blockholders trade aggressively to compete for profits, as in a Cournot oligopoly. Total quantities (here, trading volumes) are higher than under monopoly, so more information is impounded in prices. Multiple blockholders thus serve as a commitment device to reward or punish the manager ex post for his actions.

The co-ordination problems and externalities created by splitting a block play oppos-

[1]Prior papers on blockholder trading focus on the "Wall Street Rule" (the possibility of blockholder exit), rather than additional purchases. For example, Hirshman's (1970) book is titled "Exit, Voice, and Loyalty", and the models of Admati and Pfleiderer (2008) and Edmans (2008) only analyze block disposal, not enhancement. Although the blockholder can buy as well as sell in this paper, we use the term "exit" to describe the blockholder's influence on managerial decisions through her trading (in either direction), to be consistent with prior literature.

ing roles in "voice" and "exit." For "voice", the externalities are positive: intervention improves the value of other shareholders' stakes, but this effect is not internalized by the individual blockholder. Since these externalities are positive, there is "too little" intervention with multiple blockholders. For "exit", the externalities are negative. Higher trading volumes reveal more information to the market maker, leading to a less attractive price for other informed traders. Blockholders trade "too much" from the viewpoint of maximizing combined profits. However, firm value does not depend on trading profits as they are a mere transfer from liquidity traders to blockholders. Instead, "too much" trading is beneficial as it increases price informativeness and induces effort ex ante. The 2007 hedge fund crisis is a prominent example of the substantial price changes that result from multiple investors trading in the same direction.

We derive an interior solution for the optimal number of blockholders that maximizes firm value. This optimum arises from a trade-off between voice and exit: fewer blocks maximize intervention incentives, but more blocks increase trading. Therefore, it is increasing in the value created by managerial effort and decreasing in the value created by blockholder intervention. If blockholders are passive and non-interventionist, as is the case for most mutual funds, a large number is optimal. By contrast, if investors contribute significantly to the firm's operations, such as venture capitalists, concentrated ownership is efficient. The optimal number is also increasing in the manager's alignment with the stock price, since this augments the importance of stock price informativeness for the manager's effort choice.

In the core model, blockholders are automatically informed about firm value. We extend the model to allow for costly information acquisition. In equilibrium some blockholders may decide to stay uninformed, because their trading profits are insufficient to justify monitoring. Since uninformed blockholders do not engage in exit, and reduce intervention by diluting ownership, they unambiguously reduce firm value. Thus, the optimal number of blockholders is bounded above, to ensure that competition in trading is sufficiently low that trading profits are adequate to motivate all blockholders to acquire information. If trading profits (net of monitoring costs) increase, this bound is weakened and so the optimal number of blockholders rises. This in turn occurs if market liquidity and the blockholders' informational advantage increase, and monitoring costs fall.

While the core model assumes that blockholder and manager efforts are perfect substitutes, with independent effects on firm value, an additional extension analyzes complementarities. One case involves negative complementarities, where firm value depends on the higher of the output levels of the two parties rather than the combined output level. This may occur if the blockholders correct managerial shirking: firm value can be high even if the manager does not work, as long as the blockholders exert effort. Since only the higher output level matters, the optimum is determined entirely by the more effective action, and ignores trade-off considerations with the less effective action. The optimal number of blockholders is therefore either very low (if blockholder effort is relatively effective) or very high (if managerial effort is relatively effective).

An opposite case is perfect positive complementarities, where firm value depends on the minimum output level. Since managerial effort is only productive if it is accompanied by high blockholder effort (and vice versa), the optimal number of blockholders balances the output levels of the manager and blockholders. The effect of effort productivity changes direction: the optimal number is now decreasing in the effectiveness of managerial effort and increasing in the effectiveness of blockholder effort. If managerial effort is ineffective, a high number of blockholders is necessary to "boost" managerial output so that it is at a similar level to blockholder output.

We show that the firm value optimum may differ from the socially optimal number of blockholders that maximizes total surplus (firm value net of effort costs), and the private optimum that would be chosen by the blockholders if they retraded their stakes to maximize their combined net payoffs. However, the comparative statics with respect to the effectiveness of manager and blockholder effort are the same for all three optima. This is important for the paper's empirical predictions, since the private optimum is most likely to be observed in reality. We close by discussing these empirical implications, in particular those most specific to our model. Most previous justifications of the multiple blockholder structure argues that it reduces private benefit consumption. Here, rather than extracting rents, blockholders play a positive role, by directly improving firm value through intervention and indirectly inducing managerial effort through trading. The model thus generates predictions for how the optimal number of blockholders depends on the effectiveness of manager and blockholder effort.

More generally, the model suggests a different way of thinking about the interaction between multiple blockholders that can give rise to new avenues for empirical research. Prior models perceive them as entities that compete for private benefits, and so existing empirical studies of multiple blockholders have focused on rent extraction (e.g. Laeven and Levine (2007)). Our paper suggests that future analyses may be motivated by conceptualizing them as informed traders, competing for trading profits. Blockholders therefore impact price efficiency, and their value added depends on microstructure factors such as liquidity. Two recent examples of such papers are Smith and Swan (2008) and Gallagher, Gardner and Swan (2008), which show that trading by multiple blockholders improves firm value by disciplining management.

Finally, a number of empirical papers use total institutional ownership as a measure of market efficiency, since institutions are typically more informed than retail investors. However, price efficiency requires not only that investors be informed, but that they impound their information into prices. Therefore, the number of informed shareholders is also a relevant factor. Similarly, total institutional ownership is often employed as a proxy for corporate governance, but the structure of such ownership is also an important determinant.

This paper is organized as follows. Section 2 reviews related literature. Section 3 presents the model and analyzes the effect of blockholder structure on both "voice" and "exit". Section 4 derives the optimal number of blockholders that maximizes firm value, total surplus, and the blockholders' payoff. Section 5 extends the model to analyze costly information acquisition, complementarities and differences in the manager's contract, Section 6 considers empirical implications, and Section 7 concludes. The Appendix contains all proofs not in the main text.

2 Literature Review

The vast majority of blockholder models involve the large shareholder adding value through direct intervention, or "voice" as termed by Hirshman (1970). This can involve implementing profitable investment projects and strategies, or overturning an inefficient managerial action. In Shleifer and Vishny (1986), Admati, Pfleiderer, and Zechner (1994), Maug (1998), Kahn and Winton (1998) and Mello and Repullo (2004), a larger block is unambiguously more desirable as it reduces the free-rider problem and maximizes incentives to intervene.

By contrast, Burkart, Gromb and Panunzi (1997) show that the optimal block size is finite if blockholder intervention can deter managerial initiative ex ante. Bolton and von Thadden (1998) and Faure-Grimaud and Gromb (2004) achieve a finite optimum through a different channel, as too large a block reduces free float. While these papers only consider a single shareholder[2], Pagano and Roell (1998) point out that if the finite optimum is lower than the total amount of external financing required, the entrepreneur will need to raise funds from additional shareholders. Although this leads to a multiple blockholder structure, the extra blockholders play an entirely passive role: they are merely a "budget-breaker" to provide the remaining funds. Replacing the additional blockholders by creditors or dispersed shareholders would have the same effect. In this paper, all blockholders play an active role. In Winton (1993), as in our model, all investors actively monitor, and total monitoring is highest under a single large shareholder owing to the free-rider problem. A multiple blockholder structure arises as investors face wealth constraints, rather than from concerns over price efficiency.

Attari, Banerjee and Noe (2006), Faure-Grimaud and Gromb (2004), and Aghion, Bolton and Tirole (2004) feature a blockholder who can only intervene and a speculative agent who can only trade. In the first paper, the speculative investor may sell even in

[2]Bolton and von Thadden (1998) mention that their model might be extended to incorporate more than one non-atomistic shareholder, but suspect that it is "dominated either by full dispersion or by a [single blockholder] structure." Hence they do not derive multiple blockholders as being optimal.

the absence of negative information, to reduce stock prices and trigger intervention by the blockholder. In the last two papers, more accurate prices induce "voice", but the blockholder does not trade and thus has no effect on price efficiency.

Two recent papers by Admati and Pfleiderer (2008) and Edmans (2008) analyze an alternative mechanism through which blockholders can add value: "exit". Informed trading causes prices to more accurately reflect fundamental value, in turn inducing the manager to undertake actions that enhance value.[3] Lowenstein (1988) argues that governance through exit is particularly important and common among U.S. investors, since they often face legal and institutional hurdles to intervention (see, e.g., Black (1990), Bebchuk (2007), and Becht et al. (2008)). Admati and Pfleiderer and Edmans both consider a single blockholder and do not feature "voice". To our knowledge, this is the first theory that analyzes both governance mechanisms of exit and voice, and the tradeoffs between them.

Most existing theories of multiple blockholders focus on their consumption of private benefits and the resulting control contests. In Zwiebel (1995), the final shareholding structure represents the outcome of a power struggle as blockholders compete to extract rents. Here, the number of blockholders is optimally chosen to maximize firm value. In Bennedsen and Wolfenzon (2000), Müller and Wärneryd (2001), Bloch and Hege (2003), Maury and Pajuste (2005) and Gomes and Novaes (2006), blockholder structure is also optimal, but designed to limit private benefit consumption. By studying different blockholder actions, our model generates a number of different empirical predictions. Here, blockholders create positive value rather than extracting rents, and so the relative effectiveness of blockholder and manager effort affects the optimal shareholding structure. In addition, the implications regarding the effect of blockholder structure on market efficiency, and the impact on firm value of microstructure features such as liquidity, are specific to a model in which blockholders exert governance through trading.

Like us, Noe (2002) models multiple blockholders as enhancing firm value rather than extracting private benefits. Owing to the free-rider problem, each blockholder has an insufficient stake to justify intervention, but supplements her return through trading profits. However, blockholder exit does not exert governance, since stock price informativeness has no effect on managerial effort.[4]

[3]In Holmstrom and Tirole (1993), Calcagno and Heider (2007) and Ferreira, Ferreira and Raposo (2008), price efficiency is also desirable as it helps monitor management, but is not affected by blockholder structure. In Fulghieri and Lukin (2001), efficient prices reduce the cost of raising funds for a high-quality firm.

[4]Similarly, Maug (1998, 2002), Kahn and Winton (1998), Mello and Repullo (2004), Brav and Mathews (2008), and Kalay and Pant (2008) allow the blockholder either to intervene or to sell her stake (in the last two papers, the intervention occurs through voting). However, exit again does not exert governance, and so these papers are theories of voice only. Duan (2007) empirically studies the choice between exit and voice (through voting).

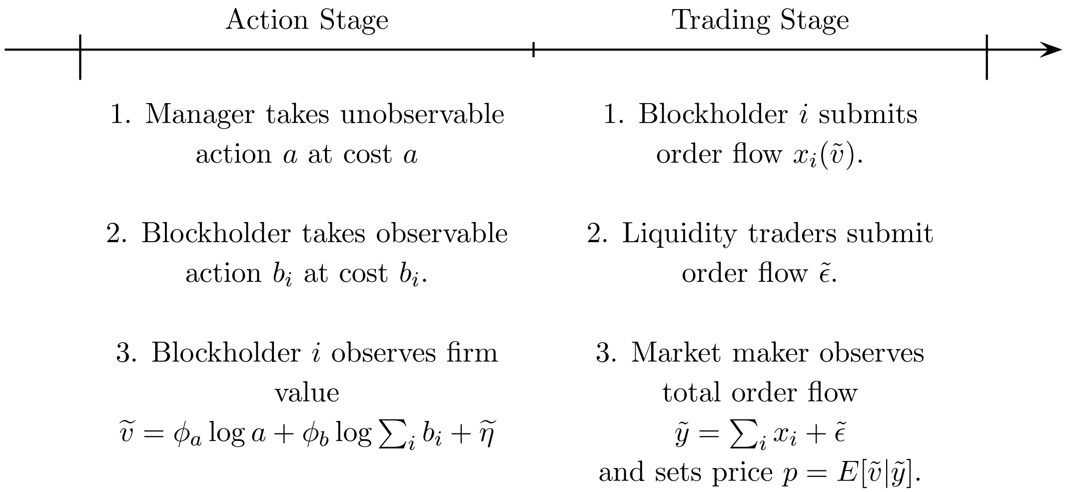

Figure 1: Timeline of the model

Finally, Bolton and Scharfstein (1996) also demonstrate that free-rider problems among investors can improve firm value. A multiple creditor structure can dominate a single lender, since the resulting co-ordination problems hinder efficient renegotiation in default. This deters the manager from strategically defaulting, and thus makes creditors more willing to lend. In our paper, the benefits of co-ordination problems manifest through informed trading and the effect on stock prices.

3 Model and Analysis

Our model consists of a game between the manager, a market maker and the I blockholders of the firm. The game has two stages, and the timeline is given in Figure 1.

In the first stage, the manager and blockholders take actions that affect firm value. Firm value is given by

$$\tilde{v} = \phi_a \log a + \phi_b \log \sum_i b_i + \widetilde{\eta}, \tag{1}$$

where $a \in [0, \infty)$ represents the action taken by the manager, $b_i \in [0, \infty)$ represents the action taken by blockholder i, and $\tilde{\eta}$ is normally distributed noise with mean zero and variance σ_η^2. The manager incurs personal cost a when taking action a, while each blockholder i incurs personal cost b_i when taking action b_i.[5] The manager's action is

[5]Firm value depends on the logarithm of the combined blockholder effort level, and the action has a linear cost to each blockholder. This functional form ensures that adding blockholders does not change the available technology (in addition, it leads to substantial tractability). The common assumption of a quadratic cost and a linear effect of b_i on $\tilde{v}$ is inappropriate here: with a convex cost function, the blockholders' technology would improve if there are multiple small blockholders, since each would be operating at the low marginal cost part of the curve. A single blockholder would be able to reduce monitoring costs by dividing herself up into multiple small "units", and increase total effort. Instead,

broadly defined to encompass any decision that improves firm value but is personally costly, such as exerting effort or forgoing pet projects. Similarly, the blockholder's action can involve exerting effort (e.g. advising the manager) or choosing not to extract private benefits.[6] The parameter ϕ_a (ϕ_b) measures the productivity of manager (blockholder) effort. We use the term "effort" to refer to a and b_i and "output" to refer to $\phi_a \log a$ and $\phi_b \log \sum_i b_i$, i.e. effort scaled by its productivity.

In the core model, the manager's and blockholders' actions are perfect substitutes, i.e. have independent effects on firm value. The productivity of the manager's effort does not depend on the level of blockholder effort, and vice-versa. This benchmark case appears to be a plausible specification for most firms and manager and blockholder actions: for example, extraction of private benefits by blockholders is detrimental to firm value regardless of the manager's effort. However, in some situations, there may be positive or negative complementarities between the manager's and blockholders' actions. These are analyzed in Section 5.2.

Action a is privately observed by the manager, while actions b_i are publicly observed. The assumption that a is private is a feature of any moral hazard problem. By contrast, the assumption that b_i is public is made purely for tractability. The effects of I on competition in trading and free-rider problems in intervention are independent of whether or not b_i is observable.[7]

We normalize the number of shares outstanding to 1. The risk-neutral manager owns α shares of the firm, and each risk-neutral blockholder holds β/I shares, where $\alpha + \beta < 1$.[8] Our model focuses exclusively on the optimal number of blockholders (I) among which a given level of concentrated ownership is divided, and thus holds the amount of concentrated ownership (β) constant. This separates our paper from previous literature that analyzes the optimal β. For example, Shleifer and Vishny (1986) and Maug (1998) show that a higher β raises incentives to intervene, but this must be traded off against the potential reduction in managerial initiative (Burkart, Gromb and Panunzi (1997)) and free float (Bolton and von Thadden (1998), Faure-Grimaud and

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the linear cost means that the monitoring technology is constant, and so there are no mechanical reduction in monitoring costs from splitting a block.

[6]See Barclay and Holderness (1989) for a description of the private benefits that blockholders can extract. Unlike in earlier theories of multiple blockholders, here blockholders do not compete (with either each other or the manager) to consume private benefits.

[7]If b_i is public, the analysis becomes significantly more complex as it would involve mixed strategies. As in Maug (1998, 2002), each blockholder will randomize between intervention and non-intervention, and the market maker's pricing rule will reflect this.

[8]We could also extend the model by introducing managerial risk aversion and endogenizing α. Then, the increased price efficiency that results from a greater number of blockholders reduces the risk imposed by aligning the manager with equity value. The optimal α is greater, further inducing managerial effort. Since the effect of price efficiency on α is featured in Holmstrom and Tirole (1993) and Calcagno and Heider (2007) and further reinforces the effects in this paper, we hold α constant.

Gromb (2004), Edmans (2008)). In this model, free float is fixed at $1-\alpha-\beta$ and plays no role.

In the second stage of the game, the blockholders, noise traders, and a market maker trade the firm's equity. As in Admati and Pfleiderer (2008), each blockholder is assumed to observe firm value $\tilde{v}$ perfectly, while noise traders are uninformed. The blockholders' superior information can be motivated by a number of underlying assumptions. Their large stakes may give them greater access to information: given their voting power, management will be more willing to meet with them. Even if blockholders have the same access to information as other investors, they may be more informed as they have stronger incentives to engage in costly analysis of this information. For example, equity analysts and mutual funds undertake detailed analysis of public information to form their own financial projections and valuations. Edmans (2008) microfounds this relationship between block size and informedness. If there are short-sales constraints (or any non-trivial short-sales costs), blockholders can sell more if information turns out to be negative. Since information is more useful to them, they have a greater incentive to acquire it in the first place.[9] Our results are qualitatively unchanged if each blockholder obtains an imperfect signal of $\tilde{v}$: we only require that blockholders have superior information to atomistic investors.[10]

A number of empirical studies indeed find that blockholders are better informed than other investors and impound their information into prices through trading. Parrino, Sias and Starks (2003) and Chen, Harford and Li (2007) find that blockholders have superior information about negative firm prospects, which they use to vote with their feet. Sias, Starks and Titman (2006) show that such blockholder trading has a causal effect on stock prices; similarly, Scholes (1972) and Mikkelson and Partch (1985) demonstrate that the negative stock price reaction to secondary block distributions is due to information, rather than the sudden increase in supply or a reduction in expected blockholder monitoring.

After observing $\tilde{v}$, each blockholder submits a market order $x_i(\tilde{v})$. Noise traders submit market orders with a normally distributed net quantity $\tilde{\epsilon}$, with mean zero and

[9] Blockholders' ability to intervene does not preclude them from trading on information. Investors can exert voice even in the absence of a board seat (which might subject them to insider trading rules): for example, institutional investors often "jawbone" management into adopting a particular corporate strategy or cutting back on projects, but can still trade freely. In Maug (1998, 2002), Kahn and Winton (1998), and Mello and Repullo (2004), the blockholder can also both trade and intervene.

[10] We could also allow signal precision to be increasing in the blockholder's individual stake and thus fall with I. This does not change any of the results as long as signal precision does not decline sufficiently rapidly with I to outweigh the beneficial effect of greater I on competition in trading. The results are in the Online Appendix. The core model's assumption that signal precision is independent of I does not mean that introducing additional blockholders increases the amount of information in the economy. A single blockholder already has a perfect signal of fundamental value. Instead, the results arises entirely from competition in trading.

variance σ_ε^2, where ε and η are independent. We use the term "liquidity" to refer to the standard deviation of noise trader demand, σ_ε. After observing total order flow $\tilde{y} = \sum_i \widetilde{x}_i + \tilde{\epsilon}$, the market maker determines the price $\tilde{p}$ and trades the quantity necessary to clear the market. Due to perfect competition, the market maker sets $\tilde{p}$ so that he earns zero profits, i.e. the price equals expected firm value given the order flow.

The manager's objective is to maximize the market value of his shares less the cost of effort, i.e. $\alpha\tilde{p} - a$. Each blockholder's objective is to maximize her trading profits, plus the fundamental value of her shares, less her cost of effort.

We solve for the equilibrium of the game by backward induction.

3.1 The Trading Stage

To proceed by backward induction, we take the decisions a of the manager and b_i of the blockholders as given. (In equilibrium, these conjectures will be correct and equal the actions derived subsequently in Proposition 3.) The trading stage of the game is similar to the speculative trading model of Kyle (1985) and its extensions to multiple informed investors.[11]

Proposition 1 *(Trading Equilibrium): The unique linear equilibrium of the trading stage is symmetric and has the form:*

$$x_i(\tilde{v}) = \gamma(\tilde{v} - \phi_a \log a - \phi_b \log \textstyle\sum_i b_i) \quad \forall i \tag{2}$$

$$p(\tilde{y}) = \phi_a \log a + \phi_b \log \textstyle\sum_i b_i + \lambda\tilde{y}, \tag{3}$$

where

$$\lambda = \frac{\sqrt{I}}{I+1}\frac{\sigma_\eta}{\sigma_\epsilon} \tag{4}$$

$$\gamma = \frac{1}{\sqrt{I}}\frac{\sigma_\epsilon}{\sigma_\eta}, \tag{5}$$

and a and b_i are the market maker's and blockholders' conjectures regarding the actions. Each blockholder's trading profits are given by

$$\frac{1}{\sqrt{I}(I+1)}\sigma_\eta\sigma_\epsilon. \tag{6}$$

Proof If the market maker uses a linear pricing rule of the form $p(y) = \mu + \lambda y$, blockholder i maximizes:

$$E[(\tilde{v} - \mu - \lambda y)x_i \,|\, \tilde{v} = v] = (v - \mu - \lambda \sum_{j \neq i} x_j)x_i - \lambda x_i^2.$$

[11] See, for example, Kyle (1984), Admati and Pfleiderer (1988), Holden and Subrahmanyam (1992), and Foster and Viswanathan (1993).

This maximization problem yields

$$x_i(v) = \frac{1}{\lambda}[v - \mu - \lambda \sum_j x_j(v)] \quad \forall i.$$

The strategies of the blockholders are symmetric and we thus have

$$x_i(v) = \frac{1}{(I+1)\lambda}(v - \mu) \quad \forall i.$$

The market maker takes the blockholders' strategies as given and sets

$$p(y) = E[\tilde{v}|y]. \tag{7}$$

Using the normality of $\tilde{v}$ and $\tilde{y}$ yields

$$\lambda = \frac{\sqrt{I}}{I+1}\frac{\sigma_\eta}{\sigma_\epsilon},$$

$$\mu = \phi_a \log a + \phi_b \log \textstyle\sum_i b_i.$$

From this we obtain:

$$x_i(v) = \frac{1}{\sqrt{I}}\frac{\sigma_\epsilon}{\sigma_\eta}(v - \phi_a \log \widehat{a} - \phi_b \log \textstyle\sum_i b_i) \quad \forall i,$$

$$p(y) = \phi_a \log a + \phi_b \log \textstyle\sum_i b_i + \displaystyle\frac{\sqrt{I}}{I+1}\frac{\sigma_\eta}{\sigma_\epsilon} y,$$

as required. Blockholder i's trading profits equal $x_i\,(p - v)$ and can computed immediately using the above expressions. ■

Trading profits are increasing in σ_η and σ_ϵ, as σ_η reflects the blockholders' informational advantage and σ_ϵ represents their ability to profit from information by trading with liquidity investors. In addition, aggregate blockholder trading profits are decreasing in the number I of blockholders. This is because multiple blockholders compete as in a Cournot oligopoly. Each blockholder chooses her trading volume to maximize individual profits. A higher volume reveals more information and makes the price less attractive to all informed traders, but she ignores this negative externality and so trades in excess of the level that would maximize combined blockholder profits.

While greater trading volumes reduce aggregate profits, they also impound more information into prices. Our definition of price informativeness is $\mathrm{E}\left[\frac{d\tilde{p}}{dv}\right]$, the expected change in price for a given change in firm value. This definition is particularly relevant for our setting as it captures the incentives for an agent compensated according to the stock price to improve fundamental value. It will thus later be used to derive the manager's optimal action. The common measure used in the microstructure literature is $\left(\mathrm{Var}(\tilde{v}) - \mathrm{Var}(\tilde{v}|\tilde{p})\right) / \mathrm{Var}(\tilde{v})$, which measures the proportion of the variance of $\tilde{v}$ that is captured by prices. The next lemma states that these measures are identical.

Lemma 1 *The following two measures of price informativeness are equivalent:*

1. $(\mathrm{Var}(\tilde{v}) - \mathrm{Var}(\tilde{v}|\tilde{p})) \,/\, \mathrm{Var}(\tilde{v})$.
2. $\mathrm{E}\left[\frac{d\tilde{p}}{dv}\right]$.

Proof Using the formula for the conditional variance of a bivariate normal distribution

$$\mathrm{Var}(\tilde{v}|\tilde{p}) = (1 - \mathrm{Corr}(\tilde{v}, \tilde{p})^2)\,\mathrm{Var}(\tilde{v}),$$

we have

$$(\mathrm{Var}(\tilde{v}) - \mathrm{Var}(\tilde{v}|\tilde{p})) \,/\, \mathrm{Var}(\tilde{v}) = \mathrm{Corr}(\tilde{v}, \tilde{p})^2. \tag{8}$$

Since, in equilibrium, the price is a linear function of $\tilde{v}$ and $\tilde{\epsilon}$,

$$\mathrm{E}\left[\frac{d\tilde{p}}{dv}\right] = \frac{\mathrm{Cov}(\tilde{v}, \tilde{p})}{\mathrm{Var}(\tilde{v})}.$$

From the law of iterated expectations and (7),

$$\mathrm{Var}(\tilde{p}) = \mathrm{Cov}(\tilde{v}, \tilde{p}).$$

Therefore,

$$\mathrm{Corr}(\tilde{v}, \tilde{p})^2 = \mathrm{E}\left[\frac{d\tilde{p}}{dv}\right]. \tag{9}$$

Combining (8) and (9) gives Lemma 1. ■

The next proposition calculates price informativeness in the equilibrium derived in Proposition 1.

Proposition 2 *Price informativeness, as defined by either of the above measures, is equal to* $I/(I+1)$.

Proof The result follows from equations (2), (3), (4), and (5). ■

Both measures of price informativeness are increasing in I; in the extreme, as I approaches infinity, prices become fully informative. On the other hand, in the monopolistic Kyle model ($I = 1$), the blockholder fully internalizes the negative effect of a higher trading volume on profits. She limits her order, and so prices reveal only one-half of the insider's private information.

The positive link between the number of blockholders and price informativeness does not arise because a greater number of informed agents mechanically leads to an increase in the amount of information in the market. Indeed, a single blockholder already has a perfect signal of fundamental value; since she faces no trading constraints, she could theoretically impound this entire information into prices. The result arises instead from competition in trading.

As in the Kyle model, liquidity σ_ε has no effect on price informativeness. From equation (5), greater noise trading allows blockholders to trade more aggressively. This increase in informed trading exactly counterbalances the effect of increased noise and leaves price informativeness unchanged. In Section 5.1 we show that liquidity becomes relevant under costly information acquisition.

3.2 The Action Stage

We now solve for the actions of the manager and the blockholders in the first stage.

Proposition 3 *(Optimal Actions): The manager's optimal action is*

$$a = \phi_a \alpha \left(\frac{I}{I+1} \right) \tag{10}$$

and the optimal action of each blockholder is

$$b_i = \phi_b \beta \left(\frac{1}{I} \right)^2 . \tag{11}$$

Proof The manager maximizes the market value of his shares, less the cost of effort:

$$E \left[\alpha \widetilde{p} - a \right] . \tag{12}$$

When setting the price $\widetilde{p}$, the market maker uses his conjecture for the manager's action a. Therefore, the manager's actual action affects the price only through its influence on $\tilde{v}$, and thus blockholders' order flow. The manager's first-order condition is given by:

$$\alpha \left(\mathrm{E} \left[\frac{d\tilde{p}}{dv} \right] \right) \left(\frac{\phi_a}{a} \right) - 1 = 0. \tag{13}$$

From Proposition 2, his optimal action is therefore

$$a = \alpha \left(\frac{I}{I+1} \right) \phi_a .$$

Each blockholder maximizes her trading profits, plus the fundamental value of her shares, less her cost of effort. From (6), the blockholder's trading profits do not depend her first-stage action.[12] Therefore, blockholder i simply chooses b_i to maximize the fundamental value of her shares, less her cost of effort:

$$E \left[\left(\frac{\beta}{I} \right) \widetilde{v} - b_i \right] . \tag{14}$$

[12] This is because the blockholder's action is publicly observable. Informed trading profits depend on the blockholder's relative information advantage, and this is unaffected by a publicly observable variable. See Maug (1998, 2002) and Kahn and Winton (1998) for models where the blockholder's action is unobservable.

The optimal action of blockholder i is

$$b_i = \phi_b \beta \left(\frac{1}{I}\right)^2. \tag{15}$$

■

The manager's action a is the product of three variables: the effectiveness of effort ϕ_a, his equity stake α, and price informativeness $\frac{I}{I+1}$. It is thus increasing in I as a higher I augments price informativeness. The intuition is as follows. Greater price informativeness (a higher $\mathrm{E}\left[\frac{d\tilde{p}}{dv}\right]$) means that the stock price more closely reflects the firm's fundamental value, and consequently the manager's effort. Therefore, the manager is more willing to bear the cost of working. In effect, blockholder trading rewards managerial effort ex post, therefore inducing it ex ante. The dynamic consistency of this reward mechanism depends on the number of blockholders. Critically, trading occurs *after* the manager has taken his action, at which point the action cannot be undone and shareholders are concerned only with maximizing their trading profits. A single blockholder optimizes her profits by limiting her order, at the expense of price informativeness. Therefore, the promise of rewarding effort by bidding up the price to fundamental value is not credible. By contrast, multiple blockholders trade aggressively, augmenting price informativeness, and thus constitute a commitment device to reward the manager ex post for his actions. While such aggressive trading is motivated purely by the private desire to maximize individual profits in the presence of competition, it has a social benefit by eliciting managerial effort.[13]

In sum, multiple blockholders lead to greater trading volumes. This both reduces aggregate profits and impounds more information into prices. Since firm value is increasing in price informativeness (as it induces effort ex ante) and independent of trading profits (which are a pure transfer from atomistic shareholders to blockholders), higher trading volumes lead overall to an increase in firm value. A number of empirical papers use total institutional ownership as a measure of market efficiency, since institutions have greater information. However, price efficiency depends not only on the amount of information held by investors, but the extent to which this information is impounded into prices. The latter in turn depends on the number of informed shareholders.[14] Similarly, many studies use total institutional ownership as a proxy for corporate governance, but the structure of such ownership is also an important determinant.

[13]Fishman and Hagerty (1995) also show that introducing additional informed traders is a commitment to trading more aggressively. They use this result to show that, if there are multiple informed agents, a specific informed agent will sell her information to other traders, rather than only exploiting it herself.

[14]Oehmke (2008) shows that competition between prime brokers in liquidating collateral reduces sale proceeds and may encourage hedge funds to concentrate collateral with a single broker. Boehmer and Kelley (2008) find empirically that competition among institutional traders increases price informativeness.

As in earlier models, combined blockholder effort $\sum_i b_i$ is decreasing in I, owing to the free-rider problem. Therefore, there is a trade-off between the intervention and trading effects.

4 The Optimal Number of Blockholders

This section derives the optimal number of blockholders. We start by deriving the optimal number that maximizes firm value, and later analyze the social optimum (that maximizes total surplus, taking into account the costs borne by the manager and blockholder) and the private optimum (that maximizes the total payoff to blockholders).

Proposition 4 *(Firm Value Optimum): The number I^* of blockholders that maximizes firm value is:*

$$I^* = \frac{\phi_a - \phi_b}{\phi_b}.^{15} \tag{16}$$

Proof From Proposition 3, expected firm value is:

$$E[\widetilde{v}] = \phi_a \log\left[\phi_a \alpha \left(\frac{I}{I+1}\right)\right] + \phi_b \log\left[\phi_b \beta \left(\frac{1}{I}\right)\right]. \tag{17}$$

The first-order condition with respect to I is given by:

$$\frac{\phi_a - \phi_b - \phi_b I}{I + I^2} = 0. \tag{18}$$

$\hat{I} = (\phi_a - \phi_b)/\phi_b$ satisfies the first order condition. Since the left hand side of (18) is positive for $I < \hat{I}$ and negative for $I > \hat{I}$, I^* is indeed a maximum. ■

The optimal number of blockholders solves the trade-off between the positive effect of more blockholders on managerial effort, and the negative effect on blockholder intervention. The optimum is therefore increasing in ϕ_a, the productivity of the manager's effort, and declining in ϕ_b, the productivity of blockholder intervention. In Section 6, we discuss potential empirical proxies for these variables.

While Proposition 4 is concerned with maximizing firm value, the social optimum maximizes total surplus, which also takes into account the costs of the manager's and blockholders' actions. Informed trading profits do not affect total surplus, since they are a transfer from liquidity traders to the blockholders. In theory, the social optimum would be chosen by a social planner. If the noise traders are the firm's atomistic shareholders

[15] In reality, the number of blockholders must be a strictly positive integer. To economize on notation, we ignore such technicalities when stating I^*. If $\frac{\phi_a - \phi_b}{\phi_b} < 1$, the optimal number is 1. If $\frac{\phi_a - \phi_b}{\phi_b}$ is a non-integer, the optimal number is found by comparing (17) under the two adjacent integers.

(as in Kahn and Winton (1998) and Bolton and von Thadden (1998)), it will also be chosen by the initial owner when taking the firm public, since IPO proceeds will equal total surplus. The owner will have to compensate the blockholders (in the form of a lower issue price) for their expected intervention costs, and the manager for his effort in the form of a higher wage. Trading profits have no effect on IPO proceeds: while blockholders will pay a premium in expectation of trading gains, small shareholders will demand discounts to offset their future losses.

Proposition 5 *(Social Optimum): The number I^*_{soc} of blockholders that maximizes total surplus is the unique positive solution to*

$$\frac{\phi_a}{I\,(I+1)} - \frac{\phi_b}{I} - \frac{\phi_a \alpha}{(I+1)^2} + \frac{\phi_b \beta}{I^2} = 0, \tag{19}$$

which may be higher or lower than I^*. Moreover, I^*_{soc} is increasing in ϕ_a and β, and decreasing in ϕ_b and α.

Proof Total surplus is given by:

$$\phi_a \log\left[\phi_a \alpha \left(\frac{I}{I+1}\right)\right] + \phi_b \log\left[\phi_b \beta \left(\frac{1}{I}\right)\right] - \phi_a \alpha \left(\frac{I}{I+1}\right) - \phi_b \beta \frac{1}{I}. \tag{20}$$

Taking first-order conditions yields (19). The Appendix proves that there is a unique positive solution and that it maximizes (20). It also addresses the comparative statics.
■

Compared to equation (17), equation (20) contains two additional terms. Increasing the number of blockholders raises the cost of managerial effort, but reduces the combined cost of blockholder monitoring. The social optimum may thus be higher or lower than the number that maximizes firm value. If β rises, total blockholder costs $\phi_b \beta \frac{1}{I}$ become more important in the social welfare function, and so I^*_{soc} rises to reduce these costs by lowering intervention. Conversely, a rise in α increases the importance of the manager's costs and thus lowers I^*_{soc}. The comparative statics with respect to ϕ_a and ϕ_b are the same as in Proposition 4.

Finally, we analyze the privately optimal division of β that would maximize blockholders' combined payoffs. In other words, we ask the question: if blockholders in aggregate hold β of the firm, do they have incentives to split or combine stakes to achieve the number that maximizes either firm value or total surplus?

Proposition 6 *(Private Optimum): The number I^*_{priv} of blockholders that maximizes total blockholders' payoff is the unique positive solution to*

$$\beta\left[\frac{\phi_a}{I\,(I+1)} - \frac{\phi_b}{I} + \frac{\phi_b}{I^2}\right] - \frac{(I-1)}{2\sqrt{I}(I+1)^2}\sigma_\eta \sigma_\varepsilon = 0, \tag{21}$$

which may be higher or lower than I^*, and higher or lower than I^*_{soc}. Moreover, I^*_{priv} is increasing in ϕ_a and β, and decreasing in ϕ_b and $\sigma_\eta \sigma_\varepsilon$.

Proof Total blockholders' payoff is given by:

$$\beta\left\{\phi_a \log\left[\phi_a\alpha\left(\frac{I}{I+1}\right)\right] + \phi_b \log\left[\phi_b\beta\frac{1}{I}\right]\right\} - \phi_b\beta\frac{1}{I} + \frac{\sqrt{I}}{I+1}\sigma_\eta\sigma_\varepsilon. \tag{22}$$

Taking first-order conditions yields (21). The Appendix proves that there is a unique positive solution and that it maximizes (22). It also addresses the comparative statics. ■

The blockholders' objective function differs from firm value in three ways. They only enjoy β of any increase in firm value; bear the costs of intervention; and are concerned with informed trading profits. Increasing I above I^* therefore has an ambiguous effect: it reduces the combined costs of intervention, but also reduces total trading profits by exacerbating competition. Therefore, as with the social optimum, the private optimum may be higher or lower than the number that maximizes firm value. An increase in β augments blockholders' monitoring costs and I^*_{priv}. If $\sigma_\eta\sigma_\varepsilon$ rises, trading profits become more important in the objective function and so shareholders combine blocks to reduce competition.

The blockholders' objective function also differs from the social welfare function in three ways. Blockholders are concerned with informed trading profits and only β of firm value, but ignore the cost of managerial effort. Again, the sum of these three effects is ambiguous. Increasing I above I^*_{soc} would both reduce total blockholder costs and total trading profits.

The comparative statics with respect to ϕ_a and ϕ_b are the same as in Propositions 4 and 5. This is particularly important since blockholders may trade away from the structure chosen to maximize firm value or IPO proceeds, and so the private optimum is most likely to be observed empirically (see also Maug (1998) and Edmans (2008)).

5 Extensions

5.1 Costly Information Acquisition

In the core model, the blockholders are endowed with private information about firm value $\tilde{v}$. In this subsection, we assume that blockholders are initially uninformed but can learn $\tilde{v}$ by paying a cost c in the first stage of the game. Blockholders that do not pay this cost will remain uninformed in the second stage.

To solve this modified version of the model, we again use backward induction.

Proposition 7 *(Equilibrium With Costly Information): Let J be the number of blockholders that acquire information in the first stage of the game. Then in the unique linear equilibrium of the trading stage, the $I - J$ uninformed blockholders do not trade. The J*

informed blockholders submit demands as in (2) and the market maker sets the price as in (3) with

$$\lambda = \frac{\sqrt{J}}{J+1}\frac{\sigma_\eta}{\sigma_\epsilon} \tag{23}$$

$$\gamma = \frac{1}{\sqrt{J}}\frac{\sigma_\epsilon}{\sigma_\eta}. \tag{24}$$

In the first stage of the game, the manager's optimal action is

$$a = \phi_a \alpha \left(\frac{J}{J+1}\right) \tag{25}$$

and the optimal action of each blockholder is

$$b_i = \phi_b \beta \left(\frac{1}{I}\right)^2. \tag{26}$$

The number J of blockholders that acquire information is

$$J = \min\{I, n\},$$

where n satisfies

$$\frac{1}{\sqrt{n}(n+1)}\sigma_\eta \sigma_\epsilon = c.$$

Proposition 7 shows that when the number of blockholders I is sufficiently large (greater than n), some blockholders choose not to acquire information. If all blockholders became informed, competition in trading is sufficiently fierce that individual trading profits are insufficient to recoup the monitoring cost c. Hence, in equilibrium, some blockholders remain uninformed and do not participate in the trading stage of the game, earning zero trading profits.

We now analyze the optimal number of blockholders that maximizes firm value. We first observe that it is never optimal to have I greater than n. If $I > n$, then from Proposition 7, some blockholders will not acquire information in equilibrium. Uninformed blockholders do not trade and thus have no effect on governance through exit. Moreover, they dilute ownership and reduce incentives to engage in voice. Uninformed blockholders are thus unambiguously detrimental to firm value, and so the optimum involves no such blockholders. This leads to the next proposition.

Proposition 8 *(Firm Value Optimum With Costly Information): The optimal number I^*_{costly} of blockholders that maximizes firm value with costly information acquisition is equal to*

$$I^*_{costly} = \min\left(\frac{\phi_a - \phi_b}{\phi_b}, n\right). \tag{27}$$

If $n < \frac{\phi_a - \phi_b}{\phi_b}$, I^*_{costly} and firm value are increasing in σ_η and σ_ε and decreasing in c. If $n \geq \frac{\phi_a - \phi_b}{\phi_b}$, I^*_{costly} and firm value are independent of σ_η, σ_ε and c.

The optimal number I^*_{costly} of blockholders with costly information acquisition is weakly increasing in σ_η and σ_ϵ and weakly decreasing in c. The intuition is as follows. If $n < \frac{\phi_a - \phi_b}{\phi_b}$, then the optimum with costless information acquisition I^* is so large that competition in trading reduces individual informed trading profits below the cost of monitoring. Some blockholders thus choose to remain uninformed, and their existence reduces firm value. The optimum is therefore n, the maximum number of blockholders under which competition is sufficiently low that trading profits are high enough for all blockholders to become informed. A fall in the cost of information acquisition c, an increase in the informational advantage σ_η, and a rise in liquidity σ_ε all lead to an increase in trading profits (net of monitoring costs) Higher net profits in turn raise n, as they allow greater competition in trading to be sustained before net profits become negative. This in turn increases I^*_{costly} towards I^*, and thus raises firm value.

By contrast, if $n > \frac{\phi_a - \phi_b}{\phi_b}$, net trading profits are sufficiently high that all blockholders become informed. The analysis is as in the core model of Section 4, where the optimum depends only on the effectiveness of manager and blockholder effort. The constraint that the number of blockholders is sufficiently low to induce information acquisition is not binding. Changes in net trading profits, and thus changes in σ_η, σ_ε and c, have no effect on the optimal number of blockholders or firm value.

5.2 Complementarities

In the core model, the manager's and blockholders' actions are perfect substitutes, with independent effects on firm value. This appears to be a reasonable assumption for most firms and actions; for example, extraction of private benefits by blockholders reduces firm value regardless of the manager's effort. However, in specific circumstances, there may be complementarities between the manager's and blockholders' efforts. If complementarities are positive (negative), the marginal productivity of one party's action is increasing (decreasing) in the effort level of the other party. This subsection extends the core model to these cases.

Positive complementarities arise if manager and blockholder outputs are mutually interdependent. For example, venture capital investors often have expertise in devising an effective strategy, which is then executed by the manager. Both strategy formulation and implementation are necessary for the firm to become successful, and so venture capital models typically feature positive complementarities.

With positive complementarities, blockholders are "allies" of the manager, providing him with specialist advice. The opposite case of negative complementarities arises if blockholders are "adversaries" of the manager, preventing rent extraction. Thus, they are most productive if managerial effort is low, i.e. the manager is pursuing pet projects. This case is most likely in mature firms, where the optimal strategy is often clear to the manager. Inefficiencies arise not because the manager is unaware of the correct course of action and needs blockholders' advice, but because he has private incentives to depart from the efficient action. For example, managers of "cash cows" often know that they should return excess cash to shareholders, but may choose instead to reinvest it inefficiently. Where effort is taken to mean "working" rather than "forgoing private benefits", complementarities may also be negative in mature firms. In such companies, there is often a limited set of value-enhancing actions that can be taken, so blockholder and manager efforts would be duplicative.

We start by analyzing perfect negative complementarities, where firm value depends only on the maximum output level of the manager and blockholders:[16]

$$\widetilde{v} = \max\left[\phi_a \log a, \phi_b \textstyle\sum_i b_i\right] + \widetilde{\eta}. \tag{28}$$

The optimal actions can no longer be derived independently. The manager's optimal action depends on his conjecture $\widehat{b}_i$ for the blockholders' actions. Blockholder i's optimal action depends on her conjecture for the manager's effort ($\widehat{a}$) and for the actions of the other blockholders ($\widehat{b}_j$, $j \neq i$). We use the Nash equilibrium solution concept, where each party chooses the optimal action given his/her conjectures, and all conjectures are correct.

Proposition 9 *(Negative Complementarities): The manager's optimal action is*

$$a = \begin{cases} \phi_a \alpha \frac{I}{I+1} & \text{if } \alpha \frac{I}{I+1}\left(\phi_a \log\left[\phi_a \alpha \frac{I}{I+1}\right] - \phi_b \log \sum_i \widehat{b}_i\right) \geq a \\ 0 & \text{if } \alpha \frac{I}{I+1}\left(\phi_a \log\left[\phi_a \alpha \frac{I}{I+1}\right] - \phi_b \log \sum_i \widehat{b}_i\right) < a. \end{cases} \tag{29}$$

Similarly, blockholder i's effort level is:

$$b_i = \begin{cases} \phi_b \beta \left(\frac{1}{I}\right)^2 & \text{if } \frac{\beta}{I}\left(\phi_b \log\left(\phi_b \beta \frac{1}{I}\right)^2 - \left[\phi_a \log \widehat{a} - \phi_b \log \sum\limits_{j \neq i} \widehat{b}_j\right]\right) \geq b_i \\ 0 & \text{if } \frac{\beta}{I}\left(\phi_b \log\left(\phi_b \beta \frac{1}{I}\right)^2 - \left[\phi_a \log \widehat{a} - \phi_b \log \sum\limits_{j \neq i} \widehat{b}_j\right]\right) < b_i. \end{cases} \tag{30}$$

The number of blockholders I^* that maximizes firm value is

$$I^* = \begin{cases} \infty & \text{if } \phi_a \log(\phi_a \alpha) \geq \phi_b \log(\phi_b \beta) \\ 1 & \text{if } \phi_a \log(\phi_a \alpha) < \phi_b \log(\phi_b \beta) \end{cases}. \tag{31}$$

[16] An alternative way to model complementarities is to use a constant elasticity of substitution production function, e.g. $\widetilde{v} = \left[(\phi_a \log a)^\rho + (\phi_b \log \sum_i b_i)^\rho\right]^{1/\rho} + \widetilde{\eta}$. Such a production function turns out to be intractable with a logarithmic functional form; in turn this specification was necessary for the tractability of the core model. We therefore use a second common method.

In the core model of perfect substitutes, firm value depends on both manager and blockholder efforts. Since the optimal shareholder structure must trade-off both, I^* is typically an interior solution. Here, firm value depends only on the maximum output level and there are no trade-off concerns. If managerial effort is relatively productive, I^* should be chosen exclusively to maximize the potency of exit and completely ignores voice; thus the optimal number of blockholders is infinite. By contrast, if blockholder effort is relatively productive, I^* is at its minimum value of 1.

In the core model, I^* is smoothly increasing in ϕ_a. Here, ϕ_a has a discontinuous effect. If $\phi_a \log(\phi_a \alpha) < \phi_b \log(\phi_b \beta)$, I^* is independent of ϕ_a. A small increase in ϕ_a has zero effect on I^*: since blockholder effort is still relatively more productive, I^* continues to be exclusively determined by voice, irrespective of the effectiveness of managerial effort. However, when ϕ_a rises above the level for which $\phi_a \log(\phi_a \alpha) = \phi_b \log(\phi_b \beta)$, I^* jumps from 1 to ∞. For $\phi_a \log(\phi_a \alpha) \geq \phi_b \log(\phi_b \beta)$, I^* is already exclusively determined by exit considerations, and so further increases in managerial productivity have no effect on I^*. Similarly, changes in ϕ_b have either a zero or infinite effect on I^*.

In sum, negative complementarities lead to more extreme results than the core model. The optimal number of blockholders is a corner solution. ϕ_a and ϕ_b have the same directional effect as in the core model, but their impacts are discontinuous.

The opposite case is perfect positive complementarities. We analyze a Leontief production function where firm value depends only on the minimum output level of the manager and blockholders, i.e.

$$\widetilde{v} = \min\left[\phi_a \log a, \phi_b {\textstyle\sum_i} b_i\right] + \widetilde{\eta}. \tag{32}$$

We again apply the Nash equilibrium solution concept.

Proposition 10 *(Positive Complementarities): The manager's optimal action is*

$$a = \min\left(\phi_a \alpha \left(\frac{I}{I+1}\right), \exp\left(\frac{\phi_b}{\phi_a} \log {\textstyle\sum_i} \widehat{b}_i\right)\right). \tag{33}$$

Similarly, blockholder i's effort level is:

$$b_i = \begin{cases} \phi_b \beta \left(\frac{1}{I}\right)^2 & \text{if } \phi_a \log \widehat{a} \geq \phi_b \log\left[\phi_b \beta \left(\frac{1}{I}\right)^2\right] + \phi_b \log \sum\limits_{j \neq i} \widehat{b}_j \\ \exp\left(\frac{\phi_a}{\phi_b} \log \widehat{a} - \log \sum\limits_{j \neq i} \widehat{b}_j\right) & \\ \quad \text{if } \phi_b \log \sum\limits_{j \neq i} \widehat{b}_j \leq \phi_a \log \widehat{a} < \phi_b \log\left[\phi_b \beta \left(\frac{1}{I}\right)^2\right] + \phi_b \log \sum\limits_{j \neq i} \widehat{b}_j & \\ 0 & \text{if } \phi_a \log \widehat{a} < \phi_b \log \sum\limits_{j \neq i} \widehat{b}_j \end{cases}. \tag{34}$$

The number I^* of blockholders that maximizes firm value is the unique positive solution to

$$\frac{I^2}{I+1} = \frac{\phi_b \beta}{\phi_a \alpha} \exp\left(\phi_b - \phi_a\right). \tag{35}$$

I^* is increasing in ϕ_b and β, and decreasing in ϕ_a and α.

As with the core case, the optimal number of blockholders I^* is typically an interior solution, i.e. involves multiple, but finite, blockholders. However, the comparative statics with respect to ϕ_a and ϕ_b are *opposite* to the core case. In the core case, I^* is increasing in ϕ_a. If managerial effort becomes more productive, it becomes increasingly important in the trade-off between exit and voice, and so I^* rises to enhance exit. With perfect positive complementarities, the optimal number of blockholders must balance the levels of manager and blockholder outputs. If ϕ_a rises, managerial effort is more effective and so it is not necessary to "boost" it via a high I. Instead, I should be used to enhance blockholder effort so that it becomes sufficiently high to complement the manager's output. This involves reducing I.

5.3 General Compensation Contract

In the core model, the manager's payoff stems from the market value of his shares, $\alpha\widetilde{p}$, as in Holmstrom and Tirole (1993). In a more general setting, the manager can be compensated according to the fundamental value $\widetilde{v}$ as well as the market value $\widetilde{p}$, for instance using stock with a long vesting period. We thus generalize the manager's objective function to

$$E\left[\alpha\left(\omega p + (1-\omega)v\right) - a\right].$$

$\omega > 0$ is a standard assumption in the literature, which can be motivated by a number of underlying factors. These include takeover threat (Stein (1988)), concern for managerial reputation (Narayanan (1985), Scharfstein and Stein (1990)), or the manager expecting to sell his shares for $\widetilde{p}$ before $\widetilde{v}$ is realized, e.g. to finance consumption (Stein (1989)).[17] The core model has $\omega = 1$.

Proposition 11 *(General Compensation Contract): The number I^* of blockholders that maximizes firm value is the larger root of*

$$\frac{\phi_a \omega}{(I+1-\omega)(I+1)} - \frac{\phi_b}{I} = 0 \tag{36}$$

if equation (36) has solutions. In this case, I^* is increasing in ω. If (36) has no solutions, $I^* = 1$.

[17] Kole (1997) shows that vesting periods are short in practice, perhaps because long vesting periods would subject the manager to excessive risk.

Proof Proceeding as in the main model, we have

$$a = \alpha\phi_a \left[1 - \frac{\omega}{I+1}\right]. \tag{37}$$

Firm value is given by:

$$E[v] = \phi_a \ln\left[\phi_a\alpha\left[1 - \frac{\omega}{I+1}\right]\right] + \phi_b \ln\left[\phi_b\beta\frac{1}{I}\right]. \tag{38}$$

The first-order condition is given by (36). The Appendix proves that, if there are two roots, the larger root yields a maximum. It also addresses the comparative statics. ■

As in the core model, the optimal number of blockholders represents a trade-off between the positive effect of greater blockholders on "exit", and the negative effect on "voice." The effect of I on stock price efficiency is more important when the manager is more closely aligned with the stock price, and so the optimal number of blockholders I^* increases with the manager's short-term concerns ω.

6 Empirical Implications

The paper is motivated by the empirical observation that many firms are held by multiple small blockholders, in contrast to some earlier theories that advocate highly concentrated ownership. The results in the paper generate a number of additional empirical implications, over and above its initial motivation. However, we emphasize that empirical testing will have to overcome a number of challenges. First, although the model generates clear, closed-form predictions for the optimal number of blockholders in terms of certain variables, a number of these parameters (such as the effectiveness of blockholder and manager effort) are difficult to measure directly. The key challenge for empiricists is to come up with accurate proxies. Second, while the model predicts that these variables have a causal impact on blockholder structure, it may be that additional factors outside the model have an effect on both. The same issue applies to the model's implications for the effect of blockholder numbers on price efficiency. Therefore, documenting correlations will be insufficient to support the model; identification of causal effects will require careful instrumentation.

The core mechanism in this paper is that multiple blockholders can exert governance through passive trading in addition to active intervention. Gallagher, Gardner and Swan (2008) find that the threat of disciplinary exit from multiple blockholders leads to superior subsequent firm performance. They use a measure of portfolio churning to ensure they are identifying governance through exit rather than voice. Smith and Swan (2008) test whether institutional trading is successful at restraining executive compensation. Multiple moderately-sized investors with frequent trading have greatest effect; institutional concentration only matters insofar as it affects trading activity.

The majority of the empirical predictions concern the factors that determine blockholder structure. In the paper, we considered different criteria for the optimal number of blockholders. In practice, sometimes the social optimum may be observed, specially if the firm has recently undergone an initial public offering, or lock-ups prevent blockholders from re-trading from the initial structure. For most firms, it is most likely that the private optimum will be observed. Importantly, both optima share the same predictions for ϕ_a and ϕ_b: they are increasing in the productivity of the manager's effort and declining in the productivity of blockholder intervention.

The magnitude of ϕ_b depends on the nature of blockholders' expertise. Using the terminology of Dow and Gorton (1997), if blockholders have forward-looking ("prospective") information about optimal future investment decisions or strategic choices, direct intervention is particularly valuable and ϕ_b is high. For example, venture capital financiers are typically expert in managing start-up businesses and their effort directly affects the firm's prospects. Indeed, venture capitalists often hold highly concentrated stakes, which they retain even after the firm goes public and the "exit" governance mechanism becomes available.

On the other hand, mutual funds and insurance companies typically lack specialist expertise on new strategic directions but instead are skilled at gathering backward-looking ("retrospective") information to evaluate the effect of past decisions on firm value. Their primary benefit is to impound the effects of prior managerial effort into the stock price. In such a case, ϕ_b is low and I^* is high. Similarly, Section 5.1 shows that if information is costly, the optimal number of blockholders is decreasing in the monitoring cost c. Institutions skilled at gathering retrospective information have a low c, further reinforcing the prediction that I^* is high. Indeed, as firms mature, active venture capitalist investors are typically replaced by passive institutional shareholders, and the number of blockholders usually increases. However, as emphasized at the start of this section, this could be for reasons outside the model. As firms mature, they typically become larger; if blockholder wealth constraints limit the number of dollars they can invest in a firm (e.g. Winton (1993)), this will lead to more dispersed ownership. Therefore, the above empirical observation is only tentative support for the model; a formal test will have to control for factors such as firm size.

Another determinant of ϕ_b is blockholders' control rights and thus ability to intervene (holding constant the size of their individual stakes).[18] Black (1990), Bebchuk (2007) and Becht et al. (2007) note that U.S. shareholders face substantial legal and institutional hurdles to intervention, compared to their foreign counterparts. This reduces ϕ_b, thus increasing I^*, and is consistent with smaller and more numerous blockholders in the U.S.

[18] In reality, control rights will also be increasing in the size of each blockholder's individual stake β/I. This will reinforce the negative effect of I on intervention currently in this paper.

The manager's effectiveness ϕ_a will be higher if he is more talented. Talent can be measured directly using managerial characteristics, such as education, experience or past performance, or proxied by firm size (Gabaix and Landier (2008)). Firm size may also affect ϕ_a since many managerial actions can "rolled out" across the entire firm; for example, if the CEO designs a new method to reduce production costs, this can be applied firmwide. In such a case, managerial effort has a multiplicative effect on firm value (Edmans, Gabaix and Landier (2008)) and so ϕ_a is greater in large firms.

The above predictions concern the core model, where manager and blockholder outputs have independent effects on firm value, which likely applies in most situations. However, start-up firms may feature positive complementarities, for instance if venture capital investors formulate strategy and the management implements it. Typically, ϕ_a will be significantly greater than ϕ_b: the manager is able to add greater value than blockholders, given his close proximity to firm operations. In such a case, Section 5.2 predicts that I^* is lower under positive complementarities than in the core case of perfect substitutes. This may explain the concentrated blockholder structure in early-stage firms. Moreover, in such firms, the manager often has a significant equity stake (high α) which gives him strong incentives to exert effort. From equation (35), I^* should be low to ensure blockholder effort is also high. This reinforces the predictions for venture capital-financed firms discussed above.

By contrast, negative complementarities may occur in mature firms where the agency costs of free cash flow are potentially high: the manager may have private incentives to reinvest excess cash, which can be prevented by direct blockholder intervention. Since ϕ_a will typically be higher than ϕ_b, the model predicts that such firms should be held by many blockholders, reinforcing the earlier predictions for mature firms.

The theory also suggests that governance through exit is most important where the manager's short-term concerns ω are highest. Therefore, the number of blockholders should be higher when the manager's stock and options have shorter vesting periods, or takeover defenses are weaker. Again, simple cross-sectional correlations will be insufficient to support this prediction, since blockholders can plausibly affect the compensation contract.

The above predictions are specific to a model where blockholders create positive value in a firm (rather than competing to extract rents), either directly by intervening (hence the importance of ϕ_b) or indirectly by inducing managerial effort (hence the importance of ϕ_a and ω). Additional distinguishing predictions stem from the model's focus on governance through exit, which generates a number of implications related to microstructure. One category concerns the determinants of the optimal number of blockholders. Section 5.1 predicts that this number is increasing in liquidity σ_ε and uncertainty σ_η. A second category of predictions relates to the effect of I on certain outcome variables. First, measures of informed trading and price efficiency should be increasing in the number of blockholders. In addition, since the empirically observed I may differ from the firm value optimum, the model derives predictions for the effect of I on firm value.[19] The raw prediction that firm value is first increasing, then decreasing in the number of blockholders is also generated by other theories of multiple blockholders. However, our model also predicts that these effects are strongest where liquidity is highest – i.e. the loss in firm value from having inefficiently few blockholders is greatest where there is most noise trader demand. Similarly, while multiple blockholder theories would also predict that the event-study reaction to an unanticipated splitting of a block should be high if the initial number of blockholders is low, this paper predicts that this effect should be increasing in liquidity. Finally, our model shows that liquidity is beneficial for firm value. While many other papers also generate this positive relationship (e.g. Holmstrom and Tirole (1993), Maug (1998), Faure-Grimaud and Gromb (2004), Edmans (2008)), here the specific mechanism is through increasing the number of blockholders.

7 Conclusion

Why are so many firms held by multiple blockholders when such a shareholding structure generates free-rider problems in monitoring? This paper offers a potential explanation. The same co-ordination issues that hinder intervention increase blockholders' effectiveness in exerting governance through an alternative governance mechanism: exit. Multiple blockholders act competitively in their trading behavior, impounding more information into the stock price. This in turn induces higher managerial effort, particularly if the manager has high stock price concerns.

The optimal number of blockholders depends on the relative productivity of managerial and blockholder effort. If outputs are perfect substitutes, the optimum is decreasing in the effectiveness of blockholder intervention and increasing in the potency of managerial effort. It is therefore high if blockholders are mutual funds that gather retrospective rather than prospective information, and low if they are venture capital investors. If there are negative complementarities, changes in productivity can have discontinuous effects, switching the optimal number of blockholders from a very low to a very high level (or vice-versa). However, if complementarities are positive, the productivity parameters have opposite effects on the optimal shareholder structure. If blockholder effort is unproductive, concentrated blockholders are necessary to augment it to a sufficient level to complement the manager's effort.

[19] If I is always at the firm value optimum, there should be no relationship between I and firm value. However, the empirically observed I is likely to be the private optimum, which differs from the firm value optimum. Moreover, the private optimum may shift for exogenous reasons, such as a blockholder suffering a liquidity shock or a change in management.

The paper suggests a number of potential avenues for future research. On the empirical side, the model generates a number of empirical predictions for the determinants of blockholder structure. Testing such predictions is non-trivial, since some of the key explanatory variables (productivity of effort) may be difficult to measure precisely, and these variables may be jointly determined along with blockholder structure. A quite separate empirical implication is that the number of sizable shareholders is an important determinant of both governance and investor informedness, in addition to their total ownership. On the theoretical side, the paper has assumed symmetric blockholders and the analysis has focused on their optimal number. It would be interesting to extend the analysis to asymmetric blockholders, for instance to examine the optimal distribution of shares between a fixed number of blockholders. Similarly, while we have focused our study on the efficient number of blockholders, the model can be expanded to consider the simultaneous determination of the manager's stake and total blockholder ownership.

More broadly, the model suggests a new way of thinking about the interactions between multiple blockholders: as competing for trading profits, rather than private benefits. Therefore, future corporate finance models of multiple blockholders could incorporate more complex effects currently analyzed in asset pricing models of many informed traders. The present paper assumes a single trading period, but in reality there may be multiple periods in which information may arrive and blockholders may trade. Trading profits, and thus incentives to acquire costly information, then depend not only on the quality of information but its timeliness. A blockholder who receives information late may find that the price has already moved, reducing her trading profits. In addition, in the present paper, blockholders trade on information only. If blockholders are subject to liquidity shocks (as in Faure-Grimaud and Gromb (2004)), the addition of multiple trading rounds may give incentives for other blockholders to "front-run" and sell in advance of an anticipated forced liquidation. This may increase the potency of governance through exit, but reduce incentives to engage in interventions with long-run benefits.

A Appendix

Proof of Proposition 5 (Social Optimum)

Putting equation (19) under a common denominator yields

$$\frac{\phi_a I\left(I+1\right)-\phi_b I\left(I+1\right)^2-\phi_a \alpha I^2+\phi_b \beta\left(I+1\right)^2}{I^2\left(I+1\right)^2}=0. \tag{39}$$

Equation (19) is thus a cubic, and has at most three roots. The function is discontinuous at $I=-1$ and approaches $-\infty$ either side of $I=-1$ (since the $-\frac{\phi_a \alpha}{(I+1)^2}$ term dominates). It is also discontinuous at $I=0$ and approaches $+\infty$ either side of $I=0$ (since the $\frac{\phi_b \beta}{I^2}$ term dominates). It is continuous everywhere else.

As $I \rightarrow -\infty$, the $-\frac{\phi_b}{I}$ term in equation (19) dominates, and so the function asymptotes the x-axis from above. Since it approaches $-\infty$ as I rises to -1, and is continuous between $I=-\infty$ and $I=-1$, there must be one root between these two points. Similarly, since the function tends to $+\infty$ as I rises from just above -1 to just below 0, and is continuous between these two points, there must be a second root within this interval. As $I \rightarrow +\infty$, the $-\frac{\phi_b}{I}$ term in equation (19) again dominates, and so the function asymptotes the x-axis from below. Since the function tends to $+\infty$ as I approaches 0 from above, and is continuous between $I=0$ and $I=+\infty$, there must be a third root ($\widetilde{I}$) between these two points. There can be no other positive roots, since there are two negative roots and three roots in total. The positive root is a local maximum, since the gradient is negative for $I<\widetilde{I}$ and positive for $I>\widetilde{I}$.

Let $F\left(I,\theta\right)$ denote the left-hand side of equation (39), where θ is a vector of parameters ϕ_a, ϕ_b, α, β. I^*_{soc} is defined by $F=0$. Differentiating with respect to θ gives

$$\frac{\partial F}{\partial \theta}+\frac{\partial F}{\partial I}\frac{\partial I}{\partial \theta}=0.$$

Since the gradient F is negative just below I^*_{soc} and positive just above I^*_{soc}, $\frac{\partial F}{\partial I}|_{I=I^*_{soc}}<0$. Therefore, the sign of $\frac{\partial I}{\partial \theta}$ equals the sign of $\frac{\partial F}{\partial \theta}$, which in turn is the cross-partial derivative of total surplus (20) with respect to I and θ. This generates the comparative statics with respect to α, β, ϕ_a and ϕ_b.

Proof of Proposition 6 (Private Optimum)

Equation (21) can be rewritten

$$2\beta\left(-\phi_b\sqrt{I}+\frac{\phi_a}{\sqrt{I}}+\frac{\phi_b}{I^{3/2}}\right)-\frac{I-1}{I+1}\sigma_\eta\sigma_\varepsilon=0.$$

Let

$$F(I)=2\beta\left(-\phi_b\sqrt{I}+\frac{\phi_a}{\sqrt{I}}+\frac{\phi_b}{I^{3/2}}\right)-\frac{I-1}{I+1}\sigma_\eta\sigma_\varepsilon.$$

We need only consider $I \geq 1$. Since $2\beta\left(-\phi_b\sqrt{I} + \frac{\phi_a}{\sqrt{I}} + \frac{\phi_b}{I^{3/2}}\right)$ is decreasing in $I \in [1,\infty)$ and $\frac{I-1}{I+1}\sigma_\eta\sigma_\varepsilon$ is increasing in $I \in [1,\infty)$, $F(I)$ is decreasing in $[1,\infty)$. Then since $F(\infty) < 0$ and $F(1) > 0$, there exists a unique root of $F(I) = 0$ in $[1,\infty)$.

The comparative statics results follow from taking the cross-partial derivatives of the objective function. The cross-partial with respect to I and β is $\frac{\phi_a}{I(I+1)} - \frac{\phi_b}{I} + \frac{\phi_b}{I^2}$, which is positive from equation (21). The other cross-partial derivatives can be immediately signed.

Proof of Proposition 7 (Equilibrium With Costly Information)

The only difference from the previous analysis is that in the action stage of the game, blockholder i now simultaneously chooses her action b_i and whether to become informed.

We proceed by backwards induction. Let J be the number of blockholders that acquire information. In the trading stage, uninformed blockholders cannot expect to make profits and thus do not trade. Therefore, only the J informed blockholders trade and the equilibrium is similar to the one derived in Proposition 1.

Now in the action stage of the game, the manager must choose an action a. Using the same arguments as in Proposition 3, the manager's optimal action is

$$a = \phi_a\alpha\left(\frac{J}{J+1}\right). \tag{40}$$

Blockholders must choose actions b_i and whether to become informed. These decisions can be taken independently since informed trading profits are independent of b_i (which is public), and the choice of b_i depends only on blockholder i's stake β/I. The optimal action of each blockholder is thus

$$b_i = \phi_b\beta\left(\frac{1}{I}\right)^2. \tag{41}$$

From equation (6), if there are I informed blockholders, then each blockholder's trading profits are given by:

$$\frac{1}{\sqrt{I}(I+1)}\sigma_\eta\sigma_\epsilon.$$

A blockholder will acquire information if and only if her trading profits are higher than c. This gives the number J of blockholders that decide to become informed in equilibrium.

Proof of Proposition 8 (Firm Value Optimum With Costly Information)

Let n and $J(I)$ be as given in Proposition 7. Using the results of Proposition 3, the expected firm value is

$$E[\widetilde{v}] = \phi_a \log\left[\phi_a\alpha\left(\frac{J(I)}{J(I)+1}\right)\right] + \phi_b \log\left[\phi_b\beta\left(\frac{1}{I}\right)\right]. \tag{42}$$

We wish to maximize the above expression with respect to I. Since $J(I) = n$ for $I \geq n$, it is never optimal to increase I beyond n since it reduces the second term in the firm value while keeping the first term constant. Therefore, $I^*_{costly} \leq n$.

When $I \leq n$, $J(I) = I$ and the problem is the same as in Proposition 4. From (16) we obtain the desired result.

Proof of Proposition 9 (Negative Complementarities)

Deriving $\widetilde{p}$ as in the main model and solving the manager's objective function, he will choose either $a = \phi_a \alpha \frac{I}{I+1}$ or $a = 0$. If $\phi_a \log \left[\phi_a \alpha \frac{I}{I+1}\right] < \phi_b \log \sum_i \widehat{b}_i$, exerting $a = \phi_a \alpha \frac{I}{I+1}$ will have no effect on $\widetilde{p}$ and so the manager will choose $a = 0$. Even if $\phi_a \log \left[\phi_a \alpha \frac{I}{I+1}\right] \geq \phi_b \log \sum_i \widehat{b}_i$, it is not automatic that the manager will exert effort. Exerting effort increases $\widetilde{p}$ not by $\frac{I}{I+1} \phi_a \log \left[\phi_a \alpha \frac{I}{I+1}\right]$, as in the core model, but by only

$$\frac{I}{I+1} \left(\phi_a \log \left[\phi_a \alpha \frac{I}{I+1} \right] - \phi_b \log \sum_i \widehat{b}_i \right)$$

because blockholder effort "supports" firm value even if $a = 0$. Hence the manager choose $a = \phi_a \alpha \frac{I}{I+1}$ only if

$$\alpha \frac{I}{I+1} \left(\phi_a \log \left[\phi_a \alpha \frac{I}{I+1} \right] - \phi_b \log \sum_i \widehat{b}_i \right) \geq a.$$

and so the optimal a is as given by (29). Blockholder i's effort level is derived similarly.

There are two candidates for a Nash equilibrium:

$$\begin{cases} a = 0, b_i = \phi_b \beta \left(\frac{1}{I}\right)^2 \\ a = \phi_a \alpha \frac{I}{I+1}, b_i = 0 \ . \end{cases}$$

Firm value is thus either $\phi_a \log \left[\phi_a \alpha \frac{I}{I+1}\right]$ or $\phi_b \log \left[\phi_b \beta \frac{1}{I}\right]$. The former is monotonically increasing in I, and maximized at $\phi_a \log (\phi_a \alpha)$ for $I = \infty$. The latter is monotonically decreasing in I, and maximized at $\phi_b \log (\phi_b \beta)$ for $I = 1$. Thus I^* is as given in equation (31).

Proof of Proposition 10 (Positive Complementarities)

The manager will not exert effort above the level for which

$$\phi_a \log a = \phi_b \log \sum_i \widehat{b}_i,$$

i.e.

$$a = \exp \left(\frac{\phi_b}{\phi_a} \log \sum_i \widehat{b}_i \right).$$

This derives the optimal a as given in equation (33). Similarly, blockholder i will not exert effort above the level for which

$$\phi_b \log b_i = \phi_a \log \widehat{a} - \phi_b \log \sum_{j \neq i} \widehat{b}_j,$$

i.e.

$$b_i = \exp\left(\frac{\phi_a}{\phi_b} \log \widehat{a} - \log \sum_{j \neq i} \widehat{b}_j\right).$$

A Nash equilibrium requires the following three conditions to hold:

$$\begin{aligned} \phi_b \log I b_i = \phi_a \log a. \\ a \leq \phi_a \alpha \left(\frac{I}{I+1}\right) \\ b_i \leq \phi_b \beta \left(\frac{1}{I}\right)^2. \end{aligned}$$

If the first condition was violated, then the party producing the higher output would gain by reducing effort. The two inequality conditions represent the maximum levels of effort that the manager and blockholders will exert, given the marginal cost of effort.

Out of the continuum of potential Nash equilibria, we seek the one that maximizes firm value. Since firm value is increasing in both a and b_i, it is clear that at least one incentive compatibility constraint will bind. If neither constraint binds, then all parties are exerting suboptimal effort. We could raise the effort levels of all parties while maintaining the equality condition and violating neither constraint.

We now show that, in fact, *both* constraints will bind. Consider the case where $b_i = \phi_b \beta \left(\frac{1}{I}\right)^2$. (Starting with $a = \phi_a \alpha \left(\frac{I}{I+1}\right)$ leads to the same result). Then we have

$$\begin{aligned} \phi_b \log \left[\phi_b \beta \left(\frac{1}{I}\right)\right] = \phi_a \log a \\ a = \exp\left(\frac{\phi_b}{\phi_a} \log \left[\phi_b \beta \left(\frac{1}{I}\right)\right]\right). \end{aligned}$$

Recall that we also require $a \leq \phi_a \alpha \left(\frac{I}{I+1}\right)$. Hence firm value is optimized by solving:

$$\max_I \exp\left(\frac{\phi_b}{\phi_a} \log \left[\phi_b \beta \left(\frac{1}{I}\right)\right]\right) \text{ s.t. } \exp\left(\frac{\phi_b}{\phi_a} \log \left[\phi_b \beta \left(\frac{1}{I}\right)\right]\right) \leq \phi_a \alpha \left(\frac{I}{I+1}\right).$$

The constraint will bind, and so we obtain

$$\phi_a \log \left[\phi_a \alpha \left(\frac{I}{I+1}\right)\right] = \phi_b \log \left[\phi_b \beta \left(\frac{1}{I}\right)\right]. \tag{43}$$

The firm value optimum setting I to ensure all parties exert their "full" effort levels. The intuition is as follows. Consider a Nash equilibrium where the blockholders are exerting their full effort (i.e. $b_i = \phi_b \beta \left(\frac{1}{I}\right)^2$), and the manager is not (i.e. $a < \phi_a \alpha \left(\frac{I}{I+1}\right)$). b_i is thus constrained by I via the equation $b_i = \phi_b \beta \left(\frac{1}{I}\right)^2$, and so firm value rises if I is reduced to relax this constraint and allow b_i to rise. Unlike in the core model, we do not have the side-effect that reducing I decreases a. I only determines the upper bound to a, not its level. Since $a < \phi_a \alpha \left(\frac{I}{I+1}\right)$, the upper bound is not a constraint anyway. Rather than declining, a will rise to accompany the increase in b_i and ensure that $\phi_b \log I b_i = \phi_a \log a$ still holds.

From equation (43), the optimal number of blockholders is determined implicitly by:

$$\frac{I^2}{I+1} = \frac{\phi_b \beta}{\phi_a \alpha} \exp\left(\phi_b - \phi_a\right) = Z.$$

Using the quadratic formula, the unique positive solution is

$$\underline{I} = \frac{Z + \sqrt{Z^2 + 4Z}}{2},$$

which is increasing in ϕ_b and β, and decreasing in ϕ_a and α.

Proof of Proposition 11 (General Compensation Contract)

Putting the derivative (36) under a common denominator yields

$$F(I, \omega) = \frac{\phi_a \omega I - \phi_b (I + 1 - \omega)(I+1)}{I (I + 1 - \omega)(I+1)}.$$

It is therefore a quadratic, and has at most two roots. As $I \to -\infty$, the $\phi_b (I + 1 - \omega)(I+1)$ term dominates and so the function (36) asymptotes the x-axis from above. As I approaches -1 from below, the function approaches ∞ and is discontinuous at $I = -1$. For $-1 < I < 1 - \omega$, F is negative because the $(I + 1 - \omega)$ term in the denominator now turns negative. It approaches $-\infty$ at $I = -1$ and $I = \omega - 1$. For $\omega - 1 < I < 0$, F is positive since the $(I+1)$ term in the denominator now turns positive. It approaches ∞ at $I = \omega - 1$ and $I = 0$. Hence, there are no roots for $I < 0$.

As I approaches 0 from above, $F \to -\infty$. As $I \to \infty$, F asymptotes the x-axis from below. Therefore, the function either has zero or two roots. If F has no roots, it is negative for all $I > 0$ and so the optimal number of blockholders is its minimum value of 1. If it has two roots, the upper root I_u is the maximum since the derivative is positive below I_u and negative above I_u. As in the proof of Proposition 5, the cross-partial is sufficient to determine the sign of $\frac{\partial I}{\partial \omega}$. This cross-partial is given by

$$\frac{\partial^2 E[v]}{\partial I \partial \omega} = \frac{\phi_a}{(I + 1 - \omega)^2} > 0.$$

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