MANAGEMENT SCIENCE
DOI 10.1287/mnsc.1080.0961ec
pp. ec1–ec28

informs®

e-companion
ONLY AVAILABLE IN ELECTRONIC FORM

Electronic Companion—"Dynamic Cost Reduction Through Process Improvement in Assembly Networks" by Fernando Bernstein and A. Gürhan Kök, *Management Science*, DOI 10.1287/mnsc.1080.0961.

Dynamic Cost Reduction through Process Improvement in Assembly Networks

Fernando Bernstein • A. Gürhan Kök

Fuqua School of Business, Duke University, Durham, NC 27708-0120

fernando@duke.edu • *gurhan.kok@duke.edu*

Appendix A – Algorithm

This section presents a simple algorithm to find the optimal investment path for the case of a single firm and the equilibrium investment paths in the decentralized assembly system. The algorithm is denoted by Algorithm $(M, \{z_t^n\})$, where M denotes the number of firms and $\{z_t^n \in \Re^M : t = 1, ..., T;\ n = 1, ..., T - t\}$ are the input parameters. We begin with the case of a single firm.

Algorithm $(1, \{x_t^n\})$

1. Set $\bar{x}_T^* = x_T^1$
2. For $t = T - 1, ..., 1$
3.   For $n = 1, ..., T - t$
4.   If $x_t^n < \bar{x}_{t+n}^*$, then $\bar{x}_t^* = x_t^n$ and go to 7.
5.   Next n
6. $\bar{x}_t^* = x_t^{T-t+1}$
7. Next t

The idea behind this algorithm is to compute, in each period t, the optimal investment strategy for that period assuming that the problem starts in that period. The algorithm produces, for each period t, a unique investment quantity $\bar{x}_t^* = x_t^n$ for some $1 \le n \le T - t + 1$. The outcome of the algorithm can then be used to derive the optimal investment path as follows:

- Invest in period 1 up to the level given by $x_1^* = \bar{x}_1^* = x_1^{n_1}$, for some n_1.

- Invest in period $n_1 + 1$ up to the level given by $x^*_{n_1+1} = \bar{x}^*_{n_1+1} = x^{n_2}_{n_1+1}$, for some n_2.
- ...

The next result states that the algorithm indeed produces the optimal investment path. The proof is based on the result in Proposition 2 that shows that for any given period t at most one of the inequalities $x^n_t < \bar{x}^*_{t+n}$, $n = 1, ..., T - t$, in Step 4 is satisfied. This n (if any) implies that it is optimal to invest in process improvement in periods t and $t + n$, but not in between. Otherwise, the firm must end process improvement activities for this product after period t.

Proposition 9 *Algorithm* $(1, \{x^n_t\})$ *produces the optimal investment path.*

In order to show that a similar algorithm produces an equilibrium of the dynamic investment game, we need to verify that (3) and (4) continue to hold in the decentralized assembly setting. It is straightforward to verify that an equilibrium vector x^c satisfies the properties in Lemma 4, while Lemma 2 shows that (3) holds. Then, a proof similar to that of Proposition 9 shows that Algorithm $(M, \{\hat{x}^n_t\})$ produces an equilibrium of the dynamic investment game in closed-loop strategies. Specifically, if the uniqueness conditions (10) hold, then this algorithm produces the unique open-loop equilibrium of the game. Otherwise, it produces the largest open-loop equilibrium, preferred by all firms. As for the case of a single firm, the outcome of Algorithm $(M, \{\hat{x}^n_t\})$ can be used to derive the equilibrium investment path as follows:

- All suppliers invest in period 1 up to the level given by $x^c_1 = \bar{x}^c_1 = \hat{x}^{n_1}_1$, for some n_1.
- All suppliers invest in period $n_1 + 1$ up to $x^c_{n_1+1} = \bar{x}^c_{n_1+1} = \hat{x}^{n_2}_{n_1+1}$, for some n_2.
- ...

The complexity of these algorithms is $O(T^2)$, essentially $T\,(T-1)\,/2$ comparisons, after the quantities $\{x^n_t\}$ are computed. Computing $\{x^n_t\}$ requires solving a game with M variables for each t and n. Because these games are supermodular by Proposition 3, a tatonnement algorithm starting from large initial values converges to the largest equilibrium in a number of steps that is polynomial in M.

Appendix B – Closed-Loop Equilibria

This section provides a brief numerical study that explores the number of closed-loop equilibria that may arise under cost-contingent contracts. We use the example in Section 5.1 as the base case. The parameters in the base case are $M = 2$, $T = 2$, $\beta_i = 1.1$ and $\beta_j = 1.3$, $k_1 = k_2 = k = 4$, $\delta = 0.9$, $v_{0i} = v_{0j} = 1$, $a = (10, 10)$, and $b = (1, 0.09)$. We refer to the two suppliers as suppliers i and j. Because this is an example with two periods and two suppliers, there are four possible types of equilibria:

(i) Suppliers i and j invest in both periods

(ii) Supplier i invests in period 1 and supplier j invests in both periods

(iii) Supplier j invests in period 1 and supplier i invests in both periods

(iv) Suppliers i and j invest in the first period only.

While equilibria of types (i) and (iv) imply synchronized investments by the suppliers, those of types (ii) and (iii) imply asynchronized investments. In the base case, there are three closed-loop equilibria of the type (i), (ii), and (iii), respectively. In equilibria of type (ii), supplier i invests more in the first period than it does in the synchronized equilibria (i), which in turn increases the investment by supplier j in both periods due to complementarity of investments.[8] A similar observation applies to the case of an equilibrium of type (iii). In other words, in a two-period setting and under closed-loop strategies a supplier can increase the overall investment levels by committing to a higher investment level in an earlier period.

The table below presents a set of examples that illustrate how the number and type of equilibria may change as the parameters of the system vary one at a time.

[8] In a two-period setting, the investment decisions for supplier j in periods 1 and 2 are complementary to the first-period investment decision of supplier i when this firm only makes an investment in the first period.

Parameter changed	Equilibrium type (*i*)	(*ii*)	(*iii*)	(*iv*)
$b_2 = 0.11$				✓
$b_2 = 0.10$	✓	✓	✓	
$b_2 = 0.09$ (base case)	✓	✓	✓	
$b_2 = 0.08$	✓			
$b_2 = 0.07$	✓			
$\beta_j = 1.1$	✓	✓	✓	
$\beta_j = 1.3$ (base case)	✓	✓	✓	
$\beta_j = 1.5$	✓	✓	✓	
$k = 2$	✓	✓	✓	
$k = 4$ (base case)	✓	✓	✓	
$k = 6$				✓
$\delta = 0.85$	✓			
$\delta = 0.9$ (base case)	✓	✓	✓	
$\delta = 0.95$				✓
$a_2 = 10$ (base case)	✓	✓	✓	
$a_2 = 9$				✓
$a_2 = 8$				✓

The first set of examples correspond to changes in b_2 (higher b_2 implies lower demand in the second period). With high b_2 neither firm invests in the second period and only an equilibrium of type (*iv*) arises. With low b_2, demand is higher in the second period, making it more attractive for suppliers to invest in the second period as well – therefore, an equilibrium of type (*i*) arises. With intermediate values of b_2, however, there exist multiple equilibria. In addition to a type-(*i*) equilibrium, asynchronized equilibria arise in which one supplier invests in the first period and the other invests in both periods. Varying the learning rate β_j does not seem to affect the number and type of equilibria. We also observe that as the unit cost of investment k increases, the discount rate δ increases, or the second-period demand intercept a_2 decreases, only equilibria of type (*iv*) arises – in all of these cases, investing in the second period becomes less attractive.

Appendix C – Proofs

Proposition 1

Proof. We show inductively that $\pi_t^*(x_{t-1})$ is increasing and concave in x_{t-1}. The property holds for period $T+1$. Assume now that it holds for period $t+1$, i.e., that $\pi_{t+1}^*(x_t)$ is increasing and concave in x_t. Then, $\pi_t(x_t) - k(x_t - x_{t-1}) + \delta\pi_{t+1}^*(x_t)$ is concave and

$$\pi_t^*(x_{t-1}) = \begin{cases} \pi_t(x_t^u) - k(x_t^u - x_{t-1}) + \delta\pi_{t+1}^*(x_t^u), & \text{if } x_{t-1} < x_t^u \\ \pi_t(x_{t-1}) + \delta\pi_{t+1}^*(x_{t-1}), & \text{if } x_t^u \leq x_{t-1} \end{cases}$$

Note that $\pi_t^*(x_{t-1})$ is linear with slope k for $x_{t-1} < x_t^u$. In addition, because $\pi_t(\cdot)$ is increasing and strictly concave and $\pi_{t+1}^*(\cdot)$ is increasing and concave, we have that $\pi_t^*(x_{t-1})$ is increasing and strictly concave for $x_{t-1} > x_t^u$. Moreover,

$$\frac{\partial\pi_t^*(x)}{\partial x} = \begin{cases} k, & \text{if } x < x_t^u \\ \pi_t'(x) + \delta\frac{\partial\pi_{t+1}^*(x)}{\partial x}, & \text{if } x_t^u < x \end{cases}$$

Since x_t^u solves $\pi_t'(x_t) - k + \delta\frac{\partial\pi_{t+1}^*(x_t)}{\partial x} = 0$, we have that π_t^* is differentiable at x_t^u, and $\frac{\partial\pi_t^*(x_t^u)}{\partial x} = k$. In addition, since $\pi_t^*(\cdot)$ is strictly concave for $x > x_t^u$, we have that $\pi_t'(x) + \delta\frac{\partial\pi_{t+1}^*(x)}{\partial x} < k$ for $x > x_t^u$. Thus, $\frac{\partial\pi_t^*(x)}{\partial x} = k$ for $x \leq x_t^u$ and $\frac{\partial\pi_t^*(x)}{\partial x} < k$ for $x > x_t^u$. ■

Lemma 1

Proof. Consider the case $n < T - t_0$. If no investments are made in periods t_0+1 through t_0+n-1, then $x_{t_0}^* > x_t^u$ and $\frac{\partial\pi_t^*(x_{t_0}^*)}{\partial x} = \pi_t'(x_{t_0}^*) + \delta\frac{\partial\pi_{t+1}^*(x_{t_0}^*)}{\partial x}$ for $t = t_0+1, ..., t_0+n-1$. In addition, $x_{t_0}^* = x_{t_0}^u$ solves

$$0 = \pi_{t_0}'(x_{t_0}^*) - k + \delta\frac{\partial\pi_{t_0+1}^*(x_{t_0}^*)}{\partial x} = \pi_{t_0}'(x_{t_0}^*) - k + \delta\pi_{t_0+1}'(x_{t_0}^*) + \delta^2\frac{\partial\pi_{t_0+2}^*(x_{t_0}^*)}{\partial x} = ... =$$

$$\sum_{t=t_0}^{t_0+n-1}\delta^{t-t_0}\pi_t'(x_{t_0}^*) - k + \delta^n\frac{\partial\pi_{t_0+n}^*(x_{t_0}^*)}{\partial x} = \sum_{t=t_0}^{t_0+n-1}\delta^{t-t_0}\pi_t'(x_{t_0}^*) - k + \delta^n k,$$

where the last equality holds since $x_{t_0}^* \leq x_{t_0+n}^u$. Then, $x_{t_0}^* = x_{t_0}^u = x_{t_0}^n$.

The case with $n = T - t_0$ follows similarly.

Following a similar reasoning as above, we have that

$$0 = \pi_{t_0}'(x_{t_0}^n) - k + \delta\frac{\partial\pi_{t_0+1}^*(x_{t_0}^n)}{\partial x} = \pi_{t_0}'(x_{t_0}^n) - k + \delta\pi_{t_0+1}'(x_{t_0}^n) + \delta^2\frac{\partial\pi_{t_0+2}^*(x_{t_0}^n)}{\partial x} = ... =$$

$$\sum_{t=t_0}^{t_0+n-(l+1)} \delta^{t-t_0}\pi_t'(x_{t_0}^n) - k + \delta^{n-l}\frac{\partial \pi_{t_0+n-l}^*(x_{t_0}^n)}{\partial x} \leq \sum_{t=t_0}^{t_0+n-(l+1)} \delta^{t-t_0}\pi_t'(x_{t_0}^n) - k + \delta^{n-l}k,$$

where the inequality follows from Proposition 1. This implies that $\sum_{t=t_0}^{t_0+n-(l+1)} \delta^{t-t_0}\pi_t'(x_{t_0}^{n-l}) \leq \sum_{t=t_0}^{t_0+n-(l+1)} \delta^{t-t_0}\pi_t'(x_{t_0}^n)$, which is equivalent to $x_{t_0}^n \leq x_{t_0}^{n-l}$. ■

Lemma 4 *Suppose that in the optimal investment path a positive investment is made in period t_0, for some $1 \leq t_0 < T$. In addition, suppose that another positive investment is made in period $t_0 + n$, for $t_0 + n \leq T + 1$. Then, (a)*

$$\sum_{t=t_0}^{t_0+n-1} \delta^{t-t_0}\pi_t'(x_t^*) = \begin{cases} (1-\delta^n)k, & \text{if } 1 \leq n < T+1-t_0 \\ k, & \text{if } n = T+1-t_0. \end{cases}$$

(b) For some $l < n-1$,

$$\sum_{t=t_0+l}^{t_0+n-1} \delta^{t-t_0-l}\pi_t'(x_{t_0}^*) < \begin{cases} (1-\delta^{n-l})k, & \text{if } 1 \leq n < T+1-t_0 \\ k, & \text{if } n = T+1-t_0. \end{cases}$$

Proof. (a) Suppose that the $i+1$ periods of optimal investment in the optimal investment path, starting with period t_0, are t_0, t_0+n_1,..., $t_0+n_1+...+n_i$, and that $n = n_1+...+n_j$, for some $j \leq i$. Let $m = n_1+...+n_i$. Note that the optimal investment levels are $x_{t_0}^{n_1}$ at t_0, $x_{t_0+n_1}^{n_2}$ at t_0+n_1, .., $x_{t_0+m}^{T-t_0-m+1}$ at t_0+m. Then,

$$\begin{aligned}
\sum_{t=t_0}^{T} \delta^{t-t_0}\pi_t'(x_t^*) &= \sum_{t=t_0}^{t_0+n_1-1} \delta^{t-t_0}\pi_t'(x_{t_0}^{n_1}) + \sum_{t=t_0+n_1}^{t_0+n_1+n_2-1} \delta^{t-t_0}\pi_t'(x_{t_0+n_1}^{n_2}) \\
&\quad +... + \sum_{t=t_0+m}^{T} \delta^{t-t_0}\pi_t'(x_{t_0+m}^{T-t_0-m+1}) \\
&= \sum_{t=t_0}^{t_0+n_1-1} \delta^{t-t_0}\pi_t'(x_{t_0}^{n_1}) + \delta^{n_1}\sum_{t=t_0+n_1}^{t_0+n_1+n_2-1} \delta^{t-t_0-n_1}\pi_t'(x_{t_0+n_1}^{n_2}) \\
&\quad +... + \delta^m \sum_{t=t_0+m}^{T} \delta^{t-t_0-m}\pi_t'(x_{t_0+m}^{T-t_0-m+1})
\end{aligned}$$

By the definition in (2),

$$\begin{aligned}
\sum_{t=t_0}^{t_0+n_1-1} \delta^{t-t_0}\pi_t'(x_{t_0}^{n_1}) &= (1-\delta^{n_1})\,k, \\
\sum_{t=t_0+n_1}^{t_0+n_1+n_2-1} \delta^{t-t_0-n_1}\pi_t'(x_{t_0+n_1}^{n_2}) &= (1-\delta^{n_2})k, \cdots \\
\sum_{t=t_0+m}^{T} \delta^{t-t_0-m}\pi_t'(x_{t_0+m}^{T-t_0-m+1}) &= k.
\end{aligned}$$

Hence, we get the following telescopic sum

$$\begin{aligned}\sum_{t=t_0}^{T}\delta^{t-t_0}\pi_t'(x_t^*) &= (1-\delta^{n_1})k+\delta^{n_1}(1-\delta^{n_2})k+...+\delta^{m}k \\ &= k\end{aligned}$$

Similarly,

$$\begin{aligned}\sum_{t=t_0}^{t_0+n-1}\delta^{t-t_0}\pi_t'(x_t^*) &= \sum_{t=t_0}^{t_0+n_1-1}\delta^{t-t_0}\pi_t'(x_{t_0}^{n_1})+\sum_{t=t_0+n_1}^{t_0+n_1+n_2-1}\delta^{t-t_0}\pi_t'(x_{t_0+n_1}^{n_2})+...+\sum_{t=t_0+n-n_j}^{t_0+n-1}\delta^{t-t_0}\pi_t'(x_{t_0+n-n_j}^{n_j}) \\ &= (1-\delta^{n_1})k+\delta^{n_1}(1-\delta^{n_2})k+...+\delta^{n-n_j}(1-\delta^{n_j})k \\ &= (1-\delta^{n_1+...+n_j})k=(1-\delta^{n})k.\end{aligned}$$

(b) Consider the case $t_0+n<T+1$. If no investments are made in periods t_0+1 through t_0+n-1, then $x_{t_0}^*>x_t^u$ and $\frac{\partial\pi_t^*(x_{t_0}^*)}{\partial x}=\pi_t'(x_{t_0}^*)+\delta\frac{\partial\pi_{t+1}^*(x_{t_0}^*)}{\partial x}$ for $t=t_0+1,...,t_0+n-1$. Then, $x_{t_0}^*$ solves $0=\pi_{t_0}'(x_{t_0}^*)-k+\delta\frac{\partial\pi_{t_0+1}^*(x_{t_0}^*)}{\partial x}=\pi_{t_0}'(x_{t_0}^*)-k+\delta\pi_{t_0+1}'(x_{t_0}^*)+\delta^2\frac{\partial\pi_{t_0+2}^*(x_{t_0}^*)}{\partial x}=...=\sum_{t=t_0}^{t_0+n-1}\delta^{t-t_0}\pi_t'(x_{t_0}^*)-k+\delta^{t_0+n-t_0}\frac{\partial\pi_{t_1}^*(x_{t_0}^*)}{\partial x}=\sum_{t=t_0}^{t_0+n-1}\delta^{t-t_0}\pi_t'(x_{t_0}^*)-k+\delta^{t_0+n-t_0}k$, where the last equality holds since $x_{t_0}^*\le x_{t_0+n}^u$. Consider $1\le l<n-1$. Since $x_{t_0}^*>x_{t_0+l}^u$, we have that $k>\frac{\partial\pi_{t_0+l}^*(x_{t_0}^*)}{\partial x}=\pi_{t_0+l}'(x_{t_0}^*)+\delta\frac{\partial\pi_{t_0+l+1}^*(x_{t_0}^*)}{\partial x}=\sum_{t=t_0+l}^{t_0+n-1}\delta^{t-t_0-l}\pi_t'(x_{t_0}^*)+\delta^{n-l}k$. The case with $t_0+n=T+1$ follows similarly. ■

Proposition 2

Proof. We show this by complete induction. Begin with $T=2$. In this case, $\bar{x}_2^*=x_2^1$, and $\bar{x}_1^*=x_1^1$ if $x_1^1<x_2^1$, and $\bar{x}_1^*=x_1^2$ otherwise. Suppose that $x_1^1<x_2^1$ but in the optimal investment path it is optimal to invest in period 1 only. Then, Lemma 1 implies that $x_1^*=x_1^u=x_1^2$ and that $x_1^2\le x_1^1$, by part (3). At the same time,

$$\pi_1'(x_1^2)+\delta\pi_2'(x_1^2)=k=(1-\delta)k+\delta k=\pi_1'(x_1^1)+\delta\pi_2'(x_2^1)<\pi_1'(x_1^1)+\delta\pi_2'(x_1^1),$$

where the first equality follows from Lemma 4(a) and the inequality from $x_1^1<x_2^1$. This implies that $x_1^1<x_1^2$, a contradiction. We conclude that if $x_1^1<x_2^1$, it then is optimal to invest in periods 1 and 2. In that case, it follows that $x_1^*=x_1^1$ and $x_2^*=x_2^1$. If $x_1^1\ge x_2^1$, then a positive investment can only occur in period 1, and $x_1^*=x_1^2$.

Suppose now that the result holds if we consider the investment problem starting in any period $t_0+1\le T$. Consider now the investment problem starting in period t_0. Suppose first that

$x_{t_0}^1 < x_{t_0+1}^*$ – let's show that it is then optimal to invest in periods t_0 and t_0+1 and then follow the optimal investment path given by the inductive step. Suppose instead that it is optimal to invest in periods t_0 and t_0+n, for $n > 1$, and then follow the optimal investment path for the subsequent periods given by induction. This implies that $x_{t_0}^n \leq x_{t_0}^1$ by (3). Note that

$$\sum_{t=t_0}^{t_0+n-1} \delta^{t-t_0} \pi_t'(x_{t_0}^n) = (1-\delta^n)k = (1-\delta)k + \delta(1-\delta^{n-1})k = \pi_{t_0}'(x_{t_0}^1) + \delta(1-\delta^{n-1})k. \tag{11}$$

Suppose that $x_{t_0+1}^* = x_{t_0+1}^{m_1}$ with $m_1 \geq n-1$. Then, (3) implies that $x_{t_0+1}^* \leq x_{t_0+1}^{n-1}$ which, in turn, implies that $\sum_{t=t_0+1}^{t_0+n-1} \delta^{t-t_0-1} \pi_t'(x_{t_0+1}^*) \geq (1-\delta^{n-1})k$. Suppose now that the last period of positive investment before period $n-1$ from the optimal path that starts in period t_0+1 is t_0+m_1 and $x_{t_0+m_1}^* = x_{t_0+m_1}^{m_2}$ with $n - m_1 \leq m_2 \leq T - t_0 - m_1 + 1$. Then,

$$\sum_{t=t_0+1}^{t_0+m_1-1} \delta^{t-t_0-1} \pi_t'(x_t^*) + \delta^{m_1-1} \sum_{t=t_0+m_1}^{t_0+n-1} \delta^{t-t_0-m_1} \pi_t'(x_{t_0+m_1}^*) = (1-\delta^{m_1-1})k + \delta^{m_1-1} \sum_{t=t_0+m_1}^{t_0+n-1} \delta^{t-t_0-m_1} \pi_t'(x_{t_0+m_1}^{m_2})$$

$$\geq (1-\delta^{m_1-1})k + \delta^{m_1-1}(1-\delta^{n-m_1})k = (1-\delta^{n-1})k,$$

where the last inequality again follows from (3). Thus,

$$(1-\delta^{n-1})k \leq \sum_{t=t_0+1}^{t_0+n-1} \delta^{t-t_0-1} \pi_t'(x_t^*) < \sum_{t=t_0+1}^{t_0+n-1} \delta^{t-t_0-1} \pi_t'(x_{t_0}^1)$$

for the optimal investment path that starts in period t_0+1, because $x_{t_0}^1 < x_{t_0+1}^*$ and the investments are cumulative. It follows from (11) that

$$\sum_{t=t_0}^{t_0+n-1} \delta^{t-t_0} \pi_t'(x_{t_0}^n) < \sum_{t=t_0}^{t_0+n-1} \delta^{t-t_0} \pi_t'(x_{t_0}^1),$$

which implies that $x_{t_0}^1 < x_{t_0}^n$, a contradiction. Suppose now that $x_{t_0}^1 < x_{t_0+1}^*$ and it is optimal to invest in period t_0 and in no other subsequent period. That investment level would be $x_{t_0}^{T+1-t_0} \leq x_{t_0}^1$, by (3). Then,

$$\sum_{t=t_0}^{T} \delta^{t-t_0} \pi_t'(x_{t_0}^{T+1-t_0}) = k = (1-\delta)k + \delta k = \pi_{t_0}'(x_{t_0}^1) + \delta \sum_{t=t_0+1}^{T} \delta^{t-t_0-1} \pi_t'(x_t^*) < \sum_{t=t_0}^{T} \delta^{t-t_0} \pi_t'(x_{t_0}^1),$$

where $\{x_t^*\}$ is the optimal investment path starting from period t_0+1, and the last equality follows from Lemma 4(a). This again results in a contradiction. We conclude that if $x_{t_0}^1 < x_{t_0+1}^*$, then it is optimal to invest in periods t_0 and t_0+1, following then the optimal investment path given by the inductive hypothesis.

If $x^1_{t_0} \geq x^*_{t_0+1}$ but $x^2_{t_0} < x^*_{t_0+2}$ similar arguments show that it cannot be optimal to invest $x^n_{t_0}$ and then follow the optimal investment path that starts with $x^*_{t_0+n}$, for $n > 2$, by noting that

$$\sum_{t=t_0}^{t_0+n-1} \delta^{t-t_0} \pi'_t(x^n_{t_0}) = (1-\delta^n)k = (1-\delta^2)k + \delta^2(1-\delta^{n-2})k = \pi'_{t_0}(x^2_{t_0}) + \delta\pi'_{t_0+1}(x^2_{t_0}) + \delta^2(1-\delta^{n-2})k,$$

and then again showing that

$$(1-\delta^{n-2})k \leq \sum_{t=t_0+2}^{t_0+n-1} \delta^{t-t_0-1}\pi'_t(\bar{x}^*_t) < \sum_{t=t_0+2}^{t_0+n-1} \delta^{t-t_0-1}\pi'_t(x^2_{t_0}).$$

For this case, it remains to show that $x^2_{t_0} \geq x^*_{t_0+1}$. To that end, note that

$$\pi'_{t_0}(x^*_{t_0+1}) + \delta\pi'_{t_0+1}(x^*_{t_0+1}) \geq \pi'_{t_0}(x^1_{t_0}) + \delta\pi'_{t_0+1}(x^1_{t_0+1}) = (1-\delta)k + \delta(1-\delta)k = (1-\delta^2)k = \pi'_{t_0}(x^2_{t_0}) + \delta\pi'_{t_0+1}(x^2_{t_0}),$$

where the first inequality follows because $x^1_{t_0} \geq x^*_{t_0+1}$, and $x^1_{t_0+1} \geq x^*_{t_0+1}$ by (3). Then, if $x^1_{t_0} \geq x^*_{t_0+1}$ but $x^2_{t_0} < x^*_{t_0+2}$, it is optimal to invest in periods t_0 and t_0+2, following then the optimal investment path that starts in period $t_0 + 2$. ■

Proposition 3

Proof. Define $\Pi_i(x) = \pi_i(x) - k_i(x_i - 1)$ and $\gamma(x) = -\frac{1}{2(M+1)^2 b}(a - \sum_j v_j(x_j))$. The first derivative of π_i is given by

$$\frac{\partial \pi_i}{\partial x_i}(x_t) = -\frac{1}{2(M+1)^2 b}(a - \sum_j v_j(x_j))v'_i(x_i).$$

Because the cross partial derivative of each supplier i's profit function is positive, the game is supermodular:

$$\frac{\partial^2 \Pi_i(x)}{\partial x_j \partial x_i} = \tfrac{1}{2(M+1)^2 b} v'_j(x_j) v'_i(x_i) > 0 \text{ for } j \neq i.$$

Because the game is supermodular, the set of equilibria is a lattice and there exists one largest and one smallest equilibrium.

For uniqueness, we establish sufficient conditions for strict diagonal dominance of the Jacobian.

$$\begin{aligned}
\left|\frac{\partial^2 \Pi_i(x)}{\partial x_i^2}\right| - \sum_{j\neq i} \frac{\partial^2 \Pi_i(x)}{\partial x_j \partial x_i} &= -\gamma(x) v''_i(x_i) - \tfrac{1}{2(M+1)^2 b}(v'_i(x_i))^2 - \sum_{j\neq i} \tfrac{1}{2(M+1)^2 b} v'_j(x_j) v'_i(x_i) \\
&= -\tfrac{1}{2(M+1)^2 b} v'_i(x) \left(\left(a - \sum_j v_j(x_j)\right)(\beta_i + 1)(x_i)^{-1} - \sum_j \beta_j v_j(x_j)(x_j)^{-1} \right) \\
&> -\tfrac{1}{2(M+1)^2 b} v'_i(x) \left((\beta_i + 1)\, a\, (x_i)^{-1} - \sum_j \left(1 + \beta_i + \beta_j\right) v_j(x_i) \right) \\
&> -\tfrac{1}{2(M+1)^2 b} v'_i(x) \left((\beta_i + 1)\, a\, (x_i)^{-1} - \sum_j \left(1 + \beta_i + \beta_j\right) v_{0j} \right).
\end{aligned}$$

If the condition stated in the proposition holds, the term in parenthesis is non-negative, establishing uniqueness. Otherwise, it suffices to note that each supplier's profit function Π_i increases in the other suppliers' investment levels and that the assembler's profit increases in all suppliers' investment levels. Then, all firms prefer the largest equilibrium. The monotonicity results follow from the supermodularity of the game and the signs of the derivative of the equilibrium conditions with respect to those parameters. ■

Proposition 4

Proof. This follows from a simple re-formulation of our problem together with the result in Exercise 4.10 in Fudenberg and Tirole (2000). Fudenberg and Tirole state that open-loop equilibrium is in the set of closed-loop equilibria in deterministic games if the action space in each period is independent of the state variables.

In our model, the action space for firm i is constrained by the state variable, cumulative investment level in the previous period, i.e. $x_{it} \geq x_{i,t-1}$ for all i, t. However, it is possible to reformulate the problem without constraints on investment levels. Define the decision variables for firm i as the incremental investment in period t (y_{it}). Then, $x_{it} = 1 + \sum_{m=1}^{t} y_{it}$. In this formulation, the action space of this deterministic game is the same in each period and is not affected by the history. That is, $y_{it} \geq 0$ for all i, t. The result follows. ■

Theorem 1

Proof. Define

$$\gamma_t(x) = -\frac{1}{2(M+1)^2 b_t}(a_t - \sum_j v_j(x_j)). \tag{12}$$

Consider supplier i. For any fixed paths of investment x_{-i} made by the other suppliers, supplier i's problem is as the one considered in Section 4, with the intercept in period t replaced by $a_t - \sum_{j \neq i} v_j(x_{jt})$. Then, for fixed x_{-i}, the results in that section apply. Based on the definition of $\gamma_t(x_t)$, we have that $\partial \pi_{it}(x_t)/\partial x_{it} = \gamma_t(x_t) v_i'(x_{it})$. Recall that $\gamma_t(x) < 0$ and $v_i' < 0$ and increasing since v_i is convex.

Let $(x_1^c, ..., x_M^c)$ be an open-loop equilibrium. Suppose that, under this equilibrium, supplier i invests in the last period T, but there is another supplier j that does not. Denote $t_0 < T$ to be the last period in which supplier j makes an investment. Since supplier i invests in period T we have that $\gamma(x_T^c) v_i'(x_{iT}^c) = k_i$, while $x_{jT}^c = x_{j,t_0}^c$ and $\gamma(x_T^c) v_j'(x_{j,t_0}^c) < k_j$ because supplier j's unconstrained investment level in period T is lower than x^c_{j,t_0}. This implies that $(k_i/k_j)\, v'_j(x^c_{j,t_0}) > v'_i(x^c_{iT})$. Moreover, since $x^c_{i,t_0} \leq ... \leq x^c_{iT}$ and v'_i is increasing, we have that $v'_i(x^c_{it_0}) < .. < v'_i(x^c_{i,T})$. We also have from Lemma 4(a) that

$$\sum_{t=t_0}^{T} \delta^{t-t_0} \frac{\partial \pi_{jt}}{\partial x_{jt}}(x^c_t) = \sum_{t=t_0}^{T} \delta^{t-t_0} \frac{\partial \pi_{jt}}{\partial x_{jt}}(x^c_{j,t_0}, x^c_{-j,t}) = \sum_{t=t_0}^{T} \delta^{t-t_0} \gamma(x^c_t) v'_j(x^c_{j,t_0}) = k_j.$$

Because $\gamma_t(x) < 0$, we have that

$$\sum_{t=t_0}^{T} \delta^{t-t_0} \gamma_t(x^c_t) v'_i(x^c_{it}) > \sum_{t=t_0}^{T} \delta^{t-t_0} \gamma_t(x^c_t) v'_i(x^c_{iT}) > \frac{k_i}{k_j} \sum_{t=t_0}^{T} \delta^{t-t_0} \gamma_t(x^c_t) v'_j(x^c_{j,t_0}) = k_i.$$

Now consider the investment periods for supplier i, starting from the last period with positive investment before or at t_0. Suppose that these are the following $n+1$ periods $t_0 - l < t_1 < ... < t_n < T$, with $l \geq 0$. By Lemma 4(b), we have

$$\sum_{t=t_0}^{T} \delta^{t-t_0} \gamma(x^c_t) v'_i(x^c_{it}) < k_i,$$

leading to a contradiction.

We have shown so far that in the last period T, either all suppliers make a positive investment or none does. Suppose now that this is true for every period between periods $t_n < T$ and T. We now consider two possible cases. Suppose first that all suppliers make a positive investment in period t_n. Further, suppose that supplier i also makes a positive investment in period $t_n - 1$, while supplier j does not. We then have that $\gamma(x^c_{t_n-1}) v'_i(x^c_{it_n-1}) = (1-\delta) k_i$ for supplier i, while for supplier j, $x^c_{jt_n-1} = x^c_{j,t_0}$ and $\gamma_t(x^c_{t_n-1}) v'_j(x^c_{j,t_0}) < (1-\delta) k_j$, where t_0 is the last investment period for that supplier before t_n. This again implies that $(k_i/k_j)\, v'_j(x^c_{j,t_0}) > v'_i(x^c_{it})$ for all $t = t_0, ..., t_n - 1$. Because supplier j invests in periods t_0 and t_n, while it makes no investments between those two periods, we have that

$$(1 - \delta^{t_n - t_0}) k_i = \frac{k_i}{k_j} \sum_{t=t_0}^{t_n - 1} \delta^{t-t_0} \frac{\partial \pi_{jt}}{\partial x_{jt}}(x^c_t) = \frac{k_i}{k_j} \sum_{t=t_0}^{t_n-1} \delta^{t-t_0} \gamma_t(x^c_t) v'_j(x^c_{j,t_0}) < \sum_{t=t_0}^{t_n-1} \delta^{t-t_0} \gamma_t(x^c_t) v'_i(x^c_{it}).$$

Now again consider the investment periods for supplier i, starting from the last period with positive investment before or at t_0. Suppose that these are given by $t_0 - l < t_1 < ... < t_{n-1} < t_n - 1$ for $l \geq 0$. By Lemma 4(b), we have

$$\sum_{t=t_0}^{t_n-1} \delta^{t-t_0} \gamma(x^c_t) v'_i(x^c_{it}) < \left(1 - \delta^{t_n - t_0}\right) k_i,$$

leading to a contradiction.

Finally, suppose now that no suppliers invest in periods t_n through $t_n + l - 1$, and all suppliers make a positive investment in period t_n+l. In addition, suppose that supplier i makes an investment in period $t_n - 1$, while supplier j does not, and let t_0 be the last period before t_n in which supplier j makes a positive investment. Because supplier i invests in periods $t_n - 1$ and $t_n + l$, we have that

$$\sum_{t=t_n-1}^{t_n+l-1} \delta^{t-t_n+1}\gamma_t(x_t^c)v_i'(x_{i,t_n-1}^c) = (1-\delta^{l+1})k_i, \quad \text{while}$$

$$\sum_{t=t_n-1}^{t_n+l-1} \delta^{t-t_n+1}\gamma_t(x_t^c)v_j'(x_{j,t_0}^c) < (1-\delta^{l+1})k_j. \tag{13}$$

To see why the latter inequality holds, note that for fixed $x_{-j,t}^c$, supplier j's unconstrained investment level in period $t_n - 1$ maximizes the convex function $\sum_{t=t_n-1}^{t_n+l-1} \delta^{t-t_n+1}\pi_{jt}(x, x_{-j,t}^c)$ (because the next period with investment after $t_n - 1$ is $t_n + l$). Since this supplier does not invest in period $t_n - 1$, while it does in period t_0, we must have that x_{j,t_0}^c is larger than the above unconstrained maximizer, leading to the inequality in (13). Once again, this implies that $(k_i/k_j)\, v_j'(x_{j,t_0}^c) > v_i'(x_{it}^c)$ for all $t = t_0, ..., t_n - 1$. The result then follows as in the previous cases. ■

Lemma 2

Proof. Consider a vector of investment level paths x_{-i} for all suppliers but supplier i. Let $R_{i,t_0}^n(x_{-i})$ be the best response of supplier i to the vector x_{-i} for investing at time t_0 for n periods. That is, $R_{i,t_0}^n(x_{-i})$ is the solution to

$$\sum_{t=t_0}^{t_0+n-1} \delta^{t-t_0}\frac{\partial \pi_{it}(x_i, x_{-i})}{\partial x_i} = (1-\delta^n)k_i. \tag{14}$$

Clearly, $R_{i,t_0}^n(x_{-i,t_0}^{n,c}) = x_{i,t_0}^{n,c}$. Fixing $x_{-i,t_0}^{n,c}$, let $\tilde{a}_t = a_t - \sum_{j\neq i} v_j(x_{j,t_0}^{n,c})$ and consider supplier i's (single firm) investment problem with intercept values $\tilde{a}_t$. It is then optimal for supplier i to invest in period t_0 for n periods, and applying (3) in Lemma 1 to supplier i's single-firm problem with intercept values $\tilde{a}_t$, we have

$$x_{i,t_0}^{n,c} = R_{i,t_0}^n(x_{-i,t_0}^{n,c}) \leq R_{i,t_0}^{n-l}(x_{-i,t_0}^{n,c}), \quad \text{for} \quad 1 \leq l \leq n-1.$$

The same reasoning applies to all suppliers. Then, for fixed $1 \leq l \leq n-1$, we have that

$$x_{j,t_0}^{n,c} = R_{j,t_0}^n(x_{-j,t_0}^{n,c}) \leq R_{j,t_0}^{n-l}(x_{-j,t_0}^{n,c}), \quad \text{for} \quad 1 \leq j \leq M.$$

Theorem 2.5 in Vives (2000) states that the largest (or unique) equilibrium $x_{t_0}^{n-l,c}$ satisfies

$$x_{t_0}^{n-l,c} = \sup\left\{x : x \leq \left(R_{1t_0}^{n-l}(x_{-1}), ..., R_{Mt_0}^{n-l}(x_{-M})\right)\right\}.$$

This implies that $x_{t_0}^{n,c} \leq x_{t_0}^{n-l,c}$. ■

Lemma 3

Proof. We start with period 1. Suppose that the optimal investment levels are $\hat{x}_1^n$ and $\hat{y}_1^m$. If $n > m$, then $\hat{x}_1^n \leq \hat{x}_1^m \leq \hat{y}_1^m$, where the first inequality follows from Lemma 2. For investments in periods $m+1, ..., n$, note that $\hat{y}_{m+1}^{m_2} \geq \hat{y}_1^m \geq \hat{x}_1^n$. Then, the investment levels in the second problem are no smaller than those in the first problem for all periods $1, ..., n$. Consider now the case $n \leq m$. Here, $\hat{x}_1^n \leq \hat{x}_{n+1}^{n_2}$ for some n_2. If $n_2 > m-n$, then $\hat{x}_1^n \leq \hat{x}_{1+n}^{n_2} \leq \hat{x}_{1+n}^{m-n} \leq \hat{y}_{1+n}^{m-n} \leq \hat{y}_1^m$, where the second inequality follows from Lemma 2 and the last inequality follows from the fact that in the second problem it is optimal to invest in period 1 for m periods (see Proposition 9). Thus, $x_t^c \leq y_t^c$ for $t = 1, ..., m$. If $n_2 \leq m-n$, then suppose that $1 + n + n_2 + ... + n_l$ is the last period of investment before period m in the equilibrium path of the first problem. Then, $\hat{x}_1^n \leq \hat{x}_{1+n}^{n_2} \leq ... \leq \hat{x}_{1+n+...+n_l}^{n_{l+1}} \leq \hat{x}_{1+n+...+n_l}^{m-n} \leq \hat{y}_{1+n+...+n_l}^{m-n} \leq \hat{y}_1^m$, where the second to last inequality follows from Lemma 2 and the last inequality follows because it is optimal to invest in period 1 for m periods in the second problem. Thus, we conclude that $x_t^c \leq y_t^c$ for $1 \leq t \leq \max\{n, m\}$. Similar arguments apply to all subsequent periods.

Suppose that the last periods of investment for the first and second problems are t_0 and t_1, respectively. If $t_1 < t_0$, then $\hat{x}_{t_0}^{T-t_0+1} \leq \hat{y}_{t_0}^{T-t_0+1} \leq \hat{y}_{t_1}^{T-t_1+1}$ where the last inequality follows because t_1 is the last period of investment in the equilibrium path of the second problem. If $t_0 \leq t_1$, then $\hat{x}_{t_0}^{T-t_0+1} \leq \hat{x}_{t_0}^{T-t_1+1} \leq \hat{y}_{t_1}^{T-t_1+1}$, where the first inequality follows from Lemma 2. ■

Proposition 5

Proof. Based on Theorem 1 and Proposition 2, the timing of investments in the optimal investment path is determined by making comparisons of the form $\hat{x}_{t_0}^n \leq \hat{x}_{t_0+n}^m$. Recall that $\hat{x}_{t_0}^n$ solves

$$-\sum_{t=t_0}^{t_0+n-1} \delta^{t-t_0} \frac{\left(a_t - \sum_{j=1}^M v_j(\hat{x}_{jt_0}^n)\right) v_i'(\hat{x}_{it_0}^n)}{2(M+1)^2 b_t} = (1-\delta^n)k_i, \; i = 1, ..., M. \tag{15}$$

Note that $-v_i'(x_i)\sum_{j=1}^M v_j(x_j)$ and $-v_i'(x_i)$ are decreasing in x_i. This observation together with the two inequalities stated in the proposition imply that $\hat{x}_{t_0}^n \leq \hat{x}_{t_0+n}^m$. If $a_t = a$ for all t, then the second inequality in the statement of the proposition is redundant and the first inequality only depends on $\{b_t\}$ and δ.

Consider the case of constant a_t and let t_m be the last period of investment in the equilibrium investment path. It follows that it is optimal to invest in periods t_0 and $t_0+1<t_m$ if $\frac{1}{b_{t_0}} \leq \frac{1}{b_{t_0+1}}$, i.e., for the same price level demand is higher in period t_0+1. Thus, in the case of constant demand elasticity, all suppliers invest in process improvement as the market is expanding and will all stop investment in the same time period. ∎

Proposition 6

Proof. In this proof, we use the target-price contracts (TP(ρ)) and (TP) introduced in Section 5.2. Let us start with the linear demand case. Let x^{tp} be the equilibrium solution. Recall that $\hat{x}^n_{t_0}$ in (C) satisfies

$$-\sum_{t=t_0}^{t_0+n-1} \delta^{t-t_0} \tfrac{1}{2(M+1)^2 b_t}(a_t - \sum_j v_j(\hat{x}^n_t))v_i'(\hat{x}^n_{i,t_0}) = (1-\delta^n)k_i \text{ for all } i. \tag{16}$$

Define the vector $x^{n,tp}_{t_0}$ as the solution to

$$-\sum_{t=t_0}^{t_0+n-1} \delta^{t-t_0} \tfrac{1}{2(M+1)b_t}(a_t - \sum_j \tilde{v}_{jt})v_i'(x^{n,tp}_{i,t_0}) = (1-\delta^n)k_i \text{ for all } i. \tag{17}$$

Note that $(a_t - \sum_j v_j(x_{jt}))/(M+1) < (a_t - \sum_j \tilde{v}_{jt})$ for all x and t. The inequality is equivalent to $M\sum_j \tilde{v}_{it} + \left(\sum_j \tilde{v}_{it} - \sum_j v_j(x_{jt})\right) < Ma_t$, and it holds because $0 \leq \tilde{v}_{it} \leq v_{i0}$, $0 \leq v_{it}(x) \leq v_{i0}$ for all x, and $a_t > 2\sum_j v_{j0}$ for all t. This implies that the left-hand-side of (17) evaluated at $\hat{x}^n_{i,t_0}$ is less than that of (16). Thus, $x^{n,tp}_{i,t_0} > \hat{x}^n_{i,t_0}$ for any t_0 and n. (The right hand side of the above equations is k_i instead of $(1-\delta^n)\,k_i$ if $t+n = T+1$.)

Because the results in (3) and Lemma 4 apply to decentralized settings under target-price contracts, Algorithm $(1, \{x^{n,tp}_{it}\})$ can be used for $i = 1, ..., M$ to obtain x^{tp}. Then, Lemma 3 implies that $x^{tp} > x^c$.

Next, we show the same result with the nonlinear demand functions considered. With $d_t(p) = ((a_t - p)/b_t)^\eta$, solving the Stackelberg equilibrium we get $m_{it} = (a_t - \sum_j v_j(x_{jt}))\eta/((1+\eta)(M+\eta))$, $m_{At} = (a_t - \sum_j v_j(x_{jt}))/(1+\eta)$, and $\pi_{it}(x_t) = b^{-\eta}((a_t - \sum_j v_j(x_{jt}))\eta/((1+\eta)(M+\eta)))^{\eta+1}$. In this case, $\hat{x}^n_{t_0}$ in (C) satisfies

$$-\sum_{t=t_0}^{t_0+n-1} \delta^{t-t_0} (b_t(\eta+1))^{-\eta} (\eta/(\eta+M))^{\eta+1}(a_t - \sum_j v_j(\hat{x}^n_{jt}))^\eta v_i'(\hat{x}^n_{i,t_0}) = (1-\delta^n)k_i \text{ for all } i. \tag{18}$$

It can be verified that under TP, $q_t^c = (b_t(\eta+1))^{-\eta}\,(\eta/(\eta+M))^{\eta}(a_t - \sum_j \tilde{v}_{jt})^{\eta}$. Then, under TP or TP($\rho$), $x_{t_0}^{n,tp}$ is the solution to

$$-\sum_{t=t_0}^{t_0+n-1} \delta^{t-t_0} q_t^c v_i'(x_{i,t_0}^{n,tp}) = (1-\delta^n)k_i \text{ for all } i,$$

The comparison of the left-hand-sides of the first order conditions for (C) and TP is equivalent to the following inequality. $(a_t - \sum_j v_j(x_{jt}))\eta/\,(\eta+M) < (a_t - \sum_j \tilde{v}_{jt})$. The inequality is equivalent to $(\eta+M)\sum_j \tilde{v}_{it} - \eta\sum_j v_j(x_{jt}) < Ma_t$, and it holds for all x and t because $0 \le \tilde{v}_{it} \le v_{i0}$, and $a_t > (\eta+M)v_{i0}$ for all t by the concavity condition. Thus, $x_{i,t_0}^{n,tp} > \hat{x}_{i,t_0}^{n}$ for any t_0 and n, which implies that $x^{tp} > x^c$.

With the constant elasticity demand function $d_t(p) = b_t^{-1}p^{-1/a_t}$, solving the Stackelberg equilibrium we get $m_{it} = a_t \sum_j v_j(x_{jt})\,(1-a_t)^{-1}\,(1-a_tM)^{-1}$, $m_{At} = a_t \sum_j v_j(x_{jt})\,(1-a_t)^{-1}$, and $\pi_{it}(x_t) = a_t b_t^{-1}(\sum_j v_j(x_{jt})\,(1-a_t)^{-1}\,(1-a_tM)^{-1})^{1-1/a_t}$. In this case, $\hat{x}_{t_0}^{n}$ in (C) satisfies

$$-\sum_{t=t_0}^{t_0+n-1} \delta^{t-t_0}\,(1-a_tM)^{-1}\,b_t^{-1}(\sum_j v_j(\hat{x}_{jt}^{n})\,(1-a_t)^{-1}\,(1-a_tM)^{-1})^{-1/a_t} v_i'(\hat{x}_{i,t_0}^{n}) = (1-\delta^n)k_i \text{ for all } i. \tag{19}$$

It can be verified that under TP or TP(ρ), $q_t^c = b_t^{-1}(\sum_j \tilde{v}_{jt}\,(1-a_t)^{-1}\,(1-a_tM)^{-1})^{-1/a_t}$. Then, $x_{t_0}^{n,tp}$ is the solution to

$$-\sum_{t=t_0}^{t_0+n-1} \delta^{t-t_0} q_t^c v_i'(x_{i,t_0}^{n,tp}) = (1-\delta^n)k_i \text{ for all } i,$$

The comparison of the left-hand-sides of the first order conditions for (C) and TP is equivalent to the following inequality. $(\sum_j v_j(x_{jt}^c))^{-1/a_t}\,(1-aM)^{-1} < (\sum_j \tilde{v}_{jt})^{-1/a_t}$, equivalently $(1-aM)^{a_t}\sum_j v_j(x_{jt}^c) < \sum_j \tilde{v}_{jt}$. Because $(1-aM) < 1$, the inequality holds if $\tilde{v}_{jt} \le v_{jt}(x^c)$ for all j,t. Hence $x^{tp} > x^c$ for TP, and for TP(ρ) for large enough ρ, but may not hold for ρ close to 0.

With the exponential demand function, solving the Stackelberg equilibrium we get $m_{it} = m_{At} = a$ and $\pi_{it}(x_t) = b_t^{-1}a_t\exp(-\,(M+1) - a_t^{-1}\sum_j v_j(x_{jt}))$. In this case, $\hat{x}_{t_0}^{n}$ in (C) satisfies

$$-\sum_{t=t_0}^{t_0+n-1} \delta^{t-t_0} b_t^{-1}\exp(-\,(M+1) - a_t^{-1}\sum_j v_j(x_{jt}))v_i'(\hat{x}_{i,t_0}^{n}) = (1-\delta^n)k_i \text{ for all } i. \tag{20}$$

It can be verified that under TP, $q_t^c = b_t^{-1}\exp(-\,(M+1) - a_t^{-1}\sum_j v_j(x_{jt}^c)))$. Then, under TP,

$x_{t_0}^{n,tp}$ is the solution to

$$-\sum_{t=t_0}^{t_0+n-1} \delta^{t-t_0} q_t^c v_i'(x_{i,t_0}^{n,tp}) = (1-\delta^n)k_i \text{ for all } i,$$

which is equivalent to the condition for $\hat{x}_{t_0}^n$. This implies that, under TP, $x^{tp} = x^c$. Now, consider TP(ρ). If ρ is chosen such that $q_t > [<] q_t^c$ for all t, then $x^{tp} > [<] x^c$. ■

Proposition 7

Proof. (i) This part follows from the proof of Proposition 6. (ii) Under (TP), w_{it}, m_{At} and q_t are the same as those under (C) for all i, t, by definition. Because $\pi_{At} = m_{At} q_t$, the assembler's profit remains the same. Because $\pi_{it} = (w_{it} - v_i(x_{it}))\, q_t$ and $x_{it}^{tp} \geq x_{it}^c$, suppliers' profits remain the same in the case of exponential demand function and strictly increase in all the other demand forms considered in the paper. ■

Theorem 2

Proof. (i) Let us start with the case of linear demand. The proof is in two parts. We first show that $x^* > x^c$ and then show that $x^* > x^{tp}$. Recall that the uniqueness conditions (10) guarantee a unique solution to the first order conditions of the equilibrium. The same conditions guarantee a unique solution to the first order conditions

$$(2M+1) \sum_{t=t_0}^{t_0+n-1} \delta^{t-t_0} \gamma_t(x) v_i'(x) = (1-\delta^n)k_i, \quad \text{for} \quad i = 1, .., M,$$

implying that the centralized problem for periods t_0 through period t_0+n-1 is jointly concave (γ_t is defined in (12)). We denote by $\tilde{x}_{t_0}^n$ to the solution of this system of equations. Because $\tilde{x}_{t_0}^n$ is the unique solution to the above set of equations, it can be regarded as the unique equilibrium of an M-supplier game, in which all suppliers invest in period t_0 for n periods and where the unit investment cost is $\tilde{k}_i \stackrel{def}{=} k_i/(2M+1)$. Thus Algorithm ($M, \{\hat{x}_t^n\}$) finds the unique equilibrium under (C) and Algorithm ($M, \{\tilde{x}_t^n\}$) finds the unique optimal centralized solution. Note that $\tilde{x}_{it_0}^n > \hat{x}_{it_0}^n$ for all t_0 and n, because the equilibrium investment levels are decreasing in k_i, and $k_i > \tilde{k}_i$. By Lemma 3, we then have that $x^* > x^c$.

Next, we examine how x^{tp} compares with x^*. In order to facilitate the comparison, we introduce another target price contract. Under this contract, parameters are such that wholesale prices and quantities are set to match those in the solution to the centralized regime, that is $q_t = \frac{1}{2(M+1)b_t}(a_t - \sum_j v_j(x^*_{jt}))$. The investment levels, denoted x^a, are the solution to

$$-q_t(x^*_t)v'_i(x_{it}) - k_i + \delta\partial\pi^{tp}_{it}(x_{it})/\partial x = 0, \text{ for all } i, t. \tag{21}$$

The first order conditions for (TP) and (21) differ only in the coefficient of $v'_i(x_{it})$. Because $x^* > x^c$, we have that $q_t(x^*) > q_t(x^c)$ for all t. This implies that $x^a > x^{tp}$. We define $x^{n,a}_t$ to be the investment levels under (21) in period t_0 given that the next investment period is $t_0 + n$. Recall that $\tilde{x}^n_{i,t_0}$ satisfies

$$(2M+1)\sum_{t=t_0}^{t_0+n-1} \delta^{t-t_0}\gamma_t(\tilde{x}^n_{t_0})v'_i(\tilde{x}^n_{i,t_0}) = (1-\delta^n)k_i \text{ for all } i.$$

Replacing $\tilde{x}^n_{i,t_0}$ in the first order condition under (21), we get

$$(M+1)\sum_{t=t_0}^{t_0+n-1} \delta^{t-t_0}\gamma_t(\tilde{x}^n_{t_0})v'_i(\tilde{x}^n_{t_0}) < (1-\delta^n)k_i \text{ for all } i,$$

which implies that $x^* > x^a$. As a consequence, $x^* > x^a > x^{tp} > x^c$.

To summarize, we are able to show the result by comparing the coefficients of the right-hand-side of the first order conditions in (C), (TP) and the centralized cases, which were, in the linear demand case, $1/(M+1)$, 1, and $(2M+1)/(M+1)$, respectively. We next show the result for the nonlinear demand functions. With $d_t(p) = ((a_t - p)/b_t)^\eta$, the ratios are $\eta/(\eta+M)$, 1, $(\eta + M + \eta M)/(\eta + M)$, respectively. For the isoelastic demand function $d_t(p) = b_t^{-1}p^{-1/a_t}$, the ratios are $(1-aM)^{-1}$, 1, $1 + M/(1-aM)$, respectively. The argument presented above for the linear demand case can be replicated for these two cases as well. Similar to linear demand, these comparisons imply that $x^c < x^{tp} < x^*$. With the exponential demand function, the ratios are 1, 1, $(M+1)$, which implies that $x^c = x^{tp} < x^*$.

(ii) We now show that the investment periods are the same in cost-contingent, target-price and under centralized decisions when $a_t = a$. Let us start with the linear demand case. The proof for the cost-contingent contracts and the centralized decisions is by induction on t. The proof for period T is trivial because all firms invest in the single-period problem. Assume that the cost-contingent contracts and the centralized decisions lead to the same investment pattern from periods t_0+1 through T. Consider period t_0. If an investment is made in period t for n periods under (C), the investment level will be $\tilde{x}^n_t$ defined similar to $\hat{x}^n_t$. The comparisons $\tilde{x}^n_t < \bar{x}^*_{t+n}$ in the algorithm determine the investment periods in the optimal solution. Fix i. We need to show that $\hat{x}^n_{i,t_0} < \bar{x}^c_{i,t_0+n}$ if and only if $\tilde{x}^n_{i,t_0} < \bar{x}^*_{i,t_0+n}$. Choose m such that $\bar{x}^c_{i,t_0+n} = \hat{x}^m_{i,t_0+n}$ and $\bar{x}^*_{i,t_0+n} = \tilde{x}^m_{i,t_0+n}$. The same m satisfies both equations because of the induction assumption. We have that $\hat{x}^n_{i,t_0} < \bar{x}^c_{i,t_0+n}$ if and only if $\sum_{t=t_0}^{t_0+n-1} \delta^{t-t_0}\gamma_t(\hat{x}^n_{t_0})v'_i(\hat{x}^n_{i,t_0}) = (1-\delta^n)\,k_i$ and $\sum_{t=t_0+n}^{t_0+m-1} \delta^{t-t_0}\gamma_t(\hat{x}_{t_0})v'_i(\hat{x}^n_{i,t_0}) > (1-\delta^m)k_i$. If $t_0+m-1 = T$, then the right-hand side of the inequality shall be k_i instead of $(1-\delta^m)\,k_i$. Combining the two, we obtain

$$((1-\delta^m)/(1-\delta^n)) \sum_{t=t_0}^{t_0+n-1} \delta^{t-t_0}\gamma_t(\hat{x}^n_{t_0}) < \sum_{t=t_0+n}^{t_0+m-1} \delta^{t-t_0}\gamma_t(\hat{x}^n_{t_0}).$$

If $a_t = a$ for all t, then we have $((1-\delta^m)/(1-\delta^n)) \sum_{t=t_0}^{t_0+n-1} \delta^{t-t_0}(1/b_t) < \sum_{t=t_0+n}^{t_0+m-1} \delta^{t-t_0}(1/b_t)$, which is independent of x. Similarly, $\tilde{x}^n_{i,t_0} < \bar{x}^*_{i,t_0+n}$ if and only if $\sum_{t=t_0}^{t_0+n-1} \delta^{t-t_0}\gamma_t(\tilde{x}^n_{t_0})v'_i(\tilde{x}^n_{i,t_0}) = (1-\delta^n)\,k_i$ and $\sum_{t=t_0+n}^{t_0+m-1} \delta^{t-t_0}\gamma_t(\tilde{x}^n_{t_0})v'_i(\tilde{x}^n_{i,t_0}) > (1-\delta^m)k_i$. Combining the two, we have

$$((1-\delta^m)/(1-\delta^n)) \sum_{t=t_0}^{t_0+n-1} \delta^{t-t_0}\gamma_t(\tilde{x}^n_{t_0}) < \sum_{t=t_0+n}^{t_0+m-1} \delta^{t-t_0}\gamma_t(\tilde{x}^n_{i,t_0}),$$

which holds if and only if the condition that is independent of x holds as well. This means that the timing of investments are the same under the cost-contingent contracts and the centralized decisions. To see that the timing of investments are the same in (TP) and (C), consider the comparisons $x^{n,tp}_t < \bar{x}^{tp}_{t+n}$ in the algorithm, which holds if and only if which is equivalent to

$$((1-\delta^m)/(1-\delta^n)) \sum_{t=t_0}^{t_0+n-1} \delta^{t-t_0}\gamma_t(\hat{x}^n_{t_0}) < \sum_{t=t_0+n}^{t_0+m-1} \delta^{t-t_0}\gamma_t(\hat{x}^n_{t_0}),$$

exactly the same condition for $\hat{x}^n_{t_0} < \bar{x}^c_{t+n}$. Therefore, the algorithm will make the same decisions for all t and n for (TP) and (C), leading to the same investment periods.

For the nonlinear demand functions, let us examine the first order conditions for (C), namely, equations (18), (19), (20). The algorithm makes the comparisons of the first order conditions for (C), (TP) and the centralized cases, and the terms with a_t and $\sum_j v_j(x_{jt})$ in the sum from $t = t_0$ to $t_0 + n$ cancel out. ■

Proposition 8

Proof. (i) Solving the second and third stages of the Stackelberg game backwards, we find the profit function of all the parties in a period given the costs of the components. The expressions for the equilibrium quantities and margins are given below.

$$m_{iA} = \frac{3a_B c + a_A\left(2+c^2\right) - \left(1-c^2\right)(cv_0(1-\alpha) + (2+c\alpha)(v_{1A}+v_{2A}))}{3b(4-5c^2+c^4)}, \text{ for } i = 1, 2,$$
$$m_{iB} = \frac{3a_A c + a_B\left(2+c^2\right) - \left(1-c^2\right)(-4v_0(1-\alpha) + (2\alpha+c)(v_{1A}+v_{2A}))}{3b(4-5c^2+c^4)}, \text{ for } i = 1, 2,$$
$$m_A = \frac{2a_A + a_B c + 2cv_0(1-\alpha) + \left(c\alpha - 2 + c^2\right)(v_{1A}+v_{2A})}{4-c^2},$$
$$m_B = \frac{2a_B + a_A c - (4-2c^2)v_0(1-\alpha) + \left(c - (2-c^2)\alpha\right)(v_{1A}+v_{2A})}{4-c^2},$$
$$q_A = \frac{2a_A + a_B c + 2cv_0(1-\alpha) + \left(c\alpha - 2 + c^2\right)(v_{1A}+v_{2A})}{3(4-c^2)},$$
$$q_B = \frac{2a_B + a_A c - (4-2c^2)v_0(1-\alpha) + \left(c - (2-c^2)\alpha\right)(v_{1A}+v_{2A})}{3b(4-c^2)}.$$

The profit functions in any given period are $\pi_{it} = m_{iAt}q_{At} + m_{iBt}q_{Bt}$ and $\pi_{At} = m_{At}q_{At}$. The structure of the supplier profit functions are similar to those in the monopolist assembler setting. It can be shown that they are supermodular in (x_{1t}, x_{2t}) and that a synchronization result similar to that in Theorem 1 holds.

(ii) The proof follows from Lemma 3 and the fact that the equilibrium of the single-investment game increases in α. For any single-investment game starting in period t_0 and ending in t_0+n-1, we define and compute $x^n_{t_0}$ similar to (2). It can be shown that $\partial^2\pi_{it}/\partial\alpha\partial x_{it} > 0$ if $a_j \geq v_{10} + v_{20}$ for any i. Because the cross-partial of π_{it} is positive for periods t_0 through t_0+n-1, x^n_t is increasing in α, which implies the result for the multi-period problem by Lemma 3.

(iii) The total profit generated in Assembler A's production line is $\tilde{\pi}_{At} = m_{1At}q_{At} + m_{2At}q_{At} + m_{At}q_{At}$. The proof is similar to part (ii), except that $\partial^2\tilde{\pi}_{it}/\partial\alpha\partial x_{it} < 0$ if $a_j \geq v_{10} + v_{20}$ for any i and $c > 0$ (and equals zero if $c = 0$). This implies that x^* is decreasing in α. ■

Proposition 9

Proof. The result follows from Lemma 1 and Proposition 2. ■

Proposition 10 *The equilibrium production quantity, wholesale prices, and profits are given by* (4) – (5)*.*

Proof. The optimal prices and quantity in each period t only depend on the cost and demand parameters in that period. That is, these decisions are independent of the history of prices and quantities in periods $1, ..., t-1$, but they depend on the vector of cumulative investment levels x_t. Therefore, based on the assembler's choice of margin m_{At}, the suppliers engage in a non-cooperative pricing game, in which supplier i's profit (before investments in cost reduction) is given by

$$\pi_i(m_{it}|x_t, m_{-i,t}) = m_{it}\frac{a_t - \left(m_{At} + \sum_{j=1}^{M}(m_{jt} + v_{jt}(x_{jt}))\right)}{b_t},$$

leading to supplier equilibrium values $\{m_{jt}(m_{At}, x_t)\}_{j=1,..,M}$. Anticipating the suppliers' equilibrium margin decisions, the assembler, as the Stackelberg leader, announces its margin m_{At} to maximize

$$\pi_A(m_{At}|x_t) = m_{At}\frac{a_t - \left(m_{At} + \sum_{j=1}^{M}(m_{jt}(m_{At}, x_t) + v_{jt}(x_{jt}))\right)}{b_t}.$$

Solving for optimal m_{At} and substituting in the equilibrium $\{m_{jt}(m_{At}, x_t)\}_{j=1,..,M}$ yields the following equilibrium margins

$$m_{it} = \frac{1}{2(M+1)}(a_t - \sum_j v_j(x_{jt})), \qquad m_{At} = \frac{1}{2}(a_t - \sum_j v_j(x_{jt})).$$

The optimal assembler margin determines the wholesale and retail prices,

$$w_{it} = v_i(x_{it}) + \frac{1}{2\,(M+1)}(a_t - \sum_j v_j(x_{jt})), \quad p_t = a_t - \frac{1}{2\,(M+1)}(a_t - \sum_j v_j(x_{jt})), \text{ for } i = 1,..,M.$$

The equilibrium wholesale prices reflect the assembler's response to cost reductions under a cost-contingent contract. Indeed, a reduction in cost achieved through investment in process improvement is followed by a decrease in the wholesale price (note that w_{it} decreases if v_i does). The equilibrium margins imply the production quantity given in (4) and the profits in (5). ■

Proposition 11 *The investment game in period T is not supermodular in the starting investment levels x_{t-1}.*

Proof. In order to see why this is the case, we consider a setting with two suppliers. The investment game in the last period T is supermodular as shown in Proposition 3. Let x_T^u be the unconstrained equilibrium in period T, that is, the equilibrium that arises without consideration of the lower bounds given by the vector of initial investment levels x_{T-1}. Consider the following two vectors $(\bar{x}_{i,T-1}, \bar{x}_{j,T-1}) > (x_{i,T-1}, x_{j,T-1})$ of starting cumulative investment levels in period T, as shown in Figure 1 below. The curve over the horizontal axis in Figure 1 represents the best-response function of supplier j, while the curve on the vertical axis represents the best-response function of supplier i. Both curves are increasing because the last period's game is supermodular. Note from Figure 1 that the equilibrium vector is

$$x_T^u \quad \text{when the initial state is} \quad (x_{i,T-1}, x_{j,T-1}), \tag{22}$$

$$\bar{x}_T \quad \text{when the initial state is} \quad (\bar{x}_{i,T-1}, x_{j,T-1}), \tag{23}$$

$$\bar{\bar{x}}_T \quad \text{when the initial state is either} \quad (\bar{x}_{i,T-1}, \bar{x}_{j,T-1}) \text{ or } (x_{i,T-1}, \bar{x}_{j,T-1}). \tag{24}$$

Because the best-response functions in period T are increasing, we have that $\bar{x}_T > x_T^u$. Thus, supplier j's equilibrium profit in period T as a function of these initial investment levels satisfies

$$0 = \pi_{jT}^c(\bar{x}_{i,T-1}, \bar{x}_{j,T-1}) - \pi_{jT}^c(x_{i,T-1}, \bar{x}_{j,T-1}) < \pi_{jT}^c(\bar{x}_{i,T-1}, x_{j,T-1}) - \pi_{jT}^c(x_{i,T-1}, x_{j,T-1}).$$

The equality follows from (24). The right-hand side is positive because of (22), (23), $\bar{x}_T > x_T^u$, and the fact that π_{jT} is increasing in x_{iT}. The strict inequality implies that the last-period equilibrium profit is not supermodular as a function of the initial cumulative investment levels.

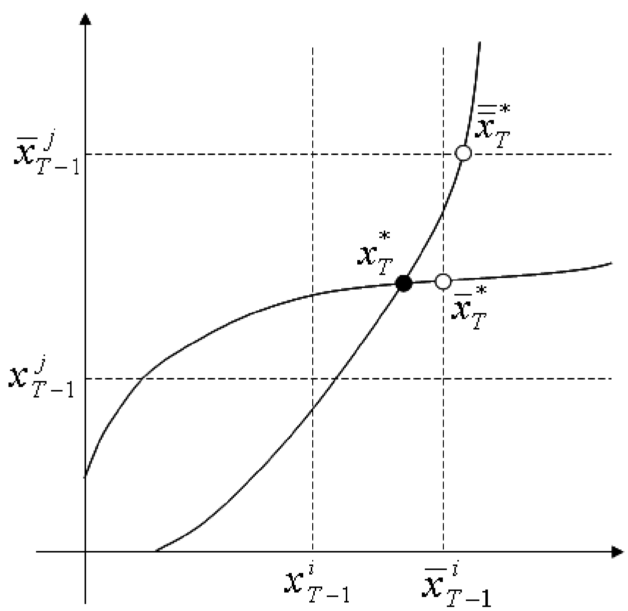

Figure 1: Supermodularity under closed-loop strategies.

■

References

Vives, X. 2000. *Oligopoly Pricing: Old Ideas and New Tools.* The MIT Press. Cambridge, MA.

Appendix D - Numerical Study

b_t=(1,0.1,0.5) for T=3, (1,0.5,0.1,0.5) for T=4, (1,0.5,0.1,0.1,0.5) for T=5.

#	k	δ	v_0	β_i	a_t	M	T	Regime	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	π_1	π_2	π_3	π_A
1	5	0.9	5	1.1	20	2	3	Cent.	6.88	7.47	7.47			6.88	7.47	7.47								83.2	83.2		345.7
1	5	0.9	5	1.1	20	2	3	C	3.05	3.33	3.33			3.05	3.33	3.33								85.3	85.3		290.3
1	5	0.9	5	1.1	20	2	3	TP	5.15	5.62	5.62			5.15	5.62	5.62								93.6	93.6		290.3
2	5	0.95	5	1.1	20	2	3	Cent.	7.62	7.62	7.62			7.62	7.62	7.62								89.4	89.4		367.4
2	5	0.95	5	1.1	20	2	3	C	3.40	3.40	3.40			3.40	3.40	3.40								91.5	91.5		310.5
2	5	0.95	5	1.1	20	2	3	TP	5.74	5.74	5.74			5.74	5.74	5.74								100.1	100.1		310.5
3	2	0.9	5	1.1	20	2	3	Cent.	10.77	11.67	11.67			10.77	11.67	11.67								99.4	99.4		361.6
3	2	0.9	5	1.1	20	2	3	C	4.88	5.30	5.30			4.88	5.30	5.30								100.6	100.6		327.5
3	2	0.9	5	1.1	20	2	3	TP	8.23	8.94	8.94			8.23	8.94	8.94								106.0	106.0		327.5
4	2	0.95	5	1.1	20	2	3	Cent.	11.91	11.91	11.91			11.91	11.91	11.91								106.1	106.1		383.7
4	2	0.95	5	1.1	20	2	3	C	5.41	5.41	5.41			5.41	5.41	5.41								107.4	107.4		348.6
4	2	0.95	5	1.1	20	2	3	TP	9.13	9.13	9.13			9.13	9.13	9.13								112.9	112.9		348.6
5	5	0.9	5	1.1	20-2(t-1)	2	3	Cent.	6.88	7.00	7.00			6.88	7.00	7.00								60.5	60.5		271.4
5	5	0.9	5	1.1	20-2(t-1)	2	3	C	3.05	3.09	3.09			3.05	3.09	3.09								62.5	62.5		218.6
5	5	0.9	5	1.1	20-2(t-1)	2	3	TP	5.15	5.21	5.21			5.15	5.21	5.21								70.3	70.3		218.6
6	5	0.95	5	1.1	20-2(t-1)	2	3	Cent.	7.19	7.19	7.19			7.19	7.19	7.19								65.0	65.0		287.8
6	5	0.95	5	1.1	20-2(t-1)	2	3	C	3.18	3.18	3.18			3.18	3.18	3.18								67.0	67.0		233.7
6	5	0.95	5	1.1	20-2(t-1)	2	3	TP	5.36	5.36	5.36			5.36	5.36	5.36								75.0	75.0		233.7
7	2	0.9	5	1.1	20-2(t-1)	2	3	Cent.	10.77	10.97	10.97			10.77	10.97	10.97								75.6	75.6		286.5
7	2	0.9	5	1.1	20-2(t-1)	2	3	C	4.88	4.95	4.95			4.88	4.95	4.95								76.8	76.8		254.1
7	2	0.9	5	1.1	20-2(t-1)	2	3	TP	8.23	8.36	8.36			8.23	8.36	8.36								81.8	81.8		254.1
8	2	0.95	5	1.1	20-2(t-1)	2	3	Cent.	11.26	11.26	11.26			11.26	11.26	11.26								80.6	80.6		303.3
8	2	0.95	5	1.1	20-2(t-1)	2	3	C	5.09	5.09	5.09			5.09	5.09	5.09								81.8	81.8		270.0
8	2	0.95	5	1.1	20-2(t-1)	2	3	TP	8.58	8.58	8.58			8.58	8.58	8.58								87.0	87.0		270.0
9	5	0.9	5	1+0.1i	20	2	3	Cent.	6.90	7.48	7.48			6.57	7.10	7.10								84.0	85.9		348.4
9	5	0.9	5	1+0.1i	20	2	3	C	3.07	3.34	3.34			3.03	3.29	3.29								86.8	87.0		295.0
9	5	0.9	5	1+0.1i	20	2	3	TP	5.18	5.64	5.64			5.00	5.42	5.42								95.2	94.8		295.0
10	5	0.95	5	1+0.1i	20	2	3	Cent.	7.64	7.64	7.64			7.24	7.24	7.24								90.2	92.2		370.2
10	5	0.95	5	1+0.1i	20	2	3	C	3.41	3.41	3.41			3.36	3.36	3.36								93.1	93.3		315.4
10	5	0.95	5	1+0.1i	20	2	3	TP	5.76	5.76	5.76			5.54	5.54	5.54								101.7	101.4		315.4
11	2	0.9	5	1+0.1i	20	2	3	Cent.	10.79	11.69	11.69			10.07	10.87	10.87								100.0	101.6		363.5
11	2	0.9	5	1+0.1i	20	2	3	C	4.89	5.31	5.31			4.73	5.12	5.12								101.8	102.1		331.0
11	2	0.9	5	1+0.1i	20	2	3	TP	8.25	8.96	8.96			7.80	8.44	8.44								107.1	107.0		331.0
12	2	0.95	5	1+0.1i	20	2	3	Cent.	11.93	11.93	11.93			11.09	11.09	11.09								106.7	108.4		385.7
12	2	0.95	5	1+0.1i	20	2	3	C	5.42	5.42	5.42			5.23	5.23	5.23								108.6	109.0		352.2
12	2	0.95	5	1+0.1i	20	2	3	TP	9.15	9.15	9.15			8.61	8.61	8.61								114.1	114.0		352.2
13	5	0.9	5	1+0.1i	20-2(t-1)	2	3	Cent.	6.90	7.02	7.02			6.57	6.68	6.68								61.3	63.0		273.9
13	5	0.9	5	1+0.1i	20-2(t-1)	2	3	C	3.07	3.10	3.10			3.03	3.07	3.07								63.9	64.0		223.0
13	5	0.9	5	1+0.1i	20-2(t-1)	2	3	TP	5.18	5.24	5.24			5.00	5.05	5.05								71.7	71.4		223.0

#	k	δ	v_0	β_i	a_t	M	T	Regime	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	π_1	π_2	π_3	π_A
14	5	0.95	5	1+0.1i	20-2(t-1)	2	3	Cent.	7.20	7.20	7.20			6.85	6.85	6.85								65.8	67.6		290.4
14	5	0.95	5	1+0.1i	20-2(t-1)	2	3	C	3.19	3.19	3.19			3.15	3.15	3.15								68.4	68.7		238.2
14	5	0.95	5	1+0.1i	20-2(t-1)	2	3	TP	5.38	5.38	5.38			5.19	5.19	5.19								76.5	76.2		238.2
15	2	0.9	5	1+0.1i	20-2(t-1)	2	3	Cent.	10.79	10.99	10.99			10.07	10.25	10.25								76.1	77.6		288.3
15	2	0.9	5	1+0.1i	20-2(t-1)	2	3	C	4.89	4.97	4.97			4.73	4.81	4.81								77.8	78.2		257.3
15	2	0.9	5	1+0.1i	20-2(t-1)	2	3	TP	8.25	8.38	8.38			7.80	7.92	7.92								82.9	82.8		257.3
16	2	0.95	5	1+0.1i	20-2(t-1)	2	3	Cent.	11.27	11.27	11.27			10.50	10.50	10.50								81.2	82.7		305.1
16	2	0.95	5	1+0.1i	20-2(t-1)	2	3	C	5.10	5.10	5.10			4.93	4.93	4.93								82.9	83.3		273.3
16	2	0.95	5	1+0.1i	20-2(t-1)	2	3	TP	8.61	8.61	8.61			8.12	8.12	8.12								88.1	88.0		273.3
17	5	0.9	5	1.1	20	3	3	Cent.	6.00	6.53	6.53			6.00	6.53	6.53			6.00	6.53	6.53			32.0	32.0	32.0	237.4
17	5	0.9	5	1.1	20	3	3	C	2.06	2.28	2.28			2.06	2.28	2.28			2.06	2.28	2.28			28.7	28.7	28.7	140.1
17	5	0.9	5	1.1	20	3	3	TP	3.98	4.42	4.42			3.98	4.42	4.42			3.98	4.42	4.42			39.4	39.4	39.4	140.1
18	5	0.95	5	1.1	20	3	3	Cent.	6.67	6.67	6.67			6.67	6.67	6.67			6.67	6.67	6.67			34.9	34.9	34.9	253.0
18	5	0.95	5	1.1	20	3	3	C	2.34	2.34	2.34			2.34	2.34	2.34			2.34	2.34	2.34			31.6	31.6	31.6	153.3
18	5	0.95	5	1.1	20	3	3	TP	4.53	4.53	4.53			4.53	4.53	4.53			4.53	4.53	4.53			42.6	42.6	42.6	153.3
19	2	0.9	5	1.1	20	3	3	Cent.	9.49	10.29	10.29			9.49	10.29	10.29			9.49	10.29	10.29			46.0	46.0	46.0	257.7
19	2	0.9	5	1.1	20	3	3	C	3.51	3.83	3.83			3.51	3.83	3.83			3.51	3.83	3.83			44.1	44.1	44.1	198.9
19	2	0.9	5	1.1	20	3	3	TP	6.79	7.42	7.42			6.79	7.42	7.42			6.79	7.42	7.42			51.3	51.3	51.3	198.9
20	2	0.95	5	1.1	20	3	3	Cent.	10.51	10.51	10.51			10.51	10.51	10.51			10.51	10.51	10.51			49.5	49.5	49.5	273.9
20	2	0.95	5	1.1	20	3	3	C	3.92	3.92	3.92			3.92	3.92	3.92			3.92	3.92	3.92			47.5	47.5	47.5	213.5
20	2	0.95	5	1.1	20	3	3	TP	7.59	7.59	7.59			7.59	7.59	7.59			7.59	7.59	7.59			54.9	54.9	54.9	213.5
21	5	0.9	5	1.1	20-2(t-1)	3	3	Cent.	6.00	6.09	6.09			6.00	6.09	6.09			6.00	6.09	6.09			20.3	20.3	20.3	182.7
21	5	0.9	5	1.1	20-2(t-1)	3	3	C	2.02	2.02	2.02			2.02	2.02	2.02			2.02	2.02	2.02			16.9	16.9	16.9	88.2
21	5	0.9	5	1.1	20-2(t-1)	3	3	TP	3.91	3.91	3.91			3.91	3.91	3.91			3.91	3.91	3.91			26.5	26.5	26.5	88.2
22	5	0.95	5	1.1	20-2(t-1)	3	3	Cent.	6.25	6.25	6.25			6.25	6.25	6.25			6.25	6.25	6.25			22.3	22.3	22.3	194.4
22	5	0.95	5	1.1	20-2(t-1)	3	3	C	2.10	2.10	2.10			2.10	2.10	2.10			2.10	2.10	2.10			19.0	19.0	19.0	97.9
22	5	0.95	5	1.1	20-2(t-1)	3	3	TP	4.07	4.07	4.07			4.07	4.07	4.07			4.07	4.07	4.07			28.8	28.8	28.9	97.9
23	2	0.9	5	1.1	20-2(t-1)	3	3	Cent.	9.49	9.65	9.65			9.49	9.65	9.65			9.49	9.65	9.65			33.2	33.2	33.2	202.0
23	2	0.9	5	1.1	20-2(t-1)	3	3	C	3.51	3.53	3.53			3.51	3.53	3.53			3.51	3.53	3.53			31.4	31.4	31.4	146.0
23	2	0.9	5	1.1	20-2(t-1)	3	3	TP	6.79	6.83	6.83			6.79	6.83	6.83			6.79	6.83	6.83			38.1	38.1	38.1	146.0
24	2	0.95	5	1.1	20-2(t-1)	3	3	Cent.	9.90	9.90	9.90			9.90	9.90	9.90			9.90	9.90	9.90			35.8	35.8	35.8	214.3
24	2	0.95	5	1.1	20-2(t-1)	3	3	C	3.64	3.64	3.64			3.64	3.64	3.64			3.64	3.64	3.64			33.9	33.9	33.9	156.8
24	2	0.95	5	1.1	20-2(t-1)	3	3	TP	7.04	7.04	7.04			7.04	7.04	7.04			7.04	7.04	7.04			40.8	40.8	40.8	156.8
25	5	0.9	5	1+0.1i	20	3	3	Cent.	6.04	6.57	6.57			5.79	6.27	6.27			5.56	5.99	5.99			33.3	34.8	36.2	243.6
25	5	0.9	5	1+0.1i	20	3	3	C	2.11	2.33	2.33			2.12	2.33	2.33			2.12	2.33	2.33			31.6	31.6	31.6	152.5
25	5	0.9	5	1+0.1i	20	3	3	TP	4.07	4.51	4.51			3.97	4.38	4.38			3.88	4.25	4.25			42.5	41.9	41.3	152.5
26	5	0.95	5	1+0.1i	20	3	3	Cent.	6.71	6.71	6.71			6.40	6.40	6.40			6.11	6.11	6.11			36.3	37.9	39.3	259.5
26	5	0.95	5	1+0.1i	20	3	3	C	2.39	2.39	2.39			2.39	2.39	2.39			2.38	2.38	2.38			34.6	34.6	34.7	166.2
26	5	0.95	5	1+0.1i	20	3	3	TP	4.62	4.62	4.62			4.48	4.48	4.48			4.35	4.35	4.35			45.9	45.2	44.7	166.2
27	2	0.9	5	1+0.1i	20	3	3	Cent.	9.53	10.33	10.33			8.95	9.67	9.67			8.42	9.07	9.07			47.0	48.3	49.5	262.1
27	2	0.9	5	1+0.1i	20	3	3	C	3.55	3.87	3.87			3.49	3.79	3.79			3.42	3.70	3.70			46.3	46.5	46.6	208.0
27	2	0.9	5	1+0.1i	20	3	3	TP	6.87	7.50	7.50			6.54	7.12	7.12			6.24	6.77	6.77			53.6	53.2	52.9	208.0

#	k	δ	v_0	β_i	a_t	M	T	Regime	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	π_1	π_2	π_3	π_A
28	2	0.95	5	1+0.1*i*	20	3	3	Cent.	10.55	10.55	10.55			9.86	9.86	9.86			9.24	9.24	9.24			50.5	51.9	53.1	278.4
28	2	0.95	5	1+0.1*i*	20	3	3	C	3.96	3.96	3.96			3.87	3.87	3.87			3.78	3.78	3.78			49.8	50.0	50.2	223.0
28	2	0.95	5	1+0.1*i*	20	3	3	TP	7.66	7.66	7.66			7.27	7.27	7.27			6.90	6.90	6.90			57.3	56.9	56.6	223.0
29	5	0.9	5	1+0.1*i*	20-2(*t*-1)	3	3	Cent.	6.04	6.13	6.13			5.79	5.88	5.88			5.56	5.63	5.63			21.5	22.8	24.0	188.5
29	5	0.9	5	1+0.1*i*	20-2(*t*-1)	3	3	C	2.08	2.08	2.08			2.10	2.10	2.10			2.10	2.10	2.10			19.6	19.6	19.5	100.2
29	5	0.9	5	1+0.1*i*	20-2(*t*-1)	3	3	TP	4.03	4.03	4.03			3.94	3.94	3.94			3.84	3.84	3.84			29.4	28.9	28.4	100.2
30	5	0.95	5	1+0.1*i*	20-2(*t*-1)	3	3	Cent.	6.30	6.30	6.30			6.03	6.03	6.03			5.77	5.77	5.77			23.6	25.0	26.3	200.5
30	5	0.95	5	1+0.1*i*	20-2(*t*-1)	3	3	C	2.16	2.16	2.16			2.17	2.17	2.17			2.17	2.17	2.17			21.8	21.7	21.7	110.3
30	5	0.95	5	1+0.1*i*	20-2(*t*-1)	3	3	TP	4.18	4.18	4.18			4.08	4.08	4.08			3.97	3.97	3.97			31.9	31.4	30.9	110.3
31	2	0.9	5	1+0.1*i*	20-2(*t*-1)	3	3	Cent.	9.53	9.70	9.70			8.95	9.10	9.10			8.42	8.56	8.56			34.2	35.4	36.4	206.1
31	2	0.9	5	1+0.1*i*	20-2(*t*-1)	3	3	C	3.55	3.58	3.58			3.49	3.51	3.51			3.42	3.45	3.45			33.5	33.6	33.7	154.5
31	2	0.9	5	1+0.1*i*	20-2(*t*-1)	3	3	TP	6.87	6.93	6.93			6.54	6.60	6.60			6.24	6.30	6.30			40.2	39.8	39.5	154.5
32	2	0.95	5	1+0.1*i*	20-2(*t*-1)	3	3	Cent.	9.95	9.95	9.95			9.32	9.32	9.32			8.76	8.76	8.76			36.7	38.0	39.1	218.5
32	2	0.95	5	1+0.1*i*	20-2(*t*-1)	3	3	C	3.69	3.69	3.69			3.61	3.61	3.61			3.54	3.54	3.54			36.0	36.2	36.3	165.6
32	2	0.95	5	1+0.1*i*	20-2(*t*-1)	3	3	TP	7.13	7.13	7.13			6.79	6.79	6.79			6.47	6.47	6.47			43.0	42.6	42.3	165.6
33	5	0.9	5	1.1	20	2	4	Cent.	6.88	7.72	7.72	7.72		6.88	7.72	7.72	7.72							89.9	89.9		369.1
33	5	0.9	5	1.1	20	2	4	C	3.05	3.45	3.45	3.45		3.05	3.45	3.45	3.45							92.0	92.0		312.1
33	5	0.9	5	1.1	20	2	4	TP	5.15	5.82	5.82	5.82		5.15	5.82	5.82	5.82							100.6	100.6		312.1
34	5	0.95	5	1.1	20	2	4	Cent.	8.02	8.02	8.02	8.02		8.02	8.02	8.02	8.02							101.5	101.5		409.8
34	5	0.95	5	1.1	20	2	4	C	3.59	3.59	3.59	3.59		3.59	3.59	3.59	3.59							103.7	103.7		350.0
34	5	0.95	5	1.1	20	2	4	TP	6.05	6.05	6.05	6.05		6.05	6.05	6.05	6.05							112.8	112.8		350.0
35	2	0.9	5	1.1	20	2	4	Cent.	10.77	12.06	12.06	12.06		10.77	12.06	12.06	12.06							106.6	106.6		385.5
35	2	0.9	5	1.1	20	2	4	C	4.88	5.48	5.48	5.48		4.88	5.48	5.48	5.48							107.9	107.9		350.3
35	2	0.9	5	1.1	20	2	4	TP	8.23	9.25	9.25	9.25		8.23	9.25	9.25	9.25							113.4	113.4		350.3
36	2	0.95	5	1.1	20	2	4	Cent.	12.52	12.52	12.52	12.52		12.52	12.52	12.52	12.52							119.3	119.3		426.9
36	2	0.95	5	1.1	20	2	4	C	5.70	5.70	5.70	5.70		5.70	5.70	5.70	5.70							120.6	120.6		390.1
36	2	0.95	5	1.1	20	2	4	TP	9.61	9.61	9.61	9.61		9.61	9.61	9.61	9.61							126.4	126.4		390.1
37	5	0.9	5	1.1	20-2(*t*-1)	2	4	Cent.	6.87	6.87	6.87	6.87		6.87	6.87	6.87	6.87							50.5	50.5		239.6
37	5	0.9	5	1.1	20-2(*t*-1)	2	4	C	3.01	3.01	3.01	3.01		3.01	3.01	3.01	3.01							52.4	52.4		187.3
37	5	0.9	5	1.1	20-2(*t*-1)	2	4	TP	5.08	5.08	5.08	5.08		5.08	5.08	5.08	5.08							60.0	60.0		187.3
38	5	0.95	5	1.1	20-2(*t*-1)	2	4	Cent.	7.21	7.21	7.21	7.21		7.21	7.21	7.21	7.21							57.1	57.1		264.4
38	5	0.95	5	1.1	20-2(*t*-1)	2	4	C	3.17	3.17	3.17	3.17		3.17	3.17	3.17	3.17							59.1	59.1		209.8
38	5	0.95	5	1.1	20-2(*t*-1)	2	4	TP	5.35	5.35	5.35	5.35		5.35	5.35	5.35	5.35							67.1	67.1		209.8
39	2	0.9	5	1.1	20-2(*t*-1)	2	4	Cent.	10.77	10.79	10.79	10.79		10.77	10.79	10.79	10.79							65.2	65.2		254.5
39	2	0.9	5	1.1	20-2(*t*-1)	2	4	C	4.85	4.85	4.85	4.85		4.85	4.85	4.85	4.85							66.4	66.4		222.4
39	2	0.9	5	1.1	20-2(*t*-1)	2	4	TP	8.19	8.19	8.19	8.19		8.19	8.19	8.19	8.19							71.3	71.3		222.4
40	2	0.95	5	1.1	20-2(*t*-1)	2	4	Cent.	11.30	11.30	11.30	11.30		11.30	11.30	11.30	11.30							72.7	72.7		280.0
40	2	0.95	5	1.1	20-2(*t*-1)	2	4	C	5.09	5.09	5.09	5.09		5.09	5.09	5.09	5.09							74.0	74.0		246.4
40	2	0.95	5	1.1	20-2(*t*-1)	2	4	TP	8.59	8.59	8.59	8.59		8.59	8.59	8.59	8.59							79.1	79.1		246.4
41	5	0.9	5	1+0.1i	20	2	4	Cent.	6.90	7.73	7.73	7.73		6.57	7.33	7.33	7.33							90.7	92.7		371.9
41	5	0.9	5	1+0.1i	20	2	4	C	3.07	3.46	3.46	3.46		3.03	3.40	3.40	3.40							93.6	93.8		317.0
41	5	0.9	5	1+0.1i	20	2	4	TP	5.18	5.84	5.84	5.84		5.00	5.60	5.60	5.60							102.2	101.9		317.0

#	k	δ	v_0	β_i	a_t	M	T	Regime	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	π_1	π_2	π_3	π_A
42	5	0.95	5	1+0.1i	20	2	4	Cent.	8.03	8.03	8.03	8.03		7.60	7.60	7.60	7.60							102.4	104.6		412.7
42	5	0.95	5	1+0.1i	20	2	4	C	3.60	3.60	3.60	3.60		3.53	3.53	3.53	3.53							105.4	105.8		355.3
42	5	0.95	5	1+0.1i	20	2	4	TP	6.07	6.07	6.07	6.07		5.82	5.82	5.82	5.82							114.5	114.2		355.3
43	2	0.9	5	1+0.1i	20	2	4	Cent.	10.79	12.08	12.08	12.08		10.07	11.22	11.22	11.22							107.2	108.9		387.4
43	2	0.9	5	1+0.1i	20	2	4	C	4.89	5.49	5.49	5.49		4.73	5.29	5.29	5.29							109.1	109.5		353.9
43	2	0.9	5	1+0.1i	20	2	4	TP	8.25	9.27	9.27	9.27		7.80	8.71	8.71	8.71							114.6	114.5		353.9
44	2	0.95	5	1+0.1i	20	2	4	Cent.	12.54	12.54	12.54	12.54		11.63	11.63	11.63	11.63							119.9	121.7		429.0
44	2	0.95	5	1+0.1i	20	2	4	C	5.71	5.71	5.71	5.71		5.49	5.49	5.49	5.49							121.9	122.3		393.9
44	2	0.95	5	1+0.1i	20	2	4	TP	9.63	9.63	9.63	9.63		9.04	9.04	9.04	9.04							127.7	127.6		393.9
45	5	0.9	5	1+0.1i	20-2(t-1)	2	4	Cent.	6.89	6.89	6.89	6.89		6.57	6.57	6.57	6.57							51.2	52.8		242.0
45	5	0.9	5	1+0.1i	20-2(t-1)	2	4	C	3.03	3.03	3.03	3.03		3.00	3.00	3.00	3.00							53.7	53.9		191.7
45	5	0.9	5	1+0.1i	20-2(t-1)	2	4	TP	5.11	5.11	5.11	5.11		4.93	4.93	4.93	4.93							61.4	61.1		191.7
46	5	0.95	5	1+0.1i	20-2(t-1)	2	4	Cent.	7.22	7.22	7.22	7.22		6.87	6.87	6.87	6.87							57.9	59.7		267.0
46	5	0.95	5	1+0.1i	20-2(t-1)	2	4	C	3.18	3.18	3.18	3.18		3.14	3.14	3.14	3.14							60.5	60.8		214.4
46	5	0.95	5	1+0.1i	20-2(t-1)	2	4	TP	5.37	5.37	5.37	5.37		5.18	5.18	5.18	5.18							68.6	68.3		214.4
47	2	0.9	5	1+0.1i	20-2(t-1)	2	4	Cent.	10.79	10.81	10.81	10.81		10.07	10.09	10.09	10.09							65.8	67.2		256.2
47	2	0.9	5	1+0.1i	20-2(t-1)	2	4	C	4.87	4.87	4.87	4.87		4.72	4.72	4.72	4.72							67.5	67.8		225.6
47	2	0.9	5	1+0.1i	20-2(t-1)	2	4	TP	8.22	8.22	8.22	8.22		7.77	7.77	7.77	7.77							72.4	72.3		225.6
48	2	0.95	5	1+0.1i	20-2(t-1)	2	4	Cent.	11.32	11.32	11.32	11.32		10.55	10.55	10.55	10.55							73.3	74.8		281.8
48	2	0.95	5	1+0.1i	20-2(t-1)	2	4	C	5.11	5.11	5.11	5.11		4.94	4.94	4.94	4.94							75.1	75.4		249.8
48	2	0.95	5	1+0.1i	20-2(t-1)	2	4	TP	8.62	8.62	8.62	8.62		8.13	8.13	8.13	8.13							80.2	80.1		249.8
49	5	0.9	5	1.1	20	3	4	Cent.	6.00	6.75	6.75	6.75		6.00	6.75	6.75	6.75		6.00	6.75	6.75	6.75		35.2	35.2	35.2	254.3
49	5	0.9	5	1.1	20	3	4	C	2.06	2.38	2.38	2.38		2.06	2.38	2.38	2.38		2.06	2.38	2.38	2.38		31.9	31.9	31.9	154.4
49	5	0.9	5	1.1	20	3	4	TP	3.98	4.60	4.60	4.60		3.98	4.60	4.60	4.60		3.98	4.60	4.60	4.60		42.9	42.9	42.9	154.4
50	5	0.95	5	1.1	20	3	4	Cent.	7.02	7.02	7.02	7.02		7.02	7.02	7.02	7.02		7.02	7.02	7.02	7.02		40.8	40.8	40.8	283.7
50	5	0.95	5	1.1	20	3	4	C	2.49	2.49	2.49	2.49		2.49	2.49	2.49	2.49		2.49	2.49	2.49	2.49		37.4	37.4	37.4	179.4
50	5	0.95	5	1.1	20	3	4	TP	4.82	4.82	4.82	4.82		4.82	4.82	4.82	4.82		4.82	4.82	4.82	4.82		49.1	49.1	49.1	179.4
51	2	0.9	5	1.1	20	3	4	Cent.	9.49	10.64	10.64	10.64		9.49	10.64	10.64	10.64		9.49	10.64	10.64	10.64		49.7	49.7	49.7	275.2
51	2	0.9	5	1.1	20	3	4	C	3.51	3.97	3.97	3.97		3.51	3.97	3.97	3.97		3.51	3.97	3.97	3.97		47.8	47.8	47.8	214.7
51	2	0.9	5	1.1	20	3	4	TP	6.79	7.69	7.69	7.69		6.79	7.69	7.69	7.69		6.79	7.69	7.69	7.69		55.2	55.2	55.2	214.7
52	2	0.95	5	1.1	20	3	4	Cent.	11.05	11.05	11.05	11.05		11.05	11.05	11.05	11.05		11.05	11.05	11.05	11.05		56.3	56.3	56.3	305.6
52	2	0.95	5	1.1	20	3	4	C	4.14	4.14	4.14	4.14		4.14	4.14	4.14	4.14		4.14	4.14	4.14	4.14		54.3	54.3	54.3	242.2
52	2	0.95	5	1.1	20	3	4	TP	8.01	8.01	8.01	8.01		8.01	8.01	8.01	8.01		8.01	8.01	8.01	8.01		62.1	62.1	62.1	242.2
53	5	0.9	5	1.1	20-2(t-1)	3	4	Cent.	5.96	5.96	5.96	5.96		5.96	5.96	5.96	5.96		5.96	5.96	5.96	5.96		15.0	15.0	15.0	159.1
53	5	0.9	5	1.1	20-2(t-1)	3	4	C	1.89	1.89	1.89	1.89		1.89	1.89	1.89	1.89		1.89	1.89	1.89	1.89		11.3	11.3	11.3	63.1
53	5	0.9	5	1.1	20-2(t-1)	3	4	TP	3.65	3.65	3.65	3.65		3.65	3.65	3.65	3.65		3.65	3.65	3.65	3.65		20.2	20.2	20.2	63.1
54	5	0.95	5	1.1	20-2(t-1)	3	4	Cent.	6.26	6.26	6.26	6.26		6.26	6.26	6.26	6.26		6.26	6.26	6.26	6.26		17.9	17.9	17.9	176.7
54	5	0.95	5	1.1	20-2(t-1)	3	4	C	2.03	2.03	2.03	2.03		2.03	2.03	2.03	2.03		2.03	2.03	2.03	2.03		14.3	14.3	14.3	77.8
54	5	0.95	5	1.1	20-2(t-1)	3	4	TP	3.93	3.93	3.93	3.93		3.93	3.93	3.93	3.93		3.93	3.93	3.93	3.93		23.9	23.9	23.9	77.8
55	2	0.9	5	1.1	20-2(t-1)	3	4	Cent.	9.47	9.47	9.47	9.47		9.47	9.47	9.47	9.47		9.47	9.47	9.47	9.47		27.6	27.6	27.6	178.1
55	2	0.9	5	1.1	20-2(t-1)	3	4	C	3.43	3.43	3.43	3.43		3.43	3.43	3.43	3.43		3.43	3.43	3.43	3.43		25.8	25.8	25.8	122.6
55	2	0.9	5	1.1	20-2(t-1)	3	4	TP	6.63	6.63	6.63	6.63		6.63	6.63	6.63	6.63		6.63	6.63	6.63	6.63		32.2	32.2	32.2	122.6

#	k	δ	v_0	β_i	a_t	M	T	Regime	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	π_1	π_2	π_3	π_A
56	2	0.95	5	1.1	20-2(t-1)	3	4	Cent.	9.93	9.93	9.93	9.93		9.93	9.93	9.93	9.93		9.93	9.93	9.93	9.93		31.3	31.3	31.3	196.7
56	2	0.95	5	1.1	20-2(t-1)	3	4	C	3.61	3.61	3.61	3.61		3.61	3.61	3.61	3.61		3.61	3.61	3.61	3.61		29.4	29.4	29.4	138.7
56	2	0.95	5	1.1	20-2(t-1)	3	4	TP	6.99	6.99	6.99	6.99		6.99	6.99	6.99	6.99		6.99	6.99	6.99	6.99		36.2	36.2	36.2	138.7
57	5	0.9	5	1+0.1i	20	3	4	Cent.	6.04	6.79	6.79	6.79		5.79	6.48	6.48	6.48		5.56	6.18	6.18	6.18		36.6	38.1	39.6	260.8
57	5	0.9	5	1+0.1i	20	3	4	C	2.11	2.42	2.42	2.42		2.12	2.42	2.42	2.42		2.12	2.41	2.41	2.41		34.9	34.9	34.9	167.3
57	5	0.9	5	1+0.1i	20	3	4	TP	4.07	4.69	4.69	4.69		3.97	4.55	4.55	4.55		3.88	4.41	4.41	4.41		46.1	45.5	45.0	167.3
58	5	0.95	5	1+0.1i	20	3	4	Cent.	7.06	7.06	7.06	7.06		6.72	6.72	6.72	6.72		6.40	6.40	6.40	6.40		42.3	44.0	45.6	290.6
58	5	0.95	5	1+0.1i	20	3	4	C	2.54	2.54	2.54	2.54		2.53	2.53	2.53	2.53		2.52	2.52	2.52	2.52		40.6	40.7	40.7	193.2
58	5	0.95	5	1+0.1i	20	3	4	TP	4.91	4.91	4.91	4.91		4.75	4.75	4.75	4.75		4.59	4.59	4.59	4.59		52.6	51.9	51.3	193.2
59	2	0.9	5	1+0.1i	20	3	4	Cent.	9.53	10.68	10.68	10.68		8.95	9.98	9.98	9.98		8.42	9.35	9.35	9.35		50.8	52.2	53.4	279.7
59	2	0.9	5	1+0.1i	20	3	4	C	3.55	4.01	4.01	4.01		3.49	3.92	3.92	3.92		3.42	3.82	3.82	3.82		50.1	50.3	50.5	224.2
59	2	0.9	5	1+0.1i	20	3	4	TP	6.87	7.77	7.77	7.77		6.54	7.36	7.36	7.36		6.24	6.99	6.99	6.99		57.6	57.2	56.9	224.2
60	2	0.95	5	1+0.1i	20	3	4	Cent.	11.09	11.09	11.09	11.09		10.35	10.35	10.35	10.35		9.68	9.68	9.68	9.68		57.4	58.9	60.2	310.4
60	2	0.95	5	1+0.1i	20	3	4	C	4.18	4.18	4.18	4.18		4.08	4.08	4.08	4.08		3.97	3.97	3.97	3.97		56.7	56.9	57.1	252.3
60	2	0.95	5	1+0.1i	20	3	4	TP	8.09	8.09	8.09	8.09		7.65	7.65	7.65	7.65		7.25	7.25	7.25	7.25		64.6	64.2	63.8	252.3
61	5	0.9	5	1+0.1i	20-2(t-1)	3	4	Cent.	6.01	6.01	6.01	6.01		5.76	5.76	5.76	5.76		5.53	5.53	5.53	5.53		16.1	17.4	18.5	164.8
61	5	0.9	5	1+0.1i	20-2(t-1)	3	4	C	1.97	1.97	1.97	1.97		1.99	1.99	1.99	1.99		2.00	2.00	2.00	2.00		14.0	14.0	13.9	75.6
61	5	0.9	5	1+0.1i	20-2(t-1)	3	4	TP	3.81	3.81	3.81	3.81		3.73	3.73	3.73	3.73		3.65	3.65	3.65	3.65		23.3	22.8	22.4	75.6
62	5	0.95	5	1+0.1i	20-2(t-1)	3	4	Cent.	6.31	6.31	6.31	6.31		6.04	6.04	6.04	6.04		5.78	5.78	5.78	5.78		19.2	20.5	21.8	182.8
62	5	0.95	5	1+0.1i	20-2(t-1)	3	4	C	2.11	2.11	2.11	2.11		2.12	2.12	2.12	2.12		2.12	2.12	2.12	2.12		17.1	17.1	17.1	90.7
62	5	0.95	5	1+0.1i	20-2(t-1)	3	4	TP	4.08	4.08	4.08	4.08		3.98	3.98	3.98	3.98		3.88	3.88	3.88	3.88		27.1	26.5	26.0	90.7
63	2	0.9	5	1+0.1i	20-2(t-1)	3	4	Cent.	9.52	9.52	9.52	9.52		8.94	8.94	8.94	8.94		8.42	8.42	8.42	8.42		28.5	29.7	30.7	182.2
63	2	0.9	5	1+0.1i	20-2(t-1)	3	4	C	3.48	3.48	3.48	3.48		3.42	3.42	3.42	3.42		3.36	3.36	3.36	3.36		27.8	27.9	28.0	130.9
63	2	0.9	5	1+0.1i	20-2(t-1)	3	4	TP	6.74	6.74	6.74	6.74		6.43	6.43	6.43	6.43		6.14	6.14	6.14	6.14		34.3	34.0	33.7	130.9
64	2	0.95	5	1+0.1i	20-2(t-1)	3	4	Cent.	9.98	9.98	9.98	9.98		9.35	9.35	9.35	9.35		8.79	8.79	8.79	8.79		32.3	33.5	34.7	201.0
64	2	0.95	5	1+0.1i	20-2(t-1)	3	4	C	3.67	3.67	3.67	3.67		3.60	3.60	3.60	3.60		3.52	3.52	3.52	3.52		31.6	31.7	31.8	147.6
64	2	0.95	5	1+0.1i	20-2(t-1)	3	4	TP	7.10	7.10	7.10	7.10		6.76	6.76	6.76	6.76		6.44	6.44	6.44	6.44		38.5	38.1	37.8	147.6
65	5	0.9	5	1.1	20	2	5	Cent.	6.88	9.66	9.81	9.81	9.81	6.88	9.66	9.81	9.81	9.81						156.5	156.5		597.1
65	5	0.9	5	1.1	20	2	5	C	3.05	4.36	4.43	4.43	4.43	3.05	4.36	4.43	4.43	4.43						159.1	159.1		526.6
65	5	0.9	5	1.1	20	2	5	TP	5.15	7.36	7.47	7.47	7.47	5.15	7.36	7.47	7.47	7.47						170.0	170.0		526.6
66	5	0.95	5	1.1	20	2	5	Cent.	9.66	10.18	10.18	10.18	10.18	9.66	10.18	10.18	10.18	10.18						181.3	181.3		681.1
66	5	0.95	5	1.1	20	2	5	C	4.36	4.60	4.60	4.60	4.60	4.36	4.60	4.60	4.60	4.60						184.1	184.1		606.0
66	5	0.95	5	1.1	20	2	5	TP	7.36	7.76	7.76	7.76	7.76	7.36	7.76	7.76	7.76	7.76						195.7	195.7		606.0
67	2	0.9	5	1.1	20	2	5	Cent.	10.77	15.07	15.29	15.29	15.29	10.77	15.07	15.29	15.29	15.29						178.1	178.1		617.3
67	2	0.9	5	1.1	20	2	5	C	4.88	6.88	6.99	6.99	6.99	4.88	6.88	6.99	6.99	6.99						179.8	179.8		573.9
67	2	0.9	5	1.1	20	2	5	TP	8.23	11.61	11.79	11.79	11.79	8.23	11.61	11.79	11.79	11.79						186.6	186.6		573.9
68	2	0.95	5	1.1	20	2	5	Cent.	15.07	15.86	15.86	15.86	15.86	15.07	15.86	15.86	15.86	15.86						204.6	204.6		702.6
68	2	0.95	5	1.1	20	2	5	C	6.88	7.25	7.25	7.25	7.25	6.88	7.25	7.25	7.25	7.25						206.3	206.3		656.3
68	2	0.95	5	1.1	20	2	5	TP	11.61	12.24	12.24	12.24	12.24	11.61	12.24	12.24	12.24	12.24						213.6	213.6		656.3
69	5	0.9	5	1.1	20-2(t-1)	2	5	Cent.	6.88	8.48	8.48	8.48	8.48	6.88	8.48	8.48	8.48	8.48						77.6	77.6		342.7
69	5	0.9	5	1.1	20-2(t-1)	2	5	C	3.05	3.75	3.75	3.75	3.75	3.05	3.75	3.75	3.75	3.75						79.9	79.9		280.0
69	5	0.9	5	1.1	20-2(t-1)	2	5	TP	5.15	6.33	6.33	6.33	6.33	5.15	6.33	6.33	6.33	6.33						89.2	89.2		280.0

#	k	δ	v_0	β_i	a_t	M	T	Regime	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	π_1	π_2	π_3	π_A
70	5	0.95	5	1.1	20-2(t-1)	2	5	Cent.	8.84	8.84	8.84	8.84	8.84	8.84	8.84	8.84	8.84	8.84						89.8	89.8		387.0
70	5	0.95	5	1.1	20-2(t-1)	2	5	C	3.92	3.92	3.92	3.92	3.92	3.92	3.92	3.92	3.92	3.92						92.2	92.2		320.6
70	5	0.95	5	1.1	20-2(t-1)	2	5	TP	6.62	6.62	6.62	6.62	6.62	6.62	6.62	6.62	6.62	6.62						102.2	102.2		320.6
71	2	0.9	5	1.1	20-2(t-1)	2	5	Cent.	10.77	13.28	13.28	13.28	13.28	10.77	13.28	13.28	13.28	13.28						96.1	96.1		360.6
71	2	0.9	5	1.1	20-2(t-1)	2	5	C	4.88	6.00	6.00	6.00	6.00	4.88	6.00	6.00	6.00	6.00						97.6	97.6		322.0
71	2	0.9	5	1.1	20-2(t-1)	2	5	TP	8.23	10.13	10.13	10.13	10.13	8.23	10.13	10.13	10.13	10.13						103.5	103.5		322.0
72	2	0.95	5	1.1	20-2(t-1)	2	5	Cent.	13.83	13.83	13.83	13.83	13.83	13.83	13.83	13.83	13.83	13.83						109.7	109.7		406.0
72	2	0.95	5	1.1	20-2(t-1)	2	5	C	6.27	6.27	6.27	6.27	6.27	6.27	6.27	6.27	6.27	6.27						111.2	111.2		365.1
72	2	0.95	5	1.1	20-2(t-1)	2	5	TP	10.57	10.57	10.57	10.57	10.57	10.57	10.57	10.57	10.57	10.57						117.5	117.5		365.1
73	5	0.9	5	1+0.1i	20	2	5	Cent.	6.90	9.68	9.82	9.82	9.82	6.57	9.08	9.21	9.21	9.21						157.7	160.6		600.8
73	5	0.9	5	1+0.1i	20	2	5	C	3.07	4.37	4.44	4.44	4.44	3.03	4.25	4.32	4.32	4.32						161.3	161.9		533.3
73	5	0.9	5	1+0.1i	20	2	5	TP	5.18	7.38	7.49	7.49	7.49	5.00	7.01	7.11	7.11	7.11						172.2	171.9		533.3
74	5	0.95	5	1+0.1i	20	2	5	Cent.	9.68	10.19	10.19	10.19	10.19	9.08	9.54	9.54	9.54	9.54						182.5	185.8		685.1
74	5	0.95	5	1+0.1i	20	2	5	C	4.37	4.61	4.61	4.61	4.61	4.25	4.48	4.48	4.48	4.48						186.4	187.1		613.3
74	5	0.95	5	1+0.1i	20	2	5	TP	7.38	7.78	7.78	7.78	7.78	7.01	7.38	7.38	7.38	7.38						198.1	197.8		613.3
75	2	0.9	5	1+0.1i	20	2	5	Cent.	10.79	15.08	15.31	15.31	15.31	10.07	13.87	14.07	14.07	14.07						179.0	181.3		619.9
75	2	0.9	5	1+0.1i	20	2	5	C	4.89	6.90	7.00	7.00	7.00	4.73	6.57	6.67	6.67	6.67						181.3	182.0		578.7
75	2	0.9	5	1+0.1i	20	2	5	TP	8.25	11.64	11.81	11.81	11.81	7.80	10.83	10.98	10.98	10.98						188.2	188.2		578.7
76	2	0.95	5	1+0.1i	20	2	5	Cent.	15.08	15.88	15.88	15.88	15.88	13.87	14.57	14.57	14.57	14.57						205.5	208.1		705.4
76	2	0.95	5	1+0.1i	20	2	5	C	6.90	7.27	7.27	7.27	7.27	6.57	6.91	6.91	6.91	6.91						208.0	208.7		661.5
76	2	0.95	5	1+0.1i	20	2	5	TP	11.64	12.26	12.26	12.26	12.26	10.83	11.38	11.38	11.38	11.38						215.4	215.3		661.5
77	5	0.9	5	1+0.1i	20-2(t-1)	2	5	Cent.	6.90	8.50	8.50	8.50	8.50	6.57	8.02	8.02	8.02	8.02						78.6	80.9		345.8
77	5	0.9	5	1+0.1i	20-2(t-1)	2	5	C	3.07	3.77	3.77	3.77	3.77	3.03	3.69	3.69	3.69	3.69						81.7	82.1		285.7
77	5	0.9	5	1+0.1i	20-2(t-1)	2	5	TP	5.18	6.36	6.36	6.36	6.36	5.00	6.08	6.08	6.08	6.08						91.1	90.8		285.7
78	5	0.95	5	1+0.1i	20-2(t-1)	2	5	Cent.	8.86	8.86	8.86	8.86	8.86	8.35	8.35	8.35	8.35	8.35						90.8	93.4		390.4
78	5	0.95	5	1+0.1i	20-2(t-1)	2	5	C	3.94	3.94	3.94	3.94	3.94	3.85	3.85	3.85	3.85	3.85						94.2	94.6		326.7
78	5	0.95	5	1+0.1i	20-2(t-1)	2	5	TP	6.65	6.65	6.65	6.65	6.65	6.35	6.35	6.35	6.35	6.35						104.2	103.9		326.7
79	2	0.9	5	1+0.1i	20-2(t-1)	2	5	Cent.	10.79	13.30	13.30	13.30	13.30	10.07	12.30	12.30	12.30	12.30						96.8	98.8		362.8
79	2	0.9	5	1+0.1i	20-2(t-1)	2	5	C	4.89	6.02	6.02	6.02	6.02	4.73	5.77	5.77	5.77	5.77						98.9	99.4		326.1
79	2	0.9	5	1+0.1i	20-2(t-1)	2	5	TP	8.25	10.16	10.16	10.16	10.16	7.80	9.51	9.51	9.51	9.51						104.9	104.8		326.1
80	2	0.95	5	1+0.1i	20-2(t-1)	2	5	Cent.	13.85	13.85	13.85	13.85	13.85	12.79	12.79	12.79	12.79	12.79						110.4	112.5		408.4
80	2	0.95	5	1+0.1i	20-2(t-1)	2	5	C	6.29	6.29	6.29	6.29	6.29	6.01	6.01	6.01	6.01	6.01						112.6	113.2		369.6
80	2	0.95	5	1+0.1i	20-2(t-1)	2	5	TP	10.60	10.60	10.60	10.60	10.60	9.91	9.91	9.91	9.91	9.91						119.0	118.9		369.6
81	5	0.9	5	1.1	20	3	5	Cent.	6.00	8.50	8.63	8.63	8.63	6.00	8.50	8.63	8.63	8.63	6.00	8.50	8.63	8.63	8.63	68.2	68.2	68.2	419.9
81	5	0.9	5	1.1	20	3	5	C	2.06	3.10	3.16	3.16	3.16	2.06	3.10	3.16	3.16	3.16	2.06	3.10	3.16	3.16	3.16	64.3	64.3	64.3	297.9
81	5	0.9	5	1.1	20	3	5	TP	3.98	6.00	6.11	6.11	6.11	3.98	6.00	6.11	6.11	6.11	3.98	6.00	6.11	6.11	6.11	78.6	78.6	78.6	297.9
82	5	0.95	5	1.1	20	3	5	Cent.	8.50	8.96	8.96	8.96	8.96	8.50	8.96	8.96	8.96	8.96	8.50	8.96	8.96	8.96	8.96	80.6	80.6	80.6	481.1
82	5	0.95	5	1.1	20	3	5	C	3.10	3.29	3.29	3.29	3.29	3.10	3.29	3.29	3.29	3.29	3.10	3.29	3.29	3.29	3.29	76.5	76.5	76.5	351.4
82	5	0.95	5	1.1	20	3	5	TP	6.00	6.37	6.37	6.37	6.37	6.00	6.37	6.37	6.37	6.37	6.00	6.37	6.37	6.37	6.37	91.9	91.9	91.9	351.4
83	2	0.9	5	1.1	20	3	5	Cent.	9.49	13.33	13.53	13.53	13.53	9.49	13.33	13.53	13.53	13.53	9.49	13.33	13.53	13.53	13.53	87.2	87.2	87.2	445.8
83	2	0.9	5	1.1	20	3	5	C	3.51	5.05	5.13	5.13	5.13	3.51	5.05	5.13	5.13	5.13	3.51	5.05	5.13	5.13	5.13	84.9	84.9	84.9	371.2
83	2	0.9	5	1.1	20	3	5	TP	6.79	9.78	9.93	9.93	9.93	6.79	9.78	9.93	9.93	9.93	6.79	9.78	9.93	9.93	9.93	94.2	94.2	94.2	371.2

#	k	δ	v_0	β_i	a_t	M	T	Regime	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	π_1	π_2	π_3	π_A
84	2	0.95	5	1.1	20	3	5	Cent.	13.33	14.04	14.04	14.04	14.04	13.33	14.04	14.04	14.04	14.04	13.33	14.04	14.04	14.04	14.04	101.2	101.2	101.2	508.6
84	2	0.95	5	1.1	20	3	5	C	5.05	5.34	5.34	5.34	5.34	5.05	5.34	5.34	5.34	5.34	5.05	5.34	5.34	5.34	5.34	98.7	98.7	98.7	429.2
84	2	0.95	5	1.1	20	3	5	TP	9.78	10.33	10.33	10.33	10.33	9.78	10.33	10.33	10.33	10.33	9.78	10.33	10.33	10.33	10.33	108.7	108.7	108.7	429.2
85	5	0.9	5	1.1	20-2(t-1)	3	5	Cent.	6.00	7.38	7.38	7.38	7.38	6.00	7.38	7.38	7.38	7.38	6.00	7.38	7.38	7.38	7.38	26.8	26.8	26.8	232.2
85	5	0.9	5	1.1	20-2(t-1)	3	5	C	2.06	2.49	2.49	2.49	2.49	2.06	2.49	2.49	2.49	2.49	2.06	2.49	2.49	2.49	2.49	23.0	23.0	23.0	120.8
85	5	0.9	5	1.1	20-2(t-1)	3	5	TP	3.98	4.82	4.82	4.82	4.82	3.98	4.82	4.82	4.82	4.82	3.98	4.82	4.82	4.82	4.82	34.5	34.5	34.5	120.8
86	5	0.95	5	1.1	20-2(t-1)	3	5	Cent.	7.71	7.71	7.71	7.71	7.71	7.71	7.71	7.71	7.71	7.71	7.71	7.71	7.71	7.71	7.71	32.4	32.4	32.4	264.0
86	5	0.95	5	1.1	20-2(t-1)	3	5	C	2.65	2.65	2.65	2.65	2.65	2.65	2.65	2.65	2.65	2.65	2.65	2.65	2.65	2.65	2.65	28.4	28.4	28.4	146.7
86	5	0.95	5	1.1	20-2(t-1)	3	5	TP	5.12	5.12	5.12	5.12	5.12	5.12	5.12	5.12	5.12	5.12	5.12	5.12	5.12	5.12	5.12	40.9	40.9	40.9	146.7
87	2	0.9	5	1.1	20-2(t-1)	3	5	Cent.	9.49	11.69	11.69	11.69	11.69	9.49	11.69	11.69	11.69	11.69	9.49	11.69	11.69	11.69	11.69	42.9	42.9	42.9	255.2
87	2	0.9	5	1.1	20-2(t-1)	3	5	C	3.51	4.30	4.30	4.30	4.30	3.51	4.30	4.30	4.30	4.30	3.51	4.30	4.30	4.30	4.30	40.7	40.7	40.7	188.7
87	2	0.9	5	1.1	20-2(t-1)	3	5	TP	6.79	8.32	8.32	8.32	8.32	6.79	8.32	8.32	8.32	8.32	6.79	8.32	8.32	8.32	8.32	48.7	48.7	48.7	188.7
88	2	0.95	5	1.1	20-2(t-1)	3	5	Cent.	12.18	12.18	12.18	12.18	12.18	12.18	12.18	12.18	12.18	12.18	12.18	12.18	12.18	12.18	12.18	49.7	49.7	49.7	288.3
88	2	0.95	5	1.1	20-2(t-1)	3	5	C	4.51	4.51	4.51	4.51	4.51	4.51	4.51	4.51	4.51	4.51	4.51	4.51	4.51	4.51	4.51	47.5	47.5	47.5	217.9
88	2	0.95	5	1.1	20-2(t-1)	3	5	TP	8.72	8.72	8.72	8.72	8.72	8.72	8.72	8.72	8.72	8.72	8.72	8.72	8.72	8.72	8.72	55.9	55.9	55.9	217.9
89	5	0.9	5	1+0.1i	20	3	5	Cent.	6.04	8.54	8.67	8.67	8.67	5.79	8.06	8.17	8.17	8.17	5.56	7.62	7.72	7.72	7.72	70.2	72.5	74.6	428.5
89	5	0.9	5	1+0.1i	20	3	5	C	2.11	3.15	3.20	3.20	3.20	2.12	3.11	3.16	3.16	3.16	2.12	3.06	3.11	3.11	3.11	68.4	68.6	68.8	315.4
89	5	0.9	5	1+0.1i	20	3	5	TP	4.07	6.09	6.19	6.19	6.19	3.97	5.83	5.93	5.93	5.93	3.88	5.59	5.68	5.68	5.68	83.0	82.2	81.5	315.4
90	5	0.95	5	1+0.1i	20	3	5	Cent.	8.54	9.00	9.00	9.00	9.00	8.06	8.47	8.47	8.47	8.47	7.62	7.99	7.99	7.99	7.99	82.7	85.4	87.7	490.4
90	5	0.95	5	1+0.1i	20	3	5	C	3.15	3.33	3.33	3.33	3.33	3.11	3.28	3.28	3.28	3.28	3.06	3.23	3.23	3.23	3.23	81.0	81.3	81.5	370.5
90	5	0.95	5	1+0.1i	20	3	5	TP	6.09	6.45	6.45	6.45	6.45	5.83	6.16	6.16	6.16	6.16	5.59	5.90	5.90	5.90	5.90	96.7	95.8	95.1	370.5
91	2	0.9	5	1+0.1i	20	3	5	Cent.	9.53	13.37	13.57	13.57	13.57	8.95	12.36	12.54	12.54	12.54	8.42	11.47	11.63	11.63	11.63	88.6	90.6	92.3	451.7
91	2	0.9	5	1+0.1i	20	3	5	C	3.55	5.09	5.17	5.17	5.17	3.49	4.92	4.99	4.99	4.99	3.42	4.76	4.82	4.82	4.82	87.95	88.286	88.61	383.84
91	2	0.9	5	1+0.1i	20	3	5	TP	6.87	9.86	10.01	10.01	10.01	6.54	9.24	9.38	9.38	9.38	6.24	8.69	8.81	8.81	8.81	97.38	96.907	96.5	383.84
92	2	0.95	5	1+0.1i	20	3	5	Cent.	13.37	14.08	14.08	14.08	14.08	12.36	12.99	12.99	12.99	12.99	11.47	12.03	12.03	12.03	12.03	102.7	104.85	106.8	515.03
92	2	0.95	5	1+0.1i	20	3	5	C	5.09	5.38	5.38	5.38	5.38	4.92	5.18	5.18	5.18	5.18	4.76	5.00	5.00	5.00	5.00	102	102.4	102.8	442.94
92	2	0.95	5	1+0.1i	20	3	5	TP	9.86	10.41	10.41	10.41	10.41	9.24	9.73	9.73	9.73	9.73	8.69	9.13	9.13	9.13	9.13	112.1	111.6	111.2	442.94
93	5	0.9	5	1+0.1i	20-2(t-1)	3	5	Cent.	6.04	7.44	7.44	7.44	7.44	5.79	7.06	7.06	7.06	7.06	5.56	6.72	6.72	6.72	6.72	28.39	30.197	31.87	239.53
93	5	0.9	5	1+0.1i	20-2(t-1)	3	5	C	2.11	2.57	2.57	2.57	2.57	2.12	2.56	2.56	2.56	2.56	2.12	2.54	2.54	2.54	2.54	26.43	26.461	26.53	136.15
93	5	0.9	5	1+0.1i	20-2(t-1)	3	5	TP	4.07	4.97	4.97	4.97	4.97	3.97	4.81	4.81	4.81	4.81	3.88	4.65	4.65	4.65	4.65	38.29	37.647	37.09	136.15
94	5	0.95	5	1+0.1i	20-2(t-1)	3	5	Cent.	7.77	7.77	7.77	7.77	7.77	7.36	7.36	7.36	7.36	7.36	6.99	6.99	6.99	6.99	6.99	34.14	36.17	38.03	271.88
94	5	0.95	5	1+0.1i	20-2(t-1)	3	5	C	2.72	2.72	2.72	2.72	2.72	2.70	2.70	2.70	2.70	2.70	2.68	2.68	2.68	2.68	2.68	32.19	32.283	32.39	163.1
94	5	0.95	5	1+0.1i	20-2(t-1)	3	5	TP	5.25	5.25	5.25	5.25	5.25	5.07	5.07	5.07	5.07	5.07	4.89	4.89	4.89	4.89	4.89	44.97	44.31	43.69	163.1
95	2	0.9	5	1+0.1i	20-2(t-1)	3	5	Cent.	9.53	11.74	11.74	11.74	11.74	8.95	10.92	10.92	10.92	10.92	8.42	10.19	10.19	10.19	10.19	44.02	45.612	47.03	260.24
95	2	0.9	5	1+0.1i	20-2(t-1)	3	5	C	3.55	4.36	4.36	4.36	4.36	3.49	4.24	4.24	4.24	4.24	3.42	4.12	4.12	4.12	4.12	43.3	43.524	43.75	199.42
95	2	0.9	5	1+0.1i	20-2(t-1)	3	5	TP	6.87	8.43	8.43	8.43	8.43	6.54	7.96	7.96	7.96	7.96	6.24	7.53	7.53	7.53	7.53	51.36	50.941	50.6	199.42
96	2	0.95	5	1+0.1i	20-2(t-1)	3	5	Cent.	12.24	12.24	12.24	12.24	12.24	11.36	11.36	11.36	11.36	11.36	10.59	10.59	10.59	10.59	10.59	50.96	52.716	54.27	293.78
96	2	0.95	5	1+0.1i	20-2(t-1)	3	5	C	4.56	4.56	4.56	4.56	4.56	4.43	4.43	4.43	4.43	4.43	4.30	4.30	4.30	4.30	4.30	50.25	50.509	50.77	229.5
96	2	0.95	5	1+0.1i	20-2(t-1)	3	5	TP	8.83	8.83	8.83	8.83	8.83	8.32	8.32	8.32	8.32	8.32	7.86	7.86	7.86	7.86	7.86	58.85	58.401	58.04	229.5