

# Black-Litterman Return Forecasts in



# Allocation **ZEPHYR ADVISOR™**

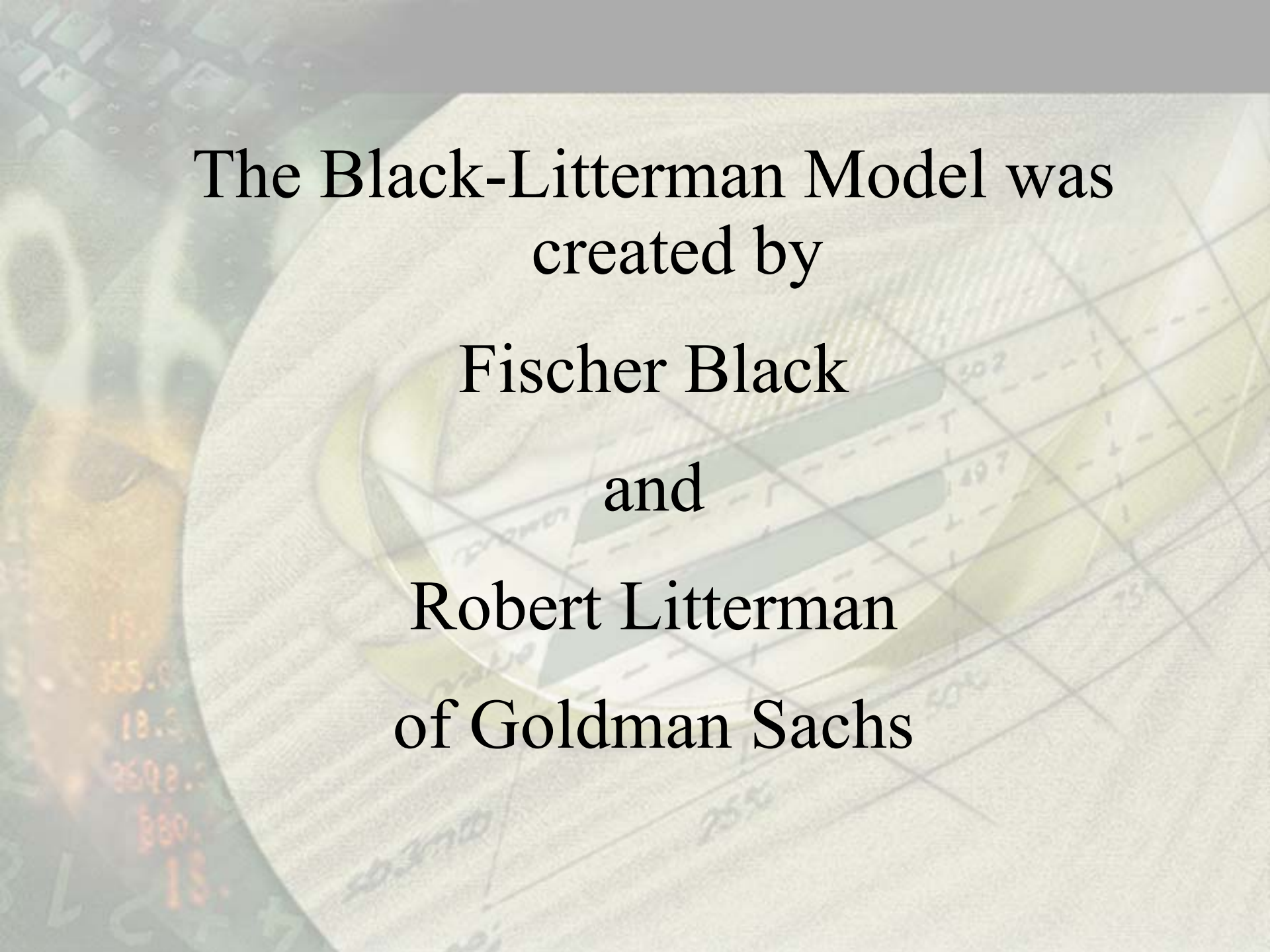
Tom Idzorek and Jill Adroque

Zephyr Associates, Inc.

September 9, 2003

# Using Black-Litterman Return Forecasts for Asset Allocation Results in Diversified Portfolios

The background of the slide is a collage of financial and technological imagery. At the top, a portion of a computer keyboard is visible. Below it, a financial chart is shown, featuring a 3D bar chart with three bars of increasing height. To the right of the chart is a table with columns labeled 'T' and 'L'. The table contains numerical values: 50.2, 49.7, and 49.7. In the bottom left corner, there is a vertical list of numbers: 18.5, 355.0, 18.5, 3598.0, 880.0, and 13.0. The overall color palette is muted, with greys, yellows, and oranges.



The Black-Litterman Model was  
created by

Fischer Black

and

Robert Litterman

of Goldman Sachs

# Allocation **ADVISOR**<sup>™</sup>

is Asset Allocation Software

Goal: To create a diversified portfolio of assets that performs well given an acceptable level of risk

How?



# Asset Allocation = Mean-Variance Optimization

- Mean-Variance Optimization was developed by Nobel Laureate Harry Markowitz in 1952
  - "Portfolio Selection." *Journal of Finance* 7, no. 1 (March 1952): 77-91.
  - *Portfolio Selection: Efficient Diversification of Investments*. 1959. Reprint. 1970
- Markowitz created a technique for creating efficient diversified portfolios

# Principles of Mean-Variance Optimization

- Diversification: The risk of a portfolio can be decreased by combining assets whose returns move in different directions under certain market conditions.
- Markowitz discovered that that an investor can reduce the volatility of a portfolio and increase its return at the same time.

# Mean-Variance Optimization (MVO)

- Inputs
  - Returns
  - Risks
  - Correlations
- Calculations--Create Efficient Frontier
- Output--Select Portfolios from the Efficient Frontier

# Forecasts are the cornerstone of good Asset Allocation

- Three forecasts are needed for MVO:
  - Expected Returns
  - Expected Risks
  - Correlations
- AllocationADVISOR currently allows you to use historical values as forecasts or to create your own forecasts

# Historical Values as Forecasts

- Expected Return = Arithmetic Mean Return
- Expected Risk = Standard Deviation
- Correlation = Pair-wise Correlation

# Historical Forecasts

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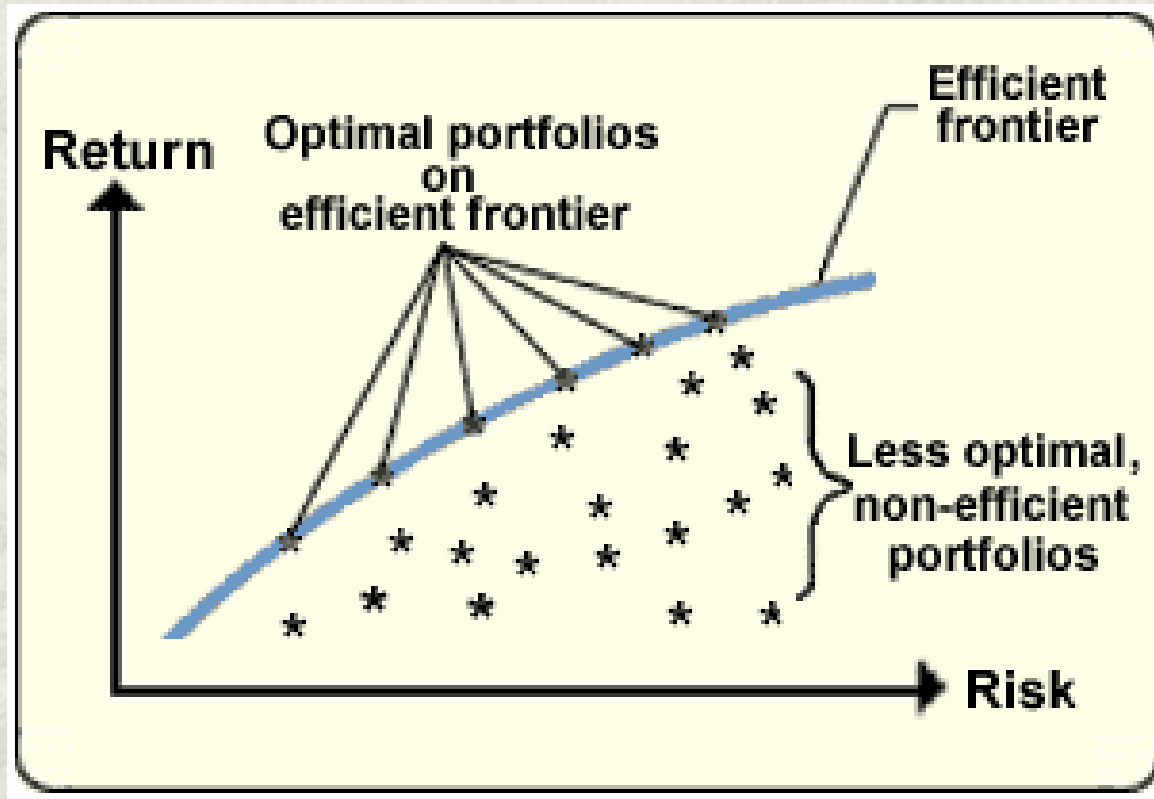
## Analysis Inputs

| <u>Assets</u>       | <u>Forecast</u> |             | <u>Date</u>  |            | <u>Constraint</u> |            |
|---------------------|-----------------|-------------|--------------|------------|-------------------|------------|
|                     | <u>Return</u>   | <u>Risk</u> | <u>Start</u> | <u>End</u> | <u>Min</u>        | <u>Max</u> |
| US Bonds            | 8.34%           | 3.43%       | 9501         | 0306       | 0%                | 100%       |
| Int'l Bonds         | 6.49%           | 8.14%       | 9501         | 0306       | 0%                | 100%       |
| US Large Growth     | 10.69%          | 20.94%      | 9501         | 0306       | 0%                | 100%       |
| US Large Value      | 12.87%          | 15.36%      | 9501         | 0306       | 0%                | 100%       |
| US Small Growth     | 7.76%           | 27.57%      | 9501         | 0306       | 0%                | 100%       |
| US Small Value      | 12.89%          | 15.66%      | 9501         | 0306       | 0%                | 100%       |
| Int'l Dev. Equity   | 4.01%           | 15.19%      | 9501         | 0306       | 0%                | 100%       |
| Int'l Emerg. Equity | 2.29%           | 24.75%      | 9501         | 0306       | 0%                | 100%       |

## Correlations

|                        | <u>1</u> | <u>2</u> | <u>3</u> | <u>4</u> | <u>5</u> | <u>6</u> | <u>7</u> | <u>8</u> |
|------------------------|----------|----------|----------|----------|----------|----------|----------|----------|
| 1. US Bonds            | 1.0000   |          |          |          |          |          |          |          |
| 2. Int'l Bonds         | 0.3972   | 1.0000   |          |          |          |          |          |          |
| 3. US Large Growth     | 0.2325   | -0.0320  | 1.0000   |          |          |          |          |          |
| 4. US Large Value      | 0.2895   | -0.0598  | 0.8242   | 1.0000   |          |          |          |          |
| 5. US Small Growth     | 0.1334   | -0.0785  | 0.8450   | 0.6979   | 1.0000   |          |          |          |
| 6. US Small Value      | 0.2150   | -0.1371  | 0.7244   | 0.8166   | 0.8681   | 1.0000   |          |          |
| 7. Int'l Dev. Equity   | 0.1879   | 0.3856   | 0.6154   | 0.5838   | 0.5475   | 0.4964   | 1.0000   |          |
| 8. Int'l Emerg. Equity | -0.0182  | -0.1144  | 0.6516   | 0.5908   | 0.6736   | 0.6197   | 0.7069   | 1.0000   |

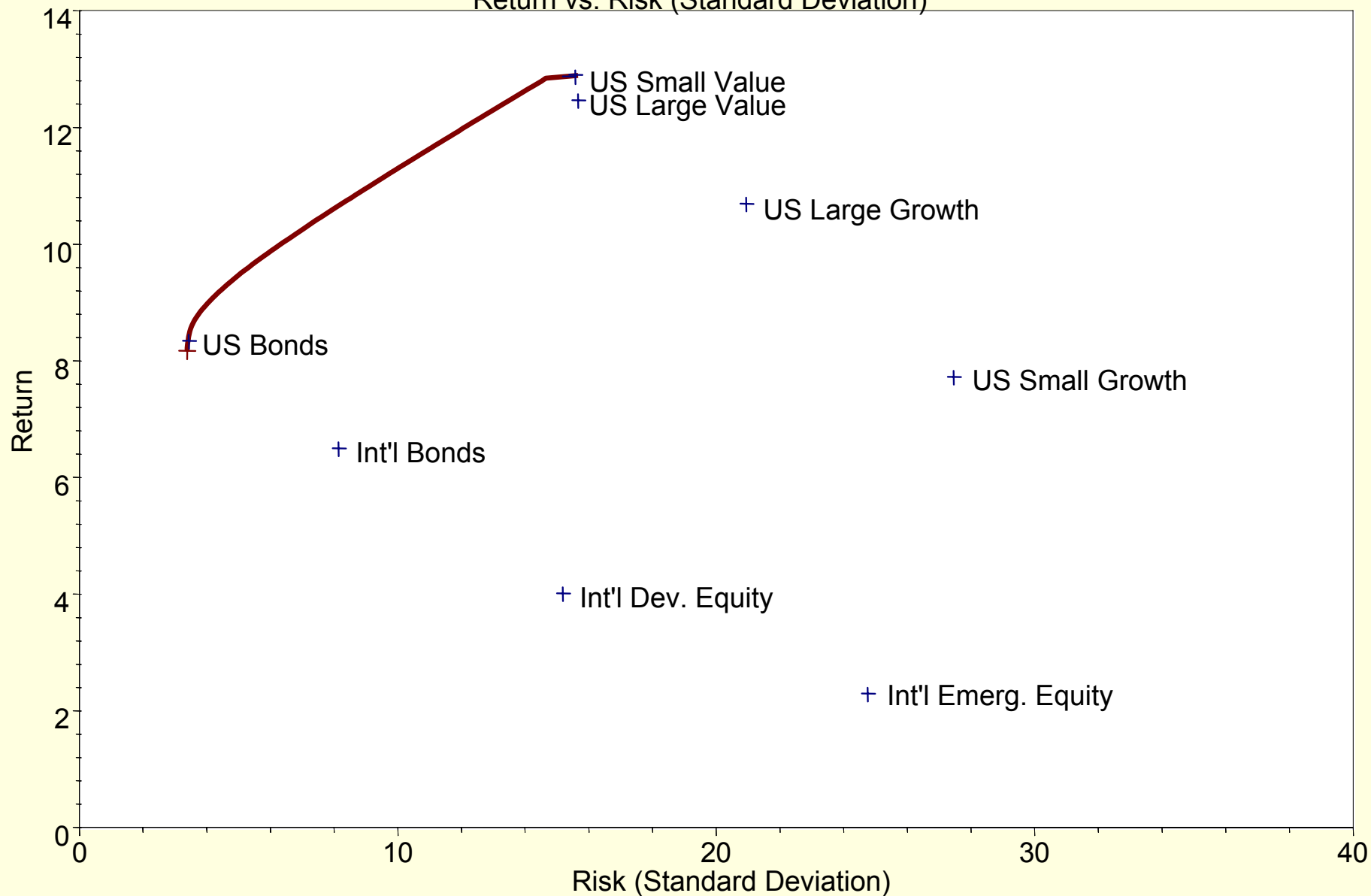
# Create Efficient Frontier



# Asset Allocation Analysis

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Efficient Frontier  
Return vs. Risk (Standard Deviation)

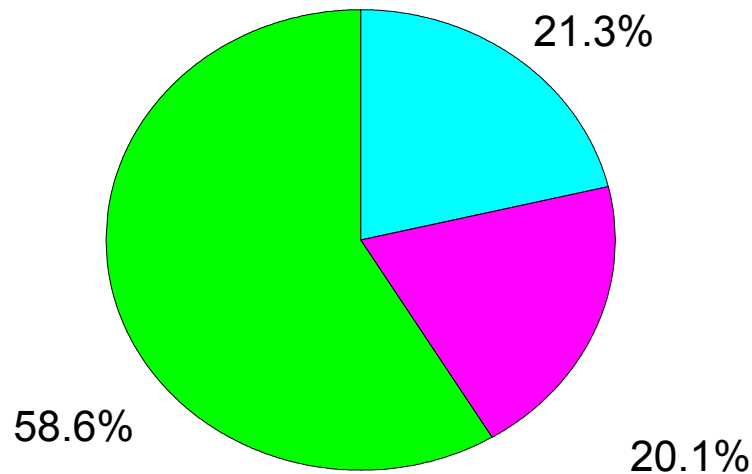


# MVO with Historical Data often yields unreasonable results

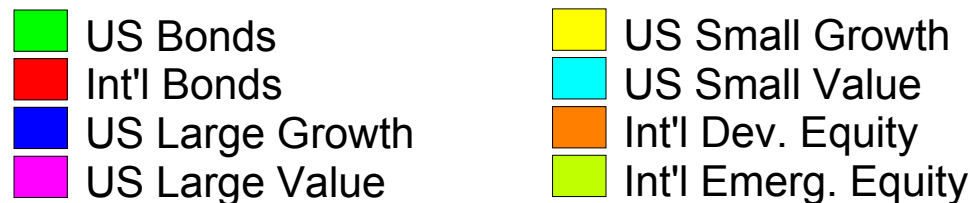
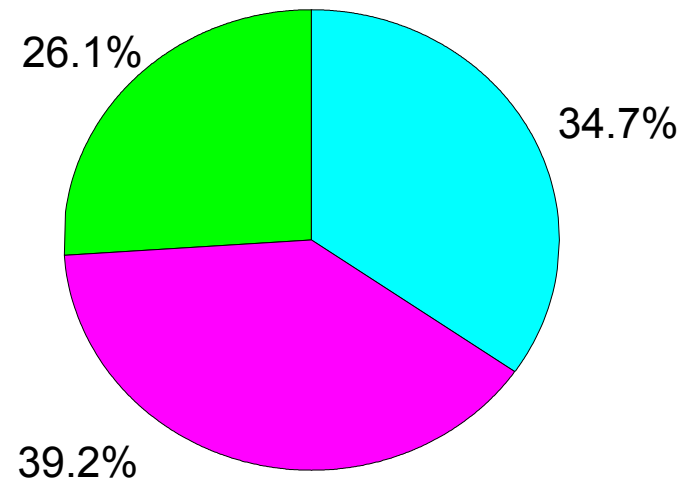
- Portfolios are often unintuitive
- Portfolios are often highly concentrated
- Portfolio allocations change drastically with small changes in the forecasts

# Most of the Portfolios Created With Historical Returns Use Three Asset Classes or Less

## Sample Portfolio 1



## Sample Portfolio 2



# Why does this happen?

- If two asset classes are similar, but one has a slightly higher forecasted return, MVO allocates everything to the asset with the higher forecasted return and nothing to the other asset.

# Why is it important?

- Unintuitive portfolios make it very tempting to “tweak” the portfolio using constraints.



- If you are going to use lots of constraints, you might as well skip using MVO in your asset allocation process.

So should we forget about MVO?

**NO**



# Long-term data can help

- Using long-term data, like that created by Global Financial Data, generally leads to more diversified portfolios than 5 or 10 years of data.

# The Black-Litterman Model creates better return forecasts to use with MVO

- Black, Fischer, and Robert Litterman. “Global Portfolio Optimization.” *Financial Analysts Journal*, September/October 1992, pp. 28-43.
- He, Guangliang, and Robert Litterman. “The Intuition Behind Black-Litterman Model Portfolios.” *Investment Management Research*, Goldman, Sachs & Company, December 1999.
- Litterman, Robert, and the Quantitative Resources Group, Goldman Sachs Asset Management. *Modern Investment Management: An Equilibrium Approach*. New Jersey: John Wiley & Sons, 2003.

Black-Litterman Return Forecasts are  
coming to

Allocation **ZEPHYR**  
**ADVISOR**<sup>™</sup>

# How does it work?

- Start with the Market Returns
- Apply your own unique views of how certain markets are going to behave
- The end result is a set of return forecasts that give rise to diversified portfolios when used with MVO

# Market Portfolio

- The Market Returns are the returns that are implied by the Market Portfolio.
- The Market Portfolio is the capitalization-weighted portfolio of the assets.

# A Simple Three Asset Example: The Market Portfolio

|              | <u>Market Cap</u> | <u>Weight</u> |
|--------------|-------------------|---------------|
| US Equity    | \$ 11,498         | 38.2%         |
| US Bonds     | \$ 8,280          | 27.5%         |
| Int'l Equity | \$ 10,350         | 34.4%         |

# Market Returns

- The Market Returns are derived from known information using Reverse Optimization:
  - Risk Premium
  - Covariance Matrix
  - Market Capitalization of the assets

# Market Returns are derived using Reverse Optimization

$$\Pi = \lambda \Sigma w_{mkt}$$

$\Pi$  ( $\Pi$ ) is the excess market returns over the risk free rate

$\lambda$  (Lambda) is the risk aversion coefficient

$\Sigma$  (Sigma) is the covariance matrix of returns

$w_{mkt}$  is the market capitalization weight of the assets

To Reverse Engineer the returns for this portfolio we need:

- The Risk Aversion Coefficient  $\lambda$
- The Covariance Matrix for the three assets  $\Sigma$
- The market weights for the three assets  $w_{mkt}$

The Risk Aversion Coefficient is the rate at which more return is required for more risk

$$\lambda = \frac{E(r) - r_f}{\sigma^2} = \frac{\text{Risk Premium}}{\text{Variance}}$$

# The Risk Aversion Coefficient ( $\lambda$ )

$$\lambda = \frac{\text{Risk Premium}}{\text{Variance}}$$

- Assume a Risk Premium of 4%
- The Historical (Long-term) Standard Deviation for the Market Portfolio is 10.84%. The Variance is the Standard Deviation Squared, 1.177%

# The Risk Aversion Coefficient

$$\lambda = \frac{\text{Risk Premium}}{\text{Variance}}$$

$$\lambda = \frac{4.00\%}{1.117\%}$$

$$\lambda = 3.404$$

# Covariance Matrix

The covariance of each pair of assets is calculated using the historical (long-term) correlations and standard deviations.

Covariance (A,B) =

Correlation (A,B) \* Std Dev (A) \* Std Dev (B)

# Covariance Matrix ( $\Sigma$ )

| <u>Covariance</u> | US Equity | US Bonds | Int'l Equity |
|-------------------|-----------|----------|--------------|
| US Equity         | 0.036     | 0.002    | 0.010        |
| US Bonds          | 0.002     | 0.003    | 0.001        |
| Int'l Equity      | 0.010     | 0.001    | 0.025        |

$$\text{Cov}(US\ Equity, US\ Equity) = 0.036$$

$$\text{Cov}(US\ Equity, US\ Bonds) = 0.002$$

$$\text{Cov}(US\ Equity, Int'l\ Equity) = 0.010$$

# Market Portfolio Weights ( $w_{mkt}$ )

Market Portfolio Weights are based on the market capitalization for each of the three assets

|              | <u>Market Cap</u> | <u>Weight</u> |
|--------------|-------------------|---------------|
| US Equity    | \$ 11,498         | 38.2%         |
| US Bonds     | \$ 8,280          | 27.5%         |
| Int'l Equity | \$ 10,350         | 34.4%         |

$$w_{\text{US Equity}} = .382$$

$$w_{\text{US Bonds}} = .275$$

$$w_{\text{Int'l Equity}} = .344$$

# Implied Excess Market Returns

$$\Pi = \lambda \Sigma w_{mkt}$$

$$\begin{aligned} \Pi_{US\ Equity} = \lambda * [ & \text{Cov}(US\ Equity, US\ Equity) * W_{US\ Equity} \\ & + \text{Cov}(US\ Equity, US\ Bonds) * W_{US\ Bonds} \\ & + \text{Cov}(US\ Equity, Int'l\ Equity) * W_{US\ Int'l\ Equity} ] \end{aligned}$$

$$\Pi_{US\ Equity} = 3.404 * [0.036 * .382 + .002 * .275 + 0.010 * .344]$$

$$\Pi_{US\ Equity} = 6.05\%$$

# Implied Total Market Return

- The Total Return is the Excess Return plus the Risk-Free Rate
- Forecast Return for US Equity =  $6.05 + 4.5 = 10.55\%$

# Market Return Forecasts

- The same calculations are made for US Bonds and Int'l Equity:
  - Forecast Return for US Equity = 10.6 %
  - Forecast Return for US Bonds = 5.2 %
  - Forecast Return for Int'l Equity = 8.9 %
- These returns can be used in AllocationADVISOR, or you can combine them with your own unique views

# Market Return Forecasts for Three Assets

Zephyr AllocationADVISOR

## Analysis Inputs

| <u>Assets</u> | <u>Forecast</u> |             | <u>Date</u>  |            | <u>Constraint</u> |            |
|---------------|-----------------|-------------|--------------|------------|-------------------|------------|
|               | <u>Return</u>   | <u>Risk</u> | <u>Start</u> | <u>End</u> | <u>Min</u>        | <u>Max</u> |
| US Equity     | 10.6%           | 19.0%       | 2601         | 0307       | 0%                | 100%       |
| US Bond       | 5.2%            | 5.5%        | 2601         | 0307       | 0%                | 100%       |
| Intl Equity   | 8.9%            | 15.7%       | 2601         | 0307       | 0%                | 100%       |

## Correlations

|                | <u>1</u> | <u>2</u> | <u>3</u> |
|----------------|----------|----------|----------|
| 1. US Equity   | 1.00     |          |          |
| 2. US Bond     | 0.19     | 1.00     |          |
| 3. Intl Equity | 0.34     | 0.17     | 1.00     |

# A more complete set of assets

|                      | <u>Market Cap</u> | <u>Weight</u> |
|----------------------|-------------------|---------------|
| Lehman US Universal  | \$ 8,280          | 19.34%        |
| Lehman Global Ex US  | \$11,184          | 26.13%        |
| Russell 1000 Growth  | \$ 5,174          | 12.09%        |
| Russell 1000 Value   | \$ 5,174          | 12.09%        |
| Russell 2000 Growth  | \$ 575            | 1.34%         |
| Russell 2000 Value   | \$ 575            | 1.34%         |
| Citigroup World x US | \$10,350          | 24.18%        |
| Citigroup EM World B | \$ 1,496          | 3.49%         |

# Market Returns

Zephyr AllocationADVISOR

## Analysis Inputs

| <u>Assets</u>       | <u>Forecast</u> |             | <u>Date</u>  |            | <u>Constraint</u> |            |
|---------------------|-----------------|-------------|--------------|------------|-------------------|------------|
|                     | <u>Return</u>   | <u>Risk</u> | <u>Start</u> | <u>End</u> | <u>Min</u>        | <u>Max</u> |
| US Bonds            | 5.07%           | 3.43%       | 9501         | 0306       | 0%                | 100%       |
| Int'l Bonds         | 5.90%           | 8.14%       | 9501         | 0306       | 0%                | 100%       |
| US Large Growth     | 12.20%          | 20.94%      | 9501         | 0306       | 0%                | 100%       |
| US Large Value      | 9.89%           | 15.36%      | 9501         | 0306       | 0%                | 100%       |
| US Small Growth     | 13.34%          | 27.57%      | 9501         | 0306       | 0%                | 100%       |
| US Small Value      | 9.21%           | 15.66%      | 9501         | 0306       | 0%                | 100%       |
| Int'l Dev. Equity   | 10.66%          | 15.19%      | 9501         | 0306       | 0%                | 100%       |
| Int'l Emerg. Equity | 12.27%          | 24.75%      | 9501         | 0306       | 0%                | 100%       |

## Correlations

|                        | <u>1</u> | <u>2</u> | <u>3</u> | <u>4</u> | <u>5</u> | <u>6</u> | <u>7</u> | <u>8</u> |
|------------------------|----------|----------|----------|----------|----------|----------|----------|----------|
| 1. US Bonds            | 1.0000   |          |          |          |          |          |          |          |
| 2. Int'l Bonds         | 0.4081   | 1.0000   |          |          |          |          |          |          |
| 3. US Large Growth     | 0.2168   | -0.0335  | 1.0000   |          |          |          |          |          |
| 4. US Large Value      | 0.2754   | -0.0602  | 0.8241   | 1.0000   |          |          |          |          |
| 5. US Small Growth     | 0.1070   | -0.0837  | 0.8445   | 0.6971   | 1.0000   |          |          |          |
| 6. US Small Value      | 0.1869   | -0.1413  | 0.7242   | 0.8160   | 0.8684   | 1.0000   |          |          |
| 7. Int'l Dev. Equity   | 0.1879   | 0.3856   | 0.6154   | 0.5838   | 0.5475   | 0.4964   | 1.0000   |          |
| 8. Int'l Emerg. Equity | -0.0182  | -0.1144  | 0.6516   | 0.5908   | 0.6736   | 0.6197   | 0.7069   | 1.0000   |

# Add your own unique views

- Investors generally have opinions, or views, about how certain markets will behave in the future.



Each view includes a measure of  
certainty



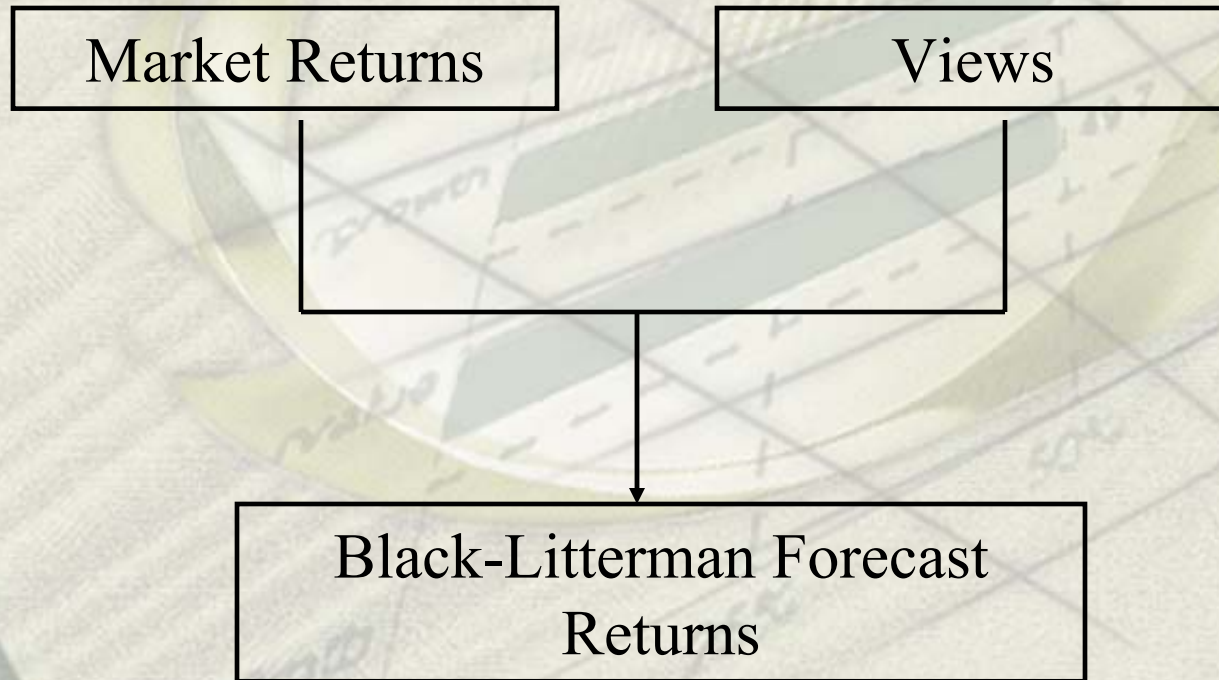
# Example of an Absolute View

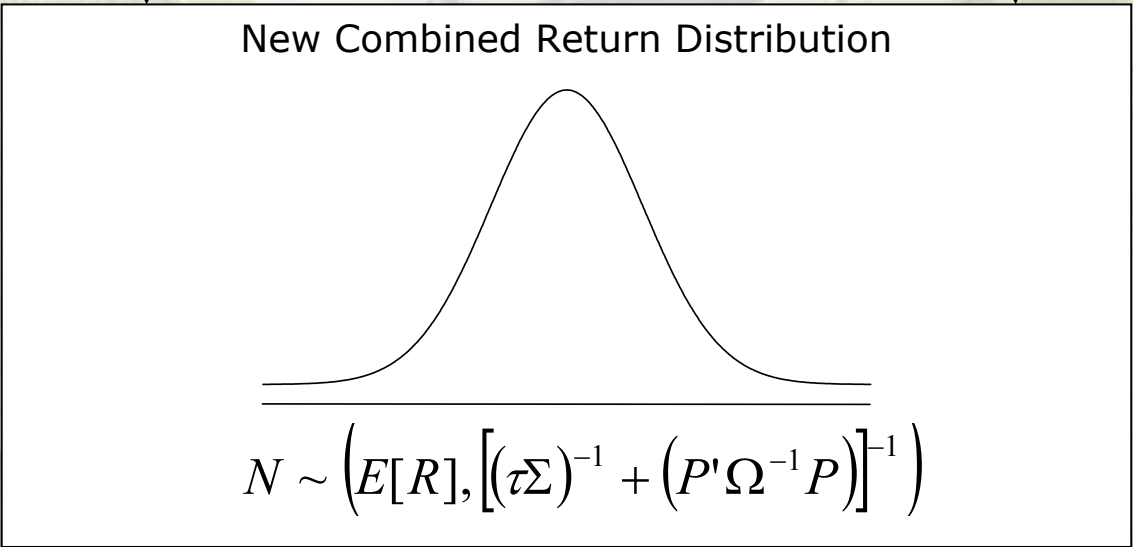
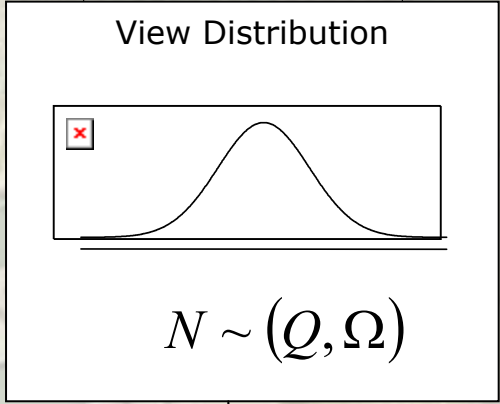
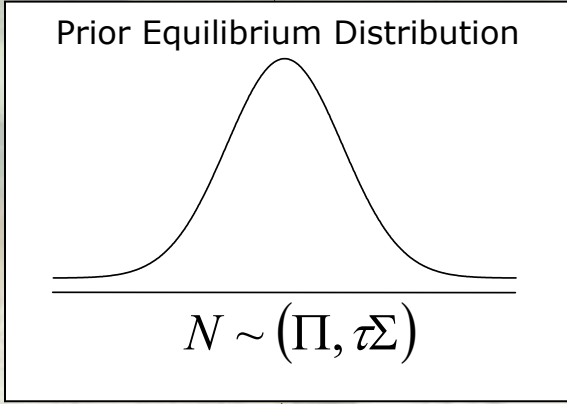
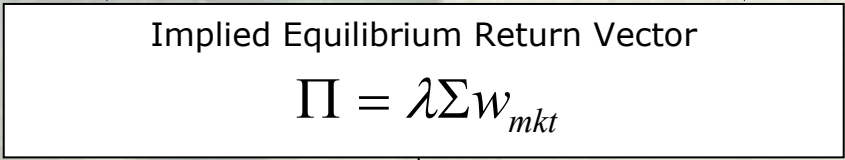
- Opinion: “I think that International Equity is going to do well.”
- View: International Developed Equity will have a return of 11%
- Confidence of View: 55%

# Example of a Relative View

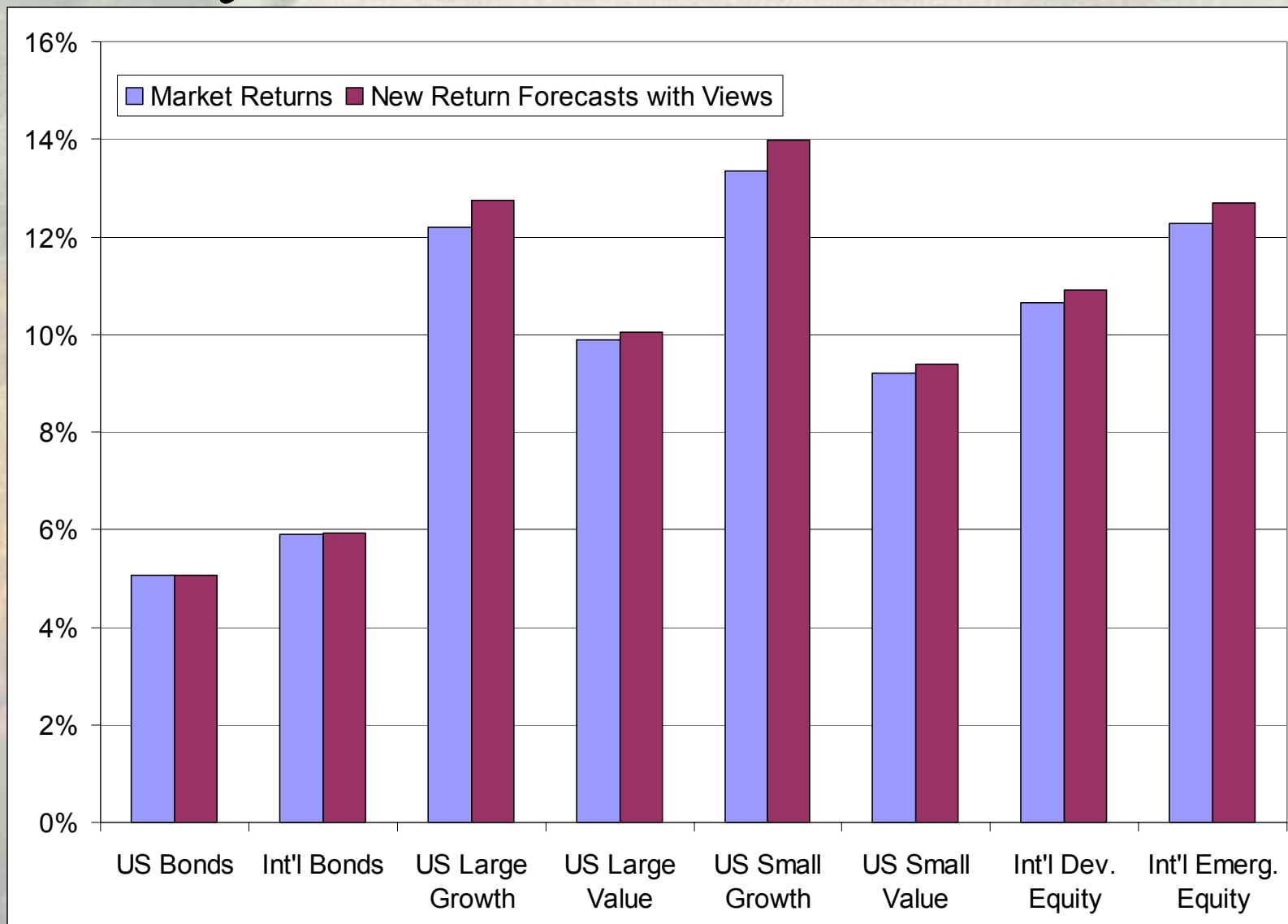
- Opinion: “I believe that Growth is going to outperform Value.”
- View: US Large Growth and US Small Growth will outperform US Large Value and US Small Value by 3%.
- Confidence of View: 80%

# Combine the Market Returns with your Views

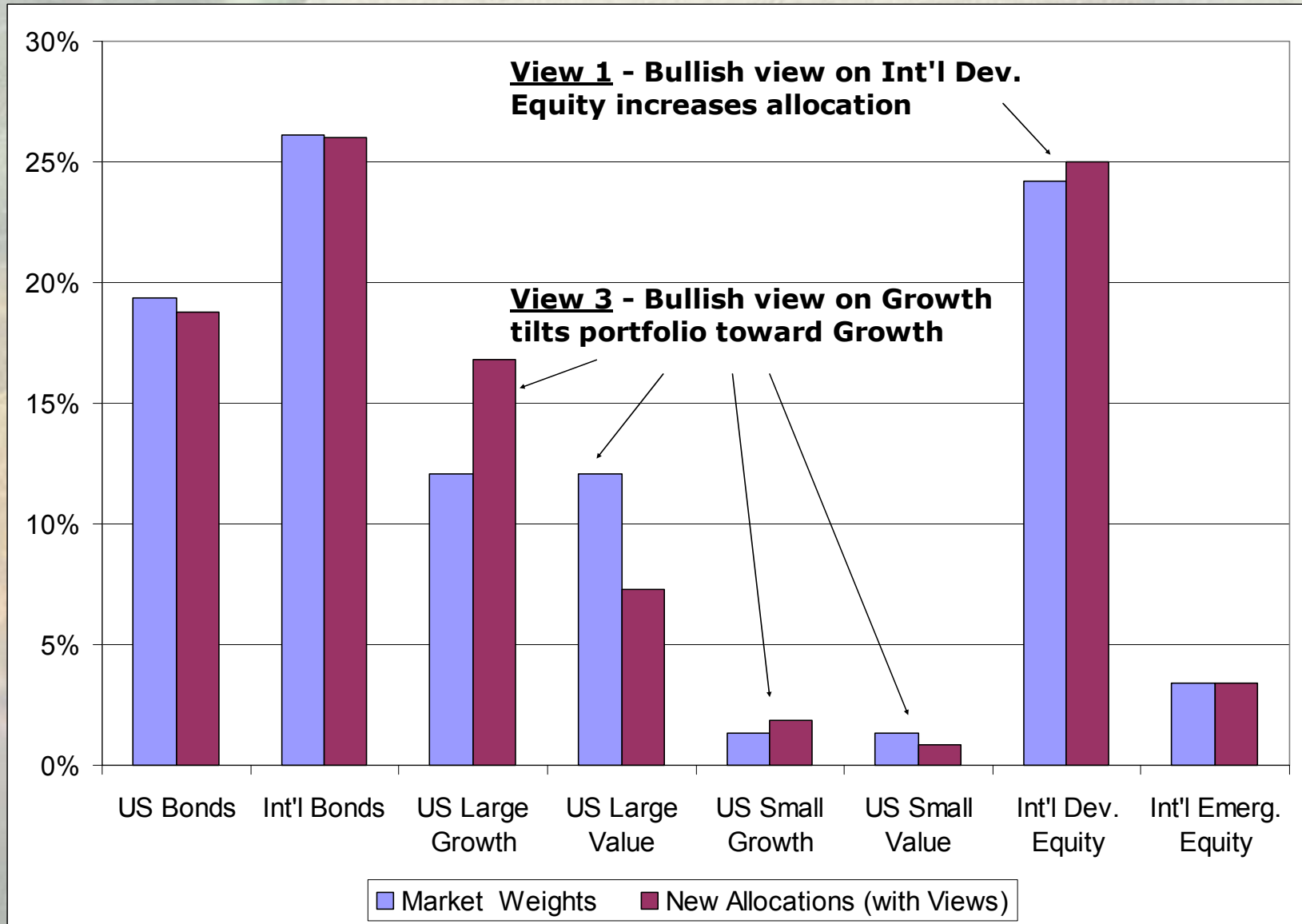




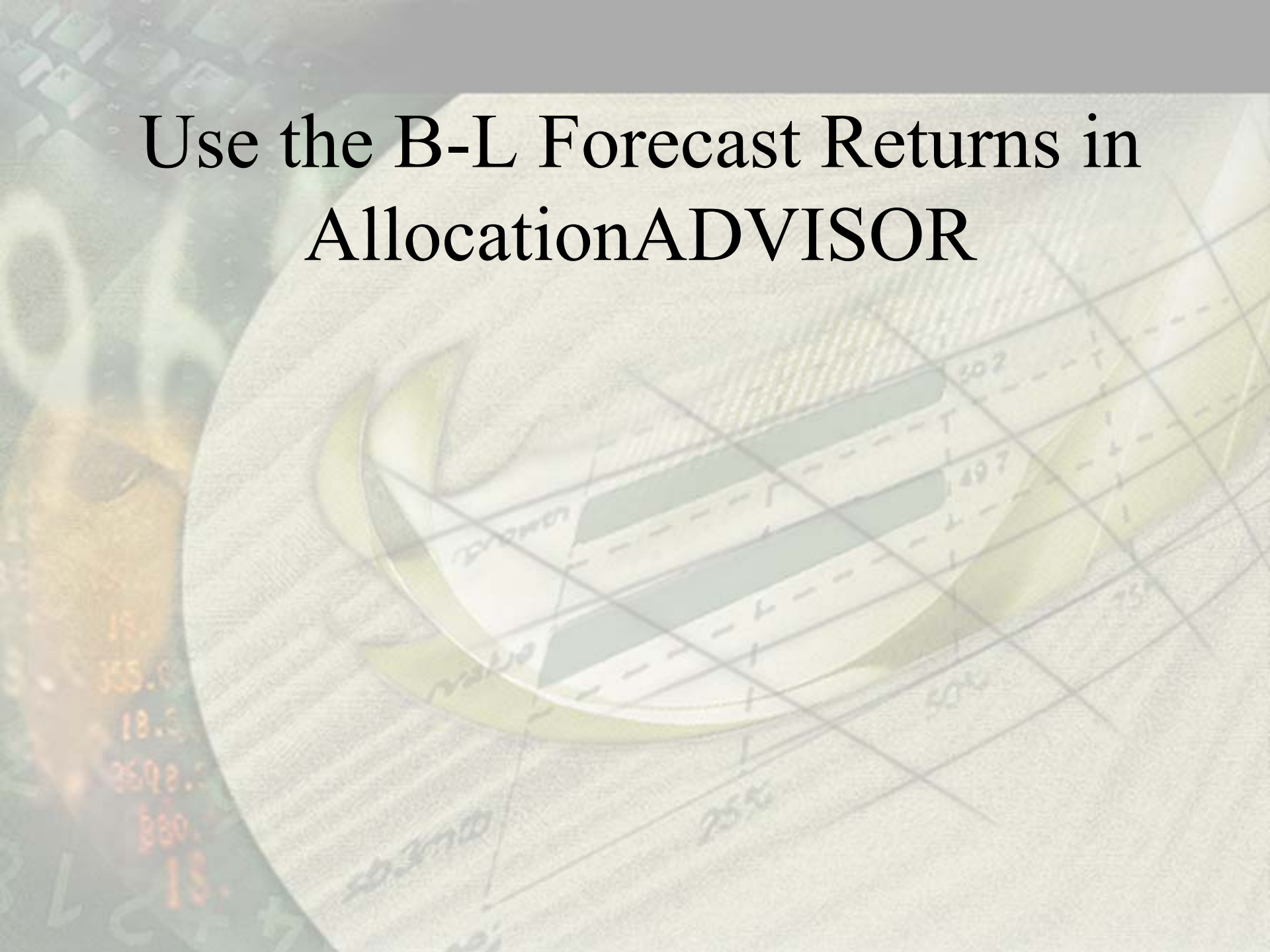
# The Views move the Return Forecasts away from the Market Returns



# And the resulting Asset Allocations will change, reflecting your views



# Use the B-L Forecast Returns in AllocationADVISOR



# Black-Litterman Forecast Returns

Zephyr AllocationADVISOR

## Analysis Inputs

| <u>Assets</u>       | <u>Forecast</u> |             | <u>Date</u>  |            | <u>Constraint</u> |            |
|---------------------|-----------------|-------------|--------------|------------|-------------------|------------|
|                     | <u>Return</u>   | <u>Risk</u> | <u>Start</u> | <u>End</u> | <u>Min</u>        | <u>Max</u> |
| US Bonds            | 5.07%           | 3.43%       | 9501         | 0306       | 0%                | 100%       |
| Int'l Bonds         | 5.92%           | 8.14%       | 9501         | 0306       | 0%                | 100%       |
| US Large Growth     | 12.76%          | 20.94%      | 9501         | 0306       | 0%                | 100%       |
| US Large Value      | 10.04%          | 15.36%      | 9501         | 0306       | 0%                | 100%       |
| US Small Growth     | 13.99%          | 27.57%      | 9501         | 0306       | 0%                | 100%       |
| US Small Value      | 9.39%           | 15.66%      | 9501         | 0306       | 0%                | 100%       |
| Int'l Dev. Equity   | 10.92%          | 15.19%      | 9501         | 0306       | 0%                | 100%       |
| Int'l Emerg. Equity | 12.70%          | 24.75%      | 9501         | 0306       | 0%                | 100%       |

## Correlations

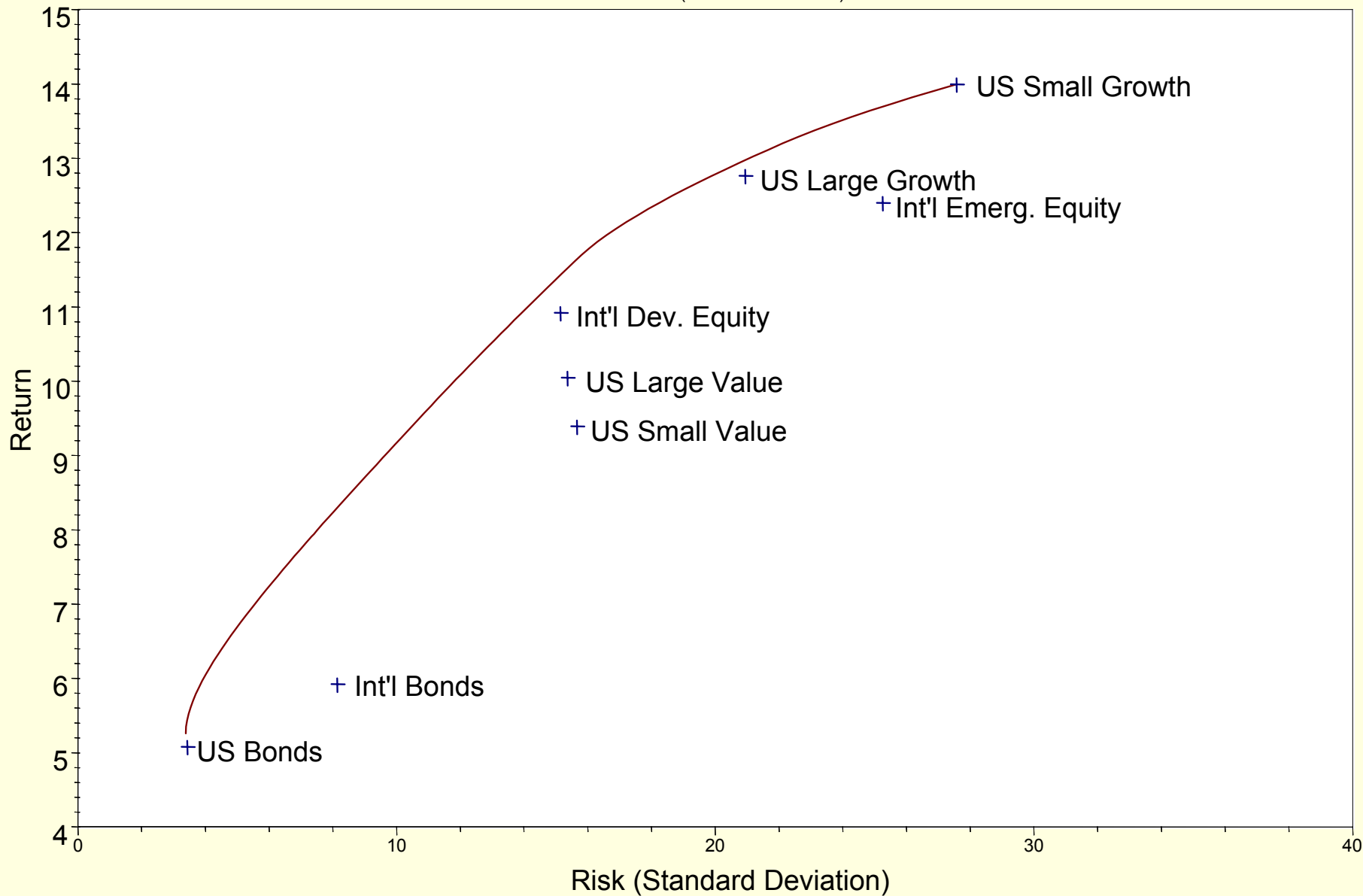
|                        | <u>1</u> | <u>2</u> | <u>3</u> | <u>4</u> | <u>5</u> | <u>6</u> | <u>7</u> | <u>8</u> |
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| 1. US Bonds            | 1.0000   |          |          |          |          |          |          |          |
| 2. Int'l Bonds         | 0.4081   | 1.0000   |          |          |          |          |          |          |
| 3. US Large Growth     | 0.2168   | -0.0335  | 1.0000   |          |          |          |          |          |
| 4. US Large Value      | 0.2754   | -0.0602  | 0.8241   | 1.0000   |          |          |          |          |
| 5. US Small Growth     | 0.1070   | -0.0837  | 0.8445   | 0.6971   | 1.0000   |          |          |          |
| 6. US Small Value      | 0.1869   | -0.1413  | 0.7242   | 0.8160   | 0.8684   | 1.0000   |          |          |
| 7. Int'l Dev. Equity   | 0.1879   | 0.3856   | 0.6154   | 0.5838   | 0.5475   | 0.4964   | 1.0000   |          |
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# Asset Allocation Analysis

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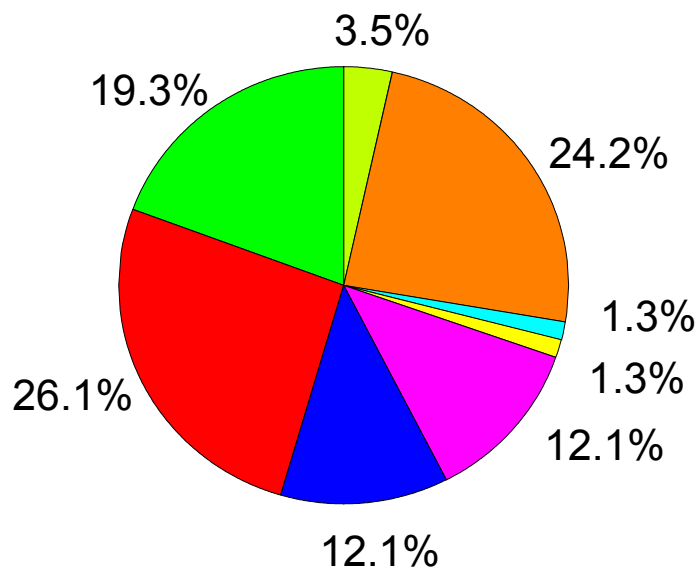
## Efficient Frontier

Return vs. Risk (Standard Deviation)

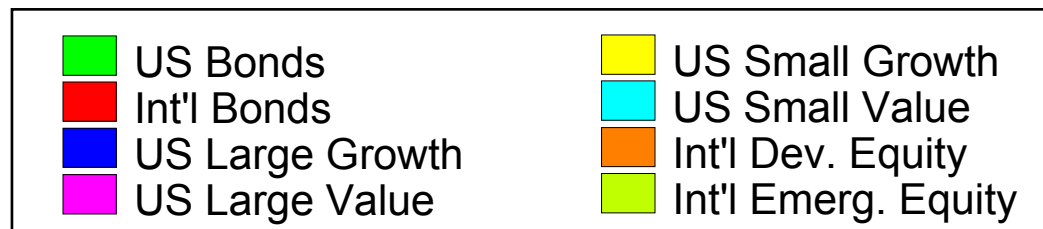
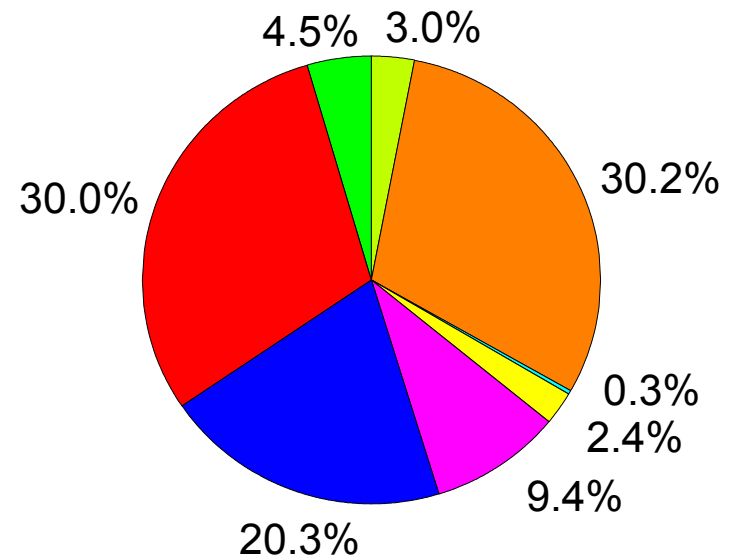


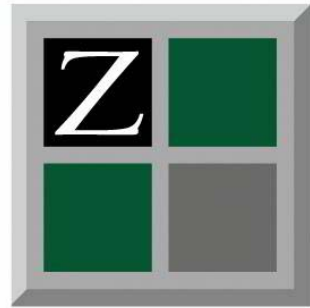
# The B-L Return Forecasts yield more diversified portfolios

## Sample Portfolio 1



## Sample Portfolio 2





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