Asymmetric Returns and Optimal Hedge Fund Portfolios

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It is now well established that the construction of optimal hedge fund portfolios requires techniques that reach well beyond traditional mean variance analysis. For example, Brooks and Kat [2002] demonstrate that various hedge fund strategies have more downside than upside risk—returns exhibit negative skew and excess kurtosis. Lo [2001] and Anson [2002] argue much the same from a conceptual perspective. Krokhmal, Uryasev, and Zrazhevsky [2002] and Signer and Favre [2002] demonstrate that assuming normality in hedge fund returns leads to portfolios that are more risky than in the case when asymmetry is explicitly considered.

If certain hedge fund strategies have more downside than upside risk, and one must take this into account in building portfolios, what is the best approach? Unfortunately, there is no easy answer. A myriad of various risk metrics and optimization approaches have been proposed as solutions. None have emerged to universal acceptance as of yet. That said, several methods appear to offer acceptable avenues for practitioners engaged in building and managing hedge fund portfolios.

In this article, I leverage off of recent work and compare various optimization techniques applied to hedge fund portfolio construction. I specifically focus on strategy allocation, as opposed to the more general problem of allocating to stocks, bonds, and hedge funds, which was the thrust of most prior research. The first section provides a brief background on the issue of asymmetric returns and the implications for optimization. In the second section, I apply common portfolio optimization techniques to the hedge fund strategy allocation problem employing Duarte's general model, which views portfolio optimization as a single problem from which other techniques fall out as special cases. The third section examines the Cornish-Fisher expansion as an efficient and promising methodology when applied to hedge fund strategies.

My major conclusion is that the incorporation of asymmetry produces significantly different hedge fund portfolios than in the situation when returns are viewed as symmetric. In particular, I find that optimal hedge fund portfolios should have up to a 30% smaller allocation to distressed debt than symmetric return models indicate. This is due to the fact that downside risk for distressed debt is unusually high. The lower allocations to distressed debt are offset by larger allocations to equity market neutral, rotational, and systematic macro strategies, which produce more positively skewed portfolios.

THE ASYMMETRIC RETURN ISSUE

As already noted, it is now well documented that various hedge fund strategies exhibit asymmetric return patterns characterized by negative skew and excess kurtosis.[1] The consequences of this are that returns sometimes surprise on the downside—much

EXHIBIT 1
Convertible Arbitrage Frequency Distribution

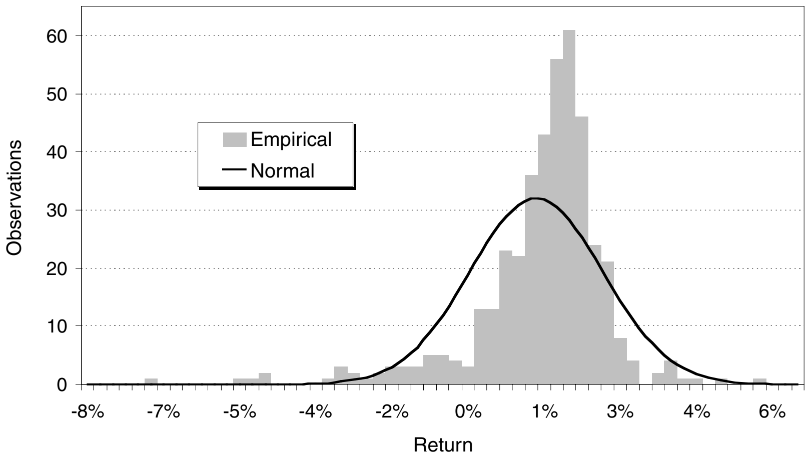


EXHIBIT 2
Hedge Fund Performance Characteristics, 1995–2002

Strategy	Index	Mean return	St. dev	Skew	Kurtosis
Aggregates/ composite	CSFB	10.6%	9.0%	.10	1.3*
	HFR	11.1%	7.9%	-.46	2.5*
	EACM	9.7%	4.6%	.24	2.9*
Equity market neutral	CSFB	10.7%	3.2%	.10	0.0
	HFR	9.9%	3.1%	.11	0.1
	EACM	6.8%	2.6%	-.78	2.2*
Convertible arbitrage	CSFB	9.7%	4.9%	-1.59*	4.0*
	HFR	9.7%	4.7%	-3.22*	17.7*
	EACM	7.8%	5.7%	-2.07*	6.2*
Equity hedge/ long biased	CSFB	11.9%	11.7%	.22	2.8*
	HFR	12.4%	12.4%	-.22	1.7*
	EACM	11.9%	11.9%	.51	2.6*
Bond hedge/fixed	CSFB	6.9%	4.0%	-3.43*	17.8*
income arbitrage	HFR	8.3%	3.3%	-1.25*	4.3*
	EACM	4.7%	5.0%	-2.79*	10.6*
Macro	CSFB	14.2%	12.9%	-.02	1.5
	HFR	11.3%	6.4%	.21	-0.3
	EACM	9.7%	8.6%	.51	0.3
Event driven	CSFB	10.1%	6.3%	-3.32*	21.0*
	HFR	10.0%	4.5%	-2.52*	14.5*
	EACM	10.2%	4.8%	-2.38*	12.4*
Merger arb	HFR	10.7%	3.9%	-2.51*	10.8*
	EACM	8.6%	4.5%	-2.08*	7.7*
Distressed debt	HFR	9.5%	5.7%	-1.73*	9.1*
	EACM	9.6%	5.4%	-1.74*	9.8*

Based on annualized monthly data reported by CSFB, HFR and EACM for 93 observations. The standard deviation of the skew is .254 and for kurtosis, .508. An asterisk denotes statistical significance from zero with 99% confidence.

more so than would be the case if the return pattern were a normal bell-shaped distribution. For this reason, when sizable losses occur, they often appear as "six standard deviation" events. The most notorious example involved Long Term Capital Management in 1998 when an unexpectedly large "blowout" in credit spreads forced the fund to collapse.[2]

To illustrate this phenomenon, Exhibit 1 shows the return distribution for convertible arbitrage hedge fund managers based on monthly data reported by Evaluation Associates Inc. (EAI) and Hedge Fund Research (HFR) from 1990 through 2002, and for CSFB's Tremont index from 1994 through 2002.[3] The chart clearly illustrates a significant departure from normality. The standard deviation for the sample series is 1.31% with an average monthly return of 0.85%. If the return distribution were normal, one would expect to see a return less than –3.1% only once every 200 months. Yet actual returns are below –3.1% once every 45 months. Furthermore, the largest reported negative return for the sample is –6.7%, which is a six standard deviation event. Such are the consequences of skew and excess kurtosis.

Negative skew and excess kurtosis are also evident from performance data for other hedge fund strategies. Importantly, returns for distressed debt, fixed income, and merger arbitrage are asymmetric and exhibit significant skew and excess kurtosis (*Exhibit 2*). For this reason, relying on standard deviation as a measure of risk results in undue confidence in the expected performance of these strategies. Furthermore, the distribution statistics imply that if one were to employ standard portfolio optimization techniques such as mean-variance analysis, there is a risk of over-allocation to these strategies if one wants to avoid downside surprise. Obviously, one needs to incorporate asymmetric return distributions when constructing hedge fund portfolios to minimize episodic performance deterioration.

This proposition does not necessarily extend to more general asset allocation prob-

EXHIBIT 3
Risk Measurement Definitions

Optimization approach	Abbreviation	γ_1	γ_2	γ_3	σ
Mean variance	MV	0	1	0	$[\Sigma_j(r_j-r)^2/m]^{1/2}$
Mean semivariance	MSV	0	0	1	$[\Sigma_j(\min\{0;r_j-r)\}^2/m]^{1/2}$
Mean downside risk	MDR	0	0	0	$[\Sigma_j(\min\{0;r_j-\nu)\}^2/m]^{1/2}$
Mean abs. deviation	MAD	1	1	0	$\Sigma_j\|(r_j-r)\|/m$
Mean abs. semideviation	MASD	1	0	1	$\Sigma_j(\min\{0;r_j-r)\}/m$
Mean abs. downside risk	MADR	1	0	0	$\Sigma_j(\min\{0;r_j-\nu)\}/m$

There are m portfolios with r_j representing the jth portfolio return and r the mean portfolio return.
Source: Duarte.

EXHIBIT 4
The Duarte Unifying Portfolio Optimization Approach

The mathematical problem is to maximize

$$(1-\lambda)\,(1^{(m)})^T\mathbf{r}/m - \lambda\gamma_1(1^{(m)})^T\boldsymbol{\sigma}/m - \lambda(1-\gamma_1)\,\boldsymbol{\sigma}^T\boldsymbol{\sigma}/m$$

subject to:

$$(1^{(m)})^T\,\mathbf{w} = c$$

$$\mathbf{Rw} = \mathbf{r}$$

$$\mathbf{u} - \mathbf{d} = r - [\gamma_2(1-\gamma_3) + \gamma_3]1^{(m)}(1^{(m)})^T\mathbf{r}/m - (1-\gamma_2)(1-\gamma_3)\mathbf{v}1^{(m)}$$

$$\boldsymbol{\sigma} = \gamma_2(1-\gamma_3)\mathbf{u} + \mathbf{d}$$

$$(1^{(m)})^T\mathbf{r}/m \geq \rho$$

$$\mathbf{w} \geq 0^{(n)},\ \boldsymbol{\sigma},\mathbf{u},\mathbf{d} \geq 0^{(m)}$$

where λ is risk aversion, $\gamma_1,\gamma_2,$ and γ_3 are binary variables that determine optimization methodology, $\mathbf{w}$ is the vector of weights, c is a wealth available, $\mathbf{R}$ is the matrix of asset returns for various scenarios, $\mathbf{r}$ is the vector of returns for the various scenarios, $\mathbf{v}$ is Fishburn's minimum acceptable return, ρ is a parameter to generate the efficient frontier, σ is the risk measure, $\mathbf{u} = \max\{\mathbf{r}_j - \mathbf{r}/m\}$ is upside deviation and $\mathbf{d} = -\min\{\mathbf{r}_j - r/m\}$ *is downside deviation.*

lems where the issue is selecting an overall allocation to hedge funds. For example, when choosing the appropriate mix of stock, bond, and hedge fund exposure in an investment portfolio, asymmetry is less a concern because aggregate return distributions for hedge fund portfolios do not always exhibit significant skew and kurtosis. The reason for this is simply that the aggregation of a dozen or more incongruent return distributions—only some of which are asymmetric—may produce a composite in which abnormalities offset. The question for hedge fund portfolio managers is how to consistently produce such portfolios.

A BRIEF REVIEW OF OPTIMIZATION METHODS

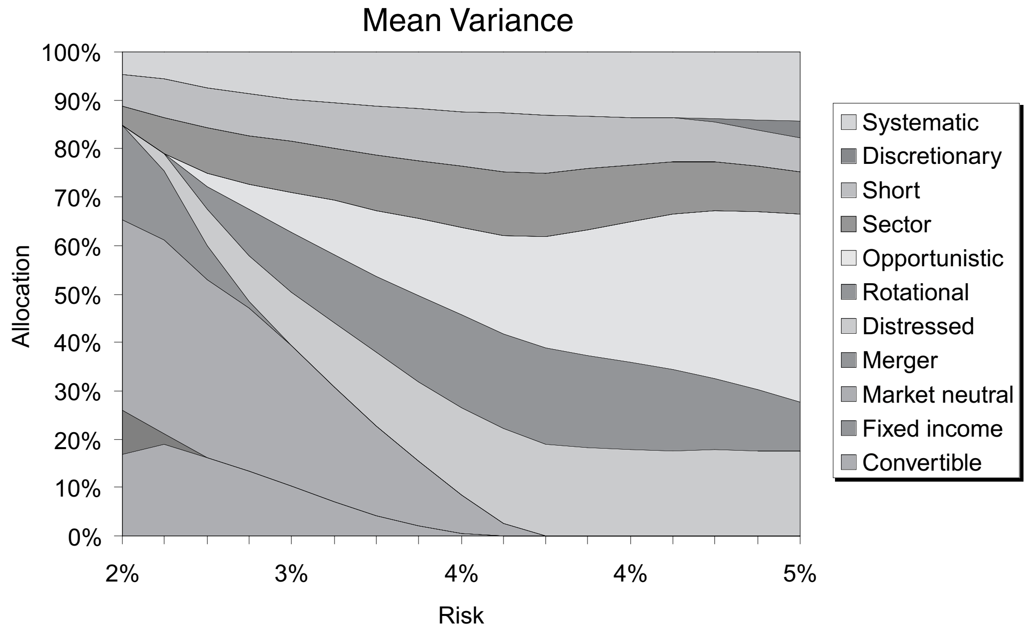
Having established the need for hedge fund portfolio optimization that allows for asymmetry, I turn to a consideration of relevant methodologies. In this regard, Duarte [1999] proposed an intriguing approach in which portfolio optimization is regarded as a general problem that includes standard optimization methods as special cases. Duarte's proposal has the particular advantage of allowing one to easily compare optimization results using different concepts of risk, including those that take into account asymmetry. It also elucidates portfolio optimization conceptually as a problem of simply choosing a risk metric.

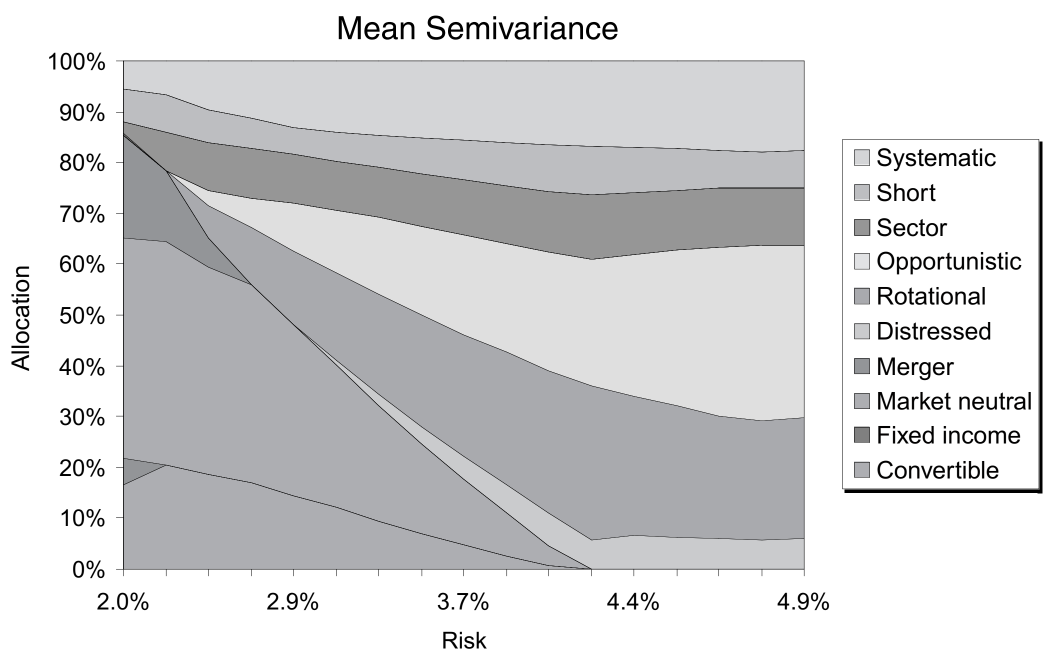
The six different risk measures in the Duarte formulation include mean variance (MV), mean semivariance (MSV), mean downside risk (MDR), mean absolute deviation (MAD), mean absolute semideviation (MASD), and mean absolute downside risk (MADR) (*Exhibits 3 and 4*). MV is the classic Markowitz [1959] approach. MSV is a downside risk metric that admits a lower half bell-shaped distribution. It was initially promoted by Marmer and Ng [1993] as suitable for portfolios with options.

The MDR method is similar to MSV, but downside deviations are calculated relative to a "minimum acceptable return." It was advocated by Fishburn [1977], Sortino and Van der Meer [1991], and Harlow [1993], as a way of producing optimal portfolios for investors where required certain returns were necessary to meet future liability requirements. The key feature common in MV, MSV, and MDR is that risk is defined as squared deviations. This means that large return divergences are penalized more severely.

For the second group of risk measures, consisting of MAD, MASD, and MADR, the major distinguishing fea-

EXHIBIT 5
Optimal Hedge Fund Portfolios

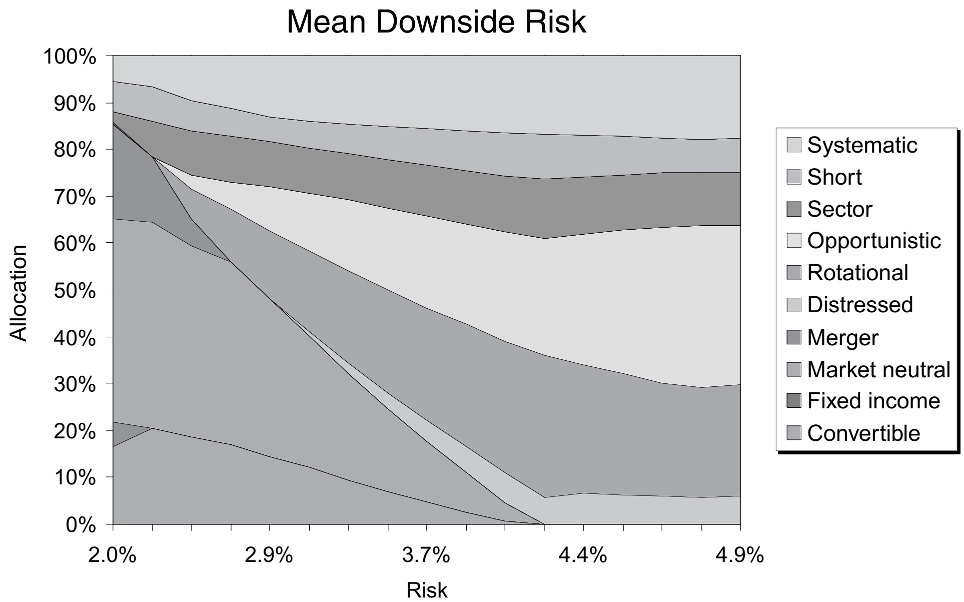


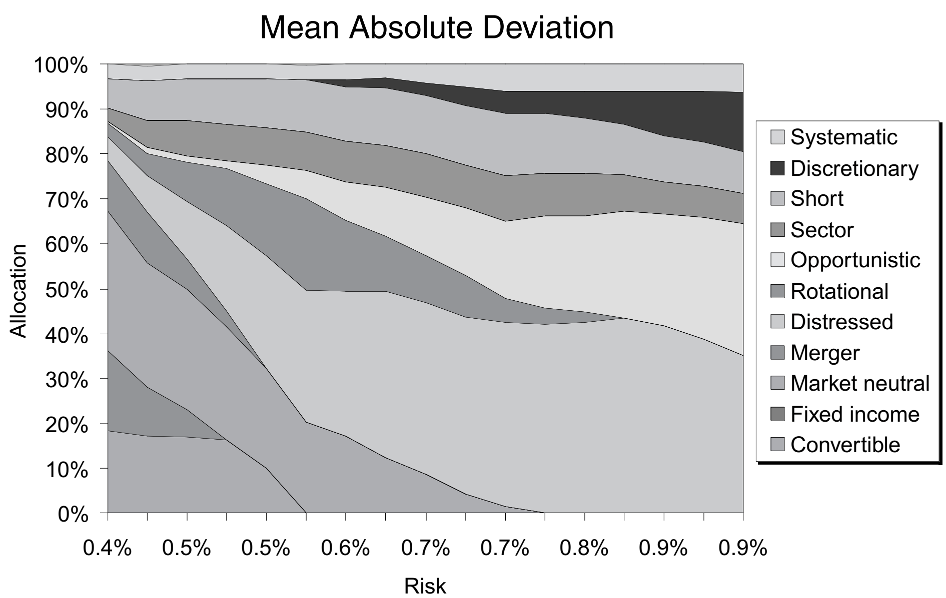


ture is that deviations are assigned no special penalty, but are weighted equally. Konno and Yamazaki [1991] advocated MAD because it is extremely fast computationally since quadratic programming is not required to obtain solutions. MASD and MADR are analogous to MSV and MDR, except that deviations are weighted equally. Duarte and Maia [1997] discuss these latter two concepts.

As for which are best suited for hedge fund portfolio optimization, clearly one must prefer those that allow for asymmetry. This would rule out MV since it presumes normal return distributions, leaving MSV and MASD as viable candidates. For these, both presume a smooth bell-shaped distribution below the mean. This could be problematic in the case of erratic distributions where there may be a clumping of returns in the lower tail, as illustrated in Exhibit 1.

E X H I B I T 5 (Continued)
Optimal Hedge Fund Portfolios





Furthermore, the MDR concept of risk employs a "minimum acceptable return." This is clearly subjective and what is a minimum acceptable return to a hedge fund portfolio manager may be very different from that of the investor. Also, the minimum acceptable return in a bull market may be significantly different from that during a bear market. The fact that this approach has not received widespread acceptance is evidence of this drawback.

As for the deviation risk measurement concepts embodied in MAD, MASD, and MADR, they too have critical limitations. Specifically, weighting very large negative downside surprises the same as small downside perturbations is at odds with the concept of convex utility. Indeed, the vast majority of investors would assign large penalties to outsized downside deviations, but probably be less averse to a small disappointment. In this regard, squaring deviations as in MV, MSV, and MDR is more intuitively appealing. Furthermore, the concept of squared deviation penalties has stood the test of time in other disciplines. For example, few question the use of "squared

EXHIBIT 6

Comparative Statistics for Various Allocation Methods

Metric	MV	MSV	MDR	MAD	CF
Strategy			--Average weight--		
Convertible arbitrage	4.5%	5.3%	7.3%	4.6%	8.4%
Fixed income	0.9%	0.7%	0.3%	2.0%	0.1%
Equity market neutral	11.5%	14.4%	17.3%	11.6%	18.5%
Merger arbitrage	2.4%	2.5%	2.3%	1.9%	2.8%
Distressed debt	30.0%	14.1%	3.5%	31.3%	0.0%
Rotational	5.6%	13.2%	19.0%	7.4%	19.0%
Opportunistic equity	18.4%	18.6%	18.4%	13.4%	17.0%
Domestic equity	0.0%	0.0%	0.0%	0.0%	0.1%
Equity sector	6.6%	10.5%	10.2%	8.1%	8.9%
Global equity	0.0%	0.0%	0.0%	0.0%	0.0%
Short sellers	9.4%	9.3%	7.3%	11.1%	4.2%
Discretionary macro	2.5%	0.4%	0.0%	4.0%	0.0%
Systematic macro	8.3%	11.2%	14.2%	4.6%	21.0%
Average portfolio					
Skew	-0.23	0.30	0.53	-0.75	0.58
Kurtosis	2.32	1.69	1.43	4.40	0.70
Deviation vs. MV	--	6.64	11.19	3.97	17.00
Sq. dev. vs. MV	--	0.78	2.13	0.18	3.86

EXHIBIT 7

Cornish-Fisher Optimal Allocations

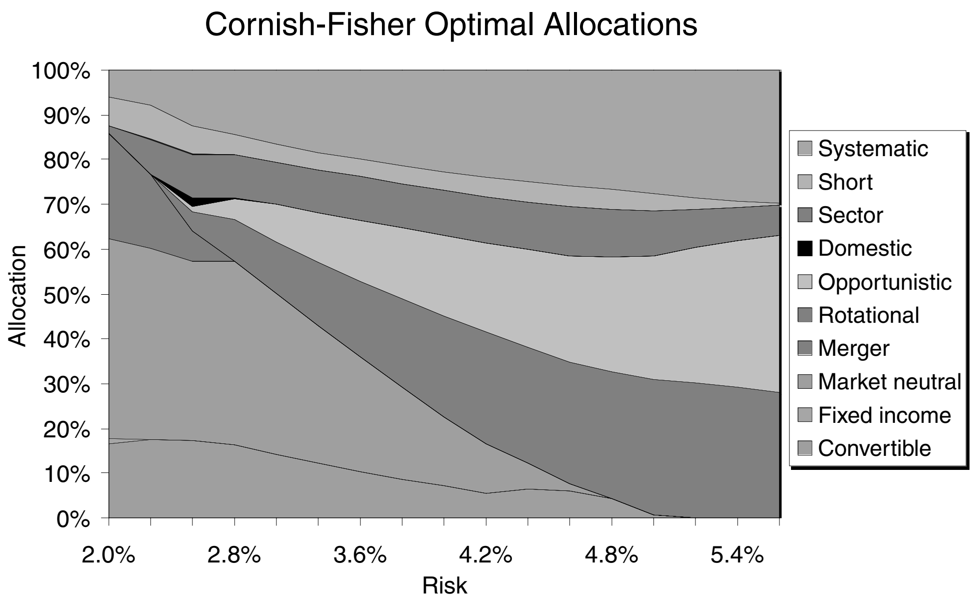


EXHIBIT 8

Summary Portfolio Statistics for MV Versus Cornish-Fisher Expansion

	Portfolio		Mean-variance				Cornish-Fisher expansion				
	Return	Std. dev.	Skew	Kur-tosis	Appar-ent CF VaR	Actual MV VaR	Std. dev.	Skew	Kur-tosis	Appar-ent MV VaR	Actual CF VaR
1	8.8%	1.9%	-0.69	1.17	-$0.5	-$0.9	2.0%	-0.11	-0.31	-$0.6	-$0.6
2	9.3%	2.0%	-0.55	0.84	-$0.6	-$0.9	2.1%	-0.03	-0.35	-$0.6	-$0.6
3	9.9%	2.2%	-0.51	1.37	-$0.7	-$1.0	2.5%	0.36	0.09	-$0.9	-$0.6
4	10.3%	2.4%	-0.47	1.78	-$0.8	-$1.2	2.8%	0.50	0.26	-$1.0	-$0.7
5	10.6%	2.6%	-0.44	2.24	-$0.9	-$1.4	3.1%	0.60	0.45	-$1.2	-$0.8
6	10.9%	2.8%	-0.41	2.48	-$1.0	-$1.6	3.3%	0.66	0.60	-$1.3	-$0.8
7	11.2%	3.0%	-0.34	2.49	-$1.1	-$1.8	3.6%	0.69	0.72	-$1.5	-$0.9
8	11.4%	3.2%	-0.27	2.44	-$1.2	-$1.9	3.8%	0.70	0.80	-$1.6	-$1.0
9	11.6%	3.4%	-0.20	2.36	-$1.3	-$2.0	4.0%	0.71	0.87	-$1.7	-$1.1
10	11.8%	3.6%	-0.14	2.52	-$1.4	-$2.2	4.2%	0.71	0.92	-$1.8	-$1.2
11	12.0%	3.8%	-0.11	2.68	-$1.6	-$2.3	4.4%	0.72	1.02	-$2.0	-$1.3
12	12.2%	4.0%	-0.03	2.70	-$1.7	-$2.4	4.6%	0.73	1.09	-$2.1	-$1.4
13	12.3%	4.2%	0.00	2.76	-$1.8	-$2.6	4.8%	0.73	1.12	-$2.2	-$1.5
14	12.5%	4.4%	0.03	2.83	-$1.9	-$2.7	5.0%	0.72	1.12	-$2.3	-$1.6
15	12.6%	4.6%	0.06	2.88	-$2.0	-$2.9	5.2%	0.72	1.13	-$2.4	-$1.7
16	12.8%	4.8%	0.09	2.92	-$2.2	-$3.0	5.4%	0.73	1.18	-$2.5	-$1.8
17	12.9%	5.0%	0.12	2.94	-$2.3	-$3.2	5.6%	0.73	1.23	-$2.7	-$1.9

Return and standard deviation are annualized. VaR is maximum monthly loss with 99% confidence.

errors" in regression analysis. Also, the need for deviation risk measures as a means of simplifying calculations has diminished with the advent of cheap and quick computing power.

While these conceptual issues are important, the primary question is the extent to which each methodology really makes a difference in practice. If they all produce similar results, then any discussion of the relative merits of alternative risk measures is moot.

COMPARATIVE OPTIMAL PORTFOLIOS

To determine the extent to which there are actual differences in allocations, I derive optimal hedge fund portfolios by strategy employing the MV, MSV, MDR, and MAD approaches. I solve for 17 portfolios that produce identical incremental returns but minimize risk for each metric. The portfolio returns vary from 8.8% to 12.9% annualized over the sample. In the case of MDR, I define cash as the minimum acceptable return. The analysis is restricted to a composite history for each strategy constructed from data reported by EAI, HFR, and CSFB.[4]

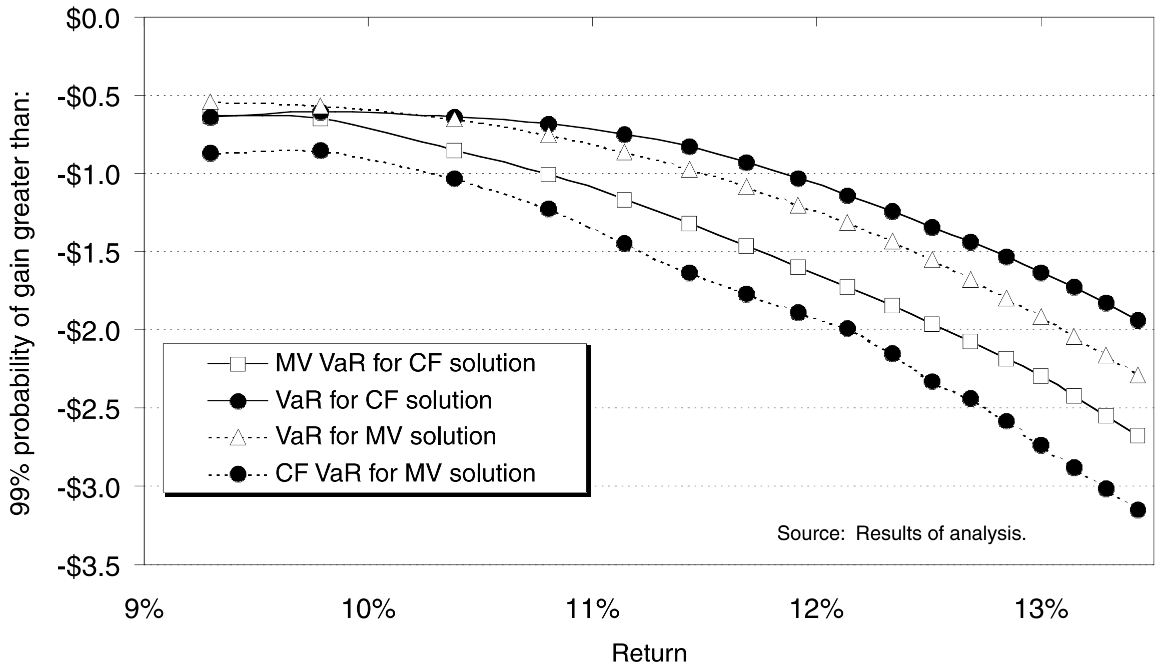
The results of the analysis are displayed in Exhibit 5. The efficient portfolios show a fairly wide array of optima, as might be expected. That said, the allocation patterns are remarkably similar. For example, in lower-return portfolios, all approaches allocate heavily to fixed income arbitrage, convertible arbitrage, and equity sector strategies. For higher-return portfolios, all techniques allocate heavily to distressed debt, opportunistic equity, and systematic macro strategies.

The most significant differences in the portfolios are illustrated in Exhibit 6, which shows average allocations for the 17 portfolios generated by each technique. The MV and MAD allocations are the most similar, with average strategy allocations varying at most only a few percent. The MSV and MDR average allocations are also very close to each other. Since these latter two approaches consider only downside risk, asymmetry considerations appear to be the key distinguishing determinant of differences.

For example, MV and MAD allocate more heavily

EXHIBIT 9

VaR for Mean-Variance Versus Cornish-Fisher Optimal Portfolios



to distressed debt—a negatively skewed strategy with significant excess kurtosis. In contrast, MSV and MDR allocate much less to distressed debt and favor rotational and systematic macro strategies, which have positive skew and low kurtosis. As a consequence, the resulting MV and MAD portfolios exhibit negative skew and excess kurtosis, while the MSV and MDR portfolios exhibit positive skew and much lower kurtosis. The application of MSV and MDR therefore serves to transform portfolios from negative skew and high kurtosis to positive skew and significantly lower kurtosis.

This is more clearly illustrated by examining which methodology produces portfolios most variant from MV. To do this, I simply calculate the average and squared deviation of portfolio weights from those obtained via the MV approach. That is, I take the difference between the allocations produced by MV versus the other methods for every strategy in each of the 17 portfolios and then sum the results. A value of zero would therefore indicate that any method's allocations are identical with MV. The results in Exhibit 6 confirm that the MAD allocation is closest to MV, followed by MSV. MDR is the most divergent.

DELTA-GAMMA APPROXIMATIONS

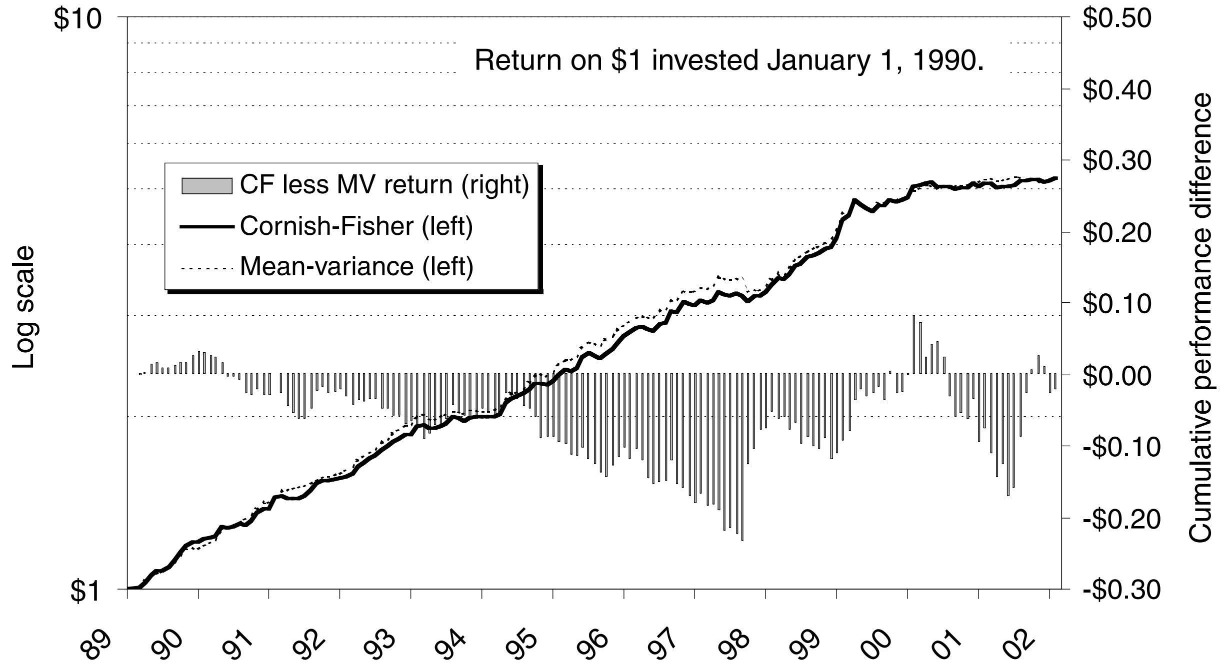
Where does this leave one? All the proposed methods for portfolio optimization possess conceptual limitations. That said, the MSV and MDR approaches definitely appear to improve portfolio characteristics by reducing the negative skew and excess kurtosis evidenced in the MV and MAD portfolios. This is accomplished with no loss of return whatsoever, since the optimization process produces identical returns for each portfolio.

The one option not considered by Duarte is to embed skew and kurtosis directly in the optimization process. While this could be done mathematically by expanding Duarte's optimization framework, numerical approaches such as value at risk (VaR) offer immediate gratification and avoid complexity.

As Jorion [2001] notes, a virtue of VaR is that it accounts for nonlinearities in distributions. The problem with VaR is that it is an empirical "black box" which lacks analytical tractability. There is no straightforward and elegant mathematical solution to the portfolio optimization problem. However, Favre and Galeano [2002] and Signer and Favre [2002] propose the use of delta-gamma approximations to improve the efficacy of VaR via the Cornish-Fisher (CF) expansion.[5] CF is simple, creative, and vastly reduces the complexity of applying

E X H I B I T 10

Cornish-Fisher Versus Mean-Variance Portfolio Return Paths



VaR. It is no more time intensive than MV and the other analytical methods included in the Duarte framework.

The CF approach involves solving:

$$min\ VAR(w) = V(r - z\sigma) \quad (1)$$

subject to the normal portfolio constraints such as requiring that the weights sum to unity and are positive, and requiring that portfolio returns equal the asset return vector multiplied by portfolio weights.[6] The CF expansion replaces the normal z value with an alternative defined as:

$$z' = z + (z^2 - 1)\sigma/6 + (z^3 - 3z)K/24 - (2z^3 - 5z)\sigma^2/36 \quad (2)$$

where s is skew and K represents kurtosis. Thus, asymmetry is considered explicitly in the optimization process.

Some analysts have urged caution in applying the CF expansion, arguing that there are situations when it does not provide a good approximation. For example, Mina and Ulmer [1999] demonstrate this is the case when a portfolio has a return distribution consistent with a pure short call or put position. However, in general this is a concern only for the upper tail of the distribution, not the lower, which is the focus of VaR. Therefore, CF appears to be highly applicable for the purpose at hand.

EMPIRICAL ANALYSIS

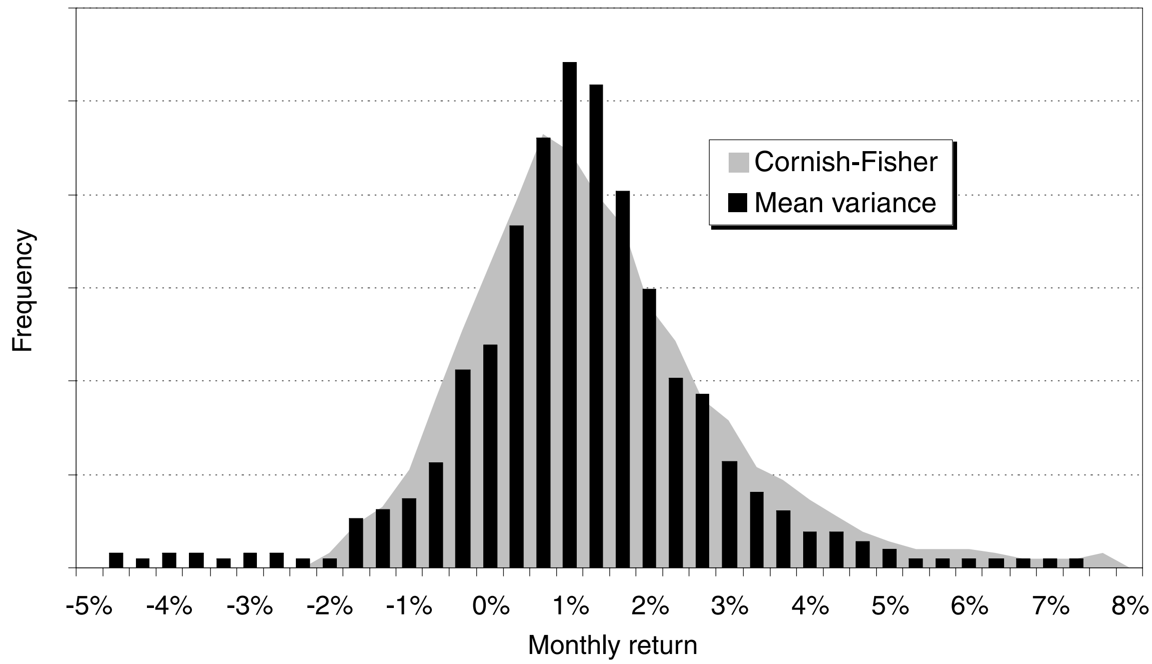
To evaluate the extent to which the CF expansion can improve upon the classical portfolio optimization techniques already reviewed, I replicate the prior analysis applying the CF expansion to hedge fund strategies. Exhibit 7 shows the resulting allocations.

As one might expect, CF optimization produces significantly different allocations from standard MV analysis. In addition, the CV allocations are distinctive from the other approaches. Indeed, the CF approach forces the distressed debt strategy allocation entirely to zero for all portfolios. This is due to its negative skew, high kurtosis, and embedded co-moment properties. In place of distressed debt, CF prefers systematic macro, rotational, and market neutral strategies. While the MSV and MDR approaches also generally favor this substitution pattern, the CF technique pushes much further in eliminating negative skew and driving down kurtosis (*Exhibits 6, 8, and 9*).

The average skew for portfolios produced via the CF expansion is 0.58. This contrasts with a –0.23 skew for the MV portfolios, 0.30 for the MSV portfolios, 0.53 for the MDR portfolio, and –0.75 for the MAD portfolios. The average weighted kurtosis for the CF portfolios is only 0.73. This is significantly lower than the 2.32, 1.69, 1.43, and 4.40 reported for the MV, MSV, MDR,

EXHIBIT 11

Return Distributions for Optimal Hedge Fund Portfolios



and MAD portfolios, respectively. In this regard, the CF expansion emerges as an optimization approach that most explicitly minimizes skew and kurtosis.

The consequences in general are smoother return streams with less of the periodic downside shocks delivered by other techniques. This is shown in Exhibit 10, which compares CF portfolio performance versus that of the optimum MV portfolio for the most aggressive hedge fund strategy portfolios.[7] If one uses the CF definition of risk, the significant downside performance experienced by some hedge fund strategies in both 1998 and in 2002 is avoided. In contrast, MV produces higher returns in prior years but then delivers larger downside shocks.[8]

This is also evident in the return frequency distribution where the skew and kurtosis-reducing attributes of the CF expansion are most evident. Exhibit 11 demonstrates this by comparing the CF return distributions for all optimum portfolios with those for MV. Clearly, CF eliminates the large downside portfolio returns resulting under MV and picks up performance on the upside.

Critically, these findings flow directly from risk definition. That is, if one defines risk as standard deviation in an MV context, then CF appears more risky—"actual" VaR off of the MV solution is less than the "apparent" VaR that MV would assign to the CF solution. Similarly, if one defines risk in a CF context, where skew and kurtosis are considered, then the VaR of the CF solutions is significantly lower than the VaR for the MV solution. Risk definition is therefore a key element in constructing efficient portfolios.

FURTHER CONSIDERATIONS AND CAVEATS

The analytical results presented here are more than pedantic. They provide a reference allocation for the blend of hedge fund strategies that performed best over the last dozen years. In this regard, they serve as a rudimentary benchmark in the same way that retrospective optimization is often naively used to establish stock and bond portfolio benchmark allocations. While I certainly do not advocate using the hedge fund strategy allocations presented here as a basis for future allocation, they nonetheless present an interesting point of departure.

Beyond historical insight, the results do in fact demonstrate some of the attractive properties of hedge fund portfolios in general. For example, in all cases the analysis shows that a multi-strategy hedge fund portfolio is desirable—no single strategy emerges as dominant and allocations to multiple strategies are always deemed appropriate. That said, it is clear that systematic macro is a particularly robust strategy, receiving significant allocations no matter what the optimization method or the amount of risk desired. In addition, allocations to opportunistic equity, rotational, equity sector, and equity market neutral are also fairly consistent, although they vary more depending on portfolio risk level.

The analysis also reveals that more aggressive hedge fund portfolios should contain larger allocations to long-biased equity and macro strategies, while lower risk portfolios should contain higher allocations to market neutral and merger arbitrage. Importantly, these conclusions are derived over a period of both a strong bull and bear market for equities. This provides a stronger testament than would otherwise be the case.

Although outside the scope of this study, of considerable interest would be the extent to which optimal hedge fund portfolios vary over time. For example, do some strategies show a pattern of secularly declining returns and higher risk, such that larger allocations were appropriate early in the sample but not later? Also, of interest would be a more systematic examination of the possible effects of measurement error in strategy definitions and the extent to which leverage has played a role in returns. Furthermore, survivor bias is largely neglected in the analysis. This may have profound effects on return distributions as well as risk, and these effects may vary by strategy.

As for actual application of the techniques explored here, they offer practitioners viable alternatives in a forward-looking context. All that is required is the usual return and covariance forecasts. The one exception is the CF approach, where explicit skew and kurtosis forecasts are necessary. This remains unexplored territory and one may be left with the simple option of extrapolating past skew and kurtosis into the future. Perhaps an error-correction approach similar to GARCH may be suitable for skew and kurtosis forecasting.

CONCLUSION

Hedge fund investors are increasingly sophisticated and are demanding the same rigor in portfolio construction that is utilized in constructing efficient portfolios of other assets. The problem is that negative skew and excess kurtosis for many hedge fund strategies make this a challenging endeavor. Successful hedge fund portfolio managers in the future are likely to be those that confront these issues creatively.

The analysis presented here demonstrates that optimization technique does matter for hedge fund strategy allocation. This is particularly the case if downside risk is to be avoided. In this respect, one can construct more efficient portfolios using the CF expansion, although other methods such as MSV (mean semivariance) and MDR (mean downside risk) offer partial solutions to the asymmetry problem.

The analytical results provide a useful reference. In particular, the analysis reveals that significant allocations to market neutral equity, convertible arbitrage, and merger arbitrage are appropriate for the low-risk hedge fund portfolios. In addition, significant allocations to rotational managers, opportunistic equity, and systematic global macro are desirable for higher risk hedge fund portfolios. The one hedge fund strategy that stands out as having a potentially paradoxical influence on portfolio returns is distressed debt. Downside-risk sensitive optimization approaches correct for the propensity of this strategy to produce portfolios with negative skew and a fat lower tail.

ENDNOTES

[1]The strategies that do not appear extremely skewed with high kurtosis include equity market neutral, macro, and long-short equity. See Brooks and Kat [2002] for more strategy by strategy details.

[2]Of course, there were other issues—such as excess leverage and a dominant position in illiquid markets. Nonetheless, the unprecedented blowout in credit spreads is often cited as a major causal factor.

[3]These indexes are described in detail on the websites: EACM.com, Hedgefundresearch.com, and Hedgeindex.com, respectively. Crerend [1998] also discusses the key aspects of the EAI index.

[4]The strategies selected are subjective and a function of available data from each vendor. I define "convertible arbitrage," "fixed income arbitrage," "merger arbitrage," and "distressed debt" as a simple average of returns reported by each vendor for these strategies. "Equity market neutral" is a simple average of returns reported by EAI and CSFB for this strategy. "Rotational" is a simple average of returns reported by EAI for event driven rotational, EAI relative value rotational, and CSFB event driven multistrategy. "Opportunistic equity" is the EAI series reported for this category. "Domestic equity" is a simple average of the same category reported by EAI, "equity hedge" as reported by HFR, and equity "long/short" reported by CSFB. "Sector" equity is the HFR series of the same name. "Global equity" is a simple average of the EAI series of the same name and the HFR and CSFB emerging market series. "Short sellers" are a simple average of all three vendors' series of the same name. "Discretionary macro" is the EAI series of the same name. "Systematic macro" is an average of the HFR and CSFB macro series.

[5]The original CF exposition dates back more than a half century. See Cornish and Fisher [1937].

[6]Note that V is the value of the portfolio, r is return, z is the normal confidence measure, and σ is the standard deviation of the portfolio return.

[7]I select the more risky portfolios to illustrate more clearly divergences in the stream of returns.

[8]On both these occasions, credit spread "blowouts" pummeled distressed debt returns, in particular. In 1998, it was lower-grade emerging market credit spreads in general that widened following the Russian debt default. In 2002, the WorldCom default pushed out credit spreads for lower-grade corporate debt.

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