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Collinearity and the Use of Latent Root Regression for Combining GNP Forecasts

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ABSTRACT

In combining economic forecasts a problem often faced is that the individual forecasts display some degree of dependence. We discuss latent root regression for combining collinear GNP forecasts. Our results indicate that latent root regression produces more efficient combining weight estimates (regression parameter estimates) than ordinary least squares estimation (OLS), although out-of-sample forecasting performance is comparable to OLS.

KEY WORDS Combining forecasts Multicollinearity Latent root regression GNP forecasts

Researchers appear to have reached agreement regarding the value of combining forecasts. Performance, measured in terms of a variety of error summary statistics, can be improved by combining multiple forecasts (e.g. Granger and Newbold, 1977; Winkler and Makridakis, 1983; Armstrong, 1985). An important unanswered question, however, regards what combination procedure to use. Procedures suggested by Bates and Granger (1969), with subsequent extensions and applications by Newbold and Granger (1974) and Winkler (1981) among others, model the forecast errors with a multinormal process, the parameters of which determine the combining weights. A number of alternative combining procedures have also been proposed, including simple averages (Makridakis and Winkler, 1983), unrestricted regressions (Granger and Ramanathan, 1984), weighting procedures based on assessments of which forecast might perform best (Bunn, 1975; Gupta and Wilton, 1987) and various *ad hoc* procedures (Ashton and Ashton, 1985).

In developing composite models using the multinormal model or related regression approaches one major problem is that the covariance matrix must typically be estimated with relatively small quantities of data. This results in unstable estimation of the covariance matrix and even more unstable estimation of the combining weights (Kang, 1986). Furthermore, for economic forecasting the problem is exacerbated by the fact that different forecasters have

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errors that are typically highly correlated; correlations above 0.8 are not at all unusual (Clemen and Winkler, 1986; Figlewski and Urich, 1983).

We explore the possibility of using latent root regression (Webster *et al.*, 1974; Gunst *et al.*, 1976) as a procedure for combining dependent forecasts. This approach provides an explicit framework for analysis of collinear data through the mathematics of latent roots and vectors. The data we analyze (GNP forecasts studied in Clemen and Winkler, 1986) display pairwise correlations of forecast errors between 0.82 and 0.96. Given these relatively high correlations as well as Kang's demonstration of the instability of the estimated weights in this data set, it seems reasonable to think that latent root regression (LRR) might improve on the performance of OLS.

In the next section we explain the regression approach to combining forecasts, followed by a brief discussion of LRR in the context of the forecast combination model. The third section describes our data set and presents empirical results. Collinearity diagnostics for the data are also discussed. The fourth section summarizes and concludes.

A REGRESSION APPROACH TO COMBINING FORECASTS

We assume that at time $t-1$ we have access to k forecasts, $f_t = (f_{1t}, \ldots, f_{kt})$, for θ_t. We can write θ_t stochastically in terms of the (possibly biased) forecasts f_{it}:

$$\theta_t = a_i + b_i f_{it} + u_{it} \tag{1}$$

where each $u_t = (u_{1t}, \ldots, u_{kt})'$ is an independent realization from a normal process with mean vector $(0, \ldots, 0)'$ and covariance matrix Σ. At time $t-1$, we have available past observations (forecasts and actual values) for time $t = 1, \ldots, t-1$. To represent these data we will adopt the following notation:

$$[\theta, \mathrm{F}] = \begin{pmatrix} \theta_1 & 1 & f_{1,1} & \ldots & f_{k,1} \\ \cdot & \cdot & \cdot & & \cdot \\ \theta_{t-1} & 1 & f_{1,t-1} & \ldots & f_{k,t-1} \end{pmatrix} \tag{2}$$

We include the vector of ones because, in general, we will be estimating regression coefficients including a constant term.

Multiply each of the different equations (1) by a factor γ_i, such that $\Sigma\gamma_i = 1$. Then combine equations (1) to obtain the following regression representation:

$$\begin{aligned} \theta_t &= \Sigma\gamma_i a_i + \Sigma\gamma_i b_i f_{it} + \Sigma\gamma_i u_{it} \\ &= \beta_0 + \beta_1 f_{1t} + \cdots + \beta_k f_{kt} + \varepsilon_t \\ &= f_t^* \beta + \varepsilon_t \end{aligned} \tag{3}$$

where

$$\beta = (\beta_0, \ldots, \beta_k)' = (\Sigma\gamma_i a_i, \gamma_1 b_1, \ldots, \gamma_k b_k)'$$
$$f_t^* = (1, f_{1t}, \ldots, f_{kt})$$

and

$$\varepsilon_t = \Sigma\gamma_i u_{it}$$

The distributional assumptions regarding u_t imply that the regression equation error terms ε_t obey standard OLS assumptions. Therefore, the OLS estimator of β is given by the familiar expression

$$\beta^* = (F'F)^{-1}F'\theta \tag{4}$$

As usual, β^* is the best linear unbiased estimator of β, and, assuming stationarity of the process through time, the forecast $\theta_t^* = f_t^*\beta^*$ is the best linear unbiased predictor of θ_t.

In the event of multicollinearity in the F matrix, β^* (and hence θ_t^*) can be inefficient. If the process is stationary, one solution to the problem of multicollinear regressors is simply to acquire more data to improve the efficiency of the estimation, thereby improving prediction performance. However, this is often not possible, especially when working with economic data. Thus, there is some motivation to consider biased estimation and prediction if the biased approach might yield a substantial improvement in terms of estimation efficiency. LRR is one such technique. The following is a brief description of the procedure, abstracted from Webster *et al.* (1974) and Gunst *et al.* (1976). We direct the interested reader to those papers for more details.

LRR seeks to identify near-singularities in the explanatory variables and to determine their predictive value. The procedure uses this information to estimate the regression parameters β by adjusting for non-predictive near-singularities. Define the matrix A to be an $n \times (k+1)$ data matrix containing standardized dependent and independent variables. The correlation matrix $(A'A)$ has latent roots λ_i and corresponding latent vectors α_i defined by

$$|A'A - \lambda_i I| = 0$$

and

$$(A'A - \lambda_i I)\alpha_i = 0.$$

Denote the elements of α_i by

$$\alpha_i' = (\alpha_{0i}, \alpha_{1i}, \ldots \alpha_{ki})$$

and

$$\alpha_i^{0\prime} = (\alpha_{1i}, \ldots \alpha_{ki})$$

That is, α_i^0 contains all of the elements of α_i except the first one. Also, define

$$\eta^2 = \Sigma(\theta_i - \theta)^2$$

The OLS estimator β^* can be written as

$$\beta^* = -\eta\Sigma c_i\alpha_i^0$$

where

$$c_i = \alpha_{0i}\lambda_i^{-1}(\Sigma\alpha_{0j}^2/\lambda_j)^{-1}$$

Values of λ_i and α_{0i} close to zero indicate a non-predictive near-singularity. As α_{0i} becomes close to zero, c_i should also be close to zero. However, since λ_i is also small, c_i may be quite large, and may have a dominant effect in the estimate β^*. Gunst *et al.* (1976) suggest setting $c_i = 0$ for $|\lambda_i| \leqslant 0.3$ and $|\alpha_{0i}| \leqslant 0.1$, thus obtaining the LRR estimate of the parameter β. Webster *et al.* (1974) and Gunst *et al.* (1976) provide detailed geometrical interpretations and discussion of this technique.

COMBINING GNP FORECASTS

Wharton Econometrics (Wharton), Chase Econometrics (Chase), Data Resources, Inc. (DRI) and the Bureau of Economic Analysis (BEA) make quarterly forecasts of many economic variables. We used their level forecasts of nominal GNP (1970–83) (obtained directly from Wharton and BEA and from the *Statistical Bulletin* published by the Conference Board for Chase and DRI) to construct growth rate forecasts (in percentage terms), and we calculated the deviations from actual growth as determined from GNP reported in *Business Conditions Digest*. Forecasts with four different horizons (one, two, three and four quarters) were analyzed. For example, the four-quarter GNP forecast predicts the percentage change for the three-month period four quarters in the future (counting the current one). We label the data by year and quarter; e.g. 75.3 refers to the third quarter of 1975. Finally, the data are divided into two periods, one for estimation and one for forecast evaluation. The estimation period runs through 1979 for each horizon, with the remaining data kept in reserve as an independent sample for forecast evaluation. Estimation periods and sample sizes are shown in Table I. For analysis of the individual forecasts, the reader is referred to Clemen and Winkler (1986) and Clemen (1986).

Our desire to try LRR as a combining technique was originally motivated by the high pairwise correlations among the individual forecasts and the instability of the estimated weights (Kang, 1986). However, while these observations suggest multicollinearity, we have no clear indication of the severity of the problem. Belsley *et al.* (1980) and Belsley (1982, 1984) have discussed diagnostics for explicit measurement of the severity of multicollinearity. We calculated variance inflation factors, condition indexes and the variance–decomposition proportions for each of the four forecast horizons. These diagnostics are reported in Table II. For condition numbers (defined as the largest of the condition indexes), the value 30 is suggested as a screen; situations with larger values are then examined more closely. All our condition numbers are between 20 and 30; thus, on the basis of this diagnostic alone our data do not appear to display severe multicollinearity. For variance inflation factors (VIFs), Montgomery and Peck (1982) suggest that values from 5 to 10 indicate severe multicollinearity. Our VIFs range up to 4.6. Variance–decomposition proportions can also be used to detect multicollinearity, which is indicated by two numbers exceeding 0.5 in any one row of the variance–decomposition table. For our forecasts, the variance–decomposition calculations reveal collinearity between (1) the DRI and BEA forecasts in the one- and two-quarter horizons, (2) the Wharton and BEA forecasts in the three-quarter one and (3) the Chase and DRI as well as the constant and BEA variables in the four-quarter horizon.

To some extent, the use of these diagnostics is problematic. For instance, condition indexes are based on eigenvalues of the sample covariance matrix, and it is unclear to what extent models built and estimated on the basis of this diagnostic might be sensitive for relatively small

Table I. Estimation periods and sample sizes for each of the four forecast horizons

Horizon	Estimation period	Sample size
1	70.4–79.4	36
2	71.1–79.4	36
3	71.2–79.4	35
4	71.3–79.4	33

sample sizes. Our own practical experience has further suggested that, while the presence of a condition index greater than 30 may be a reliable indicator of collinearity, values slightly less than 30 do not necessarily mean that effects due to collinearity will be unnoticeable. With regard to the variance–decomposition proportions, our results indicate that the one-quarter DRI and BEA forecasts appear to be associated with an ill-conditioned covariance matrix. However, the correlation coefficient between the one-quarter DRI and BEA (0.82, reported in Clemen and Winkler, 1986) is the least of the pairwise correlations for this horizon. Likewise, the correlation between Wharton and BEA errors in the two-quarter analysis (0.94) is the second-lowest of the reported pairwise correlations. Given these observations, it seems reasonable to conclude that multicollinearity, perhaps at a relatively low level, is present in our data.

Application of LRR, using the Gunst *et al.* (1976) criteria for vector deletion, produced the results shown in Table III. Details regarding the latent roots and vectors and the vector deletion patterns for each analysis are available from the authors. For comparison, OLS results are also included in Table III. Generally speaking, LRR and OLS produced coefficient estimates that are comparable in terms of signs and relative sizes. (While this comparison is a matter of degree, two exceptions are BEA in the one- and four-quarter horizons.) On the other hand, LRR generally yielded more efficient estimates of the parameters than OLS, as measured by the *t*-statistics. The coefficient estimates for the Chase and DRI forecasts are highly significant in the one-quarter horizon. In the two-quarter horizon, coefficient estimates for DRI and BEA are significant, as is the DRI coefficient estimate in the three-quarter horizon.

Table II. Multicollinearity diagnostics for GNP forecasts

Horizon	Condition indexes	Variance–decomposition proportions				
		Constant	Wharton	Chase	DRI	BEA
1	9.78	0.68	0.00	0.03	0.02	0.07
	15.80	0.04	0.01	0.00	0.56	0.63
	17.65	0.01	0.03	0.73	0.30	0.30
	20.93	0.27	0.96	0.24	0.11	0.00
	VIF		3.38	3.86	3.25	3.24
2	11.06	0.55	0.25	0.14	0.00	0.01
	12.58	0.17	0.60	0.13	0.01	0.13
	14.06	0.16	0.11	0.62	0.01	0.23
	27.41	0.11	0.04	0.11	0.98	0.63
	VIF		1.85	2.25	4.60	3.23
3	10.94	0.71	0.00	0.26	0.01	0.00
	13.69	0.23	0.42	0.44	0.01	0.01
	18.98	0.06	0.50	0.27	0.09	0.53
	22.24	0.00	0.08	0.03	0.88	0.46
	VIF		2.54	2.40	3.93	3.27
4	7.36	0.05	0.84	0.01	0.00	0.00
	11.14	0.29	0.06	0.29	0.09	0.01
	16.39	0.01	0.03	0.62	0.84	0.00
	22.86	0.65	0.06	0.07	0.07	0.98
	VIF		1.52	2.31	2.54	2.22

Table III. LRR and OLS regression results

Horizon		Constant	Wharton	Chase	DRI	BEA	R^2
1	LRR	1.30	−0.23	0.96	0.37	−0.11	0.40
			(−0.58)	(2.83)[a]	(4.93)[a]	(−1.78)	
	OLS	2.18	−0.53	0.65	0.33	0.48	0.46
			(−1.28)	(1.66)	(0.92)	(1.43)	
2	LRR	1.71	0.08	−0.25	0.41	0.63	0.24
			(0.24)	(−0.69)	(2.52)[a]	(2.31)[a]	
	OLS	1.48	0.06	−0.28	0.59	0.52	0.24
			(0.20)	(−0.76)	(0.87)	(1.10)	
3	LRR	4.17	0.16	−0.62	0.32	0.76	0.18
			(0.39)	(−1.60)	(2.56)[a]	(1.56)	
	OLS	4.17	0.21	−0.59	0.20	0.82	0.18
			(0.46)	(−1.53)	(0.34)	(1.47)	
4	LRR	8.69	−0.09	−0.60	0.96	−0.08	0.12
			(−0.40)	(−1.69)	(2.03)	(−0.36)	
	OLS	10.92	−0.06	−0.47	1.10	−0.63	0.17
			(−0.28)	(−1.10)	(2.39)[a]	(−0.96)	

Values in parentheses are t-statistics.
[a]Significance at the 0.05 level.

Table IV. Performance of combining methods for the post-estimation evaluation period shown. Performance is mean absolute relative error, where absolute relative error is defined as | (actual−forecast)/actual |

Horizon	Evaluation period	Equal weights	OLS	LRR
1	80.1–82.2	2.47	2.89	2.76
2	80.1–82.3	3.60	4.19	4.40
3	80.1–82.4	4.35	4.58	4.49
4	80.1–83.1	4.45	3.67	3.71

The true test of a forecasting procedure is how well it performs outside of the fitting data. Table IV presents the results obtained by using the estimated models to predict actual nominal GNP for the evaluation periods shown. We also include the arithmetic average (equal weights) as one of the combining procedures for use as a benchmark. The performance measure we used, mean absolute relative error, is mean absolute percentage error (MAPE) divided by 100. MAPE is a widely used forecast performance measure that allows performance comparisons among different forecast situations (see Armstrong, 1985). The results in Table IV show that OLS and LRR performed comparably. Given the similar estimates of the combining weights in the two analyses, this result is not surprising. The equal weights combination outperformed the regression model in all but the four-quarter horizon.

DISCUSSION AND SUMMARY

Our empirical results show that LRR produced more efficient parameter estimates than OLS. However, the similar out-of-sample performance of the two methods leads us to be somewhat ambivalent. In theory, LRR's more efficient estimation of parameters should result in more efficient predictors and hence better out-of-sample prediction performance. If the multicollinearity were more severe, the difference between OLS and LRR estimation efficiency would most likely be greater, and LRR's out-of-sample performance should exceed that of OLS.

In light of the data's high correlations, Kang's results and Clemen's and Winkler's (1986) results from combining these GNP forecasts using a Bayesian model, we find the comparable performance of LRR and OLS somewhat troubling. Compared to OLS, Clemen's and Winkler's Bayesian model resulted in forecasting performance improvements of about 16% in terms of mean squared error. One possible interpretation might be that Clemen's and Winkler's model, being mathematically similar to ridge regression (Lindley and Smith, 1972; Hocking, 1976), tended to counteract the dependence among the forecasts. Of course, other techniques are available for use with collinear data, notably principal components regression (Gunst, 1976) and LRR. Our motivation for trying LRR was that it differs fundamentally from ridge regression (and the related Clemen/Winkler model) in the way collinearity is handled. Where ridge regression depends on the estimation of a biasing parameter, principal components regression and LRR are estimated by the elimination of non-predictive near singularities as described above. However, our GNP forecasts appear to fall into a middle ground; they are collinear enough to cause some difficulty in the OLS analysis, but the collinearity is not severe enough to justify the use of LRR. Hopefully, our experience with LRR will provide some guidance for researchers in their studies of combinations of dependent forecasts.

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