Assessing Risk Tolerance

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A typical approach used to assess the risk tolerance parameter R for an exponential utility function $U(y) = 1-\exp(-y/R)$ is to ask the decision maker (DM) the following: "Suppose you face a gamble where you can win y with probability 0.5 or lose $y/2$ with probability 0.5. What is the largest y for which you would be just willing to take this gamble? I'm looking for the largest y that makes you just indifferent between taking the gamble or not." (See Figure 1.) It's easy enough to calculate numerically that the y that makes the DM indifferent is $y_I \approx 0.96R$. Given the general lack of precision in subjective assessments, we usually use y_I as a reasonable estimate for R.

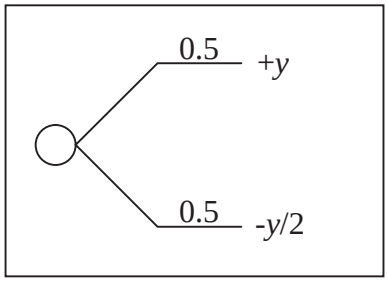

Figure 1. The conventional approach to assessing risk tolerance for an exponential utility function. Find the value of y that makes you just indifferent to taking the gamble or not.

Anyone who has taught this in class and asked students to assess R in this way knows that it is important to frame the question in a way that helps the individual think in terms of investments or lawsuits, not carnival-like lottery games. But there are other issues that we could ask about this assessment. In particular, maybe it would be easier to ask the individual to give the y^* that they would prefer. Again, it's not difficult to show (analytically this time), that this optimal y^* is given by $y^* = -(2/3)\ln(1/2)R \approx 0.46R$. Roughly, take the assessed y^* and multiply times two to get the risk tolerance. From another perspective, assessing the optimal y^* might be difficult because the expected utility function is relatively flat in the neighborhood of the optimum (von Winterfeldt & Edwards 1986).

There are also behavioral questions that we could ask. One question related to the two assessments above is whether individuals are really able to make these assessments. In particular, when we ask for the indifference y_I, perhaps the DM gives us y^* instead. Or perhaps the DM takes y^* as an anchor and adjusts toward y_I. If so, is the adjustment adequate?

Another particularly intriguing question arises from Bleichrodt, Pinto, & Wakker (2001) (hereafter BPW), the paper that won the DAS publication award last fall. In that paper, BPW suggest that when individuals respond to typical reference-lottery questions in a utility assessment, their responses are distorted in ways that are predictable according to Prospect Theory. In particular, the responses may display loss aversion if one of the reference prizes is negative and distortion of probabilities according to the probability weighting function. (BPW appeal to cumulative Prospect Theory in this respect.)

BPW do not apply their analysis to the conventional method for assessing risk tolerance, but it is possible to do this. The first result is that, if we stick with the 50-50 gamble in Figure 1, there is no positive y that will make the DM indifferent. This is because the loss-aversion coefficient that BPW use (based on data from Tverksy & Kahneman 1992), is 2.25. Thus the loss of $y/2$ is weighted so heavily that an equal chance at y can never offset the loss enough to achieve indifference. (There is also a slight asymmetric effect due to the probability weighting function.) Following through with the logic, the optimal y^* would be zero.

Suppose we modify the gamble as in Figure 2 so that there is a 2/3 chance to gain x and a 1/3 chance to lose $x/3$. Based on BPW's model, the x_I that makes the DM indifferent is about 7% greater than R: $x_I \approx 1.07R$. Likewise, the optimal x^* for this gamble is almost exactly equal to $R/2$. The similarity of these results with those for an expected-utility-maximizer (i.e., one who is not subject to BPW's distortions) using Figure 1 is serendipitous; just change the gamble to make it more attractive, thereby accounting for the Prospect-Theory effects, and use the results in essentially the same way as before.

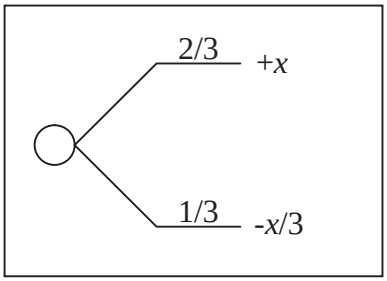

Figure 2. A different reference gamble for assessing risk tolerance.

One might ask how an EU-maximizer would respond to the gamble in Figure 2. The answer is that she would have $x_I \approx 3.21R$ and $x^* \approx 1.34R$.

The table below summarizes these results. And I hope that the message is obvious at this point: When we ask assessment questions like those described above, based on our current state of knowledge it is not entirely clear what the responses mean, and thus it is not clear what we should take the individual's "true" risk tolerance to be. Nevertheless, I offer the following observations:

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Risk Tolerance **contd.**

- BPW's empirical results are indeed compelling. In light of those results, it is easy to believe that individuals may be subject to the same distortions when assessing risk tolerance using any reference lottery.
- When I ask students to assess risk tolerance using Figure 1, some indicate a y_1 that is very small or zero. Perhaps these individuals experience BPW's distortions more than others.
- My impression is that individuals tend to give responses to the conventional assessment question that seem lower than they should be. This could be because of BPW's effects, an inclination to anchor on the optimal value, a negative reaction to gambles presented in such raw form, a combination of these, or other reasons entirely.

	Indifference		Optimum	
Gamble	EU	BPW	EU	BPW
$+y$, 0.5 $-y/2$, 0.5	$y_1 = 0.96R$	--	$y_1 = 0.46R$	$y^* = 0$
$+x$, 0.67 $-x/3$, 0.33	$x_1 = 3.21R$	$x^* = 1.07R$	$x_1 = 1.34R$	$x^* = 0.50R$

Looking at this seemingly straightforward aspect of decision-analysis practice through a behavioral lens raises interesting questions about what we get when we use our conventional risk-tolerance assessment method. And there are other approaches that we have not mentioned; even though the gambles described above are convenient, individuals might give better responses (more consistent and more accurate) when they consider more realistic lotteries, such as a combination of US treasury bills and a market index fund. Addressing these issues will require some careful behavioral studies, and I hope that someone in our field will take up the challenge!

References:

Bleichrodt, H., Pinto, J. L., & Wakker, P. P. (2001). Making descriptive use of prospect theory to improve the prescriptive use of utility. *Management Science*, 47, 1498-1514.

Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, 5, 297–323.

von Winterfeldt, D., & Edwards, W. (1986). *Decision Analysis and Behavioral Research*. Cambridge, MA: Cambridge University Press.

INFORMS, Denver

Believe it or not, it is time for us to start getting organized for the INFORMS annual meeting in Denver this coming October 24-27.

First, thanks to those of you who have already contacted us and offered to organize sessions. We feel like we are a little ahead of the game — but we want to stay there and we know time moves fast.

As we try to fill out the schedule, we'd like to be able to take into account the interests of society members to the greatest extent possible. So, if there are topics you are particularly interested in hearing about or speakers you are particularly interested in hearing, please drop us an email. While we will endeavor to take all ideas into account, your input will receive a particularly sympathetic reading if it includes an offer to organize a session or give a talk as well.

Once we've reviewed all the responses, we'll be contacting you - probably in mid-March - to firm up the details of your session. Thanks for your time, and please respond to both of us (*kara.morgan@fda.gov*, *kjenni@geomatrix.com*) and put "DAS" somewhere in your title line.

Thanks again and we look forward to seeing you in Denver!

Karen Jenni
and
Kara Morgan

Cluster co-chairs

Student Travel Support

In keeping with our objective of encouraging student participation at the national INFORMS meetings, this year DAS has funds available to support student travel for the 2004 Denver meeting. While the exact amount is yet to be determined, in the past these travel awards have been in the vicinity of $200. In addition to being a student, the winners should be presenting a paper in the DAS track of INFORMS where the student made a significant contribution to the work.

To be eligible, abstracts need to be received by **April 1**. In addition to an abstract, a statement explaining how the work fits in the scope of decision analysis is required. Both the abstract and the statement of fit should be submitted to the track chairs Kara Morgan (*Kara.Morgan@FDA.GOV*) and Karen Jenni (*KJenni@geomatrix.com*). Once papers have been accepted we will chose the winning student(s).

Please let your students know about this opportunity. Let me know if you have any questions.

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