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The Market for Mergers and the Boundaries of the Firm

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ABSTRACT

We relate the property rights theory of the firm to empirical regularities in the market for mergers and acquisitions. We first show that high market-to-book acquirers typically do not purchase low market-to-book targets. Instead, mergers pair together firms with similar ratios. We then build a continuous-time model of investment and merger activity combining search, scarcity, and asset complementarity to explain this like buys like result. We test the model by relating like-buys-like to search frictions. Search frictions and assortative matching vary inversely, supporting the model over standard explanations.

THE BOUNDARIES OF THE FIRM are constantly being drawn and redrawn in the market for corporate control. This activity surely reflects the importance of the placement of the boundaries of the firm, a topic that has received a great deal of attention in economic theory. The goal of this paper is to bridge the gap between the theory of the firm and empirical evidence on mergers, and demonstrate the significance of the theory of the firm for understanding empirical regularities in the market for mergers and acquisitions.

One of the most well-established stylized facts about merger and acquisition activity is that the typical merger involves an acquirer with high asset valuations purchasing a target with low asset valuations. This is commonly interpreted as evidence in favor of a "q-theory of mergers," in which mergers involve redeploying the assets of underperforming targets towards more profitable uses under the better management of the high-performing acquirer.[1]

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[1] This idea has its foundations in Manne (1965) who argues that low value, badly managed firms will be bought by better managed firms. It is also an extension of Tobin's q theory of investment: If mergers are simply another form of investment then high market-to-book (M/B) firms should invest by buying the assets of those firms that have low opportunities, and hence low M/B ratios. Recent work by Jovanovic and Rousseau (2002) puts this idea into a dynamic context to generate waves of activity. They show empirically that when M/B dispersion is high (implying a lot of room

In this paper, we begin by challenging this conventional wisdom with a fresh look at who buys whom. Our motivation for this challenge stems from Rhodes-Kropf, Robinson, and Viswanathan (2005). They show that while targets' assets are on average valued below acquirers', target valuations are much higher than the average firm on Compustat. Their findings suggest that "high buys less high" might be a better description of the data than "high buys low."

Our initial empirical investigation uncovers an even stronger pattern than suggested by Rhodes-Kropf et al. (2005): high buys high, moderate buys moderate, and low buys low. That is, in the market for mergers, most transactions involve high valuation firms purchasing other high valuation firms, low valuation firms acquiring other low valuation firms, and so on. In economic terms, bidders and targets are quite similar. On average, bidders and targets are less than a decile apart in market-to-book valuation. This holds regardless of whether we adjust for industry valuation effects. Thus, rather than characterizing most merger transactions as high buys low, a better description would be like buys like.

No existing theory offers guidance for why we might see such similarity in the valuation ratios of targets and acquirers. Theories based on agency (Jensen (1986)), hubris (Roll (1986)), transaction costs (Coase (1937)), their use as defensive mechanisms (Gorton, Kahl, and Rosen (2005)), or even simple beliefs in the potential for synergies suggest limited patterns in book-to-market (M/B) ratios. They do not imply convergence of bidder and target market-to-book ratios. Theories of misvaluation (Rhodes-Kropf and Viswanathan (2004) and Shleifer and Vishny (2003)) suggest target and acquirer market-to-books are high on average, but the theories do not predict that like buys like. And the q-theory of mergers (Jovanovic and Rousseau (2002)) actually suggests the opposite result: The highest market-to-book firms should acquire the lowest. It is reasonable to assume that hubris, agency or q-theory are partial or even complete motivations in some transactions. However, since they cannot explain the clustering in market-to-book ratios that we observe in the data, these theories offer limited guidance on how to explore this finding other than to suggest that further examination should dissipate the finding or, in the case of q-theory, lead us to find the opposite pattern. Since we find a robust pattern of matching in mergers it suggests that current theories do not seem to capture one of the overriding motivations in mergers.

Thus, in light of our empirical finding, the next step in our analysis is to offer a new theory of mergers, one that allows us to make predictions about when we should see firms with similar market-to-book ratios merge. Since mergers fundamentally involve redrawing the boundaries of the firm, a natural starting point for any such theory is the property rights theory of the firm (Grossman and Hart (1986), Hart and Moore (1990), Hart (1995)). The central lesson from this theory is that complementary assets should be bound together under common ownership: When there are significant complementarities between assets,

for the high M/B firms to improve the low M/B firms) then merger activity increases. This also suggests the q-theory is at work.

then placing the assets under the control of a single firm reduces the hold-up problems and underinvestment that results from the incomplete contracting. The idea that mergers reflect the desire to place complementary assets under common control is a central feature of our model. One of the basic building blocks of our analysis is that complementarities can only be realized if the assets are joined together in a firm.

Of course, to deliver predictions about market-to-book ratios, the model must also contain a number of other features. Book values reflect past investment, thus the model must allow firms to invest as well as engage in merger activity. Furthermore, market values should reflect the expected surplus that firms obtain from merger negotiations. With these features, we develop a general equilibrium merger environment in which market-to-book ratios are endogenously determined by expected future mergers and each side's bargaining power in those mergers, rather than a model in which market-to-book ratios are simply assumed to have a particular pattern.

At the core of the model is a continuous time version of a Diamond–Mortensen–Pissarides search model joined with a basic neoclassical investment model. Firms search for a merger partner with potentially complementary assets. Three key ingredients are central to our analysis.

The first two are search and scarcity. In our model, firms initiate standard investment projects and at the same time, firms search for Pareto-improving asset combinations with other firms. Firms cannot contract on the creation or distribution of the surplus generated by the asset combination: Placing assets under common control is the only way to realize the synergies from asset combinations.

When firms find an acceptable partner, they then bargain over the available surplus from merger. Whether a firm accepts or rejects a particular partner depends on whether it prefers the terms of the current offer to the expected net gains from waiting, which in turn are determined by the likelihood of future merger opportunities, as well as the expected surplus from future transactions.

Thus, the market-to-book ratio contains two parts, one from the stand-alone value of the firm and one from the net present value (NPV) of future merger activity. The NPV in the stand-alone firm arises from the skill or quality of the characteristics inherent to a firm. However, the NPV related to merger arises from the relative bargaining power of the merging firms, not from any inherent investment opportunities they bring to the newly merged firm. This is because unlike investment, a merger must be negotiated. Therefore, the benefit each party receives depends on its negotiating position, which in turn depends on each firm's ability to locate another merger partner. Since both firms are necessary for the merger, the firm with the relatively more scarce assets will more easily locate another merger partner and therefore will garner more of the merger gains. Thus, higher relative scarcity causes a firm to have a higher ex ante market-to-book ratio, regardless of whether it is the bidder or the target in a particular transaction.

The third ingredient in our model is asset complementarity. While search and scarcity work to determine how the surplus from merger is split, complementarity determines how surplus is created. We assume that gains from the merger are related to how the firms complement one another. Mergers will create greater surplus if the partners are a "better match" along one or more dimensions. Such compatibilities could arise along any dimension: better production, better technology, or better culture are just a few possibilities. For example, we assume that the best pharmaceutical company can do more with the best new drug than the second-best pharmaceutical can do. Thus, the second-best pharmaceutical firm could acquire the firm producing the best new drug, but if it did it would have to give up a larger fraction of the synergy. This element of our analysis builds on work in family and labor economics by Becker (1981), Kremer (1993), and Burdett and Coles (1997), and is related to recent work by Shimer and Smith (2000).

It is worth noting that a theory of complements is the natural opposite of the q-theory. The q-theory of mergers suggests that mergers are about substitution; the acquiring firm substitutes the target's poor management or inappropriate use of assets with superior management and direction to better extract value from those assets. In the q-theory of mergers, the most value is created by pairing the worst performing assets with the best managers. The opposite holds in our case.

That said, the fact that firms look for complementary partners does not in any way directly imply that merging firms will have similar market-to-book ratios. We show that each firm trades off its desire to merge with a better partner with the endogenously reduced bargaining power it will have in such a merger. In equilibrium, if complementarity is important, successful mergers will exhibit a high degree of complementarity. The best targets and the best acquirers have the best outside opportunities and create the most synergies. They endogenously choose to search for each other and therefore, *in equilibrium*, firms that have the highest market-to-book ratios in their respective industries will choose to merge. These market-to-book ratios are a reflection of what the market expects from future potential mergers.

Two central predictions emerge from our theory. The final step in our analysis is to test these predictions. The first prediction is simply that the like buys like effect should get stronger when we control for alternative explanations. That is, the first test of our theory is to control for obvious alternatives that could generate similar empirical patterns and see if our main result still holds. For example, two firms in the same industry may have similar market-to-book ratios just because they are in similar lines of business at the same time. If our theory does not explain the data, then controlling for this should weaken the like buys like effect, not strengthen it. Likewise, perhaps the similarity in market-to-book ratios is driven by size relatives between bidder and target—the real economic story is that firms of similar size merge, not firms with complementary assets. In this case controlling for size relatives should destroy our like buys like result.

We test this first prediction in a variety of ways. Comparing actual mergers to hypothetical mergers—pairings of randomly chosen nonmerging firms—we find that the market-to-book spreads are tighter in actual mergers than in hypothetical mergers. Among actual mergers, we find that the relative size of the bidder and target is an important determinant of the like buys like effect, but that the like buys like effect is unaffected by controlling for it. That is, the average spread between bidder and target shrinks when we control for size relatives, even though the market-to-book difference between acquirer and target loads heavily on the acquirer/target size relative. This result holds under a variety of specifications, including industry adjustments and controls for transaction size and type.

Explicitly modeling merger activity as a search process provides a second prediction. Our model predicts that when search costs are lower, assortative matching will be stronger, and the like buys like effect should be more pronounced. To test this result we develop two empirical proxies for search costs. In our model, when discount rates are low, search costs should also be low. Since this coincides with times when valuation levels are relatively high, we use valuation levels as one proxy. We find that in times of high valuations, the market-to-book spread between acquirer and target is lowest. The second proxy comes from financial market liquidity. When capital markets are liquid, search costs should also be low; the relative ease of obtaining outside funding implies that firms can afford to wait for an appropriate match to arise. We find that market-to-book spreads are lowest when debt markets are most liquid. Both these facts support a key prediction of the model, which is that assortative matching varies inversely with the cost of search.

The balance of the paper is organized as follows. First, we explore the motivating stylized facts surrounding merger activity. This is presented in Section I. Next, we build a model of merger activity based on complementarity, search, and scarcity. This is presented in Section II. In Section III, we analyze the models' predictions on patterns in market-to-book. Then, in Section IV we reexamine the empirical evidence to test the predictions of the model. Section V concludes.

I. Who Buys Whom?

This section revisits the conventional wisdom that the typical merger involves an acquirer with a high asset valuation purchasing a target with a low asset valuation. This basic finding is discussed in a great deal of prior empirical work, including Servaes (1991); Rau and Vermaelen (1998); Martin (1996); Loughran and Vijh (1997); Lang, Stulz, and Walkling (1989); Andrade, Mitchell and Stafford (2001) and many others. In their survey paper, Holmström and Kaplan (2001) argue that corporate governance issues led to the merger waves of the 1980s and 1990s. Their work implicitly squares with the "high buys low" idea inasmuch as firms with poor corporate governance have low market values and are taken over by higher valued bidders. Typically this result is couched as evidence of favorable asset redeployment; that is, that high quality managers are overtaking poorly run firms.

To examine this question, we develop a sample of 3,400 merger transactions that were announced between 1980 and 2001 between publicly listed bidders

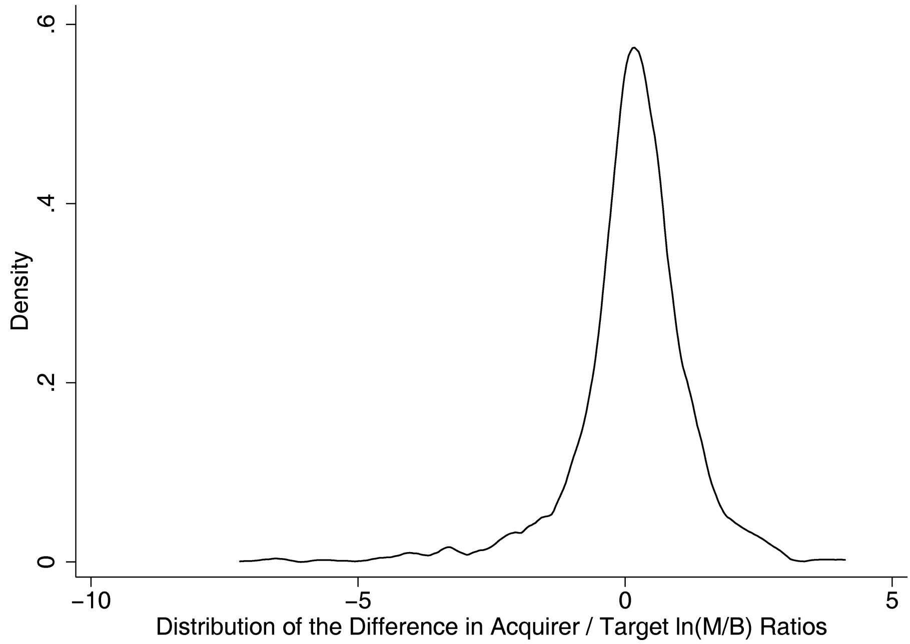

Figure 1. Distribution of Market-to-Book Spreads. This graph shows the distribution of the difference between the acquirer M/B ratio and the target M/B ratio. The area to the left-hand side of the origin on the x-axis is the 40% of the distribution for which the acquirer's M/B is lower than that of the target.

and targets in the United States. The merger data come from Securities Data Corporation's M&A database. To calculate the market-to-book ratio, we match fiscal year-end data from Compustat with CRSP market values occurring 3 months afterward. Since firms have different fiscal year end dates, this involves compensating for Compustat's year-of-record scheme, so that the year of the data corresponds to the year in which the accounting information was filed. Then, we associate this CRSP/Compustat observation with an SDC merger announcement if the announcement occurs at least 1 month after the date of the CRSP market value. If a merger announcement occurs between the fiscal year-end and 1 month after the CRSP market value, we associate the merger announcement with the previous year's accounting information. Finally, the announcement and closing dates of mergers, the method of payment (when available), and a dummy for whether the merger was withdrawn are taken from SDC and merged to the Compustat/CRSP data. This process exactly follows that in Rhodes-Kropf et al. (2005).

Figure 1 depicts the density of the difference in market-to-book valuations for bidders and targets. Positive values correspond to high buys low transactions; these occur roughly 60% of the time. The plot shows that the mean value of the difference is positive, which indicates that on average, high market-to-book acquirers purchase lower market-to-book targets. This matches Andrade, Mitchell, Stafford (2001), who report that roughly two-thirds of transactions involve an acquirer with a higher q than its target. However, the region to the left-hand side of the origin on the x-axis corresponds to the roughly 40% of the time in which the market-to-book of the acquirer is below that of the target. This low buys high result echoes the findings in Rhodes-Kropf et al. (2005).

Indeed, the evidence in Maksimovic and Phillips (2001) is consistent with this view. They show that in a substantial fraction of mergers the target firms' plants are more efficient than those of the acquirer. In these cases, the productivity of the acquirer's plants subsequently increases. Their findings suggest that asset redeployment from low q to high q firms need not be the driving force behind mergers even if mergers are driven by efficiency considerations.

To explore this issue further, we examine the joint distribution of acquirer and target market-to-book ratios. Rather than simply examining the difference, examining the bivariate distribution allows us to see which types of firms are most often involved in mergers, which speaks directly to the question of who buys whom.

In Table I, we group the population of bidders and targets into bins according to annual NYSE breakpoints of the market-to-book distribution obtained from Ken French's website. The i, j^{th} cell in Table I reports the frequency of mergers occurring between targets in decile i and bidders in decile j. The breakpoints are

Table I

Acquirer and Target Market-to-Book Ratios Using NYSE Deciles

Decile breakpoints are based on the distribution of M/B ratios for NYSE traded firms and are obtained from the website of Professor Kenneth French (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french). Each cell counts the number of mergers between 1980 and 2001 between publicly traded bidders and targets in that decile pairing. The deciles are numbered from 10 to 1 in descending order of M/B.

Pearson's χ^2 test for independence of bidder and target M/B ratios has a value of 854.91, with an associated p-value of 0.00.

The mean acquirer lies in the fourth decile, while the mean target lies in the fifth decile of the M/B distribution. The mean difference is eight-tenths of a decile.

Target	NYSE M/B Decile of Acquirer										
Decile	High	9	8	7	6	5	4	3	2	Low	Total
High	252	95	49	39	26	20	21	8	9	24	543
9	102	79	44	36	23	16	9	2	7	13	331
8	100	51	49	54	25	13	12	12	7	14	337
7	78	76	58	42	30	30	11	9	7	8	349
6	55	41	34	56	45	25	19	23	5	6	309
5	47	35	37	45	62	51	26	16	12	15	346
4	28	30	36	43	53	49	39	30	13	13	334
3	22	26	28	32	41	42	35	24	21	15	286
2	12	26	20	22	42	33	24	24	21	18	242
Low	22	20	22	26	25	33	27	34	26	45	280
Total	718	479	377	395	372	312	223	182	128	171	3,357

recomputed annually to reflect changes in the distribution of market-to-book ratios for all NYSE firms. Thus, any clustering that appears in the table is not a result of time-series clustering of merger activity.

The table illustrates a high degree of correlation between bidder and target market-to-book ratios; Pearson's χ^2 test for independence of bidder and target market-to-book ratios has a value of 854.91, with an associated p-value of 0.00. In fact, the mean difference is eight-tenths of a decile, meaning that the average transaction couples bidders and targets that are no more than one decile apart in the distribution of market-to-book ratios.

The table illustrates several interesting features of merger activity. First, it reconfirms the finding in Rhodes-Kropf et al. (2005) that, on average, although targets' assets are valued less than acquirers' assets, targets are valued more highly than the average firm. This can be seen by noting that most of the activity occurs in the upper left-hand region of the table. Thus, both acquirers and targets tend to be in the high market-to-book deciles. Second, the prevailing wisdom that high buys low is borne out by the fact that most mergers lie below the main diagonal; these correspond to mergers in which the acquirer is in a higher market-to-book decile than the target. Finally, the finding of most interest is that merger activity seems to cluster down the main diagonal. This is the like buys like diagonal. This means that in many cases, bidders and targets come from nearby points of the market-to-book distribution, indicating that if anything, although bidders have slightly higher market-to-book ratios than targets, their asset valuations are generally quite similar. This table shows why most research has focused on the high buys low result, as acquirers do tend to have slightly higher market-to-book ratios than targets. However, this table also suggests that something may be driving firms with similar market-to-book ratios to merge.

To explore this idea further, Table II reports the average bidder-target market-to-book spread by transaction type. The column labeled "mean scaled market-to-book difference" reports the bidder/target difference as a fraction of the within-industry standard deviation of market-to-book for the year of the transaction. For example, if mergers occurred between an acquirer that was one standard deviation above the mean acquiring a firm that was one standard deviation below the mean (a prototypical high buys low transaction) the scaled difference would equal two (200%).

In fact, the scaled difference is only 14% when averaged across all transactions. When the acquirer's market-to-book exceeds that of the target, the scaled difference is only 77% of a standard deviation on average. This occurs in roughly 62% of the sample. The remaining 38% of the sample has a scaled difference of negative 89% of a standard deviation.

The third bank of numbers in Table II splits the data according to whether high buys high, low buys low, etc., where we gauge whether the bidder and/or target were above or below their industry median values in a given year. When both are below the industry median, the scaled market-to-book spread is quite small, only 2% of an industry standard deviation. When high buys high, the difference is again quite small, although larger: It rises to 11.46% of a standard deviation. These low values support the idea of assortative matching,

Table II

Levels and Differences in Bidder and Target Tobin's *q*

Scaled M/B Difference is the difference between acquirer ln(Market-to-book) and target ln(Market-to-book) divided by the standard deviation of the ln(Market-to-book) for the acquirer's industry in the year of the acquisition. The units are in percent of a standard deviation, that is, 100 is one standard deviation.

	Sample Size	Percent of Sample	Mean Scaled M/B Difference
Total Mergers, 1981–2001	3,400	100%	14.58
Mean Acquirer ln(M/B): 0.8118			
Mean Target ln(M/B): 0.6816			
Target Exceeds Acquirer M/B	1,274	37%	−89.38
Acquirer Exceeds Target M/B	2,126	63%	77.16
Both Above Respective Industry Median	1,111	33%	11.46
Both Below Respective Industry Median	990	29%	2.38
Target Above, Acquirer Below	419	12%	−138.08
Acquirer Above, Target Below	880	26%	105.7

but naturally control for the fact that mergers may be occurring within or across industries at a point in time. Only when we examine transactions that involve bidders and targets on opposite sides of the median line do we find scaled market-to-book differences greater than one. It is important to note that these transactions are relatively uncommon: The most common transaction type is for both the bidder and the target to be above the industry median (high buys high). The second-most common type is for both firms to be below the industry median.

We hasten to add that these results are not isolated to a particular point in time, nor are they confounded by industry classification. As Table III demonstrates, there is no obvious clustering of the low buys high result in a given year. In particular, our results are not driven by what happened in the late 1990s, or by the merger characteristics of the 1980s.

The analysis of the entire distribution of market-to-book ratios shows that the empirical phenomenon is less clear than conventional wisdom would suggest. While it is true that a comparison of means indicates that the market-to-book of acquirers is higher than that of targets, the fact is that most of the time both firms have high market-to-book. Modest, but consistent, average differences in market-to-book between a bidder and its target mask the fact that most of the time like buys like. This pairing of firms with similar valuations implies that mergers exhibit assortative matching as predicted by our theory.

The empirical evidence on like buys like is summarized best in Figure 2. In this graph, we plot the bivariate distribution of bidder and target market-to-book ratios. To control for industry effects, we express the market-to-book ratio as a deviation from each firm's industry median. The two horizontal axes in Figure 2 are the acquirer (left-hand side) and target (right-hand side) market-to-book ratios, grouped into deciles. The vertical axis counts the number of

Table III

Time-Series of M/B Spreads

This table reports average ln(M/B) for all targets and acquirers in a given year. Count is the number of transactions. Columns headed "Target" and "Acquirer" are, respectively, annual average ln(M/B) values for targets and acquirers involved in transactions that year. Columns headed "Median(T)" and "Median(A)" report the average value of the industry median ln(M/B) for the target and acquirer, respectively.

Year	Count	Target	Acquirer	Median (T)	Median (A)
1981	37	0.22	0.43	0.21	0.12
1982	39	0.39	0.15	0.43	0.40
1983	52	0.27	0.31	0.51	0.41
1984	90	0.47	0.40	0.44	0.38
1985	104	0.49	0.46	0.64	0.64
1986	97	0.43	0.62	0.69	0.65
1987	126	0.70	0.68	0.52	0.53
1988	113	0.65	0.73	0.57	0.58
1989	110	0.61	0.62	0.56	0.55
1990	76	0.52	0.57	0.41	0.41
1991	89	0.48	0.71	0.65	0.62
1992	62	0.53	0.62	0.74	0.72
1993	97	0.71	0.97	0.77	0.76
1994	178	0.69	0.78	0.71	0.71
1995	239	0.67	0.84	0.83	0.84
1996	260	0.59	0.95	0.81	0.77
1997	358	0.72	0.86	0.99	0.97
1998	347	0.91	1.13	0.77	0.74
1999	365	0.82	1.12	0.87	0.86
2000	352	0.88	0.97	0.55	0.54
2001	208	0.71	0.83	0.50	0.44

transactions that took place between bidders and targets with that market-to-book pairing. Thus, high buys low transactions cluster at the left-hand side of the graph, and low buys high transactions cluster at the right-hand side of the graph.

Under the null hypothesis that there is no relation between bidder and target market-to-book ratio, we should expect to see a more or less uniform grid of values. On the other hand, an extreme form of the high buys low prediction would yield a graph that was peaked on the left-hand edge, where bidder valuation is highest and target valuation is lowest.

Instead, the like buys like result should appear as the bulk of merger activity lying along the main diagonal. And indeed it does. The bulk of activity involves either high buying high or low buying low: This can be seen by noting that the graph is shaped like a saddle, with peaks at the "high, high" decile and the "low, low" decile. Relatively little activity occurs in the high buys low or low buys high tails of the distribution. Instead, the fact that the bulk of merger activity lies on the main diagonal indicates that like buys like when mergers occur.

In light of this new evidence on who buys whom in mergers and acquisitions, our next task is to develop a model that can guide a deeper exploration into this

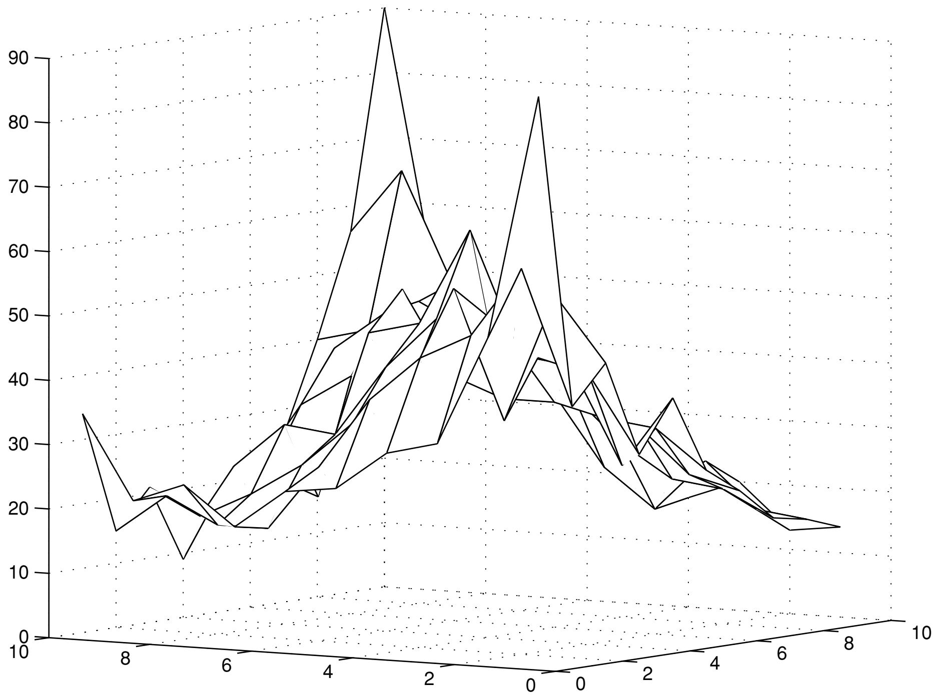

Figure 2. Industry-adjusted Ranking of Acquirer and Target M/B Ratios. This graph shows the bivariate distribution of M/B ratios in mergers, but adjusts for the industry M/B of the bidder and target. The bottom axes, which run from 0 to 10, are M/B deciles for acquirers (on the left-hand side) and targets (on the right-hand side). "10" is the highest M/B; "0" is the lowest. The vertical axis is the count of the number of transactions with acquirer and target M/B ratios falling into that bin. A first nearest neighbor smoothing approach is used to produce an empirical density. The fact that the peaks of the distribution occur at the 10/10 and 0/0 bins means that mergers are most commonly high buying high and low buying low; the saddle down the 45-degree line in the *x*–*y* plane shows that most mergers involve like buying like, regardless of their valuation level.

empirical phenomenon. The model is discussed in the next three sections. In Section IV, we return to the data to empirically explore the predictions of the theory.

II. A Search Model for the Market for Mergers

The model focuses on four different types of firms. Firms can be from either of two industries, A or B, and can be either high quality (denoted h) or low quality (denoted l). Each industry has specialized assets, denoted K_A or K_B.[2] High and

[2] Strictly speaking, it is not important for the model that the two firms be in different industries. It is only important that a firm cannot inexpensively convert its assets from one type to another. Throughout the paper we use "different industry" and "different type" interchangeably to convey the idea that a firm of one type cannot obtain other-type assets unless it acquires them from that type of firm.

low quality are represented by z_{i_h} or z_{i_l}, where $i \in A, B$ denotes industry. This parameter can represent quality in any number of firm features: management, technology, culture, etc.; the only restriction is that an increase in quality along this dimension must increase the value of the firm. In equilibrium firms with differing quality will invest in a different amount of capital. Thus, the four different types of firms in the model are $(z_{A_h}, K_{A_h}), (z_{A_l}, K_{A_l}), (z_{B_h}, K_{B_h}), (z_{B_l}, K_{B_l})$. Four types is the minimum differentiation required to demonstrate the impact of complements and substitutes in mergers.

Firms of quality level $p \in h, l$ produce output y according to the production technology

$$y_{i_p} = z_{i_p} K_{i_p}^{\alpha}, \tag{1}$$

where $i \in A, B$ indicates the firm's industry. The parameter z_{i_p} is constant across states and over time, and, for simplicity, neither firm's capital stock depreciates. The production parameter $\alpha \in (0, 1)$ implies decreasing returns to scale, which in turn guarantees that the capital stock is determinant. The particular choice of production function allows simple closed-form solutions but does not drive any results.

The price of output is exogenously determined and normalized to one. The appropriate discount rate is $r > 0$; therefore, the quantity $Y_{i_p} \equiv \frac{y_{i_p}}{r}$ represents the capitalized value of the infinite stream of future output.[3]

We allow continuous and frictionless adjustment of the capital stock. Thus, the capital stock is always set optimally. The cost of one unit of capital is normalized to one dollar. Therefore, firms choose investment given prices to maximize firm value.

Firms can also choose to merge. If firms merge then the new merged entity has quality that is a function of the individual pre-merger quality parameters, that is, $z_M(z_{A_p}, z_{B_f})$, where M denotes the merged entity and $p, f \in h, l$ represent original firm quality. After the merger is consummated, the joint production function becomes

$$y_{A_p B_f} = z_M(z_{A_p}, z_{B_f}) * \left(K_{A_p}^{\alpha} + K_{B_f}^{\alpha}\right). \tag{2}$$

The simple assumption that a merger alters only the z parameter does not drive any result. Overall the form of the production function is of limited importance as the key results depend on general changes in output due to the merger rather than on specifically how output changes.

There are gains from a merger if

$$y_{A_p} + y_{B_f} < z_M\left(K_{A_p}^{*\alpha} + K_{B_f}^{*\alpha}\right) - r\left(K_{A_p}^{*} + K_{B_f}^{*} - K_{A_p} - K_{B_f}\right), \tag{3}$$

where K_i^* is the optimal capital stock chosen if the merger occurs. Thus, gains from a merger are more likely if z_M is larger.

[3] We could also include future investment opportunities without qualitatively altering our results.

A. Complements versus Substitutes

The idea that mergers are about substitutes comes from the q-theoretic logic that tells us that good management should apply its superior skill and replace poorly performing management. Thus, high quality management should substitute for low quality management. In this case $z_M = \max[z_{A_p}, z_{B_f}]$.

What if mergers are instead about complements? In this case a high quality firm may be better off merging with another firm with high quality features rather than trying to redeploy poorly performing assets. For example, consider a firm with the best new product but no distribution channels. Should it look to merge with the best marketing and distribution firm? It seems natural that this would be the best match. No asset or feature is replacing another, but there is a clear potential for the sum to be greater than the parts. In any arena complementarities are about the ability of each part to enhance the total; good food is made better with good wine and good wine is made better with good food. Although any meal is made better with better wine, one would not typically choose to improve a basic meal with stellar wine and improve a weak wine with wonderful food. Instead, total utility is improved by matching the quality of the wine and food. We are suggesting that a similar idea may hold in mergers; total output may be greatest when the best in one area match with the best in another and simultaneously the lesser quality firms also merge.

Generally, if mergers are about complements then output is such that

$$y_{A_h B_h} + y_{A_l B_l} \geq y_{A_h B_l} + y_{A_l B_h}. \tag{4}$$

That is, total (social) output is greater if firms merge with other firms with similar quality assets. Note that under the q-theory of mergers the opposite would be true.

Complementarities suggest that pairing the best with the best and the worst with the worst produces more social surplus than if the best pairs with the worst. Condition (4) is sometimes referred to as supermodularity (see Topkis (1998) or Shimer and Smith (2000)). Note that if condition (4) holds this does not ensure that type 1 firms will merge with type 1 firms, only that it would be Pareto-improving for them to do so. Assortative matching describes this endogenous sorting and requires supermodularity.[4] We use this idea here to show how the possibility of assortative matching on assets shows up endogenously in market-to-book ratios before the merger.

Rather than stay unnecessarily general we will assume a simple functional form that implements the idea of complementarity. We assume that $z_M = z_{A_p B_f} = z^*_{A_p} z_{B_f}$ and $z_{i_p} > 1$. Many functional forms would deliver qualitatively similar results but this choice will easily and clearly demonstrate the impact of complementary mergers.[5]

[4] Shimer and Smith (2000) have shown that in general assortative matching in a search model with our assumptions requires that search costs be sufficiently low.

[5] General proofs, without this simplification, are available from the authors.

B. Incomplete Contracts and Complementary Assets

Before we continue we pause and ask *why* complementarities may be important in mergers. Our theoretical justification for this idea is rooted in the theory of the firm, which is a natural place to look for guidance on what determines mergers.

One of the central predictions of Hart and Moore (1990) and Hart (1995) is that in a world of incomplete contracts, complementary assets should be contained in a single firm where they are placed under common control. This is because the spillovers create opportunities for rent-seeking that are minimized by allocating decision rights over the use of assets to a single party. On the contrary, assets without complementarities can easily be contained in different firms, their coordination mediated through simple product market transactions. Given that mergers are costly, this suggests that firms with complementary assets should be more willing to pay the costs of merging to gain the benefits associated with coordinated ownership.

Incomplete contracts enter our model in the following way. We assume that when two parties are paired, they cannot contract on the provision and division of the surplus created by combining their assets. The only way to capture the synergies is to place the assets under common control.

This assumption is a reduced-form representation of the impact of incomplete contracts on mergers. In the spirit of Williamson (1985), Grossman and Hart (1986), Hart and Moore (1990) and others, the realization of synergies involves relationship-specific investments. Firms that choose to contract with each other instead of merging would later face hold-up problems as either firm could threaten to search for another partner. This would lead to underinvestment in relationship specific objectives, such as quality, along a particular dimension on which firms could match.

Hart and Moore (1990) show that these incomplete contracting problems are most severe between assets that are highly complementary. This implies that firms with the highest degree of complementarity have the strongest incentive to merge, since the opportunity cost of underinvestment is highest between them.

This logic therefore suggests that if we believe that incomplete contracts are important, then mergers should commonly be about complementary assets joining. We develop the remaining features of the model below and look for the impact that asset complementarity has on the results.

C. States of Nature

There are two states of nature, the No-Merger state (NM) and the Mergers are Possible state (MP), $\Sigma \in \{\Sigma^{NM}, \Sigma^{MP}\}$ with associated state intensities λ^{NM} and λ^{MP}. Since time evolves continuously, at each instant, the probability of remaining in state Σ over the next time interval Δ is $e^{-\Delta\lambda^{\Sigma}}$. The model begins in the state Σ^{NM}, which means that no profitable merger opportunities are available. Specifically, in this state the merger quality function $z_M(\cdot, \cdot)$ is assumed to be such that $z_{i_p} > z_M(z_{i_p}, z_{j_f}) \forall i, j, p, f$.

If the economy is in the Σ^{NM} state, there is probability $1 - e^{-\Delta\lambda^{NM}}$ that a positive shock will occur to $z_M(\cdot, \cdot)$. If a shock occurs, the state switches to Σ^{MP} and profitable merger opportunities are available. This shock captures the idea from Gort (1969) that exogenous factors, such as the discovery of a new technology or production process, create periods of organizational flux.

We assume that firms do not merge before the shock in preparation for the benefit because they are unaware of which type of assets they will need when the shock occurs. (It seems reasonable that firms did not try to locate an internet partner in the 1980s before various politicians allegedly invented the internet.) The nature of a shock is such that we do not know what form it will take until it occurs.

This shock alters $z_M(\cdot, \cdot)$ to allow a possible benefit from joining assets of type A and B. After the shock we will assume that parameter values are large enough to ensure condition (3) holds and mergers occur. We assume that the state of nature, the function $z_M(\cdot, \cdot)$, and whether the shock has occurred are common knowledge to all firms.

It is important to note that the increase in firm output only occurs if a firm A and a firm B merge. Firms cannot contract on the creation or distribution of the surplus generated by the asset combination: Placing assets under common control is the only way to realize the synergies from asset combinations. Furthermore, if a firm remains a stand-alone entity and simply invests in more assets, then z_i remains the same as before the shock. The idea is that firms develop different kinds of assets. The synergies in a merger occur because of the complementary nature of the assets of different firms. For example, an inventor may have a new product and another firm may have the assets that market and distribute. While we assume the frictionless adjustment of capital of type A for any firm A, we assume that type A firms cannot create the assets of type B.

Let $\pi_{i_p}^{NM}$ represent the present value of a firm in industry i with quality p in the NM state. This value includes an expectation over all possible future states of the world. As noted above it is possible that the state jumps to a state where MP. When mergers are possible we assume that firms must search for a suitable partner. The delay in finding a partner represents the actual time firms must spend looking for a partner as well as the significant time evaluating the potential from different mergers, time spent in due diligence to ensure the match is viable, and time spent negotiating. For simplicity, we let time capture all costs associated with merging; thus, if it takes longer to find a partner or the discount rate is higher, then mergers are more costly. Let $\pi_{i_p}^{MP}$ represent the present value of a firm in industry i with quality p in the MP state before it has located a potential partner and announced a deal. If a firm locates a potential partner then it must negotiate the terms of the deal. If the deal is consummated then on the announcement of a completed deal a firm's value changes to $\pi_{i_p j_f}^{M}$, where $\pi_{i_p j_f}^{M}$ represents the expected value of a merger to a firm in industry

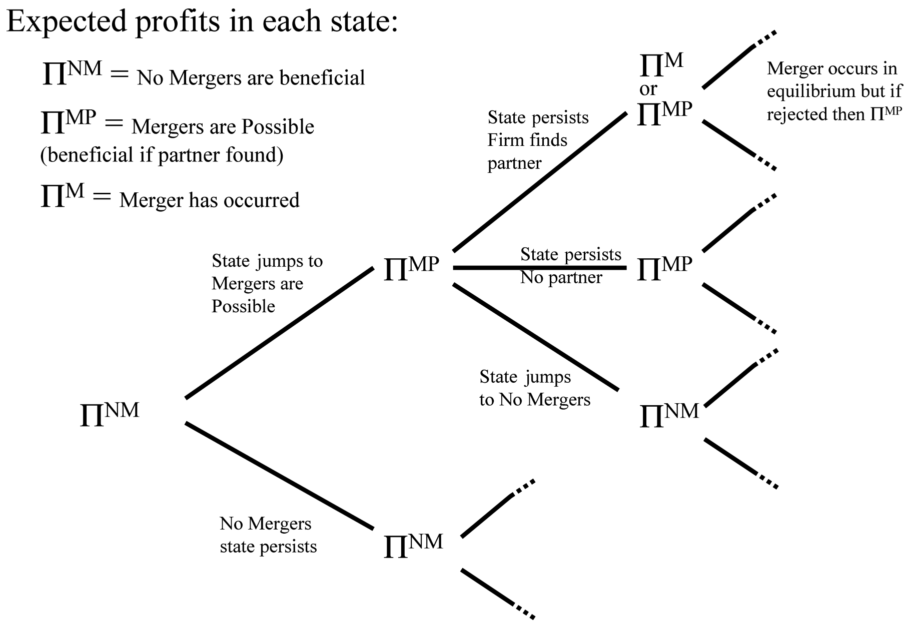

Figure 3. Extensive Form Representation of the Search Model.

i with quality p who merges with a firm in industry j with quality f. If a deal cannot be reached then firms continue to search for another potential partner. Thus, their value remains at $\pi^{MP}_{i_p}$. At any time during this search process the state may return to the NMs state and then the value of firm i_p would return to $\pi^{NM}_{i_p}$. We develop this search and negotiation more fully below.

Figure 3 depicts a game tree for the model. It shows how the economy can move from the NM state to the merger state. Once in the merger state, mergers can either occur or not occur, depending on whether merger partners are found and can agree on a deal.

We now have all of the model assumptions and can proceed to solve the model.

D. Investment When No Mergers Are Possible

The world begins in the NM state. As long as this state persists, each firm chooses its investment, I, to solve

$$\max_{I_{i_p}}\left[(K_{i_p} + I_{i_p} + \Delta z_{i_p}(K_{i_p} + I_{i_p})^{\alpha})e^{-r\Delta} - I_{i_p}\right], \tag{5}$$

where the subscript i represents either A or B and the subscript p represents either low (l) or high (h) quality. This expression describes the value of the firm as the value of assets in place, plus the value of the production that will occur over the interval Δ, minus the required investment. Since investment is frictionless, the capital stock always satisfies

$$K_{i_p}^{NM^*} = \sqrt[\alpha-1]{\frac{r}{z_{i_p}\alpha}}, \tag{6}$$

where NM^* signifies that this is the optimal capital choice in the NM state.

During any small moment of time, Δ, each firm earns $\Delta z_{i_p}(K_{i_p}^{NM^*})^\alpha$. Over this interval a shock may or may not occur. The term $\pi_{i_p}^{NM}$ represents the expected value of the NM state, and $\pi_{i_p}^{MP}$ represents the expected value if the state is such that MP. The expected value of the firm in the NM state can be written as

$$\pi_{i_p}^{NM} = \left[e^{-\Delta\lambda^{NM}}\pi_{i_p}^{NM} + (1 - e^{-\Delta\lambda^{NM}})\pi_{i_p}^{MP} + \Delta z_{i_p}\left(K_{i_p}^{NM^*}\right)^\alpha\right]e^{-r\Delta}. \tag{7}$$

This equation simply expresses the value of the firm in the NM state as the discounted expected value of the firm in each future state (the first two summands), plus interim production.

To solve for the expected value in the NM state we must find the expected value in the NM state. This requires us to examine the search negotiation process. However, to understand negotiations we must first examine the benefits that would arise from a merger.

E. The Gains from Merger

After the merger, the merged firm chooses its capital stock to solve[6]

$$\max_{I_A, I_B}\Big[\Delta z_M\left(\left(K_{A_p}^{NM^*} + I_A\right)^\alpha + \left(K_{B_f}^{NM^*} + I_B\right)^\alpha\right)e^{-r\Delta} - I_A - I_B$$
$$+ \left(K_{A_p}^{NM^*} + K_{B_f}^{NM^*} + I_A + I_B\right)e^{-r\Delta}\Big]. \tag{8}$$

The change in z changes the optimal amount of each type of asset to the following:

$$K_i^{M^*} = \sqrt[\alpha-1]{\frac{r}{z_M\alpha}}, \tag{9}$$

where the M^* signifies that this is the optimal investment in assets of type i post Merger. Note that the quality subscript on i has dropped since after the merger only z_M matters for the optimal asset choice (and not the original firm quality). We assume nothing changes in a firm in the periods after the merger, therefore the capital stock will remain the same for all future periods. Thus, the value of the merged firm is

$$\frac{z_M(\left(K_A^{M^*}\right)^\alpha + \left(K_B^{M^*}\right)^\alpha)}{r}. \tag{10}$$

[6] Remember that $K_i^{NM^*} = K_i^{MP^*}$.

The optimal capital stock in the state where mergers are possible, but before any merger has occurred, is the same as in the NM state. That is, $K_i^{NM^*} = K_i^{MP*}$. For now, we assume that firms cannot commit to a non-optimal level of capital post merger negotiations (whether or not the merger occurs) in an attempt to change the piece of the pie they receive. We will show in equilibrium (see proof of Proposition 1) that even if they could, they have no incentive to do so. Thus, the value of the merged firm is the value of the output at the optimal investment level minus the cost of the increased investment:

$$s_{A_pB_f} \equiv \frac{z_M((K_A^{M^*})^\alpha + (K_B^{M^*})^\alpha)}{r} - (K_A^{M^*} + K_B^{M^*} - K_{A_p}^{MP^*} - K_{B_f}^{MP^*}). \quad (11)$$

The set of possible agreements is $\Pi = \{(\pi^M_{A_pB_f}, \pi^M_{B_fA_p}) : 0 \leq \pi^M_{A_pB_f} \leq s_{A_pB_f}$ and $\pi^M_{B_fA_p} = s_{A_pB_f} - \pi^M_{A_pB_f}\}$, where $\pi^M_{A_pB_f}$ and $\pi^M_{B_fA_p}$ are the share of the merged firm to a player with either type of assets and quality $p, f \in \{h, l\}$.

In equilibrium, if a firm finds a partner it is possible to strike a deal as long as the utility from a deal is greater than the outside opportunity for both firms. If a firm rejects a merger partner then it remains in the MP state with expected value $\pi^{MP}_{i_p}$. Thus, $\pi^{MP}_{i_p}$ represents the utility from continuing to search and also the disagreement utility.

To determine how firms share the surplus generated by the merger we must decide on a model for the negotiations. While many different choices for the model of negotiations will work for our purposes, the simplest is the Nash bargaining solution, which in this case is just the solution to

$$\max_{(\pi^M_{A_pB_f}, \pi^M_{B_fA_p}) \in \Pi} (\pi^M_{A_pB_f} - \pi^{MP}_{A_p})(\pi^M_{B_fA_p} - \pi^{MP}_{B_f}). \quad (12)$$

The well-known solution to the bargaining problem is presented in the following Lemma.[7]

LEMMA 1: *In equilibrium the resulting merger share for a firm with assets i_p merging with a firm with assets j_f is*

$$\pi^M_{i_pj_f} = \frac{1}{2}(s_{i_pj_f} - \pi^{MP}_{j_f} + \pi^{MP}_{i_p}), \quad (13)$$

where the $\pi^{MP}_{i_p}, \pi^{MP}_{j_f}$ are the disagreement expected values and $s_{i_pj_f}$ is defined according to equation (11).

With this understanding of the merger negotiation we can now look at the state when mergers are possible and examine the search process.

[7] The generalized Nash bargaining solution is a simple extension but adds no insight and is omitted.

F. The Search for a Merger Partner

To explore the process by which potential partners locate one another and agree on a merger consideration, we combine the Nash bargaining model with the classic search model of Diamond–Mortensen–Pissarides (for examples see Diamond (1993) and Mortensen and Pissarides (1994), and for a review see Petrongolo and Pissarides (2001)).[8] This allows for a negotiated outcome that depends on each party's ability to locate another suitable merger partner. The notion of search is both realistic and important if we wish to understand patterns in merger behavior in low cost, active times versus high cost, inactive times.

If firms search for a partner, we must define the probability that they locate one. We assume that the measure of firms with assets of type A is M_A and the measure of firms assets of type B is M_B. As is standard in search models, we define $\theta \equiv M_A/M_B$. This ratio is important because the relative availability of each type of firm will determine the arrival rate of merger opportunities and therefore influence each firm's bargaining ability. This fraction represents the *relative scarcity* of each type of asset. If θ is high there are many more firms with type A assets than type B, and vice versa.

Given the availability of each type of firm, the number of negotiations per unit of time is given by the matching function $\psi(M_A, M_B)$. This function is assumed to be increasing in both arguments, concave, and homogenous of degree one. This last assumption ensures that the arrival rate of merger opportunities depends only on the relative scarcity of the assets, θ, which in turn means that the overall size of the market does not impact each firm in a different manner. Each individual firm experiences the same flow probability of finding a potential partner. Thus, the arrival rate of a merger opportunity is a Poisson process. The arrival rate of a merger is

$$\psi(M_A, M_B)/M_A = \psi\left(1, \frac{M_B}{M_A}\right) \equiv q_A(\theta) \tag{14}$$

for a firm with assets A. By the properties of the matching function, $q'_A(\theta) \leq 0$, the elasticity of $q_A(\theta)$ is between zero and unity, and q_A satisfies standard Inada conditions. Thus, firms with A assets are more likely to meet available firms with B assets if the ratio of type A to type B firms is low. From the viewpoint of a firm with assets of type B, the arrival rate of mergers is $\theta q_A(\theta) \equiv q_B(\theta)$. This differs from the viewpoint of A asset firms due to the difference in the relative scarcity of their assets. Also, note that $q'_B(\theta) \geq 0$, thus B asset firms are more likely to meet available A asset firms if the ratio of type A to type B firms is high.

During any short period of time, Δ, there is a probability that a firm with type A assets finds a merger partner, $\Delta q_A(\theta)$, and a probability that it does

[8] For a complete development of the model see Pissarides (1990). Our exposition follows this work. This combination was recently used by Inderst and Müller (2004) in examining venture investing.

not find a partner, $(1 - \Delta q_A(\theta))$, and must continue the search. During this search period there is also the probability that the MP state ends. This happens because of another technological shift that eliminates the usefulness of any more combinations of assets A and B. As noted above, the intensity of the MP state is λ^{MP}, so the probability that mergers are still viable after a search of time Δ is $e^{-\Delta\lambda^{MP}}$. If the MP state ends then each firm receives the expected value that it originally achieved at the beginning of the model in the NM state, $\pi_{i_p}^{NM}$.

We assume that the measure of each type of firm is unchanging. Therefore, $\pi_{i_p}^{MP}$ is the same at any point in time. Formally, this stationarity requires the simultaneous creation of more firms with A and B assets to replace those that merge. Let m_i denote the rate of creation of new type i firms. Stationarity requires that the inflows equal the outflows. Therefore, $m_i = q_i(\theta)M_i$. We can think of these new firms as coming from spin-offs of other combinations of assets that are no longer synergistic or from fundamental firm creation. In the context of a labor search model, this assumption would be odd, since labor models are focused on the rate of unemployment. There is no analog for mergers, since we are not interested in the "rate" that firms stay unmerged. Indeed, none of our results depend on this simplification as we could have inflows arise only from spin-offs.

We also assume for simplicity that within each type of asset firms are high or low quality with equal likelihood.

Given the discount rate r, the disagreement utility of an asset type A firm is defined by

$$\pi_{A_p}^{MP} = [\Delta q_A(\theta)\left[\frac{1}{2}\max\left(\pi_{A_pB_p}^{M}, \pi_{A_p}^{MP}\right) + \frac{1}{2}\max\left(\pi_{A_pB_f}^{M}, \pi_{A_p}^{MP}\right)\right]e^{-\Delta\lambda^{MP}} + (1 - \Delta q_A(\theta))\pi_{A_p}^{MP}e^{-\Delta\lambda^{MP}} + \pi_{A_p}^{NM}(1 - e^{-\Delta\lambda^{MP}}) + \Delta z_{A_p}\left(K_{A_p}^{NM^*}\right)^{\alpha}]e^{-r\Delta}, \tag{15}$$

where $f \neq p$. This simply expresses the value of the firm in the MP state as the discounted expected value of the firm in each future state, plus interim production. The max functions determine whether a merger will be consummated given that a potential partner is found with the same or different quality (each with a probability of 1/2). If the equilibrium merger share is greater than the value of continuing to look then the merger will occur.

III. Equilibrium Market-to-Book Ratios

A firm's market-to-book reflects its stand-alone opportunities plus the expected gains from the merger even before the merger occurs. But a firm's expected gain depends not on the total gain from the merger, but rather on the share that firm will receive. How the total merger gain is split depends on the relative bargaining power of each firm. As we will see, relative bargaining power depends on the scarcity of each firm's assets, the quality of the match, and the costs of finding another partner. Thus, we will see how patterns in market-to-book depend on industry effects and the expected quality of the match and the costs of merging.

A. Matching in Mergers

The first proposition provides the solution to the model. Since we are interested in matching we find the expected values in the different states assuming that mergers do occur (condition (3) holds) and that firms find it beneficial to wait to find a partner of similar quality ($\pi^{M}_{A_pB_f} \leq \pi^{MP}_{A_p}$). In the following proposition we can then use these solutions to find conditions under which matching will occur.

PROPOSITION 1: *Assuming condition (3) holds and* $\pi^{M}_{i_pj_f} \leq \pi^{MP}_{i_p}$, *expected profit is given by:*

$$\pi^{NM}_{i_p} = \frac{1}{\lambda^{NM}+r}\left(\lambda^{NM}\pi^{MP}_{i_p} + rY_{i_p}\right), \tag{16}$$

$$\pi^{MP}_{i_p} = \frac{(4D+q_j(\theta))Y_{i_p} + q_i(\theta)(s_{i_pj_p} - Y_{j_p})}{4D+q_i(\theta)+q_j(\theta)}, \tag{17}$$

$$\pi^{M}_{i_p} = \frac{(2D+q_i(\theta))(s_{i_pj_p} - Y_{j_p}) + (2D+q_j(\theta))Y_{i_p}}{4D+q_i(\theta)+q_j(\theta)}, \tag{18}$$

where $D = r\frac{\lambda^{MP}+\lambda^{NM}+r}{\lambda^{NM}+r}$, $i \neq j, \forall i,j \in \{A,B\}, p \in \{h,l\}$.

Proof: See Appendix.

These solutions are intuitive: They are simply weighted averages of the possible future outcomes. If, for example, the NMs state persists indefinitely ($\lambda^{NM} \to 0$) then the firm's expected profit would just be Y_{i_p}, the present value of future output as a stand-alone entity. The expected value after a merger has occurred, $\pi^{M}_{i_pj_p}$, or the expected value in the MP state, $\pi^{MP}_{i_p}$, are just a weighted average of the surplus remaining after the other firm receives its stand-alone value, $s_{i_pj_p} - Y_{j_p}$, and the firm's own stand-alone value, Y_{i_p}. The weights depend of course on the firms' relative bargaining powers, which in turn depend on the ability to find another partner and on the opportunity cost of waiting.

The market values in each state of the world calculated above assume that merging firms match on quality. The specific assumption is that if firms find a partner with different quality from their own, their expected value from continuing to search exceeds the value they would receive in the merger ($\pi^{M}_{i_pj_f} \leq \pi^{MP}_{i_p}$). The next proposition uses the solutions from Proposition 1 to determine what conditions are necessary for this assumption to be true in equilibrium.

PROPOSITION 2: *If*

$$4D[s_{i_p j_f} - Y_{j_f} - Y_{i_p}] + q_j(\theta)[s_{i_p j_f} - s_{i_f j_f}] < q_i(\theta)[s_{i_p j_p} - s_{i_p j_f}], \qquad (19)$$

for all $p \neq f,\ p, f \in h, l, \forall i \neq j,\ i, j \in \{A, B\}$, *then assortative matching will occur.*

Proof: See Appendix.

If condition (19) holds, then we say that firms are in the assortative matching regime. If so, firms will merge only if they have the same "quality", that is, they belong to the same subcategory. This result is similar to Burdett and Coles (1997); see also Eeckhout (1999).

Assortative matching in our context requires the supermodularity of synergies to be large relative to the opportunity costs of searching. Thus, for example, $s_{A_h B_h} + s_{A_l B_l}$ must be significantly greater than $s_{A_h B_l} + s_{A_l B_h}$. If mergers are complementary and condition (4) holds, then $s_{A_h B_l} - s_{A_l B_l} < s_{A_h B_h} - s_{A_l B_h}$. Therefore, if there were no costs to continuing to search and the types were equally scarce then we would be guaranteed assortative matching a la Becker (1973). However, there are two effects that alter the willingness of differing types to consummate deals.

The first effect relates to $4D[s_{i_p j_f} - Y_{j_f} - Y_{i_p}]$. The expression in brackets is the gain from a merger between firms i_p and j_f. Thus, if type p and type f firms do generate synergies it pushes them toward accepting a deal, and more synergies lead to a greater chance of a deal. This effect becomes more important the more costly it is to continue to search (larger $D = r\frac{\lambda^{MP}+\lambda^{NM}+r}{\lambda^{NM}+r}$). With high search costs the current deal looks better than going back to searching. Thus, if λ^{MP} or r increases, or if λ^{NM} decreases, then a deal between any two types of firms is more likely to occur.

The second effect relates to the scarcity of the type A and type B firms. The more scarce the type A firms are the greater $q_A(\theta)$ is. The greater $q_A(\theta)$ the more bargaining power the A asset firms have and the more A asset firms benefit from waiting for another firm with which it can produce great synergies because the firm will get all the extra production.

Proposition 2 essentially says that it must be worthwhile for a firm to continue to search for a partner who matches on quality or they will simply accept a merger with the first partner they find. Proposition 2 shows that as long as condition (19) holds firms will match on quality.

B. Patterns in Market-to-Book

Since we have determined how the synergies from the merger will be shared and how this will affect the expected value in all states of the world, we can now examine the market-to-book ratio. The market-to-book ratio at any point in time is the present value divided by its equilibrium capital stock. We first show how market-to-book ratios are affected by future merger activity and then we use this to look for patterns in market-to-book.

PROPOSITION 3: *Increasing the scarcity of a firm's assets increases the probability that the firm participates in a merger, increases the share of the synergy it receives, and increases its market-to-book ratios in all states (assuming the synergy is positive).*

Proof: See Appendix.

This proposition demonstrates that market-to-book ratios reflect future merger activity in a manner that depends on the relative scarcity of the firm's assets. This is our first key result. It tells us that the relationship between the market-to-book ratios of acquirers and targets could be anything—high buys low, low buys high, or like buys like—as it depends on the relative scarcity of the merging firms.[9] Essentially, in our analysis there are two parts to market-to-book: an investment q and a merger q. Since mergers must be negotiated, greater relative scarcity leads to higher ex ante market-to-book ratios.

Thus, even when firms are matching on quality we should not expect the absolute size of the market-to-book ratios of merging firms to be similar. Depending on industry characteristics, one type of firm could have a market-to-book significantly different from its partner even if they are matching on quality. However, this does not mean that there is no pattern in the market-to-book ratios, only that we have to be more clever in how we look for it.

This brings us to the central point of the paper. The absolute market-to-book ratios of two firms merging could be anything. However, the relative market-to-book ratios of the firms that merge depend on why the firms are merging. If mergers are generally about substitutes then firms will search for partners with quality most different from their own. Therefore, *if* mergers are about the redeployment of assets to better uses then there should be a large differential between the target and acquirer market-to-book. However, if mergers are about complements then this will be reflected in firms' ex ante market-to-book ratios in the following way.

PROPOSITION 4: *If* $z_{A_pB_f} = z_{A_p} * z_{B_f}$ *and condition (19) holds, then firms' market-to-book ratios will exhibit rank-ordering: Type A firms with the largest market-to-book will merge with the type B firms with the largest market-to-book, while type A firms with the second largest market-to-book will merge with the type B firms with the second largest market-to-book.*

Proof: See Appendix.

This endogenous outcome arises because firms trade-off their desire to merge with a better partner with the reduced bargaining power they would have in

[9] Maksimovic and Phillips (2002) make a related point. They model how the boundaries of the firm change with respect to the decision to operate in multiple segments, depending on the firm's productivity and demand shocks. Their model does not distinguish between the mergers and purchases of plants and capacity from other firms. Their model implies that following a positive demand shock better managed firms purchase assets from firms managed by less able managers, but due to differences in firm size, the relation between the market-to-book ratios and firm efficiency are not uniquely specified.

such a merger. In equilibrium mergers that complete will match on quality and this will endogenously result in similar ex ante market-to-book *ranks*, as the market recognizes this will occur ex ante. The markets' expectations are reflected in the ex ante market-to-book ratios. Thus, the potential for a socially optimal outcome that arises from complementarities in merger partners impacts bargaining power and the choice of partner and through that mechanism endogenously pushes the market-to-book rank differential of completed deals toward zero.

Thus, if mergers are about substitutes then firms would choose to merge with the type most unlike them and the market-to-book of merging firms would be far apart, whereas, if mergers are mostly about complements then one should expect the market-to-book ranks of merging firms to be similar. Clearly, some of each is occurring in the real world. However, the data suggest that complements seem to be quite important in mergers.

The intuition for the model's key result is that firms trade off searching for increased synergies with the diminished bargaining power they may possess if they face a high quality partner. In equilibrium we show that firms continue to search for a partner until they find a match on quality. Any firm would like a "better" partner, ceteris parabis, and any match may have the potential to generate some surplus, but better partners have more bargaining power. Thus, lower quality firms choose to search for lower quality partners to get a larger fraction of smaller synergies. Because high quality firms can generate more surplus by pairing with other high quality firms, lower quality firms prefer to settle for lower quality partners than to wait for a higher quality partner but face a disadvantaged bargaining position. In equilibrium, this is reflected in their market-to-book ratios: The best type A firms (who endogenously have the highest market-to-book) merge with the best type B firms, and targets and acquirers have market-to-book ratios that come from similar deciles within their industries.

C. The Effects of Search and Scarcity on Matching

Since the idea of complementarity is placed in the context of a search model the potential for firms to assortatively match depends on the search costs. Search is an important part of the model as it is both realistic and provides us with our second important testable implication. In this subsection we explore the effects of search on the matching and relative market-to-book ratios of merging firms.

The following proposition provides us with a second important prediction.

PROPOSITION 5: *If the discount rate is low then assortative matching is more likely to hold and the difference in the rank of merging firms' market-to-book ratios will be smaller.*

Proof: See Appendix.

This prediction is striking precisely because both q-theory and some known facts suggest the opposite. In low discount rate environments the market-to-book differential across firms is *greater* (as shown by Jovanovic and Rousseau (2002)) and thus, there is more opportunity for a high market-to-book firm to buy a low market-to-book firm. Hence, even if firms were matching *randomly*, the market-to-book differential of firms merging in low discount rate environments should be higher (not lower as we predict). However, in Section IV we provide empirical evidence that strongly supports this prediction. Even though the market-to-book dispersion is higher in low discount rate times, the firms that merge have smaller market-to-book differentials. This is because as search becomes less costly matching increases.

Rhodes-Kropf et al. (2005) show that waves occur when valuations are high. Thus, matching is more likely during waves. Furthermore, Harford (2005) finds that mergers that occur during waves create more value than non-wave mergers. Our theory combines these findings and suggests that mergers during waves should create more value because of the greater matching.

Overall, our theory is a "Birds of a Feather" theory of mergers that builds on the desire of better firms to get together to endogenously deliver the remarkable similarity of market-to-book ratio ranks that we see in the data. Our theory further predicts how this tightness should vary over different discount rate environments and in mergers of equals versus unequals. We propose that the desire to match on different dimensions is an important driver in mergers and acquisitions. The next section explores the empirical relevance of these predictions.

IV. Testing the Model

In this section we return to the data and explore the empirical predictions of assortative matching for merger activity. We focus our analysis on two key predictions.

First, we revisit the like buys like result controlling for alternative explanations for why we might see firms with similar market-to-book ratios merge. If the like buys like result is an artifact of another phenomenon, such as firms of similar size or industry membership merging, then when we control for these alternatives the like buys like result should dissipate. In a sense, this is a necessary condition that our model must pass if we are to regard it as a potential explanation for the empirical phenomenon we observe. We take this up in Section IV.A.

Second, we test an important implication that arises from the search feature of our model: When search costs are low, we should see more assortative matching in the merger market. This should mean that the like buys like effect is stronger in periods when search costs are low. We take this up in Section IV.B. In a sense, this tests a sufficient condition for the search component of our model. By varying the search parameter empirically, we introduce the possibility of falsifying the model by potentially observing the market-to-book spread widen as search costs drop.

But before we turn to these more formal tests, it is worth noting that some of the evidence discussed in Section I can actually be couched as evidence supporting our model. Namely, in Figure 2, when we plot the bivariate distribution of industry-adjusted market-to-book ratios for bidders and targets, we are implicitly exploring the empirical validity of Proposition 4, which predicts that the best firm from industry A should merge with the best firm from industry B even if the market-to-book across the two industries is very different. When we move from Table I, which simply expresses the market-to-book ratios of bidders and targets according to NYSE breakpoints, to Figure 2, where we express the market-to-book ratio as a deviation from each firm's industry median and rank bidders against one another instead of against a neutral benchmark like the NYSE market-to-book breakpoints, we are implicitly testing the predictions of our model.

Thus, the like buys like effect presented in Table I, which helps to motivate our empirical investigation, is actually an artifact of a much stronger phenomenon: firms are merging with other firms with a very similar *relative* rank. The fact that Figure 2 displays the pronounced saddle shape is both an expression of the stylized fact that like buys like, as well as evidence that supports Proposition 4 of our model.

A. Controlling for Alternative Explanations

We begin by asking whether the like buys like effect, which our model predicts, is not driven by some alternative explanation. The first alternative that we consider is one that arises from the work of Mitchell and Mulherin (1996) and Harford (2005), who demonstrate that mergers cluster in industries and periods of time. Rhodes-Kropf et al. (2005) show that valuation waves are an important component of this clustering. Therefore, the first alternative is that we are simply observing a by-product of the fact that because mergers occur during spikes in valuation, they naturally tend to pair firms of similar market-to-book ratios, because the valuation waves drive the log market-to-book ratios to converge.

To do this, we ask how actual mergers compare to mergers that might have occurred, but did not. In other words, we compare mergers that actually occurred with random pairings of nonmerging firms. We take each bidder and target in our sample of 3,400 transactions from the Securities Data Corporation tapes between 1980 and 2001 and replace them with a firm from the same Fama-French 12-industry classification that did not engage in a merger. This creates a matched sample of 3,400 pseudo-transactions that can be compared to the actual merger sample. Our method of constructing the comparison sample naturally controls for year and industry effects, since each year we draw a pair of firms from within the same industries as those firms that merged.

Table IV reports the comparison of actual and hypothetical mergers. In Panel A, we report some summary statistics. The first row reports the mean value of the market-to-book spread between bidder and target. It shows that in fact

Table IV

M/B Spreads in Actual and Hypothetical Mergers

This table reports comparisons between actual and hypothetical mergers. The hypothetical mergers are formed by replacing each bidder and target with another firm from their respective industries that did not merge that year. Panel A reports summary statistics for the key variables. Δ Market Equity denotes the difference in market equity between bidder and target. This is measured as the market price 3 months prior to the acquisition announcement times shares outstanding. In Panel B the dependent variable is one if the observation is a merger, zero otherwise. χ^2 values are for the test that the explanatory power of the regression is zero. ** denotes significance at the 1% level.

Panel A: Summary Statistics

Variable	Mean Value: Mergers	Mean Value: Non-mergers	t(diff)	*p*-value
Acq.(M/B) − Targ.(M/B)	0.142	0.052	2.81	0.005
|Acq.(M/B) − Targ.(M/B)|	0.732	0.999	11.51	0.000
Δ Market Equity	9,102.2	−168.6	13.28	0.000

Panel B: Probit Estimates

Variable	(1)	(2)	(3)	(4)	(5)
|Acq.(M/B) − Targ.(M/B)|	−0.0844		−0.0845	−0.0903	−0.0880
	(10.226)**		(9.685)**	(10.593)**	(9.984)**
Δ Market Equity		0.0028	0.0031		
		(4.573)**	(4.671)**		
|Δ Market Equity|				0.0031	0.0030
				(5.101)**	(5.055)**
Dummy:Acq.(M/B)>Targ.(M/B)					0.1283
					(9.723)**
Pseudo-R^2	0.016	0.033	0.049	0.029	0.040
χ^2	104.579	20.913	122.287	133.265	205.485

the pseudo mergers report a lower mean spread than the actual mergers. The difference between actual and hypothetical mergers is highly statistically significant.

The second row of Panel A gives some insight as to why this is happening. It reports the absolute value of the bidder-target market-to-book spread, which is statistically lower for actual mergers than for hypothetical mergers. Thus, the mean spread for hypothetical mergers is statistically closer to zero because the sampling procedure produces many more low buys high transactions than actually occur in the data. However, the actual magnitude of the difference between bidder and target is greater in these hypothetical transactions than in the actual mergers that occurred. Therefore, in Panel B we focus attention on the absolute value of the market-to-book spread.

The last row of Panel A reports the mean of a key independent variable, the difference in size between the bidder and target. Controlling for the relative size of the bidder and target helps us to control for another alternative explanation, which is simply that firms of a similar size merge, and that this naturally produces a similarity in market-to-book ratios. As we see, actual mergers involve a larger firm acquiring a smaller firm far more often than would occur if firms were paired randomly.

In Panel B, we report a Probit analysis of the probability of being an actual merger on the bidder/target market-to-book spread and size relatives. The key test of our model comes from examining the loading of the merger dummy on the market-to-book spread. In each of the regression specifications, the loading is negative and highly statistically significant, with t-statistics approaching 10 in every case. This indicates that the like buys like effect is significantly stronger in actual mergers than it would be if we paired non-merging firms in the same time periods and industries in which mergers are clustering. Thus, the like buys like effect is not a relic of industry clustering.

This finding holds regardless of the control variables included in the regression. As suggested by Panel A, the size relative is important for explaining which firm-pairings are actual mergers. A larger size differential between bidder and target implies a higher likelihood of merger. This holds regardless of whether we use the raw difference in log market value, or the absolute value of the difference. Since Panel A suggests that many more actual mergers involve an acquirer with a slightly higher market-to-book than the target firm, we also include a dummy for whether this is the case. As column (5) indicates, this dummy is statistically significant, but controlling for this effect does not reduce the importance of the market-to-book spread for predicting merger outcomes.[10]

B. Exploring the Role of Search

Since firms in our model must search to find partners, our model predicts that the tendency for bidder and target market-to-book ratios to converge should be strongest when search costs are low. This provides a natural way to distinguish our model from alternative models of merger activity. For example, if q-theory were the driving motivation behind most mergers, then we should expect to see market-to-book spreads widen as search costs drop.

We offer two proxies for search costs. The first involves looking directly at valuations, since high valuation periods are periods when discount rates are low, all else equal. Low discount rates imply greater patience, and patience translates into low search costs in our model. Thus, the model predicts that matching should be stronger in high valuation periods.

The second proxy for search costs is based on a measure of financial market liquidity. Search costs in our model are a metaphor for merger frictions; when

[10] In unreported tables, we regress market-to-book spreads on deal and firm characteristics and find no evidence that controlling for bidder/target size relatives or transaction amounts changes the like buys like result. These tables are available from the authors.

liquidity in capital markets is high, external capital is more readily available and merger frictions are lower. Thus, the prediction with respect to liquidity is that matching in bidder-target market-to-book ratios is stronger when liquidity is high.

B.1. Valuation Levels and Search Frictions

To explore the first prediction, we break the data into high valuation and low valuation regimes and examine bidder/target market-to-book spreads in these different regimes. The results are presented in Table V.

Panel A of Table V groups the data according to whether the merger occurred in a period of high or low valuation for the target firm. We determine high or low valuation by whether the industry median market-to-book ratio was above or below its long-run average value. Column (1) reports that the overall market-to-book dispersion is slightly higher in high valuation periods (0.89 vs. 0.82). Yet in these high valuation periods, when dispersions are higher, the spread in bidder/target market-to-book ratio is one-tenth of what it is in low valuation periods! This evidence supports the idea that assortative matching is higher in high valuation periods than in low valuation periods.

Next we break the sample into high/low valuation periods for both bidder and target. The central test comes from comparing market-to-book differentials in (low, low) valuation periods with those in (high, high) periods. In (high, high) periods, the difference in bidder and target market-to-book ratios is statistically insignificant from zero. On the other hand, in (low, low) periods, bidders have statistically lower market-to-book ratios than targets. Thus, in times of high valuation and high market-to-book dispersion, the market-to-book differential between merging firms is smaller, as predicted by the model.

B.2. Liquidity and Search Frictions

For our measure of liquidity, we use the spread of the interest rate on commercial and industrial loans over the Fed funds rate; this is sometimes known as the C&I spread and is discussed in detail in Harford (2005).[11] In particular, as Harford notes, Lown, Morgan, and Rohatgi (2000) report a high degree of correlation between the C&I spread and opinions obtained from loan officers about prevailing commercial credit standards. Lown et al. (2000) show that when the C&I spread widens, subsequent commercial loan growth drops. Thus, the C&I spread captures both actual and perceived liquidity in the loan market.

Since the C&I only varies at the time-series level, we also construct a time-series indicator of the market-to-book spread. The dependent variable in our analysis is the average each quarter of the squared difference in M/B ratios for

[11] We are grateful to Jarrad Harford for generously sharing his data with us.

Table V

Market-to-Book Differentials in High and Low Valuation Periods

In Panel A, Column (1) reports the overall industry dispersion in ln(M/B) ratios, expressed in standard deviations. Column (2) reports the median ln(M/B) spread between bidders and targets. In Panel B, targets and acquirers are in high/low valuation periods according to whether their industry is above/below its median ln(M/B) ratio. The top cell value is the average difference in ln(M/B) between bidder and target, weighted by transaction value. The middle number is the standard error, and the third number is number of transactions in the relevant valuation period.

Panel A: Target Industry Valuation Levels

	(1)	(2)
Years When M/B Above Industry Median	0.8922	0.0115
Years When Below Industry Median	0.817	0.0965

Panel B: Valuation Levels for Target and Acquirer Industries

	Target Industry	
	Low	High
Acquirer Industry Low	−0.090	−0.251
	0.04	0.11
	1239	153
Acquirer Industry High	0.054	0.006
	0.07	0.029
	206	1,471

bidders and targets. We square the difference because we are averaging over transactions within a quarter, and thus when spreads widen, we do not want the low buys high transactions to cancel out the high buys low transactions. Periods with high disparity in market-to-book ratios might be miscategorized as like buys like if the proportion of low buys high roughly matches the proportion of high buys low transactions.

We report regressions of the market-to-book spread on liquidity measures in Table VI. The central idea behind the test is that when corporate loan spreads are low, search costs should also be low. If our model is correct, then M/B spreads should also be low during these periods.

The significantly positive loading on the C&I spread indicates that M/B spreads widen when the corporate lending market is illiquid. This is exactly in line with our model, because our model predicts that when the MP state is short-lived, spreads should be wider. The data support this.

Of course, there is an alternative explanation that is in line with q-theory. Suppose that when liquidity dries up only the best mergers can afford to occur, and these are precisely the ones in which high buys low. This would also predict that M/B spreads are widest at points of market illiquidity: As liquidity grows, money gets cheaper, and progressively less attractive mergers occur, driving down the market-to-book spread.

While this story would explain the positive loading on the C&I spread, it cannot be reconciled with the loading on merger count (the number of mergers in that quarter). Merger count appears in a univariate regression in column (2). The significantly positive loading on the merger count variable indicates that as the number of mergers in a quarter increases, the marginal M/B spread grows. This is consistent with our model, inasmuch as high merger activity reflects impatience to consummate the merger, since it suggests that the M/B spread grows when firms are impatient.

Columns (4) and (5) repeat columns (1) and (3) but replace the C&I spread averaged over all loans with the C&I spread averaged only over large loans (loans in excess of $1 million). If liquidity patterns in the market for small loans were markedly different from those in the market for large loans, then we might be observing a spurious correlation. As we see in columns (4) and (5), the positive loading on the C&I spread is even stronger when we focus on the end of the market that most closely proxies the type of liquidity relevant for firms initiating mergers.

The first five columns of Table VI report regressions in which the dependent variable is the average squared difference in raw market-to-book differences. Columns (6) through (10) report regressions in which the dependent variable is the average squared difference of industry-demedianed market-to-book differences. Both sets of regressions indicate that the like buys like effect is strongest when search costs are lowest. These findings, coupled with the findings on the discount rate, illustrate that search frictions are an important determinant of the like buys like effect, and that variation in search frictions predicts variation in market-to-book spreads in the manner predicted by our theory.

C. Market Reactions

Ultimately, market reactions to merger announcements tell us about the difference between the expected value of a potential merger and the expected value of an actual, announced merger. In our model, this is simply the difference in the price before and after the merger divided by the price before the merger, $(\pi^M_{i_p j_p} - \pi^{MP}_{i_p})/\pi^{MP}_{i_p}$. Intuitively, this quantity should be affected by the magnitude of expected synergies, the probability of creating them, and a firm's ability to negotiate for the surplus.

Recent work by Song and Walkling (2000, 2005) supports these intuitive predictions. Song and Walkling (2000) find that when an acquisition is announced, the target's rival firms exhibit abnormal stock returns. They find that the magnitude of the return is systematically related to the rival's probability of later becoming an acquisition target itself. Song and Walkling (2005) show that the returns to a bidder's rivals vary monotonically with the bidder's returns: Bidders with large positive returns engender market reactions in rivals that are also positive, and bidders with large negative returns engender

Table VI

Liquidity and Market-to-Book Spreads

This table reports regressions on quarterly level data from 1986:Q3 to 2001:Q4. In columns (1)–(5), the dependent variable is the square of the M/B spread between acquirers and targets, averaged over all transactions in the quarter. In columns (6)–(10), the dependent variable is squared differences in deviations from industry medians, averaged over the quarter. The C&I spread is the spread in the corporate loan rate over the Fed funds rate. Sixty-one quarterly observations are used. Newey-west t-statistics are report beneath point estimates. * and ** denote significance at the 5% and 1% level, respectively.

	Market-to-Book Spread					Industry-Adjusted Market-to-Book Spread				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
C&I spread	1.238		1.454			0.561		0.732		
(all loans)	(3.05)**		(3.91)**			(1.76)		(2.51)*		
C&I spread				1.582	1.659				0.874	0.939
(large loans)				(3.97)**	(4.64)**				(2.80)**	(3.37)**
Merger count		0.003	0.004		0.004		0.003	0.003		0.003
		(2.00)**	(2.79)**		(2.75)**		(2.39)**	(2.80)**		(2.83)**
Constant	−1.144	0.922	−1.938	−1.197	−1.652	−0.022	0.790	−0.651	−0.285	−0.667
	(1.57)	(5.85)***	(2.63)*	(2.13)*	(3.04)**	(0.04)	(6.59)***	(1.12)	(0.64)	(1.53)
R^2	0.12		0.22	0.22	0.30	0.04		0.14	0.11	0.20

negative market reactions in rivals. These findings indicate that market reactions reflect the magnitude of expected synergies and the probability that they occur.

Scarcity, which embodies the firm's ability to negotiate for the surplus, has an ambiguous effect on the market reaction. If a firm is relatively more scarce it will capture more of the surplus from the merger, which should raise the market reaction, all else equal. However, if the market expects such a firm to be more likely to find a partner, a greater fraction of the stock price change will already be impounded into the price. Overall, the reaction of the market may be larger or smaller depending on which effect dominates. If a firm is relatively less scarce the market expects that it is unlikely to find a partner so the news of a merger would be surprising. However, a relatively less scarce firm will have weak negotiating powers and therefore also not extract much of the benefit.

This ambiguity is borne out by other empirical work. Servaes (1991) and Lang et al. (1989) argue that mergers between a high market-to-book acquirer and a low market-to-book target should have the greatest market reaction because these are better mergers. They find some support for this idea but they also find many results that seem inconsistent with this idea. For example, Servaes (1991) finds that the acquirer's q has no effect on the target's reaction. Lang et al. (1989) find that the q estimates from the year before the merger do not explain the bidder's gain, but when low q buys low q the target gains are higher. Furthermore, many of the bidder and target reactions are shown to be unrelated to differences in market-to-book. We would argue that this is because there are many different types of mergers and there is no reason that the best mergers are the ones in which the target is relatively less scarce. Of course, some mergers are redeploying assets to a better use and therefore some mergers where high buys low will have high synergies. Thus, we would not be surprised to find that some high buys low mergers have positive announcement effects.

Ultimately, our model does not capture what part of a merger is not expected by the market. For example, bidders may announce acquisition programs, changing the market's expectations and hence the bidder's market-to-book. Furthermore, in our model average reactions will be positive. The negative reaction that has been documented for acquirer stock prices may arise from the possibility that the acquirer is overvalued. This possibility is outside the scope of this model but is developed fully in Rhodes-Kropf and Viswanathan (2004) and explored empirically in Rhodes-Kropf et al. (2005). Jovanovic and Braguinsky (2004) and McCardle and Viswanathan (1994) argue that the market reaction is due to learning about the investment opportunities of the target and acquirer. The market's reaction is really about the benefits of a particular merger relative to the other potential options. What is the probability of a merger? What is the value from a merger? What do we expect each firm to extract from the merger? Overall market reactions are an interesting but difficult place to look to understand why mergers are occurring or the value they are creating.

V. Conclusion

One of the key results of the property rights theory of the firm is that complementary assets should be under common control. Thus, insofar as this theory helps us understand the boundaries of the firm, we should observe firms with complementary assets or technologies joining together, redrawing the boundaries of the firm in such a way that complementary assets are placed under the command of a single firm.

This paper develops a search model with matching and asset complementarity that ties the property rights theory of the firm to new facts about who buys whom in merger transactions. While conventional wisdom suggests that high asset value firms buy low asset value firms, we show that a more appropriate interpretation is that firms with similar asset valuations purchase one another. That is, mergers exhibit assortative matching: instead of high buys low, we see that like buys like. We argue that this assortative matching is a direct result of asset complementarity and costly search.

In our model, asset complementarity generates merger synergies, but scarcity and costly search determine how that surplus is divided between the two firms. Each firm in the merger market trades off the desire to merge with a better partner with the endogenously reduced bargaining power it would have in such a merger. In equilibrium, if complementarity is important, successful mergers will exhibit a high degree of assortative matching. The best targets and the best acquirers have the best outside opportunities and create the most synergies. Thus, the best merge with the best, the worst with the worst, and so forth.

The decision to merge balances the expected benefits of pairing with the current potential partner against the expected benefits of waiting and finding a more suitable partner. In the model, the identity of the bidder or target is determined by the fact that incomplete contracts require one party to oversee the joint assets of the newly merged firm. This party is one whose managerial talent is best suited to the merged resources of the new entity. On the other hand, the market-to-book ratios of the bidder and target are determined by the relative bargaining power of each party during the merger negotiation. A firm with relatively more scarce assets will, in general, command a larger fraction of the surplus from merger, since it will be able to effectively threaten to break off merger negotiations and find an outside offer of equal or greater value. Markets impound the expected value of this added surplus into market prices ex ante, which means that firms with relatively more scarce assets will have a higher market-to-book ratio, even if their investment opportunities are no different from those of their counterparty in the merger transaction. Thus, the difference in market-to-book ratios could be large or small, as it depends on relative scarcity even when mergers are about complements and firms match on quality.

However, with complementary asset quality, firms of similar quality endogenously choose to seek one another out. Therefore, in equilibrium firms that have the highest market-to-book ratios in their respective industries will choose to merge with one another, leaving other firms with lower market-to-books to merge together.

We test the model's empirical validity in two ways. First, we attempt to rule out alternative explanations for the like buys like effect. Assortative matching in mergers still holds after controlling for industry, time effects, deal characteristics, and bidder-target size relatives. This is a necessary condition for the empirical validity of the model, since if these stories explained the result there would be no need for the model.

A stronger test comes from varying the parameters of the model and observing if assortative matching varies in the manner predicted by theory. For this, we examine how assortative matching varies with search frictions. The model predicts that the like buys like result should strengthen when search frictions drop. The q-theory predicts the opposite. Proxying search frictions with valuation levels and the level of liquidity in the corporate lending market, we find that assortative matching is strongest when valuations are high and capital markets liquid, that is, when search frictions are low.

There are a number of fruitful avenues for future work. Our model allows us to bridge the gap between the property rights theory of the firm and the empirical evidence on merger and acquisition activity, indicating that mergers reflect conscious efforts to redraw the boundaries of the firm in a manner that best allows complementary assets to be placed under common control. But our model is largely silent on the way in which asset complementarities are assessed and realized. Developing new empirical tests along these lines may shed better light on the motives for merger activity by more fully articulating the ways in which complementarities arise.

Our work also has implications for how firms are sold. Boone and Mulherin (2007) document the fact that some firms solicit many merger offers while others negotiate with a single partner. This reflects some equilibrium in the search market in which perceived scarcity of assets and the potential desire to find optimal asset complementarities drive firms to solicit an optimal number of merger offers. Our model offers one way to think about the process by which firms go about searching for and identifying partners. But further tests relating perceived asset complementarities to the pre-announcement details of merger activity may shed additional light on the process by which firms search for appropriate partners.

More broadly, our empirical findings suggest that merger activity may be a rich area for examining the general theories and predictions from assortative matching models. The advantage of merger markets over more traditional applications of search theory, such as marriage markets, is that in merger activity the market values of the firms in question allow us to identify more cleanly matching parameters that may be difficult to observe in other settings.

Our analysis also complements theories of misvaluation, such as Rhodes-Kropf and Viswanathan (2004). In their work, mergers occur for unmodelled fundamental reasons that are confounded or magnified by the possibility of misvaluation. Instead of focusing on why firms merge, their work focuses on when they occur and what transaction medium they use. This paper instead focuses on the underlying rationale for mergers. Combining these ideas could better explain why mergers cluster in time at the industry level (Mitchell and Mulherin (1996), Harford (2005), and Rhodes-Kropf et al. (2005)).

In addition, our main empirical finding—that like buys like—casts new light on many questions in corporate finance. Do diversifying mergers exhibit stronger or weaker assortative matching? Do the long-run valuation consequences of mergers vary according to the degree of assortative matching that occurs? Are like buys like mergers more likely to shed unrelated assets around the time of the merger? Are like buys like transactions more likely to be initiated by better governed firms? Can these findings explain why some conglomerate firms trade at a discount relative to a portfolio of stand-alone firms? We leave these questions for future research.

Appendix

Proof of Proposition 1: Let $i \neq j$, $i, j \in \{A, B\}$ and p, f subscripts represent subtypes, $p, f \in \{h, l\}$ and $p \neq f$. We begin with equations (7) and (15). Given the assumption that condition (3) holds and $\pi^M_{i_p j_f} \leq \pi^{MP}_{i_p}$ equation (15) can we rewritten as

$$\pi^{MP}_{i_p} = \big[\big[1/2\Delta q_i(\theta)\pi^M_{i_p j_p} + (1 - 1/2\Delta q_i(\theta))\pi^{MP}_{i_p}\big]e^{-\Delta\lambda^{MP}} + (1 - e^{-\Delta\lambda^{MP}})\pi^{NM}_{i_p} + \Delta z_{i_p}\big(K^{NM^*}_{i_p}\big)^{\alpha}\big]e^{-r\Delta}. \tag{A1}$$

This and equation (7) can then be rearranged and written as

$$\pi^{MP}_{i_p} = c_{1i}(\Delta)\pi^M_{i_p j_p} + c_{2i}(\Delta)\pi^{NM}_{i_p} + c_{3i}(\Delta), \tag{A2}$$

where

$$c_{1i}(\Delta) = \frac{1/2\Delta q_i(\theta)\exp(-\Delta\lambda^{MP})\exp(-r\Delta)}{1 - (1 - 1/2\Delta q_i(\theta))\exp(-\Delta\lambda^{MP})\exp(-r\Delta)},$$

$$c_{2i}(\Delta) = \frac{(1 - \exp(-\Delta\lambda^{MP}))\exp(-r\Delta)}{1 - (1 - 1/2\Delta q_i(\theta))\exp(-\Delta\lambda^{MP})\exp(-r\Delta)},$$

$$c_{3i}(\Delta) = \frac{\Delta z_{i_p}\big(K^{NM^*}_{i_p}\big)^{\alpha}\exp(-r\Delta)}{1 - (1 - 1/2\Delta q_i(\theta))\exp(-\Delta\lambda^{MP})\exp(-r\Delta)}. \tag{A3}$$

and

$$\pi^{NM}_{i_p} = c_{4i}(\Delta)\pi^{MP}_{i_p} + c_{5i}(\Delta), \tag{A4}$$

where

$$c_{4i}(\Delta) = \frac{(1-\exp(-\Delta\lambda^{NM}))\exp(-r\Delta)}{1-\exp(-\Delta\lambda^{NM})\exp(-r\Delta)},$$

$$c_{5i}(\Delta) = \frac{\Delta z_{i_p}\left(K_{i_p}^{NM^*}\right)^{\alpha}\exp(-r\Delta)}{1-\exp(-\Delta\lambda^{NM})\exp(-r\Delta)}. \tag{A5}$$

Solving for $\pi_{i_p}^{MP}$ we find

$$\pi_{i_p}^{MP} = \frac{c_{1i}(\Delta)\pi_{i_pj_p}^{M} + c_{2i}(\Delta)c_{5i}(\Delta) + c_{3i}(\Delta)}{1-c_{2i}(\Delta)c_{4i}(\Delta)}. \tag{A6}$$

Using Lemma 1 and solving we find

$$\pi_{i_p}^{MP} = \frac{c_{1i}(\Delta)\frac{1}{2}s - c_{1i}(\Delta)\frac{1}{2}\pi_{j_p}^{MP} + c_{2i}(\Delta)c_{5i}(\Delta) + c_{3i}(\Delta)}{1-c_{2i}(\Delta)c_{4i}(\Delta) - \frac{1}{2}c_{1i}(\Delta)}. \tag{A7}$$

Since this equation is true for firms with both types of assets (i, j) we have two equations and two unknowns. Taking the limit as $\Delta \to 0$ and using L'Hôpital's rule we find

$$\lim_{\Delta\to 0} c_{1i}(\Delta) = \frac{1/2q_i(\theta)}{\lambda^{MP}+r+1/2q_i(\theta)}, \quad \lim_{\Delta\to 0} c_{2i}(\Delta) = \frac{\lambda^{MP}}{\lambda^{MP}+r+1/2q_i(\theta)},$$

$$\lim_{\Delta\to 0} c_{3i}(\Delta) = \frac{z_{i_p}\left(K_{i_p}^{NM^*}\right)^{\alpha}}{\lambda^{MP}+r+1/2q_i(\theta)}, \quad \lim_{\Delta\to 0} c_{4i}(\Delta) = \frac{\lambda^{NM}}{\lambda^{NM}+r},$$

$$\lim_{\Delta\to 0} c_{5i}(\Delta) = \frac{z_{i_p}\left(K_{i_p}^{NM^*}\right)^{\alpha}}{\lambda^{NM}+r}.$$

Therefore, using the fact that if $x_n \to$ x and $y_n \to$ y then $x_ny_n \to xy$ we find that

$$\pi_{i_p}^{MP} = \frac{\left(\begin{array}{l} q_i(\theta)\frac{1}{4}s_{i_pj_p} + \left(\frac{\lambda^{MP}}{\lambda^{NM}+r}+1\right)z_{i_p}(K_{i_p}^{NM^*})^{\alpha} \\ \quad - q_i(\theta)\frac{1}{4}\frac{q_j(\theta)\frac{1}{4}s_{i_pj_p} + \left(\frac{\lambda^{MP}}{\lambda^{NM}+r}+1\right)z_{j_p}\left(K_{j_p}^{NM^*}\right)^{\alpha}}{\lambda^{MP}+r+\frac{1}{4}q_j(\theta)-\lambda^{MP}\frac{\lambda^{NM}}{\lambda^{NM}+r}} \end{array}\right)}{\lambda^{MP}+r+\frac{1}{4}q_i(\theta)-\lambda^{MP}\frac{\lambda^{NM}}{\lambda^{NM}+r} - q_i(\theta)\frac{1}{4}\frac{q_j(\theta)\frac{1}{4}}{\lambda^{MP}+r+\frac{1}{4}q_j(\theta)-\lambda^{MP}\frac{\lambda^{NM}}{\lambda^{NM}+r}}}. \tag{A8}$$

Let $D = \lambda^{MP} - \lambda^{MP}\frac{\lambda^{NM}}{\lambda^{NM}+r} + r = r\frac{\lambda^{MP}+\lambda^{NM}+r}{\lambda^{NM}+r}$ and remember that output $y_{i_p} = z_{i_p}(K_{i_p}^{NM^*})^{\alpha}$. Therefore, equation (A8) can be reduced to

$$\pi_{i_p}^{MP} = \frac{(4D + q_j(\theta))Y_{i_p} + q_i(\theta)(s_{i_p j_p} - Y_{j_p})}{4D + q_i(\theta) + q_j(\theta)}. \tag{A9}$$

Using Lemma 1 we can substitute into the negotiation equilibrium, equation (13), and solve for the merger profit

$$\pi_{i_p j_p}^{M} = \frac{(2D + q_i(\theta))(s_{i_p j_p} - Y_{j_p}) + (2D + q_j(\theta))Y_{i_p}}{4D + q_i(\theta) + q_j(\theta)}. \tag{A10}$$

Finally we can substitute into equation (7)

$$\pi_{i_p}^{NM} = \frac{\lambda^{NM}}{\lambda^{NM} + r}\frac{(4D + q_j(\theta))Y_{i_p} + q_i(\theta)(s_{i_p j_p} - Y_{j_p})}{4D + q_i(\theta) + q_j(\theta)} + \frac{y_{i_p}}{\lambda^{NM} + r}. \tag{A11}$$

We must also check that $\pi_{i_p}^{MP} < \pi_{i_p j_p}^{M}$ because we assumed in the writing of equation (A1) that if a suitable partner is found a merger would occur. This reduces to checking that $0 < s_{i_p j_p} - Y_{j_p} - Y_{i_p}$. This is true as long as the merger creates value.

Finally, it is easy to show that firms have no incentive to commit to a non-optimal level of capital since $\frac{d}{dI_i}(\pi_{i_p j_p}^{M} - I_i) \leq 0$ reduces to $K_{i_p} \leq \sqrt[\alpha-1]{\frac{r}{\alpha z_{i_p}}}$. Q.E.D.

Proof of Proposition 2: Let $i \neq j$, $i, j \in \{A, B\}$ and p, f subscripts represent subtypes, $p, f \in \{h, l\}$ and $p \neq f$. A subtype p firm has a disagreement utility of $\pi_{i_p}^{MP}$ and a subtype f firm has a disagreement utility of $\pi_{j_f}^{MP}$. The synergy between a subtype p and a subtype f firm would be $s_{i_p j_f}$. For two firms subtype $p \neq f$ to be unwilling to consummate a deal it must be that $\pi_{i_p j_f}^{M} < \pi_{i_p}^{MP}$, or $s_{i_p j_f} - \pi_{j_f}^{MP} < \pi_{i_p}^{MP}$. Thus, substituting for $\pi_{j_f}^{MP}$ and $\pi_{i_p}^{MP}$ from Proposition 1 finds that different types will not merge with each other as long as

$$4D[s_{i_p j_f} - Y_{j_f} - Y_{i_p}] + q_j(\theta)[s_{i_p j_f} - s_{i_f j_f}] < q_i(\theta)[s_{i_p j_p} - s_{i_p j_f}]. \tag{A12}$$

Thus, if this condition holds then the only mergers will be between the same subtypes. It might seem that a low type firm that found a higher quality partner would always like to merge so that $\pi_{A_2 B_1}^{M} > \pi_{A_2}^{MP}$. This is not the case. Certainly a type l firm that finds a type h firm may generate more production than if it merged with a type l firm, but then the type h firm has more bargaining power. Thus, both firms may find it mutually beneficial to continue searching. (It may also seem that we must be concerned that the condition for the merger partner $\pi_{B_h A_l}^{M} > \pi_{B_h}^{MP}$ holds even though $\pi_{A_l B_h}^{M} < \pi_{A_h}^{MP}$. However, any solution to the Nash bargaining solution must give both firms more than their reservation value so $\pi_{A_l B_h}^{M} < \pi_{A_h}^{MP}$ implies $\pi_{B_h A_l}^{M} < \pi_{B_h}^{MP}$.)

Simultaneously, for two firms subtype $p = f$ to be willing to consummate a deal it must be that $\pi_{i_p j_f}^{M} > \pi_{i_p}^{MP}$, or $s_{i_p j_f} - \pi_{j_p}^{MP} > \pi_{i_p}^{MP}$. Using the equilibrium disagreement utilities and the definition of the synergy we find that the same subtypes will merge with each other as long as

$$4D[s_{i_k j_k} - Y_{i_k} - Y_{j_k}] > 0, \tag{A13}$$

which is true by the assumption that synergies are positive (condition 3). Q.E.D.

Proof of Proposition 3: Let $i \neq j, i, j \in \{A, B\}$ and p subscript represent subtypes, $p \in \{h, l\}$. The more scarce that firms with assets A are, the smaller θ is. The more scarce that firms with assets B are, the larger θ is.

The first part of the proposition is true because $q\prime_A(\theta) \leq 0$ and $q\prime_B(\theta) \geq 0$.

The second part of the proposition is shown by the following: The derivative of each firm's share with respect to θ is

$$\frac{\partial \pi^M_{i_p j_p}}{\partial \theta} = \frac{2D(s_{i_p j_p} - Y_{j_p} - Y_{i_p})(q_i'(\theta) - q_j'(\theta))}{(4D + q_i(\theta) + q_j(\theta))^2}. \tag{A14}$$

We know $q_A'(\theta) < 0$ and $q_B'(\theta) > 0$, therefore, $\frac{\partial \pi^M_{A_p B_p}}{\partial \theta} < 0$ and $\frac{\partial \pi^M_{B_p A_p}}{\partial \theta} > 0$.

The last part of the proposition is true if $\frac{\partial \pi^{NM}_{A_p}}{\partial \theta} < 0$, $\frac{\partial \pi^{MP}_{A_p}}{\partial \theta} < 0$, $\frac{\partial \pi^{M}_{A_p B_p}}{\partial \theta} < 0$, $\frac{\partial \pi^{NM}_{B_p}}{\partial \theta} > 0$, $\frac{\partial \pi^{MP}_{B_p}}{\partial \theta} > 0$, $\frac{\partial \pi^{M}_{B_p A_p}}{\partial \theta} > 0$, since capital in every state is unaffected by θ.

$$\frac{\partial \pi^{NM}_{i_p}}{\partial \theta} = \frac{\lambda^{NM}}{\lambda^{NM} + r} \frac{[4Dq_i'(\theta) + q_j(\theta) q_i'(\theta) - q_i(\theta) q_j'(\theta)](s_{i_p j_p} - Y_{j_p} - Y_{i_p})}{(4D + q_i(\theta) + q_j(\theta))^2}, \tag{A15}$$

$$\frac{\partial \pi^{MP}_{i_p}}{\partial \theta} = \frac{[4Dq_i'(\theta) + q_j(\theta) q_i'(\theta) - q_i(\theta) q_j'(\theta)](s_{i_p j_p} - Y_{j_p} - Y_{i_p})}{(4D + q_i(\theta) + q_j(\theta))^2}, \tag{A16}$$

$$\frac{\partial \pi^{M}_{i_p j_p}}{\partial \theta} = \frac{[2Dq_i'(\theta) + q_j(\theta) q_i'(\theta) - 2Dq_j'(\theta) - q_i(\theta) q_j'(\theta)](s_{i_p j_p} - Y_{j_p} - Y_{i_p})}{(4D + q_i(\theta) + q_j(\theta))^2}, \tag{A17}$$

where $D = r\frac{\lambda^{MP} + \lambda^{NM} + r}{\lambda^{NM} + r}$ and output $y_{i_p} = z_{i_p}(K^{NM^*}_{i_p})^{\alpha}$. Therefore, $\frac{\partial \pi^{NM}_{A_p}}{\partial \theta} < 0$ and $\frac{\partial \pi^{MP}_{A_p}}{\partial \theta} < 0$ if

$$\left[4Dq_A'(\theta) + q_B(\theta) q_A'(\theta) - q_A(\theta) q_B'(\theta)\right] < 0, \tag{A18}$$

and $\frac{\partial \pi^{M}_{A_p B_p}}{\partial \theta} < 0$ if

$$\left[2Dq_A'(\theta) + q_B(\theta) q_A'(\theta) - 2Dq_B'(\theta) - q_A(\theta) q_B'(\theta)\right] < 0. \tag{A19}$$

Both are true since $q_A'(\theta) \leq 0$ and $q_B'(\theta) \geq 0$. The proofs for B asset firms are parallel and are omitted. Q.E.D.

Proof of Proposition 4: Let $i \neq j$, $i, j \in \{A, B\}$ and p, f subscripts represent subtypes, $p, f \in \{h, l\}$ and $f \neq p$. The M/B ratio has two components: part that is due to the stand alone firm value and part that is from the potential gain from the merger. Remembering that $K_{i_p}^{NM^*} = K_{i_p}^{MP^*}$, we can write the portion of M/B due to potential merger gains as $(\pi_{i_p}^{MP} - Y_{i_p})/K_{i_p}^{NM^*}$. If firms could not merge their M/B ratio would be $Y_{i_p}/K_{i_p}^{NM^*} = \frac{z_{i_p}(K_{i_p}^{NM^*})^{\alpha-1}}{r} = \frac{1}{\alpha}$. Thus, with mergers, the total M/B ratio pre-merger is

$$\frac{\pi_{i_p}^{MP}}{K_{i_p}^{NM^*}} = \frac{1}{\alpha} + \frac{\pi_{i_p}^{MP} - Y_{i_p}}{K_{i_p}^{NM^*}}, \tag{A20}$$

in the MP state. And

$$\frac{\pi_{i_p}^{NM}}{K_{i_p}^{NM^*}} = \frac{\lambda^{NM}}{\lambda^{NM} + r}\left(\frac{\pi_{i_p}^{MP} + rY_{i_p}}{K_{i_p}^{NM^*}}\right), \tag{A21}$$

in the NMs state.

Proposition 2 ensures that if condition (19) holds then firms will assortatively match. So firms with the same subtype will merge, that is, A_p will merge with B_p. Given that condition (19) holds, Proposition 1 provides the equilibrium solution. Therefore, the M/B ratios can be rewritten

$$\frac{\pi_{i_p}^{MP}}{K_{i_p}^{NM^*}} = \frac{1}{\alpha} + \frac{q_i(\theta)(s_{i_p j_p} - Y_{j_p} - Y_{i_p})}{[4D + q_i(\theta) + q_j(\theta)]K_{i_p}^{NM^*}}, \tag{A22}$$

$$\frac{\pi_{i_p}^{NM}}{K_{i_p}^{NM^*}} = \frac{\lambda^{NM}}{\lambda^{NM} + r}\left((1 + r)\frac{1}{\alpha} + \frac{q_i(\theta)(s_{i_p j_p} - Y_{j_p} - Y_{i_p})}{[4D + q_i(\theta) + q_j(\theta)]K_{i_p}^{NM^*}}\right). \tag{A23}$$

Our naming convention is such that the firm with the lower p has the higher quality and M/B as long as $(s_{i_p j_f} - Y_{i_p} - Y_{j_p})/K_{i_p}^{NM}$ is an increasing function of z_{i_p}. This equation can be rewritten as

$$\left(\frac{1-\alpha}{\alpha}\right)\frac{2(z_M)^{\frac{1}{1-\alpha}} - (z_{i_p})^{\frac{1}{1-\alpha}} - (z_{j_p})^{\frac{1}{1-\alpha}}}{(z_{i_p})^{\frac{1}{1-\alpha}}}. \tag{A24}$$

So the ex ante M/B is increasing in z_{i_p} as long as

$$\frac{d}{dz_{i_p}}\left[2\left(\frac{z_M}{z_{i_p}}\right)^{\frac{1}{1-\alpha}} - \left(\frac{z_{j_p}}{z_{i_p}}\right)^{\frac{1}{1-\alpha}}\right] > 0, \tag{A25}$$

which is clearly true for $z_M = z_{i_p} * z_{j_p}$. Therefore, firms in the lowest ordinal subcategory (highest quality) have the highest M/B ratios pre-merger. Therefore, the search for complementarities results in mergers in which type A firms with the largest M/B will merge with the type B firms with the largest M/B, while type A firms with the second largest M/B will merge with the type B firms with the second largest M/B. Q.E.D.

Proof of Proposition 5: Let $i \neq j$, $i, j \in \{A, B\}$ and p, f subscripts represent subtypes, $p, f \in \{h, l\}$ and $f \neq p$. We need to show that raising the discount rate makes Condition (19) less likely to hold. This condition can be rewritten as

$$s_{i_p j_f} - Y_{i_p} - Y_{j_f} < \frac{1/2q_i(\theta)(s_{i_p j_p} - Y_{i_p} - Y_{j_p})}{2D + 1/2q_i(\theta) + 1/2q_j(\theta)} + \frac{1/2q_j(\theta)(s_{i_f j_f} - Y_{i_f} - Y_{j_f})}{2D + 1/2q_i(\theta) + 1/2q_j(\theta)}, \tag{A26}$$

which can be further written as

$$\begin{aligned} &\frac{1/2q_i(\theta)}{2D + 1/2q_i(\theta) + 1/2q_j(\theta)} \frac{\left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right)}{r^{\frac{1}{1-\alpha}}} \left(2z_{i_p j_p}^{\frac{1}{1-\alpha}} - z_{i_p}^{\frac{1}{1-\alpha}} - z_{j_p}^{\frac{1}{1-\alpha}}\right) \\ &\quad + \frac{1/2q_j(\theta)}{2D + 1/2q_i(\theta) + 1/2q_j(\theta)} \frac{\left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right)}{r^{\frac{1}{1-\alpha}}} \left(2z_{i_f j_f}^{\frac{1}{1-\alpha}} - z_{i_f}^{\frac{1}{1-\alpha}} - z_{j_f}^{\frac{1}{1-\alpha}}\right) \\ &\quad > \frac{\left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right)}{r^{\frac{1}{1-\alpha}}} \left(2z_{i_p j_f}^{\frac{1}{1-\alpha}} - z_{i_p}^{\frac{1}{1-\alpha}} - z_{j_f}^{\frac{1}{1-\alpha}}\right). \end{aligned} \tag{A27}$$

Therefore we need to show that

$$\begin{aligned} &\frac{d}{dr} \frac{1/2q_i(\theta)(s_{j_p} - Y_{i_p} - Y_{j_p})}{2D + 1/2q_i(\theta) + 1/2q_j(\theta)} + \frac{d}{dr} \frac{1/2q_j(\theta)(s_{i_f j_f} - Y_{i_f} - Y_{j_f})}{2D + 1/2q_i(\theta) + 1/2q_j(\theta)} \\ &\quad - \frac{d}{dr}(s_{i_p j_f} - Y_{i_p} - Y_{j_f}) < 0. \end{aligned}$$

Or,

$$\begin{aligned} &1/2q_i(\theta) \frac{1}{1-\alpha} \frac{\left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right)}{r^{\frac{\alpha}{1-\alpha}}} \left(2z_{i_p j_p}^{\frac{1}{1-\alpha}} - z_{i_p}^{\frac{1}{1-\alpha}} - z_{j_p}^{\frac{1}{1-\alpha}}\right) \\ &\quad + 1/2q_j(\theta) \frac{1}{1-\alpha} \frac{\left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right)}{r^{\frac{\alpha}{1-\alpha}}} \left(2z_{i_f j_f}^{\frac{1}{1-\alpha}} - z_{i_f}^{\frac{1}{1-\alpha}} - z_{j_f}^{\frac{1}{1-\alpha}}\right) \\ &\quad > \frac{1}{1-\alpha} \frac{\left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right)}{r^{\frac{\alpha}{1-\alpha}}} \left(2z_{i_p j_f}^{\frac{1}{1-\alpha}} - z_{i_p}^{\frac{1}{1-\alpha}} - z_{j_f}^{\frac{1}{1-\alpha}}\right) (2D + 1/2q_i(\theta) + 1/2q_j(\theta)) \\ &\quad - 2\frac{\delta}{\delta r} D \left(\frac{1}{1-\alpha} \frac{\left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right)}{r^{\frac{1}{1-\alpha}}} \left(2z_{i_p j_f}^{\frac{1}{1-\alpha}} - z_{i_p}^{\frac{1}{1-\alpha}} - z_{j_f}^{\frac{1}{1-\alpha}}\right) \right). \end{aligned}$$

Divide through by $\frac{1}{1-\alpha}\frac{2D+1/2q_i(\theta)+1/2q_j(\theta)}{r}$ and this equation becomes the same as equation (A27) with an additional term subtracted from the right-hand side. Since $\frac{\partial}{\partial r}D > 0$,[12] the subtracted term reduces the right-hand side. Therefore, if there is assortative matching then equation (A27) holds. Thus raising *r* makes it harder for condition (19) to hold. Q.E.D.

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[12] $\frac{\partial}{\partial r}D = \frac{\lambda^{MP}\lambda^{NM}}{(\lambda^{NM}+r)^2} + 1.$

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