# Strategic Ignorance in the Second-Price Auction

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**Abstract**

Suppose bidders may publicly choose *not* to learn their values prior to a second-price auction with costly bidding. All equilibria with truthful bidding exhibit bidder ignorance when bidders are sufficiently few. Ignorance considerations also affect the optimal reserve price.

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- One bidder's ignorance can deter others from participating in a second-price auction.
- For small $n$, every equilibrium with truthful bidding exhibits bidder ignorance.
- The reserve price affects whether bidders learn their values.
- An example illustrates how ignorance considerations can affect the optimal reserve.

## 1 Introduction

In the auction theory literature, bidders are typically assumed either to possess exogenous private information (e.g. Samuelson (1985)) or to acquire private information as a pre-condition of bidding (e.g. Levin and Smith (1994)). However, bidders need not acquire private information to compete. This paper provides an example in which bidders sometimes choose not to learn their private values, even when learning is costless! The key idea is that ignorance can serve as a *bidding deterrent*.

*Simple Example.* Consider a second-price auction with zero reserve price. Two bidders having iid private values $v_i \sim U[0,1]$ *publicly* decide whether to learn their values prior to bidding at cost $c_B \approx 0$. If both

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learn, each bids $v_i$ and earns expected payoff about $\frac{1}{6}$. If both remain ignorant, each almost always bids $\frac{1}{2}$ and earns zero expected payoff. Finally, if exactly one learns, the ignorant bidder bids $\frac{1}{2}$, the informed bids $v_i$ when $v_i > \frac{1}{2} + c_B$ and otherwise does not bid, the ignorant earns expected payoff about $\frac{1}{2} * \left(\frac{1}{2} - 0\right) = \frac{1}{4}$, and the informed earns about $\frac{1}{2} * \left(\frac{3}{4} - \frac{1}{2}\right) = \frac{1}{8}$.

Each bidder prefers ignorance if the other is informed ($\frac{1}{4} > \frac{1}{6}$), but prefers becoming informed if the other is ignorant ($\frac{1}{8} > 0$). Thus, asymmetric equilibria exist in which one bidder is always ignorant, as well as a symmetric equilibrium in which both become informed with probability $\frac{3}{5}$. □

The introduction continues with discussion of related literature. Section 2 extends the Simple Example to allow for $n \geq 2$ bidders and a reserve price. Section 3 concludes.

**Related literature.** To the best of my knowledge, this is the first paper to consider a private-value, sealed-bid auction in which bidders endogenously decide whether to learn their values and sometimes bid without learning. Of course, there is a vast literature on information acquisition in auctions. (Seminal contributions include Milgrom and Weber (1982) and Engelbrecht-Wiggans, Milgrom, and Weber (1983).) Some well-known findings in this literature shed light on when bidders may choose not to acquire private information. For example, in common-value auctions of a single object, a bidder who lacks private information earns zero expected payoff. However, when multiple identical common-value objects are sold in a uniform-price auction, an uninformed bidder can earn a profit by bidding infinity and always winning one object. Strategic ignorance can therefore arise in such settings, albeit for a very different reason.

A key finding here is that a very small bidding cost, together with strategic ignorance, can have a significant effect on the set of equilibria in the second-price auction. Similar in spirit is Campbell (1998), who explores the equilibrium implications of a small bidding cost together with pre-auction cheap talk.

# 2 Equilibrium ignorance

**Model.** $n \geq 2$ bidders have iid private values $V \sim U[\underline{v}, \underline{v}+1]$ where $\underline{v} \geq 0$. First, the seller commits to reserve price $r \geq \underline{v}$. Second, each bidder simultaneously decides whether to *costlessly* learn his private value. The set $I \subset \{1, ..., n\}$ of "ignorant bidders" then becomes common knowledge. Finally, each bidder decides whether to pay $c_B > 0$ to submit a bid ("participate") in a second-price auction with reserve $r$.

Most of the analysis treats $r$ as given, focusing on the later stages of the game played among bidders. I will also assume $c_B \approx 0$, so that costly bidding *by itself* rarely deters participation. Finally, I restrict attention to equilibria with "truthful bidding".

**Definition 1** (Truthful bidding)**.** Bidder $i$'s strategy exhibits "*truthful bidding*" if he (a) bids $v_i$ when participating informed and (b) bids $E[V]$ when participating ignorant.

**Sufficient conditions for equilibrium ignorance.** Whenever the number of bidders is sufficiently small and $c_B \approx 0$, every equilibrium with truthful bidding exhibits bidder ignorance.

**Proposition 1.** *Fix any minimal value $\underline{v}$ and reserve price $r \geq \underline{v}$, and let $R = r - \underline{v}$. There exists $n^*(R) \geq 2$ such that, when $c_B \approx 0$, (i) if $n < n^*(R)$, at least one bidder sometimes remains ignorant in every equilibrium with truthful bidding and (ii) if $n \geq n^*(R)$, an equilibrium with truthful bidding exists in which all bidders always become informed.*

*Proof.* The set of equilibria given minimal value $\underline{v}$ and reserve $r$ is isomorphic to that given $\underline{v} = 0$ and reserve $R = r - \underline{v}$. So, suppose $\underline{v} = 0$. If $R \geq \frac{1}{2}$, ignorant bidders earn zero expected payoff and there obviously exists an all-informed equilibrium for all $n \geq 2$. So, $n^*(R) = 2$. Henceforth, suppose $R < \frac{1}{2}$.

If all bidders always become informed, each bidder's expected payoff is approximately the same as if there were no bidding cost and reserve price $R$: $\frac{1}{n(n+1)} - \frac{R^n}{n} + \frac{R^{n+1}}{n+1}$. Now, suppose that just bidder 1 remains ignorant. Since others bid truthfully whenever they submit a bid, bidding $E[V]$ allows bidder 1 to win with probability *at least* $\frac{1}{2^{n-1}}$ (when $\max_{i \neq 1} V_i < E[V]$) and, in this event, pay *at most* $\max\{R, \max_{i \neq 1} V_i\}$, thereby earning positive expected payoff. So, bidder 1 surely participates. Any informed bidder whose value is less than $E[V]$ is therefore sure to lose! Such bidders will refrain from bidding to avoid paying cost $c_B$, no matter how small that cost may be, allowing bidder 1 to win at the reserve price when $\max_{i \neq 1} V_i < E[V]$. Thus, bidder 1's expected payoff when ignorant is approximately $\frac{1/2 - R}{2^{n-1}}$. Overall, ignorance is profitable for bidder 1 when all others are informed, as long as

$$\frac{1/2 - R}{2^{n-1}} > \frac{1}{n(n+1)} - \frac{R^n}{n} + \frac{R^{n+1}}{n+1}. \tag{1}$$

As the number of bidders increases, the left-hand-side of (1) decays by factor $\frac{1}{2}$, while its right-hand-side decays at a slower rate for all $n \geq 2$. Thus, (1) is only satisfied by $n$ below a threshold $n^*(R)$. Given $n < n^*(R)$, no equilibrium exists in which all bidders are always informed. On the other hand, when $n \geq n^*(R)$, bidders prefer to become informed when all others are informed, so an all-informed equilibrium exists.[1] □

**Equilibria with ignorance.** When $n < n^*(R)$ and $c_B \approx 0$, each bidder prefers to (i) remain ignorant if all others are surely informed and (ii) become informed if anyone else is surely ignorant. Thus, there exist *asymmetric equilibria* in which one bidder always remains ignorant and all others always become informed, as well as a *symmetric equilibrium* in which each bidder becomes informed with probability between zero and one. (If two or more bidders remain ignorant in the symmetric equilibrium, each randomizes whether to participate so as to earn zero expected payoff. Section 2.1 constructs the symmetric equilibrium in a special case with two bidders and a rarely-binding reserve price. This construction extends in a straightforward way to the general case with $n \geq 2$ bidders and any reserve.)

## 2.1 Discussion: reserve prices

The seller's reserve price affects the distribution of values, by influencing bidders' decisions whether to become informed. Thus, the "optimal" reserve in a standard model in which bidders automatically learn their values may no longer be optimal.

**Example.** Suppose that $n = 2$ bidders have values $V \sim U[1, 2]$ and $c_B \approx 0$. If bidders automatically learn their values, the optimal reserve price is non-binding ($r = 1$) given truthful bidding. However, here the optimal reserve price is binding ($r^* > 1$).

**Proposition 2.** *Assuming that bidders play the "symmetric equilibrium" with truthful bidding, the optimal reserve price $r^* > 1$.*

*Proof.* First, I will derive bidders' learning and participation strategies in the "symmetric equilibrium" with truthful bidding that bidders are assumed to play given reserve $r = 1 + \Delta$, where $\Delta \approx 0$. If both become informed, each earns expected payoff $\frac{V^{(1)}-V^{(2)}}{2} + O(\Delta^2) + O(c_B) \approx \frac{1}{6}$.[2] (Raising the reserve from 1 to $1+\Delta$ has a second-order effect on bidders' expected surplus when both are informed, while raising the bidding cost has a first-order effect.) If only bidder 1 becomes informed, bidder 2 always bids $E[V]$ while bidder 1 only bids when $v_1 > E[V] + c_B$, leaving bidder 2 to pay the reserve when $v_1 < E[V] + c_B$, while bidder 1 pays $E[V]$ when $v_1 > E[V] + c_B$. Overall, bidder 2's expected payoff is $\frac{1}{4} - \frac{\Delta}{2} + O(c_B)$ while

[1] An extra step (omitted to save space) is needed for the non-generic case when (1) holds with equality at $n^*(R)$.

[2] $O(x^k)$ is shorthand for any (perhaps negative) vanishing term such that $\lim_{x\to 0} \frac{O(x^k)}{x^k} < \infty$.

bidder 1's expected payoff is $\frac{1}{8} + O(c_B)$. Finally, if both remain ignorant, a symmetric continuation equilibrium exists in which each randomizes whether to participate and earns zero expected surplus. (Indifference requires that each bidder not participate with probability $O(c_B) \approx 0$.) Each bidder's probability of becoming informed, $p^*(\Delta)$, makes them indifferent to being informed:

$$p^*(\Delta) \approx \frac{1/8 - 0}{1/8 + (1/4 - \Delta/2 - 1/6)} \approx \frac{3}{5}$$
$$\frac{\mathrm{d}p^*(\Delta)}{\mathrm{d}\Delta} \approx \frac{1}{2} * \frac{1/8}{(5/24)^2} = \frac{36}{25}.$$

Expected revenue is $E\left[V^{(2)}\right] + O(\Delta^2) + O(c_B)$ conditional on both being informed, $\frac{E[V]+\Delta}{2} + O(c_B)$ conditional on one being informed, and $E[V] + O(c_B)$ conditional on both remaining ignorant, where $E[V] = \frac{3}{2}$ and $E\left[V^{(2)}\right] = \frac{4}{3}$. Since $\Delta \approx 0$, expected revenue and its derivative are approximately

$$R(\Delta) \approx 1 + p^*(\Delta)^2 * \frac{1}{3} + 2p^*(\Delta)(1 - p^*(\Delta)) * \left(\frac{1}{4} + \frac{\Delta}{2}\right) + (1 - p^*(\Delta))^2 * \frac{1}{2}$$
$$\frac{\mathrm{d}R(\Delta)}{\mathrm{d}\Delta} \approx \frac{\mathrm{d}p^*(\Delta)}{\mathrm{d}\Delta}\left[\frac{2p^*(\Delta)}{3} - \frac{1}{2}\right] + p^*(\Delta)(1 - p^*(\Delta)).$$

Since $p^*(0) \approx \frac{3}{5}$ and $\frac{\mathrm{d}p^*(\Delta)}{\mathrm{d}\Delta} \approx \frac{36}{25}$, $\frac{\mathrm{d}R(\Delta)}{\mathrm{d}\Delta} \approx \frac{36}{25}\left[\frac{2}{5} - \frac{1}{2}\right] + \frac{6}{25} > 0$ for all $\Delta \approx 0$. Thus, the seller can increase expected revenue by imposing a binding reserve price. □

# 3 Concluding remarks

This paper has explored bidders' incentives to acquire private information in a second-price auction with private values. Most auction analyses assume that any bidder who "enters" an auction automatically learns his value. This paper shows that, when bidders can *flaunt their ignorance*, they may sometimes choose to do so to deter others from bidding. Such equilibrium ignorance decreases expected total welfare and can also decrease expected seller revenue.

Fortunately, there are several ways to combat equilibrium ignorance. First and most obvious, one can require that bidders inspect the good for sale. For example, artworks at auction are typically displayed during the bidding, allowing for at least some degree of automatic inspection. Second, one can enable private learning, as when an auction-house allows for private inspection of an artwork before the sale. If bidders cannot credibly publicize their ignorance, they have no incentive to remain ignorant. Yet another approach, suggested by the analysis of Section 2.1, is to raise the reserve price. Even if a bidder expects to face no competition, becoming informed creates a valuable option *not* to win when $v_1 < r$.

As long as the reserve is high enough that this option value outweighs the deterrent benefit of ignorance, bidders will again choose to become informed.

The notion that a seller may benefit by inducing bidders to learn their private values is at least somewhat counter-intuitive. After all, when there are just two bidders with $U[0,1]$ values, the seller earns greater revenue when both bidders remain ignorant ($\frac{1}{2}$) than in the optimal auction when both are informed ($\frac{5}{12}$). However, once bidding is costly, a seller who makes it difficult for bidders to learn their private values will deter participation in the auction. This paper shows that, in some scenarios, even making learning costless may not be sufficient to spur competition.

More broadly, this paper highlights the need for future research that explores bidders' joint decision whether to bid and whether to acquire private information about the good for sale. Past research has emphasized that bidders may learn their values but not bid (e.g. Samuelson (1985)). This paper shows that bidders may bid without learning their values, a subtlety of endogenous information acquisition in auctions that has not yet received much attention in the literature.

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