Performance and Turnover in a Stochastic Partnership

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Motivation: endogenous stability

A long-lasting ("stable") relationship is essential for effective informal incentives.

Stability is often treated as exogenous, but empirical research has found that real-world partnerships display endogenous stability.

- Older partnerships tend to be more stable and more productive.

This paper develops a theory of **endogenous stability** and **dynamic performance** in partnerships, when payoffs evolve over time according to a controlled stochastic process.

Motivation: anonymous matching markets

Anonymous re-matching undermines "trust".

In non-random repeated games with re-matching, social-welfare maximizing equilibria display *endogenous* costs of forming a new partnership – all of which **fail renegotiation-proofness**.

- "burn money" [Kranton (1996), Carmichael MacLeod (1997)]
- "incubation" [Mailath Samuelson (2006)]
- ... many others ...

In stochastic games when **"first impressions"** matter, i.e. with an atomless, future-payoff-relevant initial state:

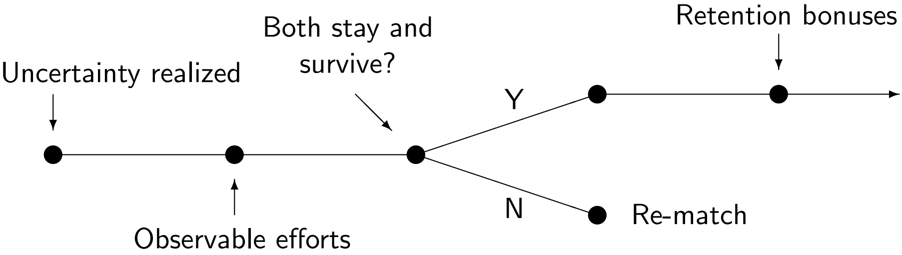
- **only "dating" maximizes social welfare**
- **dating is renegotiation-proof**
- **welfare is decreasing in exogenous matching costs**

Outline of talk

- **Model & overview of results**
- Example #1: dynamic Prisoners' Dilemma
- Example #2: repeated Prisoners' Dilemma
- Concluding remarks

Model

Each period $t = 0, 1, 2, \ldots$ in an active partnership proceeds as:



Uncertainty realized

Observable efforts

Both stay and survive?

Y

N

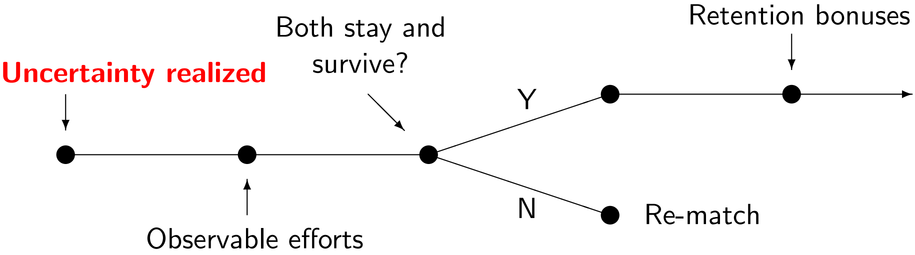
Re-match

Retention bonuses

- At birth or when re-matched, each player (“male” and “female”) is matched with one of opposite gender.
- Players “die” each period with probability $(1-\gamma)$; partnership survives with probability γ^2.
- Each player maximizes expected sum of lifetime payoffs.

Model

Each period $t = 0, 1, 2, \ldots$ in an active partnership proceeds as:



Uncertainty realized

Observable efforts

Both stay and survive?

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N

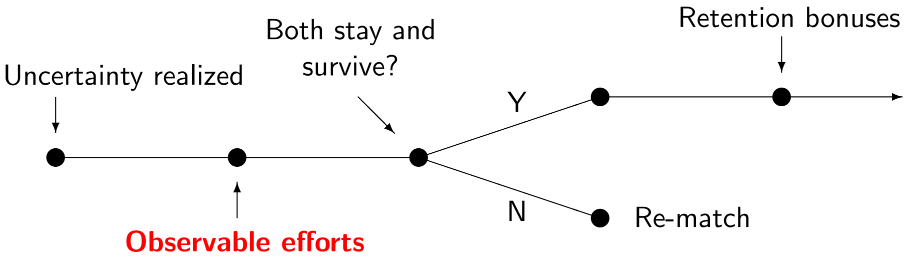
Retention bonuses

Re-match

- State $x_t \in \mathcal{X}_t$ publicly observed; $(\mathcal{X}_t, \succeq)$ partially ordered.
- Distribution of X_t depends only on $(t, x_{t-1}, \mathbf{e}_{t-1})$.
- "Persistence": $x_t' \succeq x_t$ implies $X_{t+1}|(x_t', \mathbf{e}_t)$ FOSD $X_{t+1}|(x_t, \mathbf{e}_t)$ for all $\mathbf{e}_t$.
- "First impression" X_0 includes a public randomization $Z_0 \sim U[0,1]$.

Model

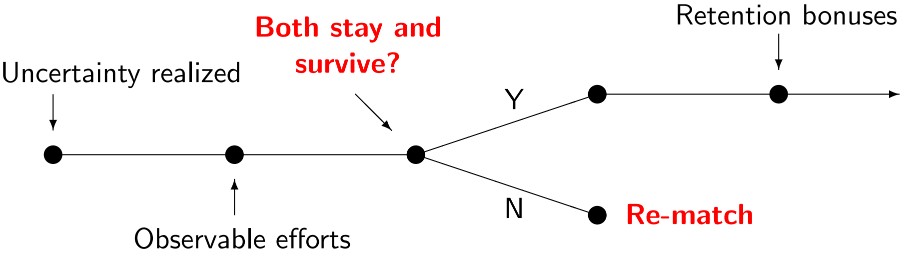
Each period $t = 0, 1, 2, \ldots$ in an active partnership proceeds as:



- Effort $e_{it} \in \mathcal{E}_t$ simultaneous then observed; $(\mathcal{E}_t, \succeq)$ finite and partially ordered with minimal element "0".
- Stage-game payoff $\pi_{it}(e_t; x_t)$ from efforts $e_t = (e_{it}, e_{jt})$ (i) weakly decreasing in e_{it}, (ii) weakly increasing in e_{jt}, (iii) satisfies weakly increasing differences in $(e_t; x_t)$, and (iv) $\pi_{it}(\mathbf{0}; x_t) = 0$ for all x_t.

Model

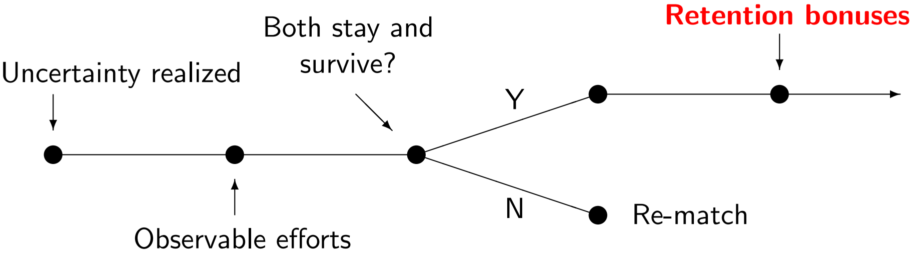
Each period $t = 0, 1, 2, \ldots$ in an active partnership proceeds as:



- The partnership ends if *either* player quits or dies.
- At partnership end, survivors may re-match at cost $m \geq 0$ [$m = 0$ *assumed for most of talk*].
- Any new match is "fresh start" in that payoff process is iid across partnerships, and one's past is unknown to new partner.

Model

Each period $t = 0, 1, 2, ...$ in an active partnership proceeds as:



- Players may make voluntary transfers at any time.
- It is without loss to restrict attention to "retention bonuses".
- *Aside*: "performance bonuses" are not in general sufficient to incentivize optimal equilibrium efforts.

Overview of Results

Joint welfare-maximizing equilibrium construction. Focus on subgame-perfect equilibrium (SPE). In spirit of APS, but additional structure yields comparative statics.

Welfare comparative statics in this eqm. Joint welfare is increasing in the state. If X_t depends only on (t, x_{t-1}), the following hold true in higher states:

- joint stage-game and joint continuation payoff are higher
- conditional stopping time of the partnership is FOSD-higher

Social-welfare maximizing equilibrium characterization in matching market with endogenous outside options. Given "meaningful first impressions",

- "dating" and "honeymoon"
- social welfare decreasing in cost of re-matching

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  - **Joint-welfare maximizing SPE**
  - Social-welfare maximizing SPE
  - Dynamics under joint-welfare maximizing SPE
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Example: dynamic Prisoners' Dilemma

Stage-game payoffs from effort take the form:

	Work	Shirk
Work	w_t, w_t	$-w_t - c_t, w_t + c_t$
Shirk	$w_t + c_t, -w_t - c_t$	0,0

$x_t = (w_t, c_t) \in \mathbf{R}^2_+$ follows an exogenous stochastic process.

Example 1: w_t iid, $c_t = c > 0$ [Ramey Watson 97]

Example 2: $\log(w_t)$ random walk, $c = 0$ [Levinthal 91]

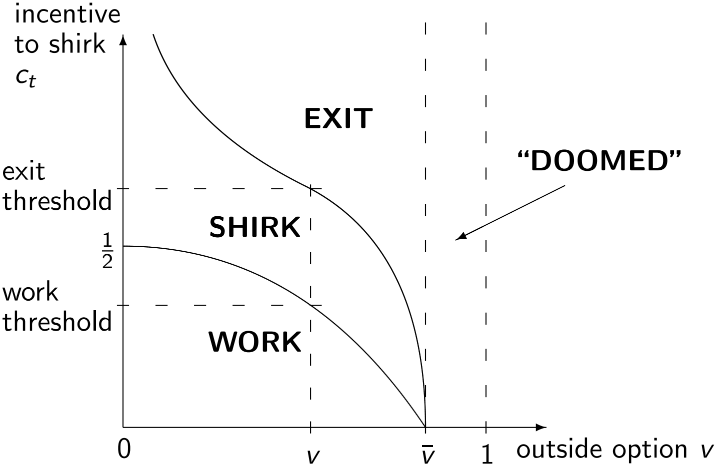
Example 3: $w_t = \frac{\sum_{s \leq t} y_s}{t}$, $y_s \sim N(w, \epsilon)$ iid, $c = 0$ [Jovanovic 79a]

I will focus here on another special case in which $\gamma^2 = \frac{1}{2}$ and

- $w_t = 1$ for all t
- $\log(c_t)$ symmetric random walk

Summary of findings

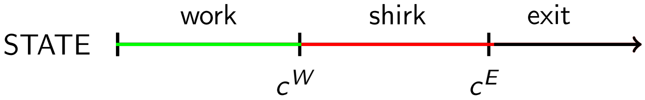
Dynamic game: $\log(c_t)$ evolves according to a random walk.



Structure of the "optimal" SPE.

Claim. Suppose that $v \in [0, 1)$. There exist $c^W \leq c^E$ such that, in a SPE that maximizes joint payoffs:

1. [**good times**] both work and stay when $c_t \leq c^W$
2. [**hard times**] both shirk and stay when $c^W < c_t \leq c^E$
3. [**exit**] both shirk and quit when $c_t > c^E$
4. [**after any deviation**] both shirk and quit



Joint-welfare maximizing SPE

Lemma 1 (Optimal exit threshold): Suppose that both players adopt work threshold c^W. Joint payoff is maximized when they also adopt exit threshold $c^E(c^W; v) = \alpha(v)c^W$, where $\alpha(v)$ is non-increasing in v. ▸ What is $\alpha(v)$?

Prop 1 (Joint-welfare maximizing SPE): Fix any outside option $v \geq 0$. $(c^{*W}(v), \alpha(v)c^{*W}(v))$-threshold equilibrium exists for some $c^{*W}(v) \geq 0$, and this SPE achieves the maximal joint payoff among all SPE.

$c^{*W}(v)$ is the "optimal work threshold"; $c^{*E}(v) = \alpha(v)c^{*W}(v)$ is the "optimal exit threshold."

Special case: worthless outside option.

Without an outside option, each player "effectively dies" when the partnership ends, i.e. discount factor $\delta = \gamma^2$.

Claim 1: $c^{*W}(0) = \sum_{t \geq 1} \delta^t \Pr(C_t \leq C_0)$.

Corollary: (a) $c^{*W}(0) = \frac{\delta}{1-\delta}$ if $C_t = C_0$ for all t. (b) $c^{*W}(0) = \frac{\delta}{2(1-\delta)}$ if $\log(C_t) = \log(C_{t-1}) + X_t$ for all $t \geq 1$, where $X_t \sim U[-\epsilon, \epsilon]$ iid and $\epsilon > 0$.

Proof sketch: Continuation payoff after time-0 efforts in state c_0 is

$$\sum_{t \geq 1} \delta^t \Pr(C_t \leq c^W | C_0 = c_0) = \sum_{t \geq 1} \delta^t \Pr(C_t / C_0 \leq c^W / c_0)$$

$$\geq (\leq) \sum_{t \geq 1} \delta^t \Pr(C_t \leq C_0) = c^{*W}(0) \text{ for all } c_0 \leq (\geq) c^W.$$

Special case: worthless outside option.

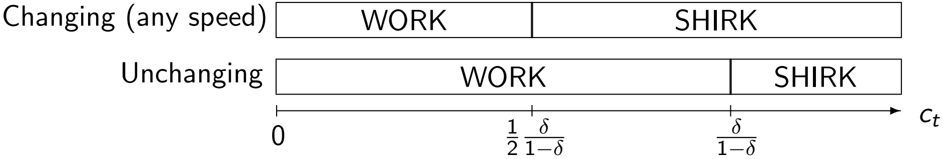


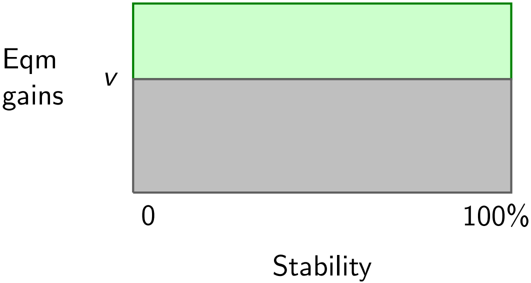
Figure: Illustration of Corollary to Claim 1.

This "discontinuity" in equilibrium outcomes arises from:

- "Discontinuity" of $\Pr(C_t \leq C_0)$
- Discontinuity of maximal SPE joint payoff w.r.t. v

Cooperation is harder in the changing game

Players cooperate in equilibrium in an even smaller set of states, compared to the unchanging case, for two main reasons:



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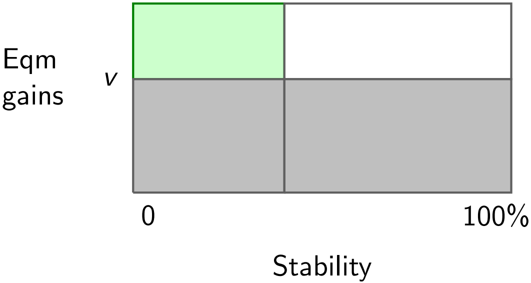
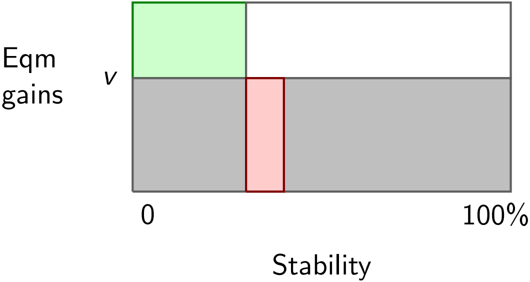
1. **Exit:** fewer future periods in which to cooperate.



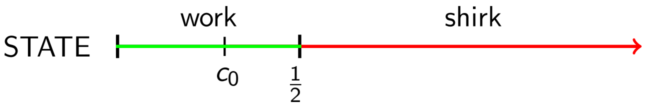
Cooperation is harder in the changing game

Players cooperate in equilibrium in an even smaller set of states, compared to the unchanging case, for two main reasons:

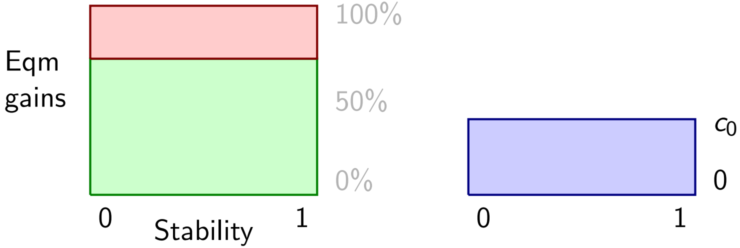
1. **Exit:** fewer future periods in which to cooperate.
2. **Hard times:** losses in periods without cooperation.



Graphical intuition why $c^W = \frac{1}{2}$ when $v = 0$, $\gamma^2 = \frac{1}{2}$



If $c_0 \leq \frac{1}{2}$, cooperation can be supported **both now and frequently in the future**.



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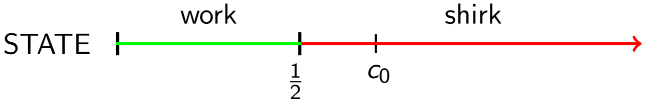
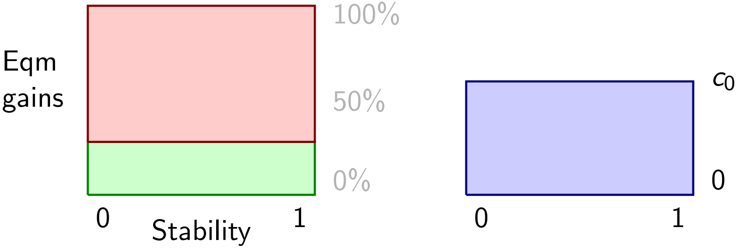


If $c_0 > \frac{1}{2}$, cooperation can be supported **neither now nor frequently in the future**.



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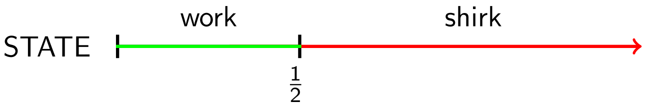
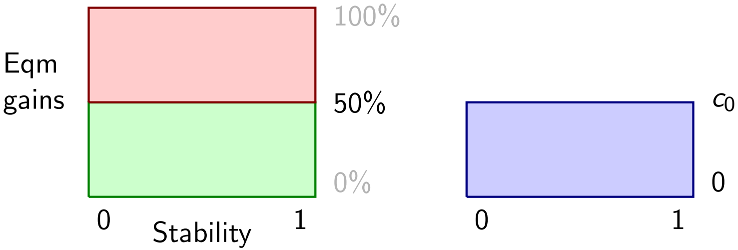
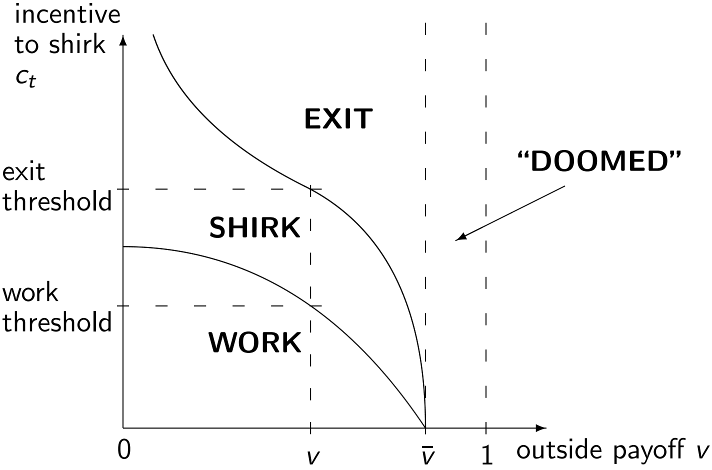


If $c_0 = \frac{1}{2}$, future gains equal current incentive to shirk.



Summary of behavior in the optimal SPE

Dynamic Prisoners' Dilemma: $\log(c_t)$ follows random walk



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Endogenous outside options

Outside option v can be "**generated by SPE play**" if there exists a SPE of the partnership game *given outside option* v in which each player's ex ante expected payoff from a new match equals v.

Let $E\left[\overline{\Pi}^{eqm}(C_0; v)\right]$ denote each player's ex ante expected payoff in the joint-welfare maximizing SPE of Proposition 1, given outside option $v \Rightarrow$ v cannot possibly be generated by SPE play *unless* $v \leq E\left[\overline{\Pi}^{eqm}(C_0; v)\right]$.

$$\overline{v} = \sup\left\{v : E\left[\overline{\Pi}^{eqm}(C_0; v)\right] \geq v\right\} \tag{1}$$

is an *upper bound* on the endogenous outside option.

Efficient benchmark

In the Dynamic PD example considered here, social welfare is maximized when both players work until parted by death, for expected lifetime payoff (and **“efficient outside option”**)

$$v^{eff} = \sum_{t\geq 0} \gamma^t = \frac{1}{1-\gamma}$$

More generally, given any outside option $v \leq v^{eff}$, social welfare is maximized when players work and stay, for expected lifetime payoff

$$\Pi^{eff}(v) = 1 + \gamma(1-\gamma)v + \gamma^2 \Pi^{eff}(v) = \frac{1+\gamma(1-\gamma)v}{1-\gamma^2}. \tag{2}$$

Observation: v^{eff} **cannot be generated by SPE play.**

Social-welfare maximizing SPE = Joint-welfare maximizing SPE?

Proposition 2: Outside option $\overline{v}$ can be generated by SPE play, in a SPE with the following path of play: (i) if the initial public randomization $z_0 \leq \frac{E\left[\overline{\Pi}^{eqm}(C_0;\overline{v})\right]-\overline{v}}{E\left[\overline{\Pi}^{eqm}(C_0;\overline{v})\right]-\gamma\overline{v}}$, then both players shirk and quit at time $t = 0$; (ii) otherwise, play proceeds as in the joint-welfare maximizing SPE of Proposition 1 given outside option $\overline{v}$.

Corollary: If $\overline{v} = E\left[\overline{\Pi}^{eqm}(C_0;\overline{v})\right]$, then **the social-welfare maximizing SPE is also the joint-welfare maximizing SPE**.

Social-welfare maximizing SPE = Joint-welfare maximizing SPE?

Proposition 3: Suppose that C_0 is atomless. Then $E\left[\overline{\Pi}^{eqm}(C_0;\overline{v})\right] = \overline{v}$ and outside option $\overline{v}$ is generated by joint-welfare maximizing SPE play.

Intuitively, C_0 means that a "first date" generates "meaningful first impressions."

▸ Why is C_0 atomless enough?

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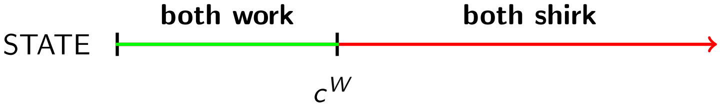
"Survivorship bias"

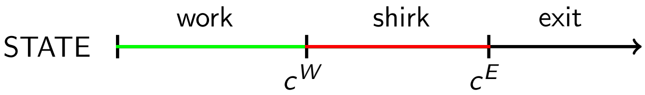


Regime persistence: *partners who have enjoyed several periods of cooperation are more likely to remain cooperative.*

Period	2	3	4	5	10	25
% work resumes	25%	16.7%	12.8%	9.8%	5.0%	2.0%

Table: Probability that cooperation *first* breaks down in period t, conditional on $c_0 = c^W$ and cooperation in periods $1, ..., t-1$.
[Assuming $v = 0$ and $\log(c_{t+1}) = \log(c_t) + \epsilon_t$ where $\epsilon_t \sim U[-1, 1]$ iid.]

“Survivorship bias”



Partnership persistence: *when* $v > 0$, *hazard of exit monotonically decreasing in age once partnership is old enough.*

Consistent with findings on the survival of organizations (Levinthal 1991), employment relationships (Topel Ward 1992), and marriages (Stevenson Wolfers 2007).

“Honeymoon effects”

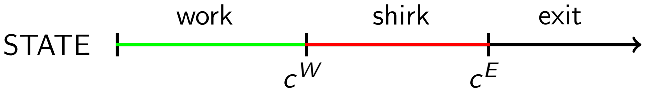
“Honeymoon effects”: If players stay matched after the “first date”, (a) players achieve maximal SPE payoff immediately and (b) depending on distributional details, players may be unlikely to break up for the first several periods of their relationship.

Effect (a) contrasts with received wisdom in literature on repeated games with re-matching, in which it is essential that new parters fail to capture all potential equilibrium gains (e.g. Kranton (1996), Carmichael MacLeod (1997)).

Effect (b) has been noted previously in non-strategic settings where productivity is subject to stochastic shocks (e.g. Fichman Levinthal (1991)).

Effect (b) consistent with findings on the survival of organizations but *not* on the survival of marriages. ▸ How to reconcile this?

Hard times



Exit is typically preceded by hard times. Players remain in a rocky relationship because of the (endogenous) option value created by the possibility of future cooperation.

Dating

When searching, players will churn through a sequence of one-period, zero-effort partnerships.

Atomless C_0 is crucial for the *unique* optimality of dating.

Otherwise, social-welfare maximizing equilibrium play can take many forms, e.g. "*burning money*", "*incubation*" / "*starting small*", etc.

Golden years

After high enough histories, the partnership may become permanent (until death).

Golden years can arise for two distinct reasons:

1. Absorbing increasing subset of the state-space.
2. High efforts in high states, combined with overwhelming positive feedback from effort.

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Example: repeated Prisoners' Dilemma

Stage-game payoffs from effort take the form:

	Work	Shirk
Work	$1, 1$	$-1 - C, 1 + C$
Shirk	$1 + C, -1 - C$	0,0

where $C > 0$ is either (i) non-random or (ii) has an atomless dist'n and is observed by both players at the start of period $t = 0$.

Costless re-matching $m = 0$ and discount factor $\delta = \frac{1}{2}$.

Non-random repeated Prisoners' Dilemma

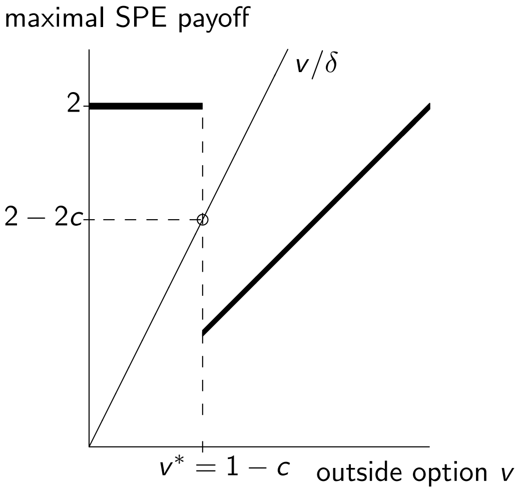

Figure: Non-random cost $c < 1$ with $m = 0$ and $\delta = 1/2$.

Example: non-random repeated Prisoners' Dilemma

Several sorts of equilibria, if played by all partnerships, generate the maximal equilibrium social welfare of $2 - 2c$ for each player:

1. **Burn money**. Each player burns $2c$ at the very start of each partnership; then both work forever.
2. **Incubation**. Players shirk and stay for T periods, then work and stay forever. Expected foregone profit of $2c$.
3. **Dating**. Players work and stay forever if the public randomization is high enough; otherwise, both shirk and quit to re-match. Expected foregone profit of $2c$.

All such equilibria are **not renegotiation-proof**.

Example: random repeated Prisoners' Dilemma

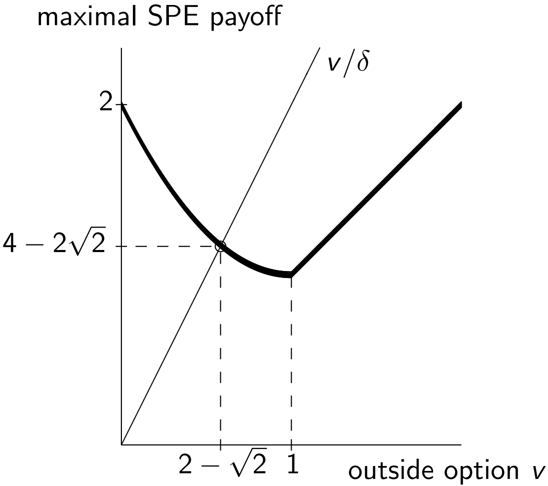

Figure: Random cost $C \sim U[0,1]$ with $m=0$ and $\delta=1/2$.

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Concluding remarks

1. The findings help clarify the conditions under which **"established relationships"** are likely to be important (e.g. Johnson McMillan Woodruff (2002)):
    - private information (e.g. Sobel (1985))
    - *not* social custom to burn money (e.g. Kranton (1996)) unless players learn nothing about productivity at first

2. This paper synthesizes elements from the literatures on **productivity shocks** and **relational contracts**, re-affirming key findings of both in a rich (yet still highly tractable) setting with both shocks and two-sided incentives:
    - survivorship bias (e.g. Jovanovic (1982))
    - honeymoon (e.g. Pissarides (1994))

Comparison / contrast with classic models of relationship dynamics

Green Porter (1984) — imperfect monitoring

- *Different implication*: during cooperative regime, the rate at which cooperation fails falls over time.

Ghosh Ray (1996), Watson (2002) — building trust (signaling)

- *Different implication*: trust not monotonically increasing in duration of partnership, though survivors tend to be those who have built up more trust.

Potential future directions

The tractability and flexibility of the model invite various extensions and suggest various explorations as future work:

- Macroeconomic volatility.
- Labor market dynamics.
- Endogenous learning.

Macroeconomic volatility

Conjecture: Partnership dynamics amplify the welfare effects of exogenous, economy-wide productivity shocks.

- Suppose economy-wide state *multiplies* stage-game payoffs, and $m > 0$ so that re-matching (or "search") is costly.
- "Better economy" ⇔ "Lower re-matching cost"
- ⇒ "choosier players"
- ⇒ (i) less cooperation in any partnership but also (ii) higher-quality matches
- Matching effect (ii) always dominates incentives effect (i)
- ⇒ lower re-matching cost improves welfare
- ⇒ welfare more than multiplicative in economy-wide state

Labor market dynamics

Conjecture: Labor market activity depends on history as well as the current state of the economy.

- Suppose matching market exhibits network externality and "break down" unless sufficiently many workers and firms want to re-match.
- In "GOOD" economy, high-quality matches abound
- ⇒ downward shock to "OK" will induce market breakdown for a while, until enough existing matches change for the worse
- In "BAD" economy, low-quality matches (or unmatched) abound
- ⇒ upward shock to "OK" induces a spurt of market activity

THANK YOU

Endogenous learning

Public learning about a payoff-relevant parameter fits within the model, where “effort” can affect how much is learned.

Conjecture: Players’ investments to learn more (and learn less) are inversely (and positively) related to the endogenous hazard of exit.

More on the “honeymoon effect”

Stinchcombe (1965) argued that partnerships can be especially unstable when they are young if, among other reasons, players are uncertain about each other’s type and *quickly learn* whether they are a good match.

This is consistent with the dynamics of my model, when one counts “dates” as relationships. Indeed, the model generates a large mass of quickly-dissolving partnerships, followed by a lull in break-ups. This pattern is found in data on American marriages.

▸ Go back

C_0 atomless implies $E\left[\overline{\Pi}^{eqm}(C_0; v)\right]$ continuous in v

$E\left[\overline{\Pi}^{eqm}(C_0; v)\right] = S(c^{*W}(v), c^{*E}(v); v)$ where

$$S(c^W, c^E; v) = \sum_{t \geq 0} \Pr\left(t \leq T(c^E)\right) \Pr\left(c_t \leq c^W | t \leq T(c^E)\right) + v \sum_{t \geq 0} \Pr\left(t = T(c^E)\right) \Pr\left(T_i^{die} > t | t = T(c^E)\right)$$

and $T(c^E)$ denotes stopping time given exit threshold c^E.

- C_0 atomless and $\log(C_t)$ random walk $\Rightarrow$ both C_t atomless and $C_t | (\max\{C_0, ..., C_{t-1}\} \leq c^E)$ atomless for all t
- $\Rightarrow$ all prob terms above are continuous in c^W, c^E
- $\Rightarrow S(c^W, c^E; v)$ continuous in c^W, c^E as well as v.
- $c^{*W}(v)$ continuous in v (Lemma 2 – not proven in talk)
- $\Rightarrow E\left[\overline{\Pi}^{eqm}(C_0; v)\right] = \sup_{c^E \geq 0} S(c^{*W}(v), c^E; v)$ is cont. in v.

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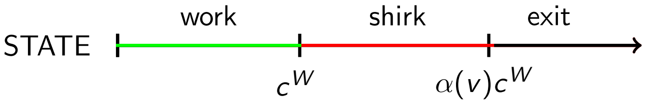
Optimal exit threshold.



work shirk exit

STATE

c^W $\alpha(v)c^W$

- $\log(C_t)$ random walk implies proportionality of work threshold c^W and optimal exit threshold $c^E(c^W; v) = \alpha(v)c^W$.
- $\alpha(v) < 1$ is possible for large enough v.
- higher v raises termination value $\Rightarrow$ optimal to end “earlier”.

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