

# Dynamics in an Evolving Partnership

David McAdams, MIT Sloan\*

September 17, 2007

## Abstract

Repeat players' ability to cooperate in equilibrium depends on how long they expect their relationship to last ("stability"), which itself may change over time. This paper develops a theory of endogenous stability and dynamic performance in partnerships with evolving payoffs, in the context of a simple repeated Prisoners' Dilemma with voluntary entry and exit. More stable partnerships are more productive and tend to generate less volatile profits in equilibrium, and partnerships tend to be least stable when they are neither new nor old. Further, players in a productive partnership prefer to face a less volatile environment while players in an unproductive partnership prefer to face a more volatile environment.

---

\*Email: [mcadams@mit.edu](mailto:mcadams@mit.edu), Post: E52-448 MIT Sloan, 50 Memorial Drive, Cambridge, MA 02142. I thank George-Marios Angeletos, Michael Grubb, Johannes Horner, Marcus Reisinger, Roberto Rigobon, Justin Wolfers, and participants at Gerzensee 2005 for stimulating conversations, and especially Robert Gibbons for his advise and encouragement. I thank MIT's Program on Innovation in Markets and Organizations for financial support. All errors or omissions are my own.

# 1 Introduction

Players in an ongoing interaction often face uncertainty regarding the fundamentals of their relationship. For example, firms engaged in a joint venture may be unsure about future payoffs within their partnership, or whether one of them will have an incentive in the future to pursue a different opportunity. Such uncertainty can make it difficult to sign complete formal contracts, especially if what might change in the relationship is difficult to communicate to an outside party.

At the same time, a long-lasting (or “stable”) relationship is crucial for the effective provision of informal incentives. If exogenous shocks may cause a partnership to end in the near future, the players in that partnership will have less incentive to work, reducing the gains from the relationship. The players then become even more likely to leave the partnership in favor of an outside option, further destabilizing their relationship and making it more difficult to cooperate.

This paper develops a theory of *endogenous stability and performance* in evolving partnerships, in the context of a simple example. Two risk-neutral players having discount factor  $\partial < 1$  decide each period  $t = 0, 1, 2, \dots$  whether to form a partnership, after which they play a symmetric Prisoners’ Dilemma with observable actions each period until one of them ends the relationship. While the partnership is active, players’ payoffs

	Work	Shirk
Work	1, 1	$-1 - d(\theta_t), 1 + d(\theta_t)$
Shirk	$1 + d(\theta_t), -1 - d(\theta_t)$	0, 0

Figure 1: Stage-game payoffs at time  $t$ , while the partnership persists.

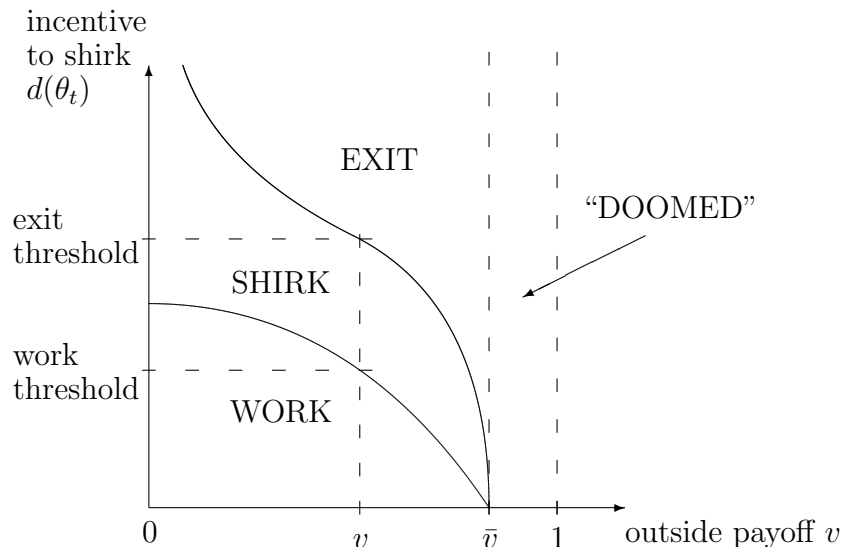


Figure 2: Summary of behavior in the optimal SPE (Theorems 1,3,4).

depend on their effort choices, as shown in Figure 1.<sup>1</sup> Before the partnership forms and after it dissolves, each player gets outside option flow payoff (or “outside payoff”)  $v \geq 0$ . The payoff-relevant state  $\theta_t$  is common knowledge at time  $t$  and evolves according to a known and exogenous random walk. Each player’s incentive to shirk at time  $t$  is  $d(\theta_t)$ , which is assumed to be positive and increasing in the state.

Players’ behavior as partners in the subgame-perfect equilibrium that maximizes joint payoffs (“optimal SPE”) is summarized by Figure 2. Since the outside payoff is fixed, the partnership can be viewed as moving up and down a vertical slice of this figure. When the region labeled “EXIT” is reached, both players shirk and quit: the partnership ends and each player receives his outside payoff in each subsequent period. Until then, both

---

<sup>1</sup>Such payoffs arise naturally in a context in which players bear all of the cost of their own effort but share equally the return to that effort. Suppose that each player generates a return equal to his cost when working alone, but generates an excess return of one when working together with the other player. The payoffs of Figure 1 then arise when the cost and return of individual effort is  $2(1 + d(\theta_t))$ .

players work and stay when in the region labeled “WORK” (also referred to as “good times”) while both shirk and stay when in the region labeled “SHIRK” (or “hard times”).

The partnership is “doomed” whenever the players’ outside payoff is close enough to the payoff from a productive partnership ( $v > \bar{v}$  in Figure 2), in the sense that the players exit the partnership immediately in every state in every equilibrium. More broadly, effective informal incentives can only be sustained in equilibrium when each player’s short-term incentive to shirk is less than a “work threshold”. This work threshold is always strictly less in a changing environment than it would be in an unchanging version of the game. In this sense, the prospect of future change makes it more difficult for players to cooperate.

Players’ entry decisions in the optimal SPE are not depicted in Figure 2. One notable feature of optimal entry is that partnerships never form during hard times. Consequently, new partnerships tend to be relatively stable and productive, a “*honeymoon effect*”.<sup>2,3</sup> At the same time, since the partnership ends once the players’ incentive to shirk first exceeds an “exit threshold”, partnerships that have lasted a long time tend to be those that have received mostly shocks that made the partnership more stable than at first. This *survivorship bias* is consistent with a broad empirical finding that, from employment (Topel and Ward (1992)) to marriage (Stevenson and Wolfers (2007)), long-lasting relationships tend to last even longer.

Jovanovic (1979b) explained the survivorship bias with a model in which a worker

---

<sup>2</sup>The honeymoon effect also arises in non-strategic settings when the attractiveness of entry follows a random walk. See Levinthal (1991) and references therein.

<sup>3</sup>By contrast, Stinchcombe (1965) argues that partnerships can be especially unstable when they begin if, among other reasons, the partners are uncertain about each other’s type and quickly learn whether they are a good match. This might partly explain why the probability of divorce in the United States is especially high during the first few years of marriage (Stevenson and Wolfers (2007)).

learns over time about the productivity of the match with his present firm and quits as soon as he becomes sufficiently pessimistic about the match.<sup>4</sup> Consequently, workers who have remained longer at the same firm are less likely to leave and more likely to be more productive. The key difference here is that partners face a moral hazard problem as well as a learning problem. Whereas the worker in Jovanovic (1979b) always enjoys the full gains from his current match, the partners in this paper must both work to enjoy those gains. Furthermore, there is a range of states – the hard times – in which both partners shirk in the optimal SPE but elect to remain together despite this failure to cooperate. Indeed, unstable partnerships tend to exhibit *endogenous volatility* of performance, as the relationship passes back and forth between good times and hard times.

The strategic nature of the partners' problem qualifies a familiar welfare comparative static regarding how players view *exogenous volatility*. In a standard real-options framework, having the option to exit allows the owner of a volatile project to enjoy the upside after positive shocks while avoiding much of the downside after negative shocks. For this reason, the option to exit is typically more valuable in more volatile environments. However, in this paper, the possibility of exit intensifies the players' moral hazard problem and thereby disrupts the performance of the partnership. Indeed, players who are currently enjoying good times are always strictly worse off in a more volatile environment, given the same current state. On the other hand, players who are currently enduring hard times are strictly better off in a more volatile environment. Intuitively, players in good times fear that a large shock will disrupt their well-functioning relationship, while players in hard times hope that a large shock will restore their poorly-functioning relationship.

---

<sup>4</sup> Also related is Jovanovic (1979a), in which the survivorship bias arises as workers who choose to remain in their current job make firm-specific investments to improve the future performance of the match.

In an appendix, I consider an extension in which players are able to stop and re-start their partnership any number of times, each at some cost. The main qualitative results of the paper extend to this setting, but re-entry *dampens* the disruptive effects of exit on players' ability to cooperate. Also, a welfare distinction arises between entry costs and exit costs. Ex ante, players always prefer lower entry costs but, depending on the distribution of the initial state  $\theta_0$ , may prefer higher exit costs. Intuitively, the reason is that entry and exit costs have different indirect effects on the strength of players' equilibrium incentives. When entry costs are lower, players expect to spend more of the future inside the partnership (after re-entry), so that each player has more to lose if he deviates from his equilibrium strategy. On the other hand, when exit costs are lower, players expect to spend more time outside the partnership.

The rest of the paper is organized as follows. The introduction continues with a discussion of some related literature. Section 2 describes the model and develops some useful preliminaries. Section 3 characterizes the optimal SPE in the special case when the outside payoff is so unattractive that players prefer to remain in the partnership permanently. Section 4 then extends the analysis to the case with an attractive outside option. Proofs are in Appendix A. Finally, Appendix B extends the analysis to a richer setting with re-entry.

### **Related literature.**

The most closely related literature is that studying “dynamic games” in which a payoff-relevant state evolves over time according to a known exogenous process, especially papers such as Haltiwanger and Harrington (1991) (HH) and Bagwell and Staiger (1997) (BS) that consider complete information games with a non-iid underlying state.<sup>5</sup> HH

---

<sup>5</sup>Also related is the substantial literature on dynamic games with an iid state, e.g. Rotemberg and Saloner (1986), as well as the growing literature that considers dynamic games with non-iid states in

and BS consider business cycle models of collusion in which demand transitions between “boom” and “bust” regimes, according to a cyclical process in HH and a Markov process in BS, respectively. A key insight of these analyses is that collusion thrives at those times when the *future* state is most likely to be conducive to collusion, an insight that is very helpful for interpreting this paper’s results as well. However, this paper’s random-walk formulation helps highlight several new insights. For example, an equilibrium rationale is provided for why firms that have colluded for several periods are less likely to compete in the near future than firms that have only colluded for a few periods.

This paper also contributes to a vast literature in the theory of repeated games that considers the extent to which informal incentives can support cooperative behavior. Among the most directly relevant are Abreu, Pearce and Stachhetti (1990) (APS) and Roth (1996). APS provides an iterative procedure by which one could compute the optimal equilibrium using numerical methods, and Roth (1996) applies this procedure to a voluntary partnership game without re-entry that is similar to the game in my basic model. Unlike the present paper, Roth does not provide an explicit characterization of the optimal equilibrium in terms of model primitives and, without such a characterization, is unable to identify most of the comparative statics developed here. However, Roth does identify one important feature of the optimal equilibrium also found here, that partners tend to exert more effort and enjoy greater profits when their partnership is less likely to end in the near future.

A literature from Bull (1987) and MacLeod and Malcomson (1989) to Levin (2003) considers how “relational contracts” can provide incentives for effort in principal-agent settings. Monetary payments play a central role in this literature, as they provide a the presence of imperfect information, e.g. Athey and Bagwell (2001), Athey, Bagwell, and Sanchirico (2004), and Horner and Jamison (2004).

means to reward observable but non-verifiable indicators of effort. By contrast, like APS and the industrial organization literature on firms' dynamic entry and exit decisions (e.g. Hopenhayn (1992) and Ericson and Pakes (1995)), I do not allow monetary payments.<sup>6</sup>

The finding that partnerships are doomed given an attractive enough outside option is similar in spirit to well-known results in the literature on self-enforcing contracts. For example, Baker, Gibbons, and Murphy (1994) show that a firm's ability to leverage subjective performance measures disappears once the firm has access to a sufficiently informative objective measure, so much so that total incentives may decrease as the quality of the objective measure increases. Similarly, Di Tella and McCullough (2002) show that total unemployment insurance may fall as the state provides more welfare support, as this outside option inhibits' families ability to provide incentives for effective self-insurance.

Since the option to exit plays an important role in the analysis, finally, the literature on so-called "option games" is tangentially related. In an option game, players' payoffs depend upon who exercises a real option (e.g. exiting a market or partnership) and when they do so. On the other hand, payoffs realized prior to exit are typically assumed to be exogenous. Consequently, the papers in this literature tend to focus on issues of strategic pre-emption or delay that arise when players prefer to be the first or last to exercise their option. See e.g. Ghemawat and Nalebuff (1985), Grenadier (2002), Solan and Vielle (2001), and Chassang (2007). Such issues are not relevant in my analysis since, by assumption, the players receive the same payoffs outside of the partnership and hence have aligned incentives regarding entry and exit. Rather, this paper focuses on whether

---

<sup>6</sup>This paper's analysis applies to partnerships such as joint ventures, in which each player's participation is essential for any of the players to enjoy a profit, and *not* to firms' decisions to enter or exit a market. See remark (i) in Section 2.

the players can cooperate *during* a partnership and thereby endogenizes the payoffs that can be realized prior to exit.

## 2 Model and preliminaries

Two risk-neutral players having discount factor  $\partial < 1$  decide when to begin and end a partnership in a stochastically evolving environment. If the partnership has not begun prior to period  $t$ , both players simultaneously decide at the beginning of that period to “join” or “stay out”. If both players join, the partnership becomes active and they proceed to play the partnership stage-game described below. Otherwise, each player receives an outside option flow payoff  $v \geq 0$  and the game proceeds to period  $t + 1$ .

*Partnership stage-game in period  $t$ .* Each period that the players’ partnership is active, the players simultaneously decide whether to “work” or “shirk”, with period- $t$  payoffs as shown in Figure 1. Then, the players simultaneously decide whether to “stay” or “quit”. If both players stay, then the partnership remains active in period  $t + 1$ . Otherwise, the partnership ends and each player gets outside payoff  $v$  in all future periods  $t + 1, t + 2, \dots$  (See remark (i).)

Actions and the current “state”  $\theta_t \in \mathbf{R}$  are perfectly observable,  $\theta_t$  becoming known at the beginning of period  $t$ . The solution-concept is subgame-perfect equilibrium (SPE), and the analysis focuses on “optimal SPE” that maximize players’ joint payoff among all SPE.

At time  $t$ , each player’s incentive to shirk if the other player works is  $d(\theta_t)$ , where  $d(\theta)$  is positive, continuous and strictly increasing in the state  $\theta$ , and unbounded above. The state follows a *random walk*  $\{\theta_t\}$ , i.e. per-period motion  $\theta_t - \theta_{t-1}$  is iid and does not depend on the history of past states or past actions. For example, one possibility is that

$d(\theta) = e^\theta$ , in which case the (persistent) percentage change in the players' incentive to shirk is iid across periods. Further, I assume that  $\theta_t - \theta_{t-1}$  has well-defined and bounded density  $f(\cdot)$  with positive density at zero. (See remark (ii).)

*Remark (i).* In an extension provided in the Appendix, qualitatively similar results are obtained when players are able to re-start their partnership after exit. More broadly, it may not be natural in many settings for each player to receive the same payoffs after exit. In a working-paper version of the paper, I establish qualitatively similar results when players' outside payoffs are themselves stochastic and evolve separately over time for each player. Second, player's payoffs after exit may depend on who initiated exit. For example, suppose that the game ends because one player quits while the other player stays. If the players were duopolists, the player who stays would enjoy becoming a monopolist and each player would have a war of attrition-like incentive to defer quitting. This paper's analysis applies more to partnerships such as joint ventures, in which each player's participation is essential for either player to enjoy any gains.

*Remark (ii).* The assumption of positive density at zero implies that the state sometimes increases by a small amount and sometimes decreases by a small amount. Thus, certain interesting degenerate cases are ruled out, such as when the state is certain not to change, or certain to increase over time, or certain to decrease over time. However, equilibrium behavior in these cases is well understood.

**Preliminaries.** Much of the analysis hinges on a few basic properties of the state's cumulative motion over time. Before proceeding, I will define an important shorthand notation and provide some interpretation.

**Definition 1** (“Average *low enough* probability”  $P(x, y)$ ). For all  $x, y \in \mathbf{R}$ , define

$$P(x, y) = \frac{\sum_{t=1}^{\infty} \partial^t \Pr(\theta_t - \theta_0 < x, \max\{\theta_1 - \theta_0, \dots, \theta_{t-1} - \theta_0\} < y)}{\sum_{t=1}^{\infty} \partial^t}$$

For an interpretation, consider an asset that pays \$1 (\$0) when the state is less (greater) than  $\theta_0 + x$  and stops generating any return once the state first strictly exceeds  $\theta_0 + y$ , where  $\theta_0$  is the initial state.  $\Pr(\theta_t - \theta_0 < x, \max\{\theta_1 - \theta_0, \dots, \theta_{t-1} - \theta_0\} < y)$  is the probability that this asset will pay \$1 in period  $t$ .  $P(x, y)$  is a weighted average of these probabilities for  $t = 1, 2, \dots$ , with weights assigned in proportion to the extent that the players discount payoffs received in those periods. Alternatively, the expected present value of the stream of payoffs generated by this asset at times  $t = 1, 2, \dots$  is equal to  $\frac{\partial}{1-\partial}P(x, y)$ , the same as a perpetuity that generates constant flow payoff  $P(x, y)$ .

### 3 Evolving partnerships with no exit

This section explores the *level and endogenous volatility of performance* in an evolving partnership, in the simplest case when the players’ outside payoff  $v = 0$ . In this case, neither player ever quits in the subgame-perfect equilibrium that maximizes their joint payoff. Even though the players face no risk that their relationship will end, they find it more difficult to cooperate than in an unchanging environment. Furthermore, partnership performance is persistent, in the sense that players who have cooperated for several periods are more likely to continue cooperating than those who have only cooperated for a few periods, and vice versa. Section 4 will consider the even more interesting case in which the players have an attractive outside option, when the *endogenous stability* of the partnership becomes an important factor.

The first and most obvious effect of a changing environment is to improve the lot of players whose current incentive to shirk is very high. Cooperation is impossible to

sustain in SPE when the players' incentive to shirk is higher than the maximal future gains from cooperation, regardless of whether the environment is fixed or changing over time. However, in a changing world, players' incentive to shirk may fall enough in the future to allow them to support some future cooperation.

The second and more interesting effect of a changing environment is that partners find it more difficult to cooperate. Intuitively, the expectation of change can drive a vicious cycle that undermines the performance of the partnership. First, players have less incentive to exert costly effort today since exogenous shocks to the environment may make it impossible to support cooperation in some future periods. And as players work less and the prospective future gains from cooperation grow smaller, the players have even less incentive to work, and so on in a self-reinforcing cycle.

What makes cooperation possible in an unchanging partnership is the fact that players can credibly promise to cooperate in *all* future periods, whenever such a promise is sufficient to support present cooperation. In a dynamic environment, such a promise is no longer credible. For example, in any period  $t$  in which  $d(\theta_t) > \frac{\partial}{1-\delta}$ , at least one player must shirk in equilibrium.<sup>7</sup> Consequently, players can only credibly promise to cooperate in those future periods when they have sufficiently little incentive to shirk. Theorem 1 illustrates the extent to which cooperation is disrupted in our dynamic setting.

**Theorem 1.** *Suppose that  $v = 0$ . On the equilibrium path of the SPE that maximizes joint payoffs, both players join at time  $t = 0$  and stay forever. Further, (a) both players work at time  $t$  when  $\theta_t \leq \theta^{W*}$  and (b) both players shirk at time  $t$  when  $\theta_t > \theta^{W*}$ , where*

---

<sup>7</sup> Since each player can guarantee zero payoff by shirking, he must strictly prefer to shirk unless his equilibrium continuation payoff is at least  $d(\theta_t)$ . Since joint continuation payoffs are bounded above by  $\sum_{t=1}^{\infty} \delta^t 2(1-0) = \frac{2\delta}{1-\delta}$ , at least one player must strictly prefer to shirk when  $d(\theta_t) > \frac{\partial}{1-\delta}$ .

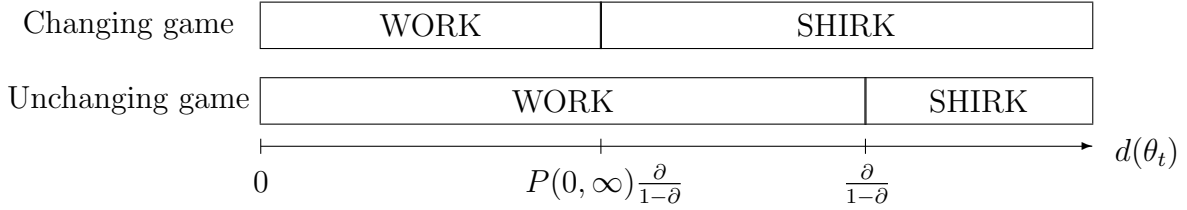


Figure 3: Behavior in the optimal SPE of Theorem 1, relative to the optimal SPE in a corresponding unchanging repeated game.

the “optimal work threshold”  $\theta^{W*}$  solves

$$\frac{d(\theta^{W*})}{\partial/(1-\partial)} = P(0, \infty) \quad (1)$$

Off the equilibrium path, both players join (if relevant) and shirk and stay regardless of the state.

### 3.1 Properties of the optimal equilibrium

**More states in which cooperation fails.** If the state were fixed so that each players’ incentive to shirk was  $d(\theta_0)$  in every period, an equilibrium would exist in which both players work in every period iff  $d(\theta_0) \leq \frac{\partial}{1-\partial}$ . This basic result is consistent with Theorem 1 since  $P(0, \infty) = \frac{\sum_{t=1}^{\infty} \partial^t \Pr(\theta_t \leq \theta_0)}{\sum_{t=1}^{\infty} \partial^t} = 1$  when  $\theta_t = \theta_0$  for all  $t$ . However, this paper restricts attention to settings in which there is some chance that the players’ incentive to shirk will increase and some chance that it will decrease over time, in which case  $0 < P(0, \infty) < 1$ . Thus, in the SPE that maximizes players’ joint payoffs, the players sometimes work but must shirk in a larger set of states than if the state were fixed. See Figure 3.

**Example 1.** Suppose that the players’ discount factor is  $\partial = 1/2$  so that  $\frac{\partial}{1-\partial} = 1$  and  $v = 0$  so that exit is unattractive. Next, let the random walk  $\{\theta_t\}$  be that generated by iid per-period motion  $\theta_{t+1} - \theta_t \sim U[-\varepsilon, \varepsilon]$  for some  $\varepsilon > 0$ . No restrictions are placed on the function  $d(\cdot)$  that maps states into players’ incentive to shirk, other than those

imposed by the model. For instance, all details of the analysis are identical if  $d(\theta) = \theta$  or if  $d(\theta) = e^\theta$ . (All that is required is that  $d(\cdot)$  is continuous and strictly increasing without bound.)

By the symmetry of the random walk,  $\Pr(\theta_t \leq \theta_0) = 50\%$  for all  $t$ . Thus,  $P(0, \infty) = 50\%$  and the optimal work threshold  $\theta^{W*}$  is characterized by an incentive to shirk  $d(\theta^{W*}) = \sum_{t=1}^{\infty} \partial^t 50\% = \frac{1}{2}$ . By contrast, if the state were unchanging, or certain to decrease over time, a SPE exists in which both players work forever as long as their incentive to shirk is less than one.

**Transitions between cooperative and uncooperative regimes.** Since exit is not an attractive option, players are forced to endure strings of periods in which cooperation breaks down and neither player works. Transitions between cooperative and uncooperative regimes arise naturally in a changing environment but have also been shown to arise in stationary settings, such as in Green and Porter (1984)'s model of a repeated game with imperfect public monitoring. In such models, players can sustain cooperation some of the time, but only by committing to transition to a punishment phase whenever the public signal suggests that someone has shirked, even if (in equilibrium) all players know that no one actually shirked. In the sequential equilibrium that maximizes players' expected payoffs, such punishment phases last long enough to deter shirking during a cooperative regime, but no longer.

Consequently, models of unchanging games with imperfect public monitoring generate a very different sort of time-path for transitions between cooperative and uncooperative regimes than in this paper's model of changing games with perfect monitoring. For example, suppose that the players both work in period 0 and both shirk in period 1. In an unchanging game with imperfect monitoring, the equilibrium that maximizes players'

expected joint payoff has the property that the players will endure a fixed number of additional periods in which no one works before returning to a cooperative footing. On the other hand, in a changing game with perfect monitoring, the players typically either cooperate again very quickly or endure a very long uncooperative regime in the optimal equilibrium. The reason is that a partnership which has been uncooperative for several periods is likely to be one that has been subjected to mostly bad exogenous shocks. To return to a cooperative footing, such a partnership must be lucky enough to receive mostly good shocks in the future.

**Example 1 continued.** Suppose that the players were just barely able to sustain cooperation at time 0 ( $\theta_0 = 1/2$ ) and entered an uncooperative regime at time 1 that has lasted through period  $t - 1$  ( $\max\{\theta_1, \dots, \theta_{t-1}\} > \theta_0$ ). The conditional likelihood of transition back to a cooperative regime at time  $t$  is  $\frac{\Pr(\theta_t \leq \theta_0, \max\{\theta_1, \dots, \theta_{t-1}\} > \theta_0)}{\Pr(\max\{\theta_1, \dots, \theta_{t-1}\} > \theta_0)}$ . For  $t = 2$ , this hazard rate is exactly 25%, while for  $t > 2$  it can be estimated by simulating the random walk. Table 1 reports the likelihood that cooperation will first resume in various periods. Most natural examples share a key feature of this example, that the hazard rate of transitions between regimes is decreasing with the duration of the current regime.<sup>8</sup>

□

**Comparative statics on welfare.** By Theorem 1, the expected present value of each player's stream of payoffs in the optimal SPE (call it  $\Pi(\theta_0, v)$ ) can be neatly described

---

<sup>8</sup>Examples can easily be concocted in which this hazard rate is not monotone. For instance, suppose that the state is very likely to either fall by slightly less than two or rise by slightly more than one, and that  $\theta_0 = \theta^*$ . Conditional on becoming uncooperative at time  $t = 1$ , the partners are much more likely to resume cooperation at time  $t = 3$  than at time  $t = 2$ .

Period	2	3	4	5	10	25
% cooperation resumes	25%	16.7%	12.8%	9.8%	5.0%	2.0%

Table 1: Probability that cooperation *first* resumes in period  $t$ , conditional on  $\theta_0 = \theta^{W^*}$  and no cooperation in periods  $1, \dots, t - 1$  (Example 1 continued).

in terms of the stochastic process. As long as  $v = 0$  so that Theorem 1 applies,

$$\Pi(\theta_0, v) = 1\{\theta_0 \leq \theta^*\} + \frac{\partial}{1 - \partial} P(\theta^{W^*} - \theta_0, \infty) \quad (2)$$

where  $\theta^{W^*}$  is defined in (1) and, like  $P(\theta^* - \theta_0, \infty)$ , depends on the underlying stochastic process  $\{\theta_t\}$ . This characterization makes it possible to develop a variety of welfare comparative statics.

Players' incentive to shirk. As one might expect, players are always better off when they have less of an incentive to shirk.

**Corollary 1.** *Suppose that  $v = 0$ .  $\Pi(\theta_0, v)$  is strictly decreasing in  $\theta_0$ .*

Volatility of the environment. To explore how exogenous volatility affects players' welfare, define a family of stochastic processes in terms of the basic process  $\{\theta_t\}$ , as follows. For all  $\sigma > 0$ , define  $\{\theta_t^\sigma\}$  by  $\theta_0^\sigma = \theta_0$  and  $\theta_{t+1}^\sigma - \theta_t^\sigma = \sigma(\theta_{t+1} - \theta_t)$ .

**Corollary 2.** *Suppose that  $v = 0$ . The optimal work threshold does not depend on the volatility of the environment.*

*Intuition for Corollary 2.* The level of the work threshold depends on the strength of the informal incentives that can be brought to bear on the players' moral hazard problem when the current state *equals* the work threshold. The strength of such incentives, in turn, depends on the likelihood that the players' short-term gain from shirking will be less in the future than it is today (in the sense formalized by Definition 1). However,

when  $\sigma' > \sigma$ , the more volatile random walk  $\{\theta_t^{\sigma'}\}$  is equally likely as the less volatile random walk  $\{\theta_t^\sigma\}$  to generate future states that are higher or lower than the current state.

**Corollary 3.** *Suppose that  $v = 0$ . (a) When  $\theta_0 < \theta^{W*}$ , the players are strictly worse off in a more volatile environment. (b) When  $\theta_0 > \theta^{W*}$ , the players are strictly better off in a more volatile environment.*

*Intuition for Corollary 3.* While greater volatility does not change the optimal work threshold, it does increase the likelihood from any given state that a shock will cause some future state to cross this threshold. Thus, players currently in hard times are better off since their partnership is more likely to return to good times, while players currently in good times are worse off since their partnership is more likely to fall into hard times.

## 4 Dynamic partnerships with entry and exit

This section enriches the analysis by considering settings in which each player prefers his outside payoff over an uncooperative partnership ( $v > 0$ ). In this case, each player prefers to end the partnership whenever the prospects for future cooperation are sufficiently bleak. The possibility of future exit amplifies the disruptive effect of a changing environment on cooperation that was explored in Section 3. As players anticipate that they may exit in the future, they have less incentive to work today. This lowers the payoff to remaining in the partnership, which gives players even more incentive to quit, and hence even more incentive to shirk, and so on in a self-reinforcing cycle.

First, I shall consider how play will proceed in the subgame-perfect equilibrium that maximizes players' joint payoff ("optimal SPE") *assuming* that the partnership is active

at time  $t = 0$ . Given a characterization of post-entry play, it will then be straightforward to determine when players will enter in the optimal SPE.

#### 4.1 Preliminaries: threshold behavior in optimal SPE

**Definition 2** (Work threshold  $\theta^W$ ). Consider any SPE. This equilibrium’s “work threshold”  $\theta^W = \inf\{\theta : \text{at least one player shirks with probability one whenever the partnership is active and } \theta_t > \theta \text{ (on or off the equilibrium path)}\}$ . If this set is empty,  $\theta^W = \infty$ .

**Definition 3** (Exit threshold  $\theta^{out}$ ). Consider any SPE. This equilibrium’s “exit threshold”  $\theta^{out} = \inf\{\theta : \text{at least one player quits with probability one whenever the partnership is active and } \theta_t > \theta \text{ (on or off the equilibrium path)}\}$ . If this set is empty,  $\theta^{out} = \infty$ .

**Lemma 1** (Work threshold). (a)  $\theta^W \leq \theta^{out}$  in any SPE. (b)  $\theta^W < \infty$  in any SPE.

**Lemma 2** (Exit threshold). If  $v > 0$ , then  $\theta^{out} < \infty$  in any SPE.

The finding of Lemma 1(a) that  $\theta^W \leq \theta^{out}$  follows from the simple observation that neither player has any incentive to work if some player is *certain* subsequently to quit, regardless of the players’ effort choices. Less obvious is the fact that, on the equilibrium path in the *optimal* SPE, both players subsequently stay in any period in which both players work.

**Lemma 3.** Consider any SPE in which, at some time  $t \geq 0$  after some history  $(h_t, \theta_t)$  when the partnership is active, both players work and then some player quits with positive probability. Another SPE exists in which both players stay whenever both work after this history, and this SPE yields weakly greater joint payoffs at this history than the original SPE.

**Definition 4** ( $(\theta^W, \theta^{out})$ -threshold strategies). The players both adopt  $(\theta^W, \theta^{out})$ -threshold strategies in a SPE if, on the equilibrium path of play when the partnership is active, (a) both work and stay whenever  $\theta_t \leq \theta^W$ , (b) both shirk and stay whenever  $\theta_t \in (\theta^W, \theta^{out}]$  (empty if  $\theta^W = \theta^{out}$ ), and (c) both shirk and quit whenever  $\theta_t > \theta^{out}$  and, off the equilibrium path, (d) both shirk and quit regardless of the state.

By Lemma 1(a) and Theorem 2, it is without loss to restrict attention to SPE in which both players adopt  $(\theta^W, \theta^{out})$ -threshold strategies for some  $\theta^W \leq \theta^{out}$ .

**Theorem 2.** *Consider any SPE having work and exit thresholds  $\theta^W, \theta^{out}$  (but in which players do not necessarily adopt  $(\theta^W, \theta^{out})$ -threshold strategies). Another SPE exists in which the players adopt  $(\theta^W, \theta^{out'})$ -threshold strategies, for some  $\theta^{out'} \geq \theta^{out}$ . Furthermore, the players' joint payoff is weakly higher in this SPE than the original one, at all histories reached on the equilibrium path.*

## 4.2 Doomed partnerships

In an unchanging repeated context, potentially profitable partnerships are always viable in all low enough states. More precisely, suppose that  $v < 1$  so that the future return to a permanently cooperative relationship exceeds the future return of the players' outside option. A SPE exists in which both players cooperate forever as long as the players' incentive to defect is sufficiently small, namely  $d(\theta_0) \leq \frac{\partial(1-v)}{1-\partial}$ . By contrast, in a changing environment, potentially profitable partnerships may be “doomed”.

**Definition 5** (Viable, doomed). A partnership is “viable in state  $\theta$ ” if there exists some SPE with exit threshold  $\theta^{out} \geq \theta$  and “doomed” if every SPE has exit threshold  $\theta^{out} = -\infty$ .

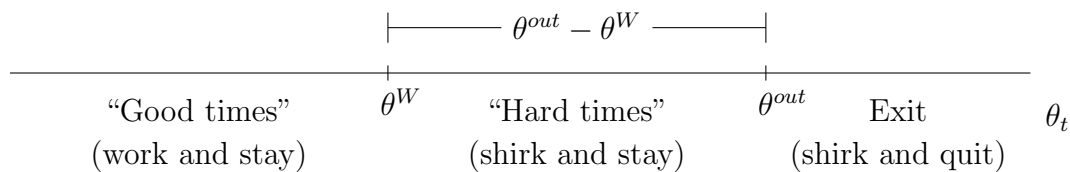


Figure 4: Thresholds and behavioral regimes in an optimal SPE (Theorems 2,4).

**Theorem 3.** *The partnership is doomed whenever the outside payoff  $v > \bar{v}$ , where*

$$\bar{v} = \frac{P(0,0)}{1 - \partial(1 - P(0,0))} \in (0, 1) \quad (3)$$

*Intuition for Theorem 3.* Suppose that  $v > 0$  so that the players have an incentive to exit from any partnership in which the prospects of future cooperation are sufficiently bleak. In the *last* period of the partnership (if reached), neither player has any incentive to work and each player earns a zero payoff, strictly less than what would have been earned had the partnership ended earlier. In a tenuous partnership – that will certainly fold if the players’ incentive to shirk rises any further – the risk that the players will have to endure a period of low future payoffs is especially acute. To compensate for this risk and induce the players in such a tenuous partnership to stay, the partnership must be sufficiently more profitable than the outside option when the players cooperate.

The effect documented in Theorem 3 becomes less significant as players become more patient, in which case only the most marginally-profitable relationships are doomed ( $\lim_{\partial \rightarrow 1} \frac{P(0,0)}{1 - \partial(1 - P(0,0))} = 1$ ). This is to be expected, given that the folk theorem applies to stochastic games, see e.g. Dutta (1995).

### 4.3 Optimal SPE when the partnership is not doomed

Suppose that  $v \in (0, \bar{v})$  so that the partnership is neither permanent nor doomed.

**Theorem 4.** *Suppose that the partnership is active at time  $t = 0$  and that  $0 < v < \bar{v}$ , where  $\bar{v}$  was defined in (3). In the SPE that maximizes joint payoffs, both players adopt  $(\theta^{W^*}, \theta^{out^*})$ -threshold strategies, where  $(\theta^{W^*}, \theta^{out^*})$  is the unique solution to the following system of equations:*

$$v = \frac{P(\theta^{W^*} - \theta^{out^*}, 0)}{1 - \partial(1 - P(0, 0))} \quad (4)$$

$$\frac{d(\theta^{W^*})}{\partial/(1 - \partial)} = P(0, \theta^{out^*} - \theta^{W^*}) - v\partial P(\theta^{out^*} - \theta^{W^*}, \theta^{out^*} - \theta^{W^*}) - v(1 - \partial) \quad (5)$$

*Proof sketch of Theorem 4.* Condition (4) characterizes the optimal distance  $\Delta^* = \theta^{E^*} - \theta^{W^*}$  between the work and exit thresholds, and follows from standard real-options logic. (An active partnership can be thought of as a project that generates a stochastic return of zero or one, depending on whether the state lies below or above the work threshold.) More novel is condition (5), which characterizes the optimal work threshold given  $\Delta^*$ .

Suppose that the initial state *equals* the work threshold,  $\theta_0 = \theta^{W^*}$ , in which case each player must at least weakly prefer to work. If the other player works, shirking and quitting yields payoff  $1 + d(\theta^{W^*}) + v\frac{\partial}{1 - \partial}$ , while working and staying yields one plus the players' equilibrium continuation payoff. In particular, at each future time  $t > 0$ , the partnership will be (a) active with both players working with probability  $\Pr(\theta_t - \theta_0 \leq 0, \max\{\theta_1 - \theta_0, \dots, \theta_{t-1} - \theta_0\} \leq \Delta^*)$ , (b) active with both players shirking with probability  $\Pr(\theta_t - \theta_0 > 0, \max\{\theta_1 - \theta_0, \dots, \theta_{t-1} - \theta_0\} \leq \Delta^*)$ , and (c) inactive with probability  $\Pr(\max\{\theta_1 - \theta_0, \dots, \theta_{t-1} - \theta_0\} > \Delta^*)$ . Thus, each player's continuation payoff is

$$\begin{aligned} & \sum_{t=1}^{\infty} \partial^t \Pr(\theta_t - \theta_0 \leq 0, \max\{\theta_1 - \theta_0, \dots, \theta_{t-1} - \theta_0\} \leq \Delta^*) \\ & + v \sum_{t=2}^{\infty} \partial^t \Pr(\max\{\theta_1 - \theta_0, \dots, \theta_{t-1} - \theta_0\} > \Delta^*) \end{aligned} \quad (6)$$

(Since the players stay at the end of period  $t = 0$ , the earliest time at which the partnership might be inactive is  $t = 2$ .) By Definition 1, the first sum in (6) equals  $\frac{\partial}{1-\partial}P(0, \Delta^*)$  while the second sum equals  $\frac{\partial}{1-\partial}v\partial(1 - P(\Delta^*, \Delta^*))$ . Thus, each player's continuation payoff can be simplified as

$$\frac{\partial}{1-\partial}(P(0, \Delta^*) + v\partial(1 - P(\Delta^*, \Delta^*))) \quad (7)$$

All together, the players can only prefer to work and stay if

$$1 + \frac{\partial}{1-\partial}(P(0, \Delta^*) + v\partial(1 - P(\Delta^*, \Delta^*))) \geq 1 + d(\theta^{W^*}) + v\frac{\partial}{1-\partial}$$

which reduces to (5).

**Optimal entry and the “honeymoon”.** Theorem 4 characterizes optimal SPE play in the subgame after both players agree to join the partnership at time  $t = 0$ . Standard arguments then establish that entry in the optimal SPE has a threshold character, with the partnership forming when the state first falls below an entry threshold  $\theta^{in}$ , and allow one to compute this threshold in terms of model primitives. (To save space, I omit the details.) In particular, the optimal entry threshold is always strictly below the optimal work threshold, i.e.  $\theta^{in} < \theta^W$ .<sup>9</sup> Thus, newly-formed partnerships in equilibrium enjoy a “honeymoon” of at least one period in which both players work and neither quits.

#### 4.4 Properties of the optimal equilibrium

**Even more states in which cooperation fails.** The players' short-term incentive to shirk at the optimal work threshold is strictly lower when  $v > 0$  than in a permanent

---

<sup>9</sup> The players earn  $v > 0$  at time  $t = 0$  and retain the valuable option to enter at a later date if they stay out. Thus, each player strictly prefers not to enter as long as the state exceeds the work threshold, since both players would shirk and earn zero immediately after entry.

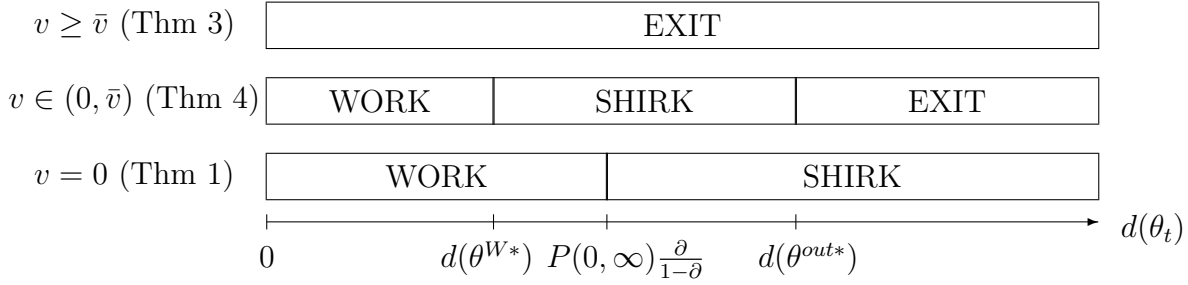


Figure 5: Comparison of behavior in the optimal SPE of Theorems 1,3,4.

partnership.<sup>10</sup> Intuitively, this additional failure to cooperate in equilibrium is due to a combination of factors. First, players cannot be punished for shirking as effectively when their outside option is more attractive. At the same time, players cannot be rewarded for working as effectively when there is some chance that the game will end soon regardless of their current action.

**Partnerships break down before they break up.** The optimal equilibrium exit threshold is *strictly* higher than the optimal equilibrium work threshold. (Since  $v < \bar{v}$ , (4) requires  $\theta^{out*} - \theta^{W*} > 0$ .) In other words, cooperation in a partnership can break down *before* the partners break up. During such “hard times”, players’ flow payoffs from the partnership are strictly less than what they could have received from their outside option.

Players endure hard times near the end of a relationship, rather than exiting sooner, because of the *option value* associated with waiting to exit. However, this option value does not arise as usual from exogenous variation in the productivity of the partnership itself. Indeed, suppose that the players acted as a single decision-maker, seeking to maximize their joint payoff. To such players, the option to exit later is worthless. Since

<sup>10</sup>By definition,  $P(0, y) < P(0, \infty)$  for all  $y < \infty$  and  $P(y, y) > 0$  for all  $y \geq 0$ . Thus, the threshold defined in (5) is strictly less than that defined in (1).

the maximum productivity of the partnership is assumed not to vary over time in our model, such players will either end the game immediately or cooperate forever.

The option to exit later becomes valuable, in equilibrium, because of the endogenous variability of players' behavior. In an unstable relationship, partnership payoffs are relatively volatile as the partnership transitions relatively frequently between hard times and good times. By contrast, in a stable relationship, confident partners produce steady, non-volatile returns.

These features of equilibrium behavior appear consistent with an interesting fact about marital separation in the United States. The National Survey of Families and Households (NSFH) of 1987-1988 asked about two thousand individuals who had experienced marital separation relatively recently to evaluate their experience.<sup>11</sup> When comparing their overall happiness “now, compared to the year before you separated”, 57.8% described their current happiness as “much better” while only 2.9% described it as “much worse.”

One possible explanation of this survey result, consistent with this paper's analysis, is that spouses' expectations regarding the future performance of a marriage (relative to their outside options) changes over time, so that there is an option value of staying married.<sup>12</sup> This option value could arise from exogenous variation in the fundamentals of the marriage (“perhaps my wife will get a raise”) and/or from endogenous variation in spousal behavior (“perhaps my husband will stop cheating on me”). This paper's analysis shows how strategic behavior can *amplify* the option-value effects due to exogenous

---

<sup>11</sup> All respondents had experienced a separation since January 1, 1977. See [www.ssc.wisc.edu/nsfh](http://www.ssc.wisc.edu/nsfh), accessed May 16, 2007.

<sup>12</sup>Other plausible explanations have nothing to do with uncertainty about how the marriage will perform, such as selection bias (if happier individuals are more likely to be surveyed) or choice-supportive bias (if respondents tend to remember the past in ways that help justify their decisions).

variation (“perhaps I will get a raise, after which my husband will stop cheating on me”).

**Long-lasting relationships tend to last even longer.** In equilibrium, long-lived partnerships will tend to be those in which players’ incentive to shirk has decreased more than average. This creates a tendency for longer-lived partnerships to be less likely to dissolve than more newly-formed matches. Of course, this paper is not the first to suggest that relationships’ tenure may be driven in part by a selection effect. For example, Jovanovic (1979b) considers a worker who learns over time about the productivity of the match with his present firm and quits as soon as he becomes sufficiently pessimistic about the match. Consequently, workers who have remained longer at the same firm are more likely to be more productive. This paper differs from Jovanovic (1979b) since the performance of the match is endogenous.

Match performance is also endogenous in Jovanovic (1979a), but through a distinct channel. In that paper, the worker faces a learning problem and must decide whether to invest to increase the future returns of his present match. In equilibrium, he only invests if the match is likely to be sufficiently long-lasting, since then the investment has time to pay off. In this paper, the players face a moral hazard problem and must decide whether to work to increase the current return to their match. In equilibrium, the players only work if the match is likely to be sufficiently long-lasting, since then informal incentives are relatively strong.

**Comparative statics on welfare.** By Theorem 4, the expected present value of each player’s stream of payoffs in the optimal SPE when the partnership is active at time  $t = 0$  (call it  $\Pi(\theta_0, v)$ ) can again be neatly described in terms of the stochastic process.

As long as  $v \in (0, \bar{v})$  so that Theorem 4 applies,

$$\Pi(\theta_0, v) = 1\{\theta_0 \leq \theta^W\} + \frac{\partial}{1 - \partial} (P(\theta^W - \theta_0, \theta^{out} - \theta_0) + v(1 - P(\theta^{out} - \theta_0, \theta^{out} - \theta_0))) \quad (8)$$

where  $(\theta^W, \theta^{out})$  themselves depend on  $v$  and the stochastic process through (4,5).

This characterization allows me to show that the qualitative features of equilibrium derived in Section 3 extend to settings with exit, and to establish some additional comparative statics results.

Players' incentive to shirk. As before, players are better off when their incentive to shirk is less, i.e. in lower states. Intuitively, when the current state is lower, the relationship is likely to last longer *and* spend a larger fraction of its expected future tenure in productive good times.

**Corollary 4.** *Suppose that  $v > 0$ .  $\Pi(\theta_0, v)$  is strictly decreasing in  $\theta_0$  over all  $\theta_0 < \theta^{out}$ .*

Volatility of the environment. As in Corollary 2, the optimal work threshold does not depend on volatility  $\sigma$  of the stochastic process. (See Section 3.1 for the definition of volatility used here.) Furthermore, the distances between the optimal thresholds all scale with  $\sigma$ . Let  $\theta^{in*}(\sigma), \theta^{W*}(\sigma), \theta^{out*}(\sigma)$  be the optimal entry, work, and exit thresholds as a function of the volatility  $\sigma$ , with shorthand  $\theta^{in*} = \theta^{in*}(1)$  and so on.

**Corollary 5.** *Suppose that  $v > 0$ . For all  $\sigma > 0$ , (a)  $\theta^{W*}(\sigma) = \theta^{W*}$ , (b)  $\theta^{out*}(\sigma) = \theta^{out*} + (\sigma - 1)(\theta^{out*} - \theta^{W*})$ , and (c)  $\theta^{in*}(\sigma) = \theta^{in*} + (\sigma - 1)(\theta^{in*} - \theta^{W*})$ .*

As in Corollary 3, the players strictly prefer a less volatile environment when the current state is less than the work threshold and strictly prefer a more volatile environment when the current state is greater than the work threshold.

**Corollary 6.** *Suppose that  $v > 0$ .  $\Pi(\theta_0, v)$  is strictly decreasing in volatility  $\sigma$  for all  $\theta_0 < \theta^{W*}$  and weakly increasing in  $\sigma$  for all  $\theta_0 > \theta^{W*}$ .*

Note that, since players only form a partnership in good times ( $\theta^{in*} \leq \theta^{W*}$ ), Corollary 6 implies that players in newly-formed partnerships always prefer to face a less volatile environment.

Increasing the attractiveness of exit.

An increase in the outside payoff has an ambiguous effect on player welfare. A higher outside payoff always makes the players better off when their partnership is sufficiently unstable (i.e. close to the exit threshold) but can make them worse off if their partnership is sufficiently stable. Intuitively, when exit becomes more attractive, players have an incentive to exit sooner, reducing the expected future gains from cooperation. Players then have less incentive to work, disrupting partnership performance. Corollary 7 illustrates this effect when the outside payoff is increased so much so as to doom the partnership (Theorem 3).

**Corollary 7.** *Suppose that  $v < \bar{v}$ . There exists  $\theta'(v) < \theta''(v) < \theta^{out*}(v)$  such that (a) given any initial state  $\theta_0 \geq \theta''(v)$ , the players strictly prefer outside payoff  $\bar{v}$  than  $v$  and (b) given any initial state  $\theta_0 \leq \theta'(v)$ , the players strictly prefer outside payoff  $v$  than  $\bar{v}$ .*

## 5 Concluding Remarks

This paper has shown how optimal equilibrium play takes a relatively simple and tractable form in a model of partnerships with evolving stage-game payoffs. This model is a very simple one, however, and there are several interesting directions for future research.<sup>13</sup> In

---

<sup>13</sup>While I believe that the main qualitative results derived here will continue to hold, it would also be interesting to extend the present analysis to settings with a much more general partnership game, to allow players' outside payoffs to evolve over time as well as stage-game payoffs, and to allow these outside payoffs to depend on who initiated exit.

particular, in future work I hope to extend the present analysis to a setting with costly learning.

Suppose à la Jovanovic (1979a) that a public signal about the true payoffs of the partnership is revealed every period, the informativeness of which depends on how much the players have chosen to pay in “learning costs”. However, unlike in the learning-by-doing literature, suppose that the partnership need not be active for the players to learn. In equilibrium, when will the players choose to learn about match quality? Since learning introduces volatility into the perceived fundamentals of the relationship, this paper’s welfare comparative statics (Corollary 6) suggest an intriguing conjecture. Players will pay to observe precise signals about their potential relationship prior to becoming partners but seek *not* to learn more about each other once they have become partners, except possibly when their relationship is “on the rocks”, in which case they will again seek to learn as precisely as possible where they truly stand.

## A Proofs

### A.1 Proof of all Corollaries

*Proof of Corollary 1.* Immediate from inspection of (2), since  $P(x, \infty)$  is strictly increasing in  $x$ . □

*Proof of Corollary 2.* Let  $P(x, y; \sigma) = \frac{\sum_{t=1}^{\infty} \delta^t \Pr(\theta_t^\sigma - \theta_0^\sigma \leq x, \max\{\theta_1^\sigma - \theta_0^\sigma, \dots, \theta_{t-1}^\sigma - \theta_0^\sigma\} \leq y)}{\sum_{t=1}^{\infty} \delta^t}$  denote the “average low enough probability” for the case with volatility  $\sigma \neq 1$ , with  $P(x, y; 1) = P(x, y)$  as in Definition 1. Adapting Theorem 1, the optimal work threshold  $\theta^{W^*}(\sigma) = \frac{\partial}{\partial x} P(0, \infty; \sigma)$ . By definition,  $P(x, y; \sigma) = P(x/\sigma, y/\sigma)$ . In particular,  $P(0, \infty; \sigma) = P(0, \infty)$  for all  $\sigma$ , so  $\theta^{W^*}(\sigma) = \theta^{W^*}$  for all  $\sigma$ . □

*Proof of Corollary 3.* Generalizing (2), let  $\Pi(\theta_0, v, \sigma)$  denote the expected present value of each player's payoffs in an active partnership facing outside payoff  $v$  and volatility  $\sigma$  (here for any  $v = 0$ ):

$$\Pi(\theta_0, v, \sigma) = 1\{\theta_0 \leq \theta^{W^*}(\sigma)\} + \frac{\partial}{1 - \partial} P(\theta^{W^*}(\sigma) - \theta_0, \infty; \sigma) \quad (9)$$

$$= 1\{\theta_0 + (\sigma - 1)(\theta_0 - \theta^{W^*}) \leq \theta^{W^*}\} + \frac{\partial}{1 - \partial} P((\theta^{W^*} - \theta_0)/\sigma, \infty) \quad (10)$$

$$= \Pi(\theta_0 + (\sigma - 1)(\theta_0 - \theta^{W^*}), v, 1) \quad (11)$$

(10) follows from (9) since  $\theta^{W^*}(\sigma) = \theta^{W^*}$  for all  $\sigma$  (Corollary 2) and since  $\theta_0 \leq \theta^{W^*}$  iff  $\theta_0 + (\sigma - 1)(\theta_0 - \theta^{W^*}) \leq \theta^{W^*}$  for all  $\sigma > 0$ . If  $\theta_0 < \theta^{W^*}$ , Corollary 1 implies that both players are worse off when  $\sigma > 1$  than when  $\sigma = 1$ , since then  $\theta_0 + (\sigma - 1)(\theta_0 - \theta^{W^*}) > \theta_0$ . Conversely, if  $\theta_0 > \theta^{W^*}$ , both players are better off when  $\sigma > 1$ .  $\square$

*Proof of Corollary 4.* This result is proven later in the Appendix, as Lemma 4. (This result is *not* immediate from inspection of (8).)  $\square$

*Proof of Corollary 5.* Let  $\theta^{W^*}(\sigma)$ ,  $\theta^{in^*}(\sigma)$ ,  $\theta^{out^*}(\sigma)$  denote the optimal thresholds as a function of outside payoff  $v$  and volatility  $\sigma$ . By Theorem 4, the work and exit thresholds are the unique solution to the follow system of equations,<sup>14</sup>

$$\begin{aligned} v &= \frac{P(-\Delta(\sigma), 0; \sigma)}{1 - P(0, 0; \sigma)} = \frac{P(\frac{-\Delta(\sigma)}{\sigma}, 0)}{1 - P(0, 0)} \\ \frac{d(\theta^{W^*}(\sigma))}{\partial/(1 - \partial)} &= P(0, \Delta(\sigma); \sigma) - vP(\Delta(\sigma), \Delta(\sigma); \sigma) \\ &= P\left(0, \frac{\Delta(\sigma)}{\sigma}\right) - vP\left(\frac{\Delta(\sigma)}{\sigma}, \frac{\Delta(\sigma)}{\sigma}\right) \end{aligned}$$

where  $\Delta(\sigma) = \theta^{out^*}(\sigma) - \theta^{W^*}(\sigma)$ . Given that  $\theta^{W^*}$  and  $\Delta = \theta^{out^*} - \theta^{W^*}$  solve this system when  $\sigma = 1$ , the solution when  $\sigma \neq 1$  is  $\theta^{W^*}(\sigma) = \theta^{W^*}$  and  $\Delta(\sigma) = \frac{\Delta}{\sigma}$ . In particular,

---

<sup>14</sup>If  $v \geq \bar{v}$ , then these equations have no finite solution but rather  $\theta^{W^*}(\sigma) = \theta^{out^*}(\sigma) = -\infty$  for all  $\sigma$ . Corollary 5 is trivially satisfied in this case, since no player ever enters so  $\theta^{in^*}(\sigma) = -\infty$  as well.

the work threshold does not depend on the volatility. Thus, as in the proof of Corollary 3, each player's payoff  $\Pi(\theta_0, v, \sigma) = \Pi(\theta_0 + (\sigma - 1)(\theta_0 - \theta^{W*}), v, 1)$ . Thus, given that the optimal exit rule when  $\sigma = 1$  is to exit as soon as the state first exceeds  $\theta^{out*}$ , the optimal exit rule when  $\sigma \neq 1$  must be to exit as soon as the state first exceeds  $\theta^{out*}(\sigma) + (\sigma - 1)(\theta^{out*} - \theta^{W*})$ . Similarly, the optimal entry rule must be to enter as soon as the state is less than  $\theta^{in*}(\sigma) + (\sigma - 1)(\theta^{in*} - \theta^{W*})$ .  $\square$

*Proof of Corollary 6.* Since the work threshold does not depend on  $\sigma$  and the exit and entry thresholds scale with  $\sigma$  (Corollary 5), we can repeat the argument in the proof of Corollary 3 to show that  $\Pi(\theta_0, v, \sigma) = \Pi(\theta_0 + (\sigma - 1)(\theta_0 - \theta^{W*}), v, 1)$ . Now, again as in the proof of Corollary 3, the desired welfare conclusions follow from the fact that players' payoffs are strictly decreasing in the state (Corollary 4).  $\square$

*Proof of Corollary 7.* Let  $\theta'$  be the solution to  $\Pi(\theta', \theta^{W*}(v), \theta^{out*}(v)) = \frac{\bar{v}}{1-\delta}$  and let  $\theta'' = \theta^{out*}(v) - \varepsilon$ . Such  $\theta'$  exists since  $\Pi(\theta_0, \theta^{W*}(v), \theta^{out*}(v))$  is continuous (inspect 8) and decreasing in  $\theta_0$  (Corollary 4), and since  $v \leq \bar{v}$  implies  $\theta^{W*}(v) > -\infty$  and hence  $\lim_{\theta_0 \rightarrow -\infty} \Pi(\theta_0, \theta^{W*}(v), \theta^{out*}(v)) = \frac{1}{1-\delta}$ .

Since the players are indifferent between staying and exiting at state  $\theta^{out*}(v)$ , they are clearly made better off when the outside payoff is increased. By a continuity argument, they must strictly prefer for the outside payoff to increase from  $v$  to  $\bar{v}$  in all states greater than  $\theta'' = \theta^{out*}(v) - \varepsilon$ , for some  $\varepsilon > 0$ . On the other hand, in all states less than  $\theta'$ , the players enjoy a payoff (arbitrarily) close to  $\frac{1}{1-\delta}$  given outside payoff  $v$  versus a payoff of only  $\frac{\bar{v}}{1-\delta}$  given the higher outside payoff  $\bar{v}$ , since in that case the partnership is doomed.  $\square$

## A.2 Proof of Theorems 1,4

The proofs of Theorems 1,4 are very similar, and combined to save space.

By Theorem 2, it is without loss of generality to restrict attention to (symmetric)  $(\theta^W, \theta^{out})$ -threshold strategies. From initial state  $\theta_0$ , each player's continuation payoff  $V(\theta_0; \theta^W, \theta^{out})$  when both players adopt  $(\theta^W, \theta^{out})$ -threshold strategies takes the form

$$\begin{aligned} & \sum_{t=1}^{\infty} \partial^t \left( Pr(\theta_t \leq \theta^W, \max\{\theta_1, \dots, \theta_{t-1}\} \leq \theta^{out} | \theta_0) + v Pr(\max\{\theta_1, \dots, \theta_{t-1}\} > \theta^{out} | \theta_0) \right) \\ &= \frac{\partial}{1 - \partial} P(\theta^W - \theta_0, \theta^{out} - \theta_0) + \partial v \sum_{t=1}^{\infty} Pr(\max\{\theta_1, \dots, \theta_t\} > \theta^{out} | \theta_0) \\ &= \frac{\partial}{1 - \partial} (P(\theta^W - \theta_0, \theta^{out} - \theta_0) + \partial v (1 - P(\theta^{out} - \theta_0, \theta^{out} - \theta_0))) \end{aligned} \quad (12)$$

*The optimal exit threshold.* Consider a hypothetical social planner that is able to dictate when the partnership end, but takes as given that both players will work whenever  $\theta_t \leq \theta^W$  and both shirk whenever  $\theta_t > \theta^W$ . Such a social planner will maximize the players' joint payoff by ending the partnership as soon as the state exceeds a threshold  $\theta^{out}(\theta^W)$ , where

$$2V(\theta^{out}(\theta^W); \theta^W, \theta^{out}(\theta^W)) = 2v \frac{\partial}{1 - \partial} \quad (13)$$

or, if  $v = 0$ , never end the partnership. When the state is equal to this threshold, the players' joint payoff is the same regardless of whether the partnership remains active (left-hand-side of (13)) or ends (right-hand-side of (13)). Substituting (12) and solving for  $v$  in terms of  $\theta^W$  and  $\theta^{out}(\theta^W)$  yields

$$v = \frac{P(\theta^W - \theta^{out}(\theta^W), 0)}{1 - \partial(1 - P(0, 0))} \quad (14)$$

(14) reduces to (4) when  $\theta^W = \theta^{W*}$  and  $v \in (0, \bar{v})$ , and yields  $\theta^{out*} = \infty$  when  $v = 0$ .

Thus, the exit thresholds defined in Theorems 1,4 are socially optimal, conditional on the work thresholds defined in those Theorems.

Further, the players have appropriate incentives to implement this optimal exit threshold. Since they receive equal payoffs, each player is indifferent between staying or quitting at the exit threshold. Finally, by Lemma 4,  $V(\theta, \theta^W, \theta^{out}(\theta^W)) \geq V(\theta^{out}(\theta^W); \theta^W, \theta^{out}(\theta^W))$  for all  $\theta \leq \theta^{out}(\theta^W)$ . So, given that the work threshold is  $\theta^W$ , each player strictly prefers for the partnership to continue when the state is less than  $\theta^{out}(\theta^W)$  and strictly prefers for it to end when the state exceeds  $\theta^{out}(\theta^W)$ .

*The optimal work threshold.* Since  $v < 1$ , both players are always better off when the work threshold is higher. What then is the highest possible equilibrium work threshold, assuming that the exit threshold is chosen optimally to satisfy (14)? When the state equals the work threshold, each player must at least weakly prefer to work given continuation play, i.e.

$$\frac{\partial}{1 - \partial} v + d(\theta^W) \leq V(\theta^W; \theta^W, \theta^{out}(\theta^W)) \quad (15)$$

Since  $V(\theta^{out}(\theta^W); \theta^W, \theta^{out}(\theta^W)) = v \frac{\partial}{1 - \partial}$ , Lemma 4 implies that  $V(\theta; \theta^W, \theta^{out}(\theta^W)) > V(\theta^W; \theta^W, \theta^{out}(\theta^W))$  for all  $\theta < \theta^W$ . Thus, both players are willing to work whenever  $\theta_t \leq \theta^W$ , as long as (15) is satisfied. Further, at the optimal work threshold, (15) must hold with equality; otherwise, an even higher work threshold could be supported in equilibrium. With equality, (15) reduces to (1) when  $v = 0$  and to (5) when  $v \in (0, \bar{v})$ . This completes the proof of both Theorems.  $\square$

### A.3 Proof of Lemma 1

*Proof of (a):*  $\theta^W \leq \theta^{out}$ . Consider any SPE with exit threshold  $\theta^{out}$ . If  $\theta_t > \theta^{out}$ , then some player must exit with probability one regardless of the players' work/shirk choices. Consequently, both players must shirk whenever  $\theta_t > \theta^{out}$ , so that  $\theta^W \leq \theta^{out}$  in this equilibrium.

*Proof of (b):*  $\theta^W < \infty$ . The future joint gains from present cooperation are bounded above by  $\sum_{t=1}^{\infty} \partial^t(2-0) = \frac{2\partial}{1-\partial}$ . Thus, if  $d(\theta) > \frac{\partial}{1-\partial}$ , then at least one player must strictly prefer to shirk regardless of continuation play. Thus,  $\theta^W \leq \bar{\theta} < \infty$  in every SPE, where  $\bar{\theta}$  solves  $d(\bar{\theta}) = \frac{\partial}{1-\partial}$ .  $\square$

## A.4 Proof of Lemma 2

Let  $V_{12}(\theta)$  be the maximum expected joint continuation payoff that can be realized in *any* SPE given initial state  $\theta_0 = \theta$ .<sup>15</sup> Since each player receives continuation payoff  $\frac{\partial v}{1-\partial}$  after exit, at least one player must quit in state  $\theta$  unless  $V_{12}(\theta) \geq \frac{2\partial v}{1-\partial}$ . As shown in the proof of Lemma 1(b), joint payoff is zero in any period in which the partnership is active and  $d(\theta) > \frac{\partial}{1-\partial}$ . So,

$$V_{12}(\theta) \leq 2\partial \Pr\left(d(\theta_1) < \frac{\partial}{1-\partial} \mid \theta_0 = \theta\right) + 2\partial E[V_{12}(\theta_1) \mid \theta_0 = \theta]$$

Define  $\tilde{\theta}$  by the condition  $\Pr\left(d(\theta_1) < \frac{\partial}{1-\partial} \mid \theta_0 = \tilde{\theta}\right) = \frac{v}{2}$ . Thus,

$$V_{12}(\tilde{\theta}) \leq \partial v + 2\partial E[V_{12}(\theta_1) \mid \theta_0 = \tilde{\theta}]$$

$V_{12}(\tilde{\theta}) \geq \frac{2\partial v}{1-\partial} = 2\partial v + 2\partial \frac{v\partial}{1-\partial}$  therefore requires that

$$E[V_{12}(\theta_1) \mid \theta_0 = \tilde{\theta}] \geq \frac{2\partial v}{1-\partial} + v \tag{16}$$

Since joint payoffs are bounded above by  $\frac{2\partial}{1-\partial}$  each period,  $V_{12}(\theta_1) - \frac{2\partial v}{1-\partial} \leq \frac{2\partial(1-v)}{1-\partial}$  for all  $\theta_1$ . Thus, (16) requires that

$$\Pr\left(V_{12}(\theta_1) > \frac{2\partial v}{1-\partial} + v \mid \theta_0 = \tilde{\theta}\right) \geq \frac{v(1-\partial)}{2\partial(1-v)}$$

---

<sup>15</sup>To avoid confusion, I use  $V$ -notation to refer to continuation payoffs and  $\Pi$ -notation to refer to (total) payoffs, i.e. current stage-game payoffs plus continuation payoffs.

Finally, define  $\Delta$  by  $\Pr(\theta_1 - \theta_0 > \Delta) = \frac{v(1-\delta)}{2\delta(1-v)}$ . We conclude that  $V_{12}(\tilde{\theta}) \geq \frac{2\delta v}{1-\delta}$  is only possible if  $V_{12}(\tilde{\theta} + \Delta) \geq \frac{2\delta v}{1-\delta} + v$  and, by induction,  $V_{12}(\tilde{\theta} + K\Delta) \geq \frac{2\delta v}{1-\delta} + Kv$  for all  $K$ . This yields a contradiction since joint continuation payoffs are bounded above by  $\frac{2\delta}{1-\delta}$  when  $v \leq 1$ .  $\square$

## A.5 Proof of Lemma 3

Suppose that, conditional on history  $(h_t, \theta_t)$ , there is a positive probability that both players will work and then at least one of the players will quit.

*Preliminaries.* Let  $\Pi_i(A_1, A_2)$  and  $V_i(A_1, A_2)$  denote, respectively, player  $i$ 's (total) payoff and the time- $t$  present value of all payoffs received after the work-shirk decisions of period  $t$  (including outside payoffs gotten in period  $t$ ), when the players' work-shirk decisions are  $(A_1, A_2) \in \{W, S\}^2$ . ( $W, S$  stand for work and shirk.) I will refer to  $V_i(A_1, A_2)$  as player  $i$ 's "continuation payoff after  $(A_1, A_2)$ ". Since each player can achieve continuation payoff  $\theta_t$  by quitting, note that  $V_i(A_1^*, A_2^*) \geq \theta_t$  for all  $i$ . Also, it must be that  $\sum_i V_i(A_1^*, A_2^*) > 2\theta_t$ . To see why, suppose for the sake of contradiction that  $\sum_i V_i(A_1^*, A_2^*) = 2\theta_t$ . Since  $V_i(A_1, A_2) \geq \theta_t$  for all  $i, (A_1, A_2)$ , it must be that  $V_i(A_1, A_2) = \theta_t$  for all  $i, (A_1, A_2)$ . But if continuation payoffs do not depend on current work-shirk decisions, each player must shirk since this is the dominant strategy of the stage-game. Also, since  $\sum_i V_i(A_1^*, A_2^*) > 2\theta_t$ , the game must proceed to period  $t + 1$  with positive probability after the players choose  $(A_1^*, A_2^*)$ . (If some player were certain to quit after  $(A_1^*, A_2^*)$ , then each player would always get outside payoff  $\theta_t$  and  $\sum_i V_i(A_1^*, A_2^*) = 2\theta_t$ .) Finally, let  $X_i(A_1^*, A_2^*)$  denote the time- $t$  present value of all payoffs received beginning in period  $t + 1$ , conditional on work-shirk decisions  $(A_1^*, A_2^*)$  and then both players staying in period  $t$ . Since each  $V_i(A_1^*, A_2^*)$  is a weighted average of  $X_i(A_1^*, A_2^*)$  and  $\theta_t$ ,  $\sum_i V_i(A_1^*, A_2^*) > 2\theta_t$  implies  $\sum_i X_i(A_1^*, A_2^*) > 2\theta_t$ .

*What if neither player quits for sure?* Suppose that, conditional on both players working, there is a positive probability that neither player will quit. In this case, each player must at least weakly prefer to stay after both work, so it is an equilibrium for each player to stay for certain instead. This restores the “stay if both work” property at this history. It remains to consider histories at which both players work with positive probability and, conditional on both players working, some player is certain to quit. So, in particular,  $(W, W) \neq (A_1^*, A_2^*)$ .

*Both players must mix between work and shirk.* Since someone is certain to quit after  $(W, W)$ ,  $\Pi_i(W, W) = 1 + \theta_t$  for all  $i$ . Suppose that some player (say player 1) were to work with probability one. Since  $\Pi_2(W, S) \geq a + \theta_t > \Pi_2(W, W)$ , player 2 would strictly prefer to shirk. By presumption, however, both players work with positive probability. So, each player must mix and hence be indifferent between working and shirking. Finally, considering three further sub-cases.

*First case: joint payoffs are highest when both players shirk.* Suppose  $\sum_i \Pi_i(S, S) = \max_{A_1, A_2} \sum_i \Pi_i(A_1, A_2)$ . Change continuation play after the work-shirk decision as follows: for all  $(A_1, A_2)$ , make continuation play after  $(A_1, A_2)$  the same as what occurs in the original equilibrium after  $(S, S)$ . Since continuation play is the same regardless of players’ work-shirk decision, a new equilibrium exists in which both players shirk. Since the players now always achieve joint payoff  $\sum_i \Pi_i(S, S)$ , the joint payoff from history  $(h_t, \theta_t)$  must be at least weakly greater than that in the original equilibrium. Also, clearly, since the players never work, the “stay if both work” property is trivially restored.

*Second case: joint payoffs are highest when one player works.* Suppose  $\sum_i \Pi_i(W, S) = \max_{A_1, A_2} \sum_i \Pi_i(A_1, A_2)$ . (The argument is identical for the case in which player 1 shirks instead of player 2.) Change continuation play after the work-shirk decision as follows:

make continuation play after  $(W, W)$  the same as after  $(W, S)$ ; and make continuation play after  $(S, W)$  the same as after  $(S, S)$ . I claim that a new equilibrium exists in which player 1 always works and player 2 always shirks. By construction, continuation play only depends on player 1's decision to work or shirk, so clearly player 2 must shirk. Furthermore, player 1 must work given that player 2 shirks. To see why, note that player 1 worked with positive probability in the original equilibrium, despite the fact that  $\Pi_1(W, W) = 1 + \theta_t < \Pi_1(S, W)$ . For working to have been a best response, it must be that  $\Pi_1(W, S) > \Pi_1(S, S)$ . And, since continuation play after  $(W, S)$  and  $(S, S)$  are unchanged, player 1 must still prefer to work given that player 2 shirks under the new strategies. Since the players now always achieve joint payoff  $\sum_i \Pi_i(W, S)$ , the joint payoff from history  $(h_t, \theta_t)$  must be at least weakly greater than that in the original equilibrium. Also, since the players never *both* work, the “stay if both work” property is trivially restored.

*Last case: joint payoffs are highest when both players work.* The last case is when  $\sum_i \Pi_i(W, W) = \max_{A_1, A_2} \sum_i \Pi_i(A_1, A_2)$ . Change continuation play after the work-shirk decision as follows: make both players stay whenever both work; make continuation play after  $(W, W, \textit{stay}, \textit{stay})$  the same as after  $(A_1^*, A_2^*, \textit{stay}, \textit{stay})$ . (Recall that both players must stay with positive probability after  $(A_1^*, A_2^*)$  and that each player's continuation payoff after  $(A_1^*, A_2^*, \textit{stay}, \textit{stay})$  is *at least*  $V_i(W, W) = \theta_t$ .) Also, change the likelihood that each player works so that their work-shirk choices constitute a (possibly degenerate) mixed-strategy equilibrium given this new continuation play. In particular, each player must work weakly more frequently.<sup>16</sup> Since both players stay whenever both work, this

---

<sup>16</sup>In the original mixed strategies, player 1 must work with probability  $\frac{\Pi_2(S, W) - \Pi_2(S, S)}{\Pi_2(S, W) - \Pi_2(S, S) + \Pi_2(W, S) - \Pi_2(W, W)}$  in order for player 2 to be indifferent. Modified continuation play in the way that we have, the new work-work payoff  $\Pi'_2(W, W) \geq \Pi_2(W, W)$  while all other payoffs

new equilibrium restores the “stay if both work” property.

To complete the proof, we need to show that it yields weakly higher joint payoffs than the original equilibrium. In fact, each player’s individual payoff is weakly higher. Consider player 1. Since he works with positive probability in both the old and new equilibrium, his equilibrium payoff equals his expected payoff from working, a weighted average of  $\Pi_1(W, W)$  and  $\Pi_1(W, S)$  in the old equilibrium and of  $\Pi'_1(W, W) > \Pi_1(W, W)$  and  $\Pi_1(W, S)$  in the new equilibrium. Since player 2 shirks less frequently in the new equilibrium, it suffices to show that  $\Pi_1(W, W) \geq \Pi_1(W, S)$ . Note that  $\Pi_2(W, W) = 1 + \theta_t < a + \theta_t \leq \Pi_2(W, S)$ . (The first inequality is since some player quits for sure after work-work in the original equilibrium. The second inequality is since each player must get continuation payoff at least  $\theta_t$ .) However, by presumption,  $\sum_i \Pi_i(W, W) = \max_{A_1, A_2} \sum_i \Pi_i(A_1, A_2)$ , so  $\Pi_1(W, W) \geq \Pi_1(W, S) + (a - 1) > \Pi_1(W, S)$ .  $\square$

## A.6 Proof of Theorem 2

The original SPE can be viewed as generating a *stochastic joint return* equal to 2 whenever the partnership is active and both players work and 0 whenever the partnership is active and both players do not work, and  $2v$  after the partnership has been stopped, where  $v \in (0, 1)$ . As a thought experiment, consider the problem faced by a (female) social planner who seeks to maximize the (male) players’ joint payoff *and decides when to stop the partnership*, but must take as given the partners’ effort choices in any subgame in which the partnership remains active. (The partners are *not* strategic players in this thought experiment.)

As a first case, suppose that the social planner must take as given that the partners are unchanged. In a new mixed-strategy equilibrium with these modified payoffs, player 1 must work with probability  $\min \left\{ 1, \frac{\Pi_2(S, W) - \Pi_2(S, S)}{\Pi_2(S, W) - \Pi_2(S, S) + \Pi_2(W, S) - \Pi_2(W, W)} \right\}$ .

effort choices are as specified in the original SPE – on and off the equilibrium path. When the social planner implements the optimal stopping rule, the players’ joint payoff will clearly be weakly higher than in the original SPE.

As a second case, suppose that the partners both work whenever  $\theta_t \leq \theta^W$  and both shirk whenever  $\theta_t > \theta^W$ , where  $\theta^W$  is the work threshold of the original SPE. The return generated by the partnership is now always weakly higher than given effort choices in the original SPE. (In the original SPE, a return of zero is earned whenever the state exceeds  $\theta^W$ , by definition of  $\theta^W$ .) Thus, when the social planner implements the optimal stopping rule for this new returns process, the players’ joint payoff will be weakly higher than in the original SPE. In particular, since the stochastic process driving the state is a random walk, the optimal stopping rule will be a threshold rule, in which the partnership is stopped when the state first exceeds  $\theta^{out'}$ , where  $\theta^{out'} \geq \theta^{out}$ .

To complete the proof, we need to show that it is a SPE for both players to work and stay when  $\theta_t \leq \theta^W$ , shirk and stay when  $\theta_t \in (\theta^W, \theta^{out'}]$ , and shirk and exit when  $\theta_t > \theta^{out'}$ . First, since these strategies are symmetric, each player receives the same continuation payoff at all histories. Thus, each player has an incentive for the partnership to continue iff the social-planner would have wanted the partnership to continue. In particular, the players find it to be a best response to implement the optimal stopping rule, staying when  $\theta_t \leq \theta^{out'}$  and quitting when  $\theta_t > \theta^{out'}$ .

Finally, I need to show that the players’ decision to work when  $\theta_t \leq \theta^W$  is a best response. Note that each player’s continuation payoff given initial state  $\theta_0 = \theta$  and the partnership continues to the next period takes the compact form

$$\begin{aligned}
 V(\theta; \theta^W, \theta^{out'}) &= \sum_{t=1}^{\infty} \partial^t \left( \Pr(\theta_t \leq \theta^W, \max\{\theta_1, \dots, \theta_{t-1}\} \leq \theta^{out'}) + v \Pr(\max\{\theta_1, \dots, \theta_{t-1}\} > \theta^{out'}) \right) \\
 &= P(\theta^W - \theta_0, \theta^{out'} - \theta_0) + v(1 - P(\theta^{out'} - \theta_0, \theta^{out'} - \theta_0))
 \end{aligned} \tag{17}$$

By construction of  $\theta^{out'}$ ,  $V(\theta^{out'}; \theta^W, \theta^{out'}) = v \frac{\partial}{1-\partial}$ . Thus, Lemma 4 implies that  $V(\theta; \theta^W, \theta^{out'})$  is strictly decreasing in  $\theta$ . Thus, it suffices to check that working is a best response when  $\theta = \theta^W$ .

**Lemma 4.** *Suppose that  $V(\theta^{out}; \theta^W, \theta^{out}) = v \frac{\partial}{1-\partial}$  and  $\theta^W \leq \theta^{out}$ . Then  $V(\theta; \theta^W, \theta^{out})$  is strictly decreasing in  $\theta$ .*

By the definition of  $\theta^W$ , there exists a sequence of states  $\theta^k \nearrow \theta^W$  such that, for each  $k$ , there exists some history in the original SPE at which the state was  $\theta^k$  and both players chose to stay with positive probability. Since each player's minimax continuation payoff is  $v \frac{\partial}{1-\partial}$  (achieved if he quits immediately) no player will work unless his expected continuation payoff is at least  $v \frac{\partial}{1-\partial} + d(\theta^k)$ . Thus, the players' joint expected continuation payoff in the original SPE must have been at least  $2(v \frac{\partial}{1-\partial} + d(\theta^k))$ . As we showed earlier, however, joint continuation payoffs are weakly higher under the new threshold strategies, and each player gets the same continuation payoff. Thus, when the state is  $\theta^k$ , each player's continuation payoff  $V(\theta^k; \theta^W, \theta^{out'}) \geq v \frac{\partial}{1-\partial} + d(\theta^k)$ , and he prefers to work given that shirking will lead to an immediate end to the partnership. Finally, since  $P(x, y)$  is continuous in  $x, y$ ,  $V(\theta; \theta^W, \theta^{out'})$  is continuous in  $\theta$ . Thus,  $\lim_{k \rightarrow \infty} V(\theta^k; \theta^W, \theta^{out'}) = V(\theta^W; \theta^W, \theta^{out'}) \geq v \frac{\partial}{1-\partial} + d(\theta^W)$  and we are done.  $\square$

## A.7 Proof of Theorem 3

The result is obvious when  $v > 1$ , so I shall focus on the case in which  $0 < v \leq 1$ .

By Lemma 2(b),  $\theta^{out} < \infty$  in any SPE. Suppose for the sake of contradiction that  $v > \bar{v}$  and  $\theta^{out} > -\infty$ . By definition of  $\theta^{out}$ , there must exist some SPE such that both players stay with positive probability given initial state  $\theta_0 = \theta^{out}$ .<sup>17</sup> However, each

<sup>17</sup>There must exist some SPE such that both players stay with positive probability given initial state  $\theta_0 \in \{\theta^1, \theta^2, \dots\}$  where  $\lim_{k \rightarrow \infty} \theta^k = \theta^{out}$ . The stated result then follows from a continuity argument.

player is willing to stay only if he expects at least  $\frac{v\partial}{1-\partial}$  continuation payoff, conditional on both players staying. In particular, at least one player must quit if the expected joint continuation payoff is less than  $\frac{2v\partial}{1-\partial}$ .

To reach a contradiction and establish that  $\theta^{out} = -\infty$  in all SPE, then, it suffices to show that the players' expected joint continuation payoff is less than  $\frac{2v\partial}{1-\partial}$  in any SPE, *when the initial state is equal to that SPE's exit threshold  $\theta^{out}$* . By Lemma 3, it suffices for this purpose to restrict attention to SPE in which at least one player shirks in the last period of the partnership.

Fix a SPE, and let  $T^{out}$  be the (random) stopping time of the partnership in this equilibrium, conditional on initial state  $\theta_0 = \theta^{out}$  and both players' staying at time 0. By Lemma 3, the players joint payoff is zero at time  $T^{out}$ , at most two at every time  $t < T^{out}$ , and  $2v$  at every time  $t > T^{out}$ . Thus, the players' expected joint continuation payoff, conditional on initial state  $\theta_0 = \theta^{out}$  is *at most*

$$\begin{aligned} V_{12}(\theta^{out}) &\leq \sum_{t=1}^{\infty} \partial^t (2 \Pr(T^{out} > t) + 2v \Pr(T^{out} < t)) \\ &= \frac{2\partial}{1-\partial} \sum_{t=1}^{\infty} \Pr(T^{out} = t) ((1 - \partial^{t-1}) + v\partial^t) \end{aligned} \quad (18)$$

By definition of the exit threshold, the partnership is certain to stop as soon as the state first exceeds  $\theta^{out}$ , and possibly sooner. Thus,  $\Pr(T^{out} \leq t | \theta_0 = \theta^{out}) \geq \Pr(\max\{\theta_1 - \theta_0, \dots, \theta_t - \theta_0\} > 0)$  for all  $t$ . Next, note that  $v\partial^t - \partial^{t-1} < 0$  is increasing in  $t$  given  $v \leq 1$ . Together, these facts and (18) imply

$$\begin{aligned} V_{12}(\theta^{out}) &\leq \frac{2\partial}{1-\partial} \sum_{t=1}^{\infty} \Pr(\theta_t - \theta_0 > 0, \max\{\theta_1 - \theta_0, \dots, \theta_{t-1} - \theta_0\} \leq 0) ((1 - \partial^{t-1}) + v\partial^t) \\ &= \sum_{t=1}^{\infty} \partial^t (2 \Pr(\max\{\theta_1 - \theta_0, \dots, \theta_t - \theta_0\} \leq 0) + 2v \Pr(\max\{\theta_1 - \theta_0, \dots, \theta_{t-1} - \theta_0\} > 0)) \\ &= \frac{2\partial}{1-\partial} (P(0, 0) + v\partial(1 - P(0, 0))) \end{aligned}$$

So, some player strictly prefers to exit when at the exit threshold – leading to a contradiction – unless  $P(0,0) + v\partial(1 - P(0,0)) \geq v$ , or  $v \leq \frac{P(0,0)}{1-\partial(1-P(0,0))} < 1$ .  $\square$

## A.8 Proof of Lemma 4

The key assumption is that  $V(\theta^{out}; \theta^W, \theta^{out}) = v \frac{\partial}{1-\partial}$ , so that players are indifferent between quitting and staying when at the exit threshold.

*First step:*  $V(\theta; \theta^W, \theta^{out}) \geq V(\theta^{out}; \theta^W, \theta^{out})$ . Suppose for the sake of contradiction that  $V(\hat{\theta}; \theta^W, \theta^{out}) < V(\theta^{out}; \theta^W, \theta^{out})$  for some  $\hat{\theta} < \theta^{out}$ . Let  $t(x) = \min\{t : X_t > x\}$  denote the time at which the game will end given initial state  $\theta_0 = \theta^{out} - x$ . ( $t(x)$  is random since it depends on the path  $\mathbf{X} = (X_1, X_2, \dots)$ .) Conditional on  $(X_1, X_2, \dots, X_{t(0)})$  and initial state  $\theta_0 \leq \theta^{out}$ , each player's expected payoff is

$$V(\theta_0; \theta^W, \theta^{out}; \mathbf{X}) \equiv \sum_{t < t(0)} \partial^t 1_{\{X_t \leq \theta^W - \theta_0\}} + \partial^{t(0)} V(\theta_0 + X_{t(0)}; \theta^W, \theta^{out}) \quad (19)$$

Since the indicator  $1_{\{X_t \leq \theta^W - \theta\}}$  is non-decreasing in  $\theta^W - \theta$  and  $V(\theta; \theta^W, \theta^{out}) = v \frac{\partial}{1-\partial}$  for all  $\theta > \theta^{out}$ ,

$$V(\theta^{out}; \theta^W, \theta^{out}; \mathbf{X}) - V(\hat{\theta}; \theta^W, \theta^{out}; \mathbf{X}) \leq \partial^{t(0)} \left( V(\theta^{out}; \theta^W, \theta^{out}) - V(\hat{\theta} + X_{t(0)}; \theta^W, \theta^{out}) \right) \quad (20)$$

Averaging over all paths,

$$\begin{aligned} V(\theta^{out}; \theta^W, \theta^{out}) - V(\hat{\theta}; \theta^W, \theta^{out}) &\leq E_{t(0)} \left[ \partial^{t(0)} \left( V(\theta^{out}; \theta^W, \theta^{out}) - \sup_{\theta^1 \in (\hat{\theta}, \theta^{out}]} V(\theta^1; \theta^W, \theta^{out}) \right) \right] \\ &\leq \partial \left( V(\theta^{out}; \theta^W, \theta^{out}) - \sup_{\theta^1 \in (\hat{\theta}, \theta^{out}]} V(\theta^1; \theta^W, \theta^{out}) \right) \end{aligned} \quad (21)$$

where the last inequality follows from the fact that  $t(0) \geq 1$ . Thus,  $V(\hat{\theta}; \theta^W, \theta^{out}) - V(\theta^{out}; \theta^W, \theta^{out}) = \Delta < 0$  requires that  $V(\theta^1; \theta^W, \theta^{out}) - V(\theta^{out}; \theta^W, \theta^{out}) < \frac{\Delta}{\partial}$  for some

$\theta^1 \in (\hat{\theta}, \theta^{out})$ . Repeating this argument, there must exist  $(\theta^2, \theta^3, \theta^4, \dots)$  such that, for all  $k$ ,  $V(\theta^k; \theta^W, \theta^{out}) - V(\theta^{out}; \theta^W, \theta^{out}) < \frac{\Delta}{\partial^k}$  for some  $\theta^k \in (\theta^{k-1}, \theta^{out})$ . This is a contradiction since  $V(\theta; \theta^W, \theta^{out}) - V(\theta^{out}; \theta^W, \theta^{out}) \geq -v \frac{\partial}{1-\partial}$ .

*Second step:*  $V(\theta; \theta^W, \theta^{out}) > V(\theta'; \theta^W, \theta^{out})$  for all  $\theta < \theta' \leq \theta^{out}$ . Let  $t' = t(\theta^{out} - \theta')$  be shorthand for the time at which the game ends given initial state  $t'$ . Conditional on  $(X_1, X_2, \dots, X_{t'})$ , each player's expected payoff given initial state  $\theta_0 \leq \theta'$  is

$$V(\theta_0; \theta^W, \theta^{out}; \mathbf{X}) \equiv \sum_{t < t'} \partial^t 1_{\{X_t \leq \theta^W - \theta_0\}} + \partial^{t'} V(\theta_0 + X_{t'}; \theta^W, \theta^{out})$$

Repeating the logic used to derive (20),

$$V(\theta'; \theta^W, \theta^{out}; \mathbf{X}) - V(\theta; \theta^W, \theta^{out}; \mathbf{X}) \leq \partial^{t'} (V(\theta^{out}; \theta^W, \theta^{out}) - V(\theta + X_{t'}; \theta^W, \theta^{out}))$$

Thus, continuing the logic used to derive (21),

$$V(\theta'; \theta^W, \theta^{out}) - V(\theta; \theta^W, \theta^{out}) \leq \partial \left( V(\theta^{out}; \theta^W, \theta^{out}) - \sup_{\theta^1 \in (\theta^{out} - \varepsilon, \theta^{out}]} V(\theta^1; \theta^W, \theta^{out}) \right) \leq 0$$

The first inequality holds since  $\theta + X_{t'} > \theta_E - (\theta' - \theta) = \theta^{out} - \varepsilon$  by definition; the second inequality follows from the first step of the proof.  $\square$

## B Extension allowing for re-entry

In this section, I extend the analysis by allowing for re-entry. If the state stops evolving after exit, the players never have an equilibrium incentive to renew their relationship. So, to focus on the case of interest, I shall assume that the state continues to follow the exogenous process  $\{\theta_t\}$  (with  $\theta_t$  becoming common knowledge at the beginning of period  $t$ ), regardless of whether the partnership is active or inactive. Thus, the analysis presented here is more relevant to situations in which the state evolves due to exogenous

changes in the environment, rather than to a process of “learning-by-doing” as in Roth (1996).<sup>18</sup>

*The partnership game with re-entry.* As in the basic model of Section 2, the players decide whether to work or shirk and then whether to stay or quit when the partnership is active. In addition, the players also decide whether to “join” or “stay out” at the beginning of each period when the partnership is inactive. The partnership remains voluntary: both players must decide to join for the partnership to be activated, while just one of them must decide to quit for it to be de-activated. Second, each player pays an “exit cost”  $C^{out} \geq 0$  any time that the partnership is de-activated and an “entry cost”  $C^{in} \geq 0$  any time that it is activated. (The players could activate their partnership at the beginning of a period and then de-activate it at the end, in which case each player would pay  $C^{in} + C^{out}$  that period.) All other assumptions are the same.

**Preliminaries.** The SPE that maximizes players’ joint payoff has a threshold character: players (re-)start the partnership whenever the state is less than an “entry threshold”  $\theta^{in}$ , (re-)stop the partnership whenever the state exceeds an “exit threshold”  $\theta^{out}$ , and both work when the partnership is active and the state is less than a “work threshold”  $\theta^W$ .

The proof that the optimal SPE has this threshold character is similar to that of Theorem 2, and omitted to save space. However, please note that the optimal threshold need *not* be ordered  $\theta^{in*} \leq \theta^{W*} \leq \theta^{out*}$  in general.

Players’ expected payoffs in such equilibria depend upon crossing-properties of the

---

<sup>18</sup>Roth (1996) considers a learning model in which a public signal about the productivity of the match is revealed each period that the partnership is active. The state corresponds to expected productivity conditional on the history of signals, and stops evolving after exit.

underlying stochastic process. A few definitions are helpful:

**Definition 6.** For given entry and exit thresholds  $\theta^{in}, \theta^{out}$  (whose notation is suppressed), let  $T(in)$  and  $T(out)$  denote the set of times at which the partners will have entered or exited at least once, and most recently entered or exited, respectively:

$$T(in) \equiv \{t > 0 : \exists t' \leq t : \theta_{t'} \leq \theta^{in} \text{ and } \theta_{t''} \leq \theta^{out} \forall t'' \in (t', t]\}$$

$$T(out) \equiv \{t > 0 : \exists t' \leq t : \theta_{t'} > \theta^{out} \text{ and } \theta_{t''} > \theta^{in} \forall t'' \in (t', t]\}$$

*Discussion:* Suppose that the partnership is active at time  $t = 0$ . Then, by definition, the players will choose whether to work or shirk in all periods  $t \in T(in) \cup \{0, 1, \dots, \min T(out)\}$  and earn their outside payoff  $v$  in all other periods. Similarly, if the partnership is inactive at time  $t = 0$ , then the players will choose whether to work or shirk in all periods  $t \in T(in)$  and earn their outside payoff  $v$  in all other periods. Consequently, given a profile of behavioral thresholds  $(\theta^{in}, \theta^W, \theta^{out})$ , players' payoffs depend not only on the current state but whether the partnership is active or inactive.

**Equilibrium payoffs, when partnership currently active.** Consider any realized path of states  $(\theta_0, \theta_1, \theta_2, \dots)$ . Assuming that the partnership is active at time 0 (which requires  $\theta_0 \leq \theta^{out}$ ), the players will at time  $t \geq 0$  (a) both work for payoff 1 if  $t \notin T(out)$  and  $\theta_t \leq \theta^W$ , (b) both shirk for payoff 0 if  $t \notin T(out)$  and  $\theta_t > \theta^W$ , (c) exit or remain inactive for payoff  $v$  if  $t \in T(out)$ , (d) exit for (additional) payoff  $-C^{out}$  if  $t - 1 \notin T(out)$  and  $t \in T(out)$ , (e) enter for (additional) payoff  $-C^{in}$  if  $t - 1 \in T(in)$  and  $t \in T(out)$ . Putting this together, when the partnership is on initially, with initial state  $\theta_0 = \theta \leq \theta^{out}$ ,

each player's equilibrium *continuation* payoff takes the form

$$\begin{aligned}
V(\theta, in; \theta^{in}, \theta^W, \theta^{out}) &= \sum_{t=1}^{\infty} \partial^t \left( \Pr(t \notin T(out) \text{ and } \theta_t \leq \theta^W | \theta_0 = \theta) \right. \\
&\quad + v \Pr(t \in T(out) | \theta_0 = \theta) - C^{out} \Pr(t-1 \notin T(out) \text{ and } t \in T(out)) \\
&\quad \left. - C^{in} \Pr(t-1 \in T(out) \text{ and } t \in T(in)) \right) \tag{22}
\end{aligned}$$

with corresponding (total) payoff  $\Pi(\theta, in; \theta^{in}, \theta^W, \theta^{out}) = 1\{\theta \leq \theta^W\} + V(\theta, in; \theta^{in}, \theta^W, \theta^{out})$ .

**Equilibrium payoffs, when partnership currently inactive.** Equilibrium payoffs when the partnership is inactive are similar, except that the players earn  $v$  until period  $\min T(in)$  rather than earning zero or one until  $\min T(out)$ . Namely, for all  $\theta > \theta^{in}$ ,

$$\begin{aligned}
V(\theta, out; \theta^{in}, \theta^W, \theta^{out}) &= \sum_{t=1}^{\infty} \partial^t \left( \Pr(t \in T(IN) \text{ and } \theta_t \leq \theta^W | \theta_0 = \theta) \right. \\
&\quad + v \Pr(t \notin T(in) | \theta_0 = \theta) - C^{out} \Pr(t-1 \in T(in) \text{ and } t \in T(out)) \\
&\quad \left. - C^{in} \Pr(t-1 \notin T(in) \text{ and } t \in T(in)) \right) \tag{23}
\end{aligned}$$

with corresponding (total) payoff  $\Pi(\theta, out; \theta^{in}, \theta^W, \theta^{out}) = v + V(\theta, out; \theta^{in}, \theta^W, \theta^{out})$ .

**Optimal work threshold.** For any given entry and exit thresholds, the optimal work threshold must be such that players are indifferent between working and shirking when the state is equal to the work threshold, even when shirking causes the partnership to turn off forever:

$$d(\theta^{W*}) + v \frac{\partial}{1 - \partial} = V(\theta^{W*}, in; \theta^{in*}, \theta^{W*}, \theta^{out*}) \tag{24}$$

Similarly, the players must be indifferent between keeping the partnership active (inactive) and making it inactive (active) when the state is equal to the exit (entry) threshold.

That is,

$$\Pi(\theta^{out*}, out; \theta^{in*}, \theta^{W*}, \theta^{out*}) = \Pi(\theta^{out*}, in; \theta^{in*}, \theta^{W*}, \theta^{out*}) + C^{out} \quad (25)$$

$$V(\theta^{in*}, in; \theta^{in*}, \theta^{W*}, \theta^{out*}) = V(\theta^{in*}, out; \theta^{in*}, \theta^{W*}, \theta^{out*}) + C^{in} \quad (26)$$

The system of equations (24,25,26) has a unique and finite solution as long as  $C^{out} \leq \frac{v}{1-\delta}$  and  $C^{in} \leq \frac{1-v}{1-\delta}$ . Otherwise, the solution is still unique but involves infinite entry and/or exit thresholds. In particular, the players never exit when  $C^{out} > \frac{v\delta}{1-\delta}$ , in which case  $\theta^{out*} = -\infty$ , and never re-enter when  $C^{in} > \frac{1-v}{1-\delta}$ , in which case  $\theta^{in*} = \infty$ .

**Optimal exit and entry thresholds.** The partnership can be viewed as an asset that generates a per-period per-player return of one when it is active and the state is less than  $\theta^W$ , zero when it is active and the state is greater than  $\theta^W$ , and  $v$  when it is inactive. Thus, for a fixed work threshold, the computation of the optimal entry and exit thresholds is straightforward.

Theorem 5 summarizes this discussion. (The formal proof is omitted to save space.)

**Theorem 5.** *Suppose that  $C^{out}, C^{in} \geq 0$ . There exists  $\theta^{in}, \theta^W, \theta^{out}$  such that, on the equilibrium path of the SPE that maximizes joint payoffs, (a) the partnership becomes active if  $\theta_t \leq \theta^{in}$ , becomes inactive if  $\theta_t > \theta^{out}$ , and otherwise remains in its current activity status and (b) if active during period  $t$ , both players work if  $\theta_t \leq \theta^W$  and both shirk if  $\theta_t > \theta^W$ .*

## B.1 Properties of the optimal SPE.

**Players prefer lower states.** As found in Corollary 4, players are strictly better off in lower states, when they have less incentive to shirk (whether their partnership is currently active or currently inactive).

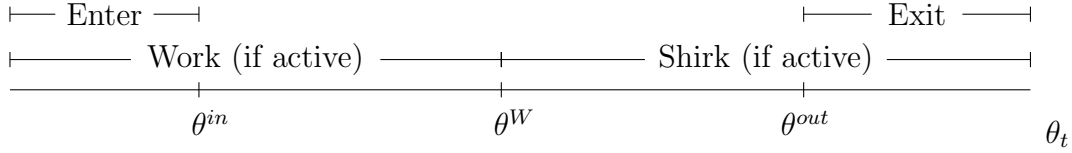


Figure 6: Behavioral thresholds given sufficiently costly entry and/or exit.

**When do players prefer a more or less volatile environment?** As found in Corollary 5, the optimal work threshold does not depend on the volatility of the environment, while the distances between the optimal entry, work, and exit thresholds all scale with volatility. Consequently, the players prefer a more volatile environment iff the current state exceeds the work threshold (whether their partnership is currently active or currently inactive). In particular, whenever a partnership is active and both players work, those players would prefer to face a less volatile environment. Further, as long as entry is at all costly, there will still be a “honeymoon effect”, i.e.  $\theta^{W*} \geq \theta^{in*}$  so that both players work in any period when they activate their relationship. Consequently, in any newly-formed partnership, the players would prefer to face a less volatile environment.

**Must players shirk just before exit?** Less clear is whether  $\theta^{W*} \leq \theta^{out*}$  when  $C^{in} + C^{out} > 1$ . If both players work in this period but are likely enough to shirk in the next period, the players will be better off de-activating their relationship. This never happens in the model without re-entry since both players shirk this period in anticipation of quitting. However, with re-entry it is at least conceivable that the future gains from cooperation after might be enough to support working in a period when the partnership will be de-activated. On the other hand, as long as  $C^{in} + C^{out}$  is large enough, it is possible to show that  $\theta^{W*} \leq \theta^{out*}$ . (This is the case illustrated in Figure 6.)

**Players prefer lower entry costs.** Less costly re-entry leads players to anticipate more future interaction on the equilibrium path of play. Consequently, players work in a larger set of states when re-entry costs fall, increasing the performance of the partnership. In this sense, lower re-entry costs mitigate the destabilizing effects of exit. On the other hand, lower exit costs continue to have an ambiguous effect on player welfare.

## References

- ABREU, D., D. PEARCE, AND E. STACCHETTI (1990): “Toward a Theory of Discounted Repeated Partnerships,” *Econometrica*, 58(5), 1041–1063.
- ATHEY, S., AND K. BAGWELL (2001): “Optimal Collusion with Private Information,” *RAND Journal of Economics*, 32, 428–465.
- ATHEY, S., K. BAGWELL, AND C. SANCHIRICO (2004): “Collusion and price rigidity,” *Review of Economic Studies*, 71, 317–350.
- BAGWELL, K., AND R. STAIGER (1997): “Collusion over the business cycle,” *RAND Journal of Economics*, 28(1), 82–106.
- BAKER, G., R. GIBBONS, AND K. MURPHY (1994): “Subjective Performance Measures in Optimal Incentive Contracts,” *Quarterly Journal of Economics*, 109(4), 1125–1156.
- BULL, C. (1987): “The Existence of Self-Enforcing Implicit Contracts,” *Quarterly Journal of Economics*, 102(1), 147–160.
- CHASSANG, S. (2007): “Fear of miscoordination and the robustness of cooperation in dynamic global games with exit,” manuscript.

- DI TELLA, R., AND R. MACCULLOCH (2002): “Informal Family Insurance and the Design of the Welfare State,” *Economic Journal*, 112, 481–503.
- DUTTA, P. (1995): “A Folk Theorem for Stochastic Games,” *Journal of Economic Theory*, 66, 1–32.
- ERICSON, R., AND A. PAKES (1995): “Markov-Perfect Industry Dynamics: A Framework for Empirical Work,” *Review of Economic Studies*, 62(1), 53–82.
- GHEMAWAT, P., AND B. NALEBUFF (1985): “Exit,” *RAND Journal of Economics*, 16(2), 184–194.
- GREEN, J., AND R. PORTER (1984): “Noncooperative Collusion under Imperfect Price Information,” *Econometrica*, 52(1), 87–100.
- GRENADIER, S. (2002): “Option Exercise Games: An Application to the Equilibrium Investment Strategies of Firms,” *Review of Financial Studies*, 15, 691–721.
- HALTIWANGER, J., AND J. HARRINGTON (1991): “The Impact of Cyclical Demand Movements on Collusive Behavior,” *RAND Journal of Economics*, 22(1), 89–106.
- HOPENHAYN, H. (1992): “Entry, Exit, and Firm Dynamics in Long Run Equilibrium,” *Econometrica*, 60(5), 1127–1150.
- HORNER, J., AND J. JAMISON (2004): “Collusion with (Almost) No Information,” manuscript.
- JOVANOVIC, B. (1979a): “Firm-Specific Capital and Turnover,” *Journal of Political Economy*, 87(6), 1246–1260.
- (1979b): “Job Matching and the Theory of Turnover,” *Journal of Political Economy*, 87(5), 972–990.

- LEVIN, J. (2003): “Relational Incentive Contracts,” *American Economic Review*, 93, 835–857.
- LEVINTHAL, D. (1991): “Random Walks and Organizational Mortality,” *Administrative Science Quarterly*, 36, 397–420.
- MACLEOD, B., AND J. MALCOMSON (1989): “Implicit Contracts, Incentive Compatibility, and Involuntary Employment,” *Econometrica*, 57(2), 447–480.
- ROTEMBERG, J., AND G. SALONER (1986): “A Supergame-Theoretic Model of Business Cycles and Price Wars During Booms,” *American Economic Review*, 76, 380–407.
- ROTH, D. (1996): “A Theory of Partnership Dynamics,” *Games and Economic Behavior*, 12, 95–112.
- SOLAN, E., AND N. VIELLE (2001): “Quitting Games,” *Mathematics of Operations Research*, 26(2), 265–285.
- STEVENSON, B., AND J. WOLFERS (2007): “Marriage and Divorce: Changes and their Driving Forces,” *Journal of Economic Perspectives*, 21(2), 27–52.
- STINCHCOMBE, A. (1965): “Social Structure and Organizations,” in *Handbook of Organizations*, ed. by J. March. Rand McNally.
- TOPEL, R. H., AND M. P. WARD (1992): “Job Mobility and the Careers of Young Men,” *Quarterly Journal of Economics*, 107(2), 439–479.