

Private Value Perturbations and Informational Advantage in Common Value Auctions

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Abstract

We analyze the value of being better informed than one's rival in a two bidder, second price common value auction. Standard models of these auctions do not pin down relative bidding postures, making analysis difficult. We show that by adding small amounts of private value information, a unique equilibrium can be restored. Additional common value information affects a bidder's payoff both directly, by increasing his information rent, and indirectly, by shifting the relative bidding posture of his opponent. Although the latter effect need not be positive, we establish broad conditions under which bidders with more information do better than their rivals. We turn to information acquisition and release and show that the desire to influence relative bidding postures can lead sellers to create new information rents (by releasing information privately to only one bidder) and bidders to forego information rents (instead choosing to gather information that a rival already has).

1 Introduction

Auction theory is largely silent on what to expect when bidders are unequally informed about the common value of an object for sale. This is due in part to the fact that second-price common value auctions often present multiple equilibria. When bidders are equally well informed, selecting the symmetric equilibrium appears reasonable, but when some bidders know more than others, there is no obvious or compelling way to choose the "right" equilibrium. This paper resolves the problem by introducing small private value perturbations to the bidders' common value for the object. As these perturbations vanish, bidding converges to a unique equilibrium of the unperturbed model. The value of being better informed than one's rival takes a subtle form in this equilibrium: it is not having more precise information *per se* that is valuable, but rather, having information that is difficult for rivals to free ride on.

In an unperturbed second price auction, the value of better information is unclear because of a strong free-riding effect. Because the winning bidder's price is set by the losing bidder's bid, the losing bidder's private information will typically be fully revealed by the auction price. Thus, when a bidder, in formulating a best response to her rival's strategy, considers the event of winning with a particular signal at a particular price, she is effectively conditioning on full information about the common value. Given this, there is no particular reason that a poorly informed bidder needs to bid cautiously. On the other hand, her inference about the common value upon winning at a particular price will be more pessimistic when her rival's strategy is more aggressive, and this *will* lead her to bid more cautiously. By simultaneously making one bidder's strategy more aggressive and the other's more cautious, one can trace out a continuum of equilibria in the unperturbed model, and none of these equilibria depend on the informational asymmetry between the two bidders.

In contrast, when there is a private value component to each bidder's signal, free-riding off a rival's bid becomes more difficult. When a bidder conditions on paying a particular price, she must infer what portion of that price reflects a "premium" for her rival's private value for the object. She should be prepared to win at this price if and only if her own private value for the object exceeds that premium. This leads to a more stringent equilibrium condition: at any price at which the bidders might tie, the expected share of private value reflected in the two bidder's bids must be the same. Under some regularity conditions on the signal distributions, this *private value perturbation (PVP)* condition pins down a unique equilibrium.

With this selection result in hand, we can analyze the payoff implications when bidders are unequally informed about the common value. Not surprisingly, additional common value information tends to increase a bidder's information rent, and with it, her payoff. However, additional information also affects a bidder's strategic posture relative to her rival; in particular, we show that in a particular sense, it tends to make her bidding less volatile. Under reasonable conditions, this strategic shift is also to her advantage – in this case superior information implies a superior payoff. We also study incentives to acquire and release (in the case of the seller) additional information. We show that a seller can gain by releasing information privately held by the better informed bidder to the less informed bidder in order to help her catch up. More surprisingly, a seller can also gain by giving providing the less informed bidder with more private information – in this case, the improvement in the strategic balance between the bidders more than compensates for the additional information rent that the seller gives away.

The paper draws on two strands of the auction literature. The first relates to common value auctions with asymmetrically informed bidders. Here most of the work has focused on first price auctions, often looking at the case in which a single informed bidder competes with several relatively uninformed bidders whose information is in some sense a coarsening of the informed bidder's information (Wilson (1967), Englebrecht-Wiggans, Milgrom, and Weber (1983), Hendricks, Porter, and Wilson (1994), among others). More recently, Laskowski and Slonim (1999) and Kagel and Levin (1999) characterize approximate first price equilibria in

models in which better information corresponds to a lower variance signal about the common value. Campbell and Levin (2000) explicitly solve a parametric model and provide examples in which having better information hurts a bidder.

For second price auctions, Milgrom and Weber (1982) and Einy, Haimanko, Orzach, and Sela (2000) show that as in the first price auction, when one bidder has information that encompasses everything known by another bidder, the latter bidder cannot make a profit. Krishna and Morgan (1998) and Mares (2000) deal with asymmetries that arise when *ex ante* symmetric bidders pool their information and bid jointly. Both papers resolve a multiplicity of equilibria by imposing some type of symmetric bidding despite the fact that the bidders are not symmetric. Perhaps the closest paper to ours is Parreiras (2002), who also uses a perturbation approach to select an equilibrium of a second price auction with asymmetrically informed bidders. However, his approach is based on the assumption there is a small first price component to the auction (rather than a private value component), and his results are quite different.

We also build on past work on auctions with both private and common values. In a seminal paper, Milgrom and Weber (1982) treat the case of symmetric bidders when a bidder's information about its private and common value can be summarized by a one-dimensional signal. Relaxing these assumptions has proved difficult, in large part because the basic toolbox of auction theory relies heavily on finding a natural ordering of bidders' signals. The lack of such an ordering when signals are multi-dimensional can cause problems for the existence and characterization of equilibrium as shown by Jackson (1999). In order to avoid this difficulty, I rely on a relatively special model of private and common values that also been used by Goeree and Offerman (1999) among others. The next section describes the model and partially characterizes equilibria. Section 3 looks at the limit in which private values are small and develops the equilibrium selection result. Section 4 studies the comparative statics of this equilibrium and presents results on the value of information, and information release and acquisition. Finally, Section 5 concludes with a discussion of possible extensions.

2 The Model

There is a single object to be sold to one of two buyers, 1 and 2, in a second price auction with a reserve price of 0. A buyer's valuation for the object is $v_i = v + \varepsilon z_i$ where εz_i is a private value component and v is common to both bidders. The common value is modeled as the sum of a number of independent components: $v = \sum_{i=1}^n y_i$, which are distributed i.i.d. according to the continuously differentiable density f_y . The private value terms z_1 and z_2 are also random variables drawn independently from continuously differentiable densities g_1 and g_2 with compact support Z , while ε is a constant scale factor. We assume that z_i has mean μ_i and variance σ_i^2 . Our principal focus will be on the case in which the scale factor ε is small, so that the private component of valuations is small relative to the common value. Prior to the

auction, each bidder observes her private value and a subset of the components of the common value. We assume that each bidder knows which of these components her rival has observed.

The bidders' information partitions divide the common value components into four sets: those observed by Bidder 1 or Bidder 2 exclusively, those observed by both bidders, and those that neither observes. Suppose that Bidder i observes m_i components exclusively and that x_i is the sum of their values. Because of the independence among components, x_i is a sufficient statistic for Bidder i 's private information about v . Furthermore, notice that a component that both bidders or neither observes plays no strategic role in the auction: the bidders will simply shift their bidding functions up by its value, if it is known, or by its expected value, if it is unobserved. Given both of these facts, we will proceed for the time being with the following, simpler, model specification: $\{v = x_1 + x_2; v_i = v + \varepsilon z_i\}$, where x_i is distributed according as the sum of m_i draws from f_y . (We will write $x_i \sim f_i$, where f_i is the m_i -fold convolution of f_y with itself.) This is similar to the sum of independent signals specification used by Goeree and Offerman (1999) who show that a bidder's strategic behavior depends only on the sum $s_i = x_i + \varepsilon z_i$. (Because z_i provides no additional information about the common value, s_i is a sufficient statistic for Bidder i 's private information about v_i .) In the sequel we will save time by simply considering the bidder's signal to be s_i .¹ We will return to the full specification of the model in Section 4 where it will help to clarify our comparative statics results.

As a prelude to examining equilibrium in the model with private values, it is helpful to briefly review the set of Nash equilibria in the case with pure common values; we do this in Section 2.1. Then, in Section 2.2, equilibria with private and common values are characterized.

2.1 The Common Value Benchmark

Here we assume that $\varepsilon = 0$, so the setting is one of purely common values, with $v = s_1 + s_2$. The following lemma and its proof provide a sense of why multiple equilibria arise in this setting. Before proceeding to the lemma, let us introduce some terminology. A bidding function maps signals to bids: $b_i : X \rightarrow \mathfrak{R}^+$. We restrict attention throughout to strictly increasing, continuously differentiable bidding functions. The set of bids used by a bidder – that is, the image of b_i – is denoted $B_i \equiv b_i(X)$, and the inverse bidding function is denoted $\phi_i : B_i \rightarrow X$. We will focus on equilibria for which $B_1 \cap B_2$ is nonempty so that each bidder has a positive chance of winning the auction. We can also define a bidder's surplus function $Y_i(p; s_i, \phi_{-i})$. This is the expected surplus earned by a bidder i with signal s_i who wins at a price p , given that her rival's (inverse) bidding strategy is ϕ_{-i} . In the case of pure common values, the surplus function is

$$\begin{aligned} Y_i(p; s_i, \phi_{-i}) &= E(v \mid s_i, s_{-i} = \phi_{-i}(p)) - p \\ &= s_i + \phi_{-i}(p) - p \end{aligned}$$

¹Essentially, this is because separating x_i and εz_i provides the bidder with no additional information about his value v_i .

In what follows, we will sometimes write $Y_i(p; s_i)$ or just $Y_i(p)$ when the missing arguments are clear from the context. Under some regularity conditions, each bidder has a well-known dominant strategy, namely, to bid so that its surplus function is positive for all prices less than its bid, zero when the price equals its bid (that is, when it ties the other bidder), and negative for all higher prices. This leads to an equilibrium condition: conditional on tying at some bid b , both bidders must believe that the expected value of the object is equal to b . Since the bidders can infer their respective signals, this condition simply requires that the expectation of v conditional on signals $\phi_1(b)$ and $\phi_2(b)$ equals b . With just one equation to identify two inverse bidding functions, we are left with a degree of indeterminacy in equilibrium.

Lemma 1 *Any pair of increasing inverse bidding functions (ϕ_1, ϕ_2) that satisfy $\phi'_i < 1$ and $\phi_1(p) + \phi_2(p) = p \forall p \in B_1 \cap B_2$ constitute a Nash equilibrium of the auction with pure common values.*

The requirement that $\phi'_i < 1$ suffices to guarantee that the surplus function is decreasing in the price paid, ensuring that the strategy described above is in fact dominant. Lemma 1 admits a range of equilibria in which one bidder bids relatively more aggressively and the other more cautiously.

Example

Suppose that Bidder 1 observes one common value component distributed $N(20, 1)$ and Bidder 2 observes 10 components, each distributed $N(2, 1)$ (so $x_1 \sim N(20, 1)$ and $x_2 \sim N(20, 10)$ and $v \sim N(40, 11)$). Among others, any strategies that are linear in signals, $b_i = k_i x_i$ for which $\frac{1}{k_1} + \frac{1}{k_2} = 1$ holds represent an equilibrium. For example, the strategies $b_1 = 11x_1$, $b_2 = 1.1x_2$ qualify; in this equilibrium, Bidder 1 wins virtually almost all of the time and earns an expected surplus of around 18.

Equilibria like this one rely on aggressive bidding by one bidder to create a more severe winner's curse for the other bidder who knows that if he has won, his rival's signal must have been quite low. This encourages the second bidder to bid cautiously, alleviating the winner's curse faced by the strong bidder and justifying his aggressive strategy. The equilibrium in the example has no relationship to the distributions of the bidders' signals; in particular, the bidder who appears to have better information about the common value does very poorly.

2.2 Private Value Perturbations

Now we turn to the case in which valuations have a private value component ($\varepsilon > 0$). In this case, upon winning, a bidder must worry about how much of the price he is paying reflects his rival's private benefit from winning the object rather than common value to both of them. In what follows, the bidders' values are assumed to be *ex ante* symmetric (that is, $\mu_1 = \mu_2 = 0$) in order to focus on the effects of asymmetries in bidder information.² Furthermore, we adopt the

²When one bidder has a higher *ex ante* expected value than the other, one can demonstrate a strong advantage for this bidder as suggested in Klemperer (1998). Once we have assumed $\mu_1 = \mu_2 = \mu$, the additional assumption

normalization $\sigma_1^2 + \sigma_2^2 = 1$ (subsuming the magnitudes of the private values into ε) and define $\eta = \sigma_1^2/\sigma_2^2$ as the relative amount of private value information held by Bidder 1. In the rest of this section, we will develop a characterization of equilibrium with private values.

As before, we consider bid functions that are strictly increasing in signals, and hence, invertible. Suppose that Bidder 2's bid function and inverse are given by b_2 and ϕ_2 and consider Bidder 1's best response. His expected value for the object conditional on winning at a price b is given by

$$E(v_1 | s_1, s_2 = \phi_2(b)) = s_1 + E(x_2 | s_2 = \phi_2(b))$$

and his expected net surplus upon winning and paying b is now equal to

$$Y_1(b; s_1) = s_1 + E(x_2 | s_2 = \phi_2(b)) - b$$

Notice that because ϕ is increasing, it doesn't matter whether we write these expressions as conditioning on Bidder 2's bid b or her signal $\phi_2(b)$. While in the pure common values case, the middle term in this expression, $E(x_2 | s_2 = \phi_2(b))$, is simply equal to Bidder 2's signal $\phi_2(b)$, with private values, Bidder 1 must infer how much to discount Bidder 2's signal to account for her private value component.

As before, we are interested in conditions that ensure that $Y_1(b; s_1)$ is decreasing in b , because this in turn will ensure that there is a single b^* that separates the prices at which Bidder 1 would like to win ($p < b^*$) from the prices at which it would prefer to lose ($p > b^*$). In this case, determining Bidder 1's optimal strategy will be relatively straightforward. With pure common values, it sufficed to constrain the slope of ϕ_2 , but with private values, an additional assumption is required.

A1. f_i and g_i are strictly log-concave.

Lemma 2 (in the appendix) shows that A1 implies the following two additional properties: $d(E(x_i | s_i)/ds_i) < 1$ (good news is not *too* good), and $dE(x_i | s_i)/ds_i > 0$ (good news is not bad news).

The first of these properties is needed to rule out situations like the one in Figure 1(b). Here, Bidder 1 has no simple separating strategy because the intervals she would like to win on, $(0, b^i)$ and (b^{ii}, b^{iii}) , are disjoint. Situations like this one can arise when a bidder can infer large changes in the common vs. private value *composition* of a rival's signal on the basis of small changes in that signal.³ The second of these properties guarantees that a higher rival signal is always good news about the common value; this need not always be the case.⁴

that $\mu = 0$ is an innocuous normalization; if μ were non-zero, the bidding functions would simply shift by a constant amount.

³For example, suppose that the common component x_2 is believed to be either low, $U(0, 2)$, or high, $U(3, 5)$, and the private value component εz_2 is either -0.5 or 0.5 with equal probability. Then a signal of $s_2 = 2.5 \pm |\delta|$ implies that the private value must be ∓ 0.5 with certainty, so $E(x_2 | s_2)$ jumps from 2 to 3 as s_2 crosses 2.5.

⁴To illustrate, in the example from the previous footnote, $E(x_2 | s_2)$ equals s_2 for $s_2 \in (0.5, 1.5)$ and $s_2 - 0.5$

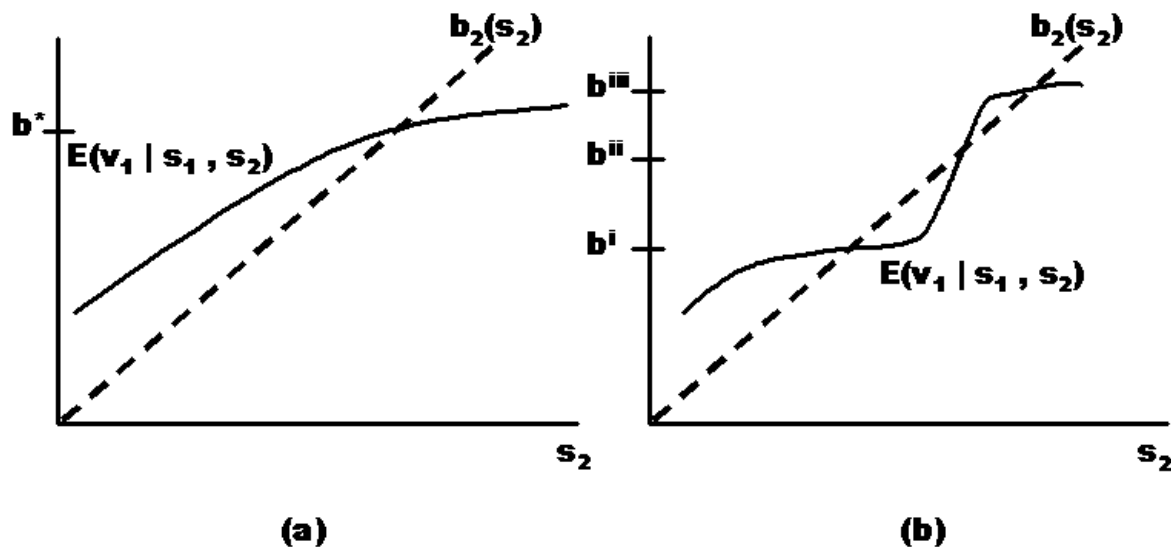


Figure 1: Expected value and price for Bidder 1 as a function of s_2 .

If Bidder 1's surplus does decrease monotonically with the price at which it wins, then its optimal bid is given by the unique solution to

$$E(v_1 | s_1, s_2 = \phi_2(b)) = s_1 + E(x_2 | s_2 = \phi_2(b)) = b$$

which can be rewritten as

$$s_1 + \phi_2(b) - \varepsilon E(z_2 | s_2 = \phi_2(b)) = b$$

or equivalently

$$\phi_1(b) + \phi_2(b) - \varepsilon E(z_2 | s_2 = \phi_2(b)) = b$$

That is, he is willing to pay up to the sum of his signal and 2's signal (which he can infer), minus the expected private value component of 2's signal. Of course, the same argument is valid for Buyer 2 whose inverse bid function should therefore satisfy

$$\phi_1(b) + \phi_2(b) - \varepsilon E(z_1 | s_1 = \phi_1(b)) = b$$

Combining these two expressions yields the following equilibrium conditions

$$\lambda(b) = E(z_1 | s_1 = \phi_1(b)) = E(z_2 | s_2 = \phi_2(b)) \quad (1)$$

$$\phi_1(b) + \phi_2(b) = b + \varepsilon \lambda(b) \quad (2)$$

for $s_2 \in (1.5, 2)$. As s_2 increases through 1.5, the possibility that it overestimates the common value due to a high private value component becomes a certainty, so a higher signal s_2 is bad news for Bidder 1.

It may be that there are some signals s_2 that always imply better news about z_2 than z_1 , regardless of the value of s_1 , and conversely; the equilibrium conditions do not apply to such signals. Let us define

Proposition 1 *Suppose that A1 is satisfied. There exists a unique Nash equilibrium, which is characterized by (1) and (2).*

Proof. First we show that there is a unique pair of strategies satisfying (1) and (2); then we will show that these strategies represent an equilibrium. $E(z_i | s_i)$ is continuous and strictly increasing by A1, and maps onto Z . Thus, applying the condition $E(z_1 | s_1) = E(z_2 | s_2) = z$ for each $z \in Z$, identifies a unique, one-to-one and increasing mapping ρ from signals s_1 to the complementary signal $s_2 = \rho(s_1)$ for which (1) is satisfied. Then, for each s_1 , the unique bid that satisfies (2) is given by $b_1(s_1) = s_1 + \rho(s_1) - \varepsilon E(z_2 | s_2 = \rho(s_1))$, and similarly for s_2 . Note that $b'_i > 1$ (because ρ is increasing and $\varepsilon E(z_i | s_i)$ has slope less than 1 by A3).

To show that these strategies are an equilibrium, note that each bidder's surplus function can now be written as

$$Y_i(p; s_i, \phi_{-i}) = s_i + E(x_{-i} | s_{-i} = \phi_{-i}(p)) - p$$

Differentiating with respect to p indicates that this surplus function is strictly decreasing:

$$\begin{aligned} \frac{d}{dp} Y_i(p; s_i, \phi_{-i}) &= \phi'_{-i}(p) \frac{d}{ds_{-i}} E(x_{-i} | s_{-i} = \phi_{-i}(p)) - 1 \\ &< 0 \end{aligned}$$

The inequality follows from A1 and because $b'_{-i} > 1 \Rightarrow \phi'_{-i} < 1$. At this point, the proof proceeds analogously to that of Lemma 1. ■

In words, because each buyer bids its expected value conditional on tying with its rival, if there is a bid at which the buyers sometimes tie, then they must have the same expected value for the object at that bid. Subtracting off the common value component, this means that the portion of its rival's bid that each bidder rationally expects to represent private value must be the same for the two bidders. This will restrict the scope for either bidder to be arbitrarily aggressive, as each must assess how likely it is that its rival's willingness to drive up the price reflects a private rather than a common benefit.

3 The Common Value Limit

So far there has been no presumption that private values represent a small fraction of the object's total value. In principle, Proposition 1 allows environments in which common and private values both matter to varying degrees to be studied. However, because equilibria will typically depend on the private value distributions g_i , it is difficult to draw general conclusions.

In the remainder of the paper, we focus on the special case in which private values are of negligible size. It turns out that as private values shrink, the relative amounts of private value information, as measured by the variance of g_1 relative to g_2 , continue to play a substantial role in the bidding. This observation leads us to a sharper characterization of equilibria that are robust to *private value perturbations (PVP)* in the common value limit.

The strategy will be to develop a more explicit formulation of the equilibrium condition (1), and then take Taylor series approximations of this condition. Taking limits as ε goes to zero leads to the PVP condition. First, the expected private value component of a rival's signal from (1) can be expressed:

$$E(z_1 | s_1 = s) = \frac{\int z f_1(s - \varepsilon z) g_1(z) dz}{\int f_1(s - \varepsilon z) g_1(z) dz}$$

We will take Taylor series expansions in the numerator and denominator of this expression as follows. First the numerator:

$$\begin{aligned} \int z f_1(s - \varepsilon z) g_1(z) dz &= f_1(s) \int z g_1(z) dz \\ &\quad - \varepsilon f_1'(s) \int z^2 g_1(z) dz \\ &\quad + o(\varepsilon) \\ &= -\varepsilon \sigma_1^2 f_1'(s) + o(\varepsilon) \end{aligned}$$

As is usual, $o(\varepsilon)$ indicates terms that vanish at a strictly faster rate than ε , and the assumption that $\mu_1 = 0$ is used twice. Next, for the denominator:

$$\begin{aligned} \int f_1(s - \varepsilon z) g_1(z) dz &= f_1(s) \int g_1(z) dz \\ &\quad - \varepsilon f_1'(s) \int z g_1(z) dz \\ &\quad + o(\varepsilon) \\ &= f_1(s) + o(\varepsilon) \end{aligned}$$

Combining these steps, we have

$$E(z_1 | s_1 = s) = -\varepsilon \sigma_1^2 \frac{f_1'(s)}{f_1(s)} + o(\varepsilon)$$

and in a similar fashion, for the private component of Buyer 2's signal we have

$$E(z_2 | s_2 = s) = -\varepsilon \sigma_2^2 \frac{f_2'(s)}{f_2(s)} + o(\varepsilon)$$

Therefore, we can rewrite the equilibrium condition (1) as

$$\begin{aligned}\sigma_1^2 \frac{f_1'(s_1)}{f_1(s_1)} &= \sigma_2^2 \frac{f_2'(s_2)}{f_2(s_2)} + o(\varepsilon^0) \quad \text{where} \\ s_1 &= \phi_1(b) \\ s_2 &= \phi_2(b)\end{aligned}$$

In the limit, as private information becomes negligible, the equilibrium bidding functions must satisfy

$$\sigma_1^2 \frac{f_1'(s_1)}{f_1(s_1)} = \sigma_2^2 \frac{f_2'(s_2)}{f_2(s_2)} \quad (3)$$

for any s_1 and s_2 for which Bidders 1 and 2 submit the same equilibrium bid. Even as private values vanish, the ratio of the amount of private value information held by the two bidders remains significant.

Definition 1 *A Nash equilibrium of the common value auction is the unique η - Private Value Perturbed (PVP) equilibrium if (3) is satisfied for every pair of signals such that $b_1(s_1) = b_2(s_2)$, and for $\sigma_1^2/\sigma_2^2 = \eta$.*

The PVP criterion contributes to analysis of the common value auction in two ways. First, it prunes out all those equilibria that do not satisfy (3) for *any* private value perturbation η . For example, if common value information has the distribution $f_1(x) = f_2(x) = 2x$, $x \in [0, 1]$, then one can infer from (3) that all PVP equilibria involve only bids that are linear in signals; by comparison, Lemma 1 admits many other pairs of increasing functions as equilibria. Second, it shows that across a narrower class of equilibria, the indeterminacy of the second price auction is robust. The PVP equilibrium will typically depend sensitively on the relative amounts of private information held by the two bidders, even though those amounts are assumed to be negligible. Equation (3) provides a framework for investigating exactly how the auction outcome should be expected to respond to small changes in the private values of the bidders.

To illustrate the intuition underlying (3), let us return to the example from Section 2.1. Recall that Bidder 2 observes ten of the common value components vs. one for Bidder 1. Suppose that $\eta = 1$ (equal amounts of private value information). With some algebra, one can show that the unique equilibrium bidding functions are given by

$$\begin{aligned}b_1 &= 40 + 11(x_1 - 20) \\ b_2 &= 40 + 1.1(x_2 - 20)\end{aligned}$$

Here the signal distributions play an important role in anchoring the bidding functions around the “neutral news” value prediction of $x_1 = 20$, $x_2 = 20$, $v = 40$. A bidder with bad news relative to the priors about his signal (in this case, $x_i < 20$) never outbids or ties a rival with good news (a higher than expected signal). This appealing result follows directly from the

private value noise: when a “bad news” signal bidder ties a rival with a “good news” signal, the first bidder should infer that some portion of that good news probably reflects a high private value draw for his rival rather than common value. Similarly, the rival should infer that the first bidder’s low signal may partially reflect bad luck with his private value. Consequently, the first bidder should bid a bit more cautiously and his rival should bid a bit higher, breaking the tie.

Furthermore, the PVP condition dictates how much a competitor’s bid “overreacts” to information that is more positive or negative than this neutral news anchor. This overreaction reflects the extra information about the common value revealed by tying a higher or lower rival signal. This reaction is muted by the fact that a higher rival signal may simply reflect a higher private value for the rival rather than a higher common value, so the smaller the ratio of private to common value information in this rival’s signal is, the larger the “overreaction” by the first bidder will be. In this case, Bidder 1 is much more sensitive to implied changes in b_2 because the signal-noise (common value to private value) ratio in s_2 is relatively high. The consequence is that Bidder 1 reacts more sensitively to both good and bad news than Bidder 2 does: Bidder 1 almost always wins when his signal is above average, and almost always loses otherwise. His expected surplus is approximately 0.4, versus about 3.8 for Bidder 2. These impressionistic results – that bidders are anchored by neutral news, and that, holding private values constant, bidders with less common value information tend toward high-low bidding – will be developed further in the next section.

4 Comparative Statics

This section applies the PVP condition to develop general results on how information affects the bidders’ prospects in the auction (both in terms of surplus and probability of winning) and policy choices by the seller. In order to study the effect of incremental changes in information, we return to the original interpretation of the model in which each composite signal x_i summarizes m_i individual signals, each reflecting a component of the common value. In this context, a bidder is said to be better informed about the common value than her rival if she observes more of its components (or equivalently, if there are fewer components about which she is uncertain). Formally, equilibrium bids should be thought of as including a constant term, comprising the value of any jointly observed components and the expected values of any jointly unobserved components, added to each of the bidding functions characterized above. This constant will be omitted in what follows, as it plays no role in either bidder’s probability of winning or expected surplus.

The PVP condition characterizes the relative aggressiveness of the two bidders point by point, based on how *locally* informative their priors are. In order to compare the bidding functions globally, some regularity conditions are required to ensure that these local comparisons hold uniformly.

A2. f_y is logconcave, symmetric, and “smoothly informative.”

Before explaining what is meant by “smoothly informative,” we formalize the notion of “good news” and “bad news.”

Definition 2 *A common value signal x_i drawn from the distribution f_i is said to be good (bad) news if $f'_i(x_i) < 0$ (> 0). If $f'_i(x_i) = 0$, it is neutral news. If $F_1(x_1) = F_2(x_2)$, then Bidders 1 and 2 are said to have relatively equal news.*

This definition of good news, like the PVP condition, is entirely local: it indicates that x_i is higher than prior beliefs *in the neighborhood of x_i* would have suggested. When prior beliefs are single-peaked, as they are under A2, this local definition takes on a global character: all signals above some threshold are good news, and all lower signals are bad news.⁵ Symmetry permits us to identify this threshold with the average signal. Of these conditions, logconcavity is the crucial one, while symmetry is convenient insofar as it will allow some unambiguous payoff implications to be derived.

The definition of “relatively equal news” captures one measure of how favorable a bidder’s signal is *relative* to prior beliefs about that signal. For example, imagine that prior beliefs are that x_1 is distributed on $[0, 1]$ and that x_2 is distributed on $[2, 3]$. How should we think about an equilibrium in which $b_1(1) = b_2(2) = 3$? One could argue that Bidder 1 is the more aggressive bidder because he ties Bidder 2 with a smaller signal, but this would be misleading: after all he manages to tie only when he receives the best possible news – and Bidder 2 the worst possible news – that either could have expected *ex ante*. Comparing the percentiles of the bidders’ signals provides some indication of whether they are high or low relative to their anticipated levels. Toward this end, the tying function γ is defined as follows: $b_1(F_1^{-1}(\gamma(u_2))) = b_2(F_2^{-1}(u_2))$. In words, $u_1 = \gamma(u_2)$ says that when Bidder 1 receives a signal in the u_1^{th} percentile of F_1 and Bidder 2’s signal is in the u_2^{th} percentile of F_2 , their bids will tie. If $\gamma(u_2) = u_2$, then the bidders have identical bid distributions and are equally likely to win the auction. One advantage of formulating the tying function in terms of the percentiles of the signal distributions is that we will be able to answer certain questions about how often a particular bidder outbids his rival without having to worry about the cardinal values of those bids.

As a step in this direction, we define a measure of how sensitively bids respond to signals. Note that under A2, $u_1 = u_2 = \frac{1}{2}$ is a neutral news percentile at which the bidders must tie. Bidder 1 is said to be more *volatile* than Bidder 2 if $|\gamma(u_2) - \frac{1}{2}| < |u_2 - \frac{1}{2}|$ for all $u_2 \neq \frac{1}{2}$. In words, if whenever the bidders have relatively equal news, Bidder 1 bids higher than Bidder 2 if the news is good and lower if the news is bad, then he more volatile. A more volatile bidder wins with probability greater than $\frac{1}{2}$ when both bidders have good news signals, and loses more than half the time when both signals are bad news. Furthermore, if there are two strategy profiles b_i and b'_i with associated tying functions γ and γ' , we will say that Bidder 2 is relatively

⁵Note that f_1 and f_2 inherit the logconcavity and symmetry of f_y .

more volatile under b_i than b'_i if $|\gamma(u_2) - \frac{1}{2}| > |\gamma'(u_2) - \frac{1}{2}|$ for all $u_2 \neq \frac{1}{2}$ – that is, if she ties more extreme signals for Bidder 1 under b_i .

Finally, “smoothly informative” is a property related to the Fisher information of f_y . The score function of a density f_i is commonly defined as its log derivative, $\rho_i(x) = f'_i(x)/f_i(x)$. We define h_i as the mapping from percentiles of F_i to the corresponding value of the score function: $h_i(u) = \rho_i(F_i^{-1}(u))$. Note that the Fisher information of f_i can be written

$$\int_0^1 h_i(u)^2 du \tag{4}$$

If f_1 represents more components of the common value than f_2 , then one can show that the Fisher information of f_1 is strictly less than that of f_2 ; that is, $h_1(u)^2 < h_2(u)^2$ in expectation⁶ (Since f_1 represents the uncertain part of Bidder 2’s belief about v , and *vice versa*, this is consistent with our description of Bidder 1 as better informed.) We say that the density f_y is *smoothly informative* if a related pointwise condition holds: whenever f_i represents the m_i -fold convolution of f_y with $m_1 > m_2$, we have $h_1(u)^2 < h_2(u)^2$ for *all* u for which $h_1(u)$ and $h_2(u)$ are nonzero. Logconcave densities that fail to be smoothly informative tend to be those whose score functions change abruptly; for example, uniform distributions have score functions that blow up at the boundaries of their support. In contrast, normal distributions are smoothly informative.

Lemma 2 *Under A2, Bidders 1 and 2 are (unconditionally) equally likely to win the auction. Conditional on their signals, if one bidder receives good news and the other bad news about the common value, the bidder with good news always wins. If both bidders receive good (bad) news, the more volatile bidder is more (less) likely to win.*

Lemma 3 *Suppose that $\eta = 1$ (symmetric levels of private value information). Then the bidder with fewer signals about the common value is more volatile in the PVP equilibrium.*

Lemma 2 formalizes some of the statements made earlier. A more volatile bidder wins relatively more often when she has good news and loses relatively more often when she has bad news. When the signal distributions are symmetric, these effects exactly cancel each other, and so the two bidders are equally likely to win.

The logic behind Lemma 3 is essentially as described in the example from the previous section. In forming its bid, each bidder must estimate the components of the common value about which its rival has private information. This estimate is informed by the bidder’s prior belief about these unobserved components and by the inferred signal of its rival. As long as these inferred signals are noiseless, the bidders can afford to ignore their priors entirely. Private values effectively add noise to the common value signal that can be inferred from a rival’s bid, forcing each bidder to shift weight back onto its prior belief about the unknown components.

⁶See Blachman 1965?

The magnitude of this shift depends on how precise prior beliefs about the common value are and how noisy the rival signal is. The bidder with fewer common value signals has a relatively less precise prior, so with equal levels of noise ($\eta = 1$), he leaves more weight on his rival's inferred signal, making his bidding more volatile.

Next we turn to the effect of information acquisition on bidding. In this model, one acquires more information by observing additional components of the common value. When Bidder 1 acquires an additional signal y_i about some component i , we will call this *duplicative* learning if this is a signal that Bidder 2 already has, and *expansive* learning if it is information that Bidder 2 does not have.

Proposition 2 *Learning about an additional component of the common value makes a bidder relatively less volatile, and its rival relatively more volatile (regardless of whether this learning is expansive or duplicative).*

Proof. Appendix ■

The payoff implications of acquiring additional common value information involve two factors. First, the acquirer either earns an information rent on the additional information (if learning is expansive) or reduces her rival's information rent (if learning is duplicative). Second, the additional information will make her opponent's bidding posture more volatile. This means that she will tend to win less often (and pay higher prices if she does win) when her opponent draws good news, but she will win more often and pay lower prices when her opponent draws bad news. Under the assumption that good news and bad news are symmetric (A2), she roughly breaks even on the marginal cases – the wins that used to be losses and the losses that used to be wins. However, the price effect is to her benefit, because she wins more often when her opponent has bad news.

Proposition 3 *Learning about an additional component of the common value increases a bidder's payoff and reduces her opponent's payoff. If the private value perturbations are symmetric ($\eta = 1$), then the bidder who is better informed about the common value earns a higher payoff.*

Proof. ■

To get clean results about the effect of information on the seller's revenue, we will specialize further and assume f_y to be normally distributed. Without further loss of generality, we assume that $f_y \sim N(0, 1)$.⁷ We can write the two bidders' composite signals as $x_i = \sqrt{m_i}z_i$, where z_i is also distributed as a standard normal. In this case, the PVP condition takes a particularly simple form:

$$\eta \frac{z_1}{\sqrt{m_1}} = \frac{z_2}{\sqrt{m_2}}$$

⁷Because both bidders price the mean value of v into their bids, this is equivalent to measuring bids and seller revenue relative to $E(v)$. Thus negative bids and revenue should not be cause for concern.

and the bidding functions are linear in signals:

$$\begin{aligned} b_1(x_1) &= \left(\frac{m_1 + m_2\eta}{m_1} \right) x_1 \\ b_2(x_2) &= \left(\frac{m_2 + m_1/\eta}{m_2} \right) x_2 \end{aligned}$$

In this model, information release by the seller can be thought of as the announcement, to one bidder privately, or publicly to both bidders, of the value of one of the components of the common value. First consider the public release of information about a component that neither bidder previously observed. This will have no effect on expected revenue: where both bidders previously bid the component at its expected value, now they will bid it at its actual value, and by independence, bidding on the components that they privately observe will not change. If information release is to affect revenues it must change the exclusive information held by the bidders. It can do this in one of two ways: the seller can release information to Bidder 2 about a component that Bidder 1 already observes, or it can release information privately to Bidder 2 about a component that neither bidder would otherwise observe. In the spirit of the discussion of information acquisition, these will be called duplicative and expansive information release, respectively.

With normal signals, the seller's revenue can be characterized in closed form.

Proposition 4 *The seller's expected revenue is*

$$E(v) - \frac{1}{\sqrt{2\pi}}(m_1/\sqrt{\eta} + m_2\sqrt{\eta})\sqrt{\frac{m_1/\eta + m_2\eta}{m_1m_2}} \quad (5)$$

When the bidders have symmetric levels of private value information ($\eta = 1$), expected revenue can be written

$$E(v) - \frac{2}{\sqrt{\pi}}\sqrt{\bar{m}}\left(\frac{\bar{m}}{\sqrt{m_1m_2}}\right)$$

where $\bar{m} = (m_1 + m_2)/2$.

Proof. After normalizing by the realized values of all commonly known components and the expected values of all commonly unknown components, the bids can be written $(\frac{m_1+m_2\eta}{\sqrt{m_1}}z_1, \frac{m_1/\eta+m_2}{\sqrt{m_2}}z_2)$ where z_1 and z_2 are standard normals. The deviation of the seller's revenue from $E(v)$ is then $E(\min(\frac{m_1+m_2\eta}{\sqrt{m_1}}z_1, \frac{m_1/\eta+m_2}{\sqrt{m_2}}z_2))$. With a bit of algebra, one can show that if z_1 and z_2 are standard normals, then

$$E(\min(k_1z_1, k_2z_2)) = -\frac{\sqrt{k_1^2 + k_2^2}}{\sqrt{2\pi}}$$

The result then follows by substitution for k_1 and k_2 . ■

The impact of asymmetric amounts of common value information is most transparent when the private value perturbations are symmetric ($\eta = 1$). In this case, the seller's shortfall below

the *ex ante* expected value of the object increases with the average amount of exclusive information \bar{m} held by each bidder, just as would be expected if common value information were symmetric. However, there is a second effect that depends on the ratio of the arithmetic and geometric means of the bidders' exclusive information. As the distribution of exclusive information becomes more unequal, holding \bar{m} constant, $\sqrt{m_1 m_2}$ falls, as do the seller's revenues. Thus, the seller does best when any exclusive information is distributed equally across the two bidders. More generally, we have the following.

Corollary 1 *Fix the average number of exclusive signals \bar{m} and let $m_1 = \lambda \bar{m}$ and $m_2 = (1 - \lambda) \bar{m}$. The seller's revenue is greatest when the bidder with more information about her private value also has more information about the common value. (That is, the revenue-maximizing level of λ is greater than $\frac{1}{2}$ iff. $\eta > 1$.)*

The proof, which requires performing fairly simple comparative statics with (5), is omitted. It is worth pointing out that small asymmetries in the private value perturbations can have a dramatic effect on the seller's desired distribution of common value information. For example, when $\eta = 2$, variation in private value accounts for twice as much variation in Bidder 1's bids as in Bidder 2's. (In both cases, this is still a negligible fraction of the total variation in the bids.) If the seller could choose how to distribute a fixed number $2\bar{m}$ of common value signals exclusively to the two bidders, it would prefer to distribute roughly 80% of them to Bidder 1.

Next consider a seller facing unequally informed bidders ($m_1 < m_2$, with $\eta = 1$). If the seller is able to erode Bidder 2's advantage in exclusive information by releasing some of it to Bidder 1 (i.e., decreasing m_2), this will always increase revenue. What is perhaps more surprising is that creating new information rents for Bidder 1 through expansive information release (increasing m_1) will also increase profits when Bidder 2's advantage is great enough ($m_2 > 2m_1$). Furthermore, if the imbalance is still a bit greater ($m_2 > (2 + \sqrt{3})m_1$), giving Bidder 1 expansive information is actually superior to giving him duplicative information.

5 Concluding Remarks

One possible complaint against private value perturbation approach is that it does not provide a true selection result because the set of equilibria that are robust to *some* private value perturbation σ_1/σ_2 is typically still large. In order to get a precise equilibrium prediction, one must know *which* private value perturbation best represents the situation. There are two responses to this criticism. First, even if the appropriate ratio σ_1/σ_2 is not known, the PVP condition still puts relatively stringent restrictions on the form of equilibrium bidding functions. For example, "sunspot" equilibria in which the bidders switch from cautious to aggressive strategies (and *vice versa*) at arbitrary prices and arbitrarily often will not survive. This may be valuable for empirical work, as the number of free parameters to estimate is reduced from a continuum to just one. Second, a selection that does not depend on the relative amounts of

private value information held by the bidders runs the risk of throwing out information that is actually important in determining auction outcomes.

Finally, a note of caution. A theme that emerges from this paper as well as several others in the theoretical literature (e.g., Klemperer (1998), Parreiras (2002)) is that second price auction outcomes can be very sensitive to small asymmetries among bidders when values are “almost” common. Here it is small asymmetries in the amount of private value information that modulate the effect of much larger asymmetries in information about the common value. However, empirical and experimental work consistently reveals that bidders account for the information held by others, but not as fully as they should. One hopes that the effects of differential information identified in the theory would show up qualitatively, if perhaps less powerfully, in the data; experimental work on auctions with differential information would be valuable in determining whether this is true.

Appendix: Proofs

Lemma 1

Suppose that ϕ_1 and ϕ_2 satisfy the conditions of the lemma and consider whether ϕ_1 is a best response to ϕ_2 for an arbitrary signal s'_1 . There are two possibilities: either $b' = b_1(s'_1) \in B_2$ (Bidder 1 sometimes ties with Bidder 2 given signal s'_1), or $b' \notin B_2$ (Bidder 1 always wins or always loses with signal s'_1). If $b' \in B_2$, then Bidder 1’s expected surplus conditional on winning and paying a price equal to her bid is

$$\begin{aligned} Y_1(b'; s'_1, \phi_2) &= s_1 + \phi_2(b') - b' \\ &= \phi_1(b') + \phi_2(b') - b' \\ &= 0 \end{aligned}$$

Furthermore, $Y_1(p; s'_1, \phi_2)$ is strictly decreasing in p (because $\phi'_2 < 1$), so Bidder 1’s surplus function is positive (negative) when she wins at prices less than (greater than) b' . By bidding b' , she wins if and only if her expected surplus is positive, so this is a best response.

Alternatively, suppose that $b' < b_2 \forall b_2 \in B_2$ (so Bidder 1 never wins with signal s_1). Let $b^* = \inf\{b \in B_1 \cap B_2\} = \inf\{b \in B_2\} > b'$ be the lowest price at which Bidder 1 could win. (b^* exists because we have assumed $B_1 \cap B_2$ nonempty, and the second equality follows from the continuity of b_1 .) Of course, $\phi_1(b^*) > s_1$. By assumption, we have

$$\begin{aligned} Y_1(b^*; \phi_1(b^*), \phi_2) &= \phi_1(b^*) + \phi_2(b^*) - b^* \\ &= 0 \end{aligned}$$

Thus Bidder 1’s surplus at any price $p \geq b^*$ for which it has a positive chance of winning is

strictly negative:

$$\begin{aligned} Y_1(p; s_1, \phi_2) &< Y_1(p; \phi_1(b^*), \phi_2) \\ &\leq Y_1(b^*; \phi_1(b^*), \phi_2) \\ &= 0 \end{aligned}$$

It follows that bidding $b' < b^*$ and never winning is a best response. The argument for the case in which $b' > b_2 \forall b_2 \in B_2$ (so that Bidder 1 always wins with signal s_1) is virtually identical and is omitted. Since the choice of s_1 was arbitrary, ϕ_1 is a best response to ϕ_2 . An identical argument establishes that ϕ_2 is a best response to ϕ_1 .

Lemma 4 *Assumption A1 implies $0 < d(E(x_i | s_i)/ds_i) < 1$.*

Proof. i. We will drop unnecessary subscripts. Choose arbitrary s and $s' > s$. There exists some x^* such that $f(x^* | s) = f(x^* | s')$. Then because the MLRP holds for x and s , we have

$$\frac{f(x | s')}{f(x^* | s')} > \frac{f(x | s)}{f(x^* | s)}$$

for all $x > x^*$, and hence $f(x | s') > f(x | s)$ for all $x > x^*$. Similarly, we have $f(x | s') < f(x | s)$ for all $x < x^*$. But this suffices to show that $F(x | s') < F(x | s)$ from which it follows that $E(x | s') > E(x | s)$.

ii. Because $E(x | s) = s - \varepsilon E(z | s)$, it will be enough to show that $E(z | s)$ is increasing in s , which can be proved just as for i.

iii. We will show that g log-concave implies that $f(x' | s)/f(x | s)$ is increasing in s whenever $x' > x$. First, note that

$$\frac{f(x' | s)}{f(x | s)} = \frac{f(x')}{f(x)} \frac{g(\frac{s-x'}{\varepsilon})}{g(\frac{s-x}{\varepsilon})}$$

so if $s' > s$,

$$\frac{f(x' | s')}{f(x | s')} > \frac{f(x' | s)}{f(x | s)} \Leftrightarrow g(\frac{s'-x'}{\varepsilon})g(\frac{s-x}{\varepsilon}) > g(\frac{s-x'}{\varepsilon})g(\frac{s'-x}{\varepsilon})$$

But $\frac{s'-x'}{\varepsilon} + \frac{s-x}{\varepsilon} = \frac{s-x'}{\varepsilon} + \frac{s'-x}{\varepsilon}$ and $s-x'$ and $s'-x$ bracket $s-x$ and $s'-x'$, so the inequality holds by log-concavity of g . Finally, we can apply i. to show A2. The proof that log-concavity of f implies A1 is very similar and is omitted. ■

Proposition 1(ii) (Existence)

The proof is by construction. A1 ensures that $\lambda_i(s_i) = E(z_i | s_i)$ is strictly increasing in s_i . Let Z_i be the image of λ_i and let Z be the closure of $Z_1 \cap Z_2$. For signals s_1 and s_2 such that

$\lambda_1(s_1) \in Z$ and $\lambda_2(s_2) \in Z$ we implicitly define the maps Q_1 and Q_2 :

$$\begin{aligned}\lambda_2(Q_2(s_1)) &= \lambda_1(s_1) \\ \lambda_1(Q_1(s_2)) &= \lambda_2(s_2)\end{aligned}$$

That is, $Q_2 = Q_1^{-1}$ links pairs of signals that map to the same value of $E(z_i | s_i)$; λ_i increasing ensures that Q_i exists and is increasing. The domain of Q_{-i} is denoted $S_i = \lambda_i^{-1}(Z)$. Let $U_i = \lambda_i^{-1}(Z_i \setminus Z)$ be the set of signals for i that cannot be paired with a signal for $-i$ mapping to the same value of z . In our construction, a signal in U_i will either be non-competitive (if $\lambda_i(s_i) < \inf\{z \in Z\}$) or will always win (if $\lambda_i(s_i) > \sup\{z \in Z\}$). Continuity ensures that if U_1 includes non-competitive signals, then U_2 does not (and similarly for signals that always win). Let the bidding functions be defined by

$$b_i(s) = \begin{cases} \tilde{b}_i(s) & s \in U_i \\ s + Q_{-i}(s) - \varepsilon \lambda_{-i}(Q_{-i}(s)) & s \in S_i \end{cases}$$

where the only constraint on \tilde{b}_i is that it has slope greater than one and satisfies smooth pasting conditions at the boundaries of U_i and S_i . Notice that for $s \in S_i$ the bidding function can be written $b_i(s) = s + E(x_{-i} | s_{-i} = Q_{-i}(s))$. Therefore, A2 and Q_{-i} increasing ensure that $b'_i(s) > 1$. By construction, the bid functions overlap precisely for $B_1 \cap B_2 = b_1(S_1) = b_2(S_2)$. Note that $b_1(s_1) = b_2(Q_2(s_1))$. Furthermore, because the bid functions are strictly increasing, $b_1(s_1) = b_2(s_2)$ only if $s_2 = Q_2(s_1)$. Thus for any s_1 and s_2 that tie, we have $\lambda_1(s_1) = \lambda_2(Q_2(s_1)) = \lambda_2(s_2)$, so (1) holds. For tying signals s_1 and s_2 with $b_1(s_1) = b_2(s_2) = b$ and $\lambda_1(s_1) = \lambda_2(s_2) = z_b$, we have

$$\begin{aligned}b &= s_1 + s_2 - \varepsilon z_b \\ &= \phi_1(b) + \phi_2(b) - \varepsilon \lambda(b)\end{aligned}$$

so (2) holds. Therefore the constructed bidding functions satisfy the conditions in i) for a Nash equilibrium.

Proposition 2

Suppose that initially Bidders 1 and 2 observe m_1 and m_2 components exclusively (with $m_1, m_2 > 1$). If Bidder 1 observes one additional signal, the exclusive information of the bidders becomes \tilde{m}_1 and \tilde{m}_2 , where $(\tilde{m}_1, \tilde{m}_2)$ equals $(m_1 + 1, m_2)$ if the learning is expansive and $(m_1, m_2 - 1)$ if the learning is duplicative. Write $f_y^{(m)}$ for the p.d.f. of the sum of m draws from f_y . For a fixed private value ratio η , let (b_1, b_2) be the PVP equilibrium generated by $f_1 = f_y^{(m_1)}$ and $f_2 = f_y^{(m_2)}$ and $(\tilde{b}_1, \tilde{b}_2)$ the equilibrium generated by $\tilde{f}_1 = f_y^{(\tilde{m}_1)}$ and $\tilde{f}_2 = f_y^{(\tilde{m}_2)}$. We would like to show that Bidder 1 is less volatile under $(\tilde{b}_1, \tilde{b}_2)$ than under (b_1, b_2) . The PVP

equilibrium condition is given by

$$h_2(u_2) = \eta h_1(\gamma(u_2))$$

for the original information structure and by

$$\tilde{h}_2(u_2) = \eta \tilde{h}_1(\tilde{\gamma}(u_2))$$

for the information structure with additional learning by Bidder 1. Consider duplicative learning, so that $\tilde{h}_1 = h_1$. Under logconcavity, h_2 and \tilde{h}_2 are decreasing, and by symmetry they are positive on $[0, 1/2)$ and negative on $(1/2, 1]$. Because f_y is smoothly informative, we also have $|\tilde{h}_2(u)| > |h_2(u)|$ (except at $u = 1/2$). By the equilibrium conditions, we have $|\eta h_1(\tilde{\gamma}(u))| > |\eta h_1(\gamma(u))|$ for all $u \neq 1/2$. Because h_1 is also decreasing and positive (negative) on $[0, 1/2)$ ($(1/2, 1]$), we must have $\tilde{\gamma}(u) < \gamma(u)$ for $u < 1/2$ and $\tilde{\gamma}(u) > \gamma(u)$ for $u > 1/2$. But this means that Bidder 2 is relatively more volatile (and Bidder 1 relatively less volatile) under the new information structure.

On the other hand, if learning is expansive, we have $\tilde{h}_2 = h_2$ and $|\tilde{h}_1(u)| < |h_1(u)|$. Because the equilibrium conditions now require $|\tilde{h}_1(\tilde{\gamma}(u))| = |h_1(\gamma(u))|$ and because γ and $\tilde{\gamma}$ are increasing in u , we once again must have $\tilde{\gamma}(u) < (>) \gamma(u)$ if $u < (>) 1/2$.

Proposition 3

Suppose Bidder 1 acquires duplicative information and that the distribution of exclusive information changes from (m_1, m_2) to $(\tilde{m}_1, \tilde{m}_2) = (m_1, m_2 - 1)$. We can express Bidder 1's expected equilibrium payoff under the original information structure in terms of the percentiles of the two signal distributions. For signal percentiles u_1 and u_2 , the corresponding signals are $F_1^{-1}(u_1)$ and $F_2^{-1}(u_2)$, and the common value is $v = F_1^{-1}(u_1) + F_2^{-1}(u_2)$. Bidder 1 wins if $u_1 > \gamma(u_2)$ in which case the price is Bidder 2's bid:

$$E(v | u_2, u_1 = \gamma(u_2)) = F_1^{-1}(\gamma(u_2)) + F_2^{-1}(u_2)$$

and Bidder 1's surplus is

$$(F_1^{-1}(u_1) - F_1^{-1}(\gamma(u_2)))$$

His expected payoff is therefore

$$\int_0^1 \int_0^1 1(u_1 > \gamma(u_2))(F_1^{-1}(u_1) - F_1^{-1}(\gamma(u_2))) du_1 du_2 \quad (6)$$

while under the new information structure it is

$$\int_0^1 \int_0^1 1(u_1 > \tilde{\gamma}(u_2))(F_1^{-1}(u_1) - F_1^{-1}(\tilde{\gamma}(u_2))) du_1 du_2 \quad (7)$$

Note that because the extra information is duplicative, there is no information rent effect for Bidder 1 (F_1 does not change); the only change in his payoff is due to the change in bidding postures (γ vs. $\tilde{\gamma}$). Because the change induces Bidder 2 to bid more volatily, Bidder 1 wins more often, and at lower prices, when Bidder 2 draws bad news and wins less often and at higher prices when Bidder 2 draws good news. The strategy of the proof is to match up pairs of signals for Bidder 2 (one good news and one bad news) and show that the former effect outweighs the latter. Each pair is of the form $(u_2, 1 - u_2)$, where $u_2 < 1/2$. Toward this end, write (6) as

$$\int_0^{1/2} \left\{ \int_0^1 \mathbf{1}(u_1 > \gamma(u_2))(F_1^{-1}(u_1) - F_1^{-1}(\gamma(u_2))) du_1 + \int_0^1 \mathbf{1}(u_1 > \gamma(1 - u_2))(F_1^{-1}(u_1) - F_1^{-1}(\gamma(1 - u_2))) du_1 \right\} du_2$$

Writing (7) analogously, the difference between the two payoffs can be expressed as $\int_0^{1/2} \psi(u_2) du_2$, where

$$\begin{aligned} \psi(u_2) &= \int_0^1 \mathbf{1}(u_1 > \tilde{\gamma}(u_2))(F_1^{-1}(u_1) - F_1^{-1}(\tilde{\gamma}(u_2))) du_1 \\ &\quad - \int_0^1 \mathbf{1}(u_1 > \gamma(u_2))(F_1^{-1}(u_1) - F_1^{-1}(\gamma(u_2))) du_1 \\ &+ \int_0^1 \mathbf{1}(u_1 > \tilde{\gamma}(1 - u_2))(F_1^{-1}(u_1) - F_1^{-1}(\tilde{\gamma}(1 - u_2))) du_1 \\ &\quad - \int_0^1 \mathbf{1}(u_1 > \gamma(1 - u_2))(F_1^{-1}(u_1) - F_1^{-1}(\gamma(1 - u_2))) du_1 \end{aligned}$$

Below, we develop alternatives expressions for $\psi(u_2)$ for arbitrary $u_2 \in [0, 1/2)$. By Proposition 2 we have $\tilde{\gamma}(u_2) < \gamma(u_2) < \gamma(1 - u_2) < \tilde{\gamma}(1 - u_2)$, so $\psi(u_2)$ can be written

$$\begin{aligned} \psi(u_2) &= \int_{\tilde{\gamma}(u_2)}^{\gamma(u_2)} F_1^{-1}(u_1) - F_1^{-1}(\tilde{\gamma}(u_2)) du_1 \\ &\quad + \int_{\gamma(u_2)}^1 F_1^{-1}(\gamma(u_2)) - F_1^{-1}(\tilde{\gamma}(u_2)) du_1 \\ &\quad - \int_{\gamma(1-u_2)}^{\tilde{\gamma}(1-u_2)} F_1^{-1}(u_1) - F_1^{-1}(\gamma(1 - u_2)) du_1 \\ &\quad - \int_{\tilde{\gamma}(1-u_2)}^1 F_1^{-1}(\tilde{\gamma}(1 - u_2)) - F_1^{-1}(\gamma(1 - u_2)) du_1 \\ &= \int_{\tilde{\gamma}(u_2)}^{\gamma(u_2)} F_1^{-1}(u_1) du_1 - (\gamma(u_2) - \tilde{\gamma}(u_2))F_1^{-1}(\tilde{\gamma}(u_2)) \\ &\quad + (1 - \gamma(u_2))(F_1^{-1}(\gamma(u_2)) - F_1^{-1}(\tilde{\gamma}(u_2))) \\ &\quad - \int_{\gamma(1-u_2)}^{\tilde{\gamma}(1-u_2)} F_1^{-1}(u_1) du_1 + (\tilde{\gamma}(1 - u_2) - \gamma(1 - u_2))F_1^{-1}(\gamma(1 - u_2)) \\ &\quad - (1 - \tilde{\gamma}(1 - u_2))(F_1^{-1}(\tilde{\gamma}(1 - u_2)) - F_1^{-1}(\gamma(1 - u_2))) \end{aligned}$$

Now we appeal to symmetry. Symmetry of f_1 and f_2 implies that γ and $\tilde{\gamma}$ are symmetric about $1/2$; that is, $\gamma(u) = 1 - \gamma(1 - u)$ and similarly for $\tilde{\gamma}$. Furthermore, symmetry of F_1 implies that $F_1^{-1}(u) + F_1^{-1}(1 - u) = 2F_1^{-1}(1/2)$. Using these facts, we have $F_1^{-1}(\tilde{\gamma}(1 - u_2)) - F_1^{-1}(\gamma(1 - u_2)) = F_1^{-1}(\gamma(u_2)) - F_1^{-1}(\tilde{\gamma}(u_2))$. Substituting in, and then combining the first and third terms and

the second and fourth terms, we obtain

$$\begin{aligned} \psi(u_2) &= \int_{\tilde{\gamma}(u_2)}^{\gamma(u_2)} F_1^{-1}(u_1) du_1 - \int_{\gamma(1-u_2)}^{\tilde{\gamma}(1-u_2)} F_1^{-1}(u_1) du_1 + (\gamma(u_2) - \tilde{\gamma}(u_2))(2\bar{x} - F_1^{-1}(\gamma(u_2)) - F_1^{-1}(\tilde{\gamma}(u_2))) \\ &\quad + (\tilde{\gamma}(1-u_2) - \gamma(1-u_2))(F_1^{-1}(\gamma(u_2)) - F_1^{-1}(\tilde{\gamma}(u_2))) \end{aligned}$$

The second line above is strictly positive for $u_2 \in (0, 1/2)$. Using the change of variables $u'_1 = 1 - u_1$, the second integral in the first line may be written $\int_{\tilde{\gamma}(u_2)}^{\gamma(u_2)} 2F_1^{-1}(1/2) - F_1^{-1}(u'_1) du'_1$. Changing the dummy variable back to u_1 and combining terms, the first line becomes

$$2(\gamma(u_2) - \tilde{\gamma}(u_2)) \left\{ \frac{1}{\gamma(u_2) - \tilde{\gamma}(u_2)} \int_{\tilde{\gamma}(u_2)}^{\gamma(u_2)} F_1^{-1}(u_1) du_1 - \frac{F_1^{-1}(\gamma(u_2)) + F_1^{-1}(\tilde{\gamma}(u_2))}{2} \right\}$$

But $F_1^{-1}(u)$ is concave for $u < 1/2$ (the curvature of $F_1^{-1}(u)$ and $F_1(F_1^{-1}(u))$ have opposite signs, and for logconcave, symmetric f_1 , $F_1'' = f_1'$ is positive below its median), so the term in brackets is just the average of a concave function over an interval minus the average of the function at the endpoints of the interval; this is positive by Jensen's inequality. With both lines positive, $\psi(u_2)$ must be strictly positive for all u_2 in the interior of $[0, 1/2]$, and consequently the change in Bidder 1's payoff under the new information structure, $\int_0^{1/2} \psi(u_2) du_2$, is strictly positive as well. This proves the first assertion of the proposition for duplicative learning.

For expansive learning, suppose that the new information structure is $(\bar{m}_1, \bar{m}_2) = (m_1 + 1, m_2)$. This time, Bidder 1's signal distribution changes to reflect his greater number of exclusive signals; we will write $\bar{f}_1 = f_y^{(\bar{m}_1)}$ for this new distribution. Bidder 1's payoff under the new information structure then becomes

$$\int_0^1 \int_0^1 1(u_1 > \bar{\gamma}(u_2)) (\bar{F}_1^{-1}(u_1) - \bar{F}_1^{-1}(\bar{\gamma}(u_2))) du_1 du_2 \quad (8)$$

Consider the hypothetical situation in which we set the tying function to $\bar{\gamma}$, reflecting less volatile bidding by Bidder 1, without changing Bidder 1's signal distribution. In this scenario, Bidder 1's expected payoff is

$$\int_0^1 \int_0^1 1(u_1 > \bar{\gamma}(u_2)) (F_1^{-1}(u_1) - F_1^{-1}(\bar{\gamma}(u_2))) du_1 du_2 \quad (9)$$

Compare (9) to (7). We have shown in the previous part of the proof that if Bidder 1 is less volatile under $\bar{\gamma}$ than γ , then (9) is greater than Bidder 1's original payoff. Therefore, it will suffice to show that (8) is greater than (9). But by a standard result from statistics (see Shantikumar, 1993), if f_1 and f_y are logconcave, and \bar{f}_1 is their convolution, then $\bar{F}_1^{-1}(b) - \bar{F}_1^{-1}(a) > F_1^{-1}(b) - F_1^{-1}(a)$ for any percentiles $a, b \in [0, 1]$ with $a < b$. Therefore, whenever the integrands are non-zero, we have $u_1 > \bar{\gamma}(u_2)$, and consequently $\bar{F}_1^{-1}(u_1) - \bar{F}_1^{-1}(\bar{\gamma}(u_2)) > F_1^{-1}(u_1) - F_1^{-1}(\bar{\gamma}(u_2))$, so (8) is indeed greater than (9).

To see that learning additional information reduces one's opponent's payoff, observe that

learning by Bidder 2 can be represented as a change in the information structure from $(\tilde{m}_1, \tilde{m}_2)$ to (m_1, m_2) or from (\bar{m}_1, \bar{m}_2) to (m_1, m_2) ; both of these changes reduce Bidder 1's payoff.

To see that Bidder 1's payoff is greater than that of Bidder 2 if $m_1 > m_2$ and $\eta = 1$, consider moving to $(m'_1, m'_2) = (m_1, m_1)$ by incrementally giving Bidder 2 more information. Bidder 1's payoff decreases, and Bidder 2's increases, at each step. Furthermore, their payoffs must be equal at (m'_1, m'_2) by symmetry, so Bidder 1 must have the higher payoff under the original information structure.

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