Collusion under Sales Monitoring*

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Abstract

Collusion under imperfect monitoring is explored when firms' prices are private information and their quantities are public information; an information structure consistent with several recent price-fixing cartels such as the citric acid and lysine cartels. For a class of symmetric oligopoly games, it is shown that symmetric equilibrium punishments cannot sustain any collusion for any discount factor. A preliminary analysis is provided into how asymmetric punishments can sustain collusion. The asymmetric punishment characterized has the firm with sales exceeding its quota compensating the firm with sales below its quota. In practice, cartels have performed such transfers through sales among the cartel members.

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... if I'm assured that I'm gonna get 67,000 tons [of lysine sales] by the year's end, we're gonna sell it at the prices we agreed to and I frankly don't care what you sell it for. [Terrance Wilson of Archer Daniels Midland from the March 10, 1994 meeting of the lysine cartel.]

And that total for us for the year, calendar year is 68,000; 68,334. 68,334 and our target was 67,000 plus alpha. Almost on target. [Mark Whitacre of Archer Daniels Midland from the January 18, 1995 meeting of the lysine cartel.][1]

1 Introduction

Many price-fixing cartels, such as the lysine cartel, involve firms selling to industrial buyers. As price can be settled through private negotiation, it is not typically observable. In such cases, compliance with the collusive agreement is based on firms' sales. Indeed, cartels can go to great lengths to ensure that sales are public information among the cartel members. In the citric acid cartel, for example, firms hired an international accounting firm to independently audit sales reports (Connor, 2001). The objective of this paper is to explore collusion when monitoring is imperfect and, contrary to the standard formulation of Green and Porter (1984), prices are private information and firms' quantities are public information.[2]

Our main result is an impossibility result. For a general class of symmetric demand structures, no collusion can be sustained by symmetric punishments. Assuming inelastic market demand, sufficient conditions for collusion to be unattainable is that the probability distribution over firm demand depends only on the difference in firms' prices or the ratio of firms' prices. This specification includes some common ones including the Hotelling model and Cabral and Riordan (1994). The rough intuition for the result is as follows. Intuitively, one would expect punishment to occur when market share is sufficiently skewed. Suppose, for example, punishment occurs when one of the firm's market shares exceeds 70%. A firm that considers charging a price below the collusive price raises the probability that its market share exceeds 70% - which makes punishment more likely - but lowers the probability that the other firm's market share exceeds 70% - which makes punishment less likely. What we show is that the probability increase of the first event exactly equals the probability decrease of the second event so that the probability of a punishment remains unchanged. Since symmetry implies the punishment payoff to this firm is the same in both events, its continuation payoff is unaffected by its price. From this we conclude that an equilibrium price for the infinite horizon game must be the same as that for the stage game. We then show that our theorem is tight by considering a demand system outside of the specified class and show that collusion can be supported with symmetric punishments.

The conclusion we draw from this result is not that firms cannot collude but

[1] These quotes are from the video transcript of "The International Lysine Cartel at Work, 3/28/00" provided by the U.S. Department of Justice, Antitrust Division.

[2] Interestingly, this information structure is the one that Stigler (1964) originally described when he raised the issue of imperfect monitoring.

rather the importance of treating apparent deviators differently from apparent non-deviators. Indeed, many price-fixing cartels such as the citric acid and sodium gluconate cartels have deployed asymmetric punishments. The next step in this project is then to characterize asymmetric punishments. Though our research is early in this regard, we do have a preliminary result showing how asymmetric punishments with side payments can sustain collusion. Some recent cartels have engaged in side payments and used inter-firm sales among the cartel members as a way in which to execute them.

2 Model

Consider an infinitely repeated duopoly game in which firms make simultaneous price decisions. Cost functions are common and linear and, without loss of generality, cost is zero. Demand is fixed at m units and discrete. This structure conforms well with many of the price-fixing cases as they often involve firms colluding in the price they charge industrial buyers and market demand is often highly inelastic. Though total demand is fixed, firm demand is stochastic. Let

$$\phi : \{0, 1, \ldots, m\} \times \Re^2 \rightarrow [0, 1]$$

be the probability function on firm 1's demand. $\phi(b; p_1, p_2)$ is the probability that firm 1 sells b units given its price is p_1 and its rival's price is p_2, where $p_1, p_2 \in \Re$. As total demand is fixed at m units, $\phi(m - b; p_1, p_2)$ is the probability that firm 2's demand is b. One can either imagine that products are differentiated or that they are homogeneous but buyer-specific shocks, which may be independent or correlated, influence their demand in each period.

We make three assumptions on the probability distribution on firm demand.

A1 ϕ is continuously differentiable with respect to p_1 and p_2.

A2 $\phi(b; p', p'') = \phi(m - b; p'', p')\ \forall b \in \{0, 1, \ldots, m\}, \forall (p', p'') \in \Re^2$.

A3 $\frac{\partial \phi(b;p,p)}{\partial p_1} + \frac{\partial \phi(b;p,p)}{\partial p_2} = 0\ \forall b \in \{0, 1, \ldots, m\}, \forall p \in \Re$.

A1 is standard and A2 imposes symmetry. A3 is the key restriction though is satisfied in many models. It holds, for example, when ϕ depends only on the price difference, $p_1 - p_2$, or the price ratio, p_1/p_2. For example, suppose $\exists \xi : \{0, 1, \ldots, m\} \times \Re \rightarrow [0, 1]$ such that

$$\phi(b; p_1, p_2) = \xi(b; \Delta)\ \forall b \in \{0, 1, \ldots, m\}, \forall (p_1, p_2) \in \Re^2,$$

where $\Delta \equiv p_1 - p_2$. Then

$$\frac{\partial \phi(b; p, p)}{\partial p_1} + \frac{\partial \phi(b; p, p)}{\partial p_2} = \frac{\partial \xi(b; 0)}{\partial \Delta} - \frac{\partial \xi(b; 0)}{\partial \Delta} = 0,$$

so that A3 holds. An example from the literature that conforms to this specification is an m-buyer generalization of Cabral and Riordan (1994). The probability that firm

1 sells to a particular buyer equals $F(p_2 - p_1)$ where $F : \Re \to [0,1]$ is continuously differentiable and non-decreasing and F' is symmetric around zero. Buyers' decisions as to whom to buy from are *iid* which implies that a firm's demand is binomially distributed,

$$\phi(b; p_1, p_2) = \left(\frac{m!}{b!(m-b)!}\right) F(p_2 - p_1)^b (1 - F(p_2 - p_1))^{m-b},$$

and thus depends only on the price difference. Or suppose $\exists \xi : \{0, 1, \ldots, m\} \times \Re \to [0,1]$ such that

$$\phi(b; p_1, p_2) = \xi(b; \Lambda) \, \forall b \in \{0, 1, \ldots, m\}, \forall (p_1, p_2) \in \Re^2.$$

where $\Lambda \equiv p_1/p_2$. Then

$$\begin{aligned} \frac{\partial \phi(b; p, p)}{\partial p_1} + \frac{\partial \phi(b; p, p)}{\partial p_2} &= \left(\frac{\partial \xi(b; 1)}{\partial \Lambda}\right)\left(\frac{1}{p_1}\right) - \left(\frac{\partial \xi(b; 1)}{\partial \Lambda}\right)\left(\frac{p_1}{p_2^2}\right) \\ &= \left(\frac{1}{p}\right)\left[\frac{\partial \xi(b; 1)}{\partial \Lambda} - \frac{\partial \xi(b; 1)}{\partial \Lambda}\right] = 0. \end{aligned}$$

Finally, a discrete choice model in which consumer indirect utility is linear in price will also work. In that case, the utility to consumer j from buying the product of firm i is $V_i^j - p_i$ so that firm 1's product is bought iff:

$$V_1^j - \beta p_1 > V_2^j - \beta p_2 \Leftrightarrow \frac{V_1^j - V_2^j}{\beta} > p_1 - p_2.$$

Note that our assumptions do not require that demand be decreasing in price. In our stochastic formulation of demand, the natural way in which to encompass that property is to assume that a higher price implies a first-order stochastic dominance shift in a firm's probability distribution over its demand:

$$\text{If } p_1'' > p_1' \text{ then} \sum_{b=0}^{k} \phi(b; p_1', p_2) \leq \sum_{b=0}^{k} \phi(b; p_1'', p_2) \;\; \forall k \in \{0, 1, \ldots, m\}.$$

Though we need not impose such a restriction, our examples will satisfy it.

There is an infinite horizon and each firm's payoff is the expected present value of its profit stream where the common discount factor is $\delta \in (0,1)$. The information structure is one of imperfect monitoring as firms' price decisions are private information. This conforms to the industrial buyer case in which price is negotiated between a seller and a buyer and thus is not publicly posted (or if it is, as with list prices, the posted prices are not necessarily meaningful). Given that it is common knowledge that market demand is fixed and each firm observes its demand then realized quantities are common knowledge. It is sufficient to think of a history at the start of period t, denoted h^{t-1}, to be a sequence of quantities sold for firm 1. A firm's strategy is then an infinite sequence of price functions, $\{\rho_i^t(\cdot)\}_{t=1}^{\infty}$, where $\rho_i^t : \{0, 1, \ldots, m\}^{t-1} \to \Re$. As described in the Introduction, this is consistent with some cartels such as the lysine cartel where compliance was determined by sales.

3 An Impossibility Result

In exploring collusion in this setting, it is natural to first consider equilibria that take full advantage of the symmetry of the model. For a particular strategy profile, let

$$v_i^t(\cdot) : \{0, 1, \ldots, m\}^{t-1} \to \Re$$

denote the continuation payoff starting at t. A *symmetric Nash equilibrium* is a Nash equilibrium in which continuation payoffs are identical when the history is symmetric. Of course, if m is not even then the only symmetric history is the initial null history. When m is even, it also includes those histories in which each firm had demand of $m/2$ in every period. A stronger notion of symmetry but still quite compelling is that if the history is permuted with respect to the firms then the payoffs are permuted as well so the identity of the firm doesn't matter. In defining a permutation, a history at the start of period t is the sequence of firms' quantities, $\{(q_1^\tau, q_2^\tau)\}_{\tau=1}^{t-1} = \{(b^\tau, m - b^\tau)\}_{\tau=1}^{t-1}$. Representing this history as the sequence of firm 1's quantities, $\{b^\tau\}_{\tau=1}^{t-1}$, its permutation is $\{m - b^\tau\}_{\tau=1}^{t-1}$. An *exchangeable Nash equilibrium* is a Nash equilibrium in which

$$v_1^t\left(b^1, \ldots, b^{t-1}\right) = v_2^t\left(m - b^1, \ldots, m - b^{t-1}\right) \ \forall \left(b^1, \ldots, b^{t-1}\right) \in \{0, 1, \ldots, m\}^{t-1}, \ \forall t.$$

A more restrictive but commonly imposed property is that of strong symmetry. A *strongly symmetric Nash equilibrium* is a Nash equilibrium in which continuation payoffs are identical for all histories:

$$v_1^t\left(b^1, \ldots, b^{t-1}\right) = v_2^t\left(b^1, \ldots, b^{t-1}\right) \ \forall \left(b^1, \ldots, b^{t-1}\right) \in \{0, 1, \ldots, m\}^{t-1}, \ \forall t.$$

Let this common continuation payoff be denoted $v^t(\cdot)$.

Our first main finding is an impossibility result. Under strong symmetry and exchangeability, collusion is not sustainable regardless of the discount factor.

Theorem 1 *Assuming A1-A3, the set of strongly symmetric exchangeable Nash equilibrium outcomes for the infinite horizon game coincides with the set of symmetric Nash equilibrium outcomes for the stage game.*

Proof. Consider a strongly symmetric exchangeable Nash equilibrium which, given the current history, gives both firms a payoff of $v(b) \equiv v^{t+1}\left(h^{t-1}, b\right)$ if the current period demand for firm 1 is b. Furthermore, suppose the current history is symmetric (or is the null history) so exchangeability implies:

$$v(b) = v(m - b) \, \forall b \in \{0, 1, \ldots, m\}.$$

Firm 1's expected payoff from pricing at p_1^t is

$$\sum_{b=0}^{m} \phi\left(b; p_1^t, p^*\right) \left[p_1^t b + \delta v(b)\right],$$

where the strategy profile calls for firms to price at p^*. By A1, a necessary condition for equilibrium is:

$$\sum_{b=0}^{m}\left(\frac{\partial\phi\left(b;p^*,p^*\right)}{\partial p_1^t}\right)\left[p_1^t b+\delta v\left(b\right)\right]+\sum_{b=0}^{m}\phi\left(b;p^*,p^*\right)b=0,$$

which we will rearrange to

$$\sum_{b=0}^{m}\left(\frac{\partial\phi\left(b;p^*,p^*\right)}{\partial p_1^t}\right)p_1^t b+\sum_{b=0}^{m}\phi\left(b;p^*,p^*\right)b+\sum_{b=0}^{m}\left(\frac{\partial\phi\left(b;p^*,p^*\right)}{\partial p_1^t}\right)\delta v\left(b\right)=0. \tag{1}$$

Our method of proof is to show that the third term is zero for if that is the case then p^* must satisfy

$$\sum_{b=0}^{m}\left(\frac{\partial\phi\left(b;p^*,p^*\right)}{\partial p_1^t}\right)p_1^t b+\sum_{b=0}^{m}\phi\left(b;p^*,p^*\right)b=0$$

which is a necessary condition for a symmetric Nash equilibrium for the stage game.

We want to show that

$$\sum_{b=0}^{m}\left(\frac{\partial\phi\left(b;p,p\right)}{\partial p_1}\right)\delta v\left(b\right)=0, \tag{2}$$

where we've dropped some extraneous notation. When m is even, it can be presented as

$$\sum_{b=0}^{\frac{m}{2}-1}\left[\left(\frac{\partial\phi\left(b;p,p\right)}{\partial p_1}\right)\delta v\left(b\right)+\left(\frac{\partial\phi\left(m-b;p,p\right)}{\partial p_1}\right)\delta v\left(m-b\right)\right]+\left(\frac{\partial\phi\left(\frac{m}{2};p,p\right)}{\partial p_1}\right)\delta v\left(\frac{m}{2}\right)=0.$$

Using exchangeability, this becomes

$$\sum_{b=0}^{\frac{m}{2}-1}\left[\left(\frac{\partial\phi\left(b;p,p\right)}{\partial p_1}\right)+\left(\frac{\partial\phi\left(m-b;p,p\right)}{\partial p_1}\right)\right]\delta v\left(b\right)+\left(\frac{\partial\phi\left(\frac{m}{2};p,p\right)}{\partial p_1}\right)\delta v\left(\frac{m}{2}\right)=0.$$

Analogously, one can derive when m is odd,

$$\sum_{b=0}^{\frac{m-1}{2}}\left[\left(\frac{\partial\phi\left(b;p,p\right)}{\partial p_1}\right)+\left(\frac{\partial\phi\left(m-b;p,p\right)}{\partial p_1}\right)\right]\delta v\left(b\right)=0.$$

Therefore, a sufficient condition for (2) to be true is:

$$\frac{\partial\phi\left(b;p,p\right)}{\partial p_1}+\frac{\partial\phi\left(m-b;p,p\right)}{\partial p_1}=0,\ \forall b\in\left\{0,1,\ldots,m\right\}. \tag{3}$$

Note that if m is even and (3) holds $\forall b\in\left\{0,1,\ldots,\frac{m}{2}-1\right\}$ then $\frac{\partial\phi\left(\frac{m}{2};p,p\right)}{\partial p_1}=0$ since the sum of the probabilities is a constant.

To prove (3), first note that an implication of A2 is

$$\frac{\partial\phi\left(m-b;p,p\right)}{\partial p_1}=\frac{\partial\phi\left(b;p,p\right)}{\partial p_2}. \tag{4}$$

It follows from A3 that

$$\frac{\partial\phi\left(b;p,p\right)}{\partial p_2}=-\frac{\partial\phi\left(b;p,p\right)}{\partial p_1} \tag{5}$$

Substituting (5) into (4) yields

$$\frac{\partial\phi\left(b;p,p\right)}{\partial p_1}+\frac{\partial\phi\left(m-b;p,p\right)}{\partial p_1}=0,$$

which completes the proof. ■

In thinking about punishment for perceived non-compliance in this setting, one would expect it to occur when market shares are sufficiently skewed; either firm 1's sales are too much or too little. The former is consistent with firm 1 having undercut the collusive price and the latter with firm 2 having done so. Strong symmetry implies that the punishment entails identical behavior, regardless of who had the higher market share, and exchangeability implies that the punishment depends only on how skewed is market share. In such a situation, Theorem 1 shows that no collusion can be sustained.

This result hinges on the fact that when firm 1 sets a price marginally below the collusive price, it reduces the likelihood of having a low demand (say, $b' < m/2$) and, at the same time, raises the probability of having a high demand (say, $m-b'$). The proof shows that the ensuing reduction in the probability of b' is exactly equal to the rise in the probability of $m-b'$ so that the probability of b' or $m-b'$ remains constant for a marginal change in price. Since the payoff is the same for b' and $m-b'$, the probability distribution over the continuation payoff is unaffected. If p^* is to be an equilibrium then it must maximize expected current profit since, at the margin, the effect of price on the expected continuation payoff is zero. This implies the equilibrium price must be the same as that for a Nash equilibrium for the stage game.[3]

4 An Example of Collusion with Symmetric Punishments

To show that Theorem 1 is tight, this section proves that collusion can be sustained with strongly symmetric exchangeable punishments if A1 and A3 do not hold. Consider the Hotelling line model defined on $[0,1]$ with firm 1 located at 0 and firm 2 at 1. In each period, two customers arrive so $m=2$. With probability α, their locations are independently and uniformly distributed over $[0,1]$. Call that event A.

[3]This result is related to the work of Blume and Heidhues (2003) and Skrzypacz and Hopenhayn (2004) who explore collusion in repeated auctions. They show that symmetric punishments cannot sustain collusion when $m=1$ but that is not very surprising since there are only two outcomes - either firm 1 wins the unit or firm 2 wins the unit - in which case symmetry immediately implies that the continuation payoff must be independent of the history.

With probability $(1-\alpha)$, one customer's location is uniformly distributed over $\left[0, \frac{1}{2}\right]$ and the other's is uniformly distributed over $\left[\frac{1}{2}, 1\right]$. Call that event B. Denote the location of customer $i \in \{1, 2\}$ by ε_i. Assuming transportation costs are 1, customer i buys from firm 1 iff

$$\varepsilon_i \leq \frac{1}{2}\left(1-(p_1-p_2)\right).$$

In event A, the probability that firm 1 gets one unit of demand is

$$\frac{1}{2}\left(1-p_1+p_2\right)\left(1-p_2-p_1\right)=\frac{1}{2}-\frac{1}{2}\Delta^2$$

where $\Delta \equiv p_1 - p_2$. More generally, the probability distribution over firm 1's demand under event A is

$$\phi_A(b;\Delta)=\begin{cases} \frac{1}{4}(1-\Delta)^2 & \text{if } b=0 \\ \frac{1}{2}-\frac{1}{2}\Delta^2 & \text{if } b=1 \\ \frac{1}{4}(1+\Delta)^2 & \text{if } b=2 \end{cases}$$

Under event A, A1-A3 are satisfied.[4]

Under event B, when the customers have somewhat opposite tastes, the probability that customer 1 buys from firm 1 is:

$$\Pr(\text{customer 1 buys from firm 1})=\begin{cases} 1 & \text{if } p_1 \leq p_2 \\ 1-(p_1-p_2) & \text{if } p_1 > p_2 \end{cases}$$

Analogously, the probability that customer 2 buys from firm 2 is:

$$\Pr(\text{customer 2 buys from firm 2})=\begin{cases} 1 & \text{if } p_1 \geq p_2 \\ 1-(p_2-p_1) & \text{if } p_1 < p_2 \end{cases}$$

Therefore, the probability of splitting the market is

$$\phi_B(1;\Delta)=\begin{cases} 1-(p_2-p_1) & \text{if } p_1 < p_2 \\ 1 & \text{if } p_1 = p_2 \\ 1-(p_1-p_2) & \text{if } p_1 > p_2 \end{cases}$$

and the derivative is

$$\phi_B'(1;\Delta)=\begin{cases} 1 & \text{if } p_1 < p_2 \\ -1 & \text{if } p_1 > p_2 \end{cases}$$

where again we have symmetry, but because of the kink in the probability function, the derivative is not zero close to $\Delta = 0$.[5]

[4] We have committed a slight abuse of notation by now having the argument of the probability function be $p_1 - p_2$ rather than (p_1, p_2).

[5] An alternative interpretation can be provided for this demand specification. Suppose customer 1 knows that he prefers firm 1's product and customer 2 knows he prefers firm 2's product. Each then buys his more preferred product with probability one when that product's price is weakly lower. In that case, the product of firm 1 (2) weakly dominates that of firm 2 (1) for customer 1 (2) in price-product trait space. However, if the product with more desired traits has a higher price then a consumer has to evaluate the price-product trait trade-off. If we think of this evaluative process as being stochastic (either in reality or from the perspective of the firms), this two-stage decision process generates the demand structure in our example.

Combining the two events, the probability of splitting the market is:

$$\phi(1;\Delta)=\begin{cases}\alpha\left(\frac{1}{2}-\frac{1}{2}\Delta^2\right)+(1-\alpha)(1+\Delta) & \text{if } p_1<p_2\\ \alpha\left(\frac{1}{2}-\frac{1}{2}\Delta^2\right)+(1-\alpha) & \text{if } p_1=p_2\\ \alpha\left(\frac{1}{2}-\frac{1}{2}\Delta^2\right)+(1-\alpha)(1-\Delta) & \text{if } p_1>p_2\end{cases}$$

in which case the derivative is

$$\phi'(1;\Delta)=\begin{cases}1-\alpha-\alpha\Delta & \text{if } p_1<p_2\\ -1+\alpha-\alpha\Delta & \text{if } p_1>p_2\end{cases}\tag{6}$$

$$\begin{aligned}\phi'\left(1;0^+\right) &= -1+\alpha\\ \phi'\left(1;0^-\right) &= 1-\alpha\end{aligned}$$

Given the lack of differentiability at $\Delta=0$, A1 and A3 are not satisfied.

The probability of firm 1 obtaining 2 customers is

$$\phi(2;\Delta)=\begin{cases}\alpha\left(\frac{1}{4}(1-\Delta)^2\right)+(1-\alpha)(-\Delta) & \text{if } p_1<p_2\\ \alpha\left(\frac{1}{4}(1-\Delta)^2\right) & \text{if } p_1=p_2\\ \alpha\left(\frac{1}{4}(1-\Delta)^2\right) & \text{if } p_1>p_2\end{cases}$$

with derivative

$$\phi'(2;\Delta)=\begin{cases}-\alpha\frac{1}{2}(1-\Delta)-(1-\alpha) & \text{if } p_1<p_2\\ -\alpha\frac{1}{2}(1-\Delta) & \text{if } p_1>p_2\end{cases}$$

$$\begin{aligned}\phi'\left(2;0^+\right) &= -\frac{1}{2}\alpha\\ \phi'\left(2;0^-\right) &= \frac{1}{2}\alpha-1\end{aligned}$$

For the stage game, a firm's problem is

$$\max_{p_1} p_1\left(\phi(1;p_1-p_2)+2\phi(2;p_1-p_2)\right)$$

$\widehat{p}$ is a symmetric Nash equilibrium if the following two inequalities hold:

$$\begin{aligned}\phi(1;0)+2\phi(2;0)+\widehat{p}\left(\phi'\left(1;0^+\right)+2\phi'\left(2;0^+\right)\right) &\leq 0\\ \phi(1;0)+2\phi(2;0)+\widehat{p}\left(\phi'\left(1;0^-\right)+2\phi'\left(2;0^-\right)\right) &\geq 0\end{aligned}$$

This can be rewritten as:

$$\begin{aligned}1+\widehat{p}(-1+\alpha-\alpha) &\leq 0\Leftrightarrow 1\leq p\\ 1+\widehat{p}(1-\alpha+\alpha-2) &\geq 0\Leftrightarrow 1\geq p\end{aligned}$$

The unique symmetric static Nash equilibrium is then $\widehat{p} = 1$ with expected profits equal to 1.

Now consider the following strongly symmetric exchangeable strategy profile. Firms start in the collusive phase and, in the collusive phase, both price at p^*. If they split the market, they remain in the collusive phase which means they price at p^* next period. If they do not split the market then with probability γ they switch to the static Nash equilibrium forever (that is, pricing at 1 thereafter) and with probability $(1-\gamma)$ remain in the collusive phase. As $\phi(1;0) = 1 - \frac{1}{2}\alpha$, the (per period) value from this scheme is defined recursively by

$$V = (1-\delta)\, p^* + \delta \left(\left(1 - \frac{1}{2}\alpha\gamma\right) V + \frac{1}{2}\alpha\gamma \right), \tag{7}$$

where recall the per period payoff to the static Nash equilibrium is 1. Solving for the collusive value,

$$V = \frac{2(1-\delta)\, p^* + \delta\alpha\gamma}{2 - 2\delta + \delta\alpha\gamma}. \tag{8}$$

The problem faced by a firm in the collusive phase is:

$$\max_{p_1} (1-\delta)\, p_1 \left(\phi(1;\Delta) + 2\phi(2;\Delta)\right) + \delta \left[\left(1 - (1-\phi(1;\Delta))\gamma\right) V + (1-\phi(1;\Delta))\gamma \right],$$

or equivalently

$$\max_{p_1} (1-\delta)\, p_1 \left(\phi(1;\Delta) + 2\phi(2;\Delta)\right) + \delta\gamma\phi(1;\Delta)(V-1) + \delta V - \delta\gamma(V-1).$$

p^* is an equilibrium if

$$\begin{aligned}
(1-\delta)\left[\phi(1;0) + 2\phi(2;0) + p^*\left(\phi'\left(1;0^+\right) + 2\phi'\left(2;0^+\right)\right)\right] + \delta\gamma\phi'\left(1;0^+\right)(V-1) &\leq 0 \\
(1-\delta)\left[\phi(1;0) + 2\phi(2;0) + p^*\left(\phi'\left(1;0^-\right) + 2\phi'\left(2;0^-\right)\right)\right] + \delta\gamma\phi'\left(1;0^-\right)(V-1) &\geq 0
\end{aligned}$$

As before, this can be rewritten as:

$$\begin{aligned}
(1-\delta)\left(1 + p^*(-1)\right) - (1-\alpha)\,\delta\gamma(V-1) &\leq 0 \\
(1-\delta)\left(1 + p^*(-1)\right) + (1-\alpha)\,\delta\gamma(V-1) &\geq 0
\end{aligned}$$

or

$$\begin{aligned}
1 - (1-\alpha)\left(\frac{\delta}{1-\delta}\right)\gamma(V-1) &\leq p^* \\
1 + (1-\alpha)\left(\frac{\delta}{1-\delta}\right)\gamma(V-1) &\geq p^*
\end{aligned}$$

The highest price p^* that can be supported is then

$$p^* = 1 + (1-\alpha)\left(\frac{\delta}{1-\delta}\right)\gamma(V-1) \tag{9}$$

Substituting (9) into (8),

$$V = (1-\delta)\left(1 + (1-\alpha)\left(\frac{\delta}{1-\delta}\right)\gamma (V-1)\right) + \delta\left(\left(1 - \frac{1}{2}\alpha\gamma\right)V + \frac{1}{2}\alpha\gamma\right),$$

and the solution with respect to V is:

$$\begin{cases} 1 & \text{if} \quad \delta + \delta\gamma - \frac{3}{2}\alpha\delta\gamma - 1 \neq 0 \\ \mathbb{R} & \text{if} \quad \delta + \delta\gamma - \frac{3}{2}\alpha\delta\gamma - 1 = 0 \end{cases}$$

By selecting γ so that

$$\delta + \delta\gamma - \frac{3}{2}\alpha\delta\gamma - 1 = 0, \tag{10}$$

any value for V can be achieved as firms can induce any value for p^*. Another way in which to see this result is that we have two equations, (7) and (9), and two unknowns, V and p^* :

$$\begin{aligned} V &= \frac{2p^* + \left(\frac{\delta}{1-\delta}\right)\alpha\gamma}{2 + \left(\frac{\delta}{1-\delta}\right)\alpha\gamma} \\ p^* &= 1 + (1-\alpha)\left(\frac{\delta}{1-\delta}\right)\gamma (V-1). \end{aligned} \tag{11}$$

Solving $\delta + \delta\gamma - \frac{3}{2}\alpha\delta\gamma - 1 = 0$ for $\frac{\delta}{1-\delta}$, one gets $\frac{2}{\gamma(2-3\alpha)}$. Substituting this into (11), these two equations become

$$\begin{aligned} V &= \frac{(2-3\alpha)p^* + \alpha}{2(1-\alpha)} \\ p^* &= 1 + \left(\frac{2(1-\alpha)}{2-3\alpha}\right)(V-1). \end{aligned}$$

In $V \times p^*$ space, these two functions coincide so that any p^* is a solution.

As (10) is equivalent to

$$\gamma\left(1 - \frac{3}{2}\alpha\right) = \frac{1-\delta}{\delta},$$

a necessary condition is $\alpha < \frac{2}{3}$. Given $\alpha < \frac{2}{3}$ and in light of $\gamma \in (0,1]$, it is also necessary that

$$\delta \geq \frac{1}{2 - \frac{3}{2}\alpha}.$$

To sum up, if

$$\alpha < \frac{2}{3} \text{ and } \delta \geq \frac{1}{2 - \frac{3}{2}\alpha}$$

then $\exists\gamma \in (0,1]$ such that (10) holds which implies that any price can be sustained by a strongly symmetric exchangeable subgame perfect equilibrium.

With Theorem 1, a slight undercutting of the collusive price did not alter the probability distribution over the continuation payoff which meant that firms would set price to maximize current expected profit. This property does not hold here, however. Using (6), note that if firm 1 prices slightly below the collusive price, the marginal effect on the probability of splitting the market is $1-\alpha$ so that there is a first-order decrease in the probability of that event. This means there is a first-order increase in the probability of the extreme event of one firm selling to both buyers and thus an increase in the probability of the low punishment payoff. This allows firms to support collusion as long as α is sufficiently small - so the marginal effect of price undercutting on the probability of an extreme sales event is sufficiently large - and firms are sufficiently patient.

5 Collusion with Asymmetric Punishments

In this section, we provide a preliminary result characterizing how asymmetric punishments with side payments can sustain collusion for the model satisfying A1-A3. Research is in progress to provide a fuller characterization of such equilibria; both with and without side payments.

Assume there are two buyers: $m = 2$.[6] There are two firms and, in each period, firms simultaneously choose price. The probability that firm 1 sells to a particular buyer equals $F(p_2 - p_1)$ where $F : \Re \rightarrow [0,1]$ is twice continuously differentiable and non-decreasing. Furthermore, F' is symmetric around zero. Buyers' decisions as to whom to buy from are *iid* which implies that a firm's demand is binomially distributed. Each firm has constant marginal cost of c.

Consider the following symmetric strategy profile which allows for side payments between firms. Recall that b^t is the number of units sold by firm 1 in period t.

- If in the collusive state in period t then set $p_i^t = p^*$ and
  - if $b^t = 1$ then remain in the collusive state in period $t+1$
  - if $b^t = 2$ then go to the type 1 punishment state in period $t+1$
  - if $b^t = 0$ then go to the type 2 punishment state in period $t+1$
- If in the type i punishment state in period t then firm i pays z to firm $j(\neq i)$.
  - If firm i pays z to firm j then
    - switch to the collusive state in period t with probability γ
    - play the static Nash equilibrium forever with probability $1-\gamma$
  - If firm j does not pay z to firm j then play the static Nash equilibrium forever.

[6] We strongly conjecture that the ensuing result extends to the general m-buyer case.

Let v and w denote the collusive payoff and the static Nash equilibrium payoff, respectively. The Nash equilibrium price is

$$c + \frac{1}{2F'(0)}$$

and

$$w = \frac{1}{2F'(0)(1-\delta)}. \tag{12}$$

When in the collusive state, the payoff faced by firm 1 is

$$\begin{aligned} &2F(p_2 - p_1) F(p_1 - p_2)(p_1 - c + \delta v) \\ &+F(p_2 - p_1)^2 [2(p_1 - c) + \delta(-z + \gamma v + (1-\gamma) w)] \\ &+F(p_1 - p_2)^2 \delta [z + \gamma v + (1-\gamma) w]. \end{aligned} \tag{13}$$

Evaluating the first-order condition at a common price of p and solving for p yields:

$$p = c + \frac{1}{2F'(0)} + \delta z. \tag{14}$$

The collusive payoff is then the expression in (13) evaluated at the price in (14) which, after some manipulation, can be shown to equal

$$v = \frac{1 + F'(0)[2\delta z + \delta(1-\gamma) w]}{F'(0)[2 - \delta(1+\gamma)]}. \tag{15}$$

The condition which ensures that firm i wants to transfer z in the type i punishment state is

$$-z + \gamma v + (1-\gamma) w \geq w \Leftrightarrow \gamma(v - w) \geq z. \tag{16}$$

Substituting (15) into (16):

$$\begin{aligned} \gamma\left(\frac{1 + F'(0)[2\delta z + \delta(1-\gamma) w]}{F'(0)[2 - \delta(1+\gamma)]} - w\right) &\geq z \Leftrightarrow \\ \gamma\big(1 - 2(1-\delta) F'(0) w\big) &\geq F'(0)[2 - \delta(1+3\gamma)] z. \end{aligned} \tag{17}$$

Using (12) in (17):

$$0 \geq F'(0)[2 - \delta(1+3\gamma)] z. \tag{18}$$

Since $z > 0$ then a necessary and sufficient condition for (18) is

$$\delta \geq \frac{2}{1+3\gamma}.$$

If $\gamma > \frac{1}{3}$ then the strategy profile is a subgame perfect equilibrium with

$$p^* = c + \frac{1}{2F'(0)} + \delta z$$

as long as δ is sufficiently close to 1.

In sum, collusion can be sustained by a punishment strategy in which a firm with above-normal sales pays the other firm. This is, of course, an asymmetric punishment and is sustainable as long as firms are sufficiently patient. In practice, the transfer z can be implemented by having the firm with excess sales buy output from the other firm at an inflated price. Several recent price-fixing cartels engaged in various forms of side payments including the citric acid cartel of 1991-95 (Connor, 2001) and the graphite electrodes cartel of 1992-97 (Levenstein, Suslow, and Oswald, 2004).

References

[1] Blume, Andreas and Paul Heidhues, "Modeling Tacit Collusion in Auctions," U. of Pittsburgh, pdf copy, February 2003.

[2] Cabral, Luis and Michael Riordan, "The Learning Curve, Market Dominance, and Predatory Pricing, " *Econometrica*, 62 (1994), 1115-1140.

[3] Connor, John M., *Global Price Fixing: Our Customers are the Enemy,* Boston: Kluwer Academic Publishers, 2001.

[4] Green, Edward J. and Robert H. Porter, "Noncooperative Collusion under Imperfect Price Information," *Econometrica*, 52 (1984), 87-100.

[5] Leveinstein, Margaret C., Valerie Y. Suslow, and Lynda J. Oswald, "Contemporary International Cartels and Developing Countries: Economic Effects and Implications for Competition Policy," *Antitrust Law Journal*, 71 (2004), 801-852.

[6] Skrzypacz, Andrzej and Hugo Hopenhayn, "Tacit Collusion in Repeated Auctions," *Journal of Economic Theory*, 114 (2004), 153-169.

[7] Stigler, George J., "A Theory of Oligopoly," *Journal of Political Economy*, 72 (1964), 44-61.