

Antitrust in Innovative Industries*

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1. Introduction

This paper is concerned with the effects of antitrust policy in markets in which innovation is important. Traditionally, antitrust analyses have largely ignored issues of innovation.¹ Yet, over the last two decades, intellectual property and innovation have come to play increasingly important roles in the economy. In the wake of these changes, and sparked by the recent *Microsoft* case, a number of commentators have expressed the concern that traditional antitrust analysis might be poorly suited to maximizing welfare in so-called “new economy” industries in which innovation is crucial to a firm’s success. As Evans and Schmalensee [2002] put it,

“...firms engage in dynamic competition *for the market* — usually through research-and development (R&D) to develop the ‘killer’ product, service, or feature that will confer market leadership and thus diminish or eliminate actual or potential rivals. Static price/output competition on the margin *in the market* is less important.”

In fact, the effects of antitrust policy on innovation are poorly understood. In the *Microsoft* case, for example, the most significant issue in evaluating the welfare effects of Microsoft’s allegedly anticompetitive practices was almost surely their effect on innovation. Microsoft argued that while a technological leader like Microsoft may possess a good deal of static market power, this is merely the fuel for stimulating dynamic competition, a process that it argued works well in the software industry. The government, in contrast, argued that Microsoft’s practices prevented entry of new firms and products, and therefore would both raise prices and retard innovation.² How to reconcile these two views, however, was never fully clear in the discussion surrounding the case.

On closer inspection, these two conflicting views reveal a fundamental tension in the effects of antitrust policy on innovation. Policies that protect new entrants from incumbents raise a successful innovator’s initial profits and may thereby encourage innovation, as the government argued. But new entrants hope to become the new Microsoft, and will want to engage in the same sorts of entry-disadvantaging behaviors should they succeed.

¹Issues of innovation have been considered when discussing “innovation markets” in some horizontal merger cases in which there was a concern that a merger might reduce R&D competition. See, e.g., Gilbert and Sunshine [1995].

²For further discussion, see e.g., Evans and Schmalensee [2002], Fisher and Rubinfeld [2000], Gilbert and Katz [2001], and Whinston [2001].

And so protective policies, by lowering the profits of incumbency, may instead retard innovation as Microsoft alleged.³

In this paper, we study the effects of antitrust policy in innovative (“dynamically competitive”) industries. We do so using models in which innovation is a continual process, with new innovators replacing current incumbents, and holding dominant market positions until they are themselves replaced. Although a great deal of formal modeling of R&D races has occurred in the industrial organization literature (beginning with the work of Loury [1979] and Lee and Wilde [1980]; see Reinganum [1989] for a survey), this work has typically analyzed a single, or at most a finite sequence, of innovative races.⁴ Instead, our models are closer to those that have received attention in the recent literature on growth (e.g., Grossman and Helpman [1991], Aghion and Howitt [1992], Aghion et al. [2001]). The primary distinction between our analysis and the analysis in this growth literature lies in our explicit focus on how antitrust policies affect equilibrium in such industries.⁵

The paper is organized as follows. In Section 2, we introduce and analyze a simple stylized model of antitrust in an innovative industry. This simple model, in which there is a single potential entrant conducting R&D with a strictly convex R&D cost function, captures antitrust policy as affecting the profit flows that an incumbent and a new entrant can earn in competition with each other, as well as the profits of an uncontested incumbent. Although a more protective antitrust policy — one that protects entrants at the expense of incumbents — may increase a new entrant’s profits, it also affects the

³To see this more clearly, consider the following question: If profits are necessary for spurring innovation as Microsoft argued, does this mean that practices that enhance a dominant firm’s ability to protect its monopoly position will spur innovation? In answering this question, we need to distinguish between a policy that restricts Microsoft’s behavior and a policy that restricts the behavior of all dominant software producers. The former type of policy is sure to increase the innovation incentives of today’s potential entrants. However, the relevant question concerns the latter type of policy, which may not increase innovative activity because today’s potential entrants are spurred precisely by the hope of becoming the next Microsoft.

⁴One exception is O’Donohue et al. [1998] who use a continuing innovation model to examine optimal patent policy. In Section 4.6 we discuss the relation of our analysis to their paper.

⁵The growth literature often considers how changes in various parameters will affect the rate of innovation, sometimes even calling such parameters measures of the degree of “antitrust policy” (e.g., Aghion et al. [2001] refer to the elasticity of substitution as such a measure). Here we are much more explicit than is the growth literature about what antitrust policies toward specific practices do. This is not a minor difference, as our results differ substantially from those that might be inferred from the parameter changes considered in the growth literature. As one example, one would get exactly the wrong conclusion if one extrapolated results showing that more inelastic demand functions lead to more R&D (e.g., Aghion and Howitt [1992]) to mean that allowing an incumbent to enhance its market power through long-term contracts leads to more R&D.

profitability of continuing incumbents (the next Microsofts). Since successful entrants become continuing incumbents, both of these effects matter for the incentive to innovate. Disentangling the two effects is a central focus of the paper.

Using this simple stylized model, we develop some general insights into the effect of antitrust policies on the rate of innovation. We do so by characterizing equilibrium in terms of “innovation benefit” and “innovation supply” functions, which provides a very simple approach to comparative statics in these settings since policy changes involve shifts only in the innovation benefit function. We show that a more protective antitrust policy “front-loads” an innovative new entrant’s profit stream, and that this feature tends to increase the level of innovative activity by potential entrants to the industry. We also note how restrictions on particular kinds of activities (R&D-reducing behaviors and voluntary deals) alter innovation and how the degree of market growth can alter the effects of antitrust policy.

In Section 3 we extend our approach to comparative statics to substantially more general innovation benefit and supply settings. The extension to innovation supply, for example, allows us to consider supply settings with constant returns to scale R&D technologies, with N potential entrants, with free entry, or with “free entry with a limited idea” as in work by Fudenberg and Tirole [2000] and O’Donohue et al. [1998]. We show using our innovation benefit and supply approach that in each case, or in any other satisfying several basic properties, the condition characterizing comparative statics is the same.

With the comparative statics results of Section 3 in hand, in Section 4 we develop applications to specific antitrust policies. First, we study a model of long-term (exclusive) contracts and show that a more protective antitrust policy necessarily stimulates innovation and raises both aggregate and consumer welfare. Next, we consider two examples of voluntary deals, (nonexclusive) licensing by the incumbent of the entrant’s innovation and price collusion. We show using our comparative statics results that these necessarily increase the rate of innovation. In the first, welfare necessarily increases, while in the second the welfare impact depends on whether innovation is initially socially excessive or insufficient. In each of these first three applications, a policy that increases an entrant’s initial profits necessarily increases innovation. This is not true in our last two applications. The first is a model of compatibility choice in industries characterized by network externalities. Here we identify cases in which innovation necessarily increases when incumbents are forced to make their products more compatible with those of future entrants, as well as cases in which innovation may decline. The second is an extension

of our long-term contracting model to the case of uncertain innovation size when there is a cost of rapidly implementing new innovations. We show that in this situation, a more protective policy may retard innovation. The key new feature in this model is that the antitrust policy has a “selection effect,” altering the set of innovations that enter the market. In such a setting, a minor innovation brings small profits to its creator, but may destroy a lot of profits for the continuing incumbent who is displaced. As a result, a more protective policy may reduce innovation incentives by substantially decreasing the profit associated with becoming a continuing incumbent.

The analysis of Sections 2-4 makes the strong assumption that only potential entrants do R&D. While useful for gaining understanding, this assumption is rarely descriptive of reality. In Section 5, we turn our attention to models in which both incumbents and potential entrants conduct R&D. Introducing incumbent investment has the potential to substantially complicate our analysis by making equilibrium behavior depend on the level of the incumbent’s lead over other firms. We study two models in which we can avoid this state dependence. In one model, the previous leading technology is assumed to enter the public domain whenever the incumbent innovates. In this model, the incumbent does R&D solely to avoid displacement by a rival. In our second model, the profit improvement from a larger lead is assumed to be linear in the size of the lead and potential entrants are assumed to win all “ties,” which again leads the incumbent’s optimal R&D level to be stationary. In this model, the incumbent does R&D to improve its profit flows until the time that it is displaced by a rival. Interestingly, in both models there are a wide range of circumstances in which a more protective policy can increase the innovation incentives of *both* the incumbent and potential entrants.

In the policies considered in Sections 2-5 antitrust policy affects entrant and incumbent profit flows, which shifts only the innovation benefit functions. In Section 6 we consider antitrust policies that have other effects. We first consider policies that instead involve shifts in innovation supply. These include policies that limit behaviors aimed at raising rival’s R&D costs, such as buying up needed R&D inputs or engaging in costly litigation to challenge entrants’ patents. We then consider predatory activities, which alter an entrant’s probability of survival, and cannot be modeled in our simple innovation benefit/innovation supply framework.

Finally, Section 7 concludes.

2. A Stylized Model of Antitrust in Innovative Industries

We begin by developing a stylized model of continuing innovation. Our aim is to develop a model that yields some general insights into the effect of antitrust policies on the rate of innovation, and that we can apply to a number of different antitrust policies in the remainder of the paper. In this section, we develop the simplest possible version of this model, which we substantially generalize in the next section.

The model has discrete time and an infinite horizon. There are two firms who discount future profits at rate $\delta \in (0, 1)$. In each period, one of the firms is the “incumbent” I and the other is the “potential entrant” E . In the beginning of each period, the potential entrant chooses its R&D rate, $\phi \in [0, 1]$; the cost of R&D is given by a strictly convex function $c(\phi)$.⁶ The R&D of the potential entrant yields an innovation — which we interpret to be a particular improvement in the quality of the product — with probability ϕ . If the potential entrant innovates, it receives a patent, enters, competes with the incumbent in the present period, and then becomes the incumbent in the next period, while the previous incumbent then becomes the potential entrant. In this sense, this is a model of “winner-take-all” competition. While the patent provides perfect protection (forever) to the innovation itself, the other firm may overtake the patent holder by developing subsequent innovations.

We will be interested in the effects of an antitrust policy α that affects the incumbent’s competition with an entrant who has just received a patent. To this end, we denote the incumbent’s profit in competition with a new entrant by $\pi_I(\alpha)$, and the profit of the entrant by $\pi_E(\alpha)$, which we assume are differentiable functions of α . We let $\pi'_E(\alpha) > 0$, so that a higher α represents a policy that is more “protective” of the entrant in the sense that it raises the profit of the entrant in the period of entry. Less clear, however, is the overall effect of an increase in α on the incentive to innovate, since an increase in α will alter as well the value of becoming a continuing incumbent. We also denote by the differentiable function $\pi_m(\alpha)$ the profit of an incumbent who faces no competition in a period. (In Section 4, when we consider specific applications, we show how these values can be derived from an underlying model of the product market.)

We examine stationary Markov perfect equilibria of the infinite-horizon game using the dynamic programming approach. Let V_I denote the expected present discounted profits of an incumbent, and V_E those of a potential entrant (both evaluated at the

⁶Note that $c(\cdot)$ must be convex if the entrant can randomize over its R&D strategy. We assume *strict* convexity to simplify exposition in the simplest version of the model.

beginning of a period). Then, since innovation occurs with probability ϕ , these values should satisfy

$$V_I = \pi_m(\alpha) + \delta V_I + \phi [\pi_I(\alpha) - \pi_m(\alpha) + \delta (V_E - V_I)], \quad (2.1)$$

$$V_E = \delta V_E + \phi [\pi_E(\alpha) + \delta (V_I - V_E)] - c(\phi). \quad (2.2)$$

Also, a potential entrant's choice of ϕ should maximize its expected discounted value. Letting $w \equiv \pi_E(\alpha) + \delta (V_I - V_E)$ denote the expected discounted benefit from becoming a successful innovator—what we shall call the *innovation prize*—the optimal innovation level is

$$\Phi(w) = \arg \max_{\phi \in [0,1]} \{\phi w - c(\phi)\}.$$

Note that the maximizer is unique since the objective function is strictly concave. Note also that $\Phi(\cdot)$ is continuous (by Berge's Theorem of the Maximum) and nondecreasing (by the Monotone Selection Theorem of Milgrom and Shannon [1994]). Function $\Phi(\cdot)$ gives the innovation decision of the entrant as a function of the innovation prize w , and its graph gives us an “innovation supply curve”, which we label IS in Figure 2.1.

Consider now the determinants of the innovation prize w . Subtracting (2.2) from (2.1), solving for $(V_I - V_E)$, and substituting its value into $w \equiv \pi_E + \delta (V_I - V_E)$, allows us to express the innovation prize as $w = W(\phi, \alpha)$, where

$$\begin{aligned} W(\phi, \alpha) &= \pi_E(\alpha) + \delta \left[\frac{\phi \pi_I(\alpha) + (1 - \phi) \pi_m(\alpha) - \phi \pi_E(\alpha) + c(\phi)}{1 - \delta + 2\delta \phi} \right] \\ &= \frac{[1 - \delta(1 - \phi)] \pi_E(\alpha) + \delta [\phi \pi_I(\alpha) + (1 - \phi) \pi_m(\alpha) + c(\phi)]}{1 - \delta + 2\delta \phi}. \end{aligned} \quad (\text{IB})$$

This equation defines an “innovation benefit curve” — the value of the innovation prize as a function of the innovation rate ϕ . In Figure 2.2 we graph it (labeled IB) along with the IS curve. Points where the two curves intersect represent equilibrium values of (ϕ, w) .

Note that the IS curve does not depend on α at all. As seen in Figure 2.2, if α shifts the IB curve up (down) at all values of ϕ , then it increases (decreases) the equilibrium

Figure 2.1: The Innovation Supply Curve

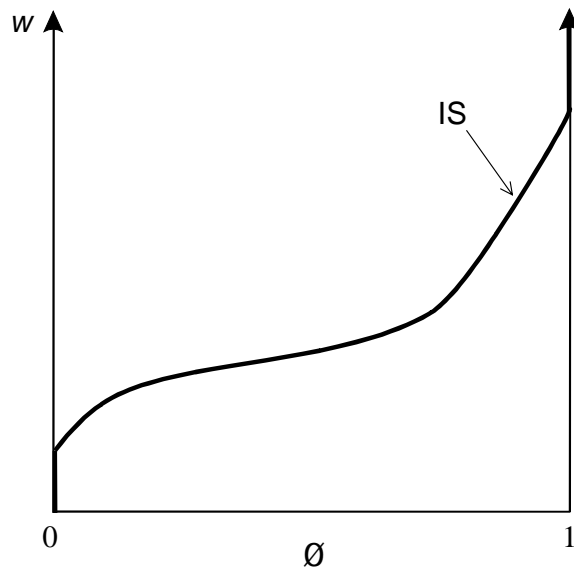


Figure 2.1:

Figure 2.2: Equilibrium and Comparative Statics

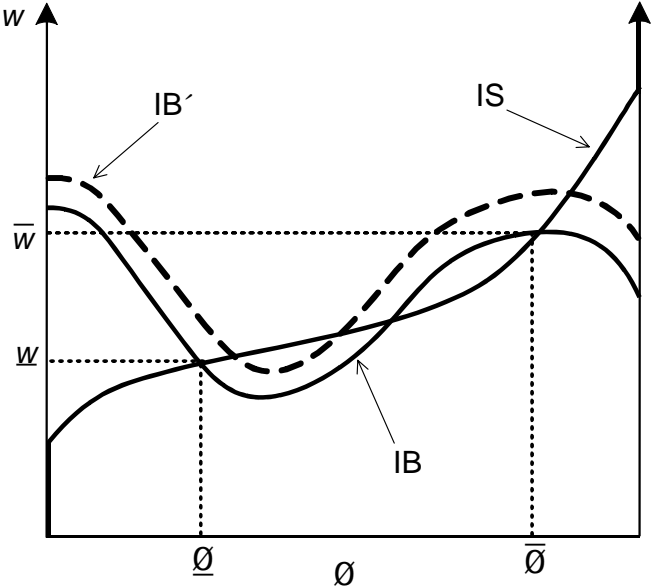


Figure 2.2:

innovation rate in the “largest” and “smallest” equilibria (denoted by $\underline{\phi}$ and $\bar{\phi}$ respectively in Figure 2.2). This can be established formally using comparative statics results of Milgrom and Roberts [1994], which we do in the next section. When there is a unique equilibrium, this result implies determinate comparative statics. Also, as evident in Figure 2.2, the same local comparative statics can be shown (using the Implicit Function Theorem) of any “stable” equilibrium (where the IB curve cuts the IS curve from above) if the IB function is shifted up or down in a neighborhood of the equilibrium. In what follows, we will say that a change in policy “increases (decreases) innovation” whenever these comparative statics properties hold. Differentiating $W(\phi, \alpha)$ with respect to α , we see that the protectiveness of antitrust policy increases (decreases) innovation if

$$\pi'_E(\alpha) + \delta \left[\frac{(1 - \phi)\pi'_m(\alpha) + \phi\pi'_I(\alpha)}{1 - \delta(1 - \phi)} \right] \geq (\leq) 0 \quad (2.3)$$

for any $\phi \in [0, 1]$.

Condition (2.3) indicates how to sort through the potentially conflicting effects of antitrust policy on innovation incentives that arise from the policy’s dual effects on a successful entrant’s initial profits and on its returns from achieving incumbency. It shows that a change in policy encourages (discourages) innovation precisely when it raises (reduces) the incremental expected discounted profits over an innovation’s lifetime: the first term on the left side of (2.3) is the change in an entrant’s profit in the period of entry due to the policy change, while the second term is equal to the change in the value of a continuing incumbent (the numerator is the derivative of the flow of expected profits in each period of incumbency conditional on still being an incumbent; the denominator captures the “effective” discount rate, which includes the probability of displacement), and thus of the entrant’s value once it is itself established as the incumbent.

In interpreting condition (2.3), it is helpful to think about the case in which the monopoly profit π_m is independent of the antitrust policy α , so that $\pi'_m(\alpha) = 0$. In this case, condition (2.3) tells us that innovation increases (decreases) if

$$\pi'_E(\alpha) + \left[\frac{\delta\phi}{1 - \delta(1 - \phi)} \right] \pi'_I(\alpha) \geq (\leq) 0. \quad (2.4)$$

Thus, innovation increases if a weighted sum of $\pi'_E(\alpha)$ and $\pi'_I(\alpha)$ increases, where the weight on $\pi'_E(\alpha)$ exceeds the weight on $\pi'_I(\alpha)$ due to discounting ($\delta < 1$). As illustrated in

Figure 2.3: Effects of Increased Protection of Entrants on Innovation

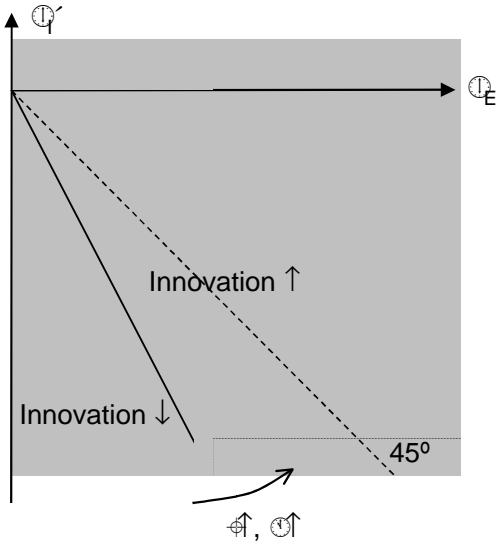


Figure 2.3:

Figure 2.3, this implies that a more protective antitrust policy raises innovation whenever $\pi'_I(\alpha) + \pi'_E(\alpha) \geq 0$; that is, provided that an increase in α does not lower the joint profits of the entrant and the incumbent in the period of entry. Intuitively, observe that a successful innovator earns $\pi_E(\alpha)$ when he enters, and earns $\pi_I(\alpha)$ when he is displaced. A more protective antitrust policy that raises π_E and lowers π_I has a *front loading effect*, effectively shifting profits forward in time. Since the later profits π_I are discounted, this front loading of profits necessarily increases the innovation prize provided that the joint profit $\pi_I + \pi_E$ does not decrease.

Observe also that the weight on $\pi'_I(\alpha)$ is increasing in ϕ and in δ . Thus, as depicted in Figure 2.3, the larger is δ or ϕ , the more likely is a more protective policy to reduce innovation. For ϕ , this is so because larger ϕ moves forward the expected date when the entrant will itself be displaced. For δ , this is so because with larger δ the discounted

value of the profits in the period in which the entrant is displaced are greater. In the limit, as $\delta \rightarrow 1$, the amount by which the joint profit $\pi_E + \pi_I$ can be dissipated while still encouraging innovation converges to zero: in this limiting case, the cost of a one dollar reduction in the value π_I that the entrant will receive when he is ultimately displaced is exactly equal to the gain from receiving a dollar more in the period in which he enters.

Even before we look at specific applications, this stylized model offers a number of other general insights:

2.1. Innovation-Deterring Activities

Incumbents may undertake otherwise unprofitable activities to make entry less attractive and thereby reduce potential entrants' R&D intensities. Such actions reduce the incumbent's current payoff, but lower the probability that the incumbent is displaced in the future. In Section 4, for example, we study two such activities, long-term exclusive contracts and, in markets characterized by network externalities, incompatibility with new versions of the product. A change in policy α that restricts such activities would not only raise π_E [$\pi'_E(\alpha) > 0$] but also — holding the rate of innovation ϕ fixed — *raise* a continuing incumbent's expected profit [$(1 - \phi)\pi'_m(\alpha) + \phi\pi'_I(\alpha) \geq 0$]. By (2.3), this will increase innovation.

2.2. Voluntary Deals

Incumbents and entrants may reach voluntary deals with each other. As one example, the incumbent might license the entrant's innovation to serve its existing customers better. As another quite different example, the incumbent and the entrant might agree to collude in their pricing to consumers. (We discuss both of these examples further in Section 4.) Were the innovation rate held fixed, allowing voluntary deals would — by definition — raise both parties' payoffs.⁷ By (2.3), such deals should therefore increase innovation.

2.3. Market Growth

Up to this point, and in the remainder of the paper, we focus on a stationary setting. It is worth noting, however, that the front-loading feature of protective antitrust policies

⁷This conclusion depends on the fact that we have only two parties negotiating. With more than two active firms, profits for some or all parties may fall when voluntary deals are allowed (see Segal [1999]).

suggests that the rate of market growth may alter the impact of antitrust policy on innovation. As an illustration, imagine that profits in period 1 are instead $\beta\pi_I(\alpha)$, $\beta\pi_E(\alpha)$, and $\beta\pi_m(\alpha)$ for $\beta \leq 1$, and are $\pi_I(\alpha)$, $\pi_E(\alpha)$, and $\pi_m(\alpha)$ from period 2 on. Following a similar derivation to that above, a change in α will now increase (decrease) the period 1 innovation rate ϕ_1 if

$$\beta\pi'_E(\alpha) + \delta \left[\frac{(1-\phi)\pi'_m(\alpha) + \phi\pi'_I(\alpha)}{1-\delta(1-\phi)} \right] \geq (\leq) 0.$$

Thus, the greater is market growth (the lower is β), the less likely is it that period 1 innovation will increase. For example, when $\beta < 1$ and $\pi'_m(\alpha) = 0$, an increase in α may reduce innovation even when joint profits upon entry increase ($\pi'_I(\alpha) + \pi'_E(\alpha) \geq 0$).

3. Comparative Statics for More General Innovation Supply and

Benefit

The fact that the comparative statics argument above did not depend on the particular shapes of the innovation supply and innovation benefit curves suggests that we can substantially generalize it. For one thing, we can generalize the innovation benefit curve by allowing all three of the profits π_I , π_E , and π_m to be affected by the rate of innovation ϕ . (This may happen because the price at which consumers purchase a durable good or a long-term contract may depend on their expectation of the innovation rate, as in the examples studied in Subsections 4.1 and 4.5.) Denoting these profits by $\pi_I(\alpha, \phi)$, $\pi_E(\alpha, \phi)$, and $\pi_m(\alpha, \phi)$, we see that the above argument continues to hold if we reinterpret the derivatives in (2.3) as being partial derivatives with respect to α holding ϕ fixed.

We can also allow for alternative models of innovation supply, such as having more than one potential entrant engage in R&D, or even allowing free entry (i.e., infinitely many potential entrants). (We still do not allow the incumbent to do R&D, we discuss relaxing this assumption in Section 5.) We define the industry's "innovation rate" ϕ as the probability that the incumbent technology is displaced with an innovation. For a symmetric industry, ϕ also determines the firms' *individual* R&D investments, the probability $u(\phi)$ that a *given* potential entrant becomes a new incumbent (i.e., moves "up"), and a potential entrant's R&D cost $c(\phi)$. The expected present discounted profits

of an incumbent (V_I) are then still described by

$$V_I = \pi_m(\alpha) + \delta V_I + \phi [\pi_I(\alpha) - \pi_m(\alpha) + \delta (V_E - V_I)], \quad (\text{VI}^*)$$

$$V_E = \delta V_E + u(\phi) [\pi_E(\alpha) + \delta (V_I - V_E)] - c(\phi). \quad (\text{VE}^*)$$

Subtracting, we can express the innovation prize $w = \pi_E + \delta (V_I - V_E)$ with the following function:

$$W(\phi, \alpha) = \frac{[1 - \delta(1 - \phi)] \pi_E(\alpha, \phi) + \delta [\phi \pi_I(\alpha, \phi) + (1 - \phi) \pi_m(\alpha, \phi) + c(\phi)]}{1 - \delta + \delta(\phi + u(\phi))}. \quad (\text{IB}^*)$$

As for the innovation supply curve, which describes the entrants' R&D response to a given innovation prize, our comparative statics results hold as long as the curve is described by a correspondence $\Phi(w)$ satisfying the following three properties:

(IS1): $\Phi(\cdot)$ is nonempty- and convex-valued;

(IS2) $\Phi(\cdot)$ has a closed graph;

(IS3) Any selection from $\Phi(\cdot)$ is nondecreasing (i.e., if $w' > w$, $\phi' \in \Phi(w')$, and $\phi \in \Phi(w)$, then $\phi' \geq \phi$).

For example, if $\Phi(\cdot)$ is a function (i.e., single-valued), (IS1) is vacuous, (IS2) means that the function is continuous, and (IS3) means that it is nondecreasing. Other correspondences satisfying (IS1)-(IS3) are obtained by taking a nondecreasing function and “filling in” its jumps, as illustrated in Figure 3.1.

Properties (IS1)-(IS3) ensure that antitrust policy affects the largest and smallest equilibrium innovation rates in the same direction in which it shifts the innovation benefit curve, which we can determine by partially differentiating $W(\phi, \alpha)$:

Proposition 3.1. *If the innovation supply correspondence $\Phi(\cdot)$ satisfies (IS1)-(IS3) and the innovation benefit function $W(\phi, \alpha)$ is continuous in ϕ , then the largest and smallest*

Figure 3.1: An IS Curve Satisfying (IS1) - (IS3)

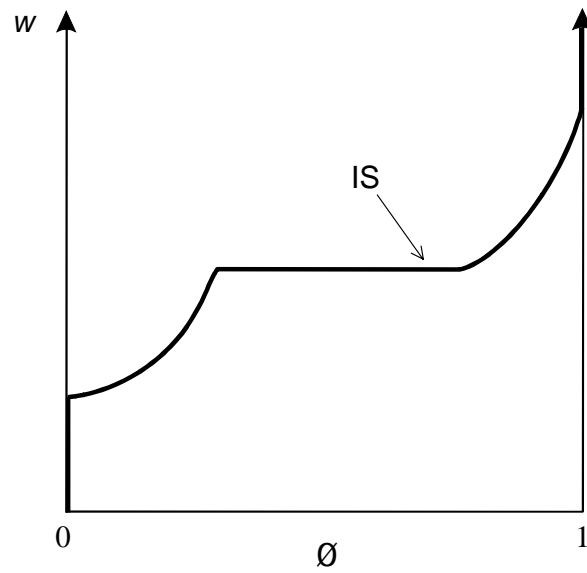


Figure 3.1:

equilibrium innovation rates exist, and both these rates are nondecreasing (nonincreasing) in α , the protectiveness of antitrust policy, if

$$\pi'_E(\alpha, \phi) + \delta \left[\frac{(1 - \phi)\pi'_m(\alpha, \phi) + \phi\pi'_I(\alpha, \phi)}{1 - \delta(1 - \phi)} \right] \geq (\leq) 0 \quad (3.1)$$

for any $\phi \in [0, 1]$.

Proof. The equilibrium innovation rates are the fixed points of the composite correspondence $\gamma(\alpha, \cdot) \equiv \Phi(W(\alpha, \cdot))$. (IS1)-(IS3) and the continuity of $W(\alpha, \cdot)$ imply that $\gamma(\alpha, \phi) = [\gamma_L(\alpha, \phi), \gamma_H(\alpha, \phi)] \neq \emptyset$, and that the correspondence $\gamma(\alpha, \cdot)$ is continuous but for upward jumps (Milgrom and Roberts 1994). Furthermore, if $W(\phi, \alpha)$ is nondecreasing (nonincreasing) in α , then so are $\gamma_L(\alpha, \phi)$ and $\gamma_H(\alpha, \phi)$. Corollary 2 of Milgrom and Roberts [1994] then establishes that the largest and smallest fixed points of $\gamma(\alpha, \cdot)$ exist and are nondecreasing (nonincreasing) in α . ■

If there are multiple equilibria, this result does allow some equilibria to move in the direction opposite to that predicted by Proposition 3.1. However, the same comparative statics hold for any locally unique equilibrium as long as the IB curve crosses the IS curve from above at the equilibrium, which is a condition needed to ensure the stability of the equilibrium under adjustment dynamics:⁸

Proposition 3.2. *Suppose that the innovation supply correspondence $\Phi(\cdot)$ satisfies (IS1)-(IS3) and the innovation benefit function $W(\phi, \alpha)$ is continuous in ϕ . Suppose in addition that for all $\alpha \in [\underline{\alpha}, \bar{\alpha}]$ there is a unique equilibrium innovation rate $\phi(\alpha)$ on an interval $[\underline{\phi}, \bar{\phi}]$ and that the IB curve crosses the IS curve from above on this interval.⁹ Then $\phi(\alpha)$ is nondecreasing (nonincreasing) if condition (3.1) holds for all $\alpha \in [\underline{\alpha}, \bar{\alpha}]$ and $\phi \in [\underline{\phi}, \bar{\phi}]$.*

Proof. $\phi(\alpha)$ is a unique fixed point of the composite correspondence $\gamma(\alpha, \cdot) \equiv \Phi(W(\alpha, \cdot))$ on $[\underline{\phi}, \bar{\phi}]$, and so the proof of Proposition 3.1 applies. ■

⁸Indeed, suppose that the industry in period t adaptively expects future innovation rate to be ϕ_{t-1} , and so chooses innovation rate $\phi_t \in \Phi(W(\alpha, \phi_{t-1}))$. The “crossing from above” condition then ensures that the adaptive dynamics starting at any disequilibrium $\phi \in [\underline{\phi}, \bar{\phi}]$ converges monotonically to the equilibrium; conversely, if we have “crossing from below,” then the dynamics must leave the interval $[\underline{\phi}, \bar{\phi}]$, so the equilibrium is not (Lyapunov) stable. Finally, using the observation of Echenique (2002), we can see that under (IS1)-(IS3) the adaptive dynamics starting at *any* equilibrium following an increase in α must be nondecreasing (nonincreasing) whenever (3.1) holds for all $\phi \in [0, 1]$.

⁹That is, $\Phi(W(\alpha, \phi)) \subset (\phi, \bar{\phi}]$ for $\phi \in [\underline{\phi}, \phi(\alpha))$ and $\Phi(W(\alpha, \phi)) \subset [\underline{\phi}, \phi)$ for $\phi \in (\phi(\alpha), \bar{\phi}]$.

We now provide three examples in which (IS1)-(IS3) hold and so Propositions 3.1 and 3.2 determine the comparative statics effect of antitrust policy on the rate of innovation.

3.1. Example: One entrant with constant returns R&D technology

Suppose that we still have one entrant but the R&D cost function is $c(\phi) = c \cdot \phi$ for $c > 0$. In this case, the innovation supply is no longer a function: $\Phi(w) = 0$ if $w < c$, $[0, \infty]$ if $w = c$, and 1 if $w > c$. However, since it satisfies (IS1)-(IS3), Propositions 3.1 and 3.2 tell us that the same comparative statics apply.

3.2. Example: More than one potential entrant

Suppose that there are $N > 1$ potential entrants in any period (in addition to the single incumbent). In the beginning of each period, each potential entrant i independently chooses its R&D rate $\psi_i \in [0, 1]$; the cost of R&D is given by a convex function $\gamma(\psi_i)$. The R&D of a given potential entrant i yields a discovery with probability ψ_i (we assume that the discoveries are independently realized). We shall focus on symmetric equilibria, in which all potential entrants choose the same equilibrium level of R&D, denoted by ψ . In this case, the likelihood that at least one of the N potential entrants makes a discovery is given by $\phi = [1 - (1 - \psi)^N]$. Thus, in a symmetric equilibrium with aggregate innovation rate ϕ , the potential entrants' individual R&D choices are $\psi_N(\phi) = 1 - (1 - \phi)^{1/N}$.

Among the potential entrants who make a discovery, only one may receive the patent for the discovery. Denote by $r_N(\psi)$ the probability that a given potential entrant receives a patent, conditional on it making a discovery, when all other potential entrants are doing R&D at level ψ . We assume that $r_N(\cdot)$ is a strictly decreasing function.¹⁰ A potential entrant who is successful at receiving a patent enters and competes with the incumbent in the present period, and then becomes the incumbent in the next period, while the previous incumbent then becomes a potential entrant.

Note that in the symmetric equilibrium, the probability that a given entrant becomes the incumbent is $u(\phi) = \psi_N(\phi) r_N(\psi_N(\phi))$, and the entrant's R&D cost is $c(\phi) =$

¹⁰When the patent is awarded randomly to one of the successful innovators,

$$r(\psi) = \sum_{k=0}^{N-1} \binom{N-1}{k} \psi^k (1-\psi)^{N-1-k} = \frac{1 - (1-\psi)^N}{\psi N}.$$

$\gamma(\psi_N(\phi))$. The innovation benefit curve is then given by substituting these expressions in (IB*) above. As for the innovation supply, the equilibrium individual innovation rate solves the following equilibrium condition for a given value of w

$$\psi = \arg \max_{\psi' \in [0,1]} \{\psi' r_N(\psi) w - c(\psi')\}. \quad (3.2)$$

As shown in Lemma 3.1 in the Appendix, this describes a unique equilibrium, which is a continuous and nondecreasing function of w . These properties are preserved for the *aggregate* equilibrium innovation rate ϕ , and so Propositions 3.1 and 3.2 above apply to this model.

3.3. Example: Free Entry

In some circumstances it may be more appropriate to assume that there is free entry into R&D competition.¹¹ This assumption can be interpreted as a limiting case of a very large number N of potential entrants each of whom engages in an infinitesimal amount of R&D while the aggregate innovation rate is positive. An innovator's conditional probability of getting a patent can then be written using expressions in footnote 5 as

$$\bar{r}(\phi) = \lim_{N \rightarrow \infty} r_N(\psi_N(\phi)) = \lim_{N \rightarrow \infty} \frac{\phi}{(1 - (1 - \phi)^{1/N}) N} = -\frac{\phi}{\ln(1 - \phi)},$$

which is a continuous decreasing function of $\phi \in [0, 1]$. The first-order condition for each potential entrant to choose a positive infinitesimal R&D is $w\bar{r}(\phi) = \gamma'(0)$, which determines the innovation supply function $\Phi(w) = \bar{r}^{-1}(\gamma'(0)/w)$ for $w > \gamma'(0)$, and $\Phi(w) = 0$ for $w \leq \gamma'(0)$. Since this is a continuous and nondecreasing function, the comparative statics is again described by Propositions 3.1 and 3.2.

¹¹The fixed N model is the appropriate model when there are a limited number of firms with the capability of doing R&D in an industry (perhaps because of complementary assets they possess due to participation in related industries).

3.4. Example: Free Entry with a Limited Idea

In the models of Fudenberg Tirole [2000] and O’Donohue et al. [1998], there are infinitely many potential entrants, but in each period only one of them is randomly drawn to receive an “idea” that enables him to invest in R&D. This potential entrant then observes a randomly drawn implementation cost γ and chooses whether to invest in implementing the innovation. If he does, he is certain to become the next incumbent.

Observe first that the optimal strategy of potential entrants takes the form of choosing a cost threshold $\bar{\gamma}$ below which to implement their idea. This is equivalent to choosing the probability of innovation $\phi = \Pr\{\gamma \leq \bar{\gamma}\} \equiv F(\bar{\gamma})$, and the associated expected innovation cost $\int_0^{\bar{\gamma}} \gamma dF(\gamma)$. In other words, the innovation supply is equivalent to having a single potential entrant who chooses an innovation probability ϕ at an expected cost of $\int_0^{G^{-1}(\phi)} \gamma dF(\gamma)$, which is a convex function of ϕ . Thus, the innovation supply curve is determined just as in the single-firm model, and satisfies (IS1)-(IS3). For the innovation benefit curve, on the other hand, we take $c(\phi) = u(\phi) = 0$ since each potential entrant in expectation does not incur any cost and has zero chance of innovating. Once again, the comparative statics are described by Propositions 3.1 and 3.2.

4. Applications

In this section, we use the results of Section 3 to analyze the effects of a number of specific practices on the rate of innovation and social welfare. The models we consider are all versions of the “quality ladder” models introduced in the recent literature on economic growth (e.g., Aghion and Howitt [1992]; Grossman and Helpman [1991]). Our results in this section will apply for any continuous nondecreasing innovation supply function.

Before turning to these applications, we first introduce a basic quality ladder model (in which antitrust policy plays no role) to serve as a benchmark. In general, antitrust policy in our applications will have two types of welfare effects: a *direct* effect — the change in welfare that would result from the policy change were the rate of innovation held fixed — and an *indirect effect* due to the resulting change in the rate of innovation. The benchmark model makes clear the factors influencing the indirect effects.

4.1. A quality ladder model

There are at least 2 firms and a continuum of infinitely-lived consumers of measure 1 who may consume a nonstorable and nondurable good with production cost $k \geq 0$. R&D may improve the quality of this good and consumers value “generation j ” of the good at $v_j = v + j \cdot \Delta$. At any time t , one firm — the current “incumbent” — possesses a perfectly effective and infinitely-lived patent on the latest generation product j_t . Likewise, at time t there is a patentholder for each of the previous generations of the product ($j_t - 1, j_t - 2, \dots$). We assume, as in Sections 2 and 3, that at time t only firms other than the incumbent in the leading technology — the potential entrants — can invest in developing the generation $j_t + 1$ product. One implication of this assumption is that in each period t the holder of the patent on generation $j_t - 1$ is a firm other than the current incumbent, who holds the patent on the current leading generation j_t . We assume that at time t , these firms engage in Bertrand competition to make sales. Thus, $\pi_E = \pi_m = \Delta$ and $\pi_I = 0$.¹² Specializing (IB*) to this case, the innovation prize is given by

$$\begin{aligned} W(\phi) &= \left[\frac{\Delta(1 - \delta) + \delta[\phi\Delta + (1 - \phi)\Delta + c(\phi)]}{1 - \delta + \delta(\phi + u(\phi))} \right] \\ &= \left[\frac{\Delta + \delta c(\phi)}{1 - \delta + \delta(\phi + u(\phi))} \right]. \end{aligned} \tag{4.1}$$

For now we need not be specific about the nature of innovation supply; we assume only that it is described by a continuous nondecreasing innovation supply function.

Since we now have a fully-specified consumer side (unlike in Sections 2 and 3), we can compare the equilibrium innovation rate to the symmetric innovation rate that maximizes aggregate welfare.¹³ Let us begin by considering the social innovation benefit. To this end, observe that a technological advancement in period t raises gross consumer surplus in every subsequent period by Δ . The *social innovation prize* w_s is therefore equal to the present discounted value of this change, $w_s = \left(\frac{\Delta}{1 - \delta}\right)$. From (VE*) we see that $V_E \geq 0$

¹²We focus here on the undominated equilibrium in which the incumbent (who makes no sales) charges a price equal to cost and the entrant with technology $j_t + 1$ charges a price of Δ .

¹³In general, the socially optimal innovation plan may be asymmetric. We focus here on the best symmetric plan since our aim is to see how changes in the symmetric equilibrium innovation rate affect welfare.

Figure 4.1: Private and Social Innovation Benefit Curves

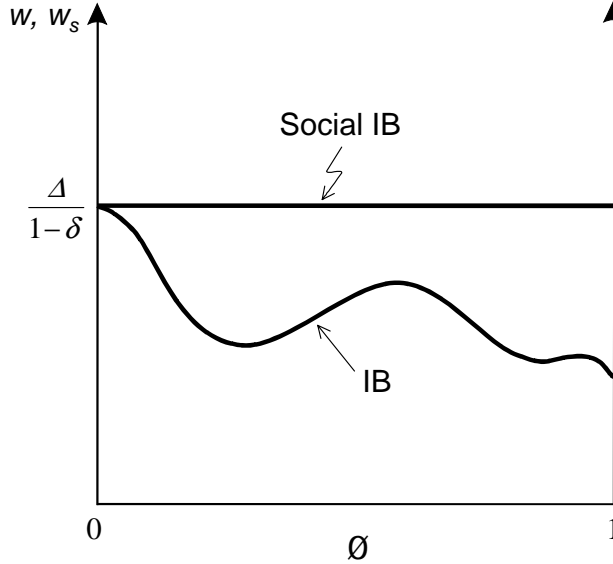


Figure 4.1:

implies $u(\phi)w - c(\phi) \geq 0$. Substituting from this inequality for $c(\phi)$ in (4.1) implies that

$$w \leq \frac{\Delta}{1 - \delta + \delta\phi}.$$

Thus, $w \leq \frac{\Delta}{1-\delta} = w_s$, so that the (private) innovation benefit curve always lies weakly below the social innovation benefit curve, as in Figure 4.1. This difference is due to the “Schumpeterian effect” that arises because an innovator is eventually replaced even though its innovation raises surplus forever.

Given the social benefit w_s (which is independent of ϕ), we can determine the socially optimal symmetric innovation rate by constructing a social innovation supply curve $\Phi_s(\cdot)$ giving the socially optimal symmetric innovation rate for a given level of the social

innovation prize. Given the relation between the social and private innovation benefit shown in Figure 4.1, it is immediate that if the social innovation supply curve coincides with the private one then the equilibrium rate of innovation must be below the socially optimal rate [note that (IS3) and the fact that w_s is independent of ϕ implies a unique socially optimal innovation rate]. This is true, for example, when there is a single potential entrant (and, hence, a single research lab) and when there is free entry with a limited idea.

In contrast, innovation may be excessive when there a fixed number $N > 1$ of potential entrants. For example, consider the case where the patent is awarded randomly among the firms who make a discovery and $c(\cdot)$ is differentiable. In this case, $r_N(\psi)$ takes the value in footnote 5. Given an innovation prize w , the socially efficient innovation rate would obtain by letting each firm internalize its contribution to social surplus by giving it the innovation prize only when it is the only successful innovator, leading to the equilibrium condition $\psi \in \arg \max_{\psi' \in (0,1)} (1 - \psi)^{N-1} \psi' - c(\psi')$. Comparing with (3.2), we see that since $r_N(\psi) > (1 - \psi)^{N-1}$, the private innovation supply exceeds the social: $\Phi_s(w) \leq \Phi(w)$, with strict inequality for all $w > c'(0)$. (Formally, the comparison follows from applying Milgrom and Roberts' [1994] Corollary 2 to compare the fixed points of the firms' private and social best-response correspondences.) This difference is due to the “business stealing effect” that arises because a potential entrant is sure to get a patent when all other firms have failed (in which case the innovation is socially useful), but also gets the patent in some cases when another firm has succeeded (in which case it is not).¹⁴

As is evident in Figure 4.1, these two differences make the comparison between the equilibrium and socially optimal rates of innovation ambiguous when there are a fixed number $N > 1$ of potential entrants. As $\delta \rightarrow 0$ the social and private innovation prizes both converge to Δ , and so only the latter innovation supply difference is present (provided that $N > 1$). As Figure 4.1 suggests (and Proposition 6.1 in Section 6 shows formally), in this case any equilibrium innovation rate will exceed the socially optimal rate. At the other extreme, when $\delta \rightarrow 1$, we have $w_s \rightarrow \infty$ while $w \leq \Delta/\phi$. Thus, as long as $\lim_{\phi \rightarrow 1} \gamma'(\phi) = \infty$, the equilibrium innovation rate will be bounded below 1, while the socially optimal innovation rate will converge to 1.

¹⁴In Aghion and Howitt [1992], two additional distortions are present: an “appropriability effect” (an incumbent monopolist captures less than his full incremental contribution to social surplus in a period) and a “monopoly distortion” effect (an incumbent produces less than the socially optimal quantity in each period). These two distortions are absent here because of our assumption of homogeneous consumer valuations and Bertrand competition.

4.2. Long-term (exclusive) contracts

To begin our discussion of specific practices, we first consider a model in which the incumbent can sign consumers to long-term contracts. This is a particular example of an R&D-detering activity.

We normalize the total number of consumers in each period to 1. Suppose that in each period t , the incumbent can offer long-term contracts to a share β_{t+1} of period $t + 1$ consumers. The contracts specify a sale in period $t + 1$ at a price q_{t+1} to be paid upon delivery. (In our simple model, this is equivalent to an exclusive contract that prevents the consumer from buying from the entrant, subject to some irrelevant issues with the timing of payments.) The antitrust policy restricts the proportion of consumers that can be offered long-term contracts: $\beta_{t+1} \leq 1 - \alpha$. We assume that the production cost k exceeds the quality increment Δ , so that an entrant cannot profitably make a sale to a customer who is bound to a long-term contract.

The timing in period t is:

- **Stage $t.1$:** Each potential entrant i observes the share of captured customers β_t and chooses its innovation rate ψ_{it} . Then innovation success is realized.
- **Stage $t.2$:** Firms name prices p_t^i to free period t consumers
- **Stage $t.3$:** Free period t consumers accept/reject these offers.
- **Stage $t.4$:** The firm with the leading technology chooses to offer to a share $\beta_{t+1} \leq 1 - \alpha$ of period $t + 1$ consumers a period $t + 1$ sales contract at price q_{t+1} to be paid upon delivery.
- **Stage $t.5$:** Period $t + 1$ consumers accept/reject these contract offers (they assume that they have no effect on the likelihood of future entry).¹⁵

We look at Markov perfect equilibria. In particular, we focus on equilibria in which potential entrants in stage $t.1$ condition their innovation choices only on the current share of captive customers β_t , and in which the choices at all other stages are stationary. (Note that since period t contracts expire at the end of that period, there is no relevant state variable affecting the contracting choice of the leading firm at stage $t.4$.)

¹⁵We assume throughout that consumers all accept if accepting is a continuation equilibrium (we do not allow consumers to coordinate). The leading firm could achieve this by, for example, committing to auction off the desired number of long-term contracts.

It is immediate that in any such equilibrium, the prices offered to free customers in any period t are $k + \Delta$ by the firm with the leading technology j_t , who wins the sale, and k by the firm with technology $j_t - 1$. Now consider a consumer's decision of whether to accept a long-term contract. If in period t the expected probability of entry in period $t + 1$ is ϕ_{t+1} , a period $t + 1$ consumer who rejects the leading firm's long-term contract offer anticipates getting the period t surplus level $\bar{v} + (j_t - 1)\Delta - k$ plus an expected gain in surplus of $\phi_{t+1}\Delta$ due to the possibility of technological advancement in period $t + 1$. Thus, he will accept the contract if and only if the price q_{t+1} satisfies $\bar{v} + j_t\Delta - q_{t+1} \geq \bar{v} + (j_t - 1 + \phi_{t+1})\Delta - k$. Hence, the maximum price the incumbent can receive in a long-term contract is $q_{t+1} = k + (1 - \phi_{t+1})\Delta$, which leaves the consumer indifferent about signing.

How many consumers will the leading firm sign up in period t ? Observe first that if the probability of entry ϕ_{t+1} were independent of β_{t+1} , then the leading firm would be indifferent about signing up an extra consumer: its period t expectation of the profit from a free consumer in period $t + 1$ is $(1 - \phi_{t+1})\Delta$, which exactly equals its maximal expected profit from a long-term contract. However, ϕ is nonincreasing in β_{t+1} , because an increase in the share of captive customers reduces the profits a successful entrant can collect in period $t + 1$.¹⁶ Therefore, the incumbent optimally signs up as many long-term customers as the antitrust constraint allows, i.e., $\beta_{t+1} = 1 - \alpha$ in every period. We can therefore fit this model into our basic model by taking

$$\begin{aligned}\pi_m(\alpha, \phi) &= \alpha\Delta + (1 - \alpha)(1 - \phi)\Delta \\ \pi_I(\alpha, \phi) &= (1 - \alpha)(1 - \phi)\Delta \\ \pi_E(\alpha, \phi) &= \alpha\Delta.\end{aligned}\tag{4.2}$$

Observe, first, that in this model an increase in α does indeed raise π_E . More significantly,

$$[\phi\pi_I(\alpha, \phi) + (1 - \phi)\pi_m(\alpha, \phi)] = (1 - \phi)\alpha\Delta + (1 - \alpha)(1 - \phi)\Delta = (1 - \phi)\Delta.$$

¹⁶Formally, the equilibrium innovation rate of a potential entrant in period $t + 1$ is $\Psi((1 - \beta_{t+1})\Delta + \delta(V_I^{t+2} - V_E^{t+2}))$ where V_I^{t+2} and V_E^{t+2} are the continuation values at the start of period $t + 2$ which are independent of β_{t+1} . By (IS3), this innovation rate is nonincreasing in β_{t+1} .

Thus, holding ϕ fixed, the expected profit of a continuing incumbent is *independent of* α . The reason is that, holding ϕ fixed, it is a matter of indifference to both the incumbent (and consumers) whether consumers accept a long-term contract. (This is a case in which an R&D-detering activity has no cost to the incumbent.) Thus, we see immediately that condition (3.1) is satisfied, and so we have:¹⁷

Proposition 4.1. *In our basic model of long-term (exclusive) contracts, restricting the use of long-term (exclusive) contracts encourages innovation.*

Consider now the welfare effects of a once-and-for-all increase in the policy α in some period τ . We assume that this intervention occurs just after stage $\tau.1$.¹⁸ The payoff effects of such a change begin in period $\tau + 1$. Note, first, that the increase raises consumer surplus: consumers are indifferent about signing exclusives when the innovation rate is held fixed, but an increase in the innovation rate delivers to them higher-quality goods at the same prices. What about the sum of consumer surplus and current incumbent (i.e., the firm with the leading technology just after stage $\tau.1$) profits? There is no direct effect of the policy change on either consumers or the current incumbent because, holding ϕ fixed, both are indifferent about whether long-term contracts are signed. What about the indirect effect caused by the increase in ϕ ? Intuitively, an innovation in period $\tau + 1$ reallocates surplus $\alpha\Delta$ from the incumbent to period $\tau + 1$ consumers. However, in subsequent periods the innovation confers an expected benefit Δ to consumers but at an expected cost to the incumbent that is less than Δ as long as the probability of future displacement is positive (i.e., $\phi > 0$).

Now consider the effects on potential entrants. Observe that in each of the examples we considered in Sections 2.2.1-2.2.3, $u(\Phi(w))w - c(\Phi(w))$ is nondecreasing in w , i.e., each potential entrant is better off if the innovation prize increases. We refer to this as the

¹⁷An alternative way of seeing the result is to observe that an increase in α raises both the joint profit upon entry $\pi_E + \pi_I$, as well as the profit of an uncontested monopolist π_m , and so must raise innovation. Joint profits upon entry are increased because the incumbent has had to offer a discount below a price of Δ to induce the captured consumers to sign (he is getting them to agree to buy a worse product than the entrant's with probability s). Uncontested monopoly profits are increased by an increase in α because, given that entry has not occurred, the incumbent is better off the fewer consumers it has signed up at discounted prices.

¹⁸We make this assumption to simplify the analysis. By doing so, the equilibrium innovation rate transitions immediately to its new steady state value and all payoff changes begin in period $\tau + 1$. If, instead, the intervention occurs at the start of the period, there would be a one period transitory effect on the innovation rate because in period τ the share of captive customers facing an entrant would be the level from before the policy change while the continuation values V_I and V_E starting in period $\tau + 1$ would be the levels in the new steady state.

Value Monotonicity Property. Since we are moving upward along the upward-sloping IS curve when α increases, the increase in ϕ caused by the increase in α must be associated with an increase in w . When the Value Monotonicity Property is satisfied, this implies that each potential entrant becomes (weakly) better off. Finally, what about the current incumbent? This turns out to be ambiguous: on one hand, the increase in ϕ speeds the incumbent's displacement. On the other hand, the value V_E that the incumbent receives when he is displaced may increase. This reasoning leads to the following result:

Proposition 4.2. *A once-and-for-all restriction on the share of long-term (exclusive) contracts that raises innovation raises consumer surplus, aggregate welfare, and (when the Value Monotonicity Property is satisfied) the values of potential entrants. The effect on the current incumbent's value is ambiguous.*

Proof. In the Appendix.

It is perhaps surprising that the welfare effect of an increase in α is necessarily positive, given that the equilibrium innovation rate may be above the first-best level due to business stealing (Section 4.1). Note, however, that long-term contracts involve an inefficiency since the incumbent makes sales of an old technology to captive customers. Thus, even if an increase in the share of captive customers brings a socially excessive innovation rate closer to the first-best level, the waste effect dominates and aggregate welfare is reduced. (Indeed, observe that potential entrants, who directly suffer from the business-stealing effect, are necessarily better off when α increases.)

4.3. Licensing of the entrant's technology

Imagine that in our long-term contracting model the incumbent can license a new entrant's technology for serving his captive consumers. Specifically, assume that the incumbent is then able to make a take-it-or-leave-it offer to these captive buyers, offering to give them instead the entrant's better product for an additional payment of Δ . The incumbent and the entrant split the gain from the agreement. We now denote the lower bound on the share of free consumers by α^* and let our policy parameter of interest $\alpha \in [0, 1]$ be the probability that such a deal is allowed. Under these assumptions, both π_I and π_E will increase, while π_m will be unaffected when α increases. As such, Propositions 3.1 and 3.2 tell us that the rate of innovation will increase if such voluntary licensing deals are allowed. Moreover, using similar reasoning to that in Proposition 4.2, we can see that

allowing such deals will also increase aggregate welfare.¹⁹

4.4. Price collusion

As another example of a voluntary deal, imagine that in our long-term contracting model the incumbent and the entrant are able to collude in their pricing to free buyers in the period in which the entrant enters, so that the incumbent now prices at $\alpha\Delta$ and the entrant at $(1 + \alpha)\Delta$, the entrant makes the sale and shares the collusion gain $\alpha\Delta$ equally with the incumbent through a side payment.²⁰ In this case, $\pi_E(\alpha) = (1 + \alpha/2)\Delta$, $\pi_I(\alpha) = \alpha\Delta/2$, and $\pi_m(\alpha) = \Delta$. The rate of innovation is increased by the degree of collusion α . In this case, however, the welfare effects are not clear. We cannot use the same type of argument as in Proposition 4.2 to show that welfare increases because the direct effect of the change on the current incumbent plus consumers is negative. Indeed, observe that there is no direct efficiency effect arising from this collusive arrangement; it is merely a transfer from consumers to the firms. Thus, the sign of the effect on aggregate welfare is determined simply by whether we were initially in a situation of over- or under-investment in R&D relative to the socially optimal symmetric rate of innovation.²¹

4.5. Compatibility in a network industry

We next consider a model of compatibility choices by a leading firm in an industry with network externalities. The model is patterned after Fudenberg and Tirole [2000] who studied limit pricing in a dynamic model.²² Overlapping generations of consumers live for two periods, and make purchases only when young. Each generation is of unit mass. The value of consumption in a period is $v + j_\tau\Delta + v(N)$ if the consumer consumes the leading quality good in period τ and this good has a “network size” of N . We follow

¹⁹As in the long-term contracting model, both the direct effect of the policy change and the indirect effect of the induced increase in innovation on the current incumbent plus consumers is positive. Since potential entrants must again be better off if the rate of innovation has increased, this implies that aggregate welfare has increased.

²⁰The entrant’s price may be constrained above by 2Δ by the existence of other firms whose technology is one step below the incumbent’s, but we assume that these entrants do not catch up with the incumbent’s technology until the next period.

²¹In a model with more general demand functions there would be an additional efficiency loss from the collusive deal because of increased pricing distortions.

²²There are several key differences: we have only one type of consumer (so limit pricing is not possible), firms make compatibility choices, and patent protection lasts forever.

the convention in the network externalities literature and assume that consumers in each generation coordinate their purchases, acting as a single agent and purchasing from a single firm. We also assume that $\Delta > v(2) - v(1)$. This implies that the firm with the leading quality will also have the leading effective quality once we include network benefits, regardless of the network sizes of the various goods.

There are many ways in which compatibility is determined in actual markets. Here we focus on one that leads to a relatively simple model that fits our framework. We assume that each firm that offers its product to consumers chooses a price p and also a compatibility level $\beta \in [0, 1]$ of this product with higher quality products. Network benefits are then determined as follows: the network size enjoyed by generation g of consumers who have bought good j is 2 if all consumers in a period consume it, is 1 if the other existing generation of consumers consumes a higher quality good, and is $1 + \beta_l$ if the other existing generation of consumers consumes a lower quality product l which has compatibility level β_l with higher quality products. That is, while consumers of high quality products can benefit from the existence of consumers who consume a lower quality product (to the extent that this good's producer allows), the reverse is not true.^{23,24} The cost of producing a product with compatibility level β is $k(\beta)$. We let $\min k(\beta) \equiv \underline{k}$.

In each period t the game proceeds as follows:

- **Stage $t.1$:** Each potential entrant i (firms other than the producer of the leading quality product) observes the purchase choice and compatibility level β_t of the current old generation of consumers. Potential entrants then make their R&D choices and innovation success is realized.
- **Stage $t.2$:** Firms choose compatibility levels β_t^i and name prices p_t^i to young consumers.
- **Stage $t.3$:** Young consumers make their purchase decisions.

Our policy parameter $\alpha \in [0, 1]$ will put a lower bound on the compatibility level

²³As an example, consider 386 and 486 chips: software designed for 386 machines also worked on 486 machines, but not the reverse.

²⁴In essence, we assume that the higher quality product can costlessly achieve as much backward compatibility as the lower quality firm allows. Were we to allow the higher quality firm a choice of whether it wants backward compatibility (at no cost), it would always choose the maximal possible level. Regarding the lower quality product, our assumptions allow its producer to commit to a compatibility level. This may be thought of as a product design choice. For example, a patented interface may prevent a new entrant from achieving backward compatibility.

that the firm with the leading technology can choose.²⁵ We focus here on equilibria in which the leading firm (whether a continuing incumbent or a new entrant) always wins the sales to young consumers, and in which on the equilibrium path the probability of entry in period $t + 1$, $\phi^*(\beta_t)$, is increasing in the compatibility level β_t chosen by this firm in the previous period t .²⁶ We let $V_I(\beta)$ denote the value of an incumbent seller who has the highest quality and yesterday sold products of quality β to today's old consumers. It will also be useful to define

$$CS(\phi) \equiv v(2) + \delta[(1 - \phi)v(2) + \phi v(1)].$$

This is the discounted expected gross consumer surplus that young consumers anticipate if they buy the product of a continuing incumbent seller (who has previously sold to today's old consumers) when the probability of entry tomorrow is ϕ .

Consider, first, the pricing, compatibility choice, and profit of a continuing incumbent who has the highest quality in a period without entry. Its relevant competitor is the firm with the next-highest quality product, the previous incumbent. The previous incumbent can offer the young consumers a surplus of²⁷

$$[v + j_{t+1}\Delta] - \Delta + [(1 + \delta)v(1) - \underline{k}].$$

The incumbent therefore sets its compatibility level equal to

$$\beta^*(\alpha) \in \arg \max_{\tilde{\beta} \geq \alpha} \{[CS(\phi^*(\tilde{\beta})) - k(\tilde{\beta}) + \Delta] - [(1 + \delta)v(1) - \underline{k}]\} + \delta V_I(\tilde{\beta}) \quad (4.3)$$

and earns $\{[CS(\phi^*(\beta^*(\alpha))) - k(\beta^*(\alpha))] + \Delta - [v(1)(1 + \delta) - \underline{k}]\}$ today from sales to current young consumers current consumers and a continuation value of $V_I(\beta^*(\alpha))$.

Now consider the pricing and compatibility choices and profit of an entrant against

²⁵We remark below on the effects of having all firms subject to this constraint.

²⁶Off the equilibrium path, when a firm other than the leading quality firm has won the sales to yesterday's young consumers, the probability of entry in these equilibria will be $\phi^*(1)$, since the leading quality firm will have no existing network, leaving entrants in the same strategic situation as when a leading quality firm makes sales but chooses to be fully compatible.

²⁷Recall that, in this equilibrium, if today's young consumers buy from the previous incumbent, then tomorrow's young consumers are certain to buy from a different firm.

a continuing incumbent who previously was the highest quality level firm and yesterday sold a product with compatibility level β_t to the current old consumers. Its relevant competitor is the continuing incumbent. The continuing incumbent can offer the young consumers a surplus of²⁸

$$[v + j_{t+1}\Delta] + [v(2) + \delta v(1) - \underline{k}].$$

The entrant will therefore set its compatibility level equal to

$$\beta^*(\alpha) \in \arg \max_{\tilde{\beta} \geq \alpha} \{[CS(\phi^*(\tilde{\beta})) - k(\tilde{\beta}) + [v(1 + \beta_t) - v(2) + \Delta] - [v(2) + \delta v(1) - \underline{k}]] + \delta V_I(\tilde{\beta})\} \quad (4.4)$$

and will earn $\{[CS(\phi^*(\beta^*(\alpha))) - k(\beta^*(\alpha))] + [v(1 + \beta_t) - v(2) + \Delta] - [v(2) + \delta v(1) - \underline{k}]\}$ from current consumers and a continuation value of $V_I(\beta^*(\alpha))$.²⁹ Of course, in equilibrium, we will have $\beta_t = \beta^*(\alpha)$.

Now consider equilibria in which the antitrust constraint binds, i.e., in which $\beta^*(\alpha) = \alpha$. For these equilibria we have:

$$\begin{aligned} \pi_m(\alpha, \phi) &= \{[CS(\phi) - k(\alpha)] + \Delta - [v(1)(1 + \delta) - \underline{k}]\} & (4.5) \\ \pi_I(\alpha, \phi) &= 0 \\ \pi_E(\alpha, \phi) &= \{[CS(\phi) - k(\alpha)] + [v(1 + \alpha) - v(2) + \Delta] - \\ & \quad [v(2) + \delta v(1) - \underline{k}]\}. \end{aligned}$$

To see the effect of a more protective antitrust policy on innovation, consider first the case in which $k(\cdot)$ is decreasing so that it is costly to block compatibility by a higher quality entrant. In this case, a firm choosing to be incompatible incurs increased production costs today to deter R&D and reduce tomorrow's likelihood of entry. As is evident from (4.5), in this case π_m and π_E both increase when α increases, while π_I remains unchanged. Propositions 3.1 and 3.2 therefore tell us that with this type of R&D-reducing behavior, innovation increases with a more protective policy.

²⁸Once again, in this equilibrium, if today's young consumers buy from the previous incumbent, then tomorrow's young consumers are certain to buy from a different firm.

²⁹>From this one can verify that the innovation rate will be nonincreasing in β_t in any model satisfying (IS1)-(IS3).

When $k(\cdot)$ is not everywhere decreasing in β , however, increased protectiveness may instead lower innovation. For example, suppose instead that $k(\cdot)$ is convex with its minimum at $\underline{\beta} \in (0, 1)$. Examining (4.5), we see that increasing α above $\underline{\beta}$ may reduce innovation: it certainly lowers π_m and may even lower π_E [if $v(1 + \alpha) - k(\alpha)$ falls].³⁰ In this case, forcing compatibility above the level that minimizes costs may reduce R&D. At such compatibility levels, increases in protectiveness no longer have the effects that arise with standard “R&D-detering activities” because we are no longer in a region where the incumbent is trading off reduced profits today for reduced R&D tomorrow.

A full investigation of the welfare effects of a more protective policy in this model is beyond our scope here. Nevertheless, it is worth noting that in this model innovation can be excessive even with only a single potential entrant (in contrast to our benchmark model). The reason is that an externality exists between the two generations of consumers in each period: when the young purchase a new entrant’s product they leave the old consumers with lower network benefits. Indeed, while new consumers have a benefit of $[\Delta - v(2) + v(1 + \beta^*(\alpha))]$ in the first period that a new entrant is in the market, the old lose $[v(2) - v(1)]$. Thus, when $\frac{\Delta}{1-\delta} < [2v(2) - v(1) - v(1 + \beta^*(\alpha))]$, an innovation lowers aggregate welfare, even ignoring R&D costs. Thus, it would not be surprising if a more protective policy that raises innovation could lower aggregate welfare here.

4.6. Uncertain innovation size and selection effects

Up to this point, we have considered models in which the nature of an innovation was fixed and antitrust policy could affect only the rate at which such innovations were discovered. In this section, we consider an extension of the long-term contracting model in which innovations are random and innovators must incur costs to bring them to market quickly. In such a setting, antitrust policy can affect not only the rate of discovery, but also the speeds with which different types of innovations make it to market. Thus, antitrust policy also involves “selection effects.” Intuitively, some innovations may bring only small benefits to their innovators, but may create large costs for the incumbents they replace. This may lead to circumstances in which more protective antitrust policies retard innovation.

To explore this possibility, we consider an extension of the long-term contracting model in which a new innovator must pay $K > 0$ to enter the market immediately. If he

³⁰If the policy instead applied to all firms, then we would have $\underline{k} = k(\alpha)$; in this case π_m and π_E would still be increasing in α , so innovation would increase with α .

does not incur this cost, he enters in the following period at no cost. We assume that the distribution of innovation sizes Δ is given by the cdf $F(\cdot)$ and for convenience we define $G(\Delta) \equiv 1 - F(\Delta)$.

To begin, observe that in this setting, if α is the share of free consumers, a new innovator will enter immediately if and only if his innovation size Δ_E satisfies $\alpha\Delta_E \geq K$, or equivalently, $\Delta_E \geq \widehat{\Delta}(K, \alpha) \equiv \frac{K}{\alpha}$.

Consider now a consumer's decision of whether to accept a contract from an incumbent whose product's value is v_I and whose innovation size was Δ_I , when the innovation rate is ϕ and the cut-off type for immediate entry is $\widehat{\Delta}$. If the consumer accepts he gets $v_I - q_{t+1}$, while if he rejects he gets $(v_I - \Delta_I - c) + \phi G(\widehat{\Delta})\Delta_I$. Hence, the incumbent will charge $q_{t+1} = c + [1 - \phi G(\widehat{\Delta})]\Delta_I$.

For given innovation sizes Δ_E and Δ_I we have the following profits for an entrant and incumbent respectively:

$$\begin{aligned}
\pi_m(\alpha, \phi, \Delta_I) &= [\alpha\Delta_I + (1 - \alpha)(1 - \phi G(\frac{K}{\alpha}))\Delta_I] & (4.6) \\
&= [1 - (1 - \alpha)\phi(1 - F(\frac{K}{\alpha}))]\Delta_I \\
\pi_I(\alpha, \phi, \Delta_I) &= [\alpha F(\frac{K}{\alpha})\Delta_I + (1 - \alpha)(1 - \phi G(\frac{K}{\alpha}))\Delta_I] \\
&= [\alpha F(\frac{K}{\alpha}) + (1 - \alpha)(1 - \phi G(\frac{K}{\alpha}))]\Delta_I \\
\pi_E(\alpha, \phi, \Delta_E) &= \text{Max}\{\alpha\Delta_E - K, 0\}
\end{aligned}$$

It is straightforward to see that condition (3.1) extends to the case of uncertain innovation, where now innovation increases (decreases) if

$$\bar{\pi}'_E(\alpha) + \delta \left[\frac{(1 - \phi)\bar{\pi}'_m(\alpha) + \phi\bar{\pi}'_I(\alpha)}{1 - \delta(1 - \phi)} \right] \geq (\leq) 0, \quad (4.7)$$

where the $\bar{\pi}$ functions are the expected profit functions (the expectation is taken with respect to the innovation size Δ). Taking expectations of (4.6), we get expected profit functions of

$$\bar{\pi}_m(\alpha, \phi) = [1 - (1 - \alpha)\phi(1 - F(\frac{K}{\alpha}))]\bar{\Delta} \quad (4.8)$$

$$\begin{aligned}\bar{\pi}_I(\alpha, \phi) &= [\alpha F + (1 - \alpha)(1 - \phi G(\frac{K}{\alpha}))]\bar{\Delta} \\ \bar{\pi}_E(\alpha, \phi) &= \int_{\frac{K}{\alpha}}^{\infty} (\alpha \Delta_E - K) f(\Delta_E) d\Delta_E,\end{aligned}$$

where $\bar{\Delta} = \int \Delta dF$.

Examining (4.8), it useful to think of a change in α as involving two effects, a direct effect of the change in α holding the cut-off type $\hat{\Delta}$ fixed, and an indirect effect of the change in $\hat{\Delta}$.

Consider the direct effect. Just as in the basic long-term (exclusive) contracting model of Section 4.2, the expected profit of a continuing incumbent, $\phi\bar{\pi}_I + (1 - \phi)\bar{\pi}_m = [1 - \phi(1 - F(\hat{\Delta}))]\bar{\Delta}$, is unaffected by a change in α holding $\hat{\Delta}$ fixed. On the other hand, the entrant's expected profit upon successful innovation, $\bar{\pi}_E$, continues to be increasing in α , just as in the basic model, although here, this effect also approaches zero as $F(\hat{\Delta}) \rightarrow 1$ since then nearly all entrants are in any case waiting for existing exclusive contracts to lapse before entering.

Now consider the effect of a decrease in the cut-off type $\hat{\Delta}$. By the envelope theorem, this has no effect on the expected profit of a successful innovator, $\bar{\pi}_E$, since the marginal type $\hat{\Delta}$ who is entering is earning zero, but reduces the expected profit of a continuing incumbent, $[1 - \phi(1 - F(\hat{\Delta}))]\bar{\Delta}$, by speeding his replacement. (Thus, in this model, signing more customers to long-term contracts — an R&D deterring activity — actually raises an incumbent's profit holding ϕ fixed.)

Whether an increase in α increases or decreases innovation depends on whether the direct or indirect effect dominates. Formally:

Proposition 4.3. *In the long-term (exclusives) model with random innovation size and costs of rapid implementation, restricting the use of long-term contracts increases (decreases) the rate of innovation ϕ if*

$$\left(\frac{\alpha}{f(\hat{\Delta})\hat{\Delta}}\right) \left(\frac{\int_{\hat{\Delta}}^{\infty} \Delta f(\Delta) \phi \Delta_E}{\bar{\Delta}}\right) \geq (\leq) \left(\frac{\delta s}{1 - \delta + \delta s}\right). \quad (4.9)$$

For example, restricting the use of long-term contracts increases the rate of innovation ϕ if $f(\hat{\Delta}) \approx 0$. In this case, the indirect effect of a change in the cut-off type is of negligible importance since there is almost no change in the likelihood of a successful innovator entering immediately. On the other hand, it lowers the rate of innovation if $\alpha > 0$, the support of Δ is bounded with $f > \underline{f} > 0$ on this support, and $F(\hat{\Delta}) \approx 1$.

When $F(\hat{\Delta})$ is close to 1, the direct effect on $\bar{\pi}_E$ approaches 0 and the indirect effect dominates, and so the innovation rate falls.

Of course, even when an increase in α causes the rate of innovation ϕ to fall, successful innovations are more likely to come into the market quickly, since the cut-off type $\hat{\Delta}$ decreases. Thus, the welfare effects of this change in innovation appear ambiguous. The following example illustrates the effect of changes in α on the rate of innovation and welfare.

Example 4.4. *Suppose there is only one potential entrant and let $c(\phi) = c\phi$. Then the innovation supply is $\Phi(w) = 0$ if $w < c$, $[0, 1]$ if $w = c$, and 1 if $w > c$. Thus, for equilibria with interior innovation rates, we must have [using (IB*)]*

$$\left(\frac{\bar{\pi}_E(\alpha)(1 - \delta + \delta\phi) + \delta [(1 - \phi)\bar{\pi}_m(\alpha) + \phi\bar{\pi}_I(\alpha)] + \delta c\phi}{1 - \delta + 2\delta\phi} \right) = c,$$

Solving for c and substituting for the expected profit functions, we have³¹

$$\begin{aligned} c &= \bar{\pi}_E(\alpha) + \left(\frac{\delta}{1 - \delta + \delta\phi} \right) [(1 - \phi)\bar{\pi}_m(\alpha) + \phi\bar{\pi}_I(\alpha)] \\ &= \int_{\frac{K}{\alpha}}^{\infty} (\alpha\Delta_E - K)f(\Delta)d\Delta_E + \left(\frac{\delta}{1 - \delta + \delta\phi} \right) [1 - \phi(1 - F(\frac{K}{\alpha}))]\bar{\Delta} \\ &= \int_{\frac{K}{\alpha}}^{\infty} (\alpha\Delta_E - K)f(\Delta)d\Delta_E + \delta \left\{ \left(\frac{1}{1 - \delta} \right) \bar{\Delta} - \left(\frac{1}{1 - \delta + \delta\phi} \right) \phi \left[(1 - F(\frac{K}{\alpha})) + \frac{\delta}{1 - \delta} \right] \bar{\Delta} \right\}. \end{aligned}$$

This equation describes an interior equilibrium innovation rate ϕ . We now assume that $\Delta \sim U[0, 1]$ and that $\alpha \in [K, 1]$. We also let $\delta = .9$ (a “period” is two years) and $K = 0.3$. Letting $\phi^*(\alpha)$ denote the equilibrium value of ϕ given α , the solid lines in Figures 5.1-5.3 graph the values of $\phi^*(\alpha)$ for $c = 0.5$, $c = 1$, and $c = 3$

To consider the welfare effects of these changes, we consider as before an intervention just after stage $\tau.1$. Once again, all of the payoff effects of this change begin in period $\tau+1$. Observe, first, that with a constant returns R&D technology, any interior equilibrium has $V_E = 0$ both before and after the change. So the only effects are those on consumers and

³¹Observe that the expression in curly brackets in the last line makes sense: the continuation payoff of an entrant starting in the period after entry is exactly equal to the present discounted social value of the innovation less the present discounted social value of the first innovation to follow it.

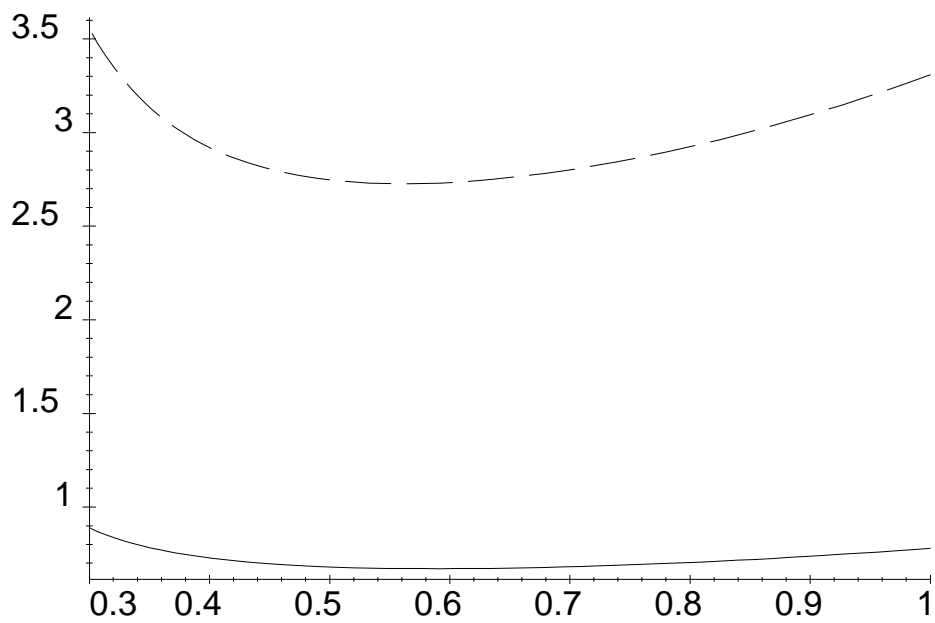


Figure 4.2: Figure 5.1: $c = 0.5$

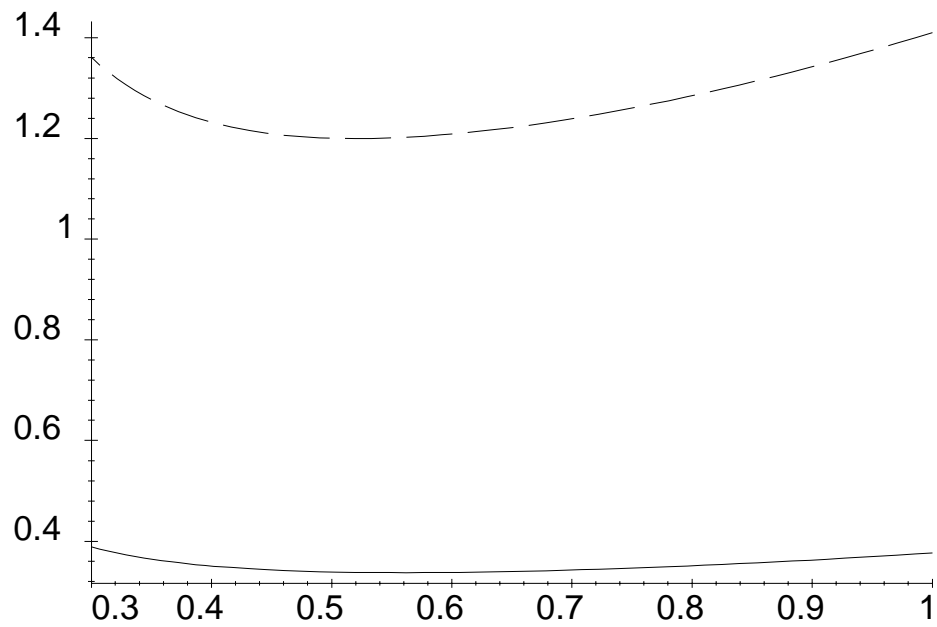


Figure 4.3: Figure 5.2: $c = 1$

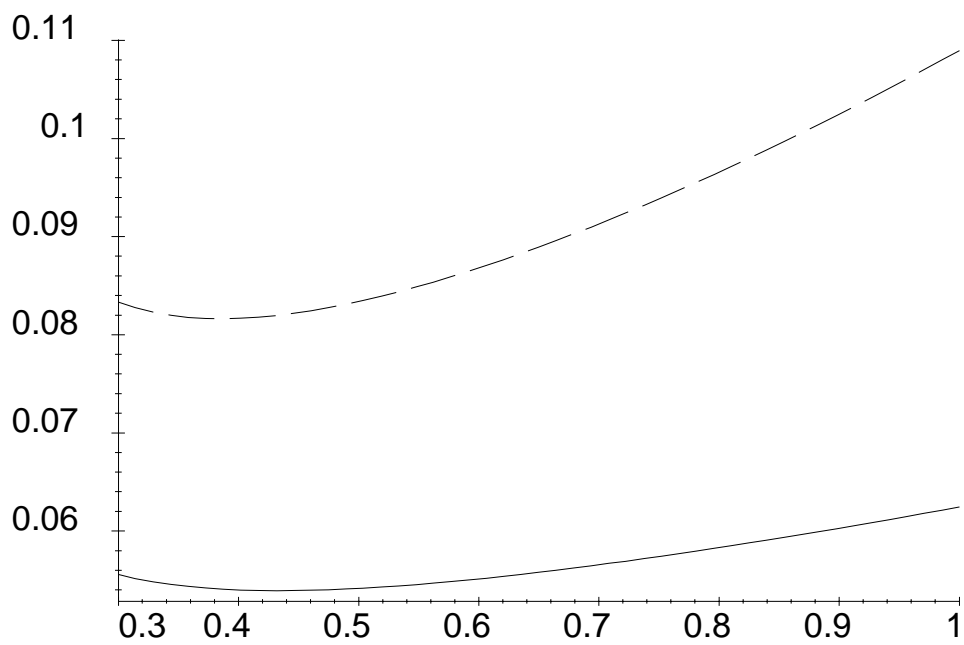


Figure 4.4: Figure 5.3: $c = 3$

the firm with the leading technology just after stage $\tau.1$ (the relevant current incumbent). When $V_E = 0$, (2.1) implies that

$$V_I = \left[\frac{1 - \phi(1 - F(\frac{K}{\alpha}))}{1 - \delta + \delta\phi} \right] \Delta_I,$$

where Δ_I is the size of this leading firm's innovation. On the other hand, the continuation payoff of consumers starting in period $\tau + 1$ is³²

$$\begin{aligned} & \left(\frac{v_I - c - \Delta_I}{1 - \delta} \right) + \left[\frac{\phi(1 - F)}{1 - \delta + \delta\phi} + \left(\frac{1}{1 - \delta} \right) \left(\frac{\delta\phi}{1 - \delta + \delta\phi} \right) \right] \Delta_I \\ & + \left(\frac{1}{1 - \delta} \right) \left(\frac{\delta\phi}{1 - \delta + \delta\phi} \right) \phi \left[\left(1 - F\left(\frac{K}{\alpha}\right) \right) + \frac{\delta}{1 - \delta} \right] \bar{\Delta}, \end{aligned}$$

where v_I is the value of the incumbent's product. Putting these together, discounted aggregate welfare starting in period $\tau + 1$ is

$$\left(\frac{v_I - c}{1 - \delta} \right) + \left(\frac{1}{1 - \delta} \right) \left(\frac{\delta\phi}{1 - \delta + \delta\phi} \right) \phi \left[\left(1 - F\left(\frac{K}{\alpha}\right) \right) + \frac{\delta}{1 - \delta} \right] \bar{\Delta}.$$

The dashed lines in Figures 4.1-4.3 graph aggregate welfare as a function of α for the cases $c = 0.5$, $c = 1$, and $c = 1.5$. In each case, the optimal policy is either being a ban on long-term contracts ($\alpha = 1$), or unrestricted contracting ($\alpha = 0.3 = K$). The optimum is unrestricted contracting when $c = 0.5$, and is no long-term contracting when $c = 1$ or when $c = 3$. (When $c = 1$, unrestricted contracting maximizes innovation, but its first-period innovation-suppressing effect tips the welfare comparison towards no long-term contracting.)

The results for this model of random innovation size are related to those of O'Donohue et al.'s [1998] study of patent policy in a model of continuing innovation. In particular, O'Donohue et al. show that "leading patent breadth," the requirement that an innovation be at least a certain minimal amount better than the current leading technology to get a patent, can increase the rate of innovation and aggregate welfare. Here, an increased

³²This can be calculated by observing that consumers start with a baseline net surplus of $(v_I - c - \Delta_I)$ in each period, and gain Δ_I from the first subsequent innovation, and $\bar{\Delta}$ from each innovation thereafter.

share of long-term contracts shifts upward the cut-off level of innovation that comes into the market rapidly. In this sense, an increase in the share of long-term contracts has effects like an increase in the leading breadth of patent protection.³³

This similarity between the effects of antitrust policy and patent policy raises the question of how optimal antitrust policy should be affected by the ability to also optimally set patent policy. While a full analysis is beyond our scope here, some insight can be gained by considering the introduction of a simple leading breadth policy into our model. Imagine, then, that we can also set directly a cut-off level Δ_C such that no innovation of size less than Δ_C can come into the market immediately. Suppose we start with an antitrust policy α that is less than 1 and an equilibrium cut-off level equal to $\hat{\Delta}$. It is clear that nothing is changed if we set $\Delta_C = \hat{\Delta}$. However, once we have done this, we change the effects of raising α . In particular, now an increase in α no longer has any effect on the set of innovations being immediately implemented; only the “direct effect” of an increase in α on profit levels remains, which we have seen causes innovation to increase. Moreover, this increase in innovation without any change in the set of innovations being immediately implemented necessarily raises welfare. Hence, the optimal antitrust policy when patent policy is available sets $\alpha = 1$. Intuitively, while antitrust policy can be used to prevent small innovations from coming to market, it does this only at the cost of introducing inefficiency when larger innovations come to market. Patent policy can achieve the same objective in this setting without this inefficiency, at least if innovation sizes are verifiable so that a leading patent breadth policy can be implemented.³⁴

³³That said, the reason leading breadth has an effect in O’Donohue et al [1998] is quite different from here. Like our model, they posit a cost of implementing an innovation (in their case, at all rather than one period earlier as here), but in their model, the rate of innovation is exogeneous. Increasing leading patent breadth therefore necessarily *reduces* the number of innovations that can enter the market without infringing an incumbent’s patent. They assume, however, that infringing innovations can be licensed to the current incumbent, who then implements them. Since increased breadth increases the length of time until the incumbent is displaced by a noninfringing innovation, the incumbent is more willing to license infringing innovations the larger is leading breadth. In the limit, as leading breadth grows infinity large, the incumbent “owns the entire quality ladder” and implements exactly the first-best set of innovations.

In contrast, in our model the rate of innovation is endogeneous. Leading breadth can have a positive welfare effect in a model such as ours for a different reason than that in O’Donohue et al.: because innovations of small size generate little profit for an entrant but can destroy large profit levels for an incumbent, R&D effort can be stimulated by excluding some small innovations from the market.

³⁴A leading breadth policy could in principle also be implemented indirectly, by requiring an innovator to pay a fee to gain access to the market, as in Llobet et al. [2000].

5. Incumbent Innovation

The analysis above imposed the strong restriction that only potential entrants engaged in R&D. Although useful for gaining insight, this assumption is clearly not representative of most settings of interest. In this section, we explore how our conclusions are affected when incumbent firms may also engage in R&D.

Allowing incumbent firms to engage in R&D has the potential to considerably complicate the analysis. In particular, once we allow for incumbent investment, we need in general to introduce a state space to keep track of the incumbent's current lead over the potential entrants. In general, the rates of R&D investment by the incumbent and its challengers may be state dependent (see, for example, Aghion et. al. [2001]).

Here we focus on two special cases in which R&D strategies are nonetheless stationary. Although clearly restrictive, these two models do have the virtue of capturing two distinct motives for incumbent R&D: (i) preventing displacement by an entrant, and (ii) increasing the flow of profits until displacement by increasing the lead over the previous incumbent.

5.1. R&D to prevent displacement

Suppose the incumbent can now do R&D denoted by ϕ_I , while the potential entrants' aggregate R&D is now denoted by ϕ_E . Similarly, the incumbent's and entrants' respective cost functions are now denoted by $c_I(\phi_I)$ and $c_E(\phi_E)$ (we allow for the fact that the cost of achieving a discovery may differ between the incumbent and the potential entrants). In this first model, we assume that if the leading quality level in period t is j_t , then quality level $j_t - 1$ is freely available to all potential producers. That is, it enters the public domain. Thus, the incumbent never has a lead greater than one step on the ladder. Thus, the only reason for an incumbent to do R&D is to try to get the patent on the next innovation in cases where at least one potential entrant has made a discovery – that is, to prevent its displacement.³⁵ With these assumptions, we need not keep track

³⁵In the usual sort of (Poisson) continuous-time model considered in the R&D literature (see, e.g., Lee and Wilde [1980], Reinganum [1989], and Grossman and Helpman [1991]), the probability of ties is zero, and so one might worry that our formulation here is dependent on a merely technical feature of the discrete-time set-up. Indeed, in such a model, the incumbent would do no R&D here. However, the usual continuous-time model relies on the implicit assumption that following an innovation, all firms reorient their R&D activity instantaneously to the next technology level. If we were to instead use a continuous-time model in which there is a fixed time period after a rival's success before which R&D for the next technology level cannot be successful, then we would get effects that parallel those in our

of any states, and there is a stationary equilibrium. Let $r_I(\phi_E)$ denote the probability that the incumbent who innovates preserves the incumbency, and $u(\phi_E, \phi_I)$ denote the probability that a given entrant innovates and then becomes the next incumbent.

Denoting by V_I and V_E the values of the incumbent and a potential entrant, we now have:

$$V_I = \pi_m(\alpha, \phi) + \delta V_I + [(1 - \phi_I)\phi_E + \phi_I(1 - r_I(\phi_E))] \{\pi_I(\alpha, \phi) - \pi_m(\alpha, \phi) - \delta(V_I - V_E)\} - c_I(\phi_I) \quad (\text{VI}^{**})$$

$$V_E = \delta V_E + u(\phi_E, \phi_I) [\pi_E(\alpha, \phi) + \delta(V_I - V_E)] - c_E(\phi_E), \quad (\text{VE}^{**})$$

where $\phi = (\phi_I, \phi_E)$.

The incumbent will choose R&D to maximize $\phi_I w_I - c_I(\phi_I)$, where

$$w_I \equiv [r_I(\phi_E) - (1 - \phi_E)] \{\pi_m(\alpha, \phi) - \pi_I(\alpha, \phi) + \delta(V_I - V_E)\} \quad (5.1)$$

is the incumbent's innovation prize, the expected incremental gain in value because the incumbent has received the patent. As for the potential entrants, they are facing innovation prize

$$w_E = \pi_E(\alpha, \phi) + \delta(V_I - V_E), \quad (5.2)$$

and will choose $\phi_E \in \Phi(w_E, \phi_I)$, where $\Phi(\cdot, \phi_I)$ is the entrants' innovation supply correspondence.

Solving (5.1) and (5.2) for $(V_I - V_E)$, we obtain (suppressing arguments of functions)

$$V_I - V_E = \frac{(1 - d)\pi_m + d\pi_I - u\pi_E - c_I + c_E}{1 - \delta + \delta(d + u)}.$$

where $d(\phi_E, \phi_I) \equiv (1 - \phi_I)\phi_E + \phi_I(1 - r_I(\phi_E))$ is now the probability that the incumbent is displaced by an entrant. Substituting, we get

$$w_I = \frac{[r_I(\phi_E) - (1 - \phi_E)]}{1 - \delta + \delta(d + u)} \{(1 + \delta u)\pi_m - (1 - \delta + \delta u)\pi_I - \delta u\pi_E + \delta(c_E - c_I)\}$$

discrete-time model (where the discount factor δ reflects how quickly R&D activity can be reoriented to the next technology level.) Thus, our discrete-time formulation captures an arguably realistic feature of the economics of R&D.

$$w_E = \frac{1}{1 - \delta + \delta(d + u)} \{ (1 - \delta + \delta d)\pi_E + \delta(1 - d)\pi_m + \delta d\pi_I + \delta(c_E - c_I) \},$$

Comparing (5.1) and (5.2) we see that when $\pi_m \geq \pi_I + \pi_E$, which captures the “efficiency effect” (that a monopoly maximizes the industry profits), the term in curly brackets in (5.1) will be larger than the entrants’ innovation prize. The countervailing effect is the “replacement effect,” which arises because the incumbent’s innovation only results in him in “replacing” himself if the entrant does not invest, while an innovating entrant can become the next incumbent. Here the replacement effect is extreme by assumption, for the innovation cannot raise the incumbent’s monopoly profits. This latter difference between incumbent and entrant innovation incentives depends on the specifics of entrant innovation supply. As an illustration, consider the following example:

Example 5.1. *Consider an extension of our long-term (exclusive) contracting model to allow for incumbent investment. We now assume that a long-term contract is a commitment by the incumbent to deliver his best current product in the next period. In this case, consumers gain Δ in surplus when the incumbent gets a new patent regardless of whether they have signed a long-term contract since the previous leading product enters the public domain. The price of a long-term contract will therefore be $q_{t+1} = k + (1 - d)\Delta$, and the profit functions can be written as*

$$\begin{aligned} \pi_m(\alpha, \phi) &= \hat{\pi}_m(\alpha, d) \equiv \alpha\Delta + (1 - \alpha)(1 - d)\Delta \\ \pi_I(\alpha, \phi) &= \hat{\pi}_I(\alpha, d) \equiv (1 - \alpha)(1 - d)\Delta \\ \pi_E(\alpha, \phi) &= \hat{\pi}_E(\alpha) \equiv \alpha\Delta. \end{aligned} \tag{5.3}$$

Thus, here the efficiency effect is zero. Now suppose we are in the free entry case. Then $r_I(\phi_E) = \bar{r}(\phi_E)$. On the other hand, each entrant makes an infinitesimal investment $\psi_E \in \arg \max_{\tilde{\psi}_E \in [0,1]} \tilde{\psi}_E \bar{r}(\phi_E) w_E - c_E(\tilde{\psi}_E)$. Comparing this to (5.1), we see that if incumbents and entrants have the same R&D cost function, the incumbent must make a zero investment. Thus, in this case, our previous analysis would go through without modification. As another example, suppose that we have either a single entrant or free entry with a limited idea and that when the entrant and incumbent both innovate they each have a $1/2$ probability of receiving the patent. Then $r_i(\phi_{-i}) = (1 - \frac{\phi_{-i}}{2})$ for $i = I, E$ and the entrant’s R&D level is $\phi_E \in \arg \max_{\tilde{\psi}_E \in [0,1]} \psi_E r_E(\phi_I) w_E - c_E(\psi_E)$. In equilibrium we must have $\phi_E \geq \phi_I$. If, in addition, the common R&D cost function has constant

returns to scale so that $c_E(\phi) = c_I(\phi) = c \cdot \phi$, then we must have $\phi_I = 0$, and so again our previous analysis would go through unchanged.

In general, when both the incumbent and potential entrants can do R&D we can distinguish between the direct and indirect effects of a change in the policy α . For the incumbent, the former captures the change in its R&D incentives holding fixed the R&D of potential entrants ϕ_E , and has the same sign as the change in w_I caused by the change in α holding (ϕ_I, ϕ_E) fixed. Similarly, the direct effect for the potential entrants has the same sign as the change in w_E caused by the change in α holding (ϕ_I, ϕ_E) fixed. The following proposition summarizes these direct effects:

Proposition 5.2. *In the model of incumbent R&D to prevent displacement, the direct effect of a more protective antitrust policy (an increase in α) on potential entrant R&D is positive (negative) if*

$$\pi'_E(\alpha, \phi) + \delta \left[\frac{(1-d)\pi'_m(\alpha, \phi) + d\pi'_I(\alpha, \phi)}{1 - \delta(1-d)} \right] \geq (\leq) 0 \quad (5.4)$$

for all α and all $d, \phi \in [0, 1]^2$. The direct effect on the incumbent R&D is positive (negative) if

$$\pi'_m(\alpha, \phi) - (1-\delta)\pi'_I(\alpha, \phi) + \delta u[\pi'_m(\alpha, \phi) - \pi'_I(\alpha, \phi) - \pi'_E(\alpha, \phi)] \geq (\leq) 0 \quad (5.5)$$

for all α and all $u, \phi_E \in [0, 1]^2$.

Observe first from (5.4) that the direct effect on entrant innovation of the policy change is exactly as before. Turning to the direct effect on the incumbent, note that the direct effects of a more protective antitrust policy on incumbent and potential entrant innovation can be *both* positive. When the efficiency effect is absent, as in the exclusives model, the direct effect on incumbent innovation will be positive if $\pi'_m(\alpha, \phi) - (1-\delta)\pi'_I(\alpha, \phi) \geq 0$. Intuitively, an increase in π_m raises the baseline profitability of being an incumbent, while an decrease in $\pi_I(\alpha, \phi)$ increases the benefit from preventing entry. For example, returning to the exclusives model, we see from (5.3) that this condition always holds there.

The direct effects are not determinative, however, of the overall change in equilibrium innovation rates, because there are interactions between the R&D levels of the incumbent

and potential entrants since the level of ϕ_i in general affects the value w_j ($i \neq j; j = I, E$). It can be seen from (5.1) and (5.2) that when $\pi'_m - \pi'_I - \pi'_E \approx 0$ (as in the exclusive contracts model, in which it is exactly zero), the level of entrant innovation increases in ϕ_I . This is enough to ensure that the overall comparative statics on ϕ_I of a local policy change in a stable equilibrium is positive when both direct effects are positive. However, even in this case, the overall effect on ϕ_E is ambiguous. One case in which this effect is clear arises in the exclusives model when we have free entry with a limited idea, $r_i(\phi_j) = 1 - \frac{\phi_j - i}{2}$, and $\alpha \approx 1$. When this is so w_E is increasing in ϕ_I . Since the direct effects are both positive, we know that both ϕ_I and ϕ_E must increase with an increase in α .

5.2. R&D to increase profit flows

We next consider a model in which rivals do not get access to the second best technology when the incumbent innovates. Thus, the incumbent can increase its flow of profits by innovating, until the time when it is displaced. Specifically, let s denote the number of steps that the incumbent is ahead of its nearest rival (this is our state variable). The variable s affects the incumbent's profit flow when entry does not occur, which we now denote by $\pi_m^s(\alpha, \phi_E)$ (it does not affect either π_I or π_E). We now make two assumptions that will imply that there is an equilibrium in which the R&D levels of the incumbent and potential entrants do not depend upon s . Specifically, we assume that $\pi_m^s(\alpha, \phi) = \mu(\alpha, \phi) + s\pi_m(\alpha, \phi)$ and that an entrant gets the patent whenever at least one entrant has made a discovery.

Example 5.3. *Consider again the extension of the long-term (exclusive) contracts model to incumbent innovation introduced in Example 5.1. A contract again is a promise to deliver the incumbent's leading technology product in the next period. To fit into our framework here, however, we change the timing of the payment in this contract, assuming instead that the payment is made when the contract is signed. Following similar reasoning*

as in Example 5.1 we see that^{36,37}

$$\pi_m^s(\alpha, \phi) = \alpha s \Delta + (1 - \alpha)[(s + \phi_I)(1 - \phi_E)\Delta - (1 - \delta)k] \quad (5.6)$$

$$= (1 - \alpha)[\phi_I(1 - \phi_E)\Delta - (1 - \delta)k] + s\Delta[\alpha + (1 - \alpha)(1 - \phi_E)] \quad (5.7)$$

$$\pi_I(\alpha, \phi) = -(1 - \alpha)k$$

$$\pi_E(\alpha, \phi) = \alpha\Delta + (1 - \alpha)\delta[k + (1 + \phi_I)(1 - \phi_E)\Delta]$$

It is clear that there is a solution in which the entrants' innovation ϕ_E and value V_E are stationary. To begin, we allow that the incumbent's R&D and value functions may depend on s : ϕ_I^s and V_I^s . In this case, we can write the value equations as

$$\begin{aligned} V_I^s &= \mu + s\pi_m + \delta V_I^s + \phi_E\{\pi_I - (\mu + s\pi_m)\} + \delta[V_E - V_I^s] \\ &\quad + \phi_I^s(1 - \phi_E)\{\pi_m + \delta(V_I^{s+1} - V_I^s)\} - c_I(\phi_I^s), \end{aligned} \quad (5.8)$$

for $s \geq 1$, and

$$V_E = \delta V_E + u(\phi_E)\left[\pi_E + \delta(V_I^1 - V_E)\right] - c_E(\phi_E). \quad (5.9)$$

The incumbent's equilibrium innovation rate satisfies

$$\phi_I^s \in \arg \max_{\psi \in [0,1]} \psi(1 - \phi_E)\{\pi_m + \delta[V_I^{s+1} - V_I^s]\} - c_I(\psi), \quad (5.10)$$

Now observe from (5.10) that ϕ_I^s will be independent of s if the difference $V_I^{s+1} - V_I^s$ is. Using (5.8) for s and $s + 1$ we see that

$$(1 - \delta)\left[V_I^{s+1} - V_I^s\right] = \pi_m - \phi_E\{\pi_m + \delta[V_I^{s+1} - V_I^s]\}$$

³⁶Observe that the price of a long-term contract, which is paid when signed in period t , is $q_t = \delta[k + (s + \phi_I)(1 - \phi_E)\Delta]$. A continuing uncontested incumbent in period t sells to free consumers (for a profit of $\alpha s \Delta$), delivers on contracts written in period $t - 1$, and writes new contracts for period $t + 1$ delivery. An incumbent who faces new entry in period t only delivers on period $t - 1$ contracts. An entrant in period t sells to free consumers (at a profit of $\alpha \Delta$), and writes new contracts for period $t + 1$ deliveries.

³⁷Note that with this change in the timing of payments, an increase in α that leaves more free consumers can lower π_E as defined here.

$$+ (1 - \phi_E) \left[\phi_I^{s+1} \{ \pi_m + \delta [V_I^{s+2} - V_I^{s+1}] \} - \phi_I^s \{ \pi_m + \delta [V_I^{s+1} - V_I^s] \} \right] - c_I(\phi_I^{s+1}) + c_I(\phi_I^s).$$

Hence, if ϕ_I^s is independent of s , there is a solution in which for all s ,

$$V_I^{s+1} - V_I^s = \frac{\pi_m(1 - \phi_E)}{1 - \delta + \delta\phi_E},$$

which is independent of s . Hence, there exists an equilibrium in which the incumbent's innovation rate is independent of s , $\phi_I^s \equiv \phi_I$, and satisfies

$$\phi_I \in \arg \max_{\psi \in [0,1]} \psi(1 - \phi_E) \left[\pi_m + \delta \left(\frac{\pi_m(1 - \phi_E)}{1 - \delta + \delta\phi_E} \right) \right] - c_I(\psi) = \arg \max_{\psi \in [0,1]} \psi \pi_m \left[\frac{1 - \phi_E}{1 - \delta + \delta\phi_E} \right] - c_I(\psi). \quad (5.11)$$

So the direct effect on ϕ_I is determined by $\pi'_m(\alpha, \phi)$.

We next solve for the entrants' innovation benefit function. Subtracting the expression for V_E from that for V_I^1 we have (omitting arguments of functions for notational simplicity)

$$[V_I^1 - V_E][1 - \delta + \delta(\phi_E + u(\phi_E))] = \pi_m + \phi_E(\pi_I - \pi_m) + \frac{\phi_I(1 - \phi_E)}{1 - \delta + \delta\phi_E} \pi_m - u(\phi_E) \pi_E - (c_I - c_E).$$

Thus, the entrants' innovation benefit function is

$$\begin{aligned} W_E &= \pi_E + \delta(V_I^1 - V_E) \\ &= \left\{ \frac{[1 - \delta(1 - \phi_E)]\pi_E + \delta(1 - \phi_E)\mu + \delta\phi_E\pi_I + \delta(1 - \phi_E) \left[1 + \frac{\phi_I}{1 - \delta + \delta\phi_E} \right] \pi_m - \delta(c_I - c_E)}{1 - \delta + \delta(\phi_E + u(\phi_E))} \right\}. \end{aligned} \quad (5.12)$$

Thus, the direct effect of α on ϕ_E is positive (negative) if

$$\pi'_E(\alpha, \phi) + \delta \left[\frac{(1 - \phi_E) \left[\mu'(\alpha, \phi) + \left(1 + \frac{\phi_I}{1 - \delta + \delta\phi_E} \right) \pi'_m(\alpha, \phi) \right] + \phi_E \pi'_I(\alpha, \phi)}{1 - \delta + \delta\phi_E} \right] \geq (\leq) 0$$

So this is similar to the case without incumbent investment except the term associated with π'_m differs due to the possibility of rising profits over time. [Observe that when $\phi_I = 0$ this expression collapses to condition (3.1).]

Proposition 5.4. *In the model of incumbent R&D to increase profit flows, the direct effect on incumbent R&D of a more protective antitrust policy (an increase in α) is positive (negative) if $\pi'_m(\alpha, \phi) \geq (\leq) 0$ for all α and all $\phi \in [0, 1]^2$, while the direct effect on potential entrant R&D is positive (negative) if*

$$\pi'_E(\alpha, \phi) + \delta \left[\frac{(1 - \phi_E) \left[\mu'(\alpha, \phi) + \left(1 + \frac{\phi_I}{1 - \delta + \delta\phi_E} \right) \pi'_m(\alpha, \phi) \right] + \phi_E \pi'_I(\alpha, \phi)}{1 - \delta + \delta\phi_E} \right] \geq (\leq) 0$$

for all α and all $\phi \in [0, 1]^2$.

For example, using (5.7) we see that in the long-term contracting model the direct effect on ϕ_I of an increase in α is always positive. On the other hand, the direct effect on ϕ_E may now be either positive or negative. When ϕ_I is close to zero, the direct effect is the same as absent incumbent innovation, and so is necessarily positive. However, when ϕ_I is large, this conclusion can be reversed. Considering now the indirect effects in the long-term contracting model, we see that increases in ϕ_E necessarily lower incumbent innovation, while increases in ϕ_I raise entrant innovation. Thus, when both direct effects are positive, we can be sure that ϕ_E increases when α rises; when instead the direct effect on entrant innovation is negative we can be sure that ϕ_I increases when α rises.

6. Other Types of Antitrust Policies

In the analysis up to this point we have considered policies that alter the profits that incumbents and entrants earn in competition with one another. Some antitrust policies have other types of effects. In this section, we briefly consider two such examples.

6.1. Predatory Activities

In some situations, antitrust may affect not only an entrant's profits in competition with the incumbent, but also the entrant's probability of survival. To focus solely on

this effect, take π_I, π_E , and π_m as fixed and suppose that a new entrant's probability of survival following its entry is $\lambda(\alpha)$, where $\lambda(\cdot)$ is increasing in α .

Now the innovation prize is

$$w = [\pi^E + \delta\lambda(\alpha)(V_I - V_E)]. \quad (6.1)$$

If $(V_I - V_E)$ were fixed, an increase in α would necessarily increase innovation. Now,

$$V_I = \pi_m + \delta V_I + \phi [\pi_I - \pi_m + \delta\lambda(\alpha)(V_E - V_I)], \quad (\text{VI}^{**})$$

$$V_E = \delta V_E + u(\phi) [\pi_E + \delta\lambda(\alpha)(V_I - V_E)] - c(\phi). \quad (\text{VE}^{**})$$

Subtracting (VE^{**}) from (VI^{**}), we can express the innovation prize with the following function:

$$W(\phi, \alpha) = \left\{ \pi_E + \left(\frac{\delta\lambda(\alpha)}{1 - \delta + \delta\lambda(\alpha)(\phi + u(\phi))} \right) [\phi\pi_I + (1 - \phi)\pi_m - u(\phi)\pi_E + c(\phi)] \right\}. \quad (6.2)$$

The fraction $\delta\lambda(\alpha)/[1 - \delta + \delta\lambda(\alpha)(\phi + u(\phi))]$ is increasing in α . Hence, provided that $(V_I - V_E)$ is positive, a more protective antitrust policy that raises the likelihood of entrant survival necessarily increases the innovation benefit.³⁸ Propositions 3.1 and 3.2 then tell us that innovation increases with this change in α .

We illustrate these effects with a simple model of predatory pricing:

6.1.1. Predatory Pricing

Consider a setting in which the entrant's probability of survival after its first production period is an increasing continuous function $\lambda(\pi_E)$ of its first-period profit. (This could be due to the entrant's financial constraints in an imperfect credit market, as in Bolton and Scharfstein [1990].) In this situation, the incumbent will be willing to price below c in the period following entry to increase the likelihood of forcing the entrant out of

³⁸For example, this will always be true whenever $V_E = 0$ (say, because of a constant returns to scale R&D technology) and π_m and π_I are non-negative. Another sufficient condition is $\phi\pi_I + (1 - \phi)\pi_m \geq u(\phi)\pi_E$ for all ϕ .

the market. To see this, consider first what the pricing equilibrium would be absent any antitrust constraint. In any such equilibrium, the entrant still wins, and consumers are indifferent between the two firms' products: the incumbent charges price p and the entrant charges price $p + \Delta$. This is an equilibrium if and only if p satisfies

$$p \leq c - [\lambda(p + \Delta - c) - \lambda(0)] (V_I - V_E) \leq p + \Delta.$$

The first inequality ensures that the incumbent prefers to lose at price p [rather than undercutting the price by ε , losing money on the sale, but increasing his chances of survival by $\lambda(p + \Delta - c) - \lambda(0)$]. The second inequality ensures that the entrant prefers to win at price $p + \Delta$. Assuming that $V_I - V_E > 0$, the middle expression is decreasing in p . Note also that the second inequality holds whenever $p \geq c - \Delta$ and the first inequality holds strictly at $p = c - \Delta$. Thus, at the highest equilibrium price p^* the first inequality binds, i.e.

$$p^* = c - [\lambda(p^* + \Delta - c) - \lambda(0)] (V_I - V_E).$$

Note that $p^* \in (c - \Delta, c)$. We focus on the equilibrium in which the incumbent charges p^* , since this strategy for the incumbent weakly dominates charging any $p < p^*$.

Now consider an antitrust policy that imposes a price floor $\alpha < c$ on the incumbent. Suppose that the floor is binding, i.e., $p^* < \alpha$. In this case, $\pi_E(\alpha) = \alpha + \Delta - c$, $\pi_I(\alpha) = 0$, and $\pi_m(\alpha) = \Delta$: thus, a higher α raises $\pi_E(\alpha)$ upon entry, does not affect $\pi_I(\alpha)$ or $\pi_m(\alpha)$, and raises $\lambda(\alpha)$. If the policy only had an effect on π_E but not on λ , then by Propositions 3.1 and 3.2 the policy would stimulate innovation. However, the policy also increases the entrant's probability of survival λ , which also increases innovation.³⁹

As in the model of long-term contracts, an increase in α holding ϕ fixed eliminates an inefficiency, here the inefficient loss of a new innovation. However, unlike the long-term contracting model, we cannot conclude that an increase in α necessarily raises aggregate welfare. To see one example in which welfare falls when α increases, suppose that the probability of survival $\lambda(\cdot)$ is constant at $\bar{\lambda}$ around $\alpha + \Delta - c$. Then a small increase in α will raise the price the entrant receives in his first period in the market (and lower consumers' payoffs in that period), have no effect on an entrant's survival probability, and

³⁹In a more general model with differentiated products, predation would make both the entrant and incumbent lose money. Thus, increasing α would raise both firms' profits as well as the entrant's probability of survival, and so would again increase innovation.

will raise the level of R&D. Because the first effect is a pure transfer, the overall effect in welfare will be determined simply by whether we have too much or too little R&D given the survival rate $\bar{\lambda}$, which can go either way just as in Section 3.1.⁴⁰ By way of contrast, if we instead have a perfectly inelastic innovation supply, α affects aggregate welfare only through an increased probability of the entrant's survival, which unambiguously raises welfare whenever $\lambda(\cdot)$ is strictly increasing.

6.2. Shifting Innovation Supply

In some cases incumbents may take actions that instead affect innovation supply. For example, an incumbent may buy up needed R&D inputs, thereby raising potential entrants' R&D costs. As another example, incumbents may engage in patent litigation claiming that an entrant's innovation infringes its own patent, raising the cost or lowering the probability of the entrant receiving a patent. Formally, we now denote the innovation supply correspondence by $\Phi(\cdot, \alpha)$. We say that an innovation supply correspondence satisfying (IS1)-(IS3) is nondecreasing (nonincreasing) in the policy parameter α if for $\alpha'' > \alpha'$ we have $\max \Phi(w, \alpha') \geq (\leq) \max \Phi(w, \alpha)$ and $\min \Phi(w, \alpha') \geq (\leq) \min \Phi(w, \alpha)$ for all w . As the following propositions establish, increases in innovation supply lead to increases in innovation in the same senses as before (we omit the proofs, which are similar to those earlier):

Proposition 6.1. *If the innovation supply correspondence $\Phi(\cdot, \alpha)$ satisfies (IS1)-(IS3) and the innovation benefit function $W(\phi, \alpha)$ is continuous in ϕ , then the largest and smallest equilibrium innovation rates exist, and both these rates are nondecreasing (nonincreasing) in α , the protectiveness of antitrust policy, if $\Phi(\cdot, \alpha)$ is nondecreasing (nonincreasing) in α for any $\phi \in [0, 1]$.*

Proposition 6.2. *Suppose that the innovation supply correspondence $\Phi(\cdot, \alpha)$ satisfies (IS1)-(IS3) and the innovation benefit function $W(\phi, \alpha)$ is continuous in ϕ . Suppose in addition that for all $\alpha \in [\underline{\alpha}, \bar{\alpha}]$ there is a unique equilibrium innovation rate $\phi(\alpha)$ on an interval $[\underline{\phi}, \bar{\phi}]$ and that the IB curve crosses the IS curve from above on this interval. Then $\phi(\alpha)$ is nondecreasing (nonincreasing) if $\Phi(\cdot, \alpha)$ is nondecreasing (nonincreasing) in α for all $\alpha \in [\underline{\alpha}, \bar{\alpha}]$ and $\phi \in [\underline{\phi}, \bar{\phi}]$.*

⁴⁰The reason we cannot use an argument like that leading to Proposition 4.2 is that the price increase has a direct negative effect on consumers plus the current incumbent; in contrast, in the long-term contracting model, all direct effects on the consumers plus the current incumbent were positive.

Thus, rightward (leftward) shifts of the innovation supply correspondence cause the rate of innovation to increase (decrease) in every stable equilibrium. Returning to the two examples mentioned above, these incumbent behaviors shift both innovation benefit and supply. If an uncontested incumbent over-buys needed R&D inputs at the end of each period, this will both raise potential entrants' R&D costs and lower π_m . Both effects cause innovation to decrease. Patent litigation, on the other hand, will not only shift the IS curve leftward but also lower both π_I and π_E . Again, both effects lower the rate of innovation.

7. Conclusion

In this paper we have studied the effects of antitrust policies in industries in which innovation is central to competitive outcomes using models of continuing innovation that are closely related to recent models in the growth theory literature. By using a stylized model with reduced form profit functions, and by characterizing the equilibrium innovation rate in terms of innovation benefit and innovation supply, we are able to develop comparative static results that apply to a wide range of market settings and antitrust policies.

In general, a tension arises in discerning the effects of antitrust policy on innovation in such settings. On the one hand, limiting incumbent behaviors that reduce the initial profit of entrants increases the incentives for R&D. But these same limitations will affect a successful entrant once it in turn becomes the next incumbent, and so could actually reduce innovation incentives. Our results show how to disentangle these two effects, and we illustrate their implications for a number of antitrust policies.

Finally, while we have used our framework to examine the effects of antitrust policies, they may prove useful for the analysis of other type of policies, such as the effects of intellectual property laws.

8. Appendix

Lemma 8.1. *The model with $N > 1$ entrants has a unique symmetric innovation equilibrium, and the equilibrium innovation rate is a continuous nondecreasing function of w .*

Proof. The symmetric equilibrium R&D rates are fixed points of correspondence $\sigma(w, \cdot) \equiv B(wr(\cdot))$, where $B(a) = \arg \max_{\psi' \in [0,1]} [a\psi' - \gamma(\psi')]$. Note that $B(a)$ is a nonempty closed interval for each a (by convexity and continuity of $\gamma(\cdot)$), and any selection from $B(\cdot)$ is nondecreasing (by the Monotone Selection Theorem of Milgrom and Shannon [1994]). Therefore, $\sigma(w, \psi) = [\sigma_L(w, \psi), \sigma_H(w, \psi)] \neq \emptyset$, with both $\sigma_L(w, \psi)$ and $\sigma_H(w, \psi)$ nondecreasing in w .

If $\sigma(w, \cdot)$ has two fixed points ψ', ψ'' with $\psi' < \psi''$, then $r(\psi') > r(\psi'')$, and since any selection from $B(\cdot)$ is nondecreasing we must have $\psi' \geq \psi''$ –contradiction. Thus, $\sigma(w, \cdot)$ has a unique fixed point, which we denote by $\Psi(w)$.

Note that correspondence $\sigma(\cdot, \cdot)$ has a closed graph, since $B(\cdot)$ has a closed graph by the Maximum Theorem, and $r(\cdot)$ is a continuous function. This in turn implies that $\sigma(w, \cdot)$ is “continuous but for upward jumps” as defined by Milgrom and Roberts [1994] (σ_L can only jump downward and σ_H can only jump upward, no matter from which direction we take $\psi \rightarrow \bar{\psi}$). Then Corollary 2 of Milgrom-Roberts applies to show that $\Psi(w)$ is nondecreasing in w .

Finally, the graph of $\Psi(\cdot)$ can be obtained by intersecting the graph of $\sigma(\cdot, \cdot)$ in the (ψ', w, ψ) space with the set described by $\psi' = \psi$ and projecting the intersection on the first axis. Since closedness is preserved under intersection and projection, we see that the graph of $\Psi(\cdot)$ is closed, and so $\Psi(\cdot)$ is a continuous function. ■

Proof of Proposition 4.2. We consider in turn the change in the payoffs of entrants, the current incumbent, consumers, and the current incumbent plus consumers.

Potential Entrants: If ϕ increases then w must have increased by (IS3). Using (VE*), we see that

$$(1 - \delta)V_E = u(\phi)w - c(\phi),$$

which implies that a potential entrant’s value V_E has weakly increased if the Value Monotonicity Property holds.

Sum of Current Incumbent and Consumers: We first compute a lower bound on the value change of the current incumbent (the firm with the leading technology just after stage $\tau.1$). A policy change just after stage $\tau.1$ changes the current incumbent’s profits only beginning in the next period. From equation (VI*) we see that we can write

$$\begin{aligned} (1 - \delta + \delta\phi)V_I &= [(1 - \phi)\pi_m + \phi\pi_I] + \delta\phi V_E \\ &= [(1 - \alpha)(1 - \phi)\Delta + (1 - \phi)\alpha\Delta] + \delta\phi V_E \end{aligned} \tag{8.1}$$

$$= (1 - \phi)\Delta + \delta\phi V_E.$$

Again, since V_E has weakly increased, a lower bound on the change in the current incumbent's value V_I starting at time $\tau + 1$ is the change in

$$\frac{\delta(1 - \phi_0)(1 - \phi)\Delta}{(1 - \delta + \delta\phi)}. \quad (8.2)$$

Now consider the consumers. Consumer welfare does not change until period $\tau + 1$ either. Since every consumer is always indifferent between signing an exclusive and being free, we can derive consumer welfare from period $\tau + 1$ on by assuming that all consumers are free. Thus, consumer welfare starting in period $\tau + 1$ is

$$\delta[(v_{j_\tau} + \phi_0\Delta - k - \Delta) + \phi\frac{\Delta}{1 - \delta}] + \delta^2[(v_{j_\tau} + \phi_0\Delta - k - \Delta) + \phi\frac{\Delta}{1 - \delta}] + \dots = \delta[(v_{j_\tau} - k - \Delta) + \phi\frac{\Delta}{(1 - \delta)^2}], \quad (8.3)$$

where v_τ is the value of the quality of the leading good at the start of period τ . This establishes that consumers are better off, since ϕ increases. Now adding (8.2) and (8.3), a lower bound on the change in the sum of consumer plus current incumbent payoffs is given by the change in

$$\frac{\delta(1 - \phi_0)(1 - \phi)\Delta}{(1 - \delta + \delta\phi)} + \delta\phi\frac{\Delta}{(1 - \delta)^2},$$

which is increasing in ϕ .

Current Incumbent: Finally, consider the current incumbent. Consider the simplest model with two firms (one potential entrant). If $V_E = 0$, which will be true if $c(\phi) = c\phi$ for some $c > 0$, then (8.1) implies that the incumbent is worse off since ϕ increases. On the other hand, suppose that ϕ is fixed at some $\bar{\phi}$ (e.g., $c(\cdot)$ is finite only at $\bar{\phi}$), then (8.1) implies that the incumbent is better off since V_E increases.

■

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