

Is Bigger Better? Customer base expansion through word of mouth reputation*

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Abstract

We develop a model of gradual reputation formation through a process of continuous investment. We consider a market in which quality is unobservable at the time of purchase so that consumers base purchasing decisions on firms' past performance - their reputation. The model has two main ingredients. First, we assume that the ability to produce high-quality products requires continuous investment. Second, we assume that as a consequence of informational frictions, such as search costs, information about firms' reputations diffuses only gradually in the market. This leads to a dual process of increase in a firm's customer base and an increase in its investment in product quality. As long as a firm continues to invest and deliver high quality, its reputation as a high quality firm grows, new customers are attracted and the firm increases in size. However, if quality deteriorates, the firm's customer base shrinks and remains stagnant until

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it is able to resurrect its reputation through successful investment. Since a good reputation is costly to acquire and takes a long time to regain once it has been lost, it becomes increasingly valuable the longer a firm's tenure as a high quality producer. Therefore, the longer its tenure, the more a firm stands to lose from tarnishing its reputation and hence the more it invests to maintain it.

Key words: Reputation, Moral hazard, Investment in quality, Firm size.

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1. Introduction

A firm's reputation is often its most valuable asset. For example, if a corporate giant like Coca Cola, McDonald's or Nike were stripped of its name - and the reputational resources associated with it - its value would be reduced to only a small fraction of what it is today. The importance of a firm's name and reputation for its balance sheet suggests that considerable managerial resources are devoted to establishing, maintaining and enhancing the value of the firm's name and reputation. The goal of this paper is to develop a modeling framework in which a firm regards its reputation as a capital asset whose value is maintained through a process of active and continuous investment.

We consider a market for a product or service whose quality is unobservable at the time of purchase. Consequently, consumers' purchasing decisions are based on what they know about a firm's past performance - the realized quality of the products it has delivered in the past. Our model has two main ingredients. First, we assume that the ability to produce high-quality products requires continuous investment in quality. Second, we assume that consumers have only limited information about the past performance of different firms. Consequently, knowledge of firms' reputations diffuses only gradually in the market. This leads to a process of gradual reputation formation, reflected by an increase in a firm's customer base, accompanied by a gradual increase in the firm's investment in quality. As long as a firm continues to invest and deliver high quality, it builds up its reputation as a high quality firm. As its reputation grows, new customers are attracted and the firm increases in size. However, if quality deteriorates, as a consequence of underinvestment or bad luck, the firm's customer base shrinks and remains stagnant until the firm is able to resurrect its reputation through successful investment. At that point the process of customer accumulation begins anew. Thus quality maintenance leads to growth of market share, while quality erosion leads to its decline.

Because reputation is costly to acquire and takes a long time to regain once it has been lost, a good reputation is more valuable to a firm the larger its customer base is. Therefore, the longer its tenure as a high quality producer - and hence the larger its size - the more a firm stands to lose from tarnishing its reputation. Consequently, in our model, the longer its tenure as a high quality producer, the more a firm invests in quality and, consequently, the higher is the quality it delivers.

The association between market tenure and/or firm size and quality, predicted by our model, seems to fit the observation that producers of high quality products with a long history in the market tend to emphasize this characteristic in their advertising. For example, the New York Times heralds the year in which it was founded on its front page and European quality beers vaunt the year in which the brand was established on their label. Similarly, advertising often seems to signal quality through market share. For example, the Hertz ad: “We’re number one.”¹

More systematic evidence supporting our results is found in empirical work on the experience rating scheme used by e-Bay. e-Bay gives buyers and sellers the opportunity to send feedbacks regarding the experience they have had with their trading partner. The feedback is in the form of a ‘positive/negative/neutral’ grade, and more extensive commentary (if desired) about the transaction. Various statistics of the results of these feedbacks are electronically posted by e-Bay, allowing future transactions to be informed by past transactions. If, as seems natural, these statistics are interpreted as a seller’s reputation, an empiricist is able to study the strategic response of buyers and sellers to reputation. One recent study that does that is Cabral and Hortacsu (2004) (see also the studies they

¹The association between investment in quality and an increase in a firm’s customer base is also found in the management literature. Prominent examples include the techniques of Total Quality Management, found in the writings of W. Edwards Deming (1986), or the related techniques of ‘six sigma.’ Although the emphasis in this literature is on the role of management in how to motivate workers, an important implication is on the contribution of such strategy to firm survival, build up of customer base, and growth of profits, which is what we stress here.

cite). Analyzing panel data on sellers, Cabral and Hortacsu find that the growth rate of a seller's transactions drops significantly following the first negative feedback from buyers. They also find that the rate of arrival of negative feedbacks increases following the first negative feedback. These findings are consistent with our theory, which predicts that sullied reputation leads to a loss of market share, and that a firm reduces its investment in quality following a negative feedback. Because the firm reduces investment in quality, low quality is more likely to result and, consequently, further negative feedbacks are likely to come in.

Brief Literature Review. Early literature on reputations in markets that focuses (like we do) on a pure moral hazard problem includes Klein and Leffler (1981), Shapiro (1983), Rogerson (1983) and Allen (1984). The basic message in this literature is that a moral hazard problem may be overcome if market interaction is repeated. An element of adverse selection is added to moral hazard in a repeated interaction framework by Kreps and Wilson (1982) and Milgrom and Roberts (1982). Most of the recent literature on reputation is in this spirit (i.e., it combines adverse selection and moral hazard). This recent literature includes Diamond (1989), Tadelis (2002), Mailath and Samuelson (2001), Watson (2002), Horner (2002), and Cabral and Hortacsu (2004).

What differentiates our approach from all these papers is that reputation in our model spreads in the market through word of mouth, or referrals - consumers tell other consumers about their experience, causing some firms to grow and other firms to decline. As a consequence of this a firm starts out small, grows gradually and changes its investment as its reputation is established. These interrelated processes of firm growth, reputation formation and the link between age, size and investment in quality represent our main contribution to the literature.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 proves that an equilibrium exists and delineates its properties. Section 4 considers extensions of the basic model. Section 5 discusses the relationship of

the model to classical issues in industry dynamics.

2. The Model

Time is discrete and the horizon is infinite. There is a continuum of firms and consumers, both of measure 1.

Firms are infinitely lived and, within each period, produce either high or low quality products. All units that a firm produces within one period are of the same quality. A firm's ability to produce high quality at any period depends on how much it invests in quality and the quality of the units it produced at the preceding period. Specifically, if a firm produced high quality products last period and if it invests x at the beginning of this period, it produces high quality products with probability $f_H(x)$. If it produced low quality last period and invests x , it produces high quality with probability $f_L(x)$.

Each period's investment is restricted to $x \in [0, \bar{x}]$ with $\bar{x} < \infty$. f_i 's are strictly concave, strictly increasing, and continuously differentiable. We assume that a given investment in quality is more effective for a firm which produced high quality last period than if it produced low quality last period. Specifically, $f_H(0) > f_L(0)$ and $f'_H(x) > f'_L(x)$ for all $x \in (0, \bar{x})$. We also assume $f_H(\bar{x}) < 1$ and $f'_L(0) = \infty$. x is a fixed cost that affects the quality of all units produced, and has no effect on variable costs. Symmetrically, a firm's variable cost of production is independent of quality. We normalize it to be zero. A firm's *state* (or *tenure*) at the beginning of a period is either 0 if it produced low quality products last period, or t , if t is the largest number of consecutive periods (starting from last period and going backwards) over which it had delivered high quality products.²

Consumers live one period and have identical downward sloping demand curves that come from expected utility maximization. Specifically, each consumer derives

²A firm's state is therefore a summary (or a "coarsening") of a firm's full history.

utility $u_H(z)$ from $z \geq 0$ units of the high quality, and utility $u_L(z)$ from z units of the low quality, product. Then, if a consumer faces probability q of getting the high quality product and a per unit price p for it, she chooses a z that maximizes

$$qu_H(z) + (1 - q)u_L(z) - pz. \quad (2.1)$$

We assume that u_i 's are strictly increasing, strictly concave, continuously differentiable, and that $u_H(0) \geq u_L(0) \geq 0$ and $u'_H(z) > u'_L(z)$ for all $z > 0$. We let $D(p; q)$ be the maximizer of (2.1), which is a consumer's demand function, and let $S(p; q)$ be the maximized value, which is a consumer's surplus.

Let

$$\Pi(p; q) \equiv pD(p; q) \quad (2.2)$$

be the per consumer period profit function (same as revenue since variable cost is assumed to be zero). We assume that $\Pi(\cdot; q)$ is single peaked for each q . We let $p(q)$ be the maximizer and $\pi(q)$ the maximized value of $\Pi(\cdot; q)$. We also let

$$s(q) \equiv S(p(q), q)$$

be a consumer's surplus under monopoly pricing.

If a consumer does not buy the product at all, the value of her outside option is zero. Since u_L is strictly increasing in z , there is a $p > 0$ so that $D(p; 0) > 0$ and, consequently, $\pi(0) > 0$. This means that even a low quality product generates positive sales, positive period profits, and a positive consumer's surplus. Finally we assume that $p(q)$, $\pi(q)$ and $s(q)$ are strictly increasing in q . Therefore, when a firm prices its product monopolistically, social surplus increases in quality, and this increase is shared by firm and consumers.

A firm's investment, x , is allowed to depend on the firm's state, denoting it by x_t for $t = 0, 1, \dots$ x_t is a firm's own private information. We let consumers' belief about x_t be y_t . In equilibrium $y_t = x_t$, but for now we keep them distinct. The

quality which is about to be realized as a result of investing x_t is known neither to the firm nor to consumers. After the product is bought and consumed, however, its quality becomes known to the firm and to consumers who bought it.

At each period a new generation of consumers of measure 1 enters the market. Each consumer lives one period. Upon exiting the market an old consumer meets with probability δ , $0 < \delta < 1$, a new consumer and tells her which firm she bought from and the product quality this firm delivered. If this firm delivered a high quality product last period, we call it an H-type, and if it delivered low quality - an L-type.³ If a new consumer does not meet an old consumer, the only way to locate a firm is by sampling a randomly selected firm, which is called “searching.” A searching consumer learns the type of the firm she finds before buying. We assume that a consumer can only search once per period and that search is costless.

A consumer who learns about a firm by meeting an old consumer has the option to search (once) as well. If she does not exercise this option, she can only buy from the firm she is referred to, called her referral firm. If she exercises this option, she can only buy from the firm she found by searching, called her search firm. There is no going back to a referral firm once this firm is turned down in favor of searching. In this environment consumers search either if they have no referral firm, or, if they expect (on average) to get a higher consumer surplus from a randomly selected firm than from their referral firm. We assume searching consumers are divided uniformly across firms, i.e., each firm receives the same measure of search consumers, which we denote by n (n is endogenously determined). By the assumption on the meeting technology, n is no less than $1 - \delta$, which is positive.

Once search activity is over and consumers are matched to firms, each firm

³The information contained in firm type is, therefore, a coarsening of the information contained in firm state.

enjoys monopoly power with respect to its captive customers. Also, all captive consumers (be they search or referral customers) of a firm hold equilibrium beliefs, which are identical, and have identical information about this firm, namely whether it delivered high or low quality products last period. As a consequence, all captive customers assign the same probability, call it q , to the event that this firm is going to deliver a high quality product. This pins down the demand curve that a firm faces, which is $D(p; q)$ multiplied by the number of captive customers that this firm serves (specified below). Given this, a firm that is believed to deliver high quality products with probability q prices its product at $p(q)$.

Given this pricing rule, let us consider a consumer's search problem. The data relevant to this problem is y_t (for $t = 0, 1, \dots$), which, as stated above, is the consumer's belief about the investment of tenure t firms, and γ_t (for $t = 0, 1, \dots$), which is the measure of tenure t firms. If the consumer has no referral firm, she is going to search for sure (because the value of her outside option is zero, while her consumer's surplus under monopoly pricing is positive). Assume, then, that the consumer has a referral firm. If this referral firm is an L type, the consumer anticipates getting $s_L = s(f_L(y_0))$ if she buys from this firm, rather than search. If the referral firm is an H type, the consumer anticipates getting $s_H = s(\bar{q})$, where⁴

$$\bar{q}(\gamma, y) = \begin{cases} \frac{1}{1-\gamma_0} \sum_{t=1}^{\infty} \gamma_t f_H(y_t) & \text{if } \gamma_0 < 1 \\ f_H(y_1) & \text{if } \gamma_0 = 1 \end{cases} . \quad (2.3)$$

Formula (2.3) follows from the fact that the consumer only knows that the firm is an H type, but not which state this firm is in. (2.3) is then the probability - averaged over all states - that an H type firm delivers high quality products. If the consumer searches, she gets an expected surplus of $\bar{s} = \gamma_0 s_L + (1 - \gamma_0) s_H$. It follows, then, that if the consumer has a referral firm of type i , $i = L, H$, she

⁴If $\gamma_0 = 1$ there are no H type firms, so there is no unambiguous way to define \bar{q} . We define \bar{q} the way we do just for concreteness. As we show in the next section, $\gamma_0 = 1$ cannot be part of an equilibrium, so any definition of \bar{q} , in particular the one we specify here, is equally good.

chooses⁵

$$\max \{s_i, \bar{s}\}. \quad (2.4)$$

We are going to assume from this point onwards that $\bar{q}(\gamma, y) > f_L(y_0)$ and thus that if a consumer has a referral firm of type H, she buys from it, while if the referral firm is of type L, the consumer searches. We show in section 3.3 that this must characterize *any* equilibrium and hence is without loss of generality.

We let b_t be the measure of customers that a state t firm serves and refer to it as its customer base. Given the above search rule, the customer base of a state 0 firm is its pro-rata share of search customers, $b_0 = n$. The customer base of a state t firm is $b_t = \delta b_{t-1} + n$ or, more explicitly,

$$b_t(n) = n \frac{1 - \delta^{t+1}}{1 - \delta}, t = 1, 2, \dots \quad (2.5)$$

Formula (2.5) follows from the fact that an H type firm starts out with n customers (when it turns from L to H). Then, in each period it loses a fraction $1 - \delta$ of, and adds a measure n to, its customer base. Therefore its customer base grows as geometric series, which is what (2.5) expresses. Note that b_t is strictly increasing in t as long as n and δ are positive.

We turn now to the formulation of a firm's problem. The data relevant to this problem is consumers' belief, y_0 and \bar{q} (which pin down the prices that firms charge) and the customer bases, b_t . Given this data, the net present value of a firm at different states, v_t , and the maximizing x_t 's are determined by the equations

$$\begin{aligned} v_0 &= b_0 \pi(f_L(y_0)) + \underset{x_0}{Max} \{-x_0 + \beta[f_L(x_0)(v_1 - v_0) + v_0]\} \\ v_t &= b_t \pi(\bar{q}) + \underset{x_t}{Max} \{-x_t + \beta[f_H(x_t)(v_{t+1} - v_0) + v_0]\}, \end{aligned} \quad (2.6)$$

where $\beta \in (0, 1)$ is the discount factor.

⁵If the two terms inside the braces of (2.4) are equal, the consumer is indifferent between searching, not searching, and randomizing between the two with any probability. This case will be relevant in the extensions section below, but here it is not.

We seek a (symmetric) steady state equilibrium, defined by the following objects:

- Investments x_t for firms at each possible state, $t = 0, 1, \dots$.
- Beliefs y_t for consumers regarding firms' investments.
- The measure γ_t of state t firms.
- The measure of search customers, n .

To ease the notation we refer to the vectors $(x_t)_{t=0}^\infty$, $(y_t)_{t=0}^\infty$, and $(\gamma_t)_{t=0}^\infty$ as x , y , and γ . (x, y, γ, n) is the tuple of endogenous variables, which is to be determined in equilibrium.

(x, y, γ, n) is an equilibrium if:

1. Firms maximize discounted profits net of investments, in accordance with (2.6), and consumers maximize utility, in accordance with (2.4).
2. Consumers' belief is correct, $y = x$.
3. γ is in a steady state with respect to the transition probabilities induced by x , i.e.,

$$\begin{aligned}\gamma_0 &= \gamma_0[1 - f_L(x_0)] + \sum_{t=1}^{\infty} \gamma_t[1 - f_H(x_t)] \\ \gamma_1 &= \gamma_0 f_L(x_0), \quad \gamma_{t+1} = \gamma_t f_H(x_t), \text{ for } t \geq 1.\end{aligned}$$

4. The measure of search customers n is consistent with the transition probabilities induced by x and with consumers' search rule:

$$n = 1 - \delta + \delta \left\{ \gamma_0 b_0(n)[1 - f_L(x_0)] + \sum_{t=1}^{\infty} \gamma_t b_t(n)[1 - f_H(x_t)] \right\}.$$

3. Analysis

In this section we prove the existence of an equilibrium and characterize its properties. More precisely, we prove the existence of an equilibrium for which $\bar{q} > f_L(y_0)$. Although it may seem constraining to impose the feature that $\bar{q} > f_L(y_0)$ from the outset, this actually simplifies the proof of existence. More importantly, it makes it easier to derive properties of the equilibrium and understand the mechanics of the model.

In greater detail, the structure of the next three subsections is as follows. First, we derive properties of the solution to a firm's problem under the assumption that $\bar{q} > f_L(y_0)$. We show that every H type firm invests more than every L type firm, and that the longer an H type's tenure, the more it invests. Therefore, if the data that a firm faces is such that $\bar{q} > f_L(y_0)$, the firm's choice of investment satisfies the same inequality. We also show that investments are no smaller than some positive lower bound, and that the equilibrium fraction of L type firms (that result from these investments) is no bigger than some upper bound that is less than one. These results are found in Section 3.1. Equipped with these results we prove - in the appendix - that an equilibrium exists. Having proven that an equilibrium exists, section 3.2 elaborates on its properties and fleshes out their empirical relevance. In section 3.3 we show that these properties characterize *any* equilibrium and are, therefore, independent of our assumption that $\bar{q} > f_L(y_0)$.

3.1. Properties of an Equilibrium

We start out by analyzing an individual firm's problem.

Taking (y, γ, n) as given, (2.6) represents a firm's objective function. By the usual dynamic programming arguments, see Stokey *et al.* (1989) (SLP, henceforth), there exists a (unique) solution v to (2.6) and a corresponding (unique)

maximizer x .⁶ Furthermore, if (y, γ, n) is such that $\bar{q} > f_L(y_0)$, all H types charge the same price, which is higher than the price charged by L types. Also, the customer base, b_t , of H types is increasing in t as per equation (2.5). This together with the corollary on page 52 of SLP imply that the solution to the firm's problem is such that v_t is strictly increasing in t . And, because v_t is strictly increasing and f'_i are strictly concave, x_t is also strictly increasing (see equation (2.6)). The next Proposition states additional properties that hold if (y, γ, n) is part of an equilibrium.

Proposition 3.1. *Assume that (x, y, γ, n) is an equilibrium and that $\bar{q} > f_L(y_0)$. Then: (i) $x_t \geq \underline{x}$, for all $t \geq 0$, where \underline{x} is some positive constant, and (ii) $\gamma_0 \leq \bar{\gamma}$, where $\bar{\gamma}$ is some positive constant, which is strictly less than 1.*

Proof: (i) We know that the measure of search customers is no less than $1 - \delta > 0$ in each period, which implies that $v_1 - v_0$ is bounded away (for all equilibria) from zero, $v_1 - v_0 \geq \delta(1 - \delta)\pi(0) > 0$. But, then, since $f'_L(0) = \infty$, the maximizer of $-x_0 + \beta f_L(x_0)(v_1 - v_0)$ is no smaller than some positive constant, call it \underline{x} . By the discussion preceding the Proposition, x_t is strictly increasing, so all x_t 's are no smaller than \underline{x} .

(ii) The steady state γ is such that $\gamma_0 = \gamma_0[1 - f_L(x_0)] + \sum_{t=1}^{\infty} \gamma_t[1 - f_H(x_t)]$ and, based on what we have shown in (i) and our assumptions about f_i 's, $1 - f_i(x_t) \leq 1 - f_L(\underline{x}) < 1$ for $i = L, H$ and $t = 0, 1, \dots$. Combining these two facts we have $\gamma_0 \leq \bar{\gamma} \equiv 1 - f_L(\underline{x}) < 1$. ■

In proving that an equilibrium exists we restrict attention to equilibria for which $\bar{q} > f_L(y_0)$, $x_t \geq \underline{x}$ and $\gamma_0 \leq \bar{\gamma}$. Proposition 3.1 tells us that a firm's best response function is consistent with these restrictions, i.e., that if we start with

⁶The state space here is the set of nonnegative integers $\{0, 1, \dots\}$, so our setting is not the same as in the text of SLP. However, following up on exercise 4.4 on page 82 in SLP, one shows that the results from SLP that we exploit here are still valid in our setting.

some (y, γ, n) tuple that satisfies these restrictions, a firm's best investment vector induces a new (y', γ', n') tuple that satisfies these restrictions as well. Therefore, if the best response function is continuous (in a suitably chosen space), one invokes a fixed point argument and shows that an equilibrium with these properties exists. This programme is carried out in the appendix.

3.2. Discussion of Equilibrium Features

We started the construction of an equilibrium by assuming that consumers buy from their referral firm if, and only if, it is of type H. Then, we analyzed the best response problem of a firm, and showed that the solution we get induces consumers to search in accordance with this rule. This means we have identified an equilibrium, or a set of equilibria, in case the fixed point is not unique. This equilibrium exhibits a host of empirically relevant properties, all of which stem from the fact that consumers follow this search rule. We now discuss these properties.

The first property is that each H type firm delivers higher quality products (on average) than each L type firm (for, otherwise, consumers' search rule would not be optimal) and, consequently, receives a higher price for its products. The second property is that H type firms keep building up their customer base, while L type firms stay small. Intuitively, since L type firms deliver low quality products (on average), they only get search customers. On the other hand, H type firms deliver high quality products, so on top of search customers (that each firm gets), they also get referral customers. Hence, H type firms serve a bigger clientele than L type firms, and this clientele increases with tenure. The third property is that each H type firm enjoys larger period profits, gross of investments in product quality, than each L type firm and, that within the class of H type firms, gross profits increase in tenure. This property follows from the first two because all H types receive the same price, which is higher than the price received by L types, and the volume of sales of H types increase in tenure. The fourth property is that

each H type firm has a larger net present value than each L type firm and that, within the class of H type firms, net present value increases with tenure. This fourth property is a direct consequence of the third one.

The fifth property is that investing zero (or any other constant) at each state is not part of an equilibrium, which implies that the dynamic equilibrium we have identified is not a simple repetition of the static equilibrium that would occur if firms were to sell the product just once. The key to this property is that what matters to investments is the continuation value *differential*. Each firm maximizes the expected value differential between being an H type and an L type, net of its investment in quality. Since, as we have indicated in the paragraph above, this differential is always positive, and since the marginal productivity of investments is infinite at zero, firms (of both types) invest a positive amount, not zero. Furthermore, since the net present value increases in tenure, so do investments. Therefore, investments cannot be constant over time, no matter what this constant is. Obviously, this result continues to hold if the marginal productivity of investment is large enough but not infinite.

An alternative way to think about these properties is as follows. The longer is the length of time over which a firm delivers a high quality product, the larger is the number of customers that are aware of this fact and, therefore, the greater is the volume of sales. On the other hand, if a firm delivers a low quality product, it (immediately) loses this favorably informed clientele, resulting in a decrease in subsequent profits. The more favorably informed customers that a firm has, the larger is the decrease in its profits. Therefore, the longer the firm has been delivering high quality product, the more it stands to lose by delivering a low quality product, which is the reason that firms invest more and more with tenure.

As noted earlier the above arguments do not imply uniqueness; there may be multiple equilibria, distinguished by different investment profiles. However, all those equilibria have in common the features described above.

3.3. Other equilibria

All these properties, intuitive though they might be, are predicated on a particular search rule (namely that a consumer searches if, and only if, her referral firm is an L type). This raises the conjecture that there may be other equilibria that are predicated on different search rules, and that exhibit, at least potentially, different properties. Contrary to this conjecture, we now show that there are no equilibria other than the ones we have identified.

Since the value of not buying the product at all is zero and since the smallest consumer surplus that a consumer captures by buying is positive (even if the product is of low quality), we can rule out the possibility where consumers don't buy the product at all. Two possibilities remain to be examined.

The first possibility is where consumers are indifferent (in equilibrium) between H types and L types and are, therefore, also indifferent between searching and buying from their referral firm, no matter what type this firm is. For the sake of concreteness, let's specify that a consumer buys from her referral firm in case she is indifferent between buying and searching.⁷ If that is the case, then in any steady state equilibrium, each firm has the same customer base. Moreover, firms of both types deliver the same average quality (for, otherwise, consumers would not be indifferent), and receive the same price. But then H types make larger net period profit. This follows because H types have to invest less than L types in order to deliver the same average quality (their f function is higher, $f_H > f_L$). But then their net present value is higher, $v_H > v_L$. But, if that is the case, the reward to delivering high quality, $v_H - v_L$, is the same for H and L types, which implies that H types invest more than L types (because $f'_H > f'_L$). Consequently, H types deliver a higher average quality product, and this contradicts the initial

⁷To be more precise one should define a firm's state here as the quality it delivered last period (either H or L) and the largest number of consecutive periods over which it had delivered this quality. Then the customer base of a firm depends on this more general concept of state.

assumption that both types deliver the same average quality.

The second possibility to contend with is where a consumer buys from a referral firm if it is an L type, but not if it is an H type. This can only happen if L types deliver higher quality on average than H types and receive, therefore, a higher price. Also, L types in this case accumulate customers, while H types don't. But this implies that the gross period profit and, consequently, the value of being an L type is higher than the value of being an H type. But, then, all firms try to be L types, which they do (costlessly) by investing zero. Given that $f_H(0) > f_L(0)$, we conclude that the average quality delivered by H types is no lower than the average quality delivered by L types. But this contradicts the initial assumption that L types deliver higher average quality. So there cannot be an equilibrium in which consumers seek L types, rather than H types.

4. Extensions

4.1. Consumers learn firms' tenure

To this point we assumed that new consumers learn the quality that a firm delivered last period but not its tenure. In this subsection we sketch an extension of the model in which new consumers learn about quality *and* tenure, i.e., they learn the state of their referral firm. We now show that the effect of this extension is that the price that a firm receives (at least in some equilibria) is strictly increasing in its tenure (before it was constant for $t \geq 1$).

When consumers learn firm types, the model and analysis are modified along the following lines. Consider first a consumer's search problem and assume the consumer's referral firm is in state t . Then, if the consumer buys from this firm, she gets the surplus

$$s_t = \begin{cases} s(f_L(y_0)) & \text{if } t = 0 \\ s(f_H(y_t)) & \text{if } t > 0 \end{cases} .$$

If the consumer searches, she gets the average surplus

$$\bar{s} = \gamma_0 s(f_L(y_0)) + \sum_{\tau=1}^{\infty} \gamma_{\tau} s(f_H(y_{\tau})).$$

The consumer chooses

$$\max\{s_t, \bar{s}\}. \quad (4.1)$$

If $s_t < \bar{s}$, the consumer searches and we indicate this by $\mu_t = 0$. If $s_t > \bar{s}$, the consumer buys from her referral firm and we indicate this by $\mu_t = 1$. If $s_t = \bar{s}$, the consumer is indifferent between buying, searching and randomizing between the two with any probability, so any value of μ_t , $0 \leq \mu_t \leq 1$, is a best choice. We denote by $\mu = (\mu_t)_{t=0}^{\infty}$ the vector of consumers' best choices, and show the dependence of μ on the data (y, γ) , using the notation

$$\mu = B(y, \gamma). \quad (4.2)$$

As this discussion shows, B is a correspondence, rather than a function.

Turning to a firm's problem, we write the customer base accumulation equation as

$$\begin{aligned} b_0 &= n \\ b_{t+1} &= b_t \delta \mu_t + n. \end{aligned}$$

A firm's period payoff is now $\pi_t(y, \gamma, \mu, n) = b_t(\mu, n)\pi(p(f_H(y_t)))$ or $b_0(n, \mu)\pi(p(f_L(y_0)))$, as the case may be, and the firm's lifetime objective is

$$F(x; y, \gamma, \mu, n) = \sum_{t=0}^{\infty} w_t(x) [\pi_t(y, \gamma, \mu, n) - x_t],$$

where $w_t(x)$ is defined in (6.2).

An equilibrium is now defined by the 5-tuple (x, y, γ, μ, n) which satisfies conditions 1-4 above along with condition (4.2).

The proof of existence proceeds now along the same lines as in the basic model, except that the fixed point argument is applied to a correspondence, not to a function. The only point worthy of mention is that we now let consumers randomize over the decision whether to search or buy from their referral firm.⁸ Randomization ensures that the payoff function of a firm, (6.1), is continuous (because the accumulation of customer base is continuous), so one is still able to apply a fixed point argument.

More interesting from the point of view of applications is the result that some of the equilibria exhibit investments that increase (strictly) in tenure and, therefore, prices that increase in tenure. To show this, we apply the fixed point argument to a closed, convex subspace of X , namely the subspace for which $x_{t+1} \geq x_t$ ($\Delta \times [1 - \delta, 1]$ remain the same). Then, the mapping in (6.3), (6.4) and (6.5) confined to this subspace has two properties. One property is that consumers' search rule is of the cutoff type: a consumer buys from her referral firm only if this firm's tenure is high enough. The second property is that the mapping returns an x that is strictly increasing. This implies the mapping has a fixed point for which $x_{t+1} > x_t$ for all $t \geq 1$. But then prices, $p(f_H(x_t))$, are strictly increasing in t . In this version of the model, then, there is another reason for investment to increase with tenure. Not only older firms have a larger customer base (as before) but they also command higher prices. Because of both these factors, older firms have more to lose by underinvesting and so invest more than younger ones.

4.2. Entry and Exit

To this point we considered an industry with a given set of firms, ruling out the possibility of entry and exit. Here we sketch how the basic model can be extended to accommodate continual entry and exit. We assume that existing firms pay a

⁸As usual, one interprets randomization either as a symmetric mixed strategy equilibrium, or as an asymmetric pure strategy equilibrium.

period fixed cost c , which is independent of tenure and/or size.⁹ On top of c , existing firms pay the fixed cost x , which *is* dependent (at least potentially) on tenure/size. We also assume that there is an infinite pool of potential entrants that make zero profits in an alternative economic activity. At the beginning of each period each potential entrant can pay c , enter, and receive a realization H with probability α , or L with probability $1 - \alpha$, where $\alpha > 0$. A new entrant gets no referrals and, accordingly, serves $b_0 = n$ customers in its initial period, the same customer base that an existing L type firm serves.

The timing of entry and exit is as follows. At the end of a period, existing firms know the realized quality of the product they just sold, and decide whether to pay c and stay, or not pay c and exit. Then, at the beginning of the next period a certain measure (endogenously determined), which we denote by e , of new entrants enter. Since an existing L type firm has a lower value than a new entrant (a new entrant has the same customer base, but a positive probability of being an H type), all L type firms exit (with the exception of a new entrant that happens to have an L realization since it has already paid c). Conversely, no H types exit; if an H type of tenure 1 exited, all entrants would not enter in the first place, which cannot be the case in equilibrium. And, a fortiori, H types of tenure greater than 1 do not exit. Thus, as long as it is type H, a firm stays in the industry and its customer base continues to grow. The first time it becomes L, it exits¹⁰. It follows, then, that the steady state measure of L types is $(1 - \alpha)e$.

We already know that, corresponding to any value of e there exists an equilibrium of the type discussed above (without entry and exit). For this to constitute a free entry equilibrium, the value of an entrant (before knowing if he is an L or

⁹Another way to pin down the measure of operating firms is to introduce a one time cost of entry, K . This approach, however, does not give rise to continual entry and exit. One can, of course, combine a one time entry cost K with a fixed cost c that has to be paid each period. No new qualitative features arise from such combination, however.

¹⁰Again, with the exception of new entrants which happen to have an L realization.

an H type) must be zero. We show in the appendix that an value of e can be found for which this is the case.

5. Conclusion

The title of this paper is "Is bigger Better?" In our model the answer to this question is in the affirmative because only those firms whose quality passes the test of time attract customers and eventually grow large. And, as a firm grows in size, it invests more and more in quality.

We conclude with a discussion of some empirical implications of our analysis. One empirical implication - at the level of the individual firm - may be termed the 'persistence of quality': an H type invests more and is therefore more likely to produce high quality than an L type. The finding by Cabral and Hortacsu (2004) in their study of e-Bay auctions - that the rate of negative feedback arrival increases twofold following the first negative feedback - seems to support this implication. In terms of our model, a seller becomes an L type when it receives its first negative feedback. Following this event it is less likely to produce high quality and therefore is more likely to receive additional negative feedbacks.

Our model also has the following two empirically relevant implications at the level of the industry. The first implication relates to the size distribution of firms. Recall that a firm's customer base grows only as long as it remains H and shrinks to the minimal size, n , as soon as the firm becomes L. Thus at any given point in time, different firms will be of different sizes depending on whether they are L or H and, if the latter, on their tenure as an H firm. The steady state size distribution, which is generated by these individual fluctuations in size, has a specific structure. Namely, since f_H is bounded away from 1, the probability that a firm of any tenure will remain H for t more consecutive periods is decreasing in t and goes to zero as t grows unboundedly large. Hence, the density of the steady

state firm size distribution is decreasing: the proportion of firms of any given size is decreasing in size.¹¹

The second implication relates to firm characteristics which determine the probability of exit from the industry. The empirical literature (see Dunne, Roberts and Samuelson (1988)) has identified firm size and age among the characteristics most strongly associated with firm turnover and, specifically, has found that the probability of exit decreases with firm size and age. The version of our model with entry and exit discussed above accounts for these facts in a very simple and direct way. Since in that version only H types survive, while L types exit at once, a firm's tenure as an H type is equivalent to its age (the number of periods since entry). And, since investment increases with tenure, the older - equivalently, the larger - a firm is, the greater the probability that it will continue to be an H type at the following period and survive. This reputational driven force contrasts with other models of industry dynamics, such as Jovanovic (1982), Hopenhayn (1992), and Ericson and Pakes (1995), in which the only characteristic of a firm is its cost and in which there is no such thing as a customer base.

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6. Appendix

Existence of Equilibrium (section 3.1)

To prove existence, it is convenient to re-formulate a firm's problem as a sequence problem. Consider, then, a firm that faces some $(y, \gamma, n) \in X \times \Delta \times [1 - \delta, 1]$, where $y \in X \equiv [\underline{x}, \bar{x}]^\infty$ is consumers' belief, $\gamma \in \Delta \equiv \left\{ \gamma \mid \sum_{\tau=0}^{\infty} \gamma_\tau = 1, \gamma_0 \leq \bar{\gamma} \right\}$ is the vector of relative proportions of firms in the different states, and $n \in [1 - \delta, 1]$ is the measure of search customers. Given this (y, γ, n) , define $\bar{q}(y, \gamma)$ as in (the second line of) (2.3). Since $\gamma_0 \leq \bar{\gamma} < 1$, \bar{q} is a well defined number in $(0, 1)$, which pins down the price, $p(\bar{q})$, that high types charge and, therefore, their per consumer profit, $\pi(\bar{q})$. The per consumer profit of low types is $\pi(f_L(y_0))$.

Let $\pi_t(y, \gamma, n) = b_t(n)\pi(\bar{q}(y, \gamma))$ be the gross period profit of a firm when it is in state t . Then, the objective of a state 0 firm, written as a sequence program, is

$$F(x; y, \gamma, n) = \sum_{t=0}^{\infty} w_t(x) [\pi_t(y, \gamma, n) - x_t], \quad (6.1)$$

where

$$w_t(x) = \frac{\beta^t f_t}{(1 - \beta) \sum_{\tau=0}^{\infty} \beta^\tau f_\tau} \quad (6.2)$$

and

$$\begin{aligned} f_0 &= 1, \\ f_t &= f_L(x_0) f_H(x_1) \cdots f_H(x_{t-1}), \quad t \geq 1. \end{aligned}$$

A state 0 firm chooses an $x \in X = [\underline{x}, \bar{x}]^\infty$ to maximize F . We endow X with the product topology. This turns X into a compact topological space by Tychonoff Theorem (see Berge, page 79). Furthermore, F is continuous in x in this topology (because of discounting). Thus, a maximizer, call it $R(y, \gamma, n)$, to F exists.

Next we endow the space $X \times X \times \Delta \times [1 - \delta, 1]$ with the product topology as well, and note that F is jointly continuous in $(x, y, \gamma, n) \in X \times X \times \Delta \times [1 - \delta, 1]$. This follows from the fact that $\gamma_0 \leq \bar{\gamma} < 1$ (see Proposition 1), which implies that \bar{q} is continuous over the relevant domain (see equation (2.3)). Given that, the Theorem of the maximum (see Berge, page 116) applies, and we conclude that $R(y, \gamma, n)$ is u.h.c. Moreover, $R(y, \gamma, n)$ is a singleton because of the strict concavity of (f_L, f_H) . As a result, $R(y, \gamma, n)$ is continuous over $X \times \Delta \times [1 - \delta, 1]$.

Let us define now a map from $X \times \Delta \times [1 - \delta, 1]$ into itself as follows

$$y' = R(y, \gamma, n) \tag{6.3}$$

$$\begin{aligned} \gamma'_0 &= \gamma_0[1 - f_L(y_0)] + \sum_{t=1}^{\infty} \gamma_t[1 - f_H(y_t)] \\ \gamma'_1 &= \gamma_0 f_L(y_0) \\ \gamma'_{t+1} &= \gamma_t f_H(y_t) \end{aligned} \tag{6.4}$$

$$n' = 1 - \delta + \delta \left\{ \gamma_0 b_0(n)[1 - f_L(y_0)] + \sum_{t=1}^{\infty} \gamma_t b_t(n)[1 - f_H(y_t)] \right\}. \tag{6.5}$$

The argument above together with the expressions for (6.4) and (6.5) show that this map is continuous. Therefore, since $X \times \Delta \times [1 - \delta, 1]$ is compact and convex, the Schauder fixed point theorem (see Berge, page 252) guarantees the existence of a fixed point. This fixed point is an equilibrium because (i) firms maximize against consumers' belief and consumers' belief is correct, both of which follow from equation (6.3), (ii) consumers' search rule is optimal, as per Proposition 3.1, and the 'physical' steady state conditions 3 and 4 are satisfied, as per (6.4) and (6.5). ■

Note that this proof of existence could be simplified by applying a fixed point argument to the pair $(\bar{q}, f_L(y_0))$. This simplification does not work, however, for the extension of the model in which consumers observe a firm's state rather than

firm type (see Section 4.1). The proof above, on the other hand, applies to both scenarios.

Existence of Free entry equilibrium (section 4.2)

The existence of an equilibrium with this property is established as follows. For every e , we select the equilibrium that gives tenure 0 firms (and hence all firms) the largest value (recall that there may be multiple equilibria). This equilibrium is well defined because equilibria, (x, γ, n) , are fixed points of a continuous map over a compact set, and the value of a firm F is a continuous function of (x, x, γ, n) . Hence, the set of equilibrium values is itself compact. Further, for sufficiently small e 's, this largest equilibrium value is non-negative, and for sufficiently large e , it is non-positive. We choose the supremum of the set of e 's, call it e^* , so that this value is non negative (by u.h.c. the value at e^* is non negative). If $e > e^*$, the value at *any* equilibrium corresponding to e is negative. Therefore e^* is consistent with free entry; even if the (largest) average equilibrium value at e^* is positive, the value of entry would drop below zero if there were more than e^* entrants.