

Finding “Lost” Profits: An Equilibrium Analysis of Patent Infringement Damages

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Abstract

We examine the impact of patent infringement damages in an equilibrium oligopoly model of process innovation where the choice to infringe is endogenous and affects market choices. Under the (currently employed) lost profits measure of damages, we find that infringement always occurs in equilibrium with the infringing firm making market choices that manipulate the resulting market profit of the patentholder. In equilibrium, infringement takes one of two forms: a “passive” form in which lost profits of the patentholder are zero, and an “aggressive” form where they are strictly positive. Even though the patentee’s profits are protected with the lost profits damage measure, innovation incentives are reduced relative to a regime where infringement is deterred.

Keywords: Patents, Lost Profits, Infringement

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1 Introduction

Patent law encourages innovation and the dissemination of knowledge by providing exclusivity in exchange for knowledge disclosures. The value of exclusivity derives from two penalties imposed on infringing parties: injunctions that stop subsequent use and damages in compensation for previous use. Because it is common for litigation to conclude after an infringer has been in the market for some time, expected damages play an important role in establishing incentives for innovation.

Since 1946 U.S. courts have largely adopted a compensatory approach to awarding damages to the patent holder as a result of patent infringement.

[Damages] have been defined by this Court as “compensation for the pecuniary loss he (the patentee) has suffered from the infringement, without regard to the question whether the defendant has gained or lost by his unlawful acts.” They have been said to constitute “the difference between his pecuniary condition after the infringement, and what this condition would have been if the infringement had not occurred.” (Yale Lock Mfg. Co. v. Sargent 117 U.S. 536, 552 [1886]).¹

The ideal damage award under this approach is the “lost profits” of the patentee which are determined by calculating patentee profits that would have occurred absent infringement. When such calculations cannot be reasonably made, damages are based on a calculation of royalties on infringer’s actual sales. The royalty payment can be thought of roughly as the amount the patentee could have received from licensing the invention to the infringer.

In this paper we examine how the lost profits measure of damages affects competition, infringement, and the incentives for innovation in a market competition between the patentee and a potential infringer. This examination involves determining a reference level for lost profits based on market competition that is consistent with equilibrium competitive choices. We focus on two questions. First, when will damages based on lost market profits deter infringement? Second, if infringement is not deterred, how are innovation incentives impacted by the lost profits approach?

We focus on a process innovation that allows the patentee to lower its costs relative to a non-innovating firm. Given the patent, the non-innovating firm chooses whether to imitate (and risk infringement). The market setting is a critical element in our analysis since the subsequent market

¹Aro Mfg. Co. v. Convertible Top Replacement Co. 377 U.S. 476 at 507 (1964).

outcomes are determined endogenously with behavior incorporating the consequences of the lost profits damage measure. We assume a best case for the enforcement regime: whenever infringement is discovered, the court assesses accurately the associated damages and prosecution does not involve transactions costs.² We then incorporate the equilibrium outcomes for infringement and market competition into a patent race to assess how the lost profit damage measure impacts innovation incentives.³

Despite the long-standing interest in the question of patent damage measures, to the best of our knowledge there is no existing equilibrium analysis of market competition and innovation. In a related paper which focuses primarily on vertical relationships, Schankerman and Scotchmer (2001) examine the effect of damage rules in a setting with “cooperative” licensing. Our focus is on horizontal competitors, so we analyze the effect of lost profit damages in terms of competitive market outcomes rather than via licensing (but see the discussion of licensing in Section 7).⁴

We find that a damage measure based exclusively on lost profits of the patentee (as a result of infringement) always results in infringement in equilibrium. Infringement occurs because a non-innovating firm always has the strategic option of “passive” infringement in which the patent is imitated but market choices mimic those in the non-infringement outcome (produce at the non-infringing quantity while taking profit gains only via lower costs). By definition, the patentholder suffers no lost profit and so the non-innovating firm strictly prefers to infringe. In some cases, infringement takes a more aggressive form where the non-innovating firm and patentee choose market positions that push lost profits to a strictly positive level.

In equilibrium, under both forms of infringement the patentee receives the same profits as if no

²We do, however, allow for uncertainty in whether the infringement is detected. This allows for weak versus strong patent protection rights and, thus, a relative assessment of incentives in the benchmark case of perfect detection.

³Merges (1997, p. 1080-1) notes that “the trend in patent law damages since the 1980s has been to allow patentees more and more latitude to describe the second order effects of the infringer’s entry and presence in the market for the patented good. ...Almost all of these damages require the patentee to spin a narrative entitled ‘what life would have been like without the infringer’...By the same token, it would seem self-evident that courts should invite evidence of second order responses by infringers under the (increasingly ornate) hypothetical scenarios being spun by patentees...”

⁴Our purpose is to illustrate how analyzing market structure illuminates the current use of legal damages. We do not address alternative damage approaches such as disgorgement of infringer profits. See, e.g., Kaplow and Shavell (1996), Blair and Cotter (1998) as well as Schankerman and Scotchmer (2001) for comparisons of various liability and damage approaches.

infringement had taken place. Thus, one might expect that the lost profits measure preserves the incentive (reward) for innovative efforts. We find, however, that basing damages on lost profits reduces the incentive to innovate relative to the benchmark case (no infringement). The explanation lies with the effect of the damage measure on infringer payoffs. In equilibrium, infringement always occurs and, at a minimum, a loser (non-innovating firm) in a patent race will have a (valuable) “passive” infringement option. Thus, as ex ante innovation incentives are based on the profit differences between being the patentee and the infringer, overall innovation incentives will be reduced.

We present the model in Section 2. Patentholder and imitator incentives are examined in Sections 3 and 4, respectively, and the equilibrium market outcomes and infringement choices are derived in Section 5. We then examine innovation incentives in a patent race in Section 6 and conclude with a discussion of our results in Section 7. Proofs are in the Appendix.

2 The Model

Our model consists of an innovator with a patent to a cost-reducing process innovation, firm i , and a potential infringer (imitator), firm j . Both firms produce a homogeneous good but with potentially different costs. Market competition is concluded before infringement damages, if any, are awarded to the patentholder. The firms are risk-neutral and maximize expected profits. We focus on a strategic setting where firms choose quantities.⁵ Prior to innovation, the status quo has the two firms competing with constant marginal costs of \bar{c} in a market with linear demand⁶

$$P(q) = \alpha - \beta q. \tag{1}$$

Thus, the prior status quo involves the traditional Cournot equilibrium outcome.

Now suppose that firm i has obtained a patent for an innovation that allows it to produce at cost c , where $c < \bar{c}$. Firm j has the option of remaining with the old technology and producing at cost \bar{c} without any risk of infringement. Denote the option for j of no-imitation by \mathcal{N} . When j chooses \mathcal{N} , we have quantity competition between firm i at cost c and firm j at cost \bar{c} ; let $\pi_i^{\mathcal{N}}$ and $\pi_j^{\mathcal{N}}$ denote the resulting profit outcomes for the firms. In the calculation of lost profits, these are the reference

⁵One could also formulate the analysis in terms of price setting, differentiated goods and a product innovation.

⁶We assume $\alpha > 2\bar{c}$ so that both firms are active (positive output). The assumption is sufficient to avoid corner cases for outputs in equilibrium outcomes and is easily relaxed.

profit levels that correspond to the hypothetical involving the market outcome in the event that no infringement had occurred.

Firm j also has the option, denoted by \mathcal{I} , of imitating firm i 's patented innovation. However, this entails a risk that the court will find infringement. We assume that imitation allows firm j to reduce costs relative to the prior technology and produce at cost s , where $s < \bar{c}$. A special case involves perfect imitation (where $c = s$), but allowing for cost differences ($s \leq c$) makes it possible to identify the different incentives of the two firms and the results for the special case follow directly from the more general analysis.⁷

If the court finds infringement, the penalty requires that the infringer make a monetary payment to firm i so that firm i earns a net payoff equal to that which would have occurred had no infringement taken place. The penalty is related to market events as follows. Given a choice of \mathcal{I} by firm j , the firms make quantity choices of q_i and q_j , respectively, for i and j . The resulting market price of P is from (1) and firm i has a realized market payoff of $(P - c)q_i$. If this is less than π_i^N , then firm j must pay i the difference. If not, then no damage penalty is assessed. Thus, the damage payment is given by $D(q_i, q_j) \equiv \text{Max}\{\pi_i^N - (P - c)q_i, 0\}$. Our penalty assumption corresponds to a lost profits calculation where licensing revenues are not included or would be zero.

We assume that the court finds infringement with probability γ when j chooses \mathcal{I} . In practice, the lack of perfect enforcement arises for a number of reasons, including (i) firm j may be able to circumvent the patent, (ii) the court may find the patent is invalid, and (iii) enforcement is not detected.⁸ The size of γ effectively indexes the strength of property rights for the patentholder and we can expect that small values of γ will make \mathcal{I} relatively more attractive for firm j . Given a choice of \mathcal{I} , the expected payoffs for each firm are given by

$$\Pi_i(q_i, q_j) = (P - c)q_i + \gamma D \tag{2}$$

$$\Pi_j(q_j, q_i) = (P - s)q_j - \gamma D, \tag{3}$$

at quantity choices q_i and q_j , the market price of $P = P(q_i + q_j)$, and lost profits damages of

⁷With $s < c$, the follower implements the innovation more effectively than the innovator. A number of studies (e.g., Schnaars [1994]) report that second-movers (e.g., non-patentholders who imitate) often achieve market dominance.

⁸One can argue that the probability is related to the underlying extent of innovation and imitation, so that γ depends on c and s . Also, penalties such as treble damages can be incorporated into γ . As we note below, such relationships are easily incorporated into the analysis. See Lemley (2001) for a general discussion of the patent system.

$D = D(q_i, q_j)$. Given a choice of \mathcal{N} , the firms earn $\pi_i^{\mathcal{N}}$ and $\pi_j^{\mathcal{N}}$. We solve for a subgame-perfect equilibrium in which j chooses between \mathcal{N} and \mathcal{I} , and then the firms simultaneously choose quantities for market competition. Finally, the court makes an infringement determination with payoffs and damages determined via (2) and (3).

3 Market Incentives of the Patentholder

How should the patentholder (innovator), firm i , choose quantity given that firm j has chosen to imitate? We might expect this to depend on the strength of the patent, indexed by γ , as well as the output expected from j . As a best-response problem, however, the choice of firm i turns out to have a simple strategic structure. The payoff in (2) reveals that i is always led to choose a quantity that maximizes the realized market payoff of $(P - c)q_i$ in response to any expected output choice of from j . Thus, the prospect of lost profits has no direct impact on the market choice of the patentholder.

Refer to Figure 1. There are two situations for firm i with respect to firm j . If j produces at a relatively high level, then the market payoff for firm i is always below the reference profit for no infringement; this is the lower curve in Figure 1. In this case, lost profits are always positive and (2) reduces to maximizing $(1 - \gamma)$ times the market payoff and γ does not matter for the optimizing choice. In this case, the best response of firm i is easily verified to be $q_i = (\alpha - c - \beta q_j) / (2\beta)$ provided the interior term is positive (and zero when it is not). When infringement is proven, firm i always gets $\pi_i^{\mathcal{N}}$ so it is optimal to choose quantity to maximize profits when infringement is not proven. Hence, the simple Cournot best response without regard to damages is optimal.

If j produces at a relatively low level, then we have the situation depicted with the upper curve in Figure 1. Now, depending on i 's response, lost profits may be positive or zero (recall the absolute value restriction on payments from i to j). If i produces below q_A or above q_B , then lost profits are strictly positive and, as before, (2) reduces to the (scaled) market payoff. For these quantity cases, i 's payoff is largest at q_A and q_B (both yield $\pi_i^{\mathcal{N}}$), so output choices below q_A or above q_B are never a best response for i . For $q_i \in (q_A, q_B)$, the market payoff exceeds $\pi_i^{\mathcal{N}}$ and we have $D = 0$ by the definition of lost profits damages. Now (2) is identically equal to the market payoff and, hence, an interior choice at point C in Figure 1 is optimal. As before, this reduces to $q_i = (\alpha - c - \beta q_j) / (2\beta)$ and, from a strategic point of view, firm i does not respond directly to patent strength (γ) or lost profits.

We formalize this argument as follows. The reference profit level of $\pi_i^{\mathcal{N}}$ corresponds to the standard

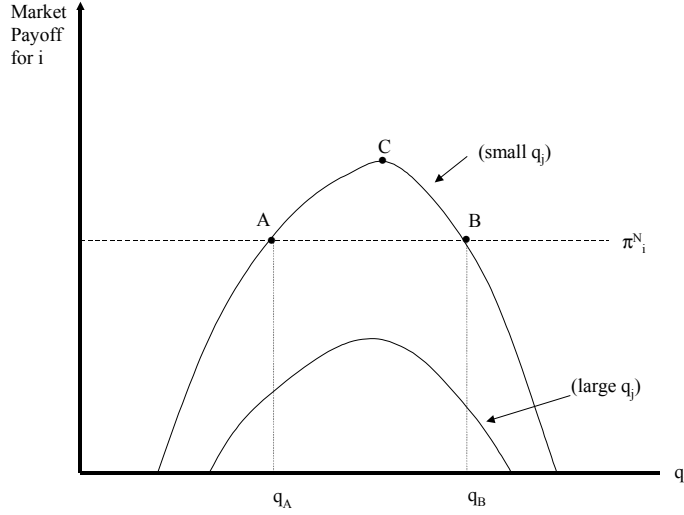


Figure 1: Patentholder Market Payoff and Lost Profit

Cournot equilibrium outcome for firm i with cost c and firm j with cost \bar{c} , and this is given by

$$q_i^N = \frac{1}{3\beta} (\alpha - 2c + \bar{c}) \quad \text{and} \quad \pi_i^N = \beta (q_i^N)^2, \quad (4)$$

$$q_j^N = \frac{1}{3\beta} (\alpha - 2\bar{c} + c) \quad \text{and} \quad \pi_j^N = \beta (q_j^N)^2. \quad (5)$$

Then, we calculate that the maximum value for market profit is above π_i^N whenever q_j is below q_j^N (as is the case with point C in Figure 1). However, as have seen, firm i 's best response is driven by the market payoff and it does not depend on whether lost profits are positive or zero. Thus, we have

Lemma 1 *The best response of firm i to q_j by firm j is given by $\phi_i(q_j) = (\alpha - c - \beta q_j) / (2\beta)$ for all $q_j \leq (\alpha - c) / \beta$, and by $\phi_i(q_j) = 0$ for larger q_j .*

Note that the strength of patent rights (γ) and the lost profit reference level have no direct effect on the patentholder's market quantity choice. Instead, the patentholder's objective always reduces to maximizing the market payoff (pure or scaled). Consequently, i 's market decision is only affected indirectly via j 's quantity choice.

4 Market Incentives of the Imitator

The strategic situation of the imitator (infringer) is considerably more complex because of the asymmetry in the payoff functions. Whether lost profits are positive or zero, the patentholder's payoff always reduces to a multiple of the market payoff. In contrast, the imitator's payoff involves his own market payoff as well as that of the patentholder whenever lost profits are positive. Moreover, the market choices of both players determine when lost profits are positive. This creates a number of subtle strategic effects.

We begin by analyzing the imitator's market incentives when lost profits are positive. Referring back to Figure 1, consider the critical points q_A and q_B where the market payoff for i crosses the reference profit of π_i^N . Fix j 's output at zero for the moment and consider the value of $[P(q_i) - c]q_i$, which corresponds to a standard monopoly payoff function. This is maximized at the monopoly output level of $q^M \equiv (\alpha - c) / (2\beta)$ with corresponding payoff $\pi^M \equiv (\alpha - c)^2 / (4\beta)$. With $c < \bar{c}$, we know that $\pi^M > \pi_i^N$ holds. Thus, we can solve for the critical points q_A and q_B where i 's payoff when $q_j = 0$ crosses π_i^N :

$$q_A = \sqrt{\frac{\pi^M}{\beta}} - \sqrt{\frac{\pi^M - \pi_i^N}{\beta}} \quad \text{and} \quad q_B = \sqrt{\frac{\pi^M}{\beta}} + \sqrt{\frac{\pi^M - \pi_i^N}{\beta}}. \quad (6)$$

Consider the imitator's best-response problem if $q_i \leq q_A$ or if $q_i \geq q_B$. Then, as shown with the dashed line in Figure 2, the market payoff for i is necessarily below π_i^N for any choice of q_j :

$$\begin{aligned} [P(q_i + q_j) - c]q_i &= (\alpha - \beta q_i - c)q_i - \beta q_j q_i \\ &\leq (\alpha - \beta q_i - c)q_i \quad \text{as } q_j \geq 0, \\ &\leq \pi_i^N \quad \text{as } q_i \notin (q_A, q_B) \text{ and } \pi^M > \pi_i^N. \end{aligned}$$

For $q_i \notin (q_A, q_B)$, lost profits are necessarily positive for any (positive) output choice by the imitator.

In contrast, for $q_i \in (q_A, q_B)$, there is a unique corresponding output level for j , denoted by Q_j for the upper solid line in Figure 2, at which lost profits cease being zero and become positive. Solving, we find

$$Q_j(q_i) = \frac{\alpha - c}{\beta} - \frac{\pi_i^N}{\beta q_i} - q_i, \quad \text{for } q_i \in (q_A, q_B). \quad (7)$$

We define Q_j to be zero outside of the interval (q_A, q_B) . The basic properties of Q_j follow directly from (7) and are summarized in

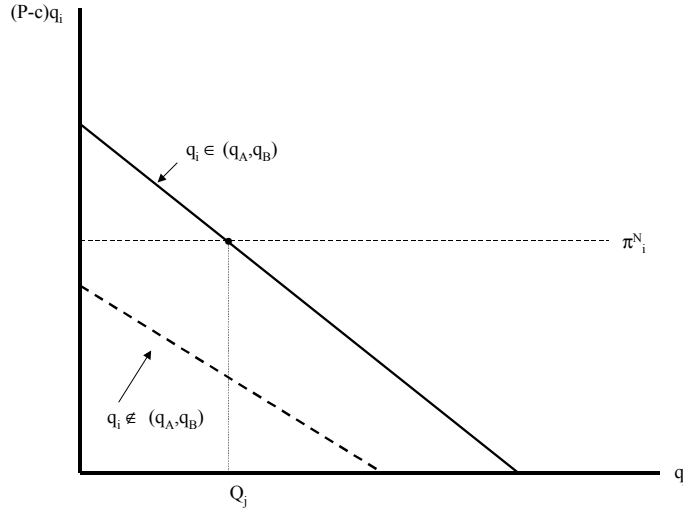


Figure 2: Imitator Output and Lost Profit

Lemma 2 For $q_i \in [q_A, q_B]$, the function $Q_j(q_i)$ satisfies (i) $Q_j(q_A) = Q_j(q_B) = 0$; (ii) $Q_j(q_i)$ is strictly concave; (iii) $Q_j(q_i)$ has a unique maximum at $q_i = q_i^N$ and $Q_j(q_i^N) = q_j^N$; (iv) the function $q_i + 2Q_j(q_i)$ is strictly increasing for $q_i \in [q_A, q_i^N]$.

Consequently, in this case the imitator can determine through its output choice whether lost profits are zero or, when positive, how large they are. With these preliminary observations in place, we now solve for the best response function of the imitator.

The first case is that of $q_i \notin (q_A, q_B)$. Since $D > 0$ for all $q_j \geq 0$, we have from (3) that $\Pi_j(q_i, q_j) = (q_j + \gamma q_i) [\alpha - \beta q_i - \beta q_j] - s q_j - \gamma c q_i - \gamma \pi_i^N$, upon substituting for P . This is a strictly concave function in q_j and, from the first-order condition, we find the unique optimal choice of

$$\varphi_j(q_i) \equiv \frac{1}{2\beta} [\alpha - s - \beta(1 + \gamma)q_i] \quad \text{if } q_i < \frac{\alpha - s}{\beta(1 + \gamma)}, \quad (8)$$

and $\varphi_j(q_i) \equiv 0$ for larger q_i . Thus, as long as the patentholder's output does not force j from the market, the impact of the lost profit penalty depends on the size of γ . As γ rises and property rights become more secure, the imitator becomes more "timid" and reduces output. When property rights vanish, at $\gamma = 0$, $\varphi_j(q_i)$ reduces to the standard Cournot best response function.

The second case arises when $q_i \in (q_A, q_B)$. A difficulty lies with determining whether j will find it profitable to produce aggressively, thereby inducing a positive lost profits penalty, or keep output

low, thereby holding lost profits to zero. As Figure 2 suggests, the output level of $Q_j(q_i)$ is critical for this choice. Formally, we have

Lemma 3 *Suppose that $q_i \in (q_A, q_B)$. If $q_i \geq (\alpha - s)/\beta$, then the best response of j is zero. If $q_i < (\alpha - s)/\beta$, then the best response of j is positive and satisfies (i) if $(\alpha - s)/\beta \leq q_i + 2Q_j(q_i)$, then the best response of j is $\phi_j(q_i) \equiv \frac{1}{2\beta}[\alpha - s - \beta q_i]$, and $D = 0$ at (q_i, q_j) ; (ii) if $q_i + 2Q_j(q_i) \leq (\alpha - s)/\beta \leq (1 + \gamma)q_i + 2Q_j(q_i)$, then the best response of j is $Q_j(q_i)$, and $D = 0$ at (q_i, q_j) ; (iii) if $(1 + \gamma)q_i + 2Q_j(q_i) \leq (\alpha - s)/\beta$, then the best response of j is $\varphi_j(q_i)$, and $D > 0$ at (q_i, q_j) .*

5 Equilibrium Market Outcomes and Infringement

We now characterize the market equilibrium choices for output, given that j has chosen to imitate and risk infringement. Then we examine j 's equilibrium infringement choice.

Proposition 1 *Suppose j chose \mathcal{I} . If $\varphi_j(q_i^N) > q_j^N$, then the unique equilibrium outcome (given imitation) is at (q_i^*, q_j^*) where $q_j^* = \varphi_j(q_i^*)$ and $q_i^* = \phi_i(q_j^*)$. Lost profits are strictly positive in equilibrium.*

In this case, the lost profit penalty is not sufficient to deter the imitator from driving the patentholder's market profit below the no-infringement reference level. This is an "aggressive" form of infringement. Solving the equations $q_j^* = \varphi_j(q_i^*)$ and $q_i^* = \phi_i(q_j^*)$, we find

$$\begin{aligned} q_i^* &= \frac{1}{\beta(3-\gamma)}(\alpha - 2c + s) \\ q_j^* &= \frac{1}{\beta(3-\gamma)}[\alpha(1-\gamma) - 2s + (1+\gamma)c]. \end{aligned}$$

The imitator produces a large quantity and the patentholder responds by reducing output (relative to the no-infringement outcome of \mathcal{N}). On balance, the patentholder's market payoff falls below the reference level and lost profits are strictly positive.

Proposition 2 *Suppose j chose \mathcal{I} . If $\varphi_j(q_i^N) \leq q_j^N$, then the unique equilibrium outcome (given imitation) is at (q_i^*, q_j^*) where $q_j^* = q_j^N$ and $q_i^* = q_i^N$. Lost profits are zero in equilibrium.*

In this case, we have a "passive" form of infringement. In fact, the patentholder produces as if no infringement occurred. The imitator, however, is infringing and producing at cost s rather than cost \bar{c} . The choice of output by the imitator is specifically set at the level which induces i to respond at the

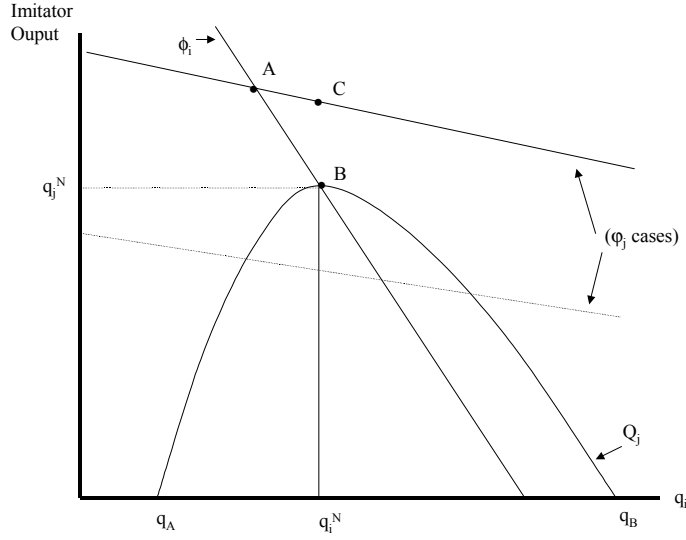


Figure 3: Market Equilibrium (given imitation)

reference output q_i^N . In other words, j produces at its own reference level for no infringement, namely, q_j^N . By doing so, no lost profits are generated. Instead, the imitator takes the gain from infringing completely in the form of reduced production costs for output q_j^N . Of course, the patentholder has a payoff of π_i^N in equilibrium for both cases.

Figure 3 illustrates the two cases for the equilibrium outcome. When φ_j is large, as with the upper solid line, the equilibrium is at point A, where φ_j and ϕ_i intersect. When it is small, the equilibrium is always at point B, where the level of lost profits is at zero.

An important question concerns which of these two cases applies in relation to the underlying structural parameters of the model: property rights (γ), the level of innovation (c) and the efficacy of imitation (s). From Propositions 1 and 2, we need only determine when $\varphi_j(q_i^N) \geq q_j^N$ occurs relative to the parameters. Substituting and simplifying, we find a dividing line between passive and aggressive infringement is determined by the cost levels c (for i) and s (for j) given by

$$\chi(c) = \bar{c} - \frac{\gamma}{3}(\alpha - 2c + \bar{c}). \quad (9)$$

Figure 4 provides a graph of χ for the typical case. First, consider how property rights determine the position of χ . In the limit as $\gamma \rightarrow 0$ and property rights vanish, we have $\chi \rightarrow \bar{c}$. Then, as is intuitively obvious, the absence of property rights (trivially) leads to aggressive infringement. As γ rises the χ line shifts down and we have two regions. Above the line, the cost of the imitator (s) is high

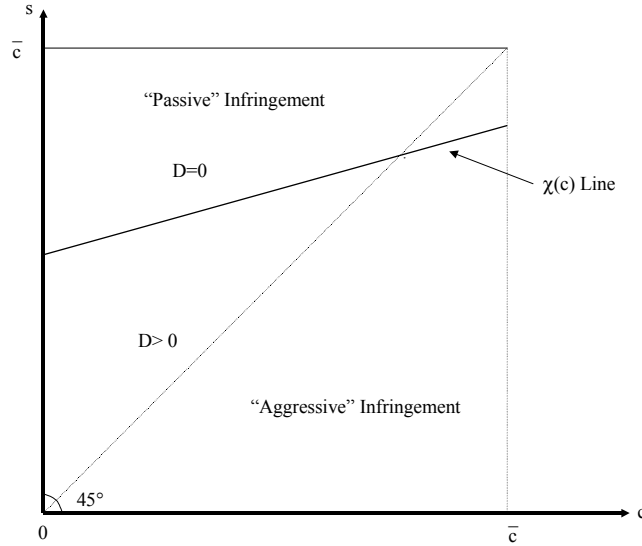


Figure 4: Equilibrium Infringement Outcomes

relative to the cost of the patentholder (c) and the prospect of lost profits is sufficiently unattractive that passive infringement is the equilibrium choice. Below the line, the costs of the imitator are lower and aggressive infringement becomes optimal.⁹

More generally, we note that the probability of infringement may depend on the extent of innovation and imitation. Then the analogue of χ is found by solving (9) under the proposed $\gamma(c, s)$ relationship.

Thus, the critical condition of $\varphi_j(q_i^N) \geq q_j^N$ for whether the market equilibrium has D at zero or positive reduces to

Proposition 3 *If $s \geq \chi(c)$, then the market equilibrium is given by Proposition 2 and $D = 0$. Otherwise, the market equilibrium is given by Proposition 1 and $D > 0$.*

The next step in the analysis is to determine whether the imitator will choose to infringe in equilibrium. Whenever lost profits are zero in equilibrium, imitation is always profitable. This is because j pursues the passive infringement strategy to produce the same output and receive the same market price as with no imitation, but production costs are lower and there is no lost profit penalty.

⁹Depending on parameters, the χ line can fall below the horizontal axis as $\gamma \rightarrow 1$ (i.e., if $4\bar{c} < \alpha$) and in this case equilibrium always involves passive infringement.

Infringement is also profitable when lost profits are positive in equilibrium. Lost profits are positive in equilibrium when j increases quantity above the passive infringement quantity. Since j can always generate profit improvement using passive infringement, active infringement will only be used if it provides yet greater rents. Therefore, no matter what the strength of property rights, the lost profits damage criterion will necessarily trigger infringement. Formally, we have

Proposition 4 *The imitator earns strictly greater profits from \mathcal{I} than from \mathcal{N} . Thus, in equilibrium, the imitator always chooses to infringe.*

6 Innovation Incentives

An objective of the patent system and of intellectual property rights regimes more generally is to encourage innovation. At the same time welfare is affected by market allocation given innovation. Determining the optimal damage measure thus depends on the effect of the measure on both dynamic and static competition.¹⁰ Because there is little consensus on the right way to model innovation, we address a more modest question: how do our results for the lost profit damage measure relate to innovation incentives?¹¹

A natural benchmark is to consider innovation incentives relative to a setting in which infringement is completely deterred and the patentholder earns the reference payoff of $\pi_i^{\mathcal{N}}$. In the equilibrium under lost profits damages, we always have infringement by the imitator but, significantly, whether infringement is passive or aggressive, the patentholder continues to earn an equilibrium payoff of $\pi_i^{\mathcal{N}}$. With the same reward to a patent, it is tempting to conclude that innovation incentives are not distorted relative to the benchmark.

This intuition, however, is misleading. While the reward to innovating is the same, the reward to *not* innovating is different. Specifically, the incentive to invest in R&D will be affected by the prospect of the reward to “failure,” namely, the option to imitate and infringe on a patented innovation. In equilibrium, this option always has positive value (Proposition 4). Thus, relative to the benchmark of

¹⁰Furthermore, such an assessment would need to examine innovation incentives that lie outside of or are substitutes for the legal system (see, e.g., Boldrin and Levine, 2003).

¹¹Ayres and Klemperer (1999) suggest a method for incrementally improving the tradeoff between dynamic and static efficiencies by inducing some limited infringement through increased use of ex post “make whole” damages over preliminary injunctions.

no infringement, the lost profits damage criterion creates a free-rider incentive. Failure has its reward too.

Innovation incentives can be explored in a variety of R&D contexts. Let us examine the incentive to innovate in a standard “memoryless” (Poisson) patent race in continuous time with two ex-ante symmetric firms, $k = 1, 2$, and interest rate r (our treatment is a variation on Reinganum [1983]). Each firm invests at the expenditure rate x_k and succeeds at innovation with instantaneous probability $h(x_k)$; the Appendix develops further the technical requirements. In the absence of a success by either firm, each earns the status quo flow profit of $\bar{\pi} \equiv (\alpha - \bar{c})^2 / (9\beta)$. The first to achieve success patents the innovation, assumes the role of the patentholder and earns a payoff with present discounted value of V_i . The other firm becomes the imitator and receives a discounted payoff of V_j . In the benchmark case, we have π_i^N and $V_i = \pi_i^N / r$ and $V_j = \pi_j^N / r$. The question of innovation incentives can be examined by computing the comparative static of equilibrium R&D with respect to V_j .

The intertemporal payoff to R&D of x_1 when the rival is at x_2 is given by

$$\begin{aligned} V_1(x_1, x_2) &= \int_0^\infty [h(x_1)V_i + h(x_2)V_j + \bar{\pi} - x_1] e^{-rt} e^{-[h(x_1)+h(x_2)]t} dt \\ &= \frac{h(x_1)V_i + h(x_2)V_j + \bar{\pi} - x_1}{r + h(x_1) + h(x_2)}, \end{aligned}$$

as follows from standard reasoning in patent-race models. It is straightforward to show that a unique equilibrium exists and is symmetric. The comparative static result is then

Proposition 5 *The equilibrium level of R&D in the patent race is decreasing in V_j , the payoff to the imitator.*

To interpret Proposition 5, note that lost profit damages involve an increase in V_j relative to the benchmark setting of no infringement. By increasing the payoff of the firm that fails to patent, R&D incentives are reduced in the patent race and both firms invest less in R&D.¹² To map the payoffs of V_i and V_j into the lost profits model, the simplest interpretation is that the duration of the infringement fight effectively runs for the market life of the innovation. Then, firm i continues to earn $V_i = \pi_i^N / r$ while firm j earns $V_j = \Pi_j(q_j^*, q_i^*) / r > \pi_j^N / r$.¹³

¹²A similar comparative static holds in an alternative model involving a two-stage game where firms invest in period 1 and then receive $\bar{\pi}$, V_i , or V_j in period 2. Also, Reinganum (1982) reports a similar comparative static for follower payoffs in a patent race with knowledge accumulation and imperfect patents (imitation).

¹³One can extend this in a number of ways. For instance, imagine that the infringement suit is resolved over a period of

7 Discussion

7.1 The Reference Level Problem and Licensing

In this paper we illustrate how damages based on lost profits in the market leads to infringement. The critical element provided by an explicit market structure is that an infringer has the (valuable) strategic option of choosing market actions that are designed to manipulate the resulting equilibrium level of lost market profits. This approach best captures situations in which the innovator and imitator are horizontal competitors and for antitrust reasons are therefore likely to have significantly limited or completely foreclosed licensing alternatives.

If licensing were feasible, the appropriate lost profits measure would be the greater of lost market profits or lost licensing revenues. With complete information and no restriction on licensing, the innovator and imitator could employ licensing to coordinate the production of maximum joint profits—one reason why antitrust is likely to be invoked—and lost profits would be based on some split of these joint profits. While the assumption of unfettered licensing is unappealing to us as a description in horizontal competitor settings, it is appropriate as a starting point for analyzing vertical relationships. Schankerman and Scotchmer (2001) take this approach in analyzing patent damages in vertical relationships and note an inherent circularity in the determination of lost profits damages: hypothetical license revenues depend on the legal damages that are avoided (via bargaining threat points) and hence depend on the reference level for lost profits. They also develop a variant of their joint profit maximizing model to consider horizontal licensing and find that the lost profits damage rule results in no infringement (even off the equilibrium path). We, on the other hand, always find infringement. The difference in results can be attributed to two major differences in the models. First, by allowing unfettered licensing Schankerman and Scotchmer’s model allows for cooperative (collusive) behavior between the firms whereas our model is essentially noncooperative. Second, the lost profits damage function in a setting with reduced-form profits is not affected by the market choices of the parties so the gaming of the damage rule inherent in our model does not occur.¹⁴

Although we do not consider settings in which licensing is feasible, our analysis does have some

time T during which the firms earn payoffs of π_i^N and $\Pi_j(q_j^*, q_i^*)$. At T , the payoffs switch to π_i^N and π_j^N with probability γ (j is found to have infringed) and to $(\alpha - 2c + s)^2 / (9\beta)$ and $(\alpha - 2s + c)^2 / (9\beta)$ with $1 - \gamma$ (no infringement).

¹⁴See Aoki and Hu (1999) for an analysis of the impact of litigation on licensing and innovation, and Baker and Mezzetti (2001) for an analysis joint ventures and enforcement.

implications for settings in which some limited licensing can occur. In our model “passive infringement” does not entail lost profit damages. Thus, an imitator that expects to choose the passive infringement quantity would have, at best, a limited incentive to buy a license from the innovator. Of course, passive infringement merely sets the floor for imitator profits, but that floor means that no license that leads to imitator profits less than that obtainable through passive infringement is feasible. Where limitations on licensing affect the maximum attainable joint profits, passive (and aggressive) infringement profit floors may make licensing unattractive.¹⁵

In practice, evidentiary considerations and murkiness in the appropriate application of economics to the creation of a hypothetical benchmark have led courts to be flexible regarding patent damage awards, oftentimes blurring distinctions among the various damage rules. When lost profits cannot be satisfactorily estimated, Section 284 of the Patent Act specifies that damages will be no less than that calculated based on reasonable royalties.¹⁶ This fallback position directly impacts the problem posed by an infringer’s strategic response to damages based on pure market lost profits discussed above. If the royalties fallback was developed primarily to make damage calculation simpler, it clearly has a salutary effect in partially patching a loophole left by a pure market lost profits doctrine. Since reasonable royalties typically leave an infringer with positive incremental profits, our notion of passive infringement extends to this situation with the reinterpretation that instead of free infringement, the passive infringer is choosing a favorable (implicit) licensing deal over aggressive infringement with its lost profit potential damages.

Although our purpose in this paper is to demonstrate how underlying market interactions affect the current patent damages regime, the analysis also applies to an alternative approach based on

¹⁵A basic licensing option, which typically invokes less antitrust concern than would unconstrained licensing, involves a lump-sum payment in return for an unrestricted right to employ the patented technology in production. Suppose such a licensing negotiation occurs in our model prior to the infringement choice. Under such a license, competition would result in the standard Cournot outcome (with the patentholder at cost c and the non-innovator at cost s). It is straightforward to show that conflicting incentives will often make a mutually acceptable license payment infeasible. For an example, take demand with $\alpha = \beta = 1$, perfect infringement detection with $\gamma = 1$, and costs with $\bar{c} = 1/4$ and $c = s = 1/8$. For more on licensing and market structure, see Anand and Khanna (2000), Gans and Stern (2000) and Jehiel and Moldovanu (2003).

¹⁶*Panduit* establishes a set of four conditions necessary for damages to be awarded based on lost profits. When these conditions are not met courts typically use a royalties-based approach to assessing damages.

disgorgement of infringer profits (see, e.g., Blair and Cotter [1998]).¹⁷ Disgorgement, like lost profits, is determined with respect to a reference level of profits absent infringement, though there the focus is on infringer profits. Passive infringement would not present as grave a concern under disgorgement, however, the patentee would now have a more complex optimization problem. For example, a patentee could scale back own output anticipating most of its profits will come from disgorgement of infringer profits.

7.2 Incomplete Information

An important element in our analysis is the attractive strategic option for the infringer to avoid lost profits damages by maintaining its pre-patent quantity level. This option makes infringement a dominant strategy and is the linchpin for our analysis. How robust is the passive infringement option to incomplete information?

In practice, costs are likely to be a source of private information. The problem is that the infringer will not know what quantity would obtain given the patent and no infringement until it has some market experience competing against the patentee. Once it has this information, the passive infringement option can be used without direct knowledge of a rival's costs because only knowledge of previous period quantity (with the patentee producing using the innovation and the infringer without that innovation) is needed. Private costs could, however, affect the boundary of the parameter region in which the passive infringement strategy is employed. This would occur through its effect on market choices, directly in market competition and indirectly in terms of the infringement damages that are anticipated.

¹⁷Blair and Cotter (1998) argue that the courts should adopt a property right damage regime (see, e.g., Calabresi and Melamed [1972] and Kaplow and Shavell [1996] for general discussions on property and liability rules) more akin to that used for trade secrets: injunctions plus damages which should be the greater of lost profits of the patentee and the incremental profits earned by the infringer. Effectively, Blair and Cotter would like the law to move back in the direction of the restitution theory for damages. They propose using lost profits when the infringer is less efficient (this is greater than disgorgement and has a greater deterrence effect) and disgorgement with more efficient infringers (this is greater than the reasonable royalties, though damages based on royalties will deter under complete information). This approach was commonly used before the 1946 patent act revision.

7.3 Weak Property Rights

The reference levels used for damages assessment are further complicated as one moves away from a binary view of infringement to a view in which firms choose from a continuum of product or process choices that have some level of associated infringement risk. Incorporation of weak property rights in our model provides some insight into this more complex problem. Proposition 3 shows that the strength of property rights impacts the attractiveness of passive versus aggressive infringement in a natural way. An interesting problem that we did not address is whether passive infringement endogenously affects a patent holder's awareness of infringement since sales levels will not suggest infringement.¹⁸ Also, with weak property rights, the innovator has a stronger incentive to choose secrecy over patenting in order to deny a competitor useful cost-reducing knowledge.¹⁹ In fact, from a "weak property rights" perspective, so-called infringement is not unambiguously bad since it isn't at all obvious that the patent holder should have exclusivity with respect to the patented technology. In this case overdeterrence of infringing innovation becomes a more serious concern.

In brief summary, the inherent complexity of market interactions when intertwined with legal sanctions does not mean that one should necessarily throw up one's hands and declare a reasonable royalty. It should, however, force an equilibrium analysis of the underlying structure of the competitive interaction.

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¹⁸Crampes and Langinier (2002) address monitoring investments regarding infringement.

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Appendix

The proofs for Lemmas 1 and 2 and for Proposition 3 are straightforward and therefore omitted.

Proof of Lemma 3: For $q_i \in (q_A, q_B)$, the payoff to j is given by

$$\Pi_j = \begin{cases} [P(q_i + q_j) - s] q_j & \text{as } q_j \leq Q_j(q_i). \\ [P(q_i + q_j)(q_j + \gamma q_i) - s q_j - \gamma c q_i - \gamma \pi_i^{\mathcal{N}}] & \end{cases}$$

This is a strictly concave objective in q_j but it has a kink at $q_j = Q_j(q_i)$. Calculating, we find

$$\frac{\partial \Pi_j}{\partial q_j} = \begin{cases} \alpha - s - \beta q_i - 2\beta q_j & \text{as } q_j \leq Q_j(q_i). \\ \alpha - s - \beta(1 + \gamma)q_i - 2\beta q_j & \end{cases}$$

By continuity and strict concavity, we know a unique solution exists. Since $\partial \Pi_j / \partial q_j|_{q_j=0} = \alpha - s - \beta q_i \leq 0 \Leftrightarrow q_i \geq (\alpha - s) / \beta$, we see that $q_j = 0$ is the solution for this case. Otherwise, in cases (i) and (iii), we see that first-order condition holds with equality, $\partial \Pi_j / \partial q_j = 0$, at the specified q_j values. In case (ii), only the kink value of $q_j = Q_j(q_i)$ satisfies the first-order condition. ■

Proof of Propositions 1 and 2: We establish Propositions 1 and 2 via a sequence of claims. First, we claim that for any cost pair (c, s) and any $\gamma \in (0, 1)$, there is no equilibrium in which j chooses to produce on ϕ_j . Recall from Lemma 3 (i) that ϕ_j is potentially a best response for j when $q_i \in (q_A, q_B)$. We know from Lemma 1 that the best response for i is always on ϕ_i . Now, recall the reference outcome \mathcal{N} , specified in (4) and (5). In \mathcal{N} , i follows the best response ϕ_i while j follows the best response of $\phi_j^{\mathcal{N}} \equiv \max\{0, (\alpha - 2\bar{c} - \beta q_i) / (2\beta)\}$; the equilibrium outcome for \mathcal{N} has i producing $q_i^{\mathcal{N}}$, and this output level also satisfies $q_i^{\mathcal{N}} = \operatorname{argmax} Q_j(q_i)$. Noting that $\phi_j > \phi_j^{\mathcal{N}}$, because $s < \bar{c}$, we see that the intersection of ϕ_i and ϕ_j occurs at a quantity \hat{q}_i that is strictly below $q_i^{\mathcal{N}}$; the corresponding quantity \hat{q}_j for j is strictly above $Q_j(\hat{q}_i)$. But this implies lost profits are positive at (\hat{q}_i, \hat{q}_j) and this implies that $\phi_j(q_i)$ is not the best response choice for j . Formally, we have $\hat{q}_i < q_i^{\mathcal{N}}$ and, by Lemma 2 (iv), this implies $\hat{q}_i + 2Q_j(\hat{q}_i) < q_i^{\mathcal{N}} + 2Q_j(q_i^{\mathcal{N}}) = (\alpha - \bar{c}) / \beta < (\alpha - s) / \beta$, where the final two steps follow from simplifying and observing that $s < \bar{c}$. Thus, Lemma 3 (i) cannot apply at \hat{q}_i .

Next, we claim there is no equilibrium in which $q_i > q_i^{\mathcal{N}}$. Suppose, by way of contradiction, that such an equilibrium does exist. Denote it by (\hat{q}_i, \hat{q}_j) . Then j must be at a best response to \hat{q}_i and this must be in the set $\{Q_j(\hat{q}_i), \phi_j(\hat{q}_i), \varphi_j(\hat{q}_i)\}$, by Lemma 3. We can rule out any $\hat{q}_i \geq (\alpha - c) / (2\beta)$ since the best response of i to any q_j is always below this level (Lemma 1). Then, in this equilibrium, we must have $(\alpha - c) / (2\beta) \geq \hat{q}_i > q_i^{\mathcal{N}}$. We can now rule out Q_j as a best response for j : ϕ_i , the best response for i intersects Q_j only one time and this occurs at $q_i^{\mathcal{N}}$. We can rule out ϕ_j for j from the

first claim above. This leaves only φ_j . Now, since ϕ_i lies strictly below Q_j for $\hat{q}_i \in (q_i^N, (\alpha - c) / (2\beta)]$, we see that $\hat{q}_j = \phi_i^{-1}(\hat{q}_i) < Q_j(\hat{q}_i)$ and $D = 0$ holds. By Lemma 3, j is not at a best response with φ_j when $D = 0$. This establishes the second claim.

We now prove Proposition 1. First, note that j will not play Q_j in any equilibrium: from $\varphi_j(q_i^N) > Q_j(q_i^N) = q_j^N > 0$, we have $(\alpha - s) / [\beta(1 + \gamma)] > q_i^N$ by definition of φ_j . We know from the text that j plays φ_j when $q_i \leq q_A$. When $q_A < q_i \leq q_i^N$, we have $(\alpha - s) / \beta > (\alpha - s) / [\beta(1 + \gamma)] > q_i^N \geq q_i$. Thus, consider which case of Lemma 3 applies: $\varphi_j(q_i^N) > q_j^N \Leftrightarrow$

$$\frac{1}{2\beta} [\alpha - s - \beta(1 + \gamma)q_i^N] > Q_j(q_i^N) \Leftrightarrow (\alpha - s) / \beta > (1 + \gamma)q_i^N + 2Q_j(q_i^N).$$

Thus, Lemma 3 (iii) applies at $q_i = q_i^N$. By Lemma 2 (iv), $(1 + \gamma)q_i + 2Q_j(q_i)$ is strictly increasing in q_i over $[q_A, q_i^N]$; therefore, Lemma 3 (iii) applies over this entire interval and j plays φ_j as a best response. Thus, we have shown that i must play ϕ_i , j must play φ_j , and any equilibrium must have $q_i \leq q_i^N$. It is then routine algebra to solve for the (unique) intersection, yielding $q_i^* = (\alpha - 2c + s) / [\beta(3 - \gamma)]$ and $q_j^* = [\alpha(1 - \gamma) - 2s + (1 + \gamma)c] / [\beta(3 - \gamma)]$, and verify directly that $q_i^* \in (0, q_i^N)$ and $q_j^* > q_j^N$.

We now prove Proposition 2. From $\varphi_j(q_i^N) < Q_j(q_i^N) = q_j^N$, we find that j cannot play φ_j in any equilibrium since the (unique) intersection of ϕ_i and φ_j now occurs at a q_i above q_i^N . Thus, j must play Q_j in any equilibrium. Thus, it only remains to verify that (q_i^N, q_j^N) are best responses. This is trivial for i since $\phi_i(q_j^N) = q_i^N$. For j , we need to verify that Lemma 3 (ii) applies. Note that $q_i^N \in (q_A, q_B)$ is clearly valid. Next, we must show $(\alpha - s) / \beta > q_i^N$. This reduces to $[2(\alpha + c) - \bar{c}] / 3 > s$; this is valid since the left-hand side is increasing in c and positive at $c = 0$, by $\alpha > 2\bar{c}$. Finally, we must verify that $q_i^N + 2Q_j(q_i^N) \leq (\alpha - s) / \beta \leq (1 + \gamma)q_i^N + 2Q_j(q_i^N)$. The inequality on the right is implied directly by $\varphi_j(q_i^N) < q_j^N$. Substituting directly for q_i^N from (5) and evaluating with Q_j via (7), this reduces to $s < \bar{c}$. Hence, j is at a best response with $Q_j(q_i^N) = q_j^N$. ■

Proof of Proposition 4: A choice of no-imitation by j leads to profits of π_j^N for j . There are two cases. First, when $\varphi_j(q_i^N) \leq Q_j(q_i^N)$ and, by Proposition 2, the equilibrium involves the market quantities from the reference outcome \mathcal{N} , we have equilibrium profits for j of

$$\begin{aligned} \Pi_j^* &= (P^N - s)q_j^N - \gamma \text{Max}\{\pi_i^N - (P^N - c)q_i^N, 0\} \\ &= (P^N - s)q_j^N \\ &= \pi_j^N + (\bar{c} - s)q_j^N, \end{aligned}$$

and a choice of \mathcal{I} by j is optimal. The second case is that of $\varphi_j(q_i^N) > Q_j(q_i^N)$ where, by Proposition 1, the equilibrium is at (q_i^*, q_j^*) . We must show that $\Pi_j^* = \Pi_j(q_j^*, q_i^*) > \pi_j^N$. Referring back

to Figure 3, note that j 's payoff is Π_j^* at point A. At point B, we have $\Pi_j(q_j^N, q_i^N) = (P^N - s)q_j^N = \pi_j^N + (\bar{c} - s)q_j^N > \pi_j^N$. Comparing to j 's payoff at point C, we have $\Pi_j(\varphi_j(q_i^N), q_i^N) > \Pi_j(q_j^N, q_i^N)$, since $\varphi_j(q_i^N) > q_j^N$ is the unique best-response for j to q_i^N . Hence, j 's payoff at point C exceeds that at point B. To complete the argument, we compare payoffs at points A and C. Index i 's output between A and C by $x \in [q_i^*, q_i^N]$ and consider j 's payoff of $\Pi_j(\varphi_j(x), x)$ as we move along φ_j between points A and C. Then

$$\begin{aligned} \frac{d}{dx}\Pi_j(\varphi_j(x), x) &= \left(\frac{\partial \Pi_j}{\partial q_j} \cdot \varphi_j'(x) + \frac{\partial \Pi_j}{\partial q_i} \right) \Big|_{(\varphi_j(x), x)} \\ &= \frac{\partial \Pi_j}{\partial q_i} \Big|_{(\varphi_j(x), x)} \\ &= \gamma(\alpha - c - 2\beta x) - \beta(1 + \gamma)\varphi_j(x), \end{aligned}$$

where the first step follows from the envelope theorem (j is at a best response) and the second by direct calculation. Note that

$$\frac{d^2}{dx^2}\Pi_j(\varphi_j(x), x) = \beta \left[(1 + \gamma)^2 / 2 - 2\gamma \right] > 0,$$

which follows from the right-hand-side being strictly decreasing in γ and equaling zero at $\gamma = 1$. Hence, by convexity,

$$\begin{aligned} \frac{d}{dx}\Pi_j(\varphi_j(x), x) &\leq \frac{d}{dx}\Pi_j(\varphi_j(x), x) \Big|_{(\varphi_j(q_i^N), q_i^N)} \\ &= \gamma(\alpha - c - 2\beta q_i^N) - \beta(1 + \gamma)\varphi_j(q_i^N) \\ &= \beta[\gamma q_j^N - (1 + \gamma)\varphi_j(q_i^N)] \\ &< 0. \end{aligned}$$

Thus, $\Pi_j(\varphi_j(x), x)$ is strictly decreasing for $x \in [q_i^*, q_i^N]$ and we have shown j 's profit at A exceeds that at C. Combining the comparison for A, B and C, we are done. ■

Proof of Proposition 5: Assume that h satisfies $h' > 0$, $h'' < 0$, $h(0) = 0 = \lim_{x \rightarrow \infty} h'(x)$, and that $h'(0)$ is sufficiently large to rule out an R&D level of zero. Assume that $V_i > \bar{\pi}/r > V_j$ and that $V_i + V_j > 2\bar{\pi}/r$; these assumptions are satisfied for the \mathcal{N} benchmark. Then, the best response for firm 1 (symmetrically for 2) to x_2 is unique, positive, and satisfies the first-order condition:

$$h'(x_1)[h(x_2)(V_i - V_j) + rV_i - \bar{\pi} + x_1] - [r + h(x_1) + h(x_2)] = 0;$$

let $x_1 = \Phi(x_2)$ denote the best-response. It is easy to verify that $\Phi' > 0$; further, an increase in V_j shifts Φ down, so that a larger failure payoff unambiguously reduces the incentive to invest in

R&D. From the first-order condition, it is straightforward to show that an equilibrium exists and all equilibria are symmetric. Thus, any equilibrium x^* must satisfy

$$F(x, V_j) \equiv h'(x) [h(x) (V_i - V_j) + rV_i - \bar{\pi} + x] - [r + 2h(x)] = 0.$$

Under the added assumption on h (see Tirole [1988, p. 416]) that

$$h''(x) [h'(x) (V_i - V_j) + rV_i - \bar{\pi} + x] + h'(x) [-1 + h'(x) (V_i - V_j)] < 0,$$

the equilibrium is unique. It is then straightforward to calculate that $\frac{dx^*}{dV_j}$ and $\frac{\partial F}{\partial V_j}$ have the same sign. Since $\frac{\partial F}{\partial V_j} = -h(x^*)h'(x^*) < 0$, we are done. ■