Regulation, Local Monopolies and Spatial Competition

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Abstract

Many regulated industries involve imperfect competition or an oligopoly market structure. In this paper we examine optimal incentive regulation for a duopoly model of spatial competition when firms have private cost information. Market structure is endogenous in this setting as regulation determines market segments for firms and output distribution across consumers in each firm's market. By varying the assignment of consumers in the fringe area between market segments, the optimal policy provides incentives by rewarding a relatively more efficient firm with a larger market, thus reducing the need to rely on incentive provision via the familiar monopoly quantity distortion within a market segment. We derive the optimal policy and assess the impact of asymmetric information relative to full information. We also examine extensions to allow for several ex ante asymmetries in firm structure.

Keywords: Incentive regulation, endogenous market structure, spatial competition

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1 Introduction

In many regulated markets we observe an imperfectly competitive or oligopolistic market structure in which firms sell products differentiated along dimensions such as location or quality. Often, these firms have local monopoly power over an identifiable "close" segment of the market and competition is concentrated on consumers who are located in "fringe" segments of the market. Local monopoly power is identified with market segments for which the products of other firms are a poor substitute while competition between firms takes place in fringe segments where no firm has a significant advantage.

Markets with these features can be found in several industries. To take a prominent example, health care involves regulated markets where quality is an important source of differentiation, leading to specialization and brand loyalty. Providers have an advantage in their specialty and with their loyal or current patients, but compete at the fringe of their specialty and for new patients. Strategic market assignment is a central issue in the debate on national health care market reform in the U.S., with California's selective contracting program having been widely emulated and often cited as a model for the country as a whole (Bamezai et al. [2003]). The California program allows the State Medicaid program as well as private insurers to negotiate price-quantity schedules with hospitals and others providers. They still contract, however, with relatively less efficient providers for beneficiaries who have no effective alternative. Selective contracting has been credited with making the rate of medical care inflation in California among the lowest in the U.S. (Zwanziger and Melnick [1988]).[1] Telecommunications markets provide further examples as, for instance, with cable and satellite television systems.[2]

These examples raise a number of questions for incentives and regulation. First, how should policy be oriented with regard to the allocation or movement of consumers between firms, and what are the

[1]This has generally been confirmed in subsequent studies; see Bamerzai et al. [2003] for references; see also Lyon [1997] for analysis of a spatial model of health care quality and Ma and McGuire [2002] for an analysis of managed care and networks of providers.

[2]The technology of a cable network may provide a significant cost advantage to a company where their network is dense; satellite networks, however, may be at an advantage for remote or less accessible locations. Of course, there may also be horizontal differentiation with respect to buyer preferences arising from programming differences (e.g., Direct TV has an exclusive license to broadcast a package of National Football League games). As discussed by Hazlett and Spitzer [1997], we also observe direct entry into cable TV markets by other cable suppliers - an "overbuild" - in locations such as San Diego, CA and Dade County (Miami), FL where it is allowed by regulation.

benefits associated with allowing or inducing competition to attract such consumers? Further, how should changes in market shares associated with the movement of consumers at the "fringe" be allowed to impact the other consumers who remain with an expanding or contracting firm? Finally, expansions and contractions in market share lead directly to questions about underlying cost structure (e.g., natural monopoly) and efficiency when regulation involves multiple suppliers. As applied to health care, these questions relate to managed care and selective contracting as policy seeks to allocate consumers to various providers and determine the terms under which patients are to be served.

In this paper, we examine the potential benefits under optimal incentive regulation from the strategic assignment of consumers in a simple Hotelling model of horizontal differentiation. The model involves a duopoly setting in which firms with market power have private cost information. With regard to policy instruments, the regulator assigns market segments, which are continuously variable in the spatial dimension, as well as prices and quantities for individual consumers. At the level of the individual consumer, we allow for variable demand so that consumption may vary across consumers.[3] We also allow for a fairly general structure of production costs and assess the role of constant, increasing and decreasing returns. Firms distribute output to consumers and costs rise with distance from the firm's location (equivalently, value falls as the horizontal quality match deteriorates); thus, each firm necessarily has a cost advantage with nearby consumers (local monopoly).

We focus on the structure of optimal incentive regulation regarding the trade-off between assignment of market segments and the allocation of production (via prices) across consumers in each firm's market segment. As a result, regulation determines the size of each firm's market and the intensity with which each market is served. Note that the allocation of production in a given market segment corresponds to a spatial extension of the Baron and Myerson [1982] framework for monopoly regulation.[4] With a spatial dimension for consumers and more than one firm, the focus of regulatory policy

[3]By contrast, when individual consumers have inelastic demands (e.g., for a unit of the good), aggregate demand over the spatial market is a constant. Variations in market assignment then shift these consumers from one firm to the other, directly determining the required change in production levels. In this case, market segments and production allocations cease to be distinct policy instruments.

[4]Absent multiple firms and potential competition, the optimal incentive policy for a single firm serving a fixed market results in a second best allocation in which price is distorted upwards and quantity downwards for individual consumers, reflecting the cost of providing incentives under incomplete information. For a recent survey and references to this literature, see Armstrong and Sappington [2002].

shifts. Now the regulator can provide an additional incentive by awarding part of a less efficient firm's market to a relatively more efficient rival, reducing the need to rely on price and quantity distortions within each market segment. Consumers who are reassigned can then benefit from being in the market of the more efficient firm while those who remain with their current firm face a smaller incentive distortion in prices.

A brief summary of the optimal policy runs as follows. Because distribution is costly, it is always optimal to award distinct market segments and, further, consumption varies with consumer location. Optimal control methods then provide a price schedule that implements the efficient distribution of output within each market segment. With regard to incentives, market size and production are reduced as a firm becomes less efficient. The other firm is awarded a larger market by reassigning consumers near the boundary between market segments (the competitive fringe) and produces more in aggregate. The reassignment of consumers also creates an externality for consumers who stay with their current firm. These welfare effects are determined by scale economies; under increasing returns, for instance, prior customers of an expanding firm will pay a lower price and consume more.

The overall impact of the distortions created by asymmetric cost information on the optimal policy for regulated market structure can be assessed via a comparison with the full information (first best) case. For the benchmark case of constant returns in production, we find that asymmetric information makes market size and production more sensitive to efficiency differences between the firms. That is, the relatively less efficient firm receives a smaller market share and produces less as compared to full information outcomes. The situation is more subtle when scale economies are present, however, and we identify several competing effects that can modify the impact of asymmetric information. Finally, in practice, we can often identify ex ante asymmetries among competing firms (e.g., "wireless" in telecommunications). While the primary analysis deals with ex ante symmetric suppliers, we extend the model in several directions to examine ex ante asymmetries in distribution costs, the degree of asymmetric cost information, and the location of firms.

Several papers in the literature on incentives and regulation address issues that arise in imperfectly competitive and oligopolistic industries, including that of endogenous market structure, our main concern in this paper.[5] In particular, McGuire and Riordan [1995] and Wolinsky [1997] also utilize

[5] Also related are the problems of regulated firms who face entry or competition from a (strategic) unregulated firm, as in Biglaiser and Ma [1995], and regulated firms who participate in outside markets, as in Anton and Gertler [1988]. In some settings, regulatory constraints, such as price caps, are imposed on an incumbent across markets, with the firm

a spatial framework to examine regulation and market structure. McGuire and Riordan employ the Laffont and Tirole [1986] framework, in which firms have private cost information and can devote effort to cost reduction, and analyze the optimal regulatory choice of a sole source (monopoly) versus dual source (duopoly, via equal division) market structure. Wolinsky examines a spatial model with private cost information and also introduces a vertical dimension to quality; the quality level, however, is chosen by the firms and can only be influenced indirectly by regulation. Taking this influence into account, Wolinsky examines the efficacy of different regulatory regimes, including the assignment of market shares. In both of these papers, individual consumers have inelastic (unit) demands for the good. Thus, variations in market shares reallocate buyers from one firm to another but there are no changes in consumption levels at the individual or aggregate level. Our analysis is then complementary as we focus on the independent trade-off between market assignment and production allocation when individual consumer demand is elastic.[6]

We present the model in Section 2 and develop the underlying economic structure of the regulatory problem in Section 3. In Section 4 we show how market segments arise and derive a measure of regulatory benefits based on an optimal distribution of each firm's aggregate output to consumers over assigned markets. The optimal policy is derived and discussed in Section 5, and the effect of asymmetric cost information on the policy is analyzed in Section 6. In Section 7 we extend the analysis to examine how optimal policy is adjusted in the presence of various ex ante asymmetries across firms. Section 8 concludes.

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operating as a monopolist in some and facing competition in others; see Armstrong and Vickers [1993], Anton, Vander Weide and Vettas [2002], and Valletti, Hoernig and Barros [2002]. Finally, endogenous market structure issues also arise with the provision of incentives to multiple potential suppliers, as in Riordan [1996], or internal units, as in Kerschbamer and Tournas [2003].

[6] As the discussion above indicates, the models also have other important differences with respect to the assumed economic structure. Several other studies examine market assignment or production quantity dimensions under incomplete information. With respect to quantity, Anton and Yao [1992] develop a model of split award auctions in procurement. Auriol and Laffont [1992] study optimal auctions with variable quantity for several information settings. Dana and Spier [1994] examine mechanism design and market structure with the discrete award choices of a monopoly, duopoly or government producer. See also the analysis of price rationing in Gilbert and Klemperer [2000].

2 The Model

Consider regulating a market with two profit-maximizing firms and the familiar Hotelling structure of horizontal differentiation. Let the set of consumers be distributed uniformly over the unit interval, $[0,1]$, and suppose that firm 0 is located at $t=0$ and firm 1 at $t=1$; consumers are identical up to their location, denoted by $t \in [0,1]$. We begin with the basic structure and objectives for the consumers, firms and regulator and then specify the information structure and regulatory framework.

2.1 Demand and Cost Structure

The inverse demand function of an individual consumer, $P(q)$, is smooth and strictly decreasing. Individual consumers surplus is then given by

$$S(q) = V(q) - qP(q), \tag{1}$$

where $V(q) \equiv \int_0^q P(x)dx$. Assume that $P(0)=\infty$, $P(\infty)=0$, and $\lim_{q\to\infty} V(q) < \infty$; these assumptions rule out corner solutions at the level of individual demand and also imply that consumers surplus aggregated over all consumers is well defined.

Production and distribution are both costly activities for the firms. Suppose each consumer $t \in [0,1]$ is to consume a quantity $q_i(t)$ from firm $i = 0,1$. Then, aggregate quantity is $Q_i = \int_0^1 q_i(t)dt$ and production costs are specified as

$$C_i = \theta_i C(Q_i), \tag{2}$$

where θ_i is a firm-specific cost parameter and $C(Q_i)$ is smooth and strictly increasing. Distribution costs for firm i, D_i, depend on how output varies across consumers. Suppose δtq is the cost of distributing q units of output to a consumer who is a distance t from the location of the firm; δ is thus the (constant) marginal cost of distributing a unit of output. If firm i distributes according to $q_i(t)$, aggregate distribution costs are

$$D_i = \delta \int_0^1 |t-i|\, q_i(t)dt, \tag{3}$$

where $|t-i|$ simply indexes the distance from each firm to a consumer at t.

As an example, suppose each consumer is served by only one firm and that all consumers receive an identical quantity of $\bar{q}$. Let $q_0(t)=\bar{q}$ for $t \in [0,B]$ and $q_0(t)=0$ for $t \in (B,1]$ where $B \in (0,1)$ denotes the "boundary" between the firms. Then, firm 0 has a "market" of size B, with aggregate production of $Q_0 = \bar{q}B$. From (2), production costs are $C_0 = \theta_0 C(\bar{q}B)$. From (3), $D_0 = \delta \int_0^B t\bar{q}dt = (\delta/2)\bar{q}B^2$ and distribution costs are convex in the size of the "market" for firm 0 (for firm 1, we replace B with $1 - B$).

An individual consumer who buys q_i generates a revenue of $q_i P(q_0 + q_1)$ for firm i. Aggregating over distribution schedules of $q_i(t)$ for $i = 0, 1$, profits for firm i are defined as

$$\Pi_i = \int_0^1 q_i(t)P(q_0(t) + q_1(t))dt - D_i - C_i - T_i. \tag{4}$$

The first term is aggregate revenue for i from all potential consumers, while C_i and D_i are from (2) and (3). T_i allows for a lump-sum tax imposed by the government (transfer, if $T_i < 0$). We assume that the regulatory objective is the sum of aggregate consumers surplus and tax revenue:

$$W = \int_0^1 S(q(t))dt + T_0 + T_1, \tag{5}$$

where $q \equiv q_0 + q_1$.[7]

2.2 Information Structure and Regulatory Framework

The regulatory instruments are the quantity schedules for each firm, $q_i(t)$, and lump-sum taxes, T_i. Prices for each consumer t are set via demand at $P(q_0(t) + q_1(t))$, and may vary with consumer locations. We focus on the adverse selection problem that arises when each firm has private cost information and assume that the value of θ_i, the firm-specific parameter in production costs, is private information of firm i. The value of θ_i is drawn from a distribution F, independent across i, with a positive and differentiable density f and a support of $[\underline{\theta}, \bar{\theta}]$. We make the standard assumption that the inverse hazard, $F(\theta)/f(\theta)$, is increasing in θ. In the model, the only elements of private information are the values of θ_i.

[7]For simplicity, we specify the objective with implicit weights of zero on firm surplus (profits) and one on lump sum transfers. Thus, relative to a weighted average of consumer and firm surplus, the regulator only values rent extraction from the firms, and tax distortions are absent with regard to lump sum payments. As discussed in Laffont and Tirole [p. 156, 1993], one can incorporate these extensions in (5) above and proceed with the same techniques employed in Section 3 below. Finally, since consumer surplus is linear in expenditure (quasi-linear preferences), the lump sum terms may also be used to achieve redistribution across consumers.

As is standard (the Revelation Principle applies to this problem), we study optimal policy via a direct mechanism in which the government solicits a cost report from each firm. A policy, Ω, is defined as a mapping from cost reports, (r_1, r_2), to policy instruments: $\Omega = \{q_i(t, r_i, r_j), T_i(r_i, r_j)\}$, for $i = 0, 1$ and $j \neq i$, where $t \in [0, 1]$, $(r_i, r_j) \in [\underline{\theta}, \bar{\theta}]^2$ and quantities are non-negative. To preserve symmetry, it is convenient to adopt the convention that the first cost-report argument in q_i and T_i is the report from firm i while the second is that from j. Regulation unfolds in the standard order: first, the government specifies a policy, Ω; next, the firms simultaneously submit cost reports, (r_i, r_j); finally, quantity schedules, lump-sum taxes and prices (via demand) are determined by evaluating the policy at the submitted cost reports.

We examine Bayesian implementation. By definition, then, a policy is incentive compatible if it is a Bayesian equilibrium for each firm to report the privately observed value of the cost parameter. Thus, a strategy of reporting $r_i = \theta_i$ is optimal for i when j is expected to report $r_j = \theta_j$. If firm i observes θ_i but reports r_i while firm j observes θ_j but reports r_j, then the mechanism selects quantity schedules and taxes for each firm based on r_i and r_j. Define revenues (net of taxes), production costs and distribution costs under Ω at these reports, respectively, by

$$R_i(r_i, r_j) = \int_0^1 q_i(t, r_i, r_j) P\left(q_i(t, r_i, r_j) + q_j(t, r_j, r_i)\right) dt - T_i(r_i, r_j), \tag{6}$$

$$C_i(\theta_i, r_i, r_j) = \theta_i C(Q_i(r_i, r_j)), \text{ where } Q_i(r_i, r_j) \equiv \int_0^1 q_i(t, r_i, r_j) dt, \tag{7}$$

$$D_i(r_i, r_j) = \delta \int_0^1 |t - i|\, q_i(t, r_i, r_j) dt \tag{8}$$

If j follows the strategy of reporting $r_j = \theta_j$, the expected profit to i from reporting r_i when θ_i is observed is

$$\pi_i(r_i \mid \theta_i) = \int_{\underline{\theta}}^{\bar{\theta}} \{R_i(r_i, \theta_j) - D_i(r_i, \theta_j) - C_i(\theta_i, r_i, \theta_j)\} dF(\theta_j). \tag{9}$$

For Ω to be incentive compatible, reporting $r_i = \theta_i$ must be optimal for i. With $\Pi_i(\theta) \equiv \pi_i(\theta \mid \theta)$, the condition for incentive compatibility (IC) is then (suppressing subscripts on r_i and θ_i)

$$\Pi_i(\theta) \geq \pi_i(r \mid \theta) \;\; \forall (r, \theta) \text{ and for } i = 0, 1. \tag{10}$$

Each firm is willing to participate in the regulated market if it earns a non-negative expected profit when the observed θ value is reported. Thus, individual rationality (IR) requires

$$\Pi_i(\theta) \geq 0 \;\; \forall \theta \text{ and for } i = 0, 1. \tag{11}$$

The regulatory objective in (5) is now given by (substituting with R_i from (6))

$$W(\Omega) \equiv E\left\{\int_0^1 V(q(t, \theta_0, \theta_1))dt - R_0(\theta_0, \theta_1) - R_1(\theta_1, \theta_0)\right\}, \tag{12}$$

where E denotes the expectation with respect to F over θ_0 and θ_1, and $q \equiv q_0 + q_1$. In (12), aggregate consumers surplus now accounts for policy choice under asymmetric information by incorporating the cost-report arguments and taking expectations over cost uncertainty. The problem of the regulator, (RP), is then given by $\max_{\Omega} W(\Omega)$ subject to (10) and (11), the incentive compatibility and individual rationality conditions, respectively.

3 Feasible Policies

In this section, we examine the conditions for incentive compatibility and individual rationality. Our treatment is brief since the results follow by adapting familiar incentive arguments to the problem at hand. We then employ the results to simplify the main problem of (RP) and discuss the economic structure of the problem.

We can characterize incentives as follows:

Lemma 1 *A policy Ω satisfies (10) and (11) if and only if*

(i) $\bar{c}_i(\theta) \equiv \int_{\underline{\theta}}^{\bar{\theta}} C(Q_i(\theta, \theta_j))dF(\theta_j)$ is non-increasing in θ;

(ii) $\Pi_i(\theta) = \Pi_i(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} \bar{c}_i(\theta')d\theta'$;

(iii) $\Pi_i(\bar{\theta}) \geq 0$.

(All proofs are in the Appendix.) The incentive structure in Lemma 1 reflects basic considerations in multi-agent adverse-selection mechanisms (e.g., Maskin and Riley [1989] and Mookherjee and Reichelstein [1992]). The function $\bar{c}_i(\theta)$ measures production cost, on average over the other firm's type, and this determines the gain or loss to reporting $r \neq \theta$ since $\pi_i(r \mid \theta) = \Pi_i(r) + (r - \theta)\bar{c}_i(r)$, as follows from the profit definitions. Were $\bar{c}_i(\theta)$ to be increasing, some type could gain by misreporting. For intuition, note that (i) necessarily holds if more efficient firms are rewarded with greater aggregate quantities, as when $Q_i(\theta, \theta_j)$ is decreasing in θ. Condition (ii) reveals that type θ earns a profit equal to the profit of the least efficient type ($\bar{\theta}$) plus an information rent (the integral). More efficient types earn higher profits since the integral is decreasing in θ; through $\bar{c}_i(\theta)$, the aggregate production of less efficient cost types (above θ) determines the magnitude of this rent. Finally, condition (iii) implies non-negative profits for the least efficient firm ($\bar{\theta}$); all other types then earn non-negative profits as $\Pi_i(\theta)$ is non-increasing.

We employ Lemma 1 to simplify the objective in (12) and the (RP) problem. Substituting for the revenue and tax terms with the accounting definition of profits, and using condition (ii) in Lemma 1 for profits, we obtain

Lemma 2 *Suppose (10) and (11) hold for Ω. Then, for $\rho(\theta) \equiv \theta + F(\theta)/f(\theta)$,*

$$\begin{aligned} W(\Omega) &= E\left\{\int_0^1 [V(q_0(t)+q_1(t)) - \delta t q_0(t) - \delta(1-t)q_1(t)]dt\right\} \\ &\quad -E\left\{\rho(\theta_0)C(Q_0)+\rho(\theta_1)C(Q_1)\right\} - \Pi_0(\bar{\theta}) - \Pi_1(\bar{\theta}). \end{aligned} \tag{13}$$

(The dependence of q_i and Q_i on the cost reports, θ_i and θ_j, has been suppressed.) Incorporating these results, the problem of the regulator (RP) reduces to

$$\max_{\{q_i(t)\}} W(\Omega) \qquad \text{subject to } \bar{c}_i(\theta) \text{ non-increasing and } \Pi_i(\bar{\theta}) \geq 0, \text{ for } i = 0, 1. \tag{14}$$

From Lemma 1, we know (10) and (11) will hold at a solution to (14).

The expression for $W(\Omega)$ has three components. The first term is the valuation of quantity net of distribution costs at the level of an individual consumer, aggregated over all consumers. The second term involves production costs for each firm and these depend on cost types and aggregate production levels. Notice that the weight on the cost function is now $\rho(\theta_i)$, which exceeds θ_i by the inverse hazard rate, reflecting the cost of providing each firm with an incentive to reveal private cost information. Finally, $\Pi_i(\bar{\theta})$ reflects the individual rationality constraint; clearly, any solution to (14) has $\Pi_i(\bar{\theta}) = 0$ and these terms will vanish in (13).

The structure of the two remaining components in (13) suggests a sequential approach for solving (RP). Focusing on the first terms in $W(\Omega)$, we see that the the cost parameters, (θ_0, θ_1), and aggregate production levels, (Q_0, Q_1), do not directly impact consumer valuations net of distribution costs. Initially, we can take them as given and solve for quantity schedules, $q_i(t)$, that maximize this component, incorporating the constraint of $Q_i = \int_0^1 q_i(t)dt$. Thus, the first step is to solve the problem of "distributing" given aggregate quantities optimally across consumers. This is analyzed below in Section 4.

The second step, analyzed in Section 5, incorporates the valuation of the aggregate quantities from the distribution problem and solves for the optimal values of the aggregates. Note that private cost information enters directly into the second component of $W(\Omega)$ in (13) via the terms $\rho(\theta_i)C(Q_i)$. Solving (RP) via two steps helps to identify the economic structure of the problem. The distribution problem leads to a price schedule over consumers, showing how given aggregates are optimally distributed and also how a market for each firm is established; this provides a measure of marginal benefit. The second step incorporates this measure and shows how marginal benefits and costs determine aggregate quantity and market size for each firm.

4 The Distribution Problem

Consider the optimal distribution of given aggregates, Q_0 and Q_1. This is given by

$$\max_{\{q_i(t)\}} \quad \int_0^1 \left[V(q_0(t)+q_i(t)) - \delta t q_0(t) - \delta(1-t)q_1(t)\right] dt, \tag{15}$$

subject to $Q_i = \int_0^1 q_i(t)dt$, for $i = 0, 1$. Recalling that the cost of distribution rises with the distance from the production location, we expect that any overlap in the quantity schedules will be inefficient: the added distribution cost generated by an overlap can be avoided by "reassigning" consumers so that only one firm supplies any individual consumer. Further, such a reassignment can be done without altering the aggregates, Q_0 and Q_1, or individual consumption levels, $q_0(t) + q_1(t)$, from their original values. Formally, we have

Lemma 3 *In any solution to (15), there is a boundary $B \in [0, 1]$ dividing the firms, such that $q_0(t) > 0$ only if $t \leq B$ and $q_1(t) > 0$ only if $t > B$.*

An optimal distribution thus establishes two "markets." Firm 0 serves consumers in $[0, B]$ while firm 1 serves those in $(B, 1]$. Now, fix a boundary and consider the distribution problem for each market. For a market of size $b > 0$ and aggregate output of $Q > 0$, this problem is given by

$$\max_{\{q(t)\}} \quad \int_0^b [V(q(t)) - \delta t q(t)]dt \qquad \text{subject to } Q = \int_0^b q(t)dt. \tag{16}$$

For firm 0, this corresponds to $b = B$ and $Q = Q_0$. For firm 1, set $b = 1 - B$ and $Q = Q_1$ and relabel consumers in the market of firm 1, $t \in (B, 1]$, with $t' \in [0, 1 - B)$. This problem is most usefully solved via optimal control, as the resulting multipliers identify the structure of prices for the optimal quantity schedule.

For a quantity schedule $q(t)$, let $p(t)$ denote the price to a consumer at location t. Recalling that $P(q)$ is the inverse demand of an individual consumer, the corresponding demand function is $v(p) \equiv P^{-1}[p]$ and, therefore, $v(p(t)) = q(t)$ relates the price and quantity schedules. Rewriting the aggregate quantity constraint in the form $Q = \int_0^b v(p(t))dt$, we have

Lemma 4 *Suppose that $Q < \int_0^b v(\delta t)dt$.*[8] *There exists a unique price $\mu = \mu(Q, b)$ such that $Q = \int_0^b v(\mu + \delta t)dt$. Let the price schedule be $p(t) = \mu + \delta t$, for $t \in [0, b]$. Then $q(t) = v((p(t))$ for $t \in [0, B]$ solves the distribution problem (16).*

The price schedule, $p(t)$, implements the distribution of Q over $t \in [0, b]$ in an intuitive fashion. The demand from each consumer t at the price $p(t)$ is then the optimal quantity of $q(t) = v(p(t))$. The price schedule is linear and, as we move away from the production location, prices rise at the rate of δ, reflecting that δ is the (constant) marginal cost of distribution.

The intercept, $\mu(Q, b)$, positions the price schedule so that aggregate demand over the market of size $[0, b]$ is equal to aggregate production. The properties of $\mu(Q, b)$ are easily computed (subscripts denote partial derivatives). As Q increases, we have $\mu_Q = \delta[v(\mu + \delta b) - v(\mu)]^{-1} < 0$ so that the price schedule shifts down in order to stimulate demand for the larger production level. As b increases, we have $\mu_b = -v(\mu + \delta b)\mu_Q > 0$. The price schedule shifts up as market size increases since Q must be spread over a larger market and, consequently, higher prices are needed to induce each individual to buy less.

In this pricing scheme, consumers at different locations are charged different prices. The efficiency rationale, of course, is that it is more costly to serve consumers as distance from the firm increases. Thus, price must rise in order to reduce quantity for these higher marginal cost consumers. If regulatory policy is so oriented, nondistorting lump-sum transfers can then be employed to smooth consumer surplus across individuals. Finally, note that the above pricing scheme does not constitute price discrimination in the traditional sense since the variation of price with location coincides with the marginal cost of distribution (Tirole [1988]).

[8] If $Q \geq \int_0^b v(\delta t)dt$, then $q(t) = v(\delta t)$ solves (16). In this case, a fraction of Q is left undistributed because the value of additional output to any consumer falls below the additional distribution cost. Since output is always costly to produce, values of Q in this range are never optimal.

The pricing scheme in Lemma 4 may not be feasible in some settings. Markets for Cable TV services provide an example as franchise agreements may limit or prohibit differential pricing (see Ch. 4 of Hazlett and Spitzer [1997]). The analysis is easily modified to address the case of a common price. Each consumer purchases $q = Q/b$ at $p = P(q)$ and, clearly, we have $\mu(Q, b) < p < \mu(Q, b) + \delta b$. Thus, distant consumers are undercharged and nearby consumers are overcharged relative to the efficient distribution. This full information distortion associated with a common price is eliminated with the optimal price schedule for the distribution problem.

The value function for (16), the distribution problem, effectively summarizes the above results for later use. Formally, the value of distributing Q optimally over a market of size b is

$$U(Q, b) \equiv \int_0^b [V(v(\mu(Q, b) + \delta t)) - \delta t v(\mu(Q, b) + \delta t)]dt. \tag{17}$$

We have $U_Q = \mu$, revealing that the value of an extra unit of aggregate production is equal to the price at the production location. Further, $U_b = V(q(b)) - q(b)P(q(b))$, so that the value of an increase in market size is given by consumers surplus at the location b, the market boundary. It is routine to verify that $U(Q, b)$ is strictly concave in (Q, b).

Finally, let us organize these results for the next step in solving (RP). For given aggregate production levels (Q_0, Q_1), an optimal distribution to consumers in $[0, 1]$ necessarily involves a boundary, B, with firm 0 serving the market $[0, B]$ and firm 1 serving $(B, 1]$. Taking B as given, we have $U(Q_0, B)$ as the value of the optimal distribution of Q_0 over $[0, B]$ and $U(Q_1, 1 - B)$ as the value for Q_1 over $(B, 1]$. Thus, the value of the optimal distribution over all consumers in $[0, 1]$ depends on Q_0 and Q_1, the aggregate production levels, and on B, the boundary dividing the two markets.

5 Production Aggregates, Market Size and Optimal Policy

We now solve for the production aggregates and market size under an optimal policy. First, we incorporate the distribution results into (13) and (14). Given a boundary B and production levels of Q_0 and Q_1, the value of the optimal distribution over $[0, 1]$ is $U(Q_0, B) + U(Q_1, 1 - B)$. The objective in (13) is then

$$W(\Omega) = E\{U(Q_0, B) + U(Q_1, 1 - B) - \rho(\theta_0)C(Q_0) - \rho(\theta_1)C(Q_1)\}; \tag{18}$$

the problem of (RP) from (14) now reduces to solving for Q_0, Q_1 and B as functions of θ_0 and θ_1 so as to maximize (18), subject to $\bar{c}_i(\theta)$ non-increasing in θ for $i = 0, 1$. We establish existence and uniqueness of an optimal policy first and then proceed with the main issue of analyzing the economic structure and properties of the solution.

5.1 Existence and Uniqueness

To analyze existence, consider pointwise maximization of the objective. Thus, let $w(Q_0, Q_1, B; \theta_0, \theta_1)$ denote the integrand in (18) under the E expectation. Maximizing w at each (θ_0, θ_1) yields production aggregates that satisfy the incentive constraint. Thus, we have

Proposition 1 *For each pair of cost parameters (θ_0, θ_1), there exist production aggregates and a boundary, (Q_0, Q_1, B), that maximize $w(Q_0, Q_1, B; \theta_0, \theta_1)$. The maximizing choices for production satisfy $Q_i(\theta_i, \theta_j) \geq Q_i(\hat{\theta}_i, \theta_j)$ for $\theta_i < \hat{\theta}_i$ and, consequently, $\bar{c}_i(\theta_i)$ is non-increasing in θ_i. Thus, (RP) has a solution and an optimal policy exists.*

Uniqueness of the optimal policy depends on the relative strength of three forces: the slope of demand, the cost diseconomies in market size due to distribution, and the potential extent of increasing returns in production. Recalling the distribution problem, the first two forces are summarized by how the price schedule in each market shifts in response to a change in aggregate production. The third, of course, depends on concavity or convexity in production costs. As a sufficient condition for uniqueness, suppose that

$$\frac{-\delta}{v(\mu)} < \rho(\theta) C''(Q), \tag{19}$$

where $\mu = \mu(Q, b)$. In this condition, the right-hand side is the slope of the marginal cost of production, scaled by $\rho(\theta)$ to account for the incentive cost of private information. The left-hand side is related to the effect of Q changes on the demand schedule for a market size of b. As Q varies, the price schedule across consumers in the market shifts downward, with μ_Q measuring the size of the shift. Since we know $\mu_Q < -\delta/v(\mu)$, condition (19) implies that market demand crosses the marginal cost curve one time from above. We then have

Proposition 2 *Suppose (19) holds for all Q, b and θ. Then the optimal policy is unique.*

Although the analysis of (19) is involved, the economic intuition is relatively simple. For any given market size, there is only one value for aggregate production where demand and marginal cost cross each other. Incorporating this, there is then only one value for the market boundary that balances costs and benefits across the two markets.[9] Uniqueness is guaranteed if there are no increasing returns in production ($C'' \geq 0$) as (19) necessarily holds in this case. Condition (19) allows for increasing returns ($C'' < 0$) as long as they are not too large relative to demand. As aggregate demand becomes steeper, (19) is more likely to hold (e.g., greater diseconomies in distribution, as with a rise in δ).

5.2 The Structure of the Optimal Policy

We now examine the economic structure of production levels and market size in the optimal policy. To simplify the exposition, suppose that (19) holds and the solution is unique. The optimal policy is then fully characterized by the Kuhn-Tucker conditions for pointwise maximization of $W(\Omega)$. We focus on the case of interior solutions for (Q_0, Q_1, B).[10] Taking the relevant partials of w, the integrand in (18), and simplifying, we find

$$Q_0 \;:\; \mu(Q_0, B) = \rho(\theta_0)C'(Q_o) \tag{20}$$

$$Q_1 \;:\; \mu(Q_1, 1-B) = \rho(\theta_1)C'(Q_1) \tag{21}$$

$$B \;:\; \mu(Q_0, B) + \delta B = \mu(Q_1, 1-B) + \delta(1-B). \tag{22}$$

The first-order conditions for quantities follow directly from $U_Q = \mu$. For the boundary, recall from Section 4 that U_b is consumers surplus at the boundary of the market. Then (22) follows by expressing U_b for each market in terms of prices. In general, the system must be solved simultaneously for the optimal (Q_0, Q_1, B) policy.

Two economic forces operate in the determination of optimal production levels and market size. First, for quantities, we have marginal cost pricing adjusted for the incentive cost of private informa-

[9] The analysis of uniqueness is complicated by the fact that w need not be concave in (Q_0, Q_1, B) since w is the sum over two markets of the net value of distribution over production cost in each market.

[10] It is straightforward to apply the Kuhn-Tucker conditions and analyze corner solutions (this is examined when we consider ex-ante asymmetries). If the range of variation in the firm-specific cost parameter is sufficiently large, then the difference between $\rho(\underline{\theta})$ and $\rho(\bar{\theta})$ can justify a "monopoly" market of $[0, 1]$ for the lower cost firm while the higher cost firm is shut down.

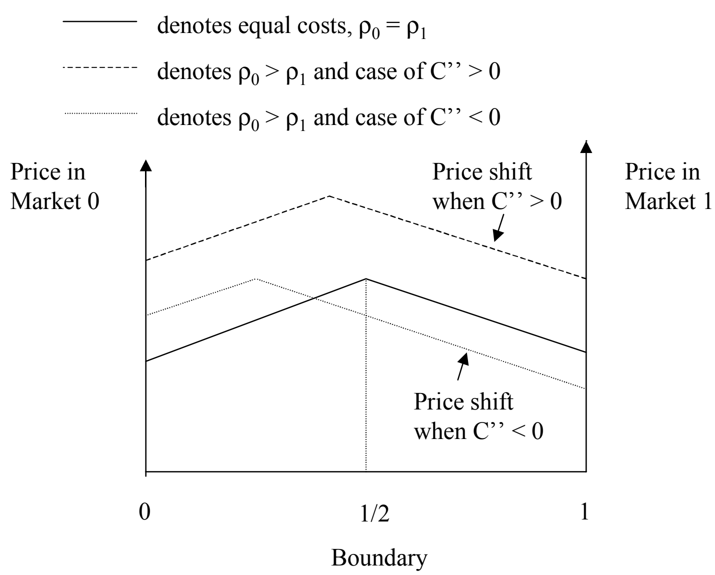

Figure 1: Price Schedules and Market Segments

tion. For a consumer at the production location of the firm ($t = 0$ or $t = 1$), distribution costs are zero and the price for this consumer then equals the underlying marginal cost of production plus the incentive cost generated by private information. Second, for market size, we have the equalization of individual consumers surplus at the boundary. Because consumers surplus is a function of price, this condition reduces to equal prices at the boundary. For firm 0, the price schedule rises linearly at the rate δ from the intercept of $\mu(Q_0, B)$, and at the optimal boundary it intersects the corresponding price schedule of firm 1.

The incentive structure of the optimal policy can be understood by considering the effect of variations in the relative production efficiency of the firms (via θ). Figure 1 illustrates how the optimal market structure responds to these variations for several cases. As a benchmark, in the simple case of equal cost draws, $\rho(\theta_0) = \rho(\theta_1)$, the optimal policy has equal market sizes ($B = 1/2$) and symmetric price schedules for the two markets.

Now suppose that firm 0 has a higher cost draw, so that $\rho(\theta_0) > \rho(\theta_1)$. If market size were to remain fixed at $B = 1/2$, then we see from (20) that price falls below marginal cost in market 0 and, therefore, a reduction in Q_0 would be warranted. In turn, this would imply an upward shift in the price schedule for market 0 and, from (22), we see that firm 1 could more efficiently supply consumers at the boundary between the firms.

The optimal adjustment of market size to the increase in $\rho(\theta_0)$ thus entails $B < 1/2$ and the relatively more efficient firm 1 is awarded part of the firm 0 market. Further, Q_0 is reduced while Q_1 is increased. As regards prices, the price schedule for market 0 shifts up. As illustrated in Figure 1, however, the shift in market 1 depends on the extent of scale economies in production. These comparative statics are summarized in

Proposition 3 *In the optimal policy, (i) the boundary $B(\theta_0, \theta_1)$ is decreasing in θ_0 and increasing in θ_1; (ii) each production aggregate $Q_i(\theta_i, \theta_j)$ is decreasing in θ_i and increasing in θ_j; (iii) the price schedule for firm i shifts up as θ_i increases, and it shifts up as θ_j increases if $C'' > 0$ (down if $C'' < 0$).*

Figure 1 illustrates these effects for the cost shift from $\theta_0 = \theta_1$ to $\theta_0 > \theta_1$. When $C'' > 0$, the market 1 price schedule shifts up. Intuitively, since Q_1 is larger we must have higher prices in market 1 as marginal cost rises with output under decreasing returns. Thus, the expansion in market 1 entails higher prices and lower quantities for consumers in the original market served by firm 1. The shifted consumers are served more efficiently by firm 1 than by firm 0, but the expansion does have a negative impact on the original consumers of firm 1. In contrast, with $C'' < 0$ the market 1 price schedule shifts down. The intuition is similar as, with Q_1 up, increasing returns means lower marginal cost and prices can fall. Thus, as the firm 1 market expands, the original consumers benefit from lower prices and higher quantities.

To summarize the incentive structure of the optimal policy, we see that as a firm has higher costs it is assigned a smaller market and produces less. The other firm gains in market size, produces more output, and the welfare effects for consumers in the original market depend directly on the nature of scale economies in production.

6 The Effect of Asymmetric Information

We now consider how asymmetric information influences the optimal policy relative to the case of full information (the first best allocation). Thus, we consider how (Q_0, Q_1, B) are set when the cost parameters are (θ_0, θ_1) and compare this to the policy at $(\rho(\theta_0),\ \rho(\theta_1))$. This isolates the incentive distortions that arise as a consequence of the adverse selection problem associated with asymmetric cost information.

To build intuition, start with the simple benchmark case of a common cost draw, $\theta_0 = \theta_1$. As we know, $B = 1/2$ and this is the same under full information (FI) and asymmetric information (AI). In this case, the effect of AI is concentrated on a downward distortion of quantities. Since $\rho(\theta_i) > \theta_i$ for each firm, (20) and (21) imply that, with B unchanged, Q_i must fall for each firm relative to the FI case. This brings demand into balance with marginal cost, which includes the added incentive cost under AI. Thus, the incentive distortions due to AI for this case take the form of quantity reductions while the boundary is unchanged.

Now consider different cost draws and take $\theta_0 > \theta_1$, so that firm 0 is relatively less efficient. From Proposition 3, both information cases have the optimal boundary below 1/2 and Q_1 exceeding Q_0. We can identify three different effects of AI on policy choices; we start with the setting of constant returns in production ($C'' = 0$) as these effects are easiest to distinguish in this case. Let B^{AI} and B^{FI} denote the optimal boundary, respectively, under AI and FI and, for convenience, adopt the normalization of $C' = 1$.[11]

Consider whether $B^{FI} \lesseqgtr B^{AI}$. We know that price in each market is balanced with marginal cost. Under constant returns, however, marginal cost does not vary with quantity. As a result, price in each market equals the relevant cost parameter and, from (24), we see that the optimal boundary is fully determined by the relative cost efficiency of the two firms:

$$B^{FI} = \frac{1}{2} - \frac{1}{2\delta}(\theta_0 - \theta_1), \quad B^{AI} = \frac{1}{2} - \frac{1}{2\delta}[\rho(\theta_0) - \rho(\theta_1)].$$

Under the hazard condition, AI increases the cost differential to $\rho(\theta_0) - \rho(\theta_1)$ from $\theta_0 - \theta_1$. Thus, $B^{AI} < B^{FI}$ and AI distorts the boundary towards 0, expanding market 1 and reducing market 0.

Figure 2 graphs the situation for market 0. First, marginal cost shifts from θ_0 up to ρ_0. Next, when the boundary is reduced from B^{FI} to B^{AI}, there is an inward shift of market 0 demand to $\mu(Q_0, B^{AI})$, as a smaller market has lower demand. We then see from Figure 2 that Q_0 is distorted downwards as a result of AI. In market 1, we have a similar cost shift from θ_1 to ρ_1; the demand shift, however, is outward since $B^{AI} < B^{FI}$ means that market 1 is larger than under FI. The net effect on

[11] That is, $B^{AI} = B(\theta_0, \theta_1)$ and $B^{FI} = B(\rho^{-1}(\theta_0), \rho^{-1}(\theta_1))$. Also, Q_i^{AI} and Q_i^{FI} and defined in a similar way.

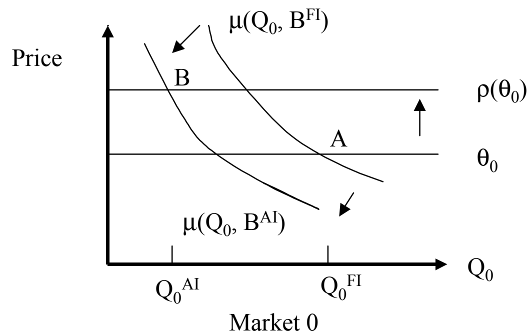

Figure 2: Effect of Asymmetric Information (AI) under Constant Returns

Q_1 is ambiguous since the cost shift pushes Q_1 down while the demand shift pushes Q_1 up.

Relative to full information, asymmetric information thus has three effects on the optimal policy choices. As illustrated with the constant returns setting, these are (i) an upward shift in marginal cost curves in each market, (ii) demand shifts in each market, in opposite directions, due to boundary adjustment, and (iii) price adjustments in each market as determined by scale economies in production. Constant returns provides a simple illustration of these effects because the cost shifts (i) directly determine the demand shifts (ii) and no price adjustment due to scale economies (iii) is necessary. In general, however, (ii) and (iii) are interdependent.

We illustrate the three effects in Figure 3 for both markets when $\theta_0 > \theta_1$ and, for the purposes of the graph, we suppose that $B^{AI} < B^{FI}$ and that $C'' < 0$ (increasing returns). With $\theta_0 > \theta_1$, the boundary is below 1/2 and market 1 has the larger production aggregate. Hence, market 0 is graphed on the left on market 1 on the right. Effect (i), the marginal cost shifts reflecting the increase from θ_i to $\rho(\theta_i)$, is depicted by the upward shift in each market. Holding the boundary fixed at B^{FI}, quantity

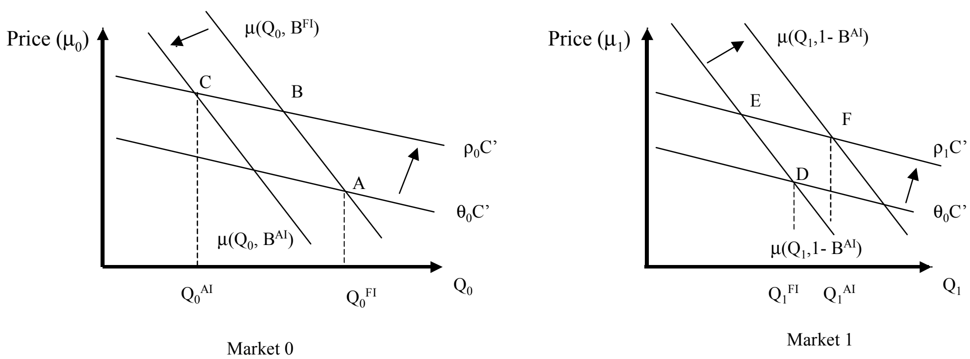

Figure 3: Effects of Asymmetric Information (AI) on Markets 0 and 1 under Increasing Returns

would fall in each market as we move up the respective market demand curves (A to B and D to E in Figure 3).

When the boundary is adjusted to B^{AI}, demand shifts in for market 0 and out for market 1. Effects (ii) and (iii) are depicted by the movement along the AI marginal cost curve for each market (B to C and E to F). The size of the demand shift, (ii), and the extent of returns to scale, (iii), determine the magnitude of quantity and price adjustment. While Q_0 must fall, Q_1 may rise or fall due to offsetting effects from (ii) and (iii); the graph depicts the case of a rise in Q_1.

The response of the optimal boundary B to AI is pivotal. Analytically, the comparison of FI to AI (θ_i to $\rho(\theta_i)$) has a discrete nature. To isolate the economic forces that drive the three effects created by AI, we can examine how the optimal boundary changes for cost-type parameters along a linear interpolation between the AI and FI type pairs.[12] This leads directly to a sufficient condition for when

[12]One could also interpret the interpolation as in Chen and Rosenthal [1996] where there is an ex-ante likelihood that

AI and incentive distortions lead to larger market size adjustments relative to FI. We have

Proposition 4 *Suppose (w.l.o.g.) that* $\theta_0 > \theta_1$ *and, hence, that* B^{AI} *and* B^{FI} *are both below* $1/2$*. Then* $B^{AI} < B^{FI}$ *holds at* (θ_0, θ_1) *if*

$$\frac{F(\theta_1)}{f(\theta_1)} C_1' \left\{ \frac{\mu_Q^1}{\mu_Q^1 - h_1 C_1''} \right\} < \frac{F(\theta_0)}{f(\theta_0)} C_0' \left\{ \frac{\mu_Q^0}{\mu_Q^0 - h_0 C_0''} \right\} \tag{23}$$

for all (h_0, h_1) *such that* $\theta_i \leq h_i \leq \rho(\theta_i)$*,* $i = 0, 1$*, and* $(h_0 - \theta_0)\left(F(\theta_0)/f(\theta_0)\right)^{-1} = (h_1 - \theta_1)\left(F(\theta_1)/f(\theta_1)\right)^{-1}$*; in (23) variables are indexed by* $i = 0, 1$ *and (implicitly) evaluated at the optimal policy for* (h_0, h_1)*. When the inequality in (23) is reversed, we have* $B^{AI} > B^{FI}$*. Furthermore, if* $B^{AI} < B^{FI}$ *then* $Q_0^{AI} < Q_0^{FI}$*, and if* $B^{AI} > B^{FI}$ *then* $Q_1^{AI} < Q_1^{FI}$*.*

Note that aggregate production must fall whenever a firm's market size is reduced. Thus, under AI, at least one of the firms must produce less. Essentially, Proposition 4 formalizes the intuition behind Figures 2 and 3. Under constant returns, (23) always holds as it collapses to the hazard property.

Consider how the optimal boundary change under AI is determined. First, fix $\theta_0 > \theta_1$ for the discussion. The first two terms in (23) reflect effect (i). The hazard rate term, which increases with θ, favors a decreasing boundary because there is a relatively larger cost shift in market 0. Scale economies determine the influence of the C' term. Under increasing returns, $C'' < 0$, we see that $C_1' < C_0'$ since $\theta_0 > \theta_1$ implies $Q_1 > Q_0$. Increasing returns thus favors a decreasing boundary as the larger production level in market 1 reinforces the hazard rate term in the cost shift. Decreasing returns, however, will offset the hazard rate term as $Q_1 > Q_0$ now implies a higher marginal cost for firm 1.

The bracketed term in (23) reflects effects (ii) and (iii). Under (19), this term is positive and it is above or below 1 as C'' is negative or positive. Consider the demand component. Because $B < 1/2$, we know from (22) that there is a positive price differential between markets, $\mu^0 - \mu^1 > 0$. As is easily verified, this implies $\mu_Q^0 < \mu_Q^1$ so that market 0, the smaller market, has a steeper demand curve. When $C'' < 0$, the bracketed term increases with μ_Q so that the demand component favors increasing the boundary. Intuitively, the bracketed term measures the price response in each market due to the cost shift, accounting for an optimal response in quantity to the cost shift. When market 1 has a larger response, as occurs under increasing returns, the price differential $(\mu_0 - \mu_1)$ narrows and

the principal observes the agent's private information prior to designing the mechanism. See also the proof technique (homotopic transformation) in Severinov [2003].

an increase in B is needed to bring consumers surplus into balance for a consumer at the boundary of the markets. Under decreasing returns, the differential widens and a decrease in B is favored. A similar intuition applies to the cost parameter term in the denominator of the bracketed term. With $h_0 > h_1$, the price differential widens (narrows) under increasing (decreasing) returns and B is pushed down (up).

The optimal boundary response to AI thus involves the interaction of several economic factors. While each factor has a systematic influence on B that derives from the structure of the larger versus the smaller market, it is the net influence of these factors that determines whether market size rises or falls as an incentive distortion associated with AI.[13] The case of constant returns provides a useful benchmark where, because the interaction of effects (i-iii) is simplified and the shift in the hazard term directly implies a widening of the price differential, we can conclude that both the market of the higher cost firm and aggregate quantity must be reduced. In general, condition (23) provides the guidelines for assessing the impact of the AI cost and demand effects on the boundary and, consequently, for setting optimal policy.

7 Ex-Ante Asymmetric Firms

In a number of settings, there are known ex-ante differences between firms. For example, the firms may employ different production technologies, as with traditional Cable TV and newer Direct Broadcast Satellite (DBS) based suppliers. In this section, we relax the assumption of ex-ante symmetry in three directions and explore briefly how policy is impacted by asymmetries in distribution costs (δ), the degree of asymmetric information (F), and the location of firms in the unit-interval (l). To keep the focus on these asymmetries, we work with the simple case of constant marginal costs and set $C' = 1$.

First, consider distribution costs and suppose δ_i is firm specific. Interpreted literally as a transportation cost, this allows one firm to have an efficiency advantage at distribution. Alternatively, viewed as horizontal differentiation, this allows for greater heterogeneity in consumer valuations across the two firms. For the analysis, note that δ_i impacts the value function for the distribution problem, and we now have $U(Q_0, B, \delta_0) + U(Q_1, 1 - B, \delta_1)$ as the value across all consumers. From the first-order

[13] It is straightforward to construct numerical examples, using (23) as a guide for setting parameter values, that demonstrate this point.

conditions, the optimal boundary is now

$$B(\theta_0, \theta_1) = \frac{\delta_1}{\delta_0 + \delta_1} + \frac{\rho(\theta_1) - \rho(\theta_0)}{\delta_0 + \delta_1}.$$

Optimal quantities are set to balance demand over each market, with $\mu(Q_0, B, \delta_0) = \rho(\theta_0)$ and $\mu(Q_1, 1 - B, \delta_1) = \rho(\theta_1)$. To sort out the effects, suppose $\delta_0 > \delta_1$ so that firm 0 is less efficient at distribution. Then, B falls and the market shifts in favor of firm 1. With equal cost draws, the market boundary is below $\frac{1}{2}$ when $\delta_0 > \delta_1$; also, a relative distribution advantage for firm 1 (small δ_0) amplifies the boundary reduction when firm 1 also has a production cost advantage ($\rho(\theta_1) < \rho(\theta_0)$). Further, Q_0 falls while Q_1 rises, reflecting the shift in market sizes. The price implementation involves extending the firm 1 price schedule towards the lower boundary (no shift); the price intercept for firm 0 rises and the schedule becomes steeper, as prices must rise to reduce demand for the smaller output and market size. The limiting case of this asymmetry, where the transportation cost for one of the firms vanishes, is explored further below in conjunction with asymmetric firm locations.

Next, consider the degree of information asymmetry across firms. The familiar notion of hazard-rate dominance, $f_0(\theta)/F_0(\theta) \leq f_1(\theta)/F_1(\theta)$, $\forall\theta \in [\underline{\theta}, \bar{\theta}]$, provides a useful comparison framework (see Laffont and Tirole [1993, p. 77] for more on the hazard rate). In our analysis, the net change is that we must employ $\rho_i(\theta) \equiv \theta + F_i(\theta)/f_i(\theta)$ for the incentive cost of each firm. The effects on the boundary and quantities then follow directly from our previous analysis of AI versus FI effects: $\rho_0(\theta) > \rho_1(\theta)$ is analogous to $\rho(\theta_0) > \rho(\theta_1)$ in the first-order conditions. Thus, the increased incentive cost with firm 0 pushes towards a smaller market with lower quantity for firm 0 and an expansion for firm 1 (at any pair of cost draws).

As an example, suppose firm 1 employs an older generation technology that has a known cost of $\bar{\theta}$ while firm 0 employs a newer technology with F_0 on $[\underline{\theta}, \bar{\theta}]$. For instance, in health care markets, firm 1 might be a hospital that employs an existing treatment or diagnosis technology while firm 0 employs a new technique with uncertain costs. Then, as the incentive cost with firm 1 is nil, this favors a shift in market allocation to the less efficient firm 1. Even though it is common knowledge that the new technology is more efficient, incentive costs make it optimal to maintain a market for firm 1. In fact, the information asymmetry can even lead to the "shelving" of the superior new technology.[14]

[14] Let F_0 be the uniform distribution. The condition for B reduces to $B = \frac{1}{2} + [\bar{\theta} + \underline{\theta} - 2\theta_0]/(2\delta)$. Then firm 0 is active only when $\theta_0 < [\delta + \bar{\theta} + \underline{\theta}]/2$; for higher θ_0, we have $B = 0$ and all of the market goes to the less efficient firm. In a different context, Wang [2000] considers oligopoly incentive regulation in a two-type model when firms have different ex ante probabilities of cost draws and finds that ex ante inefficient firms will sometimes be awarded all production.

In determining the boundary, the older firm's market is somewhat "protected" by the asymmetry in incentive costs.

Finally, let us consider the location of the firms. Suppose that firm 0 has an interior location at an $l_0 \in (0,1)$; for simplicity, maintain firm 1 at the right endpoint. We expect that firm 0 will typically serve nearby consumers on the left and to the right of l_0. Now, however, there is also the more subtle possibility that a "divided" market structure will emerge, with firm 1 serving customers who are sufficiently far from firm 0 on both sides. Extending the analysis, we find an analogue to Lemma 3 in which there may now be a lower boundary $B^L \in [0, l_0]$ as well as the previously developed upper boundary, denoted by $B^U \in [l_0, 1]$. Firm 0 serves consumers in $[B^L, B^U]$ while firm 1 serves those in $[0, B^L]$ and $[B^U, 1]$.

The distribution results in Lemma 4 apply directly to each potential market segment. The analogue to the objective in (18) then works out to be

$$U(Q_0^L, l_0 - B^L, \delta_0) + U(Q_0^U, B^U - l_0, \delta_0) - \rho(\theta_0)[Q_0^L + Q_0^U] \tag{24}$$
$$+U(Q_1^L, B^L, \delta_1) - \delta_1(1 - B^L)Q_1^L + U(Q_1^U, 1 - B^U, \delta_1) - \rho(\theta_1)[Q_1^L + Q_1^U].$$

The various terms reflect the value of distributing output across the four potential segments as well as the associated costs. The new term of $\delta_1(1 - B^L)Q_1^L$ associated with firm 1 is due to the divided market structure and reflects the cost of serving the distant segment of $[0, B^L]$ from a distance of $1 - B^L$. After accounting for this cost, firm 1 is effectively serving that segment from an origin of B^L. We are also allowing for $\delta_0 \neq \delta_1$. The analysis of the upper boundary and the associated quantities follows previous lines, so we focus here on the lower segments. For the lower boundary, firm 0 produces output such that $\mu_0^L = \rho(\theta_0)$, where $\mu_0^L \equiv \mu(Q_0^L, l_0 - B^L, \delta_0)$ is the required price from firm 0 to a consumer at the location l_0. For firm 1, we must have $\mu_1^L = \rho(\theta_1) + \delta_1(1 - B^L)$, where $\mu_1^L \equiv \mu(Q_1^L, B^L, \delta_1)$ is the required price from firm 1 to a consumer at location B^L. In addition to the incentive-cost adjustment, there is also an added price premium to reflect the cost of serving a divided market. Recalling that $S(v(p))$ is individual consumers surplus at price p, the optimal choice for the B^L boundary satisfies

$$\begin{aligned} S\left(v\left(\mu_0^L + \delta_0\left(l_0 - B^L\right)\right)\right) &= S\left(v\left(\mu_1^L + \delta_1 B^L\right)\right) + \delta_1 Q_1^L \Leftrightarrow \\ S(\rho(\theta_0) + \delta_0(l_0 - B^L)) &= S(\rho(\theta_1) + \delta_1) + \delta_1 Q_1^L, \end{aligned}$$

upon substituting the output conditions. To interpret this condition, note that if $\rho(\theta_1) + \delta_1 < \rho(\theta_0) + \delta_0 l_0$, then it makes efficiency sense for firm 1 to be the supplier for the consumer at location $t = 0$.

However, the boundary B^L is not set to equalize consumer surplus at location B^L. That would be incorrect for two reasons. First, it would ignore the added cost margin of $\delta_1 Q_1^L$ associated with firm 1 serving a distant market. Second, the marginal benefit from shifting the boundary hinges on consumer surplus for a consumer at the opposite end of the market segment relative to the origin; this is location $t = B^L$ for firm 0 while it is $t = 0$ for firm 1. Thus, the change in value for the $[0, B^L]$ market as B^L rises is the consumers surplus of the distant consumer ($t = 0$) plus the benefit associated with reducing the distance from firm 1's location to the segment origin of B^L and this is equated with the corresponding consumers surplus for the distant consumer ($t = B^L$) in the $\left[B^L, l_0\right]$ segment of firm 0.

As an application of the analysis of a divided market structure, consider the Multi-Channel Video Programming Distributor Market, defined as the video services market supplied (primarily) by Cable-TV and Direct Broadcast Satellite (DBS) providers.[15] The nature of the horizontal differentiation involves several components. Cable TV typically has superior options for local channel viewing; DBS often has superior picture quality and offers more options in specific areas such as sports, movies and music. In each case, heterogeneity across consumers for the valuation of these differences is significant. On the technology side, DBS has a clear "wireless" component in that costs of delivery do not depend directly on the consumer's geographical proximity to a cable network. To explore the intuition from the model, imagine firm 0 as cable provider with $\delta_0 > 0$ while firm 1 is a DBS provider with $\delta_1 = 0$, reflecting a "wireless" technology for distribution. The condition for the lower boundary now reduces to $B^L = l_0 - [\ \rho(\theta_1) - \rho(\theta_0)]/\delta_0$. The wireless dimension creates a natural advantage for firm 1 in supplying a consumer who has a poor horizontal match with firm 0. As a result, this force pushes to a divided market as an optimal structure.[16]

[15] According to FCC documents, in the year 2000 about 80% of MVPD subscribers, or 84.4 million households, receive cable services. DBS is now 15.4% of the MVPD market, growing at the rate of 18% from 10.1 to nearly 13 million households from June 1999 to June 2000.

[16] The B^L choice is interior to $[0, l_0]$ when $\delta_0 l_0 + \rho(\theta_0) > \rho(\theta_1) > \rho(\theta_0)$, respectively for the left and right endpoints. Note that a pure wireless technology is a convenient simplification. What matters for a divided market is that δ_1 is sufficiently small relative to δ_0.

8 Conclusion

This paper examines optimal regulation in a duopoly model of spatial competition when firms have private cost information. The key feature here is that the regulator can provide incentives by utilizing the spatial dimension, increasing or decreasing each firm's market through the assignment of consumers at the competitive fringe between the firms. Thus, the analysis focuses on the trade-off between assignment of market segments and the allocation of production across consumers in each firm's market segment. Providing incentives through market assignment reduces the need to rely on quantity distortions.

The properties of optimal regulatory policy can be summarized as follows. Because distribution is costly, it is optimal to award distinct market segments and allow consumption to vary with consumer location. To implement an efficient distribution of output in each market segment, each firm employs a price schedule that involves a fixed fee plus a variable charge based on location. Market segments and production aggregates are jointly determined by equating demand in each segment with marginal production cost, scaled upwards to account for incentive costs, and equalizing consumers surplus at the boundary between the market segments.

Market size and production both decline as a firm becomes less efficient. The other firm is awarded a larger market by reassigning customers and also produces more in aggregate. Welfare effects for prior customers of this firm vary with scale economies in production as these consumers benefit under increasing returns but consume less and pay more under decreasing returns. Compared to the full information setting, we identify three effects on regulated market structure that arise under asymmetric information. A cost effect, which depends on the inverse hazard rate, dominates and optimal policy makes market segments and production more responsive to efficiency differences between the firms in the benchmark case of constant returns in production.

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Appendix

Proof of Lemma 1: We provide only a sketch of the proof as the techniques are standard. First, suppose (i), (ii) and (iii) of the Lemma hold. Then (11) is clearly satisfied since $\Pi_i(\theta)$ is non-increasing. To verify (10), calculate $\Pi_i(\theta) - \pi_i(r \mid \theta)$ using the identity $\pi_i(r \mid \theta) = \Pi_i(r) + (r-\theta)\overline{c}_i(r)$. Now suppose (10) and (11) hold. By applying (10) at (r,θ) and at (θ,r) and then simplifying, we obtain the double inequality of $(r-\theta)\overline{c}_i(\theta) \geq \Pi_i(\theta) - \Pi_i(r) \geq (r-\theta)\overline{c}_i(r)$. Then (i) follows directly. Dividing by $(r-\theta)$, taking limits as r approaches θ and integrating then yield (ii). Finally, (11) implies (iii).■

Proof of Lemma 2: Again, we sketch the proof. First, the accounting definition of profits implies the identity $E\{\Pi_i(\theta)\} = E\{R_i(\theta_i,\theta_j) - D_i(\theta_i,\theta_j) - C_i(\theta_i;\theta_i,\theta_j)\}$. Use this to substitute for R_i in (12). Next, simplify the expression by using (8) for D_i and (ii) of Lemma 1 for Π_i. Finally, integrate by parts on the production cost terms to arrive at (13).■

Proof of Lemma 3: Let q_0 and q_1 be any given pair of quantity schedules. We will construct q_0^* and q_1^* that involve a boundary and increase the value of the objective in (15). First, define cumulative quantities on subintervals of $[0,1]$ by

$$X_0(t) = \int_0^t q_0(\tau)d\tau, \quad X_1(t) = \int_t^1 q_1(\tau)d\tau, \quad \text{and } X(t) = \int_0^t q(\tau)d\tau,$$

where $q(t) = q_0(t) + q_1(t)$. Then $Q_0 = X_0(1)$ and $Q_1 = X_1(0)$ hold in relation to aggregate production. Now, define quantity schedules by

$$q_0^*(t) = \begin{cases} q(t) & t \leq B \\ 0 & t > B, \end{cases} \qquad q_1^*(t) = \begin{cases} 0 & t \leq B \\ q(t) & t > B, \end{cases}$$

where $X(B) = Q_0$. Such a B exists since $X(0) = 0$, $X(1) = Q_0 + Q_1$, and $X(t)$ is continuous on $[0,1]$. Let X_0^*, X_1^*, and X^* be the corresponding cumulatives.

Under q_0 and q_1, the objective in (15) can be expressed as

$$\int_0^1 [V(q(t)) - \delta t q_0(t) - \delta(1-t)q_1(t)]\, dt = \int_0^1 [V(q(t)) + 2\delta X_0(t) - \delta X(t)]\, dt + \delta X_1(0). \tag{A1}$$

A similar expression holds for q_0^* and q_1^*. Since $X^*(t) = X(t)$, $X_1(0) = X_1^*(0)$, and $q^*(t) = q(t)$ by construction, the difference between the value in (15) at (q_0^*, q_1^*) and at (q_0, q_1) is, using (A1), given by $2\delta \int_0^1 [X_0^*(t) - X_0(t)]dt$. For $t \leq B$, we have $q_0^*(t) = q(t) \geq q_0(t)$ and, hence, $X_0^*(t) \geq X_0(t)$. For $t > B$, we have $X_0^*(t) = Q_0 \geq X_0(t)$. Thus, $X_0^*(t) \geq X_0(t)$ for all t and we are done.■

Proof of Lemma 4: We apply optimal control to the problem in (16). First, we find an equivalent expression for the objective function. Let $X(t) = \int_0^t q(\tau)d\tau$. Integrating by parts on $\int_0^b tq(t)dt$, the objective in (16) equals $\int_0^b [V(q(t)) - \delta bq(t) + \delta X(t)]\,dt$. Now, let $q(t)$ be the control variable; let $-X(t)$ be the state variable, which must satisfy the differential equation $\dot{X}(t) = q(t)$; the control must satisfy $q(t) \geq 0$ and the state must satisfy $X(0) = 0$ and $X(b) \leq Q$, the constraint from (16).

To establish existence of a solution, we can apply the existence theorem developed by Seierstad and Sydsaeter [1987, p. 137]. It is routine to verify that all of the conditions of the theorem are satisfied with $q(t)$ and the "multiplier" $p(t)$ as reported in Lemma 4.

There are two cases for the values of Q and b. When $Q < \int_0^b v(\delta t)dt$, the multiplier is strictly positive for all t and $X(b) = Q$ holds at the solution. The condition in Lemma 4 determines $p(0) = \mu(Q, b)$ in this case. When $Q \geq \int_0^b v(\delta t)dt$, we have $p(0) = 0$ and the solution is described in the footnote to Lemma 4.■

Proof of Proposition 1: Consider the existence of (Q_0, Q_1, B) that maximize $w(Q_0, Q_1, B; \theta_0, \theta_1)$. Clearly, w is continuous in (Q_0, Q_1, B). The value function satisfies $U(Q, b) < U(Q, 1) \leq U\left(\int_0^1 v(\delta t)dt, 1\right)$, and $\rho(\theta)C(Q)$ is strictly increasing in Q. Hence, any maximizing choice must be an element of the compact set $\left[0, \int_0^1 v(\delta t)dt\right]^2 \times [0, 1]$. Thus, there exists a maximizing choice.

Now let (Q_0, Q_1, B) be an optimal choice at (θ_0, θ_1) and let $(\hat{Q}_0, \hat{Q}_1, \hat{B})$ be an optimal choice at $(\hat{\theta}_0, \theta_1)$ where $\hat{\theta}_0 > \theta_0$. Then $w(Q_0, Q_1, B; \theta_0, \theta_1) \geq w(\hat{Q}_0, \hat{Q}_1, \hat{B}; \theta_0, \theta_1)$ and $w(\hat{Q}_0, \hat{Q}_1, \hat{B}; \hat{\theta}_0, \theta_1) \geq w(Q_0, Q_1, B; \hat{\theta}_0, \theta_1)$, by definition of optimal choices. By definition of w, adding the above two inequalities and simplifying yield $\rho(\theta_0)\left[C(\hat{Q}_0) - C(Q_0)\right] \geq \rho(\hat{\theta}_0)\left[C(\hat{Q}_0) - C(Q_0)\right]$. As $\rho(\theta_0) < \rho(\hat{\theta}_0)$ when $\theta_0 < \hat{\theta}_0$, we must have $\hat{Q}_0 \leq Q_0$ in order to satisfy the above inequality. The argument for Q_1 is analogous. By definition, it is clear that $\bar{c}_i(\theta_i)$ is non-increasing in θ_i if $Q_i(\theta_i, \theta_j)$ is non-increasing in θ_i. Finally, since the $Q_i(\theta_i, \theta_j)$ in the pointwise maximization of w satisfy the incentive constraint for $\bar{c}_i(\theta_i)$, we have a solution to (RP).■

Proof of Proposition 2: The complicating factor for uniqueness is that $w(Q_0, Q_1, B; \theta_0, \theta_1)$ is not necessarily concave in (Q_0, Q_1, B), even with the added assumption of (19). To proceed, then, consider the value of an optimal quantity choice for a given market of size b: $g(b, \rho) \equiv \max_{Q \geq 0} \{U(Q, b) - \rho C(Q)\}$. Proposition 1 directly implies that a maximizing Q exists and, hence, that $g(b, \rho)$ is well defined. The choice of Q is unique if $U(Q, b) - \rho C(Q)$ is strictly concave in Q. We have

$$\frac{\partial^2}{\partial Q^2}\{U(Q, b) - \rho C(Q)\} = \mu_Q - \rho C''(Q) = \frac{-\delta}{v(\mu) - v(\mu + \delta b)} - \rho C''(Q) < 0, \tag{A2}$$

when evaluated at $\mu(Q, b)$ where $b > 0$; the last step follows from (19) since v is strictly decreasing. If $b = 0$, then $Q = 0$ is the unique optimal choice as $U(Q, 0) = 0$ and $C(Q)$ is strictly increasing. Thus, for each $b \geq 0$ there is a unique maximizing choice of Q.

Now we show $g(b,\rho)$ is strictly concave in b. Let $H(p) \equiv S(v(p))$ denote individual consumers surplus. We have $g_{bb} = H'(\mu+\delta b)[\mu_Q \cdot \phi_b + \mu_b + \delta]$ where $\mu = \mu(Q,b)$ and ϕ_b is the comparative static of $Q = \phi(b,\rho)$, the optimal Q choice for $g(b,\rho)$. As $H' < 0$, we have $g_{bb} < 0$ provided that the term in brackets is strictly positive. With the shorthand of $v = v(\mu(Q,b))$ and $\hat{v} = v(\mu(Q,b)+\delta b)$ where $Q = \phi(b,\rho)$, we have

$$\mu_Q \cdot \phi_b + \mu_b + \delta = \mu_Q \left(\frac{-\mu_b}{\mu_Q - \rho C''} \right) + \mu_b + \delta = \frac{\hat{v}\rho C''}{\mu_Q - \rho C''} + \delta,$$

where we have calculated ϕ_b, substituted with $\mu_b = \hat{v}\mu_Q$, and simplified. Then, substituting for μ_Q and simplifying further yield

$$\mu_Q \cdot \phi_b + \mu_b + \delta = \delta \left(\frac{\delta + v\rho C''}{\delta - (\hat{v} - v)\rho C''} \right) > 0;$$

the numerator is positive by (19) and the denominator is positive by (A2). Hence, $g_{bb} < 0$.

Finally, the value of B in any optimal choice of (Q_0, Q_1, B) for $w(Q_0, Q_1, B; \theta_0, \theta_1)$ must satisfy $B \in \arg\max_b \{g(b, \rho(\theta_0)) + g(1-b, \rho(\theta_1))\}$. Proposition 1 implies such a B exists. The choice is unique as $g_{bb} < 0$ implies $g_{bb}(b, \rho(\theta_0)) + g_{bb}(1-b, \rho(\theta_1)) < 0$. Thus, we have shown that (19) implies a unique maximizing choice of (Q_0, Q_1, B) and, from Proposition 1, this maximizing choice at each (θ_0, θ_1) is the optimal policy.■

Proof of Proposition 3: This is a relatively straightforward comparative statics exercise and we limit the proof to a sketch. First, calculate the comparative statics for B by employing the value function $g(b,\rho)$ from the proof of Proposition 2. The claim then follows from $g_{bb} < 0$ and $g_{b\rho} = -[\mu_Q - \rho C'']^{-1} v(\mu+\delta b)\mu_Q C' < 0$. Let $Q = \phi(b,\rho)$ be the unique optimal Q choice for the $g(b,\rho)$ value function. The claims for $Q_i(\theta_i, \theta_j)$ then follow from $Q_i(\theta_i, \theta_j) = \phi(B(\theta_i, \theta_j), \rho(\theta_i))$.

For the price effects, the price at the boundary can be written as $\mu(\phi(1-B, \rho(\theta_1)), 1-B) + \delta(1-B)$, where $B = B(\theta_0, \theta_1)$. This is increasing in θ_0 since $g(b,\rho)$ is concave in b and B is decreasing in θ_0. The intercept for market 0 is the boundary price less δB, and this is increasing in θ_0. We find that the intercept in market 1 is increasing or decreasing in θ_0 as $\{\mu_Q[\mu_Q - \rho C'']^{-1} - 1\} \lesseqgtr 0$, where arguments are evaluated at $Q_1(\theta_1, \theta_0)$ and $B(\theta_0, \theta_1)$; this inequality reduces to $C'' \gtreqless 0$. Effects of θ_1 changes are symmetric.■

Proof of Proposition 4: Fix any pair of cost types with $\theta_0 > \theta_1$ and define $h_i(\alpha) = \theta_i + \alpha(F(\theta_i)/f(\theta_i))$ for $\alpha \in [0,1]$ and $i = 0, 1$. Let $\beta(\rho_0, \rho_1)$ denote the unique optimal boundary choice (from Proposition 2) for $\max_b [g(b, \rho_0) + g(1-b, \rho_1)]$ and then define $\gamma(\alpha) = \beta(h_0(\alpha), h_1(\alpha))$. By construction, we have $B^{AI} = B(\theta_0, \theta_1) = \gamma(1)$ and $B^{FI} = B\left(\rho^{-1}(\theta_0), \rho^{-1}(\theta_1)\right) = \gamma(0)$. Calculating, we have

$$\begin{aligned}
\gamma'(\alpha) &= \frac{\partial}{\partial h_0}\beta(h_0(\alpha), h_1(\alpha)) \cdot h_0'(\alpha) + \frac{\partial}{\partial h_1}\beta(h_0(\alpha), h_1(\alpha)) \cdot h_1'(\alpha) \\
&= [g_{bb}^0 + g_{bb}^1]^{-1}\left\{-g_{b\rho}^0 \frac{F(\theta_0)}{f(\theta_0)} + g_{b\rho}^1 \frac{F(\theta_1)}{f(\theta_1)}\right\}.
\end{aligned}$$

Here, g_{bb}^i and $g_{b\rho}^i$ must be evaluated at $b = \beta(h_0(\alpha), h_1(\alpha))$ and $\rho = h_0(\alpha)$ for $i = 0$, and at $b = 1 - \beta(h_0(\alpha), h_1(\alpha))$ and $\rho = h_1(\alpha)$ for $i = 1$. Since $g_{bb} < 0$, we can substitute for $g_{b\rho}$ with the expression from the proof of Proposition 3 to see that $\gamma'(\alpha) < 0 \Leftrightarrow$

$$v\left(\mu^1 + \delta(1-\beta)\right)\frac{F(\theta_1)}{f(\theta_1)}C'\left(Q^1\right)\left\{\frac{\mu_Q^1}{\mu_Q^1 - h_1 C''(Q^1)}\right\} < v\left(\mu^0 + \delta\beta\right)\frac{F(\theta_0)}{f(\theta_0)}C'\left(Q^0\right)\left\{\frac{\mu_Q^0}{\mu_Q^0 - h_0 C''(Q^0)}\right\},$$

where $Q^0 = \phi(\beta(h_0(\alpha), h_1(\alpha)), h_0(\alpha))$ and $\mu^0 = \mu\left(Q^0, \beta(h_0(\alpha), h_1(\alpha))\right)$ and similarly for Q^1 and μ^1. Then (23) follows upon noting that prices are equalized for each firm at the optimal boundary.

For the second claim, note that $Q_i^{AI} = \phi(B(\theta_i, \theta_j), \rho(\theta_i))$. We know that $\phi_b > 0 > \phi_\rho$. Then $\rho(\theta_i) > \theta_i$ and $B^{AI} < B^{FI}$ directly imply the claim.■