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A simple heuristic for echelon (r, nQ, T) policies in serial supply chains

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ABSTRACT

This paper proposes a simple heuristic that generates a solution for echelon (r, nQ, T) policies by sequentially solving a deterministic demand problem, a subproblem with fixed reorder intervals, and a subproblem with fixed batch sizes. For each of these problems, we further simplify the computation by solving a series of single-stage systems whose parameters are obtained directly from the original problem data. In a numerical study, we find that this heuristic outperforms an existing one in the literature.

1. Introduction

In supply chains, materials are often ordered and shipped in batches at fixed time intervals. In addition to inventory holding and customer backorder costs, two types of fixed costs, setup cost and order cost, are often incurred for inventory replenishments. The setup cost may include engineering costs associated with setting up machines, material handling costs and inspection costs. These costs are usually associated with processing a batch. The order cost may include inventory review costs, ordering costs, and shipping costs, and are often associated with each inventory reorder. For example, consider inventory replenishment through railroad transportation. A train delivers carts of materials according to a fixed schedule. Loading a cart (i.e., a batch) and dispatching a train incurs a setup cost and a shipping cost, respectively. Large retail chains, such as Wal-Mart, replenish materials in the same fashion. Items are inspected and packaged in cases, loaded onto a truck, and then delivered to a retailer site on fixed days of the week. Since these fixed costs usually account for a big portion of total supply chain operating costs, it is important to obtain efficient batch sizes and reorder intervals.

We model such an inventory replenishment practice by a periodic-review, serial inventory system, in which random customer demand occurs at stage 1; stage 1 orders from stage 2, stage 2 orders from stage 3, etc., and stage N orders from an outside supplier that has ample supply. Demand in different periods is independent, identically distributed, nonnegative, and integer-valued. Let λ denote the mean one-period demand. Each stage implements an echelon (r, nQ, T) policy, operated as follows: Stage j reviews its echelon inventory order position (inventory on hand + inventory on order + inventory at or in transit to all downstream stages - backorders at stage 1) at the beginning of every T_j period and orders according to a standard (r, nQ) policy. That is, if the echelon inventory order position is lower than or equal to the reorder point r_j, the stage orders the smallest integer-multiple of batch size Q_j that will maintain its inventory position above r_j (c.f., [1]. There is a lead time $L_j \in \mathbb{I}$ between stages $j+1$ and j, where $\mathbb{I}$ is the set of positive integers. Let $L_{[i,j]}$ be $\sum_{k=i}^{j} L_k$. A setup cost k_j is incurred for each batch Q_j ordered. (That is, the total setup cost for ordering 0, 1, 2, … batches is 0, k_j, $2k_j$, ….) A fixed ordering cost K_j is incurred for each inventory review. In addition, a linear echelon holding cost h_j per period is incurred for each unit of on-hand inventory held in echelon j (stage j and all of its downstream stages), and a linear backorder cost b per period is charged for each unit of backorders incurred at stage 1. Let $h_{[i,j]} = \sum_{k=i}^{j} h_k$. We assume that the ordering decisions between stages are synchronized, that is, whenever possible, a downstream stage places an order when its upstream stage receives a shipment (a synchronized shipping policy dominates a non-synchronized one; see [2].) Also, the batch sizes and the reorder intervals follow integer-ratio relations, that is, $T_{j+1} = n_j T_j$ and $Q_{j+1} = q_j Q_j$, $T_j, Q_j, n_j, q_j \in \mathbb{I}$, for $j = 1, \ldots, N-1$. For notational simplicity, we define $\mathbf{T_j} = (T_1, T_2 \ldots, T_j)$, $\mathbf{Q_j} = (Q_1, Q_2 \ldots, Q_j)$, $j = 1, \ldots, N$, $\mathbf{T} = \mathbf{T_N}$, and $\mathbf{Q} = \mathbf{Q_N}$.

We assume that the system starts with a plausible inventory state in which stage j's local on-hand inventory is an integer multiple of Q_{j-1}, $j = 2, \ldots, N$ (see [2,3] for an explanation). In addition, all the replenishment activities in a period occur at the beginning of the period. At stage $j > 1$, they occur in the following

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sequence: (1) An order, if any, from stage $j-1$ is received; (2) an order is placed with stage $j+1$ if the period is stage j's order period; (3) a shipment sent L_j period earlier from stage $j+1$ is received; and (4) a shipment is sent to stage $j-1$. For stage 1, order placement occurs at the beginning of stage 1's order periods, while customer demand arrives during a period. Costs are evaluated at the end of a period. The objective is to minimize the long-run average total cost per period.

With fixed $\mathbf{Q}$ and $\mathbf{T}$, Chao and Zhou [2] derived a bottom-up recursion for finding the optimal echelon reorder points. Recently, Shang and Zhou [4] revised their recursion and decomposed the average total inventory holding and backorder cost into each stage by introducing the induced-penalty cost function, i.e., the penalty cost charged to an upstream stage for holding insufficient stock. To facilitate our discussion, we summarize the decomposition below, and refer the reader to [4] for a detailed explanation.

We call the period that a stage is allowed to place an *order period*. Define

$IOP_j(t)$ = echelon inventory order position after ordering at stage j at the beginning of an order period t.

$IL_j^-(t)$ = echelon inventory level at stage j at the beginning of a period t.

Notice that $IOP_j(t)$ is only defined in order periods. Let $D(\ell)$ and $D[\ell]$ denote the total demand over ℓ and $\ell+1$ periods, respectively.

Consider an order period t for stage 1. Conditioning on $IOP_1(t)=y$, stage 1's inventory holding and backorder costs per period is

$$g_1(y,T_1)=\frac{1}{T_1}\left(\sum_{\tau=0}^{T_1-1}\mathbf{E}[h_1(y-D[L_1+\tau]) + (b+h_{[1,N]})(y-D[L_1+\tau])^-]\right).$$

Since $IOP_1(t)$ is uniformly distributed between $\{r_1+1, r_1+2,\ldots,r_1+Q_1\}$, the average inventory holding and backorder cost per period for stage 1 is

$$\mathbf{E}[g_1(IOP_1(t),T_1)]=\frac{1}{Q_1}\sum_{x=1}^{Q_1}g_1(r_1+x,T_1)\stackrel{def}{=}\widehat{g}_1(r_1,\mathbf{Q_1},\mathbf{T_1}).$$

The optimal reorder point for stage 1 is $r_1(\mathbf{Q_1},\mathbf{T_1})=\arg\min_{r_1}\widehat{g}_1(r_1,\mathbf{Q_1},\mathbf{T_1})$.

For stage $j=2,3,\ldots,N$, let $IL_j^-(t)=y$, where t is stage $j-1$'s order period. Suppose that $r_{j-1}(\mathbf{Q_{j-1}},\mathbf{T_{j-1}})$ is known. The induced-penalty cost charged to stage j is

$$g_{j-1,j}(y,\mathbf{Q_{j-1}},\mathbf{T_{j-1}})=g_{j-1}(O_{j-1}[y],\mathbf{Q_{j-2}},\mathbf{T_{j-1}}) - g_{j-1}(y-zQ_{j-1},\mathbf{Q_{j-2}},\mathbf{T_{j-1}}),$$

where

$$O_{j-1}[x]=\begin{cases}x, & x\le r_{j-1}(\mathbf{Q_{j-1}},\mathbf{T_{j-1}}),\\ x-mQ_{j-1}, & \text{otherwise.}\end{cases} \quad (1)$$

Here, $m\in\{0,\mathbb{I}\}$ such that $r_{j-1}(\mathbf{Q_{j-1}},\mathbf{T_{j-1}})+1\le x-mQ_{j-1}\le r_{j-1}(\mathbf{Q_{j-1}},\mathbf{T_{j-1}})+Q_{j-1}$; z is an integer such that $r_{j-1}(\mathbf{Q_{j-1}},\mathbf{T_{j-1}})+1\le y-zQ_{j-1}\le r_{j-1}(\mathbf{Q_{j-1}},\mathbf{T_{j-1}})+Q_{j-1}$.

Conditioning on $IOP_j(t)=y$, the inventory and penalty cost per period for stage j is

$$g_j(y,\mathbf{Q_{j-1}},\mathbf{T_j})=\frac{1}{T_j}\left(\sum_{\tau=0}^{T_j-1}\mathbf{E}\left[h_j(y-D[L_j+\tau]) + g_{j-1,j}\left(y-D\left[L_j+\left\lfloor\frac{\tau}{T_{j-1}}\right\rfloor T_{j-1}\right),\mathbf{Q_{j-1}},\mathbf{T_{j-1}}\right)\right]\right).$$

Again, since $IOP_j(t)$ is uniformly distributed, the average inventory and penalty cost per period for stage j is

$$\begin{aligned}\widehat{g}_j(r_j,\mathbf{Q_j},\mathbf{T_j}) &= \frac{1}{Q_j}\sum_{x=1}^{Q_j}g_j(r_j+x,\mathbf{Q_{j-1}},\mathbf{T_j}),\\ &= \frac{1}{Q_jT_j}\sum_{x=1}^{Q_j}\sum_{\tau=0}^{T_j-1}\mathbf{E}[h_j(r_j+x-D[L_j+\tau])] + P_j(r_j,\mathbf{Q_j},\mathbf{T_j}),\end{aligned}$$

where

$$P_j(y,\mathbf{Q_j},\mathbf{T_j})=\frac{1}{Q_jT_j}\sum_{x=1}^{Q_j}\sum_{\tau=0}^{T_j-1}\mathbf{E}\left[g_{j-1,j}\left(y+x-D\left[L_j+\left\lfloor\frac{\tau}{T_{j-1}}\right\rfloor T_{j-1}\right),\mathbf{Q_{j-1}},\mathbf{T_{j-1}}\right)\right],$$

the average induced-penalty cost per period. The optimal reorder point for stage j is $r_j(\mathbf{Q_j},\mathbf{T_j})=\arg\min_{r_j}\widehat{g}_j(r_j,\mathbf{Q_j},\mathbf{T_j})$.

With a slight abuse of notation, define $\widehat{g}_j(\mathbf{Q_j},\mathbf{T_j})=\widehat{g}_j(r_j(\mathbf{Q_j},\mathbf{T_j}),\mathbf{Q_j},\mathbf{T_j})$, the cost for stage j in which the optimal reorder point $r_j(\mathbf{Q_j},\mathbf{T_j})$ is implemented. Let the average total cost per period for stage j be

$$C_j(\mathbf{Q_j},\mathbf{T_j})=\left(\frac{K_j}{T_j}+\frac{k_j\lambda}{Q_j}+\widehat{g}_j(\mathbf{Q_j},\mathbf{T_j})\right).$$

The average system-wide cost per period is $\sum_{j=1}^{N}C_j(\mathbf{Q_j},\mathbf{T_j})$. The objective is to solve the following problem:

$$(P)\qquad \min_{\mathbf{Q},\mathbf{T}}\ \sum_{j=1}^{N}C_j(\mathbf{Q_j},\mathbf{T_j})$$
$$\text{s.t.}\quad T_{j+1}=n_jT_j,\quad Q_{j+1}=q_jQ_j,\quad j=1,\ldots,N-1.$$

Shang and Zhou [4] proposed a heuristic for (P). They first construct lower and upper bounds for the induced-penalty cost $P_j(r_j(\mathbf{Q_j},\mathbf{T_j}),\mathbf{Q_j},\mathbf{T_j})$ by regulating downstream policy parameters. By substituting these penalty bounds for the exact penalty cost function, they effectively construct bounds on the stage cost. A heuristic solution is obtained by minimizing the sum of these stage cost-bound functions subject to relaxed constraints on the batch sizes and reorder intervals.

Although the heuristic in [4] (referred to hereafter as "the SZ heuristic") is quite effective, there are two complexities involved in the computation. First, it is necessary to derive the cost-bound functions for each stage. Obtaining these cost-bound functions is quite complex because one first has to derive the induced-penalty cost function, and then regulate the policy parameters at the downstream stages. Second, it is necessary to execute a clustering algorithm, a technique that solves a separable and convex program subject to linear inequality constraints, in order to obtain the heuristic solution (see, e.g., [5] for a discussion of a clustering algorithm).

In this paper, we provide a much simpler heuristic that yet outperforms the SZ heuristic. This new heuristic can generate a solution without the two computational complexities mentioned above. From the implementation perspective, this heuristic can be implemented by a simple rule that shares the computation between the central planner and the stage managers. This contrasts with the SZ heuristic, where the central planner needs to conduct the majority of the computation.

2. The heuristic

The key idea of the heuristic is that we approximate the exact stage cost function $\widehat{g}_j(\mathbf{Q_j},\mathbf{T_j})$ by a cost function $\widehat{g}_j^h(Q_j,T_j)$ generated from a single-stage (r,nQ,T) system with the original problem data. Below we demonstrate how to construct this single-stage cost function.

Consider the information perceived by manager j under the scenario where centralized information is available. Manager j knows the echelon holding cost h_j and the real-time customer demand information. Also, since each stage manager shares a common goal of minimizing the supply chain cost, a stage manager is responsible for fulfilling the customer demand. Thus, when making the inventory decision, manager j should consider his *effective* lead time $L_{[1,j]}$, i.e., the duration between when an order is placed by stage j and when the order arrives at stage 1, assuming that stage $j+1$ has ample supply. Following the same logic, manager j should be charged a *perceived* backorder cost rate $b + h_{[j+1,N]}$ if he cannot fulfill the customer demand. (We refer the reader to [6] for an explanation of how the perceived backorder cost rate is derived.) With these perceived parameters, manager j would expect an average inventory holding and backorder cost per period

$$\widehat{g}_j^h(r_j, Q_j, T_j) = \frac{1}{Q_j}\sum_{x=1}^{Q_j} g_j^h(r_j + x, T_j),$$

where

$$g_j^h(y, T_j) = \frac{1}{T_j}\sum_{\tau=0}^{T_j-1} \mathbf{E}[h_j(y - D[L_{[1,j]} + \tau]) + (b + h_{[j,N]})(y - D[L_{[1,j]} + \tau])^-].$$

With fixed Q_j and T_j, $\widehat{g}_j^h(r_j, Q_j, T_j)$ is convex in r_j, manager j will choose the reorder point as $r_j(Q_j, T_j) = \arg\min_{r_j} \widehat{g}_j^h(r_j, Q_j, T_j)$. Let $\widehat{g}_j^h(Q_j, T_j) = \widehat{g}_j^h(r_j(Q_j, T_j), Q_j, T_j)$ and the average total perceived cost per period for stage j is $C_j^h(Q_j, T_j) = K_j/T_j + (k_j\lambda)/Q_j + \widehat{g}_j^h(Q_j, T_j)$. Our heuristic is to approximate $\widehat{g}_j(\mathbf{Q_j}, \mathbf{T_j})$ by $\widehat{g}_j^h(Q_j, T_j)$, which results in the following approximate problem to (P):

$$(AP) \qquad \min_{\mathbf{Q},\mathbf{T}} \sum_{j=1}^{N} C_j^h(Q_j, T_j)$$
$$\text{s.t.} \quad Q_{j+1} = q_j Q_j, \quad T_{j+1} = n_j T_j,\ j = 1, \ldots, N-1.$$

We propose a heuristic solution for (P) by solving (AP). The heuristic is to sequentially solve a deterministic demand model, a sub-problem with fixed reorder intervals, and a sub-problem with fixed batch sizes. For each of these problems, we further simplify the computation by solving a series of single-stage (r, nQ, T) models. We detail these steps below.

The Deterministic demand model.

We consider a deterministic demand model where the demand rate is λ, echelon holding cost rates h_j, backorder cost rate b, and order costs K_j. We solve this deterministic model to generate a set of initial reorder intervals.

To facilitate our discussion, we define *partition* and *cluster*. Let $S = \{1, 2, \ldots, N\}$. For any $i, j \in S$ with $i \le j$, the set $\{i, i+1, \ldots, j\}$ is called a cluster. A partition is the set of disjoint clusters and includes all elements in S.

Since we consider the deterministic demand model with planned backorders, h_1 should be set to

$$h_1 := \left(\frac{b}{b + h_{[1,N]}}\right) h_{[1,N]} - h_{[2,N]} \qquad (2)$$

for the subsequent analysis. Notice that the resulting h_1 may not be positive. In such a case, we need to adjust the other holding cost parameters. We refer the reader to [7] for the detailed analysis.

An efficient integer-ratio policy can be determined as follows. We first group stages into disjoint clusters $\{c(1), c(2), \ldots, c(M)\}$. Define

$$h[m] = \sum_{i\in c(m)} h_i, \qquad K[m] = \sum_{i\in c(m)} K_i, \quad m = 1, \ldots, M.$$

These clusters satisfy the following two conditions

(i) $K[1]/h[1] < \cdots < K[M]/h[M]$,

(ii) for each cluster $c(m) = \{l_1, \ldots, l_2\}$, there does not exist an l with $l_1 \le l < l_2$, so that $K[m^-]/h[m^-] < K[m^+]/h[m^+]$, where $c(m^-) = \{l_1, \ldots, l\}$ and $c(m^+) = \{l+1, \ldots, l_2\}$.

These conditions can be translated into a two-dimensional diagram that can easily identify the clusters. See [8], pp. 125-130.

Next, we use the approach in [9] to generate a common reorder interval for each cluster. That is, $T^d_{c(1)} = \sqrt{2K[1]/(h[1]\lambda)}$. For $m = 2, \ldots, M$, $T_{c(m)}$ is the solution to the following problem:

$$\min_T \left\{\frac{K[m]}{T} + \frac{h[m]\lambda T}{2}\right\}, \quad \text{s.t. } T = nT_{c(m-1)},\ n \in \mathbb{I}.$$

After all $T_{c(m)}$ are obtained, we set the initial reorder intervals as $T_i^d = T^d_{c(m)}$, for $i \in c(m)$, $m = 1, \ldots, M$.

The sub-problem with fixed reorder intervals

We fix reorder intervals to $(T_1^d, \ldots, T_N^d)$ in (AP), so the decision variables are batch sizes Q_j. We also relax the integer-ratio constraints to inequalities. Thus, the revised (AP) is

$$\min_{\mathbf{Q}} \quad \sum_{j=1}^{N} C_j^h(Q_j, T_j^d)$$
$$\text{s.t.} \quad Q_{j+1} \ge Q_j, \quad j = 1, \ldots, N-1.$$

We shall generate the heuristic batch sizes by executing the same two steps, clustering and minimization, explained below.

For the clustering step, we use k_j and h_j to identify clusters as we do in the deterministic model by replacing K_j with k_j in conditions (i) and (ii). Suppose that the resulting partition is $\{c(1), \ldots, c(Z)\}$. The stages in the cluster $c(z)$ choose a common batch size $Q_{c(z)}$ that minimizes their joint cost. That is, $Q_{c(1)} = \arg\min_Q \sum_{i\in c(1)} C_i^h(Q, T_i^d)$. For $z = 2, \ldots, Z$, we set

$$Q_{c(z)} = \arg\min_Q \sum_{i\in c(z)} C_i^h(Q, T_i^d), \quad \text{s.t. } Q = nQ_{c(z-1)},\ n \in \mathbb{I}.$$

Set $Q_i^h = Q_{c(z)}$ for $i \in c(z)$. Then, $(Q_1^h, \ldots, Q_N^h)$ will be our final heuristic batch sizes.

The sub-problem with fixed batch sizes

We fix batch sizes to $(Q_1^h, \ldots, Q_N^h)$ in (AP) and relax the reorder interval constraints. The revised (AP) is

$$\min_{\mathbf{T}} \quad \sum_{j=1}^{N} C_j^h(Q_j^h, T_j)$$
$$\text{s.t.} \quad T_{j+1} \ge T_j, \quad j = 1, \ldots, N.$$

Again, we conduct the clustering and minimization steps to obtain the heuristic reorder intervals. For the clustering step, we use the cost parameters K_j and h_j to find the partition, which is $\{c(1), \ldots, c(M)\}$ that we found in the deterministic model. The stages in the same cluster $c(m)$ choose a common reorder interval $T_{c(m)}$ that minimizes the joint cost. That is, $T_{c(1)} = \arg\min_T \sum_{i\in c(1)} C_i^h(Q_i^h, T)$. For $m = 2, \ldots, M$,

$$T_{c(m)} = \arg\min_T \sum_{i\in c(m)} C_i^h(Q_i^h, T), \quad \text{s.t. } T = nT_{c(m-1)},\ n \in \mathbb{I}.$$

The final heuristic reorder interval is $T_i^h = T_{c(m)}$ for $i \in c(m)$.

With (Q_j^h, T_j^h), one can find the best reorder point r_j^h by using the algorithm developed by Chao and Zhou [2]. The final heuristic solution for (P) is (r_j^h, Q_j^h, T_j^h), $j = 1, \ldots, N$.

3. Discussion

Identifying clusters by the ratios of fixed cost to echelon holding cost is a direct result of solving the relaxed deterministic model from Kuhn–Tucker conditions. Recently, Shang [10] considered echelon (r, nQ) policies and shows that the cost ratios can be used to generate an effective partition for the stochastic model. (The echelon (r, nQ) policies are a special case of echelon (r, nQ, T) policies by setting $T_j = 1$ for all j.) We generalize Shang's heuristic in two aspects. First, we include b in the clustering step. (When $b \to \infty$, (2) reduces to h_1, which is the original echelon holding

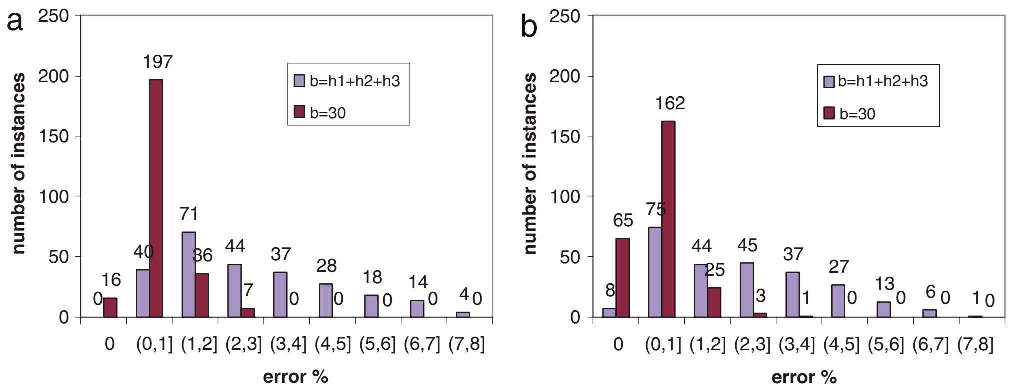

Fig. 1. (a) The error distribution of the heuristic. (b) The error distribution of [4]'s heuristic.

cost rate for stage 1 used in Shang's heuristic.) As we report in the sixth observation in Section 4, including b in clustering yields a more efficient solution. Second, we show how to apply the idea in [9] to the echelon (r, nQ, T) policy.

Our bottom-up approach for generating the heuristic batch sizes and reorder intervals is different from the approach for generating the so-called "power-of-two" policies (i.e., the batch sizes or the reorder intervals are power-of-two values; see [5,11]. Shang [9] used this approach to generate an integer-ratio solution that guarantees 94% effectiveness for the deterministic demand model. He numerically showed that such an integer-ratio solution outperforms the well-known power-of-two policy on average, if the base time period is restricted to be an integer. Here, we use the same idea to generate common batch sizes and reorder intervals for the stochastic model.

The heuristic can be applied to two commonly studied inventory policies, namely, the echelon (r, nQ) and (s, T) policies. (The (s, T) policy is a special case of the (r, nQ, T) policy by setting $Q_j = 1$ for all j.) To generate a solution for the (r, nQ) policy, all we need to do is to solve the second sub-model by replacing T_j^d with 1 in $C_j^h(Q_j, T_j^d)$. Similarly, we can solve the third sub-model to generate effective reorder intervals for the (s, T) policy by replacing Q_j^h with 1 in $C_j^h(Q_j^h, T_j)$. In fact, by setting $T_j = 1$ for all j, our heuristic is the same as the variant 1 heuristic proposed by Shang [10], with the exception of the approach for generating the common batch size. In [10], after clusters $c(z)$, $z = 1, \dots, Z$ are identified, the common batch size $Q_{c(z)}$ is obtained from the following procedure: Let $G_{c(z)}(y, Q) = \sum_{i \in c(z)} \widehat{g}_i^h(y, Q, 1)$ and $R_{c(z)}(Q) = \arg\min_y G_{c(z)}(y, Q)$. Then, $Q_{c(1)} = \arg\min_Q (\sum_{i \in c(1)} (k_i\lambda)/Q) + G_{c(1)}(R_{c(1)}(Q), Q)$. For $z = 2, \dots, Z$,

$$Q_{c(z)} = \arg\min_Q \left\{ \left(\sum_{i \in c(z)} \frac{k_i \lambda}{Q} \right) + G_{c(z)}(R_{c(z)}(Q), Q) \right\},$$

$$\text{s.t. } Q = nQ_{c(z-1)},\ n \in \mathbb{I}.$$

In other words, Shang's approach is to find the best common reorder point $R_{c(z)}(Q)$ for each cluster $c(z)$, while we allow each stage to find its best reorder point $r_j(Q, 1)$. Interestingly, the performance of our approach is similar to that of Shang's by testing the same numerical instances in Shang.

Finally, our heuristic leads to a simple rule that allows the central planner and the stage managers to share the computation when implementing the heuristic policy. The central planner only needs to (1) provide the initial reorder intervals by solving the corresponding deterministic model, (2) cluster the stages according to the cost parameters, and (3) announce that the stages in the same cluster should determine a common batch size or reorder interval. Then, the stage managers in a cluster only have to determine their best common batch size and reorder interval.

4. Numerical study

We examine the effectiveness of the proposed heuristic by using the same test bed in [4], where 512 instances are generated from the following parameters:

$$N = 3, \quad K_1, K_3 \in \{5, 50\}, \quad K_2 = 20, \quad k_1, k_3 \in \{1, 20\},$$
$$k_2 = 10, \quad h_1, h_3 \in \{0.1, 1\}, \quad h_2 = 1, \quad L_1, L_3 \in \{1, 3\},$$
$$L_2 = 2, \quad b = \{30, h_{[1,3]}\}.$$

The demand follows a Poisson distribution with $\lambda = 4$. Let C^h, C^{sz}, and C^* denote the cost of the heuristic, the cost of the SZ heuristic, and the optimal cost, respectively. We define the percentage error of the heuristic as

$$\epsilon^i\% = \frac{(C^i - C^*)}{C^*} \times 100\%, \quad i \in \{h, sz\}.$$

We provide several observations below.

(1) The average (maximum) percentage error for our heuristic among the 512 instances is 0.87% (5.92%), which is lower than 1.31% (7.67%) of the SZ heuristic. We find that both our heuristic and the SZ heuristic are more effective when b is large. For example, the percentage error for our heuristic is 1.38% when $b = h_{[1,3]}$ and is 0.36% when $b = 30$. The effectiveness of the heuristic does not seem to be related to the other parameters.
(2) In terms of the error distribution, our heuristic generates fewer optimal solutions than the SZ heuristic (53 vs. 73 instances). However, if we consider the case with the error less than 1%, our heuristic is more effective than that in [4] (384 vs. 310 instances). In particular, our heuristic seems to work well for the situation where b is small. See Fig. 1 for the detailed error distributions.
(3) Intuitively, the heuristic should perform better when K_1/h_1 and k_1/h_1 are large. This is because, under this condition, the heuristic tends to group all stages into one single cluster, resulting in the same batch size and reorder interval across all stages. This behavior is consistent with that of the optimal solution observed in [4]. Our numerical study confirms this intuition. For the 64 instances whose K_1/h_1 and k_1/h_1 are both strictly larger than the others, the average percentage error is 0.55%, which is less than the average error of 0.87%.
(4) The performance of our heuristic is highly related to whether or not an effective partition is determined, especially for the partition that generates reorder intervals. For the 5 instances whose error is greater than 5%, all of them have the following parameters: $(K_1, K_2, K_3) = (5, 20, 50)$, $(h_1, h_2, h_3) = (1, 1, 1)$, and $b = 3$. Under these parameters, three clusters are generated in the first and third steps when finding the heuristic reorder intervals. However, the optimal reorder intervals are $(T_1^*, T_2^*, T_3^*) = (6, 6, 6)$, which suggests that a single cluster

may be more preferable. Indeed, if we enforce a single cluster in the first and third steps, our heuristic generates the exact optimal reorder intervals.

(5) Our heuristic tends to generate more optimal reorder intervals than optimal batch sizes. In this test bed, our heuristic generates optimal reorder intervals and optimal batch sizes in a total of 256 and 89 instances, respectively.

(6) In our heuristic, if we ignore b in the clustering step, the resulting heuristic is less effective: The average (maximum) percentage error for the same 512 instances is 1.65% (8.37%).

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