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In this article, the authors consider the problem of outlet pricing and location in the context of unobserved spatial demand. The analysis constitutes a scenario in which capacity-constrained firms set prices conditional on their location, demand, and costs. This enables firms to develop maps of latent demand patterns across the market in which they compete. The analysis further suggests locations for additional outlets and the resultant equilibrium effect on profits and prices. Using Bayesian spatial statistics, the authors apply their model to seven years of data on apartment location and prices in Roanoke, Va. They find that spatial covariation in demand is material in outlet choice; the 95% spatial decay in demand extends 3.6 miles in a region that measures slightly more than 9.5 miles. They also find that capacity constraints can cost complexes upward of \$100 per apartment. As they predict, price elasticities and costs are biased downward when spatial covariance in demand is ignored. Costs are biased upward when capacity constraints are ignored. Using the analysis to suggest locations for entry, the authors find that properly accounting for spatial effects and capacity constraints leads to an entry recommendation that improves profitability by 66% over a model that ignores these effects.

Keywords: outlet location, pricing, spatial statistics, structural models, competition

The Role of Spatial Demand on Outlet Location and Pricing

Outlet location and pricing are of central concern to many firms. For example, the French retailer Carrefour SA added 5930 stores between 1999 and 2003, and the U.S. company Dollar General added 2426 stores in the same period (Euromonitor 2007). Paramount to expansion efficacy is the effect of outlet location on sales, prices, and profits, which are moderated by the underlying demand across regions in which a firm trades. Yet direct observation of demand across the areas in which firms trade is difficult;

not only is this demand apportioned among existing outlets located at a small (relative to the trade space) number of fixed points in space, but the latent demand also does not necessarily comport with the observed density of population. For example, the presence of complementary stores or desirable traffic patterns may elevate demand in a specific locale.

Given the central role of spatial distribution of demand in a firm's location decision, we focus on the inference of latent spatial demand. Integrating spatial statistics with a structural model of firm conduct enables us to capture a flexible distribution of latent spatial demand and use it to engage policy simulations regarding the sequential effect of locating an additional outlet on the demand, prices, and profits for the new and existing outlets. This enables us to address questions such as the following:

- Can the distribution of category demand across a given market or trade area be inferred from observations of retail sales at specific points in space? The answer to this question leads to a contour map of latent spatial demand that can be used to identify potential sites for new outlet entry or deletion.
- How do spatial demand and outlet location affect equilibrium prices and profits? We find that unobserved latent spatial

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demand leads to higher price variation over space and that a proper accounting of spatial demand effects improves the profitability of an entry recommendation by 66% over a model that ignores spatial covariance in demand and costs.

There has been considerable prior work in marketing regarding outlet sales dating back to the gravity model (see, e.g., Bucklin 1971; Ghose and Craig 1983). Our work differs from this research in several respects. First, this research typically assumes that prices are exogenous such that prices of competing outlets do not change with the location of an outlet. To the extent that firms react to the new entry by changing prices, models that ignore this could misstate potential profits. In contrast, our model captures spatial endogeneity in pricing through a structural link between random spatial demand effects and equilibrium prices. Second, such models typically do not estimate latent demand across regions (i.e., the demand apportioned to an outlet arising from a particular point in space) but rather assume that demand arises from observed differences in population across regions. In contrast, our work can accommodate unobserved sources of spatial demand. Third, we note that sales data necessary to estimate these models are often not observed, because firms sometimes keep their outlet sales data private (e.g., Wal-Mart). Our approach does not require information on outlet sales to infer latent spatial demand.¹

In this sense, our model of spatial demand is related more to analytical models of spatial location and demand in economics. Much of this work is theoretical, assumes a uniform distribution for spatial demand, and focuses on the equilibrium location of outlet location and the corresponding prices (Ansari, Economides, and Ghosh 1994; D'Aspremont, Gabszewicz, and Thisse 1979; Hotelling 1929). Recently, empirical models in economics and marketing have begun to appear that focus on solving the subgame of equilibrium prices or sales conditional on outlet location and capacity to infer latent spatial demand (Chan, Padmanabhan, and Seethuraman 2007; Davis 2001; Pinske, Slade, and Brett 2002; Thomadsen 2005, 2007; Venkataraman and Kadiyali 2005). This subgame is a reasonable starting point for the outlet location problem for several reasons. First, it must be solved before the optimal outlet location can be determined. Second, in a preponderance of markets, outlet locations are extant and fixed at the time a late entrant decides to enter, so the relevant managerial decision pertains to locating the next outlet and its capacity. Using the subgame, it is possible to explore the implications of adding an outlet on equilibrium prices and demand for the new and existing outlets. Third, competitive response latencies in constructing outlets can be large because of land acquisition, zoning, permitting, and construction. Therefore, competitive response in location may be impracticable over an intermediate duration, suggesting that a focus on the subgame is appropriate. A central innovation in these empirical models of spatial demand is that they consider observed spatial demand effects arising from

distance to some centroid, such as a population center or an airport. Our work complements the foregoing stream of research in two key ways.

First, we supplement observed spatial demand factors through spatially correlated unobserved demand effects.² Given that there are a plethora of potential spatial influences, it is unlikely that a researcher can capture all or most of them. For example, the presence of a particular employer, hotel, traffic pattern, school, restaurant, shop, family member, or friend could affect the choice of outlet and, therefore, equilibrium prices. Moreover, these influences tend to induce spatial covariance in demand shocks. The existence of spatial covariance in demand and supply shocks has several important implications for the outlet location problem.

Inclusion of unobserved spatial effects can enhance the efficiency of model estimates in information-poor environments in which covariates pertaining to spatial variation in demand are unobserved. We conjecture that such cases are more common than those of complete information. Because complete data are often prohibitively costly or unavailable, our approach affords a feasible alternative in information-poor environments.

The estimate of spatially correlated random effects in conjunction with spatial kriging yields a regional demand map that can serve as a decision aid to (1) find new locations and (2) afford insights into differences in demand across the map (e.g., a peak in a certain area might suggest an important omitted variable).

We explicitly consider the case of apartments, in which the observed population distribution cannot be used to capture latent spatial demand because the population location itself is endogenous. By including latent spatial random effects, we can capture such phenomenon. In other contexts, such as retail outlets, our model of random spatial effects can be augmented using observed distances between consumers and the outlets or demographics.

For the following four reasons, policy simulations pertaining to outlet location yield incorrect recommendations when spatial covariance is ignored. (In our simulation, the policy recommendation from a model that incorporates spatial covariance yields a locale that would increase profits by 66% over a model that ignores spatial covariation.)

1. Spatial random effects are biased toward zero when spatial covariance is ignored, leading to a downward bias in estimates of price parameter. The bias arises as a result of the small sample properties inherent in instrumental variable (IV) estimation (Altonji and Segal 1996; Buse and Moazzami 1991). We demonstrate these effects through simulation and a data application. Given that the number of outlets in a geographic trade area is often limited, the bias is consequential for the outlet location problem.
2. The downward bias in the price parameter also leads to a downward bias in the estimate for marginal costs. Because

¹By sales, we refer to the observed number of units sold at a particular outlet. By demand, we refer to the distribution of product utility across various regions. It is the distribution of demand across space that drives sales at given locales.

²We differentiate between location-specific random effects (e.g., Bayer and Timmins 2007) that are commonly employed in sorting models and spatial random effects. Spatial models allow for spatial covariance in the location-specific effects. These covariances have implications for spatial prediction and outlet location and pricing. Our work further differs from this research stream insofar as we consider the supply-side problem (price endogeneity) and capacity constraints in demand.

price is the sum of costs plus markups, for a given price, the lower estimated cost must be offset by a higher estimated markup.

3. The inclusion of spatial covariance in demand implies substitution patterns that are a function of entry location (apart from observed spatial differences). This is one approach to addressing the spatial independence-from-irrelevant-alternatives problem, in which expected share loss as a result of entry is apportioned not to proximal outlets but rather to the largest ones.
4. In the absence of spatial covariance, predictions for demand are incumbent solely on observed spatial differences and are likely to underestimate the true variation in demand. In our data, the omission of spatial covariance attenuates the standard deviation of spatial demand by a factor of 11.

Second, we consider the issue of outlet capacity because supply is not limitless. Extant models do not accommodate the possibility that capacity might be constrained (as would be the case with categories such as restaurants, hotels, or apartments). This can be problematic for several reasons:

1. In the context of policy simulations, moving an outlet (or adding a new one) cannot increase sales among extant outlets beyond their capacity.
2. Improperly accounting for this constraint leads to biased estimates for marginal costs. When demand exceeds supply, firms can raise prices with no effect on sales. The high prices associated with at-capacity firms leads to inferences of higher costs when capacity constraints are ignored. The additional error arising from poor predictions also leads to an increase in the estimated variance of the marginal cost equation in the supply-side model.
3. Our approach yields estimates of the costs of these capacity constraints (through estimates of the Kuhn–Tucker multiplier), which can be used to assess the merits of expansion.

In summary, our goal is to develop a flexible structural model of spatial demand to provide guidance to firms considering the potential location of new outlets (or changing the location of existing ones). In this sense, our work lies at the intersection of two developing research streams in marketing: empirical economics (Chintagunta et al. 2006) and spatial statistics (e.g., Bronnenberg and Mahajan 2001; Bronnenberg and Mela 2004; Bronnenberg and Sismeiro 2002). As such, we augment spatial statistics with structural models of pricing. Then, we employ these spatial methods in a different context: outlet location. Finally, given the emphasis on outlet location, we integrate capacity constraints into a spatial model.

The article proceeds as follows: We begin by developing the demand-side model. We then generalize the model to consider the pricing problem firms face in the presence of capacity constraints, latent spatial demand, and the location of other firms. Then, we outline how to estimate the model and, using these estimates, how to forecast demand prices and profits arising from the entry of a new outlet at any given location. We then describe the apartment data used to calibrate the model and follow this with both simulated and data-based results. We use the simulated data to show the biases that arise when spatial covariance and capacity constraints are omitted, and the real data show that these spatial effects lead to considerable improvements in model performance. We conclude with the results of our data application, suggested locations for additional outlets, and directions for further research.

THE MODEL

Our model presentation proceeds as follows: First, we present the consumer demand model. Specifically, we apply an individual-level, random utility, discrete choice model with spatially correlated random effects and use this model to infer the aggregate demand function. Second, we outline the equilibrium conditions of supply and demand in the context of firms that own outlets that are capacity constrained. The equilibrium conditions are derived from the profit-maximizing behavior of the firms and the utility-maximizing behaviors of the consumers. The market-clearing condition establishes the economic equilibrium of demand and supply, which leads to the structural estimation equations. These equilibrium conditions constitute a system of equations that we estimate using advances in Bayesian spatial statistics.

Demand Model: Discrete Choice

Suppose that there are J outlets for a given type of goods (e.g., apartments, car dealers, hotels, bank branches) in a given region. The location of the $j \in J$ outlet is denoted as s_j . Let the number and the set of the potential customers for a set of outlets in a region be I , $i = \{1, \dots, I\}$. Customer i 's random indirect utility for choosing outlet j at location s_j in period t is as follows:

$$(1) \quad V_{tij} = X_{ij}\beta_1 + X_{ts_j}\beta_2 - P_{ij}\gamma_i + \theta_{ts_j} + \varepsilon_{tij}.$$

In this equation, X_{ij} represents the attributes of the j th outlet, such as variety or amenities; X_{ts_j} represents the observed attributes specific to the location (e.g., the distance to a business center or a major highway); and P_{ij} is a price index for an outlet. As is common in random utility models of choice, ε_{tij} is assumed to be independently drawn from a Gumbel distribution. Consumers' sensitivities to price change are assumed to be normally distributed across the population: Let $\gamma_i = \gamma + \eta_i$, where $\eta_i \sim N(0, \sigma_\gamma^2)$. We focus on price response heterogeneity to reduce the model's dimensionality. Gowrisankaran and Rysman (2007) find that adding random effects to nonprice attributes essentially leads to no change in the model parameters and that the random effects for these attributes do not differ from zero.

The θ_{ts_j} , indexed by time t and location s_j , is the time-varying random demand shock. We decompose $\theta_{ts_j} = \theta_{s_j} + v_{ts_j}$, where θ_{s_j} is a time-invariant spatial random effect that is fixed over all the observed periods and v_{ts_j} is a spatial random effect that is independent across time.³ This decomposition implies that there is a systematic location-specific long-term effect (to capture unobserved spatial effects that do not vary over the range of the data, such as the proximity to urban amenities, distance to major employers, and school zoning) and a short-term perturbation about this mean, v_{ts_j} , to capture unobserved spatial fac-

³We considered other specifications for the temporal dependency in spatial shocks. In particular, we computed $\rho(v_{ts_j}, v_{t-1, s_j})$ for each of the J firms for each of the draws of the v_{ts_j} in the Markov chain Monte Carlo sampling chain described in the Appendix. The mean of ρ is .16, and the variance is .23. Thus, autocorrelations appear statistically small. We conjecture that the large dispersion in autocorrelations results from having a limited number of periods in our data (six periods) from which to compute them.

tors that may vary over time (e.g., construction, traffic patterns). As a result, both θ_{s_j} and v_{ts_j} may be spatially correlated. We assume that $\theta = (\theta_{s_j}, j = 1, \dots, J) \sim N[0, \sigma_\theta^2 R_J(\phi_\theta)]$ and $v_t = (v_{ts_j}, j = 1, \dots, J) \sim N[0, \sigma_v^2 R_J(\phi_v)]$, whose joint effect is as follows:

$$(2) \quad \theta_t = \theta + v_t \sim N[0, \sigma_\theta^2 R_J(\phi_\theta) + \sigma_v^2 R_J(\phi_v)].$$

The entries of the correlation matrix $R_J(\phi)$ ($\phi = \phi_\theta$ or ϕ_v) can be constructed with an isotropic exponential decay function, $\exp(-\phi \times d)$, where d is the distance between any two locations, $d = ||s_i - s_j||$; the Matérn class (Banerjee, Carlin, and Gelfand 2004); or a more general anisotropic nonparametric spatial Dirichlet process model (Duan, Guindani, and Gelfand 2007; Gelfand, Kottas, and MacEachern 2005). We note that these spatial structures are highly flexible and admit many potential latent demand surfaces.

Customer i may choose the outside good; that is, he or she may not buy in the region at all or may consider expenditures on other types of goods sold in the region. The customer's indirect utility for the outside good is given by

$$(3) \quad V_{i0} = M_t + \varepsilon_{i0}.$$

If the highest utility of choosing one outlet exceeds that of the outside good, the customer will select the highest-utility outlet.

The customer chooses the outlet with the highest utility. If we define the mean effects in the utility function as $\xi_{ij} = X_{ij}\beta_1 + X_{ts_j}\beta_2 - P_{ij}\gamma + \theta_{ts_j}$, the customer utility function in Equation 1 can be rewritten as $V_{ij} = \xi_{ij} - P_{ij}\eta_i + \varepsilon_{ij}$, where η_i and ε_{ij} capture consumer heterogeneity. We assume that the distributions for the random effects ε_{ij} and η_i are common knowledge to firms. This assumption is necessary for firms to form expectations regarding their market share. The Gumbel error assumption for ε_{ij} results in a logit choice likelihood that customer i chooses outlet j :

$$W_{ij} = \frac{e^{\xi_{ij} - P_{ij}\eta_i}}{\sum_{l=1}^J e^{\xi_{il} - P_{il}\eta_i} + e^{M_t}}.$$

We obtain the expected market share for outlet j , W_{tj} , by integrating over the remaining random effect, η_i , which yields the following:

$$(4) \quad W_{tj} = \int \frac{e^{\xi_{tj} - P_{tj}\eta_i}}{\sum_{l=1}^J e^{\xi_{tl} - P_{tl}\eta_i} + e^{M_t}} dF(\eta_i).$$

Multiplying the number of people in the market, I , by firm j 's share leads to the total expected demand, $Q_{tj} = I \times W_{tj}$. The second term in the denominator reflects the demand for the outside good. Because I , M_t , and the intercept of the additive utility cannot be separately identified at the same time, we make I and M_t constant, as we discuss in the "Data" section.

Berry, Levinsohn, and Pakes (1995, hereinafter BLP) prove that W_{tj} and ξ_{tj} is a one-to-one mapping conditioning on the distribution of η_i by a contraction mapping theorem. The natural algorithm derived from the contracting mapping to compute ξ_{tj} by setting the observed $\hat{W}_{tj} = W_{tj}$ is as follows:

$$(5) \quad \xi_t^{(g+1)} = \xi_t^{(g)} + \ln \hat{W}_t - \ln W_t \left[\xi_t^{(g)}, F(\eta_i) \right],$$

where g is the index of iterations. This algorithm inverts the vector function in Equation 5 from the observed shares \hat{W}_t to the unobserved errors ξ_t (asymptotically approached by $\hat{\xi}_t^{(g)}$). This inversion leads to a nonlinear transformation of the data, given that $F(\eta_i) = N(0, \sigma_\eta^2)$. Note that $\hat{\xi}_t$ is linear in the demand-side model parameters (β , γ , and θ), which facilitates the demand-side estimation.

Supply Model: Bertrand–Nash Game

In addition to consumers, the market comprises firms that compete on the basis of price, location, and capacity. As noted previously, we focus on the subgame of firm competition conditional on location choice and capacity. We assume that a market is composed of F firms and that each firm f has a set F_f of outlets. Each firm maximizes the total profit of its outlets F_f . We first consider firm f 's strategy. Conditioning on the prices of the outlets not belonging to f 's chain, firm f faces the following profit maximization problem:

$$(6) \quad \max \Pi_{tf} = \sum_{j \in F_f} (P_{tj} - c_{tj}) Q_{tj},$$

subject to $Q_{tj} \leq K_j$,

where c_{tj} represents the variable cost of goods sold and K_j is outlet j 's capacity constraint, which is the maximum number of goods the outlet can produce, sell, or rent. The capacity is considered rigid over the short decision frame of a firm setting prices because the expansion or contraction of an outlet's physical plant involves construction/permitting time and considerable cost. This implies that the capacity consideration is relegated to an "upper stage" of the game, in which a firm sets location and capacity before choosing price in the subgame. When a firm faces a different demand at time t as a result of setting the price P_{tj} , K_j becomes an immutable short-term constraint to which the outlet is subject.

Because the econometrician does not observe the firms' variable costs c_{tj} , we model them as

$$(7) \quad c_{tj} = Y_{tj}\beta_3 + \zeta_{ts_j},$$

where Y_{tj} is a set of cost shifters (e.g., the type of inventory) and ζ_{ts_j} are spatially correlated cost shocks. We further decompose the time-varying cost shock as $\zeta_{ts_j} = \zeta_{s_j} + e_{ts_j}^c$, where ζ_{s_j} is a time-invariant random effect that is fixed over the observed periods and $e_{ts_j}^c$ is the cost shock that is independent across time. The ζ_{s_j} effect captures the long-term cost factors that are not observed and controlled in cost shifters Y_{tj} . It can include the labor, maintenance, administrative costs, tax, insurance, and other unobserved factors, whereas $e_{ts_j}^c$ is the short-term changes in these factors. Accordingly, ζ_{s_j} and $e_{ts_j}^c$ are likely to be spatially correlated as a result of these omitted variables in the cost model. We assume the following:

$$\zeta = (\zeta_{s_j}; j = 1, \dots, J) \sim N[0, \sigma_\zeta^2 R_J(\psi_\zeta)], \text{ and}$$

$$e_t^c = (e_{ts_j}^c; j = 1, \dots, J) \sim N[0, \sigma_e^2 R_J(\psi_e)],$$

where $R_j(\psi)$ ($\psi = \psi_\zeta$ or ψ_c) can be of the same or different functional form of the correlation in spatial demand $R_j(\phi)$.

The Kuhn–Tucker conditions for this optimization problem with inequalities constraints are as follows:

$$(8) \quad \frac{\partial \Pi_{tf}}{\partial P_{tk}} = \sum_{j \in F_f} (P_{tj} - Y_{tj}\beta_3 - \zeta_{tj}) \frac{\partial Q_{tj}}{\partial P_{tk}} + Q_{tk} - \sum_{m \in F_f} \lambda_{tm} \frac{\partial Q_{tm}}{\partial P_{tk}} = 0; k \in F_f,$$

with the Kuhn–Tucker multipliers, $\lambda_{tm} \geq 0$, and

$$(9) \quad Q_{tm} = K_m, \text{ iff } \lambda_{tm} > 0, \text{ and } Q_{tm} < K_m, \text{ iff } \lambda_{tm} = 0.$$

These multipliers reflect the marginal cost of the capacity constraint on apartment profitability and reflect the value of expanding a particular outlet’s capacity by one unit.

Because $Q_{tj} = I \times W_{tj}$, the representation of Equation 8 with market shares is as follows:

$$(10) \quad \sum_{j \in F_f} (P_{tj} - Y_{tj}\beta_3 - \zeta_{tj}) \frac{\partial W_{tj}}{\partial P_{tk}} + W_{tk} - \sum_{m \in F_f} \lambda_{tm} \frac{\partial W_{tm}}{\partial P_{tk}} = 0; k \in F_f.$$

The Kuhn–Tucker conditions imply that firm f chooses optimal prices P_{tk}^* for the outlets with expected vacancy and the second-best price \hat{P}_{tm} for outlets at capacity. The term P_{tk}^* is the solution of the first-order condition in Equation 10, where $\lambda_{tk} = 0$, whereas \hat{P}_{tm} satisfies the binding capacity constraint in Equation 9. We solve P_{tk}^* , \hat{P}_{tm} , and the nonzero multiplier λ_{tm} simultaneously.

There are $|F_f|$ ($|F_f|$ denotes the number of outlets in F_f) equations arising from the first-order conditions for each firm. To facilitate explication, we define a $|F_f| \times |F_f|$ matrix, in which the (j, k) th element is

$$(11) \quad \Omega_{jk}^{(t,f)} = \frac{\partial W_{tj}}{\partial P_{tk}} = \begin{cases} -\int (\gamma + \eta_i) W_{tj} (1 - W_{tj}) dF(\eta_i), & \text{for } j = k \in F_f \\ \int (\gamma + \eta_i) W_{tj} W_{tik} dF(\eta_i), & \text{for } j \neq k \in F_f \end{cases}$$

Let the vectors $\lambda_{tf} = (\lambda_{tm}, m \in F_f)$, $P_{tf} = (P_{tj}, j \in F_f)$, $W_{tf} = (W_{tj}, j \in F_f)$, and $\zeta_{tf} = (\zeta_{tsj}, j \in F_f)$ and the submatrix $Y_{tf} = (Y_{tj}, j \in F_f)$. Note that for the outlets with spare capacity, $\lambda_{tm} = 0$, and for the outlets with full capacity, $W_{tj} = K_j/I$. With these definitions, we can rewrite the first-order conditions in Equation 10 in matrix form as follows:

$$\Omega^{(t,f)}(P_{tf} - Y_{tf}\beta_3 - \lambda_{tf}) + W_{tf} = \Omega^{(t,f)}\zeta_{tf}.$$

Likewise, for all the J outlets belonging to F firms in the market, we define the following $J \times J$ matrix whose (j, k) th element is as follows:

$$(12) \quad \Omega_{jk}^{(t)} = \begin{cases} \frac{\partial W_{tj}(P_{tj}, P_{t,-j}, X_t, \theta_t)}{\partial P_{tk}}, & \text{for } j, k \in \text{same } F_f \\ 0 & \text{otherwise} \end{cases}$$

We can write the first-order conditions for all the outlets in matrix form as

$$(13) \quad \Omega^{(t)}(P_t - Y_t\beta_3 - \lambda_t) + W_t = \Omega^{(t)}\zeta_t.$$

If $\Omega^{(t)}$ is invertible, we have the following:

$$(14) \quad P_t - \lambda_t + \Omega^{(t)-1}W_t = Y_t\beta_3 + \zeta_t.$$

In Equation 14, $B_t(\lambda_t, \gamma) \equiv \lambda_t - \Omega^{(t)-1}W_t$ indicates the firms’ markups because this expression represents the difference between a firm’s prices and its costs. Note that $\lambda_{tm} = 0$ for the outlets that have vacancy and $\lambda_{tm} > 0$ for the outlets with full occupancy. Thus, markups increase when firms are capacity constrained. The economic interpretation of this equation is that the prices of the outlets with full capacity are higher than their optimal prices. Intuitively, this suggests that firms that face demand in excess of supply will raise prices to the point at which demand is equal to capacity; lower prices serve only to decrease revenue.

Of interest, Equation 14 embeds all parameters in the supply and demand system. This suggests that both the demand-side and the cost-side parameters can be estimated, even in the absence of any demand-side data (Thomadsen 2005). Similar to BLP and others, we assume that these parameters, though unobserved to the econometrician, are known to the firm and to the consumers.

ESTIMATION

We follow a two-stage estimation approach analogous to two-stage least squares as in BLP and Nevo (2001). In the first stage, we estimate the demand-side model to obtain estimates for γ , $\Omega^{(t)}$, and W_t , which we denote as $\hat{\gamma}$, $\hat{\Omega}^{(t)}$, and \hat{W}_t . In the second stage, we treat these estimated demand-side parameters as random variables in the supply-side model to obtain the estimates for costs and the Lagrange multipliers.

Despite the loss of statistical efficiency that would be gained by using a one-stage approach to estimate the demand-side parameters (i.e., using information from both the demand and the supply models to infer the demand parameters), the two-stage approach offers several advantages over a one-stage approach. First, a two-stage approach mitigates the need to use IV estimation on the supply side.⁴ Second, the Jacobian required for joint estimation is complex, rendering estimation substantially more cumbersome. Third, should the supply-side model be misspecified, the resultant demand-side parameters will be biased (Chintagunta, Kadiyali, and Vilcassim 2006).

Demand-Side Estimation

The demand-side estimation equations for all the outlets in the market are as follows:

⁴The need for IV estimation in joint inference for γ arises because prices in the supply-side model of Equation 14 are a function of $\Omega^{(t)}[\gamma, P(\zeta_t)]$ and $W_t[\gamma, P(\zeta_t)]$, which depend on prices, which in turn depend on the cost shocks. Thus, $\Omega^{(t)}$ and W_t are endogenous to the errors in the cost equation. This suggests the need for IV estimation when they are unknown. Of note, Ω and W enter the pricing model in a nonlinear fashion. The properties of nonlinear IV estimators are not clear; as a result, we cannot be sure that an IV approach on the supply side leads to unbiased estimates for γ .

$$(15) \quad W_{ij} = \frac{Q_{ij}}{I} = \int \frac{e^{\xi_{ij} - P_{ij}\eta_i}}{\sum_{l=1}^J e^{\xi_{il} - P_{il}\eta_i} + e^{M_t}} dF(\eta_i),$$

where $Q_{ij} = K_j$ if the outlet is at capacity (e.g., a full outlet) in that period. There is no closed form for the integral in Equation 15. However, given the distribution of random price effects, $F(\eta_i)$, the integration involved to compute $W_t[\xi_t^{(g)}, F(\eta_i)]$ can be approximated with a Monte Carlo integration. The integration proceeds by simulating I_s individual η_i from the distribution $F(\eta_i)$ and approximating the integral in Equation 15, as follows:

$$(16) \quad W_{ij}(\xi_{ij}) = \frac{I_s}{\sum_{i=1}^J \sum_{l=1}^J} \frac{e^{\xi_{ij} - P_{ij}\eta_i}}{e^{\xi_{il} - P_{il}\eta_i} + e^{M_t}}.$$

Instead of using a maximum likelihood method to estimate model parameters, BLP propose an IV-based approach. An appealing aspect of this approach is that unobserved effects affect both spatial demand and prices, and the IV approach controls for this possibility by forming instruments for price. Stacking the ξ_{ij} in Equation 16 across j , we obtain the following:

$$(17) \quad \xi_t = X_t\beta_1 + X_{ts}\beta_2 - P_t\gamma + \theta + v_t.$$

The random spatial errors θ and v_t in Equation 17 can be structurally correlated with the prices P_{ij} as a result of the endogeneity of price to random spatial demand effects; prices tend to increase when demand is high.

Therefore, our goal is to compute ξ_t from Equation 16, so that we can use the orthogonality conditions between the spatial errors v_t in Equation 17 and instruments Z_t to estimate β_1 and β_2 (i.e., $E[Z_t^T v_t] = 0$). In their study, BLP prove that estimates $\hat{\xi}_t$ are computed using the iterative procedure, $\hat{\xi}_t^{(m+1)} = \hat{\xi}_t^{(m)} + \ln \hat{W}_t - \ln W_t(\hat{\xi}_t^{(m)})$, by showing that this equation is a contraction mapping. When we obtain $\hat{\xi}_t$ (approximated by $\xi_t^{(m)}$, as the sequential discrepancy between $\xi_t^{(m)}$ and $\xi_t^{(m+1)}$ approaches zero), we consider the following linear model:

$$(18) \quad v_t = \hat{\xi}_t(\gamma) - X_t\beta_1 - X_{ts}\beta_2 + P_t\gamma - \theta,$$

which is mean independent of the instruments Z_t and has a spatial covariance structure $N[0, \sigma_v^2 R_J(\phi_v)]$, given the true β_1 , β_2 , γ , and θ .⁵ We assume an exponential correlation function (Banerjee, Carlin, and Gelfand 2004) for the spatial random effects—that is, $R_J(\phi) = \exp(-\phi \times ||s - s'||)$. This function implies an exponential decay in the covariance of spatial effects over distance.⁶

Using a likelihood approach (Chernozhukov and Hong 2003; Kim 2002; Romeo 2007), we let $Z_t^T v_t \sim N[0, \sigma_v^2 Z_t^T R_J(\phi_v) Z_t]$, or equivalently,

⁵Unlike the time-varying spatial effect v_t , θ is a time-invariant spatial random effect. Accordingly, it need not be instrumented. Analogous to Nevo's (2001) use of brand-specific dummies, θ_s can be estimated using location-specific dummy variables and the prior distribution $N[0, \sigma_\theta^2 R_J(\phi_\theta)]$.

⁶Inspection of the residual variograms provides evidence that the exponential function fits the data well. Furthermore, these residuals reveal little evidence of any anisotropy in spatial decay.

$$(19) \quad Z_t^T (\hat{\xi}_t - X_t\beta_1 - X_{ts}\beta_2 + P_t\gamma - \theta) \sim N[0, \sigma_v^2 Z_t^T R_J(\phi_v) Z_t].$$

Note that this likelihood, denoted as $L(\xi_t)$, is specified over ξ_t , but the observed response variable is W_{ij} . Therefore, the likelihood for observed shares involves a nonlinear transformation of variables from W_t to $\xi_t(W_t)$. Thus, we rewrite the likelihood $L(W_t) = L[\xi_t(W_t)] |\partial \xi_t / \partial W_t|$, where the Jacobian is given by

$$(20) \quad \left| \frac{\partial \xi_t}{\partial W_t} \right| = \begin{cases} \left[\int W_{ij} (1 - W_{ij}) dF(\eta_i) \right]^{-1}, & \text{if } j = k \\ - \left[\int W_{ij} W_{ik} dF(\eta_i) \right]^{-1}, & \text{if } j \neq k \end{cases}.$$

Supply-Side Estimation

In the supply-side model, we attempt to infer both the parameters in cost function and the nonzero Lagrangian multipliers. As noted in the "Model" section, we use observed demand market share \hat{W}_{ij} to replace the expected market share W_{ij} in Equation 14. For the outlets at capacity constraint in period t , $\hat{W}_{ij} = K_j/I$ according to the Bertrand–Nash equilibrium. Using the price parameter estimates $\hat{\gamma}$ and $\hat{\sigma}_\gamma$ from the demand model, we compute $\hat{\Omega}^{(t)-1} \hat{W}_t$ and impute it into the first-order conditions:

$$(21) \quad \begin{aligned} P_t - \lambda_t + \hat{\Omega}^{(t)-1} \hat{W}_t &= Y_t \beta_3 + \zeta_t, \text{ and} \\ W_{ij}(P_t, X_t, \theta_t) &= K_j/I \text{ if } \lambda_{ij} > 0. \end{aligned}$$

The markup for the outlet chain model is $\lambda_t - \hat{\Omega}^{(t)-1} \hat{W}_t$, in which the subvector $\lambda_{1t} = 0$ for the outlets under capacity. For these outlets, $P_{1t} + [\hat{\Omega}^{(t)-1} \hat{W}_t]_1 = Y_{1t} \beta_3 + \zeta_1 + e_{1t}^c$. For the outlets at capacity, we have $\lambda_{2t} > 0$, or equivalently, $P_{2t} + [\hat{\Omega}^{(t)-1} \hat{W}_t]_2 > Y_{2t} \beta_3 + \zeta_2 + e_{2t}^c$.⁷ To use the full information available to infer the cost parameters, we supplement observed data from apartments below capacity with augmented data λ_{2t} for firms at or under capacity, forming the following likelihood:

$$(22) \quad \begin{aligned} & \left(\begin{array}{c} P_{1t} + [\hat{\Omega}^{(t)-1} \hat{W}_t]_1 - Y_{1t} \beta_3 - \zeta_1 \\ P_{2t} + [\hat{\Omega}^{(t)-1} \hat{W}_t]_2 - Y_{2t} \beta_3 - \zeta_2 - \lambda_{2t} \end{array} \right) \\ & \sim N \left(0, \sigma_c^2 \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \right), \end{aligned}$$

where we partition the covariance matrix $\sigma_c^2 R(\psi_c)$ into four submatrices. To draw the augmented parameter λ_{2t} for use in Equation 22, we use the following truncated distribution, derived in the Appendix:

$$(23) \quad N \left[\lambda_{2t} | \bar{P}_{2t} + R_{21} R_{11}^{-1} \bar{P}_{1t}, \sigma_c^2 (R_{22} - R_{21} R_{11}^{-1} R_{12}) \right] I \{ \lambda_{2t} > 0 \},$$

where $\bar{P}_{kt} = P_{kt} + [\hat{\Omega}^{(t)-1} \hat{W}_t]_k - \zeta_k - Y_{kt} \beta_3$, $k = 1, 2$.

⁷Although the condition $P_{2t} + [\hat{\Omega}^{(t)-1} \hat{W}_t]_2 > Y_{2t} \beta_3 + \zeta_2 + e_{2t}^c$ might suggest that ζ_2 and e_{2t}^c are bounded from above, we note that P_{2t} is set after firms observe ζ_2 and e_{2t}^c , and P_{2t} can be adjusted with ζ_2 and e_{2t}^c . Thus, this inequality does not place constraints on the cost shocks.

As in the demand model, we assume an exponential correlation function (Banerjee, Carlin, and Gelfand 2004) for the spatial random effects ζ and e_t^c . That is, $R_j(\psi) = \exp(-\psi \times ||s - s'||)$. We assume that the spatial random error v_t in the indirect utility function is independent of the short-term supply-side error e_t^c . This does not mean that demand and supply are independent; these are linked through the structural effect of observed effects and other factors on equilibrium prices. Note that IV estimation on the supply side is not required, because none of cost shifters in Y_t are correlated with e_t^c .

Model Inference

We use Bayesian inference to estimate the parameters in this model. The Bayesian estimation is accomplished by a Metropolis-within-Gibbs sampler, in which σ_γ^2 is sampled using the Metropolis-Hastings algorithm. The Appendix provides the full conditional distributions for the sampling chain.

IVs

Selection of instruments. In addition to price, the spatial shocks v_{ts_j} may correlate with the observed attributes X_{tj} and X_{ts_j} (e.g., amenities may be more prevalent in more desirable areas). This potential correlation suggests that the use of local attributes as instruments, as in BLP (i.e., $E[v_{ts_j}|X_{tj}, X_{ts_j}] = 0$), does not hold in the context of the outlet location problem. Instead, we consider as potential instruments Z_t (1) the proximity to work X_{ts_1} (from the same chain and from competitors), (2) attributes of distant outlets X_{dl} (e.g., clubhouse, tennis, swimming pool, gymnasium, and heat attributes from the same chain and from competitors), (3) distant cost shifters Y_{dl} (from the same chain and from competitors), and (4) lagged distant prices $P_{t-1,l}$ (from the same chain and from competitors). These are given, respectively, by the following:

$$(24) \quad Z_{tj} = \left[\begin{array}{ccc} \sum_{\substack{1 \neq j; l, j \in F_f \\ \text{dist}(l, j) > r}} X_{ts_1}, & \sum_{\substack{1 \neq j; l, j \in F_f \\ \text{dist}(l, j) > r}} X_{ts_1}, & \sum_{\substack{1 \neq j; l, j \in F_f \\ \text{dist}(l, j) > r}} X_{dl}, \\ \sum_{\substack{1 \neq j; l, j \in F_f \\ \text{dist}(l, j) > r}} X_{dl}, & \sum_{\substack{1 \neq j; l, j \in F_f \\ \text{dist}(l, j) > r}} Y_{dl}, & \sum_{\substack{1 \neq j; l, j \in F_f \\ \text{dist}(l, j) > r}} Y_{dl}, \\ \sum_{\substack{1 \neq j; l, j \in F_f \\ \text{dist}(l, j) > r}} P_{t-1,l}, & \sum_{\substack{1 \neq j; l, j \in F_f \\ \text{dist}(l, j) > r}} P_{t-1,l} & \end{array} \right],$$

where $1, j \notin F_f$ denotes that outlets l and j are not in the same outlet chain; $\text{dist}(l, j)$ is the distance between l and j ; and r is the cutoff distance for a competitor's inclusion in the summation. The considered instruments should be independent of spatial shocks and correlated with the endogenous prices. In light of this, we discuss our rationale for this set of instruments next.

Attributes as instruments. Similar to BLP and Nevo (2001), we use attributes in other locations as instruments. However, in our context, the unobserved spatial random effect v_{ts_j} may be correlated with the local observed attributes X_{tj} , X_{ts_j} , and cost shifter Y_{tj} . This correlation will decrease as outlet locations become far from one another.

Beyond a distance r (which is called the “range” effect in spatial statistics), the correlation between the instruments and the spatial shock will approach zero. Moreover, if this cutoff distance is not sufficiently large, the attributes at other outlets will affect the local apartment's choice probability in the logit demand system and therefore can be expected to correlate with current prices. In summary, with the proper choice of r , the distant attributes are uncorrelated with the errors but correlated with prices. The selection of the cutoff distance r must consider several factors. Selecting too large a distance will cause some outlets to have no instruments because no other outlets exist beyond a large distance. Conversely, selecting too small a distance will lead to high spatial correlations between the competing apartment attributes and the local spatial shocks. Given that the nature of competition differs for other locations within a chain and other locations within competing chains, we divide these instruments into a same-ownership group and a different-ownership group. An analogous logic holds to justify our choice of cost shifters and distance to points of interest as instruments.

Prices as instruments. In our data, the correlation between lagged distant own outlet prices and price is .12, and the correlation between lag distant competitor outlet prices and price is -.54, indicating that lag distant prices may be good candidates for instruments. Our finding that P_{tj} correlates positively (negatively) with P_{tk} of outlet k in its own (a competing) chain is also confirmed in our simulation. These correlations can be explained as follows: First, lag distant prices are correlated with the long-term demand shock for these locales, θ_{s_j} . Second, the long-term demand shocks are correlated with prices. An increase in demand shocks for competing distant outlets makes those competing outlets more attractive. As a result, local firms must lower prices to compete. The price rule in Equation 10 suggests that a firm adjusts the prices of all its own outlets in the same direction. Thus, there is a positive correlation between the own chain and the local outlet. In this sense, we argue that lagged distant prices for own and competing chains are correlated with current local price and are reasonable choices for instruments.

We further reason that lagged distant prices are independent of local demand shocks, v_t , because (1) distant prices are correlated with distant demand shocks and (2) distant demand shocks are independent of local demand shocks and serially independent.⁸ This logic rests on the assumption that the correlation of the demand shock does not extend as far as the effect of price on competing outlets. This assumption is likely to be true when the spatial decay is large (i.e., no spatial correlation) or the distance between the outlet and the competitive outlets used for instrumenting is sufficiently large.

Test of instruments. To explore the quality of our instruments further, we explored the orthogonality of the instruments by inspecting the plots of the marginal posterior

⁸Using lagged distant price as an instrument affords advantages over using distant price as in Nevo (2001) when capacity constraints are present. Firms at capacity raise prices, leading other firms to raise prices, implying that current distant prices might not be good instruments. Because lagged demand shocks do not affect whether capacity binds in the current period, they are not afflicted by this consideration.

distributions for $Z_t^T v_t$. We compute the 95% posterior predictive interval to assess whether it excludes 0 as a test of the orthogonality conditions. We find that all the intervals contain 0. However, this ignores the joint distribution of the orthogonality conditions. Therefore, we conducted an overidentifying test (J-test) of the quality of the instruments using the demand-side model. This test exploits the idea that the use of instruments that are correlated with the error will lead to biased parameter estimates, thus leading to poor fit for the moment conditions of instruments that are not (i.e., $Z_t^T v_t \neq 0$). Table 1 presents the results of this analysis. In each period, the J-statistic does not differ significantly from zero, indicating that, overall, the instruments are orthogonal to the model errors.

To ascertain whether the instruments are correlated with the regressors, we computed the Shea's R_p^2 statistic. The results appear in Table 1 and indicate a moderate correlation between the instruments and the regressors, comparable to or higher than that in Thomadsen (2005). In light of the range restrictions on many of our instruments, the statistic indicates that the instruments are reasonably well correlated with the regressors.

These tests also afford insights into our choice of the cutoff distance, r . Using a cutoff of 3.8 miles (which we chose to be slightly greater than the 95% spatial decay [3.6 miles] interval estimated in the "Results" section), Table 1 indicates mean independence between our instruments and v_{tsj} using the J-test. Moreover, we find that smaller distances for r do not increase the correlation between the instruments and prices (the Shea's R_p^2 does not increase), suggesting that little is to be gained by choosing a smaller distance.

SPATIAL PREDICTION AND OUTLET LOCATION DECISION

After estimates of the demand and supply model are obtained, it is possible to forecast how entry and capacity decisions at any given locale will affect the demand, prices, and profits for the existing firms and the new entrant. Such information is useful to firms that want to assess the profit potential of entering at various sites. The entering firm's decision problem consists of two steps: (1) selecting the location and (2) setting optimal price and capacity conditional on location. We can solve this decision problem by backward induction: If the location has been selected, the firm will set the price and capacity to maximize its profit. To achieve this aim, the firm must predict spatial random effects in demand and cost at the potential entry locations and then compute the resultant equilibrium sales, prices, and profits at these locations. We discuss each step in turn.

Table 1
TEST OF IVs

Period	J-Statistic	p-Value (χ^2 Under the Null Hypothesis)	Shea's R^2
2	7.38	.39	.25
3	6.86	.44	.26
4	6.93	.44	.33
5	7.25	.40	.30
6	8.02	.33	.31
7	8.31	.31	.29

Spatial Prediction of Demand and Cost Effects: Bayesian Kriging

The first goal is to forecast random effects for long-term demand shock, θ_s , and cost shock, ζ_s , over space for any new location s in the future period T . This yields a map of latent spatial demand and cost that can be used to obtain insights into the nature of the market in which the outlets compete. To conserve space, we illustrate the spatial prediction for θ_s because the same procedure applies to predict ζ_s . Using Bayesian kriging (Banerjee, Carlin, and Gelfand 2004), we assume that firms can estimate the distribution of the spatial random effects at the potential entry locations conditional on the observed prices and sales at existing outlets. Suppose that an entering firm considers building a single outlet at a new location $\tilde{s}_k \in (\tilde{s}_1, \dots, \tilde{s}_n)$ in a future period T . To select a preferred location from potential locations $(\tilde{s}_1, \dots, \tilde{s}_n)$, the firm wants to estimate the demand function at each of the considered locations \tilde{s}_k , $k = 1, \dots, n$:

$$\tilde{Q}_{Tk} = I \times \tilde{W}_{Tk}, \tag{25}$$

where

$$\tilde{W}_{Tk} = \frac{\iint \tilde{W}_{Tik} dF(\eta_i|\Theta) dF(\Theta|\text{data})}{\left[\left(e^{X_{Tk}\beta_1 + X_{T\tilde{s}_k}\beta_2 - P_{Tk}(\gamma + \eta_i) + \theta_{\tilde{s}_k} + v_{T\tilde{s}_k}} \right) \left(\sum_{l=1}^J e^{X_{Tl}\beta_1 + X_{T\tilde{s}_l}\beta_2 - P_{Tl}(\gamma + \eta_i) + \theta_{s_l} + v_{T\tilde{s}_l}} + \sum_{m=1}^n e^{X_{Tm}\beta_1 + X_{T\tilde{s}_m}\beta_2 - P_{Tm}(\gamma + \eta_i) + \theta_{s_m} + v_{T\tilde{s}_m}} + e^{M_T} \right) \right]}$$

where Θ represents all the parameters and random effects (e.g., $\beta_1, \beta_2, \gamma, \sigma_\gamma^2, \sigma_v^2, \sigma_\theta^2, \phi_\theta$, and $\theta_{s_l}, l = 1, \dots, J$) in this demand system. The Θ are sampled when we fit the model, and the integrals can be approximated using Monte Carlo simulations for η_i . However, $\theta_{\tilde{s}_k}, k = 1, \dots, n$, for the considered location are unknown as of yet. The estimation of the $\theta_{\tilde{s}_k}$ is spatial prediction. The entering firm is assumed to use Bayesian kriging to obtain the posterior distribution for latent demand $\theta_{\tilde{s}_k}$ and cost $\zeta_{\tilde{s}_k}$ at location \tilde{s}_k and the demand function \tilde{Q}_{Tk} .

We assume that the location preference error θ_s follows a Gaussian random field. Thus, for any two locations s_1 and s_2 , θ_{s_1} and θ_{s_2} have a bivariate normal distribution with the covariance calculated from the covariance function of the Gaussian process. With our choice of an exponential correlation function for the spatial random effects, $\exp(-\phi_\theta \times ||s - s'||)$, the covariance of θ_{s_1} and θ_{s_2} is given by $\sigma_\theta^2 \exp(-\phi_\theta \times ||s_1 - s_2||)$. Likewise, if there are J random variables $\theta_{s_1}, \dots, \theta_{s_J}$ associated with location s_1, \dots, s_J , the pairwise covariance of θ_{s_k} and θ_{s_j} is calculated as $\sigma_\theta^2 \exp(-\phi_\theta \times ||s_k - s_j||)$, which is also the (k, j) th entry in the covariance matrix of the multivariate normal distribution for $\theta_{s_1}, \dots, \theta_{s_J}$. We denote this matrix as $\sigma_\theta^2 R_J$.

For the spatial prediction problem in our model, the observed firms occupy the locations (s_1, \dots, s_J) and the

entering firms select $(\tilde{s}_1, \dots, \tilde{s}_n)$. The corresponding spatial random effects are $(\theta_{s_1}, \dots, \theta_{s_j})$ and $(\theta_{\tilde{s}_1}, \dots, \theta_{\tilde{s}_n})$, in which $(\theta_{s_1}, \dots, \theta_{s_j})$ can be computed from Equation 18 and the parameter draws from the Gibbs sampler. From the assumption of the spatial Gaussian process, $(\theta_{s_1}, \dots, \theta_{s_j})$ and $(\theta_{\tilde{s}_1}, \dots, \theta_{\tilde{s}_n})$ jointly have the following multivariate normal distribution:

$$(27) \quad (\theta_{s_1}, \dots, \theta_{s_j}, \theta_{\tilde{s}_1}, \dots, \theta_{\tilde{s}_n}) \sim N_{J+n}(\mathbf{0}, \sigma_\theta^2 \mathbf{R}_{J+n}),$$

in which $\sigma_\theta^2 \mathbf{R}_{J+n}$ is a $(J+n) \times (J+n)$ covariance matrix, in which the covariance of θ_{s_j} (for current locations) and $\theta_{\tilde{s}_k}$ (for future locations) is, by definition, $\sigma_\theta^2 \exp(-\phi_\theta \times \|\tilde{s}_k - s_j\|)$.

To sample $(\theta_{\tilde{s}_1}, \dots, \theta_{\tilde{s}_n})$ conditioning on the already-sampled $(\theta_{s_1}, \dots, \theta_{s_j})$, we partition $\sigma_\theta^2 \mathbf{R}_{J+n}$ into four submatrices as follows:

$$\begin{bmatrix} \sigma_\theta^2 \mathbf{R}_J & \sigma_\theta^2 \mathbf{R}_{J,n} \\ \sigma_\theta^2 \mathbf{R}_{J,n}^T & \sigma_\theta^2 \mathbf{R}_n \end{bmatrix},$$

where $\sigma_\theta^2 \mathbf{R}_{J,n}$ includes the covariances of θ_{s_j} and $\theta_{\tilde{s}_k}$ and $\sigma_\theta^2 \mathbf{R}_n$ is the covariance matrix of $(\theta_{\tilde{s}_1}, \dots, \theta_{\tilde{s}_n})$.

Conditioning on $\theta = (\theta_{s_1}, \dots, \theta_{s_j})$, $\tilde{\theta} = (\theta_{\tilde{s}_1}, \dots, \theta_{\tilde{s}_n})$ has the following distribution:

$$(28) \quad \tilde{\theta} \sim N_n(\mathbf{R}_{J,n}^T \mathbf{R}_J^{-1} \theta, \sigma_\theta^2 [\mathbf{R}_n - \mathbf{R}_{J,n}^T \mathbf{R}_J^{-1} \mathbf{R}_{J,n}]).$$

This is a n -variate conditional normal distribution with mean vector $\mathbf{R}_{J,n}^T \mathbf{R}_J^{-1} \theta$ and covariance matrix $\sigma_\theta^2 [\mathbf{R}_n - \mathbf{R}_{J,n}^T \mathbf{R}_J^{-1} \mathbf{R}_{J,n}]$. Sampling $(\theta_{\tilde{s}_1}, \dots, \theta_{\tilde{s}_n})$ from this distribution is called Bayesian spatial prediction or kriging.

Equation 28 also affords insights into the effect of spatial covariance on estimated demand. When spatial covariance is ignored $[\mathbf{R}_{J+n}(\phi) = \mathbf{I}_{J+n}]$, the conditional spatial random effects are attenuated in expectation to zero $[E(\tilde{\theta}|\theta) = \mathbf{R}_{J,n}^T \mathbf{R}_J^{-1} \theta \neq 0, \text{ if } \mathbf{R}_{J+n}(\phi) \neq \mathbf{I}_{J+n}]$.

Predicting Price and Profit

After we obtain predictions of demand and cost effects, the second step of the outlet location problem is to predict profits and prices in new locales. This problem can be formulated as follows:

$$(29) \quad \max \Pi_{Tk} = (P_{Tk} - \tilde{c}_{Tk}) \tilde{Q}_{Tk},$$

where

$$(30) \quad \tilde{c}_{Tk} = \int \left\{ Y_k \beta_3 + \zeta_{\tilde{s}_k} + e_{T\tilde{s}_k}^c \right\} dF(\beta_3, \zeta_{\tilde{s}_k}, e_{T\tilde{s}_k}^c | \text{data}).$$

There is no capacity constraint because the firm can build to demand.⁹ Thus, the entering firm maximizes its profit with respect to P_{Tk} and selects the capacity

⁹Note that there is a marginal construction cost associated with the addition of each unit of capacity. However, all the construction costs become sunk after the apartments are built. Our analysis proceeds on the assumption that the net present value of discounted profits exceeds the total construction cost.

$\tilde{K}_{Tk} = \tilde{Q}_{Tk}(P_T, X_T, \Theta_T)$. The first-order condition is as follows:

$$(31) \quad \frac{\partial \tilde{\Pi}_{Tk}}{\partial P_{Tk}} = (P_{Tk}^* - \tilde{c}_{Tk}) \frac{\partial \tilde{W}_{Tk}(P_{Tk}^*, P_T)}{\partial P_{Tk}} + \tilde{W}_{Tk}(P_{Tk}^*, P_T) = 0$$

$$\Rightarrow (P_{Tk}^* - \tilde{c}_{Tk}) \iint -(\gamma + \eta_i) \tilde{W}_{Tk}(1 - \tilde{W}_{Tk}) dF(\eta_i|\Theta) dF(\Theta|\text{data}) + \tilde{W}_{Tk} = 0.$$

The existing outlets adjust their prices accordingly, knowing the location selection \tilde{s}_k of the entering firm. Their strategic pricing problem is as follows:

$$(32) \quad \max \Pi_{Tf} = \sum_{j \in F_f} (P_{Tj} - \tilde{c}_{Tj}) \tilde{Q}_{Tj},$$

subject to $\tilde{Q}_{Tj} \leq K_j$.

Ideally, the optimal price of firm f is determined by the following Kuhn–Tucker condition:

$$(33) \quad \frac{\partial \Pi_{Tf}}{\partial P_{Tl}} = \sum_{j \in F_f} (P_{Tj} - \tilde{c}_{Tj}) \frac{\partial \tilde{W}_{Tj}}{\partial P_{Tl}} + \tilde{W}_{Tl} - \sum_{m \in F_f} \lambda_m \frac{\partial \tilde{W}_{Tm}}{\partial P_{Tl}} = 0;$$

$l \in F_f;$

$$(34) \quad l \times \tilde{W}_{Tm} = K_m, \text{ iff } \lambda_m > 0, \text{ and } l \times \tilde{W}_{Tm} < K_m, \text{ iff } \lambda_m = 0.$$

The corresponding demand $\tilde{Q}_{Tl}^* = Q_{Tl}(P_{Tl}^*, P_{Tk}^*, P_{T,-l}^*)$ may exceed the capacity K_l . The implementation of the optimization problem involves two steps: With a certain set of interim prices P_{Tl} , the demand \tilde{Q}_{Tl} is calculated; if $\tilde{Q}_{Tl} \geq K_l$, the corresponding P_{Tl} is solved from the binding constraint in Equation 34 in the next step of the optimization, and if $\tilde{Q}_{Tl} < K_l$, the corresponding P_{Tl} is solved from Equation 33 in the next step.

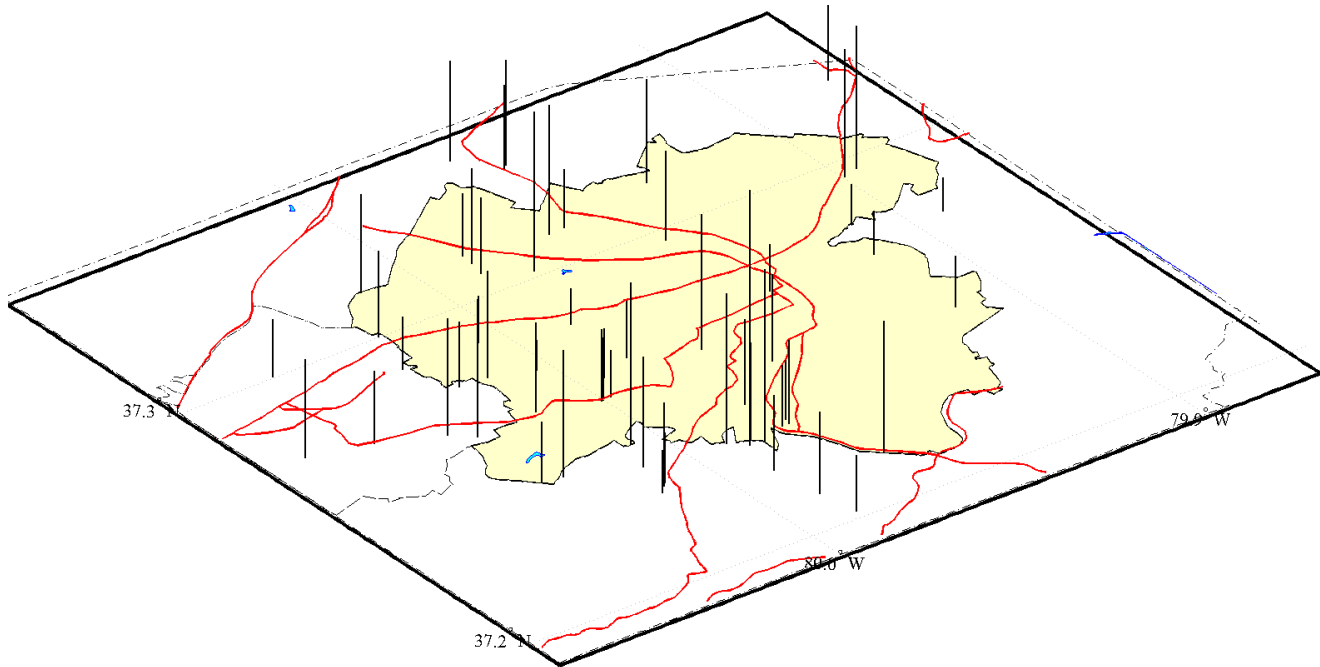
An important consideration is the uniqueness of prices in this equilibrium because multiple price equilibria would imply different profit outcomes for each equilibrium. In Web Appendix A (<http://www.marketingpower.com/jmr/april09>), we prove the existence and uniqueness of the equilibrium prices in Equation 33 by constructing a contraction mapping on a bounded set.

Equation 31 for the entering firm and Equation 33 for the existing firms constitute the strategic optimization problem that solves the new prices P_{Tl}^* , P_{Tk}^* , and the profit Π_{Tf}^* , Π_{Tk}^* , where Π_{Tk}^* is a function of location \tilde{s}_k . Optimizing Π_{Tk}^* with respect to the location \tilde{s}_k solves the optimal market entry problem. We simulate demand for the entrant using a modal configuration of attributes (across the existing outlets).

DATA

To estimate our model, we use panel data on apartment demand and prices. Apartments are a desirable category for illustrating the model because the number of outlets is sufficiently small to make estimation feasible but is sufficiently large to obtain relatively reliable estimates of spatial effects. Moreover, unlike many previous applications of spatial demand models in economics, latent demand for apartments cannot be estimated as a function of the under-

Figure 1
2005 LOCATIONS AND OCCUPANCY OF APARTMENTS IN ROANOKE, VA.



lying observed spatial distribution of the population because the population itself is endogenous. We use data from Roanoke, Va., for our analysis. This market shows good spatial coverage of apartments and little variability in supply, and a long time series of prices and vacancy rates are available for these markets. These characteristics make them ideal for our analysis.

The data for this study are provided by Real Data of Charlotte, N.C., and are detailed at www.aptindex.com. Real Data conducts an annual survey of apartments in a given market. The survey data consist of apartment attributes (e.g., whether the apartment has tennis courts, whether it has a pool, the age of the complex), addresses (which we convert to latitude and longitude), and prices. The Roanoke data cover 60 apartment complexes managed by 25 different firms and covering 7 years (1999–2005). Figure 1 depicts the location and occupancy levels of the apartments in 2005; the shortest bar corresponds to 50 rented units, and the largest corresponds to 426 rented units. The average apartment capacity of these apartments is 152 units. Eight apartments are missing several years of data, and thus we exclude them from subsequent analysis.

The apartment data are supplemented by two other data sources. First, we use the 2000 census data to attain the locations of schools and the market size, I . The census data at www.census.gov/geo/www/tiger contain the latitude and longitude of the Roanoke schools. From these data, we computed the distance to the nearest school for use as an observed spatial covariate in our analysis. Other census data (www.fedstats.gov/qf/states/51) report the number of non-home-owning households. We use this to determine I (for the Roanoke area, this was 29,489 households in

Table 2
DATA DESCRIPTIVES

<i>Attribute</i>	<i>Observations</i>	<i>M</i>	<i>SD</i>	<i>Maximum</i>	<i>Minimum</i>
Price (dollars)	364	491	102	753	290
Capacity (units)	364	163	79	636	63
Number of vacancies	364	10	17	150	0
Clubhouse	52	35%	48%	1	0
Tennis	52	32%	47%	1	0
Swimming pool	52	77%	42%	1	0
Gymnasium	52	20%	41%	1	0
Heat included	52	30%	46%	1	0
Year built	52	1976	11	2002	1952
Average distance to major employers (miles)	52	3.91	.86	5.7	2.28
Distance to nearest school (miles)	52	1.34	.74	3.05	.10
Maximum distance between apartments (miles)	1	9.6	—	—	—

2000), but our results were not particularly sensitive to this choice. Second, we determined the major employers in Roanoke from the Roanoke County Department of Economic Development (www.yesroanoke.com). For the largest employers with a single central location, we computed the mean distance from each complex to the major employers. Table 2 summarizes the variables we use in our analysis:

To construct a price index from these prices for various apartment sizes, we use the price of two-bedroom apart-

Figure 2

DISTRIBUTION OF 2005 RENTAL PRICES (\$) IN ROANOKE, VA

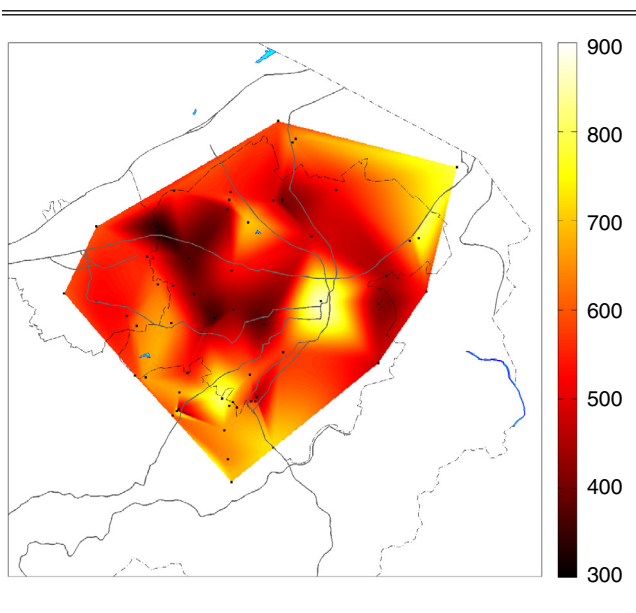
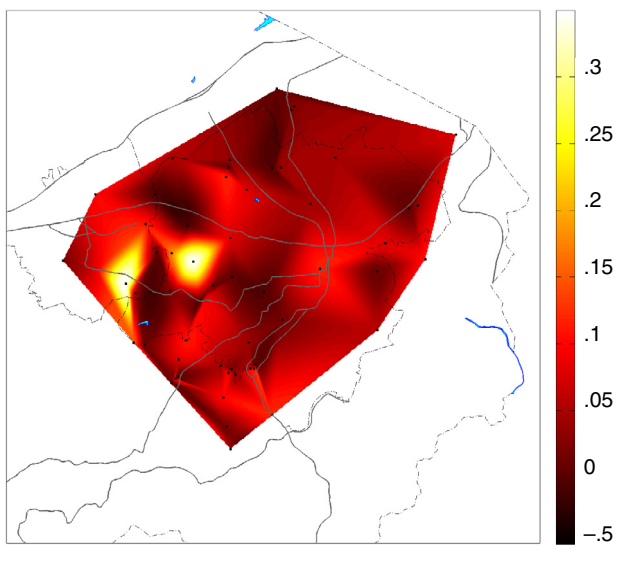


Figure 3

DISTRIBUTION OF 2005 VACANCY RATE (%) IN ROANOKE, VA.



ments (which we report in Table 2). We choose this simple approach for two reasons. First, to construct a measure that considers all apartments, we would need to create an index weight, and these weights may embed capacities, sales, or other factors that “contaminate” the pricing measure. Second, all the apartments have two-bedroom units, and the two-bedroom size is always the modal size (many apartments have only two-bedroom units). Thus, it is the most representative. When an apartment has multiple prices for the two-bedroom units, we select the median value (no

information is available on the distribution of prices for a given apartment size at a complex). During the interval of the data, average apartment prices in Roanoke increased from \$97 to \$570 per unit, though they remained constant the last three years. In the analysis, we adjust prices for inflation using Consumer Price Index figures from the Bureau of Labor Statistics. Figure 2 presents a contour map of 2005 rents in nominal dollars.

From Figure 2, we observe that the distribution of equilibrium prices is highly irregular, with the highest prices observed both near the downtown and on the periphery of town. A band of lower prices snakes from northwest to southeast. Given that prices change nonmonotonically with distance from the city center, it is desirable to capture random spatial effects rather than relying on the more commonly used approach in which demand is a linear function of distance to a population centroid, such as the center of town. The proposed spatial random-effects approach can yield highly irregular demand and price surfaces. The spatial distribution of prices in Figure 2 also suggests that there is spatial covariance in the data.

Vacancy rates averaged approximately 6%, with little change over time. In addition, total annual market capacity was fairly constant over time, with a mean of 8985 units and a standard deviation of 337 units. No apartments were being constructed as of 2005. This suggests that it is appropriate to model the subgame of prices in our data conditional on apartment locations and capacities. Figure 3 depicts a contour map of vacancy rates. Vacancy rates tend to be highest on the west side of town.

RESULTS

Simulation Results

To show (1) how the omission of spatial covariance and capacity constraints biases model estimates and (2) that our model can recover the underlying parameters, we conduct a simulation using the approach described in Web Appendix B (<http://www.marketingpower.com/jmrapril09>). Using these simulated data, we estimate three models. In Model 1, we include spatial covariance and capacity constraints. In Model 2, we omit spatial covariance. In Model 3, we omit capacity constraints. The Gibbs sampling chains proceed for 10,000 iterations in each model, though they appear to converge well before then (within 500 iterations). We discarded the first 20,000 draws to ensure convergence. Priors are set to be as noninformative as possible. We use all available attributes and prices from preceding periods as instruments. Subsequently, we report the parameter estimates and the ability of three models to recover the parameters.

Full model. As Table 3 shows, the 95% posterior predictive interval for Model 1 contains the true parameter values for all the parameters in our properly specified model, indicating that the model can recover the data-generating mechanism. Of particular interest is the recovery of the estimate for the highest Kuhn–Tucker multiplier. This parameter captures the marginal cost of the constraint to the firm; in this case, Apartment 39 in Year 8 would gain \$131 in monthly profit if it could add one apartment.

No spatial effects. Contrasting the full model to one in which we omit spatial correlation (Model 2), we observe the predicted downward bias in median price effects. The

Table 3
SIMULATION RESULTS

<i>Demand Side</i>			<i>Posterior Median (95% Posterior Prediction Interval)</i>		
<i>Parameter</i>	<i>Variable</i>	<i>Simulation Value</i>	<i>Our Model</i>	<i>No Spatial Correlation</i>	<i>No Capacity Constraints</i>
β_1	Garage	1.00	1.01 (.87, 1.12)	.94 (.85, 1.03)	—
β_2	Distance to city	-.30	-.29 (-.37, -.22)	-.28 (-.36, -.19)	—
γ	Price	.01	.0096 (.0078, .012)	.0077 (.0051, .0089)	—
σ_γ	Standard deviation γ	.002	.0017 (.0012, .0026)	.0018 (.0013, .0026)	—
σ_θ	Standard deviation θ	.40	.38 (.33, .46)	.42 (.32, .51)	—
ϕ_θ	Spatial decay θ	.40	.56 (.20, 1.02)	—	—
σ_v	Standard deviation v	.30	.29 (.25, .32)	.35 (.30, .42)	—
ϕ_v	Spatial decay v	.40	.44 (.20, .56)	—	—
	Log-marginal likelihood		-27.4	-43.9	—
<i>Supply Side</i>			<i>Posterior Median (95% Posterior Prediction Interval)</i>		
<i>Parameter</i>	<i>Variable</i>	<i>Simulation Value</i>	<i>Our Model</i>	<i>No Spatial Correlation</i>	<i>No Capacity Constraints</i>
$\beta_{3,0}$	Variable cost	150	147 (141, 154)	117 (103, 130)	173 (166, 180)
$\beta_{3,1}$	Apartment size	30	29.6 (29.0, 30.5)	29.5 (28.9, 30.6)	22.2 (19.8, 25.5)
σ_ζ	Standard deviation ζ	10	10.5 (9.6, 11.3)	10.5 (9.6, 11.2)	20.6 (17.2, 25.7)
ψ_ζ	Spatial decay ζ	.40	.43 (.20, .74)	—	1.31 (.86, 1.90)
σ_c	Standard deviation e_c	10	10.1 (9.1, 11.3)	11.3 (9.9, 13.8)	17.3 (14.6, 20.1)
ψ_c	Spatial decay	.40	.56 (.20, .87)	—	1.00 (.74, 1.47)
λ_{\max}	Kuhn-Tucker	113	112 (105, 121)	115 (107, 129)	—
	Log-marginal likelihood		-1093	-1130	-1276

magnitude of this effect in this parameter is 20%. This bias is a consequence of how correlation in the spatial errors can exacerbate small-sample bias in generalized method of moments and IV estimation (Altonji and Segal 1996; Buse and Moazzami 1991). When these correlations are high, bias is present even in fairly large samples (Altonji and Segal 1996).

We also observe a downward bias of approximately 20% in the estimate for the intercept of marginal cost, $\beta_{3,0}$. The cause of this downward bias in estimated marginal costs can be observed by inspecting Equation 14. The downward bias in the estimate for price elasticity γ lowers the expression on the left-hand side of Equation 14. To maintain this equality, estimates for the right-hand side must also be lower, leading to a downward bias in the estimates for marginal cost. Note also that there is a substantial decrease in the demand-side log-marginal likelihood from -27.4 to -43.9.¹⁰ The log-marginal likelihood on the supply side decreases from -1093 to -1130.

No capacity constraints. Contrasting the full model to one in which we omit capacity constraints, we note that cost estimates are biased upward. Firms raise prices in the presence of capacity constraints to the point at which demand is equal to capacity because lower prices yields lower per unit revenue with no attendant increase in demand. The observation of higher prices at a given level of demand leads to inferences of higher costs, consistent with Equation 14. Furthermore, the variance of the fixed and

time-varying spatial cost effects σ_ζ and σ_c are overestimated by nearly a factor of two as a result of the additional error introduced by ignoring the capacity constraints. The spatial decay is overestimated by 150%. Reflecting these biases, the log-marginal likelihood decreases considerably from -1093 to -1276.

In summary, the simulation data show that (1) our model recovers parameters well and (2) biases in parameter estimates that arise from ignoring spatial covariance and capacity constraints are in the predicted direction. Moreover, omitting spatial effects and capacity constraints has a substantial impact on model fit.

Roanoke Results

Next, we apply the data detailed in the “Data” section to estimate our model of outlet demand and pricing. We ran the sampling chain for 20,000 iterations. Inspection of the sequence of draws indicates good convergence after approximately 500 iterations, though we discarded the first 2000 iterations. Moreover, little autocorrelation in the draws was evident. In addition to estimating the full model, we provide two benchmarks: no spatial correlation and no capacity constraints. Comparison of these models with the full model yields insights into (1) the magnitude of parameter biases that can arise when capacity and spatial covariance in demand and prices are ignored and (2) the degree to which spatial covariance improves model performance.

Parameter estimates. Table 4 presents the results of our estimation. The full model is the best-fitting model, and its improvement over the model with no spatial covariance is sizable (the improvement in the log-marginal likelihood is 18 on the demand side and 22 on the supply side). This provides evidence that spatial covariance matters in practice. The improvement over a model with no capacity constraints

¹⁰The ratio of the marginal likelihood (exponential of the log-marginal likelihood) is equivalent to a Bayes factor with noninformative priors, and therefore we use this statistic for model comparison. We computed log-marginal likelihoods as in Gelfand and Dey (1994).

Table 4
ROANOKE DATA RESULTS

<i>Demand Side</i>		<i>Posterior Median (95% Posterior Prediction Interval)</i>		
<i>Parameter</i>	<i>Variable</i>	<i>Full Model</i>	<i>No Spatial Correlation</i>	<i>No Capacity Constraints</i>
$\beta_{1,1}$	Clubhouse	.31 (.06, .57)	.30 (.03, .58)	—
$\beta_{1,2}$	Tennis	.55 (.31, .80)	.58 (.35, .83)	—
$\beta_{1,3}$	Pool	.51 (.20, .78)	.45 (.14, .76)	—
$\beta_{1,4}$	Gym	.15 (–.19, .50)	–.02 (.34, .27)	—
$\beta_{1,5}$	Heat	.32 (.14, .51)	.32 (.15, .50)	—
$\beta_{2,1}$	Distance to school	.03 (–.16, .23)	.05 (–.18, .28)	—
γ	Price	.0035 (.0024, .0046)	.0032 (.0022, .0042)	—
σ_γ	Standard deviation γ	.00057 (.00030, .00084)	.00058 (.00033, .00085)	—
σ_θ	Standard deviation θ	.60 (.50, .72)	.72 (.60, .84)	—
ϕ_θ	Spatial decay θ	.84 (.39, 1.38)	—	—
σ_v	Standard deviation v_t	.11 (.10, .12)	.11 (.10, .13)	—
ϕ_v	Spatial decay v_t	2.8 (2.0, 3.73)	—	—
Log-marginal likelihood		–2.84	–20.9	—

<i>Supply Side</i>		<i>Posterior Median (95% Posterior Prediction Interval)</i>		
<i>Parameter</i>	<i>Variable</i>	<i>Full Model</i>	<i>No Spatial Correlation</i>	<i>No Capacity Constraints</i>
$\beta_{3,0}$	Variable cost intercept	432 (368, 498)	408 (346, 472)	405 (340, 465)
$\beta_{3,1}$	Heat	70.2 (33.5, 107.8)	61.6 (25.2, 98.6)	57.1 (22.3, 90.3)
$\beta_{3,2}$	Age	–7.3 (–9.3, –5.2)	–7.2 (–9.3, –5.3)	–6.7 (–8.7, –4.6)
σ_ζ	Standard deviation ζ	88.2 (75.1, 106.0)	91.0 (78.7, 108.3)	102.0 (90.3, 117.8)
ψ_ζ	Spatial decay ζ	2.8 (.46, 5.66)	—	1.9 (1.0, 5.6)
σ_c	Standard deviation e_t^c	26.6 (24.8, 28.8)	27.0 (25.4, 29.2)	34.7 (30.2, 39.7)
ψ_c	Spatial decay e_t^c	3.2 (.46, 6.00)	—	4.2 (1.6, 7.5)
λ^{\max}	Kuhn–Tucker	88.5 (48.2, 129.6)	65.2 (26.1, 118.4)	—
Log-marginal likelihood		–1362	–1384	–1503

is even more substantial, leading to a 141-point gain in the log-marginal likelihood.

Demand-side estimates. All attributes, except for the addition of a gym, play a significant role in apartment choice. The low effect of a gym may arise from many competing options, including work and office gyms and home exercise equipment. Among the attributes, pool and tennis courts each play the greatest role in apartment choice. The 95% posterior predictive interval for distance to schools includes zero, suggesting that this effect is negligible.¹¹ The small effect may reflect few school-age children among apartment dwellers or the local school zoning policies. The effective range of the median spatial decay, in which 95% of the spatial effect has decayed, is given by $3/\phi_\theta$ (Banerjee, Carlin, and Gelfand 2004), or 3.6 miles. This suggests that the demand-side spatial covariance is sizable because the maximum distance between apartments is 9.6 miles. The price parameter is positive, indicating that an increase in price lowers the likelihood of apartment choice (because this enters our likelihood function with a negative sign). Consistent with our previous findings, the price parameter is biased toward zero when spatial covariance and capacity constraints are ignored; the bias is approximately 10%.

Supply-side estimates. Table 4 indicates that heat increased the variable costs of the apartment by approxi-

mately \$70 per month per unit in 1999 dollars and that newer apartments had higher operating costs, perhaps because of greater amenities. The highest median Kuhn–Tucker multiplier was \$88 per month per unit for Apartment 37 in 2004. Among apartments at capacity, this apartment had the second-lowest costs. Because this apartment is so efficient, its capacity constraint was especially costly. In addition, apartments with high rents tended to have high costs of capacity constraints.

According to a 1998 survey of apartment managers conducted by the Institute of Real Estate Management, annual operating expenses for apartments average 42% of revenue; these costs include administrative expenses, operating expenses, maintenance expenses, tax/insurance, and payroll and amenities. For the average rent of \$485 in our data (in 1999 dollars), this implies that our estimates for variable costs should be approximately \$204 per apartment in 1999 dollars. Averaging our marginal cost estimates across apartments yields \$233 in 1999 dollars, close to the \$204 implied by the survey.

The spatial effects on the supply side (prices), with an effective range of 1.1 miles, are smaller than those on the demand side. We conjecture that costs are a more “global” variable in the sense that all apartments likely face a similar cost of capital, utilities, labor, and supplies. On the demand side, however, certain regions are more desirable than others, leading to higher spatial correlation.

As we expected, cost estimates are biased downward when spatial covariance is ignored, and the cost error variance increases when capacity constraints are ignored. We

¹¹When estimating the model, we noted that the correlation between distance to schools and distance to employers was high, so we omitted the latter because it had less explanatory power.

do not observe an upward bias in costs for the no-capacity-constraint model, perhaps because a limited number of apartments are constrained in our data. Consistent with the simulation findings, the no-capacity-constraint-cost model has a poorer fit, as indicated by higher estimates for ζ and σ_c . These parameters are, respectively, 16% and 30% too high when capacity is ignored.

Latent spatial effects. Using the median of the samples of the spatial random effects, we create contour plots of spatial random demand θ_{s_j} and cost effects ζ_{s_j} and offer a discussion of these results. Figure 4 plots the θ_{s_j} and then interpolates latent demand between these observations through triangle-based cubic interpolation.¹² This yields a map of latent demand.

Figure 4 indicates several modes of high latent demand, including just west of downtown and a more prominent mode southwest of downtown. We compare Figure 4, which computes the latent spatial demand effects, with Figure 2, which depicts vacancy. The highest demand region southwest of town not only has high latent demand but also has higher prices. Taken together, these factors imply that this modal location for demand is an ideal spot to locate a new complex.

However, selecting an optimal outlet locale must also assess the effects of competition and costs. For example, Figure 3 indicates that the mode in latent demand southwest of downtown also corresponds to higher vacancy rates because of a surfeit of apartments. Thus, the problem of

outlet location is a complex calculus involving an array of countervailing factors. We attempt to explore these trade-offs more precisely in the next section.

MANAGERIAL IMPLICATIONS

Our approach can be used to engage policy experiments pertaining to the selection of a new outlet location because it structurally links prices to outlet entry. For example, a firm could compare across available properties the effect of locating an additional outlet on demand, prices, and profits at (1) the additional outlet and (2) other outlets in the chain. In our analysis, we compare the desirability of these entry options subject to the caveat that no other outlets enter. However, even in the context of competitive response, the effect of various competitive location responses to the firm's choices of next outlet location could be simulated. This suggests that many scenarios could be used to simulate entry effects.

It might be conjectured that firms already occupy the optimal locations, so that the policy simulation is of little value. We believe that this is unlikely primarily because the ebb and flow of people into the market and changes in the attributes of various locales (e.g., new roads) over the years are likely to render the extant distribution of existing outlets suboptimal. Therefore, it is likely to be useful to conjecture what the next best location may be.

Using the procedure discussed in the "Model" section, we assess the effect of an additional apartment on equilibrium demand, prices, and profits. We begin by creating a 10×10 grid of potential apartment entry locations within the convex hull defined by extant apartment locales in Roanoke. For each location on the 100-point grid, we compute equilibrium profits associated with an entry at that location. This computation assumes a modal apartment design for the new complex—that is, using the modal feature set in the data. We execute this procedure twice, once for our full model and once for a model that omits spatial covariance and capacity constraints. This enables us to contrast the recommendation of each model. Figure 5 depicts the predicted equilibrium profits at these locations.

The top half of Figure 5 portrays the equilibrium profits for the various entry options using the full model. The areas of the solid circles correspond to predicted monthly profits, with a maximum of \$61,427, a median of \$37,025, and a minimum of \$20,356. The bottom half depicts predicted profits for the various entry options using the model with no spatial effects. The predicted profits from this model range from \$38,136 to \$35,572. The median predicted profit is \$36,971. Most strikingly, the constrained model shows a lack of variation in predicted profits because it ignores unobserved information regarding latent demand and costs from extant apartments to forecast demand and costs at new locales. Instead, it uses only observed spatial effects, such as distance to the nearest school, and these observed effects appear to be small compared with the unobserved spatial variation. The standard deviation in predicted profits is 10.8 times larger for the spatial covariance model. With this model, it might be inadvertently concluded that some of the lower-profit locations would yield a good profit. Furthermore, Figure 5 indicates several "false modes," in which the full model predicts small profits.

¹²Figure 4 interpolates latent demand for locations in which there are no apartments, based on our observed estimates for θ as well as kriged θ in a 10×10 grid.

Figure 4

LATENT SPATIAL DEMAND FOR APARTMENTS IN ROANOKE, VA.

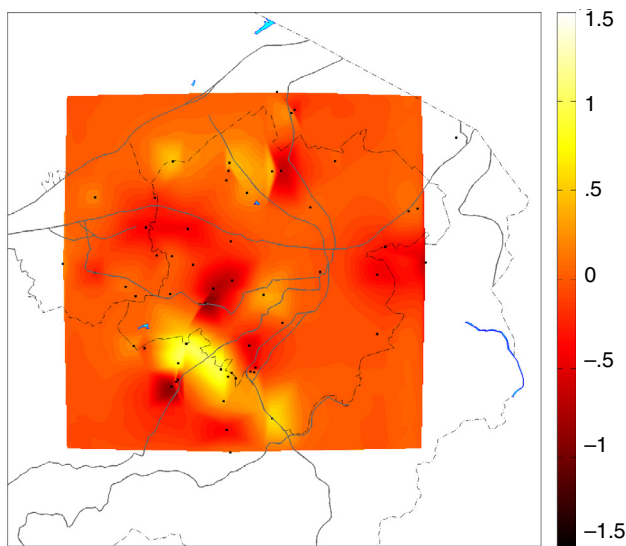
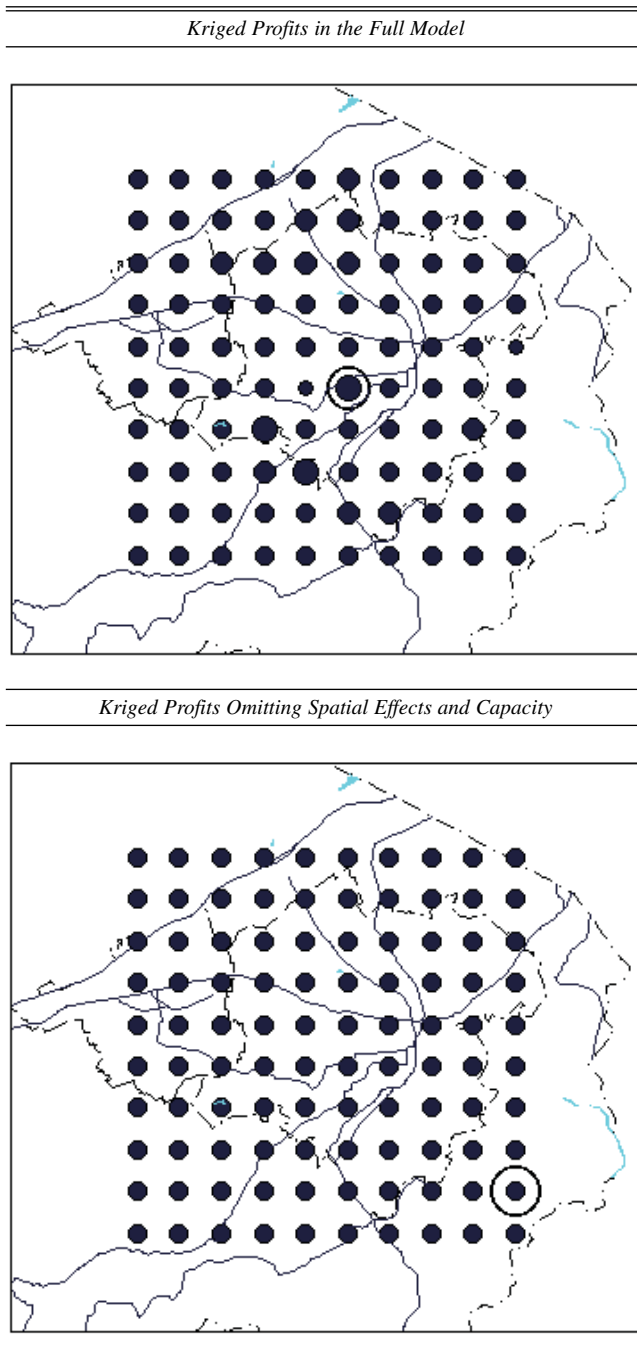


Figure 5
KRIGED PROFITS FOR POTENTIAL APARMENT LOCATIONS IN
ROANOKE, VA.



The open circles in the top and bottom halves of Figure 5 enclose the locales with the highest predicted profit in each of the respective models. For the full model, the highest predicted profit corresponds to a lesser mode in latent demand just west of downtown, as evidenced in Figure 4. The no-spatial-variation demand model recommends a locale in the far southeastern part of Roanoke. If the recommendation of the model without spatial covariance were to be adopted, expected profits would be \$37,062 (which represents the predicted profits from the full model using the recommended locale from the constrained model). Using

the recommendation from the spatial model increases expected profits to \$61,427, or 66%. Because these estimates are cash flows and because firms expand into many different cities, the profit implications of our model could be considerable.

The second-highest mode in profits is colocated with the highest mode in latent demand we discussed in the "Model" section. To provide more intuition regarding why the lesser mode in latent demand west of town is optimal and the larger mode in latent demand southwest of town is not, Figure 6 depicts the optimal locale superimposed on the spatial distributions of historical rental prices, historical vacancy rates, latent costs, and latent demands. In these distributions, a darker shade corresponds to a lower number. Black squares indicate apartment locations, and the white circle denotes optimal entry locale. The optimal location results because (1) latent demand is high, (2) latent costs are low, and (3) vacancy rates are low. Although the historical rental prices are low, this is in part offset by lower costs, suggesting that high margins are possible even in the face of lower rents. Furthermore, there is only one competing complex in this area.

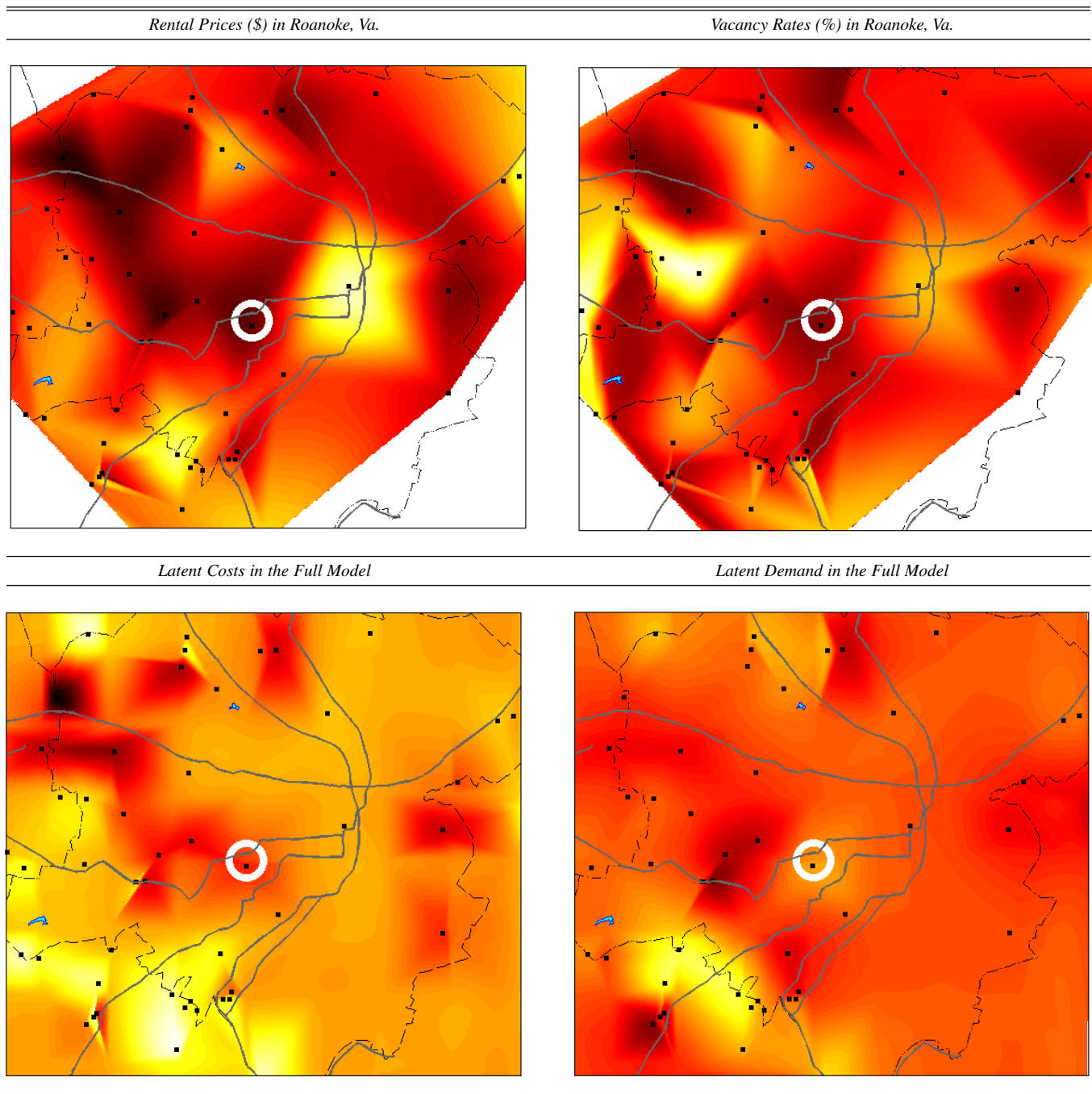
In contrast, the second-highest mode in profits and highest mode in latent demand corresponds to (1) higher latent costs, (2) higher vacancy rates, and (3) more competing chains. Thus, the presence of competition affects the recommendation for optimal location. We contend that widely employed tools for site location often neglect this aspect of the site location problem (Buckner 2004).

CONCLUSIONS

In this article, we address the problem of the firm's outlet location problem. A necessary step in this process is to solve the subgame problem of firm pricing conditional on firm locations. Firms can then use this model to simulate the effect of locating an incremental outlet on equilibrium prices, demand, and profits. Our model applies these concepts in the context of apartment data and affords recommendations about the next best location for entry.

Our work extends prior research in several respects. Unlike gravity models, we explicitly and structurally consider competitive response in prices. Unlike analytical models, our approach is data focused to enable insights into data-driven decisions in several different marketing contexts. We extend the structural model of the subgame of outlet location and prices to include, among other things, spatial correlation and capacity constraints. It is desirable to consider spatial correlation for unobserved random demand and costs effects for several reasons. First, the omission of spatial correlation in demand leads to downward biased estimates of price effects and marginal costs in the small sample sizes common in the outlet location problem. Second, spatial demand effects are noteworthy in their own right, leading to a managerially informative map of latent demand. Neglecting these effects obscures considerable spatial demand variation inherent in our data. It is also desirable to consider capacity constraints. Properly accounting for these effects mitigates an upward bias in costs, and the Kuhn-Tucker multipliers provide an estimate of the cost of the capacity constraint. Presumably, firms with higher costs would be more inclined to consider additional capacity. In addition, failure to consider capacity

Figure 6
OPTIMAL LOCATION (WHITE CIRCLE) SUPERIMPOSED ON RENT, VACANCY, COST, AND DEMAND



constraints and spatial effects leads to suboptimal recommendations for firm location.

To achieve these aims, we integrate Bayesian spatial statistics with structural models of competition in a model of outlet demand and pricing. Simulations indicate that the resultant model recovers parameters well and that ignoring spatial effects and capacity constraints leads to biased parameter estimates. We then apply this model to a novel apartment data set that includes prices, demand, and capacities over a six-year period. We find that the inclusion of spatial covariance in demand improves model fit and that

omitting spatial effects and capacity constraints leads to biased parameter estimates, as predicted.

Given that we attempt to assess the policy implications of outlet location, we use a spatial kriging approach coupled with our supply-side model to explore the impact of locating an additional outlet at potential sites of interest to a firm (or more systematically on a grid). These simulations reveal that (1) the best entry locations are related to high latent spatial demand and a dearth of nearby outlets, (2) accommodating capacity and unobserved spatial effects in policy simulations improves the profitability of recom-

mended entry locales by 66% in our data, and (3) ignoring spatial covariance leads to little variation in predicted profits, understating the actual standard deviation in predicted profits across space by a factor of 11.

Several limitations exist that also represent opportunities for further research. First, we consider the pricing subgame only and do not model the outlet location game. We do this because the subgame is an important problem in its own right and is useful for policy simulations over the intermediate term. An important extension would be to consider the entry problem as well. We believe that this would be a difficult extension because it involves a dynamic program to solve for sequential entry and prices, and it is likely that solutions to this problem would not be unique. Moreover, the distribution of latent demand can render such equilibria obsolete after a few years.

Second, many outlets sell multiple goods or have products that appeal to different markets (e.g., supermarkets, medical centers). It would be desirable to ascertain whether competition across goods leads to different outcomes for location and pricing than in the indexed single-good case we model herein.

Third, we presume that attributes such as pool and tennis are exogenous. It would be desirable to model not only the location of an outlet but also its optimal design. We believe that this exercise in combinatorial optimization would prove challenging, especially with respect to documenting the uniqueness of these equilibria.

Fourth, our model is of limited applicability when alternative channels, such as the Internet, constitute a significant portion of demand. Multichannel models (Ansari, Mela, and Neslin 2008) could be overlaid with the spatial models to develop unique insights into this setting.

Fifth, in Web Appendix A (<http://www.marketingpower.com/jmrapril09>), we establish the uniqueness of prices on the supply side under the condition that price sensitivity is negative. Given that the supply-side model is therefore a function, this result implies that it may be possible to develop a full-information likelihood-based inferential technique for supply and demand. Such an approach could lead to gains in efficiency over extant estimation approaches and yield more desirable small-sample properties.

Finally, it would be worthwhile to consider capacity a strategic variable. Because we observe only one complex change capacity in our data, it is an ideal setting to explore Bertrand–Nash competition; however, we believe that it is desirable to extend this research to the context of Cournot competition. Of note, the Bertrand and Cournot pricing outcomes align in a two-stage game in which capacity is set in Stage 1 and prices are realized conditional on capacity in Stage 2 (Kreps and Scheinkman 1983). Given that we model prices in a subgame conditional on capacity constraints, this indicates that the predicted prices in our model may be equivalent to the Cournot pricing outcomes.¹³

In summary, the approach we develop incorporates spatial effects into a model of outlet competition and affords insights into the role of unobserved spatial effects on outlet pricing and demand. We hope that our research will spark subsequent research related to these and other remaining issues.

APPENDIX: SAMPLER IN THE ESTIMATION SECTION

Demand Model

Step 1: Sample $(\tilde{\theta}, \beta_1, \beta_2, \gamma)$. The location-specific effects for an outlet are given by $\tilde{\theta} \sim N[X\alpha_1 + X_s\alpha_2, \sigma_\theta^2 R_J(\phi_\theta; d)]$, where X and X_s are time-invariant attributes. Note that $\theta = \tilde{\theta} - X\alpha_1 - X_s\alpha_2$. Let $X_{tA} = [I_J, X_t, X_{ts}, P_t]$ and $\theta_A = (\tilde{\theta}, \beta_1, \beta_2, \gamma)$, where θ_A has the multivariate normal prior $N(\mu_0, \Sigma_0)$, where

$$\mu_0 = (X\alpha_1 + X_s\alpha_2, \beta_{10}, \beta_{20}, \gamma_0)^T \text{ and } \Sigma_0 = \begin{bmatrix} \sigma_\theta^2 R_J(\phi_\theta) & 0 \\ 0 & \Sigma_0^{\beta, \gamma} \end{bmatrix}$$

The prior is conjugate to the likelihood. The full conditional for θ_A is $N(\mu_A, \Sigma_A)$, where

$$(A1) \quad \Sigma_A = \left\{ \frac{1}{\sigma_v^2} \left[\sum_{t=1}^T X_{tA}^T Z_t \right] \left[\sum_{t=1}^T Z_t^T R_J(\phi_v) Z_t \right]^{-1} \right. \\ \left. \sum_{t=1}^T Z_t^T X_{tA} + \Sigma_0^{-1} \right\}^{-1}, \text{ and}$$

$$(A2) \quad \mu_A = \Sigma_A \left\{ \frac{1}{\sigma_v^2} \left[\sum_{t=1}^T X_{tA}^T Z_t \right] \left[\sum_{t=1}^T Z_t^T R_J(\phi_v) Z_t \right]^{-1} \right. \\ \left. \sum_{t=1}^T Z_t^T \hat{\xi}_t + \Sigma_0^{-1} \mu_0 \right\}$$

Step 2. Sample α_1, α_2 , and θ . Let $\alpha_A = (\alpha_1, \alpha_2)$ and $X_A = (X, X_s)$. Given $\tilde{\theta}, \sigma_\theta^2, \phi_\theta$, and the prior $N(\mu_0^\alpha, \Sigma_0^\alpha)$ for the full conditional distribution from the posterior for α_1 and α_2 is

$$(A3) \quad N \left\{ \left[\frac{1}{\sigma_\theta^2} X_A^T R_J^{-1}(\phi_\theta) X_A + \Sigma_0^{\alpha-1} \right]^{-1} \right. \\ \left. \left[\frac{1}{\sigma_\theta^2} X_A^T R_J^{-1}(\phi_\theta) \tilde{\theta} + \Sigma_0^{\alpha-1} \mu_0^\alpha \right], \left[\frac{1}{\sigma_\theta^2} X_A^T R_J^{-1}(\phi_\theta) X_A + \Sigma_0^{\alpha-1} \right]^{-1} \right\}$$

In our Roanoke data analysis, we use independent and disperse priors for $\alpha_1, \alpha_2, \beta_1$, and β_2 : $N(0, 10^8)$.

Step 3. Sample σ_γ and $\hat{\xi}_t$; Metropolis–Hastings algorithm. Suppose that σ_γ has a truncated normal prior $N(\sigma_\gamma | 0, \sigma_0^2) I\{\sigma_\gamma > 0\}$. Use a truncated normal $N(\sigma_\gamma | \mu_p, \sigma_p^2) I\{\sigma_\gamma > 0\}$ to generate $\sigma_\gamma^{(n)}$. After recalculating $\hat{\xi}_t^{(n)}$ conditional on $\sigma_\gamma^{(n)}$ using a contraction mapping as we indicated previously, the acceptance probability for σ_γ and $\hat{\xi}_t$ is given as

$$(A4) \quad \min \left\{ \prod_{t=1}^T N \left[Z_t^T \hat{\xi}_t^{(n)} | Z_t^T X_{tA} \theta, \sigma_v^2 Z_t^T R_J(\phi_v) Z_t \right] \right. \\ \left. \frac{\partial \hat{\xi}_t^{(n)}}{\partial W_t} N \left(\sigma_\gamma^{(n)} | 0, \sigma_0^2 \right) N \left(\sigma_\gamma | \mu_p, \sigma_p^2 \right) \right\} //$$

¹³We thank an anonymous reviewer for this observation.

$$\left\{ \prod_{t=1}^T N \left[Z_t^T \hat{\xi}_t | Z_t^T X_{tA} \theta, \sigma_v^2 Z_t^T R_J(\phi_v) Z_t \right] \right. \\ \left. \left| \frac{\partial \hat{\xi}_t}{\partial \hat{W}_t} \right| N(\sigma_\gamma | 0, \sigma_\theta^2) N(\sigma_\gamma^{(n)} | \mu_p, \sigma_p^2) \right\}, 1 \Bigg\}.$$

Note that $\hat{\xi}_t$ is deterministic (obtained by a contraction mapping and Monte Carlo integration) given σ_γ .

Step 4. Sample σ_θ^2 and σ_γ^2 . We use the conjugate inverse gamma (1, 1) priors for σ_v^2 and σ_θ^2 . The posterior distributions are also inverse gamma.

Step 5. Sample ϕ_θ and ϕ_v : discrete sampler. We assume that ϕ_v (or ϕ_θ) can only take n values: (ϕ_1, \dots, ϕ_n) . For each ϕ_v (or ϕ_θ), we calculate the posterior probability

$$\left| Z_t^T R_J(\phi_i) Z_t \right|^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2\sigma_v^2} \left[\sum_{t=1}^T (\hat{\xi}_t - X_{tA} \theta_A)^T Z_t \right]^2 \right. \\ \left. \left[\sum_{t=1}^T Z_t^T R_J(\phi_i) Z_t \right]^{-1} \sum_{t=1}^T Z_t^T (\hat{\xi}_t - X_{tA} \theta_A) \right\} \\ \left\{ \text{or } |R_J(\phi_i)|^{-\frac{1}{2}} \exp \left[-\frac{1}{2\sigma_\theta^2} \theta^T R_J^{-1}(\phi_i) \theta \right] \right\}.$$

We Sample ϕ with replacement from (ϕ_1, \dots, ϕ_n) .

Cost Model

Step 1. Sample β_3 and ζ . Let $Y_{tA} = (I_J, Y_t)$, and let $\zeta_A = (\zeta, \beta_3)$, where ζ_A has the multivariate normal prior $N(\mu_0, \Sigma_0)$, where

$$\mu_0 = (0, \mu_0^{\beta_3})^T \text{ and } \Sigma_0 = \begin{bmatrix} \sigma_\theta^2 R_J(\phi_\theta) & \\ & \Sigma_0^{\beta_3} \end{bmatrix}.$$

Given λ_{2t} and γ (γ is sampled from the demand model), the likelihood is as follows:

$$(A5) \quad \exp \left\{ -\frac{1}{2\sigma_c^2} \sum_{t=1}^T [P_t - B_t(\lambda_t, \gamma) - Y_{tA} \zeta_A]^T \right. \\ \left. R_J^{-1}(\psi_c) [P_t - B_t(\lambda_t, \gamma) - Y_{tA} \zeta_A] \right\},$$

where $B_t(\lambda_t, \gamma) = \lambda_t - \hat{\Omega}^{(t-1)} \hat{W}_t$ ($\lambda_t = [\lambda_{1t}, \lambda_{2t}]^T$, where $\lambda_{1t} = 0$ and $\lambda_{2t} > 0$). The full conditional for ζ_A is $N(\mu_A, \Sigma_A)$, where

$$(A6) \quad \Sigma_A = \left[\frac{1}{\sigma_c^2} \sum_{t=1}^T Y_{tA}^T R_J^{-1}(\psi_c) Y_{tA} + \Sigma_0^{-1} \right]^{-1} \text{ and}$$

$$(A7) \quad \mu_A = \Sigma_A \left\{ \frac{1}{\sigma_c^2} \sum_{t=1}^T Y_{tA}^T R_J^{-1}(\psi_c) [P_t - B_t(\lambda_t, \gamma)] + \Sigma_0^{-1} \mu_0 \right\}.$$

In our Roanoke data analysis, we use independent and disperse priors for β_3 : $N(0, 10^8 I_p)$.

Step 2. Sample λ_{2t} : data augmentation. Let

$$\tilde{P}_t = (\tilde{P}_{1t}, \tilde{P}_{2t}) \\ = [P_{1t} + (\hat{\Omega}^{(t-1)} \hat{W}_t)_1 - Y_{1t} \beta_3 - \zeta_1, P_{2t} + (\hat{\Omega}^{(t-1)} \hat{W}_t)_2 - Y_{2t} \beta_3 - \zeta_2].$$

Because $(\tilde{P}_{1t}, \tilde{P}_{2t})$ is equivalent to $(e_{1t}^c, e_{2t}^c + \lambda_{2t})$, it follows that

$$\begin{bmatrix} \tilde{P}_{1t} \\ \tilde{P}_{2t} - \lambda_{2t} \end{bmatrix} \sim N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \sigma_c^2 \begin{bmatrix} R_{11}(\psi_c) & R_{12}(\psi_c) \\ R_{21}(\psi_c) & R_{22}(\psi_c) \end{bmatrix} \right),$$

or, equivalently,

$$(A8) \quad [\lambda_{2t} | \tilde{P}_{1t}, \tilde{P}_{2t}] \sim N \{ \tilde{P}_{2t} - R_{21}(\psi_c) R_{11}^{-1}(\psi_c) \tilde{P}_{1t}, \\ \sigma_c^2 [R_{22}(\psi_c) - R_{21}(\psi_c) R_{11}^{-1}(\psi_c) R_{12}(\psi_c)] \},$$

making use of the properties of the conditional normal and subtracting \tilde{P}_{2t} from both sides of the resultant conditional distribution for $[\tilde{P}_{2t} - \lambda_{2t} | \tilde{P}_{1t}]$. Noting that λ_{2t} is structurally greater than zero, we sample λ_{2t} from the following truncated normal distribution:

$$(A9) \quad N \{ \tilde{P}_{2t} - R_{21}(\psi_c) R_{11}^{-1}(\psi_c) \tilde{P}_{1t}, \\ \sigma_c^2 [R_{22}(\psi_c) - R_{21}(\psi_c) R_{11}^{-1}(\psi_c) R_{12}(\psi_c)] \} I \{ \lambda_{2t} > 0 \}.$$

Step 3. Sample σ_ζ^2 and σ_c^2 . We use the conjugate inverse gamma (1, 1) priors for σ_ζ^2 and σ_c^2 . The posterior distributions are also inverse gamma.

Step 4. Sample ψ_ζ and ψ_c . We use a discrete sampler. We assume that ψ_c (or ψ_ζ) can take only n values (ψ_1, \dots, ψ_n) . For each ψ_i , we calculate the posterior probability,

$$\left| R_J(\psi_i) \right|^{-\frac{1}{2}} \exp \left[-\frac{1}{2\sigma_c^2} \sum_{t=1}^T (\tilde{P}_t - \lambda_t)^T R_J^{-1}(\psi_i) (\tilde{P}_t - \lambda_t) \right] \\ \left\{ \text{or } |R_J(\psi_i)|^{-\frac{1}{2}} \exp \left[-\frac{1}{2\sigma_\zeta^2} \zeta^T R_J(\psi_i) \zeta \right] \right\}.$$

Then, we sample ψ_c (or ψ_ζ) with replacement from (ψ_1, \dots, ψ_n) .

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