

Selfish Drug Allocation for Containing an International Influenza Pandemic at the Onset

Peng Sun,^{*} Liu Yang,[†] and Francis de Véricourt [‡]

May 25, 2009

Abstract

Recent epidemiologic studies have suggested that the prophylactic use of antiviral drugs could slow down the spread of an influenza epidemic. Since drug stockpiles are presently scattered in different countries, the outbreak of an epidemic gives rise to a game in which each country must make decisions about how best to allocate its own stockpile in order to protect its population. We develop a two-period multivariate Reed-Frost model to represent the spread of the epidemic within and across countries at its onset. We consider the first two periods only to mimic the exponential growth of an epidemic in its early stage, while keeping the model tractable. Preliminary numerical studies suggest that insights from the two period model hold in general when considering the entire time horizon. Our model captures three critical sources of uncertainty: the number of initial infections, the spread of the disease, and drug efficacy. We show that for small probabilities of between-country infections, the underlying game is supermodular, Nash equilibrium exists and there is a unique one that is Pareto optimal among all existing equilibria. Further, we identify sufficient conditions under which the optimal solution of a central planner (such as the World Health Organization) constitutes a Pareto improvement over the decentralized equilibrium, suggesting that countries should agree on an allocation scheme which would benefit everyone. By contrast, when the central planner's solution does not constitute a Pareto improvement, minimizing the total number of infected persons globally requires some countries to sacrifice part of their own population, which raises intriguing ethical issues.

Keywords: Reed-Frost Model, Epidemic Control, Influenza, Supermodular Games

^{*}The Fuqua School of Business, Duke University, Durham, NC 27708-0120, USA, peng.sun@duke.edu

[†]The Fuqua School of Business, Duke University, Durham, NC 27708-0120, USA, liu.yang@duke.edu

[‡]The Fuqua School of Business, Duke University, Durham, NC 27708-0120, USA, fdv1@duke.edu

1. Introduction

Most developed countries' preparedness plan for a possible pandemic influenza requires stockpiling antiviral drugs (Rosenthal 2005). Although the efficacy of these drugs (such as Tamiflu, see Ward et al. 2005, and references therein) is not well-known, recent studies have suggested that mass prophylaxis distribution of them might be able to contain a pandemic (Ferguson et al. 2005). Even if a total extinction of the epidemic might be difficult to achieve, antivirals can buy time for a vaccine to be developed, for more drugs to be produced or for other efficient policies to be implemented. Extant imperfect vaccines (i.e., vaccines that are poor matches to virus strains that are virulent against humans) have also been shown to slow the spread of the disease (Longini et al. 2005).

The use of antivirals (or imperfect vaccines) to contain an epidemic necessitates massive administration of drugs over a very short time period, right at the onset of the first infections. However, at present drugs or vaccines are purchased mainly by the least vulnerable countries (for instance, most of world's flu vaccines are produced in Western Europe, North America and Japan, Anonymous 2005). Regions where transmissions of the H5N1 influenza A virus¹ to humans have so far taken place, such as South East Asia, do not have sufficient funds to maintain large stocks of antivirals. This implies that drugs might need to be diverted to other countries if an epidemic were to be contained (Monto 2005). (In our settings, antiviral drugs and vaccines have similar effects and we use the term "drug" to designate both of them without distinction.)

The main objective of this study is to examine how countries possessing stocks of drugs would react if an epidemic started in a country possessing little or no antiviral medication. How much of its limited supply of a drug would a country give up to contain the pandemic? How much of the drug would each country keep to guard against possible future infections? We model this problem as a game involving the countries which possess antiviral or vaccine stockpiles, by developing a mathematical model that represents the dynamics of an epidemic in its early stage in a simple, but tractable way. This model captures three important sources of uncertainty: 1) the number of initial infections; 2) the spread of the disease within and between countries; and 3) the drug efficacy. We derive insights into the success (or lack thereof) at containing an outbreak when countries hoard these drugs selfishly, that is, when they seek to protect their respective populations, even, if necessary, at the expense of others.

Our paper follows a multi-disciplinary approach as it uses a game theoretic framework with stochastic models to address the issue of containing a pandemic. To the best of our knowledge,

this study constitutes the first analysis of epidemic control where antiviral drugs or vaccines are distributed by strategic agents who seek to protect different parts of the population. By contrast, previous analyses have typically considered a central planner seeking to contain an epidemic in the whole population. This line of research usually focuses on cost-benefit analysis of public health policies (Diekmann and Heesterbeek 2000, Brandeau 2004). In particular, recent works have explored the optimal allocation of limited resources in a non-homogeneous population (see, for instance, Brandeau et al. 2003, and references therein). However, we show in this paper that ignoring strategic behaviors can lead to overoptimistic assessments of the impact of the control. That is, when decision makers are strategic, their allocation decisions might be drastically different from the centralized solution, which leads to more infections in the world compared to the centralized solution.

Recent analyzes of influenza containment efforts are based mainly on extensive and detailed numerical studies. In particular, Ferguson et al. (2005) have run simulations that show how targeted mass prophylactic use of antiviral drugs significantly increases the extinction probability of an epidemic. Longini et al. (2005) and Germann et al. (2006) further suggest that the use of inefficient vaccines instead of (or in conjunction with) antivirals would also drastically slow the spread of the disease.²

For our purpose, we represent the epidemic at an aggregate level with a two-period multidimensional Reed-Frost model. The classical Reed-Frost model assumes that susceptibles (healthy individuals who could be infected) and infectives (infected individuals who could infect susceptibles) mix homogeneously within a closed population. Thus, the number of susceptibles at the end of period $t + 1$ follows a binomial distribution with probability $(1 - a)^{Y_t}$ and sequence size X_t , where a is the within-population infection probability and Y_t and X_t are the numbers of infectives and susceptibles at time period t , respectively. The infection probability a is equal to the probability that two individuals meet during the virus' latent period multiplied by the probability of infection per contact. Note that considering a stochastic model makes the analysis more challenging. However, the future spread of the disease constitutes a very important source of uncertainty when we try to contain a pandemic at the onset. A stochastic framework therefore appears necessary in our settings.

The main simplification of this model lies in the homogeneous mixing assumption, an assumption that is made in most of the epidemiology models, including deterministic SIR models (Daley and Gani 2001). Nevertheless, the Reed-Frost model has been shown to be successful at describing epidemic dynamics (see, for instance, Daley and Gani 2001). Furthermore, homogeneous mixing is a key assumption of most tractable models of epidemics. In particular, to account

for inhomogeneity, so-called spatial models (see, for instance, Ball 1991) consider multiple locations and migration of infectives, while assuming homogenous mixing of the population in each location. The multi-dimensional Reed-Frost model developed in this paper may be considered as a discrete time approximation to the continuous time spatial model, in which the migrations and infections occur in the same time period.

We partition the world of interest into $m + 1$ countries characterized by homogenous mixing both within and between populations that have different infection probabilities. The epidemic starts in country 0 and the remaining m countries possess drug stockpiles. In addition, we consider imperfect drugs such that the infection probability decreases when the susceptible, the infective, or both receive treatment before interacting. Thus $2(m + 1)$ random variables are needed to represent the whole system, in which the number of susceptibles at the end of each period follows multivariate binomial distributions.

We also need to specify the objectives of each country in our model. For centralized resource allocation decisions, the cost-benefit analyses in the epidemiology literature often regard the goal as being to decrease the reproduction number, R_0 (for a stratified population, defined as the spectral radius of the so called “next generation matrix”, see, for example, Hill and Longini 2003), to below 1 (Longini et al. 2005). In a decentralized world, like the one that we model in this paper, it is not entirely clear what an appropriate condition for R_0 would be for each individual country, since the condition should allow for an outbreak in the world outside the considered country. Therefore, in this paper we take a different approach by considering the main objective for mass drug administration to be either the eradication of the disease or to buy time at the onset of the epidemic (as argued by Mills et al. 2006). In our case, the early stage of the pandemic should occur in the first few periods of our Reed-Frost model. We consider the first two periods only, which allows one to mimic the exponential growth of the epidemic in its early stage, while keeping the model tractable. Indeed, when considering the first period only, all countries keep their drugs for their own populations, since potential benefits have not materialized during a single time period. By contrast, numerical studies suggest that our findings for two periods hold in general when considering the entire time horizon of the epidemic, with the difference that in this case countries are willing to give up their drugs more often. In other words, the two period model appears to predict that countries act more selfishly than they would if they were to consider the final size of their population.

When the objective is to limit the impact of the pandemic, we assume that each country seeks to minimize the expected number of infections at the end of the two periods. Since population sizes are fixed in our model, this objective is equivalent to maximizing the number of susceptibles

(i.e. non-infected persons) in the two first periods. Besides the expected value objective, we also study another plausible objective for each country – minimizing the probability of the disease spreading over from other countries in the beginning (two first periods) of the epidemic. This is also equivalent to maximizing the probability that no individual within the borders is infected during the first two periods. Other objective functions may exist that estimate the total economic costs of a pandemic and its impact on the quality-adjusted life years (see, for instance, Brandeau et al. 2003, for a study on the centralized resource allocation problem). In case of an outbreak, however, we believe that the number of infected individuals and the probability of the disease spreading over from other countries are the most pressing concerns of the decision makers.

At the beginning of the first time period, an infective is discovered in country 0. All countries share a common subjective probability distribution for the number of infected persons present in the population. In particular, this distribution should take into account the possibility of multiple introductions of pandemic-capable strains as discussed by Mills et al. (2006). Based on this distribution and on the different parameters of the system (country sizes, infection probabilities and inventory levels), each country makes its drug allocation decisions. A given country (say, country i) prefers the rest of the world to contribute more drugs to country 0, so that it could keep its drug stockpile to protect its own population. On the other hand, if country 0 does not receive enough drugs, country i may choose to give up part of its drug stockpile to help contain the pandemic in other populations. Indeed, if the epidemic is not contained in the country where it first occurs, it might quickly spread to the rest of the world, dramatically increasing the chance of infection from abroad. This situation thus gives rise to a game in which each country seeks to protect its population by strategically allocating its drugs.

Several papers have considered rational individual decision making in the face of epidemics (see Philipson 2004, and references therein). Bauch and Earn (2004) have recently introduced game theory into epidemic models. Their focus, however, is on voluntary vaccination, which differs from the topic of this paper. They described a ‘commons dilemma’ such that the risk associated with receiving vaccination is certain and individual-specific, while the return is both uncertain (i.e., the individual may never come into contact with an infective) and widely distributed. Thus, rational self-interest might incline an individual to avoid vaccination, which, in turn, could prevent complete eradication of the disease. Chick et al. (2008) also developed a game theory framework to analyze the coordination of vaccine production where actors (the government and manufacturers) have different incentives. Their work contributes to the line of research concerned with the production and stock-piling of vaccines (see also Deo and Corbett

2008). More generally, the challenge of containing influenza outbreaks has received a lot of attention recently in the OR/MS literature (see Wu et al. 2005, Kornish and Keeney 2008, Cho 2008, and references therein).

Our model also links to the economics literature concerning the voluntary contributions to public goods. This area of research is primarily interested in understanding how different agents may try to enjoy a “free ride,” i.e., benefit themselves at the expense of others, while encouraging the latter to increase their own contributions to some public project or good. For instance, Marx and Matthews (2000) provide sufficient conditions under which the final public project completion time reduces to zero and free riding vanishes. In our settings, the “public project” could be the worldwide eradication of the epidemic. The “contributions” would then correspond to the number of drugs allocated to the country where the first infective appeared. However, situations occur where all players are better off not achieving the public good (i.e., not giving up drugs to country 0). In these cases, the centralized solution basically requires countries to “sacrifice” part of their population in order to save more individuals in country 0. Protecting this population is therefore not quite equivalent to a public good. In our case, the inefficiency of a decentralized solution is not necessarily due to free-riding, but rather due to the misalignment of objectives of players and that of the central planner.

Furthermore, when all contribution decisions are made at the beginning of the time horizon and for linear payoff functions, the problem of free riding reduces to a prisoners’ dilemma and no agent contributes to the public project. However, this is not the case when the project corresponds to containing a global pandemic as shown in this paper. Actually, we identify situations where countries are willing to give up all their drugs to cover part of the population where the first infectives appeared. This significant difference is due to the dynamics of an epidemic spread which induce nonlinear payoff functions that are neither concave nor convex. In particular, we show that countries either keep or give away all their drugs (and do not split their resources).

For the two-period model, we show that for small between-country infection rates, the underlying game is supermodular, which implies the existence of a unique Nash equilibrium that is Pareto optimal among all existing equilibria. This equilibrium is characterized by a subset of countries that give up all their drugs to contain the spread of the epidemic in country 0, while all other countries keep their stockpiles to protect their own populations. If a central planner, such as the World Health Organization (WHO), were to make the allocation decision in order to minimize the total average number of infectives in the world (irrespective of nationalities), all existing stockpiles would be used to supply country 0. We identify sufficient conditions under

which the optimal solution of the central planner constitutes a Pareto improvement over the decentralized equilibrium. In particular, for large populations and a high probability of disease transmission in country 0, using all existing drugs to contain the epidemic in country 0 would benefit all countries. On the other hand, when countries act selfishly and seek to protect their own populations, there could be human cost in the rest of the world. These findings raise important ethical issues, especially when the centralized decision does not constitute a Pareto improvement over the decentralized situation. In this latter case, minimizing the total number of infectives in the world requires some countries to sacrifice part of their own population. Our numerical studies show that these insights hold, in general, in the entire horizon case.

We present our epidemic model and the drug allocation game in section 2. The case where countries maximize the expected final number of susceptibles in their population is analyzed in section 3. The comparison with the decisions of a central planner follows in section 3.3. Section 4 extends some of our results to the case where countries maximize the probability of not being infected from abroad. Section 5 numerically tests some of the model assumptions. In particular, we explore how our main insights extend when considering the entire time horizon of the epidemic. We draw several conclusions and point out to future research directions in section 6

2. The Drug allocation Game

Consider $m + 1$ countries, $0, 1, \dots, m$ where country i 's population size is equal to N_i for $0 < i \leq m$ and $N_0 + 1$ for country $i = 0$. The transmission probability from a non-treated infective in country i to a non-treated susceptible in country j is denoted by $a_{ij} \geq 0$. Access to drugs is assumed to decrease susceptibility by a factor δ and infectiousness if infected by a factor ξ . For instance, recent statistical studies estimate that antiviral drugs reduce susceptibility to infection for an uninfected individual by 30%, and infectiousness for infected individuals by 60%, which corresponds to $\delta = 70\%$ and $\xi = 40\%$ (Ferguson et al. 2003, Yang et al. 2006). When both parties have used drugs, the probability of infection per contact decreases by $\sigma := \delta\xi$. In what follows, we refer to a_{ij} as the probability of between-country infection when $i \neq j$ and as the probability of within-country infection when $i = j$. We assume that the disease spreads more easily within a country than between countries, that is, $a_{ij} < a_{ii}$ for all $i \neq j$.

We consider a multi-dimensional Reed-Frost epidemic model of two periods. Let X_i^t represent the number of susceptibles in location i at time t who have not been treated, while \bar{X}_i^t represents the number of susceptibles at time t who have taken the drug previously. Similarly, \bar{Y}_i^t and Y_i^t

represent the number of infectives in location i at time t who have (\bar{Y}_i^t) and have not (Y_i^t) used the drug, respectively. Thus $Y_i^t + \bar{Y}_i^t$ individuals spread the disease during period t , before becoming symptomatic and are removed by the end of the period. The number of susceptibles and infectives therefore satisfy the following relationships,

$$X_i^t = X_i^{t-1} - Y_i^t, \quad \bar{X}_i^t = \bar{X}_i^{t-1} - \bar{Y}_i^t. \quad (1)$$

Following the Reed-Frost model assumptions, the numbers of susceptibles have binomial distributions. In a given period, susceptibles of country i can be infected by infectives of the same country and of another country j . We can then easily deduce the following conditional probabilities,

$$(X_i^{t+1} | X_i^t, \{Y_j^t, \bar{Y}_j^t\}_j) \sim \mathcal{B} \left(X_i^t, (1 - a_{ii})^{Y_i^t} (1 - \xi a_{ii})^{\bar{Y}_i^t} \prod_{j \neq i} (1 - a_{ji})^{Y_j^t} (1 - \xi a_{ji})^{\bar{Y}_j^t} \right), \quad (2)$$

$$(\bar{X}_i^{t+1} | \bar{X}_i^t, \{Y_j^t, \bar{Y}_j^t\}_j) \sim \mathcal{B} \left(\bar{X}_i^t, (1 - \delta a_{ii})^{Y_i^t} (1 - \sigma a_{ii})^{\bar{Y}_i^t} \prod_{j \neq i} (1 - \delta a_{ji})^{Y_j^t} (1 - \sigma a_{ji})^{\bar{Y}_j^t} \right), \quad (3)$$

where $\mathcal{B}(N, p)$ represent a Binomial distribution with parameters N and p .

Because of possible multiple introductions or early infections, several infected individuals might already exist in the country at the time the first case is discovered. We assume that countries share a common subjective distribution for Y , the number of infected individuals in country 0 at time $t = 0$. This implies that the multiple introductions and first infections take place in country 0 only. Thus, $Y_0^0 = Y$, $Y_i^0 = 0$ for $i > 0$ and $\bar{Y}_i^0 = 0$ for all i . The initial conditional probabilities are then equal to $(X_i^1 | X_i^0, Y) \sim \mathcal{B}(X_i^0, (1 - a_{0i})^Y)$ and $(\bar{X}_i^1 | \bar{X}_i^0, Y) \sim \mathcal{B}(\bar{X}_i^0, (1 - \delta a_{0i})^Y)$ for $i = 0, 1, \dots, m$. We further let $E_Y(\cdot)$ and $\varphi_Y(\cdot)$ be the expectation operator and characteristic function, respectively, of the random variable Y .

Each country $i > 0$ has a stockpile of drugs that can treat up to K_i susceptibles. For simplicity of exposition we assume country 0 does not have any drug stockpile, or $K_0 = 0$. Our results still hold for $K_0 > 0$. We assume that $K_i \leq N_i$, and the world's total drug supply $K := \sum_j K_j$ is less than the population of country 0, that is $K \leq N_0$. This assumption appears to hold under the current situation, where a pandemic flu is likely originating from one of the populous southeast Asian countries. Note also that similar constraints on the number of drugs are often placed in simulation of epidemics (see for example Ferguson et al. 2005). Technically, such an assumption helps to alleviate some of the difficulties in showing the supermodularity of

the game.

We denote by n_j^i the number of country j 's susceptibles, $j \geq 0$, treated by drugs from country i 's stockpile (when drugs represent antivirals and not vaccines, a unit of inventory represents the number of doses required to treat an individual during the two periods). For simplicity, we assume that the unidentified first infectives Y are not treated when drugs are distributed (similar results hold when they all receive treatment). Given all allocation decisions $\{n_j^i\}_{i,j}$ with $\sum_j n_j^i = K_i$, the initial numbers of susceptibles are equal to

$$\begin{aligned} X_i^0 &= N_i - \sum_{k=1}^m n_i^k & \text{and} & & \bar{X}_i^0 &= \sum_{k=1}^m n_i^k, & \text{for } i > 0, & \text{ and} \\ X_0^0 &= N_0 - Y - \sum_{k=1}^m n_0^k & \text{and} & & \bar{X}_0^0 &= \sum_{k=1}^m n_0^k. \end{aligned} \tag{4}$$

In the following we also use \mathbf{n} to denote the decision matrix $\{n_j^i\}_{(m+1) \times (m+1)}$ such that row vector \mathbf{n}^i represents country i 's decisions.

3. Expected Value Case

In this section, we study the case in which each country is risk neutral and seeks to minimize the average number of total infectives during the two periods. Since in our model we assume a fixed population size, the objective is equivalent to maximizing the average number of non-infected susceptibles at the end of the time horizon. In section 4, we extend our results to the case where the objective function is the probability of no infection, which might describe more accurately some countries' actual motivations.

The problem, however, is generally intractable. A country's response curves are neither convex nor concave and sometimes possess multiple local maxima. On the other hand, we are primarily interested in contaminations between countries at the onset of the epidemic for which the between-country infection rates $\{a_{ij}\}_{i \neq j}$ are small. It turns out that in this case the problem is well structured. The key step in our analysis consists in taking Taylor expansions (in the between-infection rates) of the country's average number of susceptibles in the end of period 2. Our numerical results (see Online Appendix A) suggest that the structure of the game holds even when the between-country infection rates are not too small.

3.1 Country i 's Response Curve

We next derive country i 's response curve. To that end, we first evaluate the average number of susceptibles in the end of period 2 given the initial treated and untreated population sizes

for country i . In order to simplify our mathematical expressions, we introduce the following quantities, $A_{ji} = a_{ji}[1 - (1 - a_{0j})^Y]$ and $B_{ji} = a_{ji}[1 - (1 - \delta a_{0j})^Y]$. Note that A_{ji} and B_{ji} depends on Y , but for the sake of simplicity we do not make this dependence explicit in our notation.

Proposition 1 For country $i = 0, 1, \dots, m$,

$$\begin{aligned} \mathbb{E}[X_i^2 \mid \{X_j^0, \bar{X}_j^0\}_{j \geq 0}, Y] &= X_i^0 \frac{(1 - a_{0i})^Y}{1 - A_{ii}} \prod_{j=0}^m (1 - A_{ji})^{X_j^0} (1 - \xi B_{ji})^{\bar{X}_j^0}, \\ \mathbb{E}[\bar{X}_i^2 \mid \{X_j^0, \bar{X}_j^0\}_{j \geq 0}, Y] &= \bar{X}_i^0 \frac{(1 - \delta a_{0i})^Y}{1 - \sigma B_{ii}} \prod_{j=0}^m (1 - \delta A_{ji})^{X_j^0} (1 - \sigma B_{ji})^{\bar{X}_j^0}. \end{aligned}$$

Proof : See Online Appendix B. ■

Country i seeks to maximize the average number of susceptibles in its population at the time horizon which corresponds to the following optimization problem,

$$\max_{\mathbf{n}^i} f_i(\mathbf{n}) := \mathbb{E}_Y \left[\mathbb{E} \left[X_i^2 + \bar{X}_i^2 \mid \{X_j^0, \bar{X}_j^0\}_{j \geq 0}, Y \right] \right],$$

subject to the following constraints

$$\sum_j n_j^i = K_i, \quad 0 \leq n_j^i \leq N_j - \sum_{k \neq i} n_j^k.$$

Following Eq. (4) and Proposition 1, we obtain,

$$f_i(\mathbf{n}) = \left(N_i - \sum_{k=1}^m n_i^k \right) \mathbb{E}_Y [G_1^i] + \sum_{k=1}^m n_i^k \mathbb{E}_Y [G_2^i], \quad (5)$$

where,

$$\begin{aligned} G_1^i &= \frac{(1 - a_{0i})^Y}{1 - A_{ii}} (1 - A_{0i})^{-Y} \prod_{j=0}^m (1 - A_{ji})^{N_j - \sum_{k=1}^m n_j^k} (1 - \xi B_{ji})^{\sum_{k=1}^m n_j^k}, \\ G_2^i &= \frac{(1 - \delta a_{0i})^Y}{1 - \sigma B_{ii}} (1 - \delta A_{0i})^{-Y} \prod_{j=0}^m (1 - \delta A_{ji})^{N_j - \sum_{k=1}^m n_j^k} (1 - \sigma B_{ji})^{\sum_{k=1}^m n_j^k}. \end{aligned}$$

$\mathbb{E}_Y[G_1^i]$ and $\mathbb{E}_Y[G_2^i]$ represent the proportion of susceptibles in the population who were and were not treated, respectively.

We are now ready to study country i 's optimal drug allocation given the decisions made by all other countries. As will be seen, most of our proofs are based on treating the $f_i(\mathbf{n})$ as continuous functions. Most of our result also rely on the approximation that the between-country transmission probabilities a_{kl} , $k \neq l$, are ‘‘small enough’’ or ‘‘sufficiently small.’’ Specifically, it

means that there exists an $\epsilon > 0$ such that if all $a_{kl} < \epsilon$ then the corresponding result holds. Note that the transmission probability a_{kl} reflects the probability of contact *and* infection between any infective in country k and any susceptible in country l . In practice a_{kl} should be very small.

Our first result states that country i contributes drugs to country 0 only.

Proposition 2 *For small enough between-country transmission rates a_{kl} , $k \neq l$, country i never gives drugs to another country $j \neq 0$.*

Proof : See Online Appendix C. ■

The proof (Online Appendix C) reveals the intuition behind the result: from country i 's perspective, the marginal benefit from giving drugs to country 0 always exceeds the marginal benefit from giving drugs to another country $j \neq 0$. Therefore the question is whether to use the drug within country i or in country 0.

Denote then n_i to be the number of drug units that country i decides to keep for itself, so that $K_i - n_i$ is allocated to country 0. It follows that, with $K := \sum_j K_j$,

$$X_i^0 = N_i - n_i, \quad \bar{X}_i^0 = n_i, \quad X_0^0 = N_0 - Y - K + \sum_{k=1}^m n_k, \quad \bar{X}_0^0 = K - \sum_{k=1}^m n_k.$$

With drugs from country i only going to countries i and 0, country i 's objective function becomes

$$f_i(\mathbf{n}) = (N_i - n_i)E_Y[G_1^i] + n_i E_Y[G_2^i] = N_i E_Y[G_1^i] + n_i (E_Y[G_2^i] - E_Y[G_1^i]), \quad (6)$$

where,

$$G_1^i = \frac{(1 - a_{0i})^Y}{1 - A_{ii}} (1 - A_{0i})^{N_0 - Y - K + \sum_j n_j} (1 - \xi B_{0i})^{K - \sum_j n_j} \prod_{j \neq 0} (1 - A_{ji})^{N_j - n_j} (1 - \xi B_{ji})^{n_j}, \quad (7)$$

$$G_2^i = \frac{(1 - \delta a_{0i})^Y}{1 - \sigma B_{ii}} (1 - \delta A_{0i})^{N_0 - Y - K + \sum_j n_j} (1 - \sigma B_{0i})^{K - \sum_j n_j} \prod_{j \neq 0} (1 - \delta A_{ji})^{N_j - n_j} (1 - \sigma B_{ji})^{n_j}. \quad (8)$$

(Note that $E_Y[G_2^i]$ and $E_Y[G_1^i]$ still represent the proportion of susceptibles in the population who were and were not treated, respectively, and $f_i(\cdot)$ represents the average number of susceptibles. The mathematical expressions for $f_i(\cdot)$, G_1^i and G_2^i are different than their counterparts in (5) due to the simplification of players' decision sets. We continue using the same notation here for the sake of clarity.)³

In the sequel, we assume the following technical condition holds for all countries i ,

$$N_i - (1 - \delta\xi)K_i \geq \frac{1 - \delta^2\xi}{1 - \delta}. \quad (9)$$

This assumption is not very restrictive. It merely rules out parameters such that δ and ξ are simultaneously very close to 1 and 0, respectively. Under reasonable choices of parameters, such as $\delta = 70\%$ and $\xi = 40\%$, the condition translates to $N_i > 0.72K_i + 2.7$, which is easily satisfied.

The optimal decision for country i given all other countries' drug allocations has a “bang-bang” structure as stated by the following result.

Theorem 1 *For small enough between-country transmission probabilities a_{kl} , $k \neq l$, country i 's optimal decision given all other countries' actions is always “bang-bang”: either give up everything to country 0 or give nothing. Furthermore, there is a quantity a^* defined as*

$$a^* := \frac{1 - \xi - \varphi_Y [1 - a_{00}] + \xi\varphi_Y [1 - \delta a_{00}]}{(1 - \sigma)\mathbb{E}_Y[Y]}, \quad (10)$$

such that

1. When $a_{ii} \geq a^*$, f_i is monotonically increasing in n_i and country i keeps all its drugs for itself;
2. When $a_{ii} < a^*$, f_i is convex in n_i .

Proof : See Online Appendix D. ■

Quantity a^* is a common threshold for within-country transmission probability a_{ii} for all countries i . In Online Appendix E, we provide sufficient conditions for a country to be willing to give up its drugs. The “bang-bang” structure of the optimal response is rather surprising. It greatly simplifies the action profile that we need to consider when studying equilibria of the game.

3.2 Supermodular Game and Nash Equilibrium

Theorem 1 indicates that the best strategy for country i such that $a_{ii} \geq a^*$ is degenerate with $n_i = K_i$. In other words, country i has no incentive to give up drugs to another country where the disease transmission probabilities are sufficiently low. On the other hand, when $a_{ii} < a^*$, there exist reasonable model parameters under which a country gives up all its drugs to country 0 (Online Appendix E). Hence, the drug allocation game is not degenerate and is only played by countries in which the disease transmission probability is lower than a certain threshold.

The following theorem constitutes our main result and states the existence of a unique Nash equilibrium that is Pareto optimal among all Nash equilibria.

Theorem 2 *For small enough between-country transmission probabilities a_{kl} , $k \neq l$, the game is supermodular among countries with $a_{ii} < a^*$, in which a^* is defined in Eq. (17) of Theorem 1. Further, a unique Nash Equilibrium exists that is Pareto optimal among all existing Nash equilibria. This equilibrium is such that the number of countries giving up drugs to country 0 is the highest among all existing equilibria.*

The proof depends on the supermodular structure of the game. The complete proof is in Online Appendix F. In what follows, we refer to the unique Nash Equilibrium that is Pareto optimal among all existing Nash equilibria as the “Pareto optimal Nash Equilibrium.”

According to Theorem 1, an equilibrium is characterized by a partition $S \cup \bar{S} = [1, \dots, m]$ such that $i \in S$ if and only if country i keeps all its drug, i.e. $n_i = K_i$. From Theorem 2, a Pareto optimal equilibrium exists and is characterized by the set \bar{S} with the highest cardinality.

It is worth pointing out that the existence of a Pareto optimal Nash Equilibrium does not automatically guarantee it will be the outcome of the game. As pointed out in the experimental economics literature (see, for instance, VanHuyck et al. 1990, 1991), even the Pareto dominant equilibrium that “should” be chosen according to risk dominance (Harsanyi and Selten 1988) may not occur in practice. A more recent literature on the so-called “global games” (Carlsson and Van Damme 1993) attempts to address the equilibrium selection issue by first relaxing the implicit common knowledge assumption of complete information games so that payoffs are observed with a small amount of noise. Under certain conditions, there is a unique equilibrium in the game with noise, which serves as a prediction for the underlying complete information game that contains multiple equilibria. Frankel et al. (2003), which studies supermodular games in this framework, is probably the most relevant global games literature to our problem. However, the results in Frankel et al. (2003) do not appear to directly apply to our setting.

3.3 Central Planner

Consider now a central planner, such as the WHO, whose objective is to allocate the total world drug inventory $K = \sum_i K_i$ in order to maximize the average number susceptibles at the end of period 2 in the world regardless of the country of residence. Let n_i be the amount of drugs the central planner allocates to country $i = 0, \dots, m$, with $n_0 = K - \sum_i n_i$, and define $\mathbf{n} = \{n_i\}_{i \geq 0}$ to be the corresponding drug allocation policy.

The central planner's optimization problem is, therefore,

$$\max_{\mathbf{n} \geq 0, e^\top \mathbf{n} = 0} f(\mathbf{n}) := \mathbb{E} \left[\sum_{i=0}^m X_i^2 + \bar{X}_i^2 \mid \{N_j - n_j, n_j\}_{j \geq 0} \right] \quad (11)$$

and from Proposition 1 we obtain,

$$f(\mathbf{n}) = \mathbb{E}_Y \left[\left(N_0 - Y - K + \sum_k n_k \right) G_1^0 + \left(K - \sum_k n_k \right) G_2^0 + \sum_{i=1}^m (N_i - n_i) G_1^i + n_i G_2^i \right] \quad (12)$$

where,

$$G_1^0 = \frac{(1 - a_{00})^Y}{1 - A_{00}} (1 - A_{00})^{N_0 - Y - K + \sum_k n_k} (1 - \xi B_{00})^{K - \sum_k n_k} \prod_{j \neq 0} (1 - A_{j0})^{N_j - n_j} (1 - \xi B_{j0})^{n_j}$$

$$G_2^0 = \frac{(1 - \delta a_{00})^Y}{1 - \sigma B_{00}} (1 - \delta A_{00})^{N_0 - Y - K + \sum_k n_k} (1 - \sigma B_{00})^{K - \sum_k n_k} \prod_{j \neq 0, i} (1 - \delta A_{j0})^{N_j - n_j} (1 - \sigma B_{j0})^{n_j}$$

When between-country infection rates are low, the central planner always supplies country 0 as much as possible.

Proposition 3 *For small enough between-country transmission rates a_{kl} , $k \neq l$, the optimal allocation policy uses all existing drugs for country 0, i.e. $n_0 = K$ and $n_i = 0$, $i > 0$.*

Proof : See Online Appendix G. ■

Allocating all existing drugs to country 0 maximizes the number of susceptibles at the end of period 2 in the world but might hurt some countries that could save more of their inhabitants if they acted selfishly and kept their drugs. The following results provide conditions under which supplying country 0 with all available drugs constitutes a Pareto improvement over the previous optimal decentralized equilibrium. In this case, all countries would benefit from letting the central planner to decide.

Proposition 4 *For $\{a_{kl}\}_{k \neq l}$ small enough, consider the decentralized equilibrium characterized by the set S of countries that do not give up their drugs (i.e. $i \in S \Leftrightarrow n_i = K_i$). For each $i \in S$, there is a population size threshold \hat{N}_i such that supplying country 0 with all existing drugs (i.e. $\sum_{j=1}^m n_j = K$) is a Pareto improvement over the decentralized equilibrium if for all $i \in S$,*

$$N_i > \hat{N}_i \quad \text{and} \quad K_i < \rho_i \left(K - \sum_{j \in \bar{S}} K_j \right)$$

where \hat{N}_i is independent of N_j , $j > 0$, and

$$\rho_i = \frac{a^*(1 - \sigma)}{a_{ii}(2 - \sigma - \delta)} .$$

Proof : See Online Appendix H. ■

(A cumbersome analytical expression exists for the population threshold \hat{N}_i and can be found in equation (22) in Online Appendix H.)

In other words, if countries in S are large enough and have relatively small drug inventories, giving up all existing drugs to country 0 improves the average number of non-infected susceptibles in all countries. A central planner could then try to have all countries in S agree on this policy. Without monetary enforcement, however, the centralized optimal solution may never be an equilibrium. On the other hand, the WHO may serve as a correlation center in a correlated equilibrium solution concept (Myerson 1991), which provides a coordination mechanism that potentially improves upon a Nash equilibrium in a full information game. Unfortunately correlated equilibria do not constitute such an improvement in our settings.⁴

The second condition $K_i < \rho(K - \sum_{j \in \bar{S}} K_j)$ is actually always satisfied if $\rho > 1$, which is reasonable when the epidemics spread sufficiently faster in country 0 than in the rest of the world. Hence, Proposition 4 implies that when all countries are large enough and the epidemic spreads sufficiently fast in country 0, allocating all existing drugs to country 0 constitutes a Pareto improvement over any decentralized equilibrium.

4. Alternative objective: Probability of No Infection

Existing epidemiological studies (see for instance Ferguson et al. 2005) have focused their performance analysis on the probability of epidemic extinction in a given world area. In this section, we consider the perspective of countries with drug stockpiles that decide to maximize the probability of no infection from the country where the epidemic has started. Note that this objective is neither risk seeking nor risk averse and is not well justified from a theoretical perspective. On the other hand, maximizing the probability of no infection might more accurately represent actual responses to an outbreak, given the psychological reactions and panic that most likely would occur (see for instance Lau et al. 2006).

In order to keep the model tractable, we approximate the binomial distribution of Y_i^1 by a Poisson distribution with arrival rate $\lambda_i = a_{0i}X_i^0$. This approximation is accurate when X_i^0 is large and a_{0i} is very small. Similarly, we assume that \bar{Y}_i^1 follows a Poisson distribution with

parameter $\bar{\lambda}_i = \delta a_{0i} \bar{X}_i^0$. Country i may then maximize the likelihood of no infection, which, after some algebraic simplifications, is approximately equal to

$$\begin{aligned} \mathbb{E}_Y [P(X_i^2 = X_i^0 \cap \bar{X}_i^2 = \bar{X}_i^0 \mid \{X_j^0, \bar{X}_j^0\}_j)] \approx \\ \mathbb{E}_Y \left\{ (1 - a_{0i})^{Y X_i^0} (1 - \delta a_{0i})^{Y \bar{X}_i^0} \exp \left\{ \sum_{j \neq i} [(1 - (1 - a_{0j})^Y) ((1 - a_{ji})^{X_i^0} + (1 - \delta a_{ji})^{\bar{X}_i^0} - 2) X_j^0 \right. \right. \\ \left. \left. + (1 - (1 - \delta a_{0j})^Y) ((1 - \xi a_{ji})^{X_i^0} + (1 - \sigma a_{ji})^{\bar{X}_i^0} - 2) \bar{X}_j^0 \right] \right\} \right\}. \end{aligned}$$

Country i 's optimization problem becomes, following Eq. (4),

$$\begin{aligned} \max_{\mathbf{n}^i: \mathbf{n}^i \geq 0, \sum_j n_j^i = K_i} f_i(\mathbf{n}) := \mathbb{E}_Y \left\{ (1 - a_{0i})^{Y(N_i - \mathbf{n}_i)} (1 - \delta a_{0i})^{Y \mathbf{n}_i} \right. \\ \left. \exp \left\{ \sum_{j \neq i} [(N_j - \mathbf{n}_j) (1 - (1 - a_{0j})^Y) \left((1 - a_{ji})^{N_i - \mathbf{n}_i} + (1 - \delta a_{ji})^{\mathbf{n}_i} - 2 \right) \right. \right. \\ \left. \left. + \mathbf{n}_j (1 - (1 - \delta a_{0j})^Y) \left((1 - \xi a_{ji})^{N_i - \mathbf{n}_i} + (1 - \sigma a_{ji})^{\mathbf{n}_i} - 2 \right) \right] \right\} \right\}, \end{aligned} \quad (13)$$

where $\mathbf{n}_j = \sum_{k=1}^m n_j^k$ is the total number of drugs that country j receives.

It turns out that the structure of the drug allocation game is very similar to the expected value case as shown by the following result,

Theorem 3 *For between-country transmission probabilities a_{0k} ($k > 0$) small enough,*

1. *Country i never gives drugs to country $j \neq 0$.*
2. *Country i 's optimal decision given all other countries' actions is always "bang-bang": either give up everything to country 0 or give nothing.*
3. *The game is supermodular; a unique Nash Equilibrium exists that is Pareto optimal among all existing equilibria, and has the highest number of countries giving drugs to country 0.*

Proof : See Online Appendix I.

Note also that when N_i is large enough, country i gives all its drugs to country 0 (see Proposition 6 in Online Appendix I). Hence, although the probability of no infection constitutes a rather different objective function compared to the expected value, our main results and insights hold.

5. Numerical Study

In this section we numerically explore the present results from a computational perspective. First, in section 5.1, we numerically examine the parameter settings under which having every

m	1	4	7		
N_0	10^5	10^6	10^7		
N_i	10^5	10^6	10^7		
K/N_0	10%	50%	90%		
a_{00}	$1.3/N_0$	$2/N_0$	$2.5/N_0$	$3/N_0$	
a_{ii}	a_{00}	$a_{00}/5$	$a_{00}/10$		
a_{ij}	a_{ii}	$a_{ii}/5$	$a_{ii}/10$	$a_{ii}/100$	

Table 1: Model parameters.

country give up their drugs to country 0 constitutes either the Pareto optimal Nash equilibrium or a Pareto improvement from an equilibrium. Then, section 5.2 is devoted to a simulation study of the case when countries' objectives are the expected total number of infectives in the entire time horizon of the epidemic.

In the computational study presented in this section, we fix the drug effectiveness parameters $\delta = 0.7$ and $\xi = 0.4$ (Ferguson et al. 2003, Yang et al. 2006). For simplicity, we also fix the number of initial infectives $Y = 1$.

5.1 To give or not to give?

In this section we numerically study whether having every country give up its drugs to country 0 constitutes either a Pareto equilibrium or a Pareto improvement from an equilibrium. To that end we conduct an extensive computational study with varied model parameters as indicated in Table 1.

Specifically, we vary the number of drug possessing countries, m , to be 1, 4 and 7. We also vary each country's population, positing the values 10^5 , 10^6 or 10^7 , assuming all drug possessing countries' populations are the same. The world's total drug stockpiles, K , is assumed to be 10%, 50%, or 90% of country 0's population, and evenly distributed among all drug possessing countries. Country 0's disease transmission probability a_{00} takes four different values, $1.3/N_0$, $2/N_0$, $2.5/N_0$ and $3/N_0$. If we consider country 0 being isolated from the rest of the world, $a_{00}N_0$ may be perceived as the basic reproductive ratio R_0 in the corresponding Reed-Frost model. The above choices of a_{00} reflects, roughly speaking, R_0 taking values 1.3, 2, 2.5 and 3. The within-country infection rates, a_{ii} , $i > 0$, are chosen to be a_{00} , $a_{00}/5$ or $a_{00}/10$. Finally a_{ij} takes the values 1, $1/5$, $1/10$ or $1/100$ of a_{ii} .

In total, we vary 7 model parameters. There are altogether 3,888 possible combinations of model parameters. Among the 3,888 cases, some contradict our model assumptions. For example, when $N_0 = 10^7$ and $N_i = 10^5$, the per country stockpile, $K_i = K/m$, will always

N_0	a_{ij}	a_{ii}		
		a_{00}	$a_{00}/5$	$a_{00}/10$
10^5	a_{ii}	0%	81.48%	99.07%
	$a_{ii}/5$	0%	97.22%	100%
	$a_{ii}/10$	0%	97.22%	100%
	$a_{ii}/100$	0%	99.07%	100%
10^6	a_{ii}	0%	59.09%	80.68%
	$a_{ii}/5$	0%	78.41%	81.82%
	$a_{ii}/10$	0%	78.41%	81.82%
	$a_{ii}/100$	0%	76.14%	81.82%
10^7	a_{ii}	0%	25.00%	55.77%
	$a_{ii}/5$	0%	48.08%	69.23%
	$a_{ii}/10$	0%	51.92%	57.69%
	$a_{ii}/100$	0%	51.92%	36.54%

Table 2: Percentage of “giving up” equilibria.

be greater than N_i . We therefore ignore such infeasible model parameters, which leaves 2,976 feasible combinations of model parameters. Of these, 1,586 (53.29%) of the cases are such that having all countries give their drugs to country 0 is the Pareto optimal Nash equilibrium. Of the remaining 1,390 cases where the Pareto optimal Nash equilibrium has all drug-possessing countries keep their drugs for themselves, 606 (43.60% of the 1390 cases) are such that having all countries give their drugs to country 0 constitutes a Pareto improvement over the equilibrium. Another point worth mentioning is that in the 606 cases of Pareto improvement, the model parameters of 600 cases (99.01%), when we ignore the positive a_{ij} 's, satisfy the sufficient conditions in Proposition 4. That is,

$$N_i > \frac{a_{ii}(\delta\sigma - 1) + (1 - \delta) \left[1 + N_0[1 - \varphi_Y(1 - a_{00})]/E_Y[Y] - (1 - \sigma) \left(\sum_{j \in \bar{S}} K_j a^* + K_i a_{ii} \right) \right]}{a^*(1 - \sigma)(K - \sum_{j \in \bar{S}} K_j)/K_i - a_{ii}(2 - \sigma - \delta)},$$

and

$$K_i < \frac{a^*(1 - \sigma)}{a_{ii}(2 - \sigma - \delta)} \left(K - \sum_{j \in \bar{S}} K_j \right).$$

In other words, our numerical study suggests that in roughly half of the cases, countries' selfish interests lead to the same drug allocation that a central planner would make. For 25% of the cases, countries playing the game will not supply country 0 with drugs, but negotiations before hand could yield the optimal centralized decision which consists a Pareto improvement. For the remaining 25% cases however, country 0 will not receive drugs.

Table 2 illustrates how the model parameters affect the Pareto optimal Nash equilibrium.

Each cell in Table 2 indicates the percentage of cases among the feasible combinations of parameter choices, when N_0 , a_{ii} and a_{ij} are fixed, for which having country $i > 0$ give up its drugs to country 0 is the Pareto optimal Nash equilibrium. For example, when $N_0 = 10^5$, $a_{ii} = a_{00}/10$ and $a_{ij} = a_{ii}/10$, there are a total of 108 feasible combinations of N_i , K and a_{00} , out of which 108 cases are such that having country $i > 0$ give up its drugs to country 0 is the Pareto optimal Nash equilibrium. Therefore, the corresponding cell in Table 2 is 100%(= 108/108). It is clear from Table 2 that country i is more likely to give up its drugs in the Pareto optimal Nash equilibrium when the country's a_{ii} is smaller than a_{00} , a_{ij} is smaller than a_{ii} , and N_0 is small.

We also compute the optimal solutions of the centralized planner's problem (11). In 1366 out of the 1390 cases where the Pareto optimal Nash equilibrium has all drug-possessing countries keep their drugs for themselves, the centralized optimal solution corresponds to allocating all existing drugs to country 0. In the remaining 24 cases, it is optimal for the central planner to allocate drugs according to their initial endowment (i.e. each countries uses the drugs they have). We further compare results from the Pareto optimal Nash equilibrium with the centralized optimal solution. To that end, we measure the ratio of the total expected number of infectives in the world for the decentralize case over that of the centralized solution. The geometric mean of the 1390 ratios is 1.134. That is, on average, the Pareto optimal Nash equilibrium is 13.4% worse than the centralized optimal solution. The worst 20 out of the 1390 cases have ratios above 2 (i.e. the decentralized allocation of drugs can lead to more than doubling the number of infectives compared to the central planner decision).

In short, our numerical study suggests that the decentralized equilibrium often leads to the optimal drug allocation of the central planner (in about half of the tested cases). When this is not the case, however, decentralized decision making seems to have a significant negative impact on the control of the pandemic.

5.2 Entire Time Horizon of the Pandemic

So far we have considered the two first periods of a multidimensional Reed-Frost model. Even though the main focus of this paper is on deriving analytical results and insights into the drug allocation problem, we provide preliminary simulation results for the entire time horizon, i.e. when considering the final sizes of the epidemic in multiple countries. Specifically, "final size" refers to the total number of infectives during the entire horizon of the epidemic. The main purpose of the simulation is to study the equilibrium behavior when the objective of each country

is the final size of the epidemics in the corresponding country. Note however that considering the entire time horizon of the epidemic is also problematic. Existing mathematical models and simulations of epidemics assume that transmission probabilities remain constant over the entire time horizon. However, this ignores important population behaviors such as social distancing or border closing that would certainly occur in case of a global pandemic.

We consider a world with three countries, $\{0, 1, 2\}$. Each country has a population 10^6 . We choose the transmission probability parameter $a_{00} = 2 \times 10^{-6}$. Similar to before, roughly speaking, if country 0 is isolated from the rest of the world, its basic reproductive rate is 2. The dynamics of the system is described by equations (2)-(3). If we are given the initial allocation of the drugs, which will be explained in the next paragraph, we can simulate the number of susceptibles, infectives and removals in the three countries over time. To speed up the simulation, we approximate the Binomial distribution with the corresponding Poisson distribution when the mean of the Binomial distribution is less than 0.02, and with Normal approximations otherwise. In each simulated trajectory, we stop until $y_i^t + \bar{y}_i^t = 0$ and obtain the final size of the epidemic z_i^∞ . The expected number of susceptibles in each time period t can be estimated by the average $x_i^t + \bar{x}_i^t$ across the many trajectories. The expected value of the total removal is estimated from the average z_i^∞ among the simulation trials. For simplicity, we call the expected value of the susceptible final size as the “final size”.

Assuming countries $i = 1, 2$ are identical, We vary the choices of a_{ii} and K_i such that a_{ii} takes values as $a_{00}, 0.9a_{00}, 0.5a_{00}, 0.3a_{00}, 0.1a_{00}$, and K_i takes values 0.1, 0.3 and 0.5 of the country 0’s population size N_0 . We also set the cross country transmission probability $a_{ij} = a_{0i} = a_{i0} = a_{ii}/10$ for each case. These parameters describe symmetric network structures. Online Appendix K considers “chain” network structures in which country 2 and country 0 are not directly connected. The findings are similar, except that the two period model has less predictive power if used as an approximation of the entire horizon model.

In total, we tested 18 combinations of model parameters (see Table 3). For each set combination of model parameters, we mutate the amount of drugs that countries 1 and 2 give up to country 0 through $0, 0.1, \dots, 1$ of K_i corresponding to a total of 121 cases. We simulate 10^5 trajectories in order to have an accurate estimate of the mean final size. For each set of parameters, the simulation takes more than 10 hours on a personal computer. Our numerical results indicate that a unique Pareto dominant equilibrium exists for every one of the entire horizon cases in Table 3, consistent with what is predicted by Theorem 2 for the two period case. Further, in all tested cases but one, the optimal decisions at the equilibrium either give up or keep all drugs, a result that is reminiscent of the result in Theorem 1.

		K_i					
		$0.5N_0$		$0.3N_0$		$0.1N_0$	
		Two Periods	Entire Horizon	Two Periods	Entire Horizon	Two Periods	Entire Horizon
a_{ii}	a_{00}	Keep	Give up	Keep	Keep	Keep	Keep
	$0.9a_{00}$	Keep	Give up	Keep	Keep	Keep	Keep
	$0.5a_{00}$	Keep	Give up*	Keep	Give up	Keep	Keep
	$0.4a_{00}$	Give up	Give up	Keep	Give up	Keep	Keep
	$0.3a_{00}$	Give up	Give up	Give up	Give up	Keep	Give up
	$0.2a_{00}$	Give up	Give up	Give up	Give up	Give up	Give up

Table 3: Equilibria: two period versus entire horizon models – Symmetric network structure. (*: in this equilibrium Countries $i = 1, 2$ give up $0.8K_i$ or $0.4N_0$ of drugs to Country 0.)

Table 3 compares the Pareto optimal Nash equilibrium for the two period and entire time horizon cases. In all tested cases, it turns out that Countries 1 and 2 make similar allocation decisions at the equilibrium. In Table 3, as well as later in the paper, we denote “Keep” to represent the action profile that both countries keep the drugs for their own use. Similarly, “Give up” means that both countries give up their drugs to country 0. In particular, the “Keep” and “Give up” in Table 3 indicate the corresponding action profile in the Pareto optimal Nash equilibrium. According to Table 3, when the two period model predicts that countries 1 and 2 give up their drugs to country 0, the same result holds at the equilibrium of the entire time horizon. On the other hand, countries sometimes keep their drugs with two periods, but give them up when considering the entire time horizon. This suggests that myopic decision makers would act more selfishly than they would if they considered the final size of the pandemic.

Finally, we compare in Table 4 the final sizes corresponding to the different drug allocations at equilibrium as reported in Table 3, for the finite and entire horizon cases. In most cases, the final sizes are equal since the equilibria from the two models are the same. When drug capacity K_i is very small ($0.1N_0$), or country i 's transmission rate a_{ii} is very close to a_{00} , the final sizes according to different allocation policies are still similar, reflecting the fact that it does not matter very much where the drugs are used. When a_{ii} is smaller than a_{00} ($a_{ii} \leq 0.5a_{00}$) and K_i is not too small ($K_i \geq 0.3N_0$), “Give up” and “Keep” have much more profound impact on the final size of the epidemic. For example, when $a_{ii} = 0.5a_{00}$ and $K_i = 0.5N_0$, the final size of “Keep” (dominant equilibrium from the two period model) is almost 30 times the final size of the allocation where both country give up $0.8K_i$ to country 0 (dominant equilibrium from the entire horizon model). In this case, a myopic decision makers would act more selfishly, which, in turn, would dramatically increase the number of infectives in the world.

		K_i					
		$0.5N_0$		$0.3N_0$		$0.1N_0$	
		Two Periods	Entire Horizon	Two Periods	Entire Horizon	Two Periods	Entire Horizon
a_{ii}	a_{00}	1,787,423	1,001,549	2,038,872	2,038,872	2,240,185	2,240,185
	$0.9a_{00}$	1,628,703	840,109	1,909,971	1,909,971	2,124,970	2,124,970
	$0.5a_{00}$	923,400	32,999	1,059,902	428,662	1,253,372	1,253,372
	$0.4a_{00}$	8	8	881,695	149,608	982,405	982,405
	$0.3a_{00}$	3.8	3.8	81,349	81,349	809,452	632,086
	$0.2a_{00}$	3.4	3.4	64,863	64,863	545,780	545,780

Table 4: Final sizes according to the dominant equilibria from the two period model versus the entire horizon model.

6. Conclusions

Recent epidemiological studies have suggested that the prophylactic use of antiviral drugs or imperfect vaccines could slow the spread of an influenza epidemic. When existing drug stockpiles are scattered in different countries, this situation gives rise to a game where each country seeks to protect its own population. More generally, our paper provides the first insights as to how self-interested countries would actually use their antiviral drugs to cover another population if an epidemic were to appear in the world.

Existing studies of epidemic containment make extensive use of simulations. In the presence of strategic players, this approach is very hard to generalize. For our purpose, we resorted to analytical techniques and developed a tractable multivariate Reed-Frost model where countries allocate drugs according to their narrow self-interests.

The drug allocation game may evoke a prisoners' dilemma which would then predict that all countries keep their drugs for their own population. In fact, giving up drugs to another country could be extremely difficult to justify to the public. Nevertheless, when considering epidemic dynamics and related uncertainties, our results suggest that countries might reach an equilibrium where at least some countries give up all their drug stockpiles to contain the pandemic. We consider the very beginning of a pandemic, but preliminary simulations suggest that this insight holds for the entire time horizon, with the difference that countries give up their drugs in a broader range of situations (i.e. for more sets of parameters) than for the two period case. Further, when countries have large populations and when the disease spreads quickly, implementing the optimal drug allocation of a central planner can reduce the number of infectives compared to the decentralized equilibrium in each country. In other words, an agreement between the different decision makers could reduce the scale of the epidemic in all countries. Surprisingly, many of our results hold for different objective functions (expected value

and probability of no infection), suggesting that our findings are relatively robust to varied model assumptions.

Our model simultaneously captures three critical sources of uncertainty: the number of initial infections, the spread of the disease, and drug efficacy. Future work could analyze more refined versions of the model. For instance, a fourth source of uncertainty comprises the number of initial infectives that receive treatment when drugs are distributed (at time $t = 0$). We assume for simplicity that none of these initially infected individuals are treated, although our results hold when this number is deterministic. An extension of our work could instead consider treating this number as a random variable. More general research directions include explicitly describing the distribution of drugs within and between countries (with lead times, for instance) that also consider logistic constraints. These extensions constitute important research opportunities, although thus far we have found these problems to be analytically demanding. Another extension is to model a longer time horizon and multi-period decision processes. Without closed form expressions, even a three period model is very hard to evaluate, considering the high dimensionality of the problem (e.g., each decision maker needs to consider X_i^t , \bar{X}_i^t , Y_i^t and \bar{Y}_i^t for all countries $i \geq 0$, that is, $2(m + 1)$ dimensions, at each time epoch.) In our ongoing work, the framework and findings of this paper help us derive approximations for a multi-period dynamic game that generalizes the model. These heuristics should help us numerically test and refine the insights of this paper for richer models.

So far we have focused on modelling the onset of an epidemic. We consider the first two periods only to mimic the epidemics exponential growth in its early stage, while keeping the model tractable. When the whole course of the pandemic is considered, and with the assumption that the transmission parameters remain constant over time, a pertinent objective for each country is to minimize total expected number of infected people. This would, however, require the assumption that population's behaviors do not change significantly and that countries do not close their borders during the pandemic. Nonetheless, our numerical studies suggest that our findings for two periods hold, in general, when considering the entire time horizon. On the other hand, such a game is in most cases intractable for analytical results, especially when we consider different sources of uncertainty as in our current model. However, for deterministic SIR models, each country's response curve could be inferred from the solution of a system of equations similar to Equation (2.4.7) in Daley and Gani (2001). In our view this constitutes the most promising research direction.

Our results also raise the intriguing question of how imperfect information affects the drug allocation game. Countries may not disclose their drug stockpiles or their actions. Country 0

may even not reveal the existence of a first infective. These extensions would help evaluate the value of sharing information in case of a pandemic. Furthermore, we assume that the model parameters (especially transmission probabilities a_{ij}) are common knowledge, which is necessary for analytical tractability. In practice, these parameters may be estimated, and may also constitute private information held by each country. It would then be interesting to determine mechanisms that give countries incentives to truthfully report their a_{ii} 's. Also, for political reasons, countries might have a privately known propensity to keep drugs for themselves. This situation has been studied in the arm race and negotiation literature (see Baliga and Sjoström 2004) and constitutes a valuable research direction for the drug allocation game we study in this paper.

Finally, our paper is the first to study the decentralized allocation of resources to control a pandemic. This framework appears very relevant for the containment of epidemics in other contexts. For example in a recent working paper, Chick et al. (2009) study coordination contracts for a game between governments that purchase vaccines in order to achieve an efficient cost-benefit tradeoff in their respective countries.

End Notes

1. The H5N1 influenza A virus is the virus that is the subject of greatest concern for a pandemic at present.
2. These simulations allow great precision in modeling the spatiotemporal evolution of the epidemic as well as different drug or vaccine distribution policies (e.g., ring policies or vaccination of children). The resulting estimates of the epidemic size and the probability of extinction are therefore fairly accurate. On the other hand, such detailed dynamics are analytically intractable, making structural results hard to obtain. In addition, simulations are inadequate to analyze equilibria resulting from drug distributions made by strategic players, which is the main focus of this paper. Thus, This paper focuses on deriving insights and structural results based on simplified models.
3. Given this particular structure of the game, our problem becomes similar to the seminal model proposed by Bergstrom et al. (1986) on the private provision of public goods. Our model, however, violates important assumptions made in Bergstrom et al. (1986). As a result, the analysis of Bergstrom et al. (1986) does not apply to our analysis.
4. Generally speaking the set of Nash equilibria is a subset of correlated equilibria, and the “best” correlated equilibrium may improve upon Nash. Unfortunately, the set of Nash and

correlated equilibria have the same bounds for supermodular games as indicated by Milgrom and Roberts (1990). This means that correlated equilibria do not improve in our problem.

Acknowledgement: We appreciate useful discussions with and feedback from Bill Gimson, Kimberly Lane and Fangjun Zhou at the CDC, Edward Kaplan at Yale, Duncan Simester at MIT, Padmaja Patnaik at the Family Health International (Epidemiology), Karin Yeatts at the UNC Department of Epidemiology, and seminar participants at the Duke University.

References

- Anonymous (2005). Preparing for an influenza pandemic. *The Economist*. Sep. 24, 95-96.
- Baliga, S. and Sjostrom, T. (2004). Arms Races and Negotiations. *Review of Economic Studies*, 71(2):351–369.
- Ball, F. (1991). Dynamic population epidemic models. *Mathematical Biosciences*, 107:299–324.
- Bauch, C. and Earn, D. (2004). Vaccination and the theory of games. *Proceedings of the National Academy of Sciences*, 101(36):13391–13394.
- Bergstrom, T., Blume, L., and Varian, H. (1986). On the private provision of public goods. *Journal of Public Economics*, 29:25–49.
- Brandeau, M. (2004). *Allocating Resources To control Infectious Diseases*, chapter 17, pages 443–465. Kluwer Academic Publishers.
- Brandeau, M., Zaric, G., and Richter, A. (2003). Resource allocation for control of infectious diseases in multiple independent populations: beyond cost-effectiveness analysis. *Journal of Health Economics*, 22(4):575–598.
- Carlsson, H. and Van Damme, E. (1993). Global games and equilibrium selection. *Econometrica*, 61:989–1018.
- Chick, S., Mamani, H., and Simchi-Levi, D. (2008). Supply Chain Coordination and Influenza Vaccination. Operations Research, Forthcoming.
- Chick, S., Mamani, H., and Simchi-Levi, D. (2009). Influenza vaccine supply chain with multiple agencies. working paper.
- Cho, S. H. (2008). The optimal composition of influenza vaccines subject to random production yields. UCLA Anderson School of Management, CA.
- Daley, D. J. and Gani, J. (2001). *Epidemic Modelling: An Introduction*. Cambridge University Press.

- Deo, S. and Corbett, C. (2008). Cournot competition under yield uncertainty: The case of the US influenza vaccine market. Kellogg School of Management, Evanston, IL.
- Diekmann, O. and Heesterbeek, J. (2000). *Mathematical epidemiology of infectious diseases*. Wiley.
- Ferguson, N., Mallett, S., Jackson, H., Roberts, N., and Ward, P. (2003). A population-dynamic model for evaluating the potential spread of drug-resistant influenza virus infections during community-based use of antivirals. *Journal of Antimicrobial Chemotherapy*, 51:977–990.
- Ferguson, N. M., Cummings, D., Cauchemez, S., Fraser, C., Riley, S., Meeyai, A., Iamsirithaworn, S., and Burke, D. S. (2005). Strategies for containing an emerging influenza pandemic in southeast asia. *Nature*, 437:209–214.
- Frankel, D., Morris, S., and A., P. (2003). Equilibrium selection in global games with strategic complementarities. *Journal of Economic Theory*, 108:1–44.
- Germann, T. C., Kadau, K., Longini, I. M., and Macken, C. A. (2006). Mitigation strategies for pandemic influenza in the united states. *Proceedings of the National Academy of Sciences*, 103(15):5935 – 5940.
- Harsanyi, J. and Selten, R. (1988). *A General Theory of Equilibrium Selection in Games*. MIT Press, Cambridge, MA.
- Hill, A. and Longini, I. (2003). The critical vaccination fraction for heterogeneous epidemic models. *Mathematical Biosciences*, 181:85–106.
- Kornish, L. and Keeney, R. (2008). Repeated Commit-or-Defer Decisions with a Deadline: The Influenza Vaccine Composition. *Operations Research*, 56(3):527.
- Lau, J., Tsui, H., Kim, J. H., and Griffiths, S. (2006). Perceptions about status and modes of h5n1 transmission and associations with immediate behavioral responses in the hong kong general population. *Preventive Medicine*, 43(5):406–410.
- Longini, I., Nizam, A., Xu, S., Ungchusak, K., Hanshaoworakul, W., Cummings, D., and Hal-loran, M. E. (2005). Containing pandemic influenza at the source. *Science*, 309(12):1083–1087.
- Marx, L. and Matthews, S. (2000). Dynamic voluntary contribution to a public project. *Review of Economic Studies*, 67:327–358.
- Milgrom, P. and Roberts, J. (1990). Rationalizability, learning, and equilibrium in games with strategic complementarities. *Econometrica*, 58:1255–1277.

- Mills, C., Robins, J., Bergstrom, C., and Lipsitch, M. (2006). Pandemic Influenza: Risk of Multiple Introductions and the Need to Prepare for Them. *Policy Forum*, 3(6).
- Monto, A. S. (2005). The threat of an avian influenza pandemic. *Nature*, 352(4):323–325.
- Myerson, R. B. (1991). *Game Theory: Analysis of Conflict*. Harvard University Press.
- Philipson, T. (2004). *Economic Epidemiology and Infectious Disease*, chapter 17, pages 443–465. J. Newhouse and T. Culyer. New York: North-Holland.
- Rosenthal, E. (2005). Better planning is needed for flu drugs, experts say. *The New York Times*. Oct. 19, pp8.
- Topkis, D. M. (1998). *Supermodularity and complementarity*. Princeton University Press.
- VanHuyck, J., Battalio, R., and Beil, R. (1990). Tacit coordination games, strategic uncertainty, and coordination failure. *The American Economic Review*, 80(1):234–248.
- VanHuyck, J., Battalio, R., and Beil, R. (1991). Strategic uncertainty, equilibrium selection, and coordination failure in average opinion games. *The Quarterly Journal of Economics*, 106(3):885–910.
- Ward, P., Small, I., Smith, J., Suter, P., and Dutkowski, R. (2005). Oseltamivir (Tamiflu (R)) and its potential for use in the event of an influenza pandemic. *Journal of Antimicrobial Chemotherapy*, 55:i5.
- Wu, J., Wein, L., and Perelson, A. (2005). Optimization of Influenza Vaccine Selection. *Operations Research*, 53(3):456–476.
- Yang, Y., Longini Jr, I., and Halloran, M. (2006). Design and evaluation of prophylactic interventions using infectious disease incidence data from close contact groups. *Journal of the Royal Statistical Society Series C (Applied Statistics)*, 55:317.

Online Appendix

A. Between-country transmission probabilities close to zero

The drug allocation game studied in this paper is in general intractable. However, our analytical result suggests that for a small enough between-country infection probability, countries' response curves are increasing or convex and the drug allocation game is supermodular. In the following we demonstrate that these findings hold for reasonable choices of parameters.

For simplicity we present results from a computational study involving three countries, 0, 1 and 2. We test the convexity and supermodularity of $f_i(\mathbf{n})$ (defined in (6)). We fix the population sizes of each country to be $N_0 = N_1 = N_2 = 10^5$.

In Table 5, we vary the model parameters $K_1 = K_2$ to take values 10^4 , 2×10^4 and 4×10^4 . We also vary choices of a_{00} such that if country 0 is isolated from the rest of the world and no drug is used, the proportion of uninfected people in that country at the end of the two periods takes values 99%, 90%, 70% and 50%, respectively. (This corresponds to a_{00} taking values 3.1×10^{-4} , 0.001, 0.0019 and 0.0026, respectively.) The disease transmission probability that we are interested in modelling arguably falls in this range, i.e., at least half of country 0's population (in expectation) are not infected in the first two periods of the disease.¹

For each combination of K_i and a_{00} , we vary the within-country transmission probability $a_{11} = a_{22}$ from 0 up to a_{00} , and we vary the between-country transmission probability a_{ij} from 0 up to a_{ii} . In Table 5, we report the proportion of all such choices of parameters a_{ii} and a_{ij} so that the function $f_i(\mathbf{n})$ is both convex and supermodular.²

It is clear from Table 5 that function $f_i(\mathbf{n})$ is convex and supermodular in almost all parameter settings tested (98.87% of all cases tested for Table 5). Although there do exist cases such that $f_i(\mathbf{n})$ is not convex and supermodular (the numbers below 100% in Table 5), when a_{00} is reasonably small (below 0.0019 in our case, corresponding to more than 70% survival rate during the first two periods), function f_i is always convex and supermodular. This exercise demonstrates that our main analytical results are robust with respect to reasonable choices of transmission probability parameters.

When $a_{00} = 0.0026$ and $K_i = 10,000$, corresponding to only a 50% survival rate in country

¹Were it otherwise, the severity of the disease transmission naturally would motivate societies to react in a way that reduces the transmission probability a_{00} .

²Due to extensive computation time, we checked the second order derivatives of the function $f_i(\mathbf{n})$ at points $n_i = 0, 100, 200, \dots, K_i$ for its convexity and supermodularity. A more accurate check on $n_i = 0, 1, 2, \dots, K_i$ would take months on a personal computer.

a_{00}	X_0^2/N_0	K_i		
		10,000	20,000	40,000
3.1×10^{-4}	99%	100%	100%	100%
0.001	90%	100%	100%	100%
0.0019	70%	100%	100%	100%
0.0026	50%	93.66%	94.63%	97.07%

Table 5: Percentage of convex and supermodular $f_i(\mathbf{n})$'s.

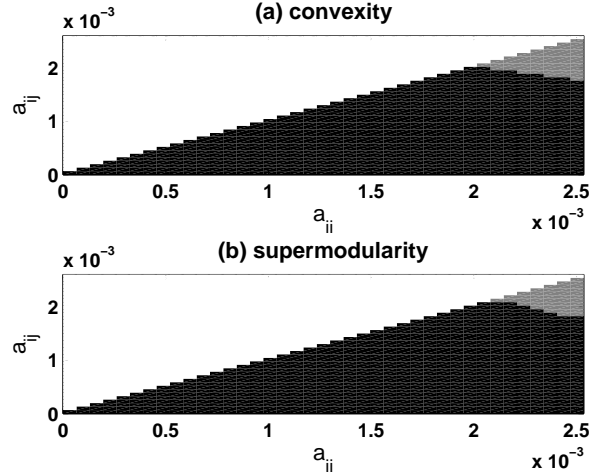


Figure 1: Choice of parameters that guarantees convexity and supermodularity of f_1 .

0, we further investigate the choices of a_{ii} and a_{kl} that make the functions f_i convex and supermodular. The black area in Figure 1(a) indicates the set of parameters a_{ii} and a_{0i} for which the corresponding functions $f_i(\mathbf{n})$ are convex in n_i . The gray area corresponds to choices of a_{ii} and a_{0i} such that f_i is not convex. The message is the same regarding supermodularity in Figure 1(b).

It is noticeable from Figure 1 that non-convexity or non-supermodularity only occur when a_{ii} and a_{ij} are relatively large. Such choices of parameters inevitably lead to low survival rates in country i during the first two time periods. This, combined with an already low (50%) survival rate in country 0, corresponds to less realistic situations. Therefore, for reasonable choices of model parameters, function f_i is indeed convex and supermodular, implying that our main results are not constrained by the “small between-country probability of infection” assumption.

B. Proof of Proposition 1

Proposition 1 For country $i = 0, 1, \dots, m$,

$$\begin{aligned} \mathbb{E}[X_i^2 \mid \{X_j^0, \bar{X}_j^0\}_{j \geq 0}, Y] &= X_i^0 \frac{(1 - a_{0i})^Y}{1 - A_{ii}} \prod_{j=0}^m (1 - A_{ji})^{X_j^0} (1 - \xi B_{ji})^{\bar{X}_j^0}, \\ \mathbb{E}[\bar{X}_i^2 \mid \{X_j^0, \bar{X}_j^0\}_{j \geq 0}, Y] &= \bar{X}_i^0 \frac{(1 - \delta a_{0i})^Y}{1 - \sigma B_{ii}} \prod_{j=0}^m (1 - \delta A_{ji})^{X_j^0} (1 - \sigma B_{ji})^{\bar{X}_j^0}. \end{aligned}$$

Proof : We have

$$\begin{aligned} \mathbb{E}[X_i^2 \mid \{X_j^0, \bar{X}_j^0\}_{j \geq 0}, Y] &= \mathbb{E}\left[X_i^1 (1 - a_{ii})^{Y_i^1} (1 - \xi a_{ii})^{\bar{Y}_i^1} \prod_{j \neq i} (1 - a_{ji})^{Y_j^1} (1 - \xi a_{ji})^{\bar{Y}_j^1}\right] \\ &= (X_i^0 \mathbb{E}[(1 - a_{ii})^{Y_i^1}] - \mathbb{E}[Y_i^1 (1 - a_{ii})^{Y_i^1}]) \times \\ &\quad \mathbb{E}[(1 - \xi a_{ii})^{\bar{Y}_i^1}] \prod_{j \neq i} \mathbb{E}[(1 - a_{ji})^{Y_j^1}] \prod_{j \neq i} \mathbb{E}[(1 - \xi a_{ji})^{\bar{Y}_j^1}] \\ &= X_i^0 \frac{(1 - a_{0i})^Y}{1 - A_{ii}} \prod_{j=0}^m (1 - A_{ji})^{X_j^0} (1 - \xi B_{ji})^{\bar{X}_j^0} \end{aligned}$$

where the second equality comes from the independence of Y_j^1 and \bar{Y}_j^1 , $j \geq 0$ and equation (1) with $(Y_j^1 \mid Y) \sim B(X_j^0, 1 - (1 - \delta a_{0j})^Y)$ and $(\bar{Y}_j^1 \mid Y) \sim B(\bar{X}_j^0, 1 - (1 - \sigma a_{0j})^Y)$. The last equality follows from Equations (23) and (25) of Lemma 4 (in Appendix J). Similarly,

$$\begin{aligned} \mathbb{E}[\bar{X}_i^2 \mid \{X_j^0, \bar{X}_j^0\}_{j \geq 0}, Y] &= \mathbb{E}\left[\bar{X}_i^1 (1 - \delta a_{ii})^{Y_i^1} (1 - \sigma a_{ii})^{\bar{Y}_i^1} \prod_{j \neq i} (1 - \delta a_{ji})^{Y_j^1} (1 - \sigma a_{ji})^{\bar{Y}_j^1}\right] \\ &= (\bar{X}_i^0 \mathbb{E}[(1 - \sigma a_{ii})^{\bar{Y}_i^1}] - \mathbb{E}[\bar{Y}_i^1 (1 - \sigma a_{ii})^{\bar{Y}_i^1}]) \times \\ &\quad \mathbb{E}[(1 - \delta a_{ii})^{Y_i^1}] \prod_{j \neq i} \mathbb{E}[(1 - \delta a_{ji})^{Y_j^1}] \mathbb{E}[(1 - \sigma a_{ji})^{\bar{Y}_j^1}] \\ &= \bar{X}_i^0 \frac{(1 - \delta a_{0i})^Y}{1 - \sigma A_{ii}} \prod_{j=0}^m (1 - \delta A_{ji})^{X_j^0} (1 - \sigma B_{ji})^{\bar{X}_j^0} \quad \blacksquare \end{aligned}$$

C. Proof of Proposition 2

Proposition 2 For small enough between-country transmission rates a_{kl} , $k \neq l$, country i never gives drugs to another country $j \neq 0$.

Proof : The first order derivative with respect to n_j^i for $j \neq i$ of country i 's the objective

function is equal to, from (5)

$$\frac{\partial f_i}{\partial n_j^i} = \left(N_i - \sum_{k=1}^m n_i^k \right) \mathbb{E}_Y \left[G_1^i \ln \frac{1 - \delta \xi A_{ji}}{1 - A_{ji}} \right] + \mathbb{E}_Y \left[\sum_{k=1}^m n_i^k G_2^i \ln \frac{1 - \delta^2 \xi A_{ji}}{1 - \delta A_{ji}} \right], \text{ when } j > 0 \quad (14)$$

$$\frac{\partial f_i}{\partial n_0^i} = \left(N_i - \sum_{k=1}^m n_i^k \right) \mathbb{E}_Y \left[G_1^i \ln \frac{1 - \delta \xi A_{0i}}{1 - A_{0i}} \right] + \sum_{k=1}^m n_i^k \mathbb{E}_Y \left[G_2^i \ln \frac{1 - \delta^2 \xi A_{0i}}{1 - \delta A_{0i}} \right]. \quad (15)$$

Eq. (26) in Lemma 5 (in Appendix J) implies that for $i \neq j$,

$$\frac{\partial f_i}{\partial n_j^i} < \frac{\partial f_i}{\partial n_0^i}. \quad (16)$$

According to the KKT conditions, at optimality \mathbf{n}^{i*} , we have

$$\begin{aligned} \left. \frac{\partial f_i}{\partial n_j^i} \right|_{\mathbf{n}^{i*}} &= \nu, \quad \text{when } 0 < n_j^{i*} < N_j - \sum_{k \neq i} n_j^k; \\ \left. \frac{\partial f_i}{\partial n_j^i} \right|_{\mathbf{n}^{i*}} &\leq \nu, \quad \text{when } n_j^{i*} = 0; \\ \left. \frac{\partial f_i}{\partial n_j^i} \right|_{\mathbf{n}^{i*}} &\geq \nu, \quad \text{when } n_j^{i*} = N_j - \sum_{k \neq i} n_j^k, \end{aligned}$$

in which ν is the Lagrangian multiplier associated with the constraint $\sum_j n_j^i = K_i$.

Since $K < N_0$, we always have $n_0^i < N_0 - \sum_{k \neq i} n_0^k$, and therefore $\partial f_i / \partial n_0^i |_{\mathbf{n}^{i*}} \leq \nu$. Following Eq. (16), we have $\partial f_i / \partial n_j^i |_{\mathbf{n}^{i*}} < \nu$, implying $n_j^{i*} = 0$ for all $j \neq 0, i$. \blacksquare

D. Proof of Theorem 1

Theorem 1 *For small enough between-country transmission probabilities a_{kl} , $k \neq l$, country i 's optimal decision given all other countries' actions is always "bang-bang": either give up everything to country 0 or give nothing. Furthermore, there is a quantity a^* defined as*

$$a^* := \frac{1 - \xi - \varphi_Y [1 - a_{00}] + \xi \varphi_Y [1 - \delta a_{00}]}{(1 - \sigma) \mathbb{E}_Y [Y]}, \quad (17)$$

such that

1. When $a_{ii} \geq a^*$, f_i is monotonically increasing in n_i and country i keeps all its drugs for itself;
2. When $a_{ii} < a^*$, f_i is convex in n_i .

Proof : For 1., the derivatives of f_i with respect to n_i is equal to, from (6)

$$\frac{\partial f_i}{\partial n_i} = \mathbb{E}_Y \left[G_2^i - G_1^i + (N_i - n_i) G_1^i \ln \frac{(1 - \xi B_{ii})(1 - A_{0i})}{(1 - A_{ii})(1 - \xi B_{0i})} + n_i G_2^i \ln \frac{(1 - \sigma B_{ii})(1 - \delta A_{0i})}{(1 - \delta A_{ii})(1 - \sigma B_{0i})} \right]. \quad (18)$$

Consider $\partial f_i / \partial n_i$ as a function of $\{a_{kl}\}_{k \neq l}$. We take the directional derivative of $\partial f_i / \partial n_i$ with respect to $\{a_{kl}\}_{k \neq l}$ along the directional positive vector \mathbf{d} . Each component of vector \mathbf{d} corresponds to a pair of countries $(k, l)_{k \neq l}$ and the corresponding transmission probability $\{a_{kl}\}$. We have, following (9),

$$\begin{aligned} \nabla_{\mathbf{d}} \mathbb{E}_Y [G_2^i - G_1^i]_{\{a_{kl}\}_{k \neq l} = 0} &= \frac{d_{0i}}{\|\mathbf{d}\|} \mathbb{E}_Y \left\{ Y \left[1 + \left(N_i - (1 - \sigma)n_i - \frac{1 - \delta\sigma}{1 - \delta} \right) a_{ii} \right] (1 - \delta) + \right. \\ &\quad (1 - \delta) \left[(N_0 - Y - K + \sum_j n_j) [1 - (1 - a_{00})^Y] + \right. \\ &\quad \left. \left. \xi (K - \sum_j n_j) [1 - (1 - \delta a_{00})^Y] \right] \right\} > 0. \end{aligned}$$

And we have the following two expressions,

$$\nabla_{\mathbf{d}} \mathbb{E}_Y \left[G_1^i \ln \frac{(1 - \xi B_{ii})(1 - A_{0i})}{(1 - A_{ii})(1 - \xi B_{0i})} \right] = \nabla_{\mathbf{d}} \mathbb{E}_Y \left[\ln \frac{(1 - \xi B_{ii})(1 - A_{0i})}{(1 - A_{ii})(1 - \xi B_{0i})} \right] = -\frac{d_{0i}}{\|\mathbf{d}\|} \mathcal{H}(a_{00}, a_{ii}), \text{ and}$$

$$\nabla_{\mathbf{d}} \mathbb{E}_Y \left[G_2^i \ln \frac{(1 - \sigma B_{ii})(1 - \delta A_{0i})}{(1 - \delta A_{ii})(1 - \sigma B_{0i})} \right] = \nabla_{\mathbf{d}} \mathbb{E}_Y \left[\ln \frac{(1 - \sigma B_{ii})(1 - \delta A_{0i})}{(1 - \delta A_{ii})(1 - \sigma B_{0i})} \right] = -\frac{d_{0i}}{\|\mathbf{d}\|} \delta \mathcal{H}(a_{00}, a_{ii}),$$

where

$$\mathcal{H}(a_{00}, a_{ii}) := \mathbb{E}_Y \left\{ [1 - (1 - a_{00})^Y] - \xi [1 - (1 - \delta a_{00})^Y] - (1 - \sigma) a_{ii} Y \right\},$$

which is positive when $a_{ii} < a^*$.

Given that $\nabla_{\mathbf{d}} \partial f_i / \partial n_i$ is a continuous function of $\{a_{kl}\}_{k \neq l}$ in the domain $D := [0, 1]^{m(m-1)}$, there is a neighborhood $N \subset D$ of the origin such that $\partial f_i / \partial n_i$ is positive and f_i is increasing in n_i for all $\{a_{kl}\}_{k \neq l} \in N$.

For 2., the second-order derivative is equal to

$$\begin{aligned} \frac{\partial^2 f_i}{\partial n_i^2} &= 2\mathbb{E}_Y \left[G_2^i \ln \frac{(1 - \sigma B_{ii})(1 - \delta A_{0i})}{(1 - \delta A_{ii})(1 - \sigma B_{0i})} - G_1^i \ln \frac{(1 - \xi B_{ii})(1 - A_{0i})}{(1 - A_{ii})(1 - \xi B_{0i})} \right] \\ &\quad + \mathbb{E}_Y \left[(N_i - n_i) G_1^i \left(\ln \frac{(1 - \xi B_{ii})(1 - A_{0i})}{(1 - A_{ii})(1 - \xi B_{0i})} \right)^2 + n_i G_2^i \left(\ln \frac{(1 - \sigma B_{ii})(1 - \delta A_{0i})}{(1 - \delta A_{ii})(1 - \sigma B_{0i})} \right)^2 \right]. \end{aligned} \quad (19)$$

The second expectation is non-negative. Following an approach similar to 1., we conclude from Eq. (28) in Lemma 6 (in Appendix J) that the first term is also non-negative for $\{a_{kl}\}_{k \neq l}$ small enough. The theorem follows directly from 1. and 2. \blacksquare

E. Condition for giving up everything to country 0

We first consider the condition in the expected value model and provide some numerical results. Then we present the corresponding result for the probability of no infection model.

We first provide the proof of Proposition 5 and some numerical results about the expected value model. Then we present the corresponding result for the probability of no infection model.

Proposition 5 *In the expected value model described in Section 3, for a_{kl} small enough, $k \neq l$, and given all other countries' actions, country i gives up all its drugs to country 0 when a_{00} and N_i are large enough compared to a_{ii} and N_0 , respectively.*

Proof : For the sake of clarity, we show the result for the case where $n_j = K_j$ for all $j \neq i$. The results hold for the more general case following a similar approach. From Theorem 1 we need only to compare country i 's objective function f_i at $n_i = 0$ and $n_i = K_i$:

$$f_i|_{n_i=0} - f_i|_{n_i=K_i} = \mathbb{E}_Y \left[N_i(G_1^0 - G_1^{K_i}) + K_i(G_1^{K_i} - G_2^{K_i}) \right]$$

where

$$G_1^{K_i} = \frac{(1 - a_{0i})^Y}{1 - A_{ii}} (1 - A_{0i})^{N_0} \prod_{j \neq 0} (1 - A_{ji})^{N_j - K_j} (1 - \xi B_{ji})^{K_j}$$

$$G_2^{K_i} = \frac{(1 - \delta a_{0i})^Y}{1 - \sigma B_{ii}} (1 - \delta A_{0i})^{N_0} \prod_{j \neq 0} (1 - \delta A_{ji})^{N_j - K_j} (1 - \sigma B_{ji})^{K_j}$$

$$G_1^0 = \frac{(1 - a_{0i})^Y}{1 - A_{ii}} (1 - A_{0i})^{N_0 - K_i} (1 - \xi B_{0i})^{K_i} \prod_{j \neq 0, i} (1 - A_{ji})^{N_j - K_j} (1 - \xi B_{ji})^{K_j} (1 - A_{ii})^{N_i}$$

For small a_{kl} , consider the directional derivative of $f_i|_{n_i=0} - f_i|_{n_i=K_i}$ as a function of a_{kl} 's along the direction $\mathbf{d} = \{d_{kl}\}_{k \neq l}$.

$$\begin{aligned} & \nabla_{\mathbf{d}}(f_i|_{n_i=0} - f_i|_{n_i=K_i})|_{\{a_{kl}\}_{k \neq l}=0} \\ &= \frac{d_{0i}}{\|\mathbf{d}\|} K_i \mathbb{E}_Y \left\{ N_i \left[(1 - (1 - a_{00})^Y) - \xi(1 - (1 - \delta a_{00})^Y) - a_{ii}(2 - \sigma - \delta)Y \right] + a_{ii}(1 - \delta\sigma)Y \right. \\ & \quad \left. - (1 - \delta) \left[Y + N_0(1 - (1 - a_{00})^Y) - K_i(1 - \sigma)a_{ii}Y \right] \right\} \\ &> \frac{d_{0i}}{\|\mathbf{d}\|} K_i \mathbb{E}_Y \left\{ N_i \left[(1 - (1 - a_{00})^Y) - \xi(1 - (1 - \delta a_{00})^Y) - a_{ii}(2 - \sigma - \delta)Y \right] - \right. \\ & \quad \left. N_0(1 - \delta)(1 - (1 - a_{00})^Y) - (1 - \delta)Y \right\}. \end{aligned}$$

It follows then that $f_i|_{n_i=0} - f_i|_{n_i=K_i} > 0$ when $a_{00} \gg a_{ii}$ and $N_i \gg N_0$. ■

We also conducted several numerical tests to demonstrate the existence of parameters that

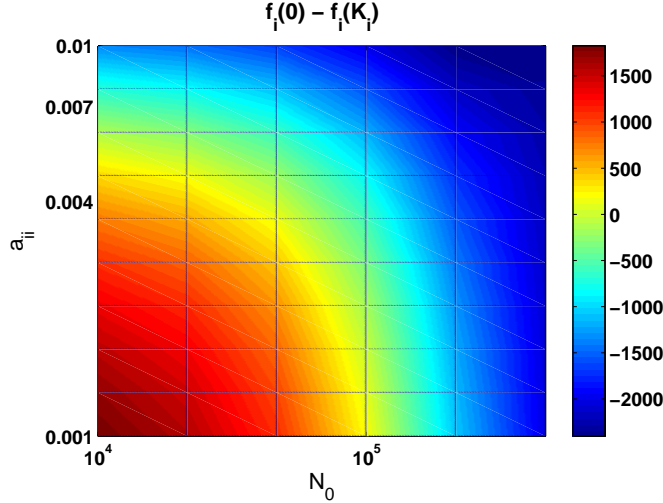


Figure 2: Whether country i gives up.

would make country i willing to give up drugs. In Figure 2, we fix $N_i = 10^5$, $K_i = 10^4$, $a_{00} = 0.01$, $a_{0i} = a_{00}/30$ and $\delta = \xi = 0.5$. By varying N_0 and a_{ii} , we see that there are reasonable choices of parameters such that the difference, $f_i(0) - f_i(K_i)$ is positive. (We also checked that the function f_i is convex for all the choices of parameters that generate Figure 2.)

It is worth noting that Proposition 5 should be interpreted in a single decision maker setting. In a game equilibrium, country i may still have the motivation to give up its drug stockpile to country 0 even if $N_i \approx N_0$, as is evident in the computational study of Section 5.2 (Table 3). Roughly speaking, the effect of other countries giving up drugs to country 0 is similar to reducing N_0 , which may motivate country i to give up to country 0 as well.

Another point worth mentioning is that Proposition 5 only provides an easily described sufficient condition for country i to give drugs to country 0. In fact, the drug stockpile K_i also plays a role, which is harder to describe, and therefore not included in Proposition 5. Following the proof in Appendix E, it is clear that the directional derivative of the difference between giving up and keeping is a concave quadratic function of K_i . Therefore, even if a_{00} and N_i are not sufficiently larger than a_{ii} and N_0 , there may still exist a continuous interval of K_i in which country i is willing to give up to country 0. Such an interval will naturally depend upon other model parameters, and therefore can not be given an intuitive description.

Next, we present the corresponding result for the probability of no infection model. Interestingly, there are some similarities to the expected value model, in terms of choice of model parameters, that makes country i willing to give up drugs.

Proposition 6 *In the probability of no infection model described in Section 4, for a_{kl} small*

enough, $k \neq l$, and given all other countries' actions, country i gives up all its drugs to country 0 if

$$N_i \gg N_0 .$$

Proof : We show the result for the case where $n_j = K_j$ for all $j \neq i$. Since country i 's optimal decision is "bang-bang", we only compare country i 's objective function f_i at $n_i = 0$ and $n_i = K_i$.

$$\begin{aligned} f_i(0) &= \mathbb{E}_Y \left\{ \exp \left[y N_i \ln(1 - \alpha_{0i}) \right. \right. \\ &\quad + (N_0 - K_i)(1 - (1 - \alpha_{00})^Y)((1 - \alpha_{0i})^{N_i} - 1) + K_i(1 - (1 - \delta\alpha_{00})^Y)((1 - \xi\alpha_{0i})^{N_i} - 1) \\ &\quad \left. \left. + \sum_{j \neq i, 0} (N_j - K_j)(1 - (1 - \alpha_{0j})^Y)((1 - \alpha_{ji})^{N_i} - 1) + K_j(1 - (1 - \delta\alpha_{0j})^Y)((1 - \xi\alpha_{ji})^{N_i} - 1) \right] \right\} \\ f_i(K_i) &= \mathbb{E}_Y \left\{ \exp \left[Y(N_i - K_i) \ln(1 - \alpha_{0i}) + y K_i \ln(1 - \delta\alpha_{0i}) \right. \right. \\ &\quad + N_0(1 - (1 - \alpha_{00})^Y)((1 - \alpha_{0i})^{N_i - K_i} + (1 - \delta\alpha_{0i})^{K_i} - 2) \\ &\quad + \sum_{j \neq i, 0} (N_j - K_j)(1 - (1 - \alpha_{0j})^Y)((1 - \alpha_{ji})^{N_i - K_i} + (1 - \delta\alpha_{ji})^{K_i} - 2) \\ &\quad \left. \left. + \sum_{j \neq i, 0} K_j(1 - (1 - \delta\alpha_{0j})^Y)((1 - \xi\alpha_{ji})^{N_i - K_i} + (1 - \sigma\alpha_{ji})^{K_i} - 2) \right] \right\} . \end{aligned}$$

For small α_{kl} , consider the directional derivative of $f_i(0)$ and $f_i(K_i)$ along the direction $\mathbf{d} = \{d_{kl}\}_{k \neq l}$.

$$\nabla_{\mathbf{d}} f_i(0) |_{\{a_{kl}\}_{k \neq l} = 0} = -\frac{d_{0i}}{\|\mathbf{d}\|} \mathbb{E}_Y \left[y N_i + (N_0 - K_i)(1 - (1 - \alpha_{00})^Y) N_i + K_i(1 - (1 - \delta\alpha_{00})^Y) N_i \xi \right]$$

$$\nabla_{\mathbf{d}} f_i(K_i) |_{\{a_{kl}\}_{k \neq l} = 0} = -\frac{d_{0i}}{\|\mathbf{d}\|} \mathbb{E}_Y \left[Y(N_i - K_i) + y K_i \delta + N_0(1 - (1 - \alpha_{00})^Y)(N_i - K_i + K_i \delta) \right] .$$

Therefore

$$\begin{aligned} \nabla_{\mathbf{d}} (f_i(0) - f_i(K_i)) |_{\{a_{kl}\}_{k \neq l} = 0} &= \frac{d_{0i}}{\|\mathbf{d}\|} K_i \mathbb{E}_Y \left\{ Y(\delta - 1) + \left[(1 - (1 - \alpha_{00})^Y) - \xi(1 - (1 - \delta\alpha_{00})^Y) \right] N_i \right. \\ &\quad \left. - (1 - (1 - \alpha_{00})^Y)(1 - \delta) N_0 \right\} , \end{aligned}$$

and the result follows for sufficiently small $\{a_{kl}\}_{k \neq l}$ ■

F. Proof of Theorem 2

Theorem 2 *For small enough between-country transmission probabilities a_{kl} , $k \neq l$, the game is supermodular among countries with $a_{ii} < a^*$, in which a^* is defined in Eq. (17) of Theorem 1. Further, a unique Nash Equilibrium exists that is Pareto optimal among all existing equilibria, and is such that the number of countries giving up drugs to country 0 is the highest among all existing equilibria.*

Proof : We study the supermodularity of f_i through its second order derivative.

$$\begin{aligned} \frac{\partial^2 f_i}{\partial n_i \partial n_j} &= \mathbb{E}_Y \left[G_2^i \ln \frac{(1 - \sigma B_{ji})(1 - \delta A_{0i})}{(1 - \delta A_{ji})(1 - \sigma B_{0i})} - G_1^i \ln \frac{(1 - \xi B_{ji})(1 - A_{0i})}{(1 - A_{ji})(1 - \xi B_{0i})} \right] \\ &\quad + \mathbb{E}_Y \left[(N_i - n_i) G_1^i \ln \frac{(1 - \xi B_{ii})(1 - A_{0i})}{(1 - A_{ii})(1 - \xi B_{0i})} \ln \frac{(1 - \xi B_{ji})(1 - A_{0i})}{(1 - A_{ji})(1 - \xi B_{0i})} \right] \\ &\quad + \mathbb{E}_Y \left[n_i G_2^i \ln \frac{(1 - \sigma B_{ii})(1 - \delta A_{0i})}{(1 - \delta A_{ii})(1 - \sigma B_{0i})} \ln \frac{(1 - \sigma B_{ji})(1 - \delta A_{0i})}{(1 - \delta A_{ji})(1 - \sigma B_{0i})} \right]. \end{aligned} \quad (20)$$

We have $\partial f_i / \partial n_j |_{\{a_{kl}\}_{k \neq l} = 0} = 0$, and for $a_{ii} < a^*$,

$$\nabla_{\mathbf{d}} \frac{\partial f_i^2}{\partial n_i \partial n_j} \Big|_{\{a_{kl}\}_{k \neq l} = 0} = \nabla_{\mathbf{d}} \mathbb{E}_Y \left[G_2^i \ln \frac{(1 - \sigma B_{ji})(1 - \delta A_{0i})}{(1 - \delta A_{ji})(1 - \sigma B_{0i})} - G_1^i \ln \frac{(1 - \xi B_{ji})(1 - A_{0i})}{(1 - A_{ji})(1 - \xi B_{0i})} \right] > 0,$$

where the positive vector $\mathbf{d} := \{d_{kl}\}_{k \neq l}$ denotes the direction for $\{a_{kl}\}_{k \neq l}$, and the inequality follows from Lemma 6 (see Appendix J). Given the continuity of $\partial f_i / \partial n_j$, for a_{kl} , $k \neq l$, small enough, we have that function f_i is supermodular when $a_{ii} < a^*$. Thus, the game among countries for which $a_{ii} < a^*$ is supermodular. From Theorem 4.2.1 in Topkis (1998), the set of equilibria forms a lattice and has therefore a smallest equilibrium \mathbf{n}^* . By definition, \mathbf{n}^* is the equilibrium with the highest number of countries that give up their drugs to country 0. Note then that for $i \neq j$

$$\frac{\partial f_i}{\partial n_j} = \mathbb{E}_Y \left[(N_i - n_i) G_1^i \ln \frac{(1 - \xi B_{ji})(1 - A_{0i})}{(1 - A_{ji})(1 - \xi B_{0i})} + n_i G_2^i \ln \frac{(1 - \sigma B_{ji})(1 - \delta A_{0i})}{(1 - \delta A_{ji})(1 - \sigma B_{0i})} \right] < 0, \quad (21)$$

where the inequality comes from Lemma 5 in Appendix J. Hence, country i 's objective function decreases in all other countries' allocation decisions n_j . It follows that \mathbf{n}^* is the unique Pareto optimal equilibrium. ■

G. Proof of Proposition 3

Proposition 3 *For small enough between-country transmission rates a_{kl} , $k \neq l$, the optimal allocation policy uses all existing drugs for country 0, i.e. $n_0 = K$ and $n_i = 0$, $i > 0$.*

Proof : Since $G_1^i|_{\{a_{kj}\}_{k \neq j}=0} = G_2^i|_{\{a_{kj}\}_{k \neq j}=0} = 1$. Denote the number of drugs country 0 receives to be

$$n_0 = K - \sum_k n_k .$$

For any fixed realization y of Y , define

$$f_0(n_0) = (N_0 - y - n_0)G_1^0(n_0) + n_0G_2^0(n_0) ,$$

where we abuse notation by defining

$$\begin{aligned} G_1^0(n) &= (1 - a_{00})^y (1 - A_{00})^{N_0 - y - n - 1} (1 - \xi B_{00})^n , \\ G_2^0(n) &= (1 - \delta a_{00})^y (1 - \delta A_{00})^{N_0 - y - n} (1 - \sigma B_{00})^{n-1} . \end{aligned}$$

It is sufficient to show that $f_0(n_0)$ is non-decreasing in $n_0 = 0, 1, \dots, K$, that is, without loss of generality, $f_0(n+1) - f_0(n) \geq 0$.

We have

$$f_0(n+1) - f_0(n) = (N_0 - y - n - 1)(G_1^0(n+1) - G_1^0(n)) + n(G_2^0(n+1) - G_2^0(n)) + (G_2^0(n+1) - G_1^0(n)) .$$

It follows that

$$\begin{aligned} G_1^0(n+1) - G_1^0(n) &= \left(1 - \frac{1 - A_{00}}{1 - \xi B_{00}}\right) G_1^0(n+1) > 0 , \\ G_2^0(n+1) - G_2^0(n) &= \left(1 - \frac{1 - \delta A_{00}}{1 - \sigma B_{00}}\right) G_2^0(n+1) > 0 , \quad \text{and} \\ G_2^0(n+1) - G_1^0(n) &= (1 - \delta a_{00})^y (1 - \delta A_{00})^{N_0 - y - n - 1} (1 - \sigma B_{00})^n \\ &\quad - (1 - a_{00})^y (1 - A_{00})^{N_0 - y - n - 1} (1 - \xi B_{00})^n > 0 , \end{aligned}$$

The result follows from the fact that $f_0(n+1) - f_0(n)$ is continuous in a_{kl} . ■

H. Proof of Proposition 4

Proposition 4 For $\{a_{kl}\}_{k \neq l}$ small enough, consider the decentralized equilibrium characterized by the set S of countries that do not give up their drugs (i.e. $i \in S \Leftrightarrow n_i = K_i$). For each $i \in S$, there is a population size threshold \hat{N}_i such that supplying country 0 with all existing drugs (i.e. $\sum_{j=1}^m n_j = K$) is a Pareto improvement over the decentralized equilibrium if for all $i \in S$,

$$N_i > \hat{N}_i \quad \text{and} \quad K_i < \rho_i \left(K - \sum_{j \in \bar{S}} K_j \right)$$

where \hat{N}_i is independent of N_j , $j > 0$, and

$$\rho_i = \frac{a^*(1 - \sigma)}{a_{ii}(2 - \sigma - \delta)} .$$

Proof : The objective function of country i for the decentralized case is equal to,

$$f_i^d = \mathbb{E}_Y [N_i G_1^d + K_i (G_2^d - G_1^d)]$$

where

$$G_1^d = \frac{(1 - a_{0i})^Y}{1 - A_{ii}} (1 - A_{0i})^{N_0 - Y - \sum_{j \in \bar{S}} K_j} (1 - \xi B_{0i})^{\sum_{j \in \bar{S}} K_j} \prod_{j \in S} (1 - A_{ji})^{N_j - K_j} (1 - \xi B_{ji})^{K_j} \prod_{j \in \bar{S}} (1 - A_{ji})^{N_j}$$

and

$$G_2^d = \frac{(1 - \delta a_{0i})^Y}{1 - \sigma B_{ii}} (1 - \delta A_{0i})^{N_0 - Y - \sum_{j \in \bar{S}} K_j} (1 - \sigma B_{0i})^{\sum_{j \in \bar{S}} K_j} \prod_{j \in S} (1 - \delta A_{ji})^{N_j - K_j} (1 - \sigma B_{ji})^{K_j} \prod_{j \in \bar{S}} (1 - \delta A_{ji})^{N_j} .$$

Similarly, the average number of survivors in country i when the central planner allocates all drugs to country 0 is equal to $f_i^c = \mathbb{E}_Y [N_i G_1^c]$ where,

$$G_1^c = \frac{(1 - a_{0i})^Y}{1 - A_{ii}} (1 - A_{0i})^{N_0 - Y - K} (1 - \xi B_{0i})^K \prod_{j=1}^m (1 - A_{ji})^{N_j} .$$

Consider the first order directional derivative $\nabla_{\mathbf{d}} (f_i^d - f_i^c) \big|_{\{a_{kj}\}_{k \neq j} = 0}$ along the direction vector $\mathbf{d} = \{d_{kj}\}_{k \neq j}$. Noting that for any given realization y of Y ,

$$\nabla_{\mathbf{d}} G_1^c \big|_{\{a_{kj}\}_{k \neq j} = 0} = - \frac{d_{0i}}{\|\mathbf{d}\|} [y + (N_0 - y - K)[1 - (1 - a_{00})^y] + \xi K [1 - (1 - \delta a_{00})^y] + (N_i - 1) y a_{ii}] ,$$

and

$$\begin{aligned}\nabla_{\mathbf{d}} G_1^d \Big|_{\{a_{kj}\}_{k \neq j} = 0} &= -\frac{d_{0i}}{\|\mathbf{d}\|} \left[y + (N_0 - \sum_{j \in \bar{S}} K_j) [1 - (1 - a_{00})^y] + \xi \sum_{j \in \bar{S}} K_j [1 - (1 - \delta a_{00})^y] \right. \\ &\quad \left. + (N_i - K_i + \sigma K_i - 1) a_{ii} y \right], \\ \nabla_{\mathbf{d}} G_2^d \Big|_{\{a_{kj}\}_{k \neq j} = 0} &= -\frac{\delta d_{0i}}{\|\mathbf{d}\|} \left[y + (N_0 - \sum_{j \in \bar{S}} K_j) [1 - (1 - a_{00})^y] + \xi \sum_{j \in \bar{S}} K_j [1 - (1 - \delta a_{00})^y] \right. \\ &\quad \left. + (N_i - K_i + \sigma K_i - \sigma) a_{ii} y \right],\end{aligned}$$

we have,

$$\begin{aligned}&\nabla_{\mathbf{d}} (f_i^d - f_i^c) \Big|_{\{a_{kj}\}_{k \neq j} = 0} \\ &= \frac{d_{0i}}{\|\mathbf{d}\|} \mathbb{E}_Y \left\{ -N_i \left[\frac{\hat{K}}{K_i} \left((1 - (1 - a_{00})^Y) - \xi (1 - (1 - \delta a_{00})^Y) \right) - a_{ii} (2 - \sigma - \delta) Y \right] + a_{ii} (\delta \sigma - 1) Y \right. \\ &\quad \left. + (1 - \delta) \left[Y + (N_0 - \sum_{j \in \bar{S}} K_j) (1 - (1 - a_{00})^Y) + \xi \sum_{j \in \bar{S}} K_j (1 - (1 - \delta a_{00})^Y) - K_i (1 - \sigma) a_{ii} Y \right] \right\} \\ &= \frac{d_{0i}}{\|\mathbf{d}\|} \left\{ -N_i \left[\frac{\hat{K}}{K_i} a^* (1 - \sigma) - a_{ii} (2 - \sigma - \delta) \right] \mathbb{E}_Y[Y] + a_{ii} (\delta \sigma - 1) \mathbb{E}_Y[Y] \right. \\ &\quad \left. + (1 - \delta) \left[\mathbb{E}_Y[Y] + N_0 (1 - \varphi_Y (1 - a_{00})) - (1 - \sigma) \mathbb{E}_Y[Y] \left(\sum_{j \in \bar{S}} K_j a^* + K_i a_{ii} \right) \right] \right\}\end{aligned}$$

where $\hat{K} = K - \sum_{j \in \bar{S}} K_j$.

It follows that $\nabla_{\mathbf{d}} (f_i^d - f_i^c) \Big|_{\{a_{kj}\}_{k \neq j} = 0} < 0$ if $N_i > \hat{N}_i$ and $K_i < \rho \hat{K}$, where

$$\hat{N}_i = \frac{a_{ii} (\delta \sigma - 1) + (1 - \delta) \left[1 + N_0 \frac{1 - \varphi_Y (1 - a_{00})}{\mathbb{E}_Y[Y]} - (1 - \sigma) \left(\sum_{j \in \bar{S}} K_j a^* + K_i a_{ii} \right) \right]}{\frac{K - \sum_{j \in \bar{S}} K_j}{K_i} a^* (1 - \sigma) - a_{ii} (2 - \sigma - \delta)} \quad (22)$$

Given $(f_i^d - f_i^c) \Big|_{\{a_{kj}\}_{k \neq j} = 0} = 0$, the results hold for $\{a_{kl}\}_{k \neq l}$ sufficiently small. \blacksquare

I. Proof of Theorem 3

Theorem 3 For between-country transmission probabilities a_{0k} ($k > 0$) small enough,

1. Country i never gives drugs to country $j \neq 0$.
2. Country i 's optimal decision given all other countries' actions is always "bang-bang": either give up everything to country 0 or give nothing.
3. The game is supermodular; a unique Pareto optimal Nash Equilibrium exists and has the

highest number of countries giving drugs to country 0.

Proof : The proof of Theorem 3 follows the proofs of Theorems 1 and 2 in Section 3. Similar to the expected value case, we first show that country i 's action can be reduced to n_i , quantity of drugs that should be kept, with $K_i - n_i$ the quantity of drugs to be given up to country 0.

Proposition 7 *When the between-country transmission probabilities a_{kl} , $k \neq l$, are small enough, country i will never give up drugs to country $j \neq 0$.*

Proof : For any fixed realization y of Y , denote function

$$f_i^{(y)}(\mathbf{n}) := (1 - a_{0i})^{y(N_i - \mathbf{n}_i)} (1 - \delta a_{0i})^{y\mathbf{n}_i} \exp \left\{ \sum_{j \neq i} \left[(N_j - \mathbf{n}_j) (1 - (1 - a_{0j})^y) ((1 - a_{ji})^{N_i - \mathbf{n}_i} + (1 - \delta a_{ji})^{\mathbf{n}_i} - 2) + \mathbf{n}_j (1 - (1 - \delta a_{0j})^y) ((1 - \xi a_{ji})^{N_i - \mathbf{n}_i} + (1 - \sigma a_{ji})^{\mathbf{n}_i} - 2) \right] \right\},$$

From (13), $f_i(\mathbf{n}) = \mathbb{E}_Y[f_i^{(Y)}(\mathbf{n})]$. The first order derivatives of $f_i^{(Y)}(\mathbf{n})$ with respect to n_j^i for $j \neq i$ are

$$\begin{aligned} \frac{\partial f_i^{(y)}}{\partial n_j^i} &= f(\mathbf{n}) \left[- (1 - (1 - a_{0j})^y) ((1 - a_{ji})^{N_i - \mathbf{n}_i} + (1 - \delta a_{ji})^{\mathbf{n}_i} - 2) \right. \\ &\quad \left. + (1 - (1 - \delta a_{0j})^y) ((1 - \xi a_{ji})^{N_i - \mathbf{n}_i} + (1 - \sigma a_{ji})^{\mathbf{n}_i} - 2) \right], \quad \text{for } j > 0, \\ \frac{\partial f_i^{(y)}}{\partial n_0^i} &= f(\mathbf{n}) \left[- (1 - (1 - a_{00})^y) ((1 - a_{0i})^{N_i - \mathbf{n}_i} + (1 - \delta a_{0i})^{\mathbf{n}_i} - 2) \right. \\ &\quad \left. + (1 - (1 - \delta a_{00})^y) ((1 - \xi a_{0i})^{N_i - \mathbf{n}_i} + (1 - \sigma a_{0i})^{\mathbf{n}_i} - 2) \right]. \end{aligned}$$

We have

$$\frac{\partial f_i^{(y)}}{\partial n_j^i} \Big|_{\{a_{kl}\}_{k \neq l} = 0} = \frac{\partial f_i^{(y)}}{\partial n_0^i} \Big|_{\{a_{kl}\}_{k \neq l} = 0} = 0.$$

The directional derivatives with respect to the positive direction \mathbf{d} at $\{a_{kl}\}_{k \neq l} = 0$ are equal to,

$$\begin{aligned} \nabla_{\mathbf{d}} \frac{\partial f_i^{(y)}}{\partial n_j^i} \Big|_{\{a_{kl}\}_{k \neq l} = 0} &= 0, \\ \nabla_{\mathbf{d}} \frac{\partial f_i^{(y)}}{\partial n_0^i} \Big|_{\{a_{kl}\}_{k \neq l} = 0} &= \frac{d_{0i}}{\|\mathbf{d}\|} \left((1 - (1 - a_{00})^y) - \xi (1 - (1 - \delta a_{00})^y) \right) (N_i - (1 - \delta)\mathbf{n}_i) > 0 \end{aligned}$$

and since $f_i^{(y)}$ is continuously twice differentiable, $\partial f_i^{(y)} / \partial n_j^i < \partial f_i^{(y)} / \partial n_0^i$ for $\{a_{kl}\}_{k \neq l}$ suffi-

ciently small. We also have that

$$\frac{\partial f_i}{\partial n_j^i} = \mathbb{E}_Y \left[\frac{\partial f_i^{(Y)}}{\partial n_j^i} \right], \quad \nabla_{\mathbf{d}} \frac{\partial f_i}{\partial n_j^i} = \mathbb{E}_Y \left[\nabla_{\mathbf{d}} \frac{\partial f_i^{(Y)}}{\partial n_j^i} \right], \quad \frac{\partial f_i}{\partial n_0^i} = \mathbb{E}_Y \left[\frac{\partial f_i^{(Y)}}{\partial n_0^i} \right], \quad \nabla_{\mathbf{d}} \frac{\partial f_i}{\partial n_0^i} = \mathbb{E}_Y \left[\nabla_{\mathbf{d}} \frac{\partial f_i^{(Y)}}{\partial n_0^i} \right].$$

The result follows then from the same argument as in the proof of Proposition 2. \blacksquare

Country i 's function becomes then $\mathbb{E}_Y [f_i^{(Y)}]$, where, for any realization y of Y ,

$$\begin{aligned} f_i^{(y)}(\mathbf{n}) &= (1 - a_{0i})^{y(N_i - n_i)} (1 - \delta a_{0i})^{y n_i} \exp \left\{ (K - s(\mathbf{n})) (1 - (1 - \delta a_{00})^y) ((1 - \xi a_{0i})^{N_i - n_i} + (1 - \sigma a_{0i})^{n_i} - 2) \right. \\ &\quad + (N_0 - y - K + s(\mathbf{n})) (1 - (1 - a_{00})^y) ((1 - \delta a_{0i})^{n_i} + (1 - a_{0i})^{N_i - n_i} - 2) \\ &\quad + \sum_{j \neq 0, i} [(N_j - n_j) (1 - (1 - a_{0j})^y) ((1 - a_{ji})^{N_i - n_i} + (1 - \delta a_{ji})^{n_i} - 2) \\ &\quad \left. + n_j (1 - (1 - \delta a_{0j})^y) ((1 - \xi a_{ji})^{N_i - n_i} + (1 - \sigma a_{ji})^{n_i} - 2) \right\}, \end{aligned}$$

where $s(\mathbf{n}) := \sum_{j=1}^m n_j$. The optimal decision has also a ‘‘bang-bang’’ structure:

Proposition 8 *For between-country transmission probabilities a_{0k} ($k > 0$) small enough, f_i is convex in n_i and country i 's optimal decision given all other countries' actions is always ‘‘bang-bang’’: either give up everything to country 0 or keep everything to itself.*

Proof : Following the proof of Theorem 1, we consider the derivatives of $f_i^{(y)}$ as follows,

$$\begin{aligned} \frac{\partial f_i^{(y)}}{\partial n_i} &= f_i^{(y)} \left\{ y (\ln(1 - \delta a_{0i}) - \ln(1 - a_{0i})) + (1 - (1 - a_{00})^y) ((1 - a_{0i})^{N_i - n_i} + (1 - \delta a_{0i})^{n_i} - 2) \right. \\ &\quad + (N_0 - y - K + s(\mathbf{n})) (1 - (1 - a_{00})^y) ((1 - \delta a_{0i})^{n_i} \ln(1 - \delta a_{0i}) - (1 - a_{0i})^{N_i - n_i} \ln(1 - a_{0i})) \\ &\quad + (K - s(\mathbf{n})) (1 - (1 - \delta a_{00})^y) ((1 - \sigma a_{0i})^{n_i} \ln(1 - \sigma a_{0i}) - (1 - \xi a_{0i})^{N_i - n_i} \ln(1 - \xi a_{0i})) \\ &\quad - (1 - (1 - \delta a_{00})^y) ((1 - \xi a_{0i})^{N_i - n_i} + (1 - \sigma a_{0i})^{n_i} - 2) \\ &\quad + \sum_{j \neq 0, i} [(N_j - n_j) (1 - (1 - a_{0j})^y) ((1 - \delta a_{ji})^{n_i} \ln(1 - \delta a_{ji}) - (1 - a_{ji})^{N_i - n_i} \ln(1 - a_{ji})) \\ &\quad \left. + n_j (1 - (1 - \delta a_{0j})^y) ((1 - \sigma a_{ji})^{n_i} \ln(1 - \sigma a_{ji}) - (1 - \xi a_{ji})^{N_i - n_i} \ln(1 - \xi a_{ji})) \right\} \\ \frac{\partial^2 f_i^{(y)}}{\partial n_i^2} &= \frac{(\partial f_i^{(y)} / \partial n_i)^2}{f_i^{(y)}} + f_i^{(y)} \left\{ 2(1 - (1 - a_{00})^y) ((1 - \delta a_{0i})^{n_i} \ln(1 - \delta a_{0i}) - (1 - a_{0i})^{N_i - n_i} \ln(1 - a_{0i})) \right. \\ &\quad - 2(1 - (1 - \delta a_{00})^y) ((1 - \sigma a_{0i})^{n_i} \ln(1 - \sigma a_{0i}) - (1 - \xi a_{0i})^{N_i - n_i} \ln(1 - \xi a_{0i})) \\ &\quad + (N_0 - y - K + s(\mathbf{n})) (1 - (1 - a_{00})^y) ((1 - \delta a_{0i})^{n_i} \ln^2(1 - \delta a_{0i}) + (1 - a_{0i})^{N_i - n_i} \ln^2(1 - a_{0i})) \\ &\quad + (K - s(\mathbf{n})) (1 - (1 - \delta a_{00})^y) ((1 - \sigma a_{0i})^{n_i} \ln^2(1 - \sigma a_{0i}) + (1 - \xi a_{0i})^{N_i - n_i} \ln^2(1 - \xi a_{0i})) \\ &\quad + \sum_{j \neq 0, i} [(N_j - n_j) (1 - (1 - a_{0j})^y) ((1 - \delta a_{ji})^{n_i} \ln^2(1 - \delta a_{ji}) + (1 - a_{ji})^{N_i - n_i} \ln^2(1 - a_{ji})) \\ &\quad \left. + n_j (1 - (1 - \delta a_{0j})^y) ((1 - \sigma a_{ji})^{n_i} \ln^2(1 - \sigma a_{ji}) + (1 - \xi a_{ji})^{N_i - n_i} \ln^2(1 - \xi a_{ji})) \right\} \end{aligned}$$

f_i is then convex in n_i if $G(a_{0i})$ is positive, where

$$G(a_{0i}) = (1 - (1 - a_{00})^y) \left((1 - \delta a_{0i})^{n_i} \ln(1 - \delta a_{0i}) - (1 - a_{0i})^{N_i - n_i} \ln(1 - a_{0i}) \right) \\ - (1 - (1 - \delta a_{00})^y) \left((1 - \sigma a_{0i})^{n_i} \ln(1 - \sigma a_{0i}) - (1 - \xi a_{0i})^{N_i - n_i} \ln(1 - \xi a_{0i}) \right).$$

Note that $G(0) = 0$ with $G'(0) = (1 - \delta) \left[(1 - (1 - a_{00})^y) - \xi (1 - (1 - \delta a_{00})^y) \right] > 0$.

We also have that

$$\frac{\partial^2 \mathbb{E}_Y [f_i^{(Y)}]}{\partial n_i^2} = \mathbb{E}_Y \left[\frac{\partial^2 f_i^{(Y)}}{\partial n_i^2} \right].$$

Therefore the result holds for small enough a_{0i} . ■

Propositions 7 and 8 show the two first parts of Theorem 3. For the last one, consider the second order derivative of

$$\frac{\partial^2 f_i^{(y)}}{\partial n_i \partial n_j} = f_i^{(y)} \left[\mathcal{G} + \frac{\partial f_i^{(y)} / \partial n_i}{f_i^{(y)}} \frac{\partial f_i^{(y)} / \partial n_j}{f_i^{(y)}} \right],$$

where

$$\mathcal{G} = (1 - (1 - a_{00})^y) \left((1 - \delta a_{0i})^{n_i} \ln(1 - \delta a_{0i}) - (1 - a_{0i})^{N_i - n_i} \ln(1 - a_{0i}) \right) \\ - (1 - (1 - \delta a_{00})^y) \left((1 - \sigma a_{0i})^{n_i} \ln(1 - \sigma a_{0i}) - (1 - \xi a_{0i})^{N_i - n_i} \ln(1 - \xi a_{0i}) \right) \\ - (1 - (1 - a_{0j})^y) \left((1 - \delta a_{ji})^{n_i} \ln(1 - \delta a_{ji}) - (1 - a_{ji})^{N_i - n_i} \ln(1 - a_{ji}) \right) \\ + (1 - (1 - \delta a_{0j})^y) \left((1 - \sigma a_{ji})^{n_i} \ln(1 - \sigma a_{ji}) - (1 - \xi a_{ji})^{N_i - n_i} \ln(1 - \xi a_{ji}) \right), \quad \text{and} \\ \frac{\partial f_i^{(y)}}{\partial n_j} = f_i^{(y)} \left\{ (1 - (1 - a_{00})^y) \left((1 - a_{0i})^{N_i - n_i} + (1 - \delta a_{0i})^{n_i} - 2 \right) \right. \\ - (1 - (1 - \delta a_{00})^y) \left((1 - \xi a_{0i})^{N_i - n_i} + (1 - \sigma a_{0i})^{n_i} - 2 \right) \\ - (1 - (1 - a_{0j})^y) \left((1 - a_{ji})^{N_i - n_i} + (1 - \delta a_{ji})^{n_i} - 2 \right) \\ \left. + (1 - (1 - \delta a_{0j})^y) \left((1 - \xi a_{ji})^{N_i - n_i} + (1 - \sigma a_{ji})^{n_i} - 2 \right) \right\}.$$

Next we verify that

$$\mathcal{J} := \mathcal{G} + \frac{\partial f_i^{(y)} / \partial n_i}{f_i^{(y)}} \frac{\partial f_i^{(y)} / \partial n_j}{f_i^{(y)}} > 0$$

for small enough $\{a_{kl}\}_{k \neq l}$.

First, it is clear that

$$\mathcal{G} \Big|_{\{a_{0i}, a_{ji}\}=0} = \frac{\partial f_i^{(y)} / \partial n_i}{f_i^{(y)}} \Big|_{\{a_{0i}, a_{ji}\}=0} = \frac{\partial f_i^{(y)} / \partial n_j}{f_i^{(y)}} \Big|_{\{a_{0i}, a_{ji}\}=0} = 0.$$

Denote

$$\mathcal{O} := (1 - (1 - a_{00})^y) - \xi(1 - (1 - \delta a_{00})^y) > 0 .$$

We then have

$$\nabla_{\mathbf{d}} \mathcal{J} = \nabla_{\mathbf{d}} \mathcal{G}_1 \Big|_{\{a_{0i}, a_{ji}\}=0} = \frac{d_{0i}}{\|\mathbf{d}\|} (1 - \delta) \mathcal{O} > 0 .$$

Therefore we have for small enough $\{a_{kl}\}_{k \neq l}$,

$$\frac{\partial^2 \mathbb{E}_Y [f_i^{(Y)}]}{\partial n_i \partial n_j} = \mathbb{E}_Y \left[\frac{\partial^2 f_i^{(Y)}}{\partial n_i \partial n_j} \right] > 0 .$$

The game is, therefore, supermodular for sufficiently small $\{a_{kl}\}_{k \neq l}$.

We also have the following directional derivative expression

$$\nabla_{\mathbf{d}} \frac{\partial f_i^{(y)} / \partial n_j}{f_i^{(y)}} \Big|_{\{a_{0i}, a_{ji}\}=0} = \begin{cases} (d_{0i} / \|\mathbf{d}\|) (1 - \delta) \mathcal{O} , & n_i > 0 \\ (d_{0i} / \|\mathbf{d}\|) \mathcal{O} , & n_i = 0 \end{cases} .$$

Thus the partial derivative $\partial f_i^{(y)} / \partial n_i > 0$, which implies the remainder of the theorem. That is, for sufficiently small $\{a_{kl}\}_{k \neq l}$, country's i objective function decreases in all other country decisions and the game has a unique Pareto optimal Nash equilibrium which has the highest number of countries giving drugs to country 0. \blacksquare

J. Supporting Lemmas and Propositions

Lemma 4 *If random variable X follows a binomial distribution with parameters N and α , then for $0 < \beta < 1$ we have,*

$$\mathbb{E}[(1 - \beta)^X] = (1 - \alpha\beta)^N \tag{23}$$

$$\mathbb{E}[X(1 - \beta)^X] = N\alpha(1 - \beta)(1 - \alpha\beta)^{N-1} \tag{24}$$

$$N\mathbb{E}[(1 - \beta)^X] - \mathbb{E}[X(1 - \beta)^X] = N(1 - \alpha)(1 - \alpha\beta)^{N-1} \tag{25}$$

Proof : When $N = 0$, the distribution of X is degenerate such that $X = 0$ with probability 1 and the result holds. Assume $N > 0$. Eq. (23) follows easily from the characteristic function of a binomial distribution. For Eq. (24), we have

$$\mathbb{E}[X(1 - \beta)^X] = (1 - \beta) \left(\mathbb{E}[(1 - \beta)^X] \right)'_{\beta} = (1 - \beta) \left((1 - \alpha\beta)^N \right)'_{\beta} = \alpha(1 - \beta)N(1 - \alpha\beta)^{N-1} .$$

where the second equality comes from (23). Eq. (25) follows directly from (23) and (24). \blacksquare

The following straightforward calculus we show the following two lemmas,

Lemma 5 *if the between-country infection rates a_{kl} , $k \neq l$, are small enough, for all $i \neq j$ and $j \neq 0$, then*

$$\frac{1 - \xi B_{ji}}{1 - A_{ji}} < \frac{1 - \xi B_{0i}}{1 - A_{0i}} \quad \text{and} \quad \frac{1 - \sigma B_{ji}}{1 - \delta A_{ji}} < \frac{1 - \sigma B_{0i}}{1 - \delta A_{0i}}. \quad (26)$$

Proof : We show the first inequality. The second one follows the same argument. Denote function

$$h(\{a_{kl}\}_{k \neq l}) := (1 - \xi B_{0i})(1 - A_{ji}) - (1 - \xi B_{ji})(1 - A_{0i}).$$

We have $h(\{0\}) = 0$. Fix any realization y of Y , take the directional derivative of h with respect to $\{a_{kl}\}_{k \neq l}$ along the directional positive vector \mathbf{d} , we have

$$\nabla_{\mathbf{d}} h|_{\{a_{kl}\}_{k \neq l} = 0} = \frac{d_{0i}}{\|\mathbf{d}\|} \left((1 - (1 - a_{00})^y) - \xi(1 - (1 - \delta a_{00})^y) \right) > 0.$$

The continuity of the function h implies the result. \blacksquare

Lemma 6 *Define*

$$\mathcal{G}_j^i(\{a_{kl}\}_{k \neq l}) := \mathbb{E}_Y \left[G_2^i \ln \frac{(1 - \sigma B_{ji})(1 - \delta A_{0i})}{(1 - \delta A_{ji})(1 - \sigma B_{0i})} - G_1^i \ln \frac{(1 - \xi B_{ji})(1 - A_{0i})}{(1 - A_{ji})(1 - \xi B_{0i})} \right],$$

where G_1^i and G_2^i are defined in (7) and (8).

Consider the directional derivatives $\nabla_{\mathbf{d}} \mathcal{G}_j^i(\{a_{kl}\} = 0)$, along the direction vector $\mathbf{d} = \{d_{kl}\}_{k \neq l}$.

We have,

$$\nabla_{\mathbf{d}} \mathcal{G}_j^i(\{a_{kl}\}_{k \neq l} = 0) = \frac{d_{0i}}{\|\mathbf{d}\|} (1 - \delta) \mathbb{E}_Y \left[(1 - (1 - a_{00})^Y) - \xi(1 - (1 - \delta a_{00})^Y) \right] > 0, \quad i \neq j, \quad (27)$$

$$\nabla_{\mathbf{d}} \mathcal{G}_i^i(\{a_{kl}\}_{k \neq l} = 0) = \frac{d_{0i}}{\|\mathbf{d}\|} (1 - \delta) \mathbb{E}_Y \left[(1 - (1 - a_{00})^Y) - \xi(1 - (1 - \delta a_{00})^Y) - Y a_{ii} (1 - \sigma) \right] > 0, \quad (28)$$

when $a_{ii} < a^*$.

Proof : The result follows from direct derivations. \blacksquare

		K_i					
		$0.5N_0$		$0.3N_0$		$0.1N_0$	
		Two Periods	Entire Horizon	Two Periods	Entire Horizon	Two Periods	Entire Horizon
a_{ii}	a_{00}	Keep	Give up	Keep	Give up	Keep	Keep
	$0.9a_{00}$	Keep	Give up	Keep	Give up	Keep	Keep
	$0.5a_{00}$	Keep	Give up*	Keep	Keep	Keep	Keep
	$0.4a_{00}$	Keep	Give up	Keep	Give up	Keep	Keep
	$0.3a_{00}$	Keep	Give up	Keep	Give up	Keep	Give up
	$0.2a_{00}$	1 Give up; 2 Keep	Give up	1 Give up; 2 Keep	Give up	1 Give up; 2 Keep	Give up

Table 6: Equilibria: two period versus entire horizon – Chain network structure. (*: in this case there are three equilibria. The dominant one has Country 1 give up $0.7K_1$ and Country 2 give up $0.8K_2$ to Country 0.)

		K_i					
		$0.5N_0$		$0.3N_0$		$0.1N_0$	
		Two Periods	Entire Horizon	Two Periods	Entire Horizon	Two Periods	Entire Horizon
a_{ii}	a_{00}	1, 604, 132	564, 948	1, 888, 778	1, 306, 988	2, 091, 401	2, 091, 401
	$0.9a_{00}$	1, 434, 154	462, 542	1, 752, 548	1, 178, 021	1, 979, 782	1, 979, 782
	$0.5a_{00}$	783, 691	1, 939	878, 480	878, 480	1, 059, 796	1, 059, 796
	$0.4a_{00}$	728, 587	4.8	764, 985	96, 126	826, 661	826, 661
	$0.3a_{00}$	688, 416	3.5	703, 669	68, 447	721, 228	556, 528
	$0.2a_{00}$	179, 311	3.3	415, 426	61, 203	601, 760	516, 789

Table 7: Final sizes according to the dominant equilibria from the two period model versus the entire horizon model.

K. Entire horizon simulation study of a chain network structure

In this section we extend the computation study in Section 5.2 by considering a “chain” network structure. Specifically, the only change from the setup in Section 5.2 is that countries 0 and 2 are not connected to each other (i.e. $a_{02} = a_{20} = 0$). Parallel to Tables 3 and 4, we obtain Tables 6 and 7, respectively.

Table 6 indicates that in most cases countries 1 and 2 keep their drugs in the two period model. This is not surprising since drugs allocated to country 0 do not affect the number of infectives in country 2 over the first two periods. Therefore country 2 does not have any incentive to give up its drugs, which further implies that country 1 is more motivated to keep its drugs. On the other hand, countries are much more willing to give-up drugs to country 0 in the entire horizon case. This suggests that myopic decision makers would act more selfishly than they would if they considered the final size of the pandemic, a message similar to the one obtained

in Section 5.2, Table 3. Note that compared to the symmetric case, when the network structure is a chain, the two period model has less predictive power as an approximation to the entire horizon model. As shown in Table 7, there are much fewer cases where the same final sizes are equal for the two models. On the other hand, when a_{ii} is small compared to a_{00} and K_i is relatively large, the difference between the final sizes according to different allocation policies can be quite dramatic. This observation is consistent with our study of the symmetric network structure in Section 5.2.