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News management and the value of firms

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The literature on asymmetric information has been concerned mainly with the problem of the informed party lying to the uninformed parties. However, in many cases, the informed party will stop short of lying but will seek to gain from private information by managing the disclosure of news. This article examines the pricing of a firm when there is such manipulation of news. I model this situation as a variant of the "persuasion game" of Milgrom and Roberts (1986) in which I can parametrize the notion of the degree of credence placed by the market on the disclosures of the informed party. An empirical hypothesis thrown up by the theory is that for otherwise identical firms, a low price/earnings ratio will be associated with a greater degree of positive skewness of the disclosure strategy.

1. Introduction

■ The literature on asymmetric information has been concerned, in the main, with the problem of the informed party misrepresenting the facts to the uninformed parties—that is, of the informed party *lying* to the uninformed. However, in many cases, such a blatant form of deception is not a viable option for the informed party, especially when such deception may later be uncovered, with severe consequences. More commonly, the informed party will stop short of lying but will attempt to gain from private information by managing the disclosure of facts. That is, they will tell the truth, but not the whole truth.

An influential strand in the literature, starting with Grossman (1981) and Milgrom (1981), has examined a special case of this problem in which the receivers of information are able to discount fully the self-interested reports of the informed party so as to reveal completely that party's type. This is the case in which the so-called unravelling argument is allowed to operate, and it has been examined by subsequent articles such as Milgrom and Roberts (1986), Farrell (1986), Okuno-Fujiwara, Postlewaite, and Suzumura (1990), and Lipman and Seppi (1992). In its simplest form, the unravelling argument rests on the assumption that the informed party has perfect information concerning the payoff-relevant state, so that the pair of strategies in which the informed party announces the most favorable report consistent with the true state and the recipient of the report "assumes the worst," given an announcement, constitutes an equilibrium. Milgrom and Roberts (1986) refer to this as the "skeptical equilibrium" and show how full revelation takes place in such settings.

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However, the unravelling argument is extremely sensitive to any uncertainty concerning what the informed party actually knows. Take a simple example. Suppose you are buying a secondhand car from a dealer and you ask the dealer whether the car has been in an accident. Suppose further that the dealer replies that he does not know. In most cases, this reply leaves open two possibilities. Either the dealer knows that the car has been in an accident but is holding back this information, or the dealer is genuinely uninformed as to whether the car has been in an accident. If both possibilities have positive probability, the unravelling argument has no force. The professed ignorance of the dealer cannot be taken to be a sure indication of inferior quality. There is a positive probability that the dealer is actually telling the whole truth. In this sense, the unravelling argument unravels when there is uncertainty concerning the information of the informed party. The market's reaction to the published accounts of firms would seem to be a prime example of a case in which there is precisely this sort of uncertainty.

In principle, it is possible to incorporate uncertainty concerning the quality of the informed party's information by expanding the space of uncertainty from the payoff-relevant state space S to the product space $S \times \wp$, where $\wp$ is the set of information partitions over S. In this way, we could incorporate uncertainty over the information partitions of the informed party. The challenge is to keep the analysis tractable. In this article I shall take up this challenge, and I exhibit a class of pricing rules that can be defined in terms of a probability distribution over the payoff-relevant state space S alone. The uncertainty over $\wp$ enters as a parameter in the distribution over S.

The setting for this article is an exchange economy in which traders face uncertainty concerning the quality of the informed party's information. Trading takes place in a "persuasion game" in the manner of Milgrom and Roberts (1986). The uncertainty over the quality of the informed party's information is captured by a set of probabilities with which the informed party receives a set of specified signals on the value of a firm. In effect, these probabilities serve as a measure of the severity of adverse selection in the market for the firm. What is of interest to us is that in equilibrium, any change in the severity of adverse selection is met with a commensurate shift in the attitude of the traders in interpreting the disclosures of the informed party. Thus, once we have solved for the equilibrium value of the firm as a function of these probabilities, we may regard these probabilities as a measure of the degree of skepticism in greeting the disclosures of the informed party.

There are two polar cases of this equilibrium. At one end is the case of extreme skepticism, as discussed by Milgrom and Roberts (1986) and Farrell (1986), in which the market "assumes the worst" and places all weight on the least favorable state for the informed party. The limiting case in the other direction is the case of naive updating by Bayes' rule on the *ex ante* probabilities. This is the case in which the disclosures are taken at face value. In general, I obtain a partial ordering of equilibrium outcomes with the above cases as the two extrema. The ordering has the interpretation "The market places more credence in the disclosures in equilibrium a than in equilibrium b."

One potential application of the results in this article is to the explanation of divergent price/earnings ratios for firms that are otherwise similar. It is well known that there is scope for "managing" the earnings figures for a firm within any given set of accounting conventions. More recently in the United Kingdom, a series of well-publicized corporate collapses has stimulated debate on the extent to which the balance sheet gives an accurate indication of a firm's underlying health. This debate has galvanized the accounting regulators into introducing ever more stringent guidelines (see, for example, *The Economist* (1991)). The controversial revelations in Smith (1992) concerning the accounting practices of several prominent firms in the United Kingdom has fueled this debate (*The Economist,* 1992). The results in this article suggest that for a set of firms that are otherwise identical, low price/earnings ratios will be associated with a greater degree of (positive) skewness

of earnings reports. This opens up the prospect of empirical investigations into the significance of news management in determining asset prices.

The next section describes the persuasion game and its solution. Section 3 deals with the comparative statics of this solution, and it draws out the observable features associated with shifts in the key parameter—the severity of adverse selection. Section 4 discusses the empirical hypothesis concerning the relationship between the earnings reports of firms and their price/earnings ratios.

2. The persuasion game

■ The setting for the game is a pure exchange economy with a finite number of states. There are N states, and the ith state is denoted by s_i. The state space is denoted by S. There is a single consumption good in this economy, and two types of assets. The *firm* is an asset $x = (x_1, x_2, \ldots, x_N)$ which pays x_i units of the consumption good in state i. I shall assume that $x_1 < x_2 < \ldots < x_N$. Thus, states with higher indices represent "good" outcomes for the owners of the firm, and states with lower indices represent "bad" outcomes. There is also a stock of the *risk-free asset*. Each unit of the risk-free asset pays one unit of the consumption good in every state.

There are three players in the game: the manager of the firm and two shareholders of the firm, called shareholder 1 and shareholder 2. The manager is risk averse and is endowed with a fraction $\epsilon > 0$ of the firm. The two shareholders are risk neutral and hold identical portfolios consisting of exactly half of the total stock of the risk-free asset, and proportion $(1 - \epsilon)/2$ of the firm.

There are $N - 1$ signals concerning the true state, indexed by the set $\{2, 3, \ldots, N\}$. I denote by σ_i the signal with index i. Each signal is a function from the state space to the set $\{0, 1\}$, where

$$\sigma_i(s) = \begin{cases} 0 & \text{if} \quad s \in \{s_1, s_2, \ldots, s_{i-1}\} \\ 1 & \text{if} \quad s \in \{s_i, s_{i+1}, \ldots s_N\}. \end{cases} \tag{1}$$

Thus, each signal has a "good" realization for the owners of the firm (namely, one), and a "bad" realization (namely, zero). The manager has superior information to the shareholders in that the manager observes a set of these signals before trade takes place, while the shareholders receive no signals. I shall denote by $\sum$ the set of signals observed by the manager. The game is played in stages, as follows.

□ **Stage 1.** Nature performs a sequence of N independent experiments in which it chooses a state in S and the set $\sum$ of signals received by the manager. Thus, in the first experiment, nature chooses the true state $s^* \in S$ according to probabilities $p = (p_1, p_2, \ldots, p_N)$, where $p \gg 0$. In experiments 2 to N, nature chooses the composition of the set $\sum$. In experiment i, nature includes σ_i in $\sum$ with probability θ_i, and excludes it with probability $1 - \theta_i$. Assume that $0 < \theta_i < 1$, for all i.

It is important to stress that the independence of nature's experiments pertains to the composition of $\sum$. The actual realization of any signal is determined by the true state, and hence will not be independent across signals.

□ **Stage 2.** The manager observes the realizations of the signals in $\sum$ implied by the true state s^*. That is, the manager observes the set

$$\{\sigma_i(s^*) \mid \sigma_i \in \textstyle\sum\}. \tag{2}$$

Given this information, the manager decides how much of this information is to be revealed to the shareholders. The manager chooses a subset R of $\sum$ and announces the set

$$\{\sigma_i(s^*) \mid \sigma_i \in R\}. \tag{3}$$

The manager's information and announcement can be represented by subsets of S. That is, the manager learns that the true state is in the set

$$I = \bigcap_{\sigma_i \in \Sigma} \{s \in S \mid \sigma_i(s) = \sigma_i(s^*)\} \tag{4}$$

and announces that the true state is in the set

$$A = \bigcap_{\sigma_i \in R} \{s \in S \mid \sigma_i(s) = \sigma_i(s^*)\}. \tag{5}$$

In the rest of the article I shall follow the convention of denoting the manager's information by I and the manager's announcement by A. For later use, I shall say that a set of states is an *interval* if it is of the form

$$\{s_i \mid j \leq i \leq k\} \tag{6}$$

for integers j and k. Note that the events I and A given in (4) and (5) are intervals, since they are the intersections of intervals.

□ **Stage 3.** Both shareholders observe the announcement of the set A by the manager. Shareholder 1 then announces a positive number V. This number has the interpretation of shareholder 1's assessment of the value of the firm in units of the risk-free asset. Following the announcement of V, the manager surrenders his share of the firm to shareholder 1 in exchange for ϵV units of the risk-free asset. Shareholder 2 is free to trade any fraction α of the firm with shareholder 1 in exchange for αV units of the risk-free asset. After all transactions have taken place, the true state is revealed, and consumption takes place. This completes the description of the game.

Before proceeding to the solution of the game, I shall comment on some of the salient features of the model. In constraining the manager to tell the truth concerning S, I am drawing on an implicit understanding that the manager's disclosures are verifiable at a later date by a third party, such as an auditor, who is able to impose a very large penalty on the manager if the earlier disclosure is exposed to be untrue. Provided that these penalties are large enough, no rational manager will violate the truth-telling condition in equilibrium. A similar rationale underlies the truth-telling constraint in the literature on "verfiable reports," which includes Milgrom (1981), Milgrom and Roberts (1986), and Okuno-Fujiwara, Postlewaite, and Suzumura (1990).

The truth-telling constraint seems particularly appropriate for disclosures in the form of financial statements. Although managers may dress up the results in ways that place a firm's prospects in the best light possible, there are well-established accounting principles that impose broad limits on what is possible. These "generally accepted accounting principles" may differ from country to country, but the practitioners are aware of them, and the regulators scrutinize firms' practices for possible violations. Above all, the ultimate sanction is the legal one against fraud. Verrecchia (1983) and Dye (1985a, 1985b) are articles in the accounting literature that employ the truth-telling constraint for precisely this reason.

I also comment on the *nature* of the message announced by the manager. In my model, the manager announces that the firm's true value lies in some interval of the payoff-relevant state space S. Taken literally, this may seem to be at variance with the way information is usually conveyed in financial accounts. However, when we consider the informational *content* of the report, rather than its superficial form, the assumption is seen to be without loss of generality. In any disclosure game between an informed and an uninformed party, the equilibrium reporting strategy will generate a partition of the relevant sample space, and the receiver of the information will update by Bayes' rule on this

partition. Hence, in terms of the informational content of the report, the description of the announcement in terms of the message space is equivalent to its description in terms of the relevant subsets of the sample space. Dye (1985b) argues for this general approach in the analysis of accounting disclosures. The only substantial assumption involved in my model is that announcements are of intervals of the set S. This is a restriction, but the corresponding gain in terms of the simplicity of the model would seem to justify its use. Future work may examine relaxing this assumption.

Let us now turn to the solution of the game. In solving this game, I appeal to a general pricing rule for the firm that can be deduced from the optimal strategies of the two shareholders. For any given announcement of V by shareholder 1, shareholder 2 compares V with the expected value of x given the manager's reporting strategy and nature's experiments. Since shareholder 2 is risk neutral, if this expected value is greater than V, she prefers the firm to the risk-free asset and buys shareholder 1's holding of the firm. If V is greater than this expected value, she prefers the risk-free asset and sells her share of the firm to shareholder 1. Since the two shareholders have identical preferences, the unique best reply for shareholder 1 is to set V equal to the expected value of x given the manager's reporting strategy and nature's experiments.

More formally, consider the product space $S \times \mathcal{A}$, where $\mathcal{A}$ is the set of all nonempty intervals of S representing all possible announcements of the manager. We can define a probability distribution over $S \times \mathcal{A}$ generated by nature's experiments and the manager's reporting strategy. Denote this distribution by μ. Thus, $\mu(s, A)$ is the probability that the true state is s and the manager announces the event A. Since the only information available to the shareholders is the manager's announcement, the shareholders calculate the expected value of x from the conditional distribution $\mu(\cdot \mid A)$. Thus, in equilibrium, if the manager announces the event A, we have $V = \sum_k x_k \mu(s_k \mid A)$. This is the pricing rule underlying the solution of the game. The bulk of the work involved in solving the game consists in deriving an explicit expression for the conditional distribution $\mu(\cdot \mid A)$.

However, one immediate conclusion we may draw is that the strategy of telling the "whole truth" is never an equilibrium strategy for the manager. The argument is as follows. Suppose, for contradiction, that the manager's equilibrium strategy is to tell the whole truth—that is, to set $A = I$ in (4) and (5). By the equilibrium pricing rule, V is set equal to the expectation of x according to $\mu(\cdot \mid A)$. However, if the manager reveals all his information, $\mu(\cdot \mid A)$ is identical to the conditional distribution on A with respect to the *ex ante* probabilities $(p_1, p_2, \ldots, p_N)$. But then, for any interval $I = \{s_i, s_{i+1}, \ldots, s_k\}$ in the range of the reporting strategy where $k < N$, the manager can do strictly better by announcing the interval $A = \{s_i, s_{i+1}, \ldots, s_k, \ldots, s_N\}$. To see this, partition A into $\{I, A \mid I\}$ and consider the conditional expectation of x on I and on $A \mid I$, respectively. Since $x_1 < x_2 < \ldots < x_N$, the latter is strictly greater. Since the conditional expectation of x on A is a convex combination of that on I and $A \mid I$ with nonzero weights, the manager can do strictly better by announcing A than by announcing I. But this contradicts the initial supposition that telling the whole truth is an equilibrium strategy. This feature of our persuasion game is in contrast to the fully revealing equilibria of Milgrom and Roberts (1986), Farrell (1986), and Okuno-Fujiwara, Postlewaite, and Suzumura (1990).

Having verified that the equilibria of our game involve at least some news management, let us focus on a class of equilibria that differ markedly in character from earlier articles. Far from the informed party revealing all available information, there is a chronic problem of news management in this class of equilibria. As with earlier articles on the persuasion game, my solution concept is sequential equilibrium, due to Kreps and Wilson (1982). My choice of solution concept is motivated by the natural requirement that the informed party cannot commit to a reporting strategy before receipt of the private information. This would seem to be an essential feature of the problem I am examining.

I introduce the term "sanitization strategy" to denote the strategy of the manager in which every "good" realization of a signal is announced but every "bad" realization is suppressed. That is, for any $\sigma_i \in \sum$,

$$\sigma_i \in R \Leftrightarrow \sigma_i(s^*) = 1. \tag{7}$$

This strategy has a representation in terms of subsets of the state space S. It maps the interval $I = \{s_i, s_{i+1}, \ldots, s_{i+k}\}$ to the announcement $A = \{s_i, s_{i+1}, \ldots, s_{i+k}, \ldots, s_N\}$. Next, I introduce the probability distribution over S that plays the central role in this article. Define the vector $\beta \equiv (\beta_1, \beta_2, \ldots, \beta_N)$ as

$$\beta_i = \begin{cases} p_1 & \text{if } i = 1 \\ p_i \prod_{k=2}^{i} (1 - \theta_k) & \text{if } i \geq 2 . \end{cases} \tag{8}$$

The probability distribution $\pi \equiv (\pi_1, \pi_2, \ldots, \pi_N)$ is defined as

$$\pi_i = \beta_i \Big/ \sum_{k=1}^{N} \beta_k. \tag{9}$$

This distribution will act as a vector of state prices on the contingent claims on the payoff-relevant states. To get a feel for the overall shape of this distribution, note from (8) that, as compared to p, the distribution π places more weight on the states with lower indices. In general, the greater is θ, the greater is the shift to the left. For any given vector of payoffs, this leftward shift of the pricing distribution will imply a fall in the price of an asset that delivers this vector of payoffs. This is the intuition behind our main result, which may be stated as follows.

Theorem 1. There exist sequential equilibria of the persuasion game in which (i) the manager employs the sanitization strategy and (ii) for any announcement $A \subseteq S$ by the manager, $V = E_\pi(x \mid A)$, the conditional expectation of x on A with respect to the distribution π.

Two comments are in order concerning the equilibrium described in Theorem 1. First, the equilibrium price of the firm is determined by a probability distribution over S only. The uncertainty surrounding the quality of the manager's information enters as a parameter in this distribution.

The second point is of more general interest. It is worthy of note that the shareholders are updating their beliefs on S by conditioning on a set of events that do not form a partition of S. In other words, the shareholders have nonpartitional information structures. Nonpartitional information structures have been examined in a series of recent articles, including Rubinstein and Wolinsky (1990), Samet (1990), Geanakoplos (1989), Brandenburger, Dekel, and Geanakoplos (1992), Shin (1993), and Morris (1992). It has been established that when information is nonpartitional, updating by Bayes' rule does not guarantee coherent belief change (see the examples in Geanakoplos (1989) and Rubinstein and Wolinsky (1990)). Additional conditions have been identified by Geanakoplos (1989) that yield coherence.

In this context, the class of equilibria identified in Theorem 1 points to a concrete example in which rational decision makers update their beliefs by conditioning on nonpartitional information structures. Thus, my result throws some light on the question of what types of nonpartitional information structures are likely to arise in practice, and it places the discussion of these structures within an economic context.

The actual proof of Theorem 1 is in two steps. First, I shall demonstrate that the above strategies form part of a Nash equilibrium. I then exhibit a belief profile that satisfies the consistency condition of Kreps and Wilson (1982). Since every information set be-

longing to the manager is reached with positive probability, this second step boils down to the task of exhibiting shareholders' beliefs at off-equilibrium information sets satisfying the consistency condition.

I begin by showing that the sanitization strategy is the manager's best reply against $V = E_\pi(x \mid A)$. The manager maximizes V in order to maximize consumption in each state. Given the information $I \subseteq S$, let A be the announcement according to the sanitization strategy, and consider any interval B that is a feasible announcement given I. I shall show that $E_\pi(x \mid A) > E_\pi(x \mid B)$ for all $B \neq A$. Since B is a feasible announcement, $I \subseteq B$, so that $I \subseteq A \cap B$. Then, partition $A \cup B$ into $\{B \mid A, A \cap B, A \mid B\}$, and let $X = B \mid A$, $Y = A \cap B$, and $Z = A \mid B$. Since $x_1 < x_2 < \ldots < x_N$, the following inequalities hold whenever these conditional expectations are defined:

$$E_\pi(x \mid X) < E_\pi(x \mid Y) < E_\pi(x \mid Z). \tag{10}$$

Since $E_\pi(x \mid B)$ is a convex combination of $E_\pi(x \mid X)$ and $E_\pi(x \mid Y)$ while $E_\pi(x \mid A)$ is a convex combination of $E_\pi(x \mid Y)$ and $E_\pi(x \mid Z)$, we have $E_\pi(x \mid A) \geq E_\pi(x \mid B)$. Thus, the sanitization strategy is a best reply for the manager against $V = E_\pi(x \mid A)$.

Now let us turn to the shareholders' strategies. I have already noted that, given shareholder 2's trading strategy, the unique best reply for shareholder 1 is to set V equal to the expected value of x given the manager's reporting strategy and nature's experiments. I shall show that this expected value is equal to $E_\pi(x \mid A)$ when the manager announces the event A.

For the probability distribution $\mu(\cdot, \cdot)$ over $S \times \mathcal{A}$ generated by nature's experiments and the manager's sanitization strategy, consider the probability that the manager announces the interval A conditional on the true state being s_j. Denote this conditional probability by

$$\mu(A \mid j). \tag{11}$$

Under the sanitization strategy, an interval is announced with positive probability only if it is of the form $\{s_k \mid i \leq k \leq N\}$. I shall attach subscripts to each possible announcement and denote

$$A_i = \{s_k \mid i \leq k \leq N\}. \tag{12}$$

Since announcements are truthful, $\mu(A_i \mid j) = 0$ for $i > j$. Thus, consider $\mu(A_i \mid j)$ for $i \leq j$. The announcement of $A_i \neq S$ implies that $\sigma_i \in \sum$. However, by the sanitization strategy, the announcement of A_i implies that $\sigma_k \notin \sum$ for all k for which $i < k \leq j$. By the sanitization strategy, $\mu(A_i \mid j)$ does not depend on whether $\sigma_k \in \sum$ for $k > j$, since these signals yield the "bad" realization whenever observed, and are suppressed. Similarly, $\mu(A_i \mid j)$ does not depend on whether $\sigma_k \in \sum$ for $k < i$, since the realizations of these signals are redundant given the observation of σ_i, in the sense that $A_i \subset A_k$. Together with the independence of nature's experiments in stage 1, we have

$$\begin{aligned}
\mu(A_i \mid j) &= \theta_i \prod_{i+1}^{j} (1 - \theta_m) && \text{if } j > i \geq 2 \\
&= \prod_{i+1}^{j} (1 - \theta_m) && \text{if } j > i = 1 \\
&= \theta_i && \text{if } j = i \geq 2 \\
&= 1 && \text{if } j = i = 1 \\
&= 0 && \text{otherwise.}
\end{aligned} \tag{13}$$

This probability can be expressed more succinctly if we define $F(i)$ and $G(i, j)$ as follows:

$$F(i) \equiv \begin{cases} \theta_i & \text{if } i \geq 2 \\ 1 & \text{if } i = 1 \end{cases} \tag{14}$$

$$G(i, j) \equiv \begin{cases} \prod_{i+1}^{j} (1 - \theta_m) & \text{if } j > i \\ 1 & \text{if } j = i \\ 0 & \text{if } j < i \,. \end{cases} \tag{15}$$

Then, $\mu(A_i \mid j)$ can be expressed as the product of these two expressions:

$$\mu(A_i \mid j) = F(i)G(i, j). \tag{16}$$

It can be verified that $\sum_i \mu(A_i \mid j) = 1$. Taking ratios of these probabilities, we have the following equation for integers $i \leq j < k$:

$$\frac{\mu(A_i \mid k)}{\mu(A_i \mid j)} = \prod_{j+1}^{k} (1 - \theta_m). \tag{17}$$

Then, by Bayes' rule,

$$\frac{\mu(k \mid A_i)}{\mu(j \mid A_i)} = \frac{\mu(A_i \mid k)p_k}{\mu(A_i \mid j)p_j} = \frac{p_k}{p_j} \prod_{j+1}^{k} (1 - \theta_m).$$

But this is equal to π_k/π_j. Together with the fact that announcements are truthful, (i.e., $\mu(k \mid A_i) > 0$ only if $k \geq i$), we can conclude that $\mu(k \mid A_i)$ is identical to the conditional probability of state s_k on the event A_i according to the prior π. Thus, the expected value of x given the announcement A_i is $\sum_k x_k \mu(k \mid A_i) = E_\pi(x \mid A_i)$.

I have shown that the strategies described in Theorem 1 form part of a Nash equilibrium. To complete the proof of Theorem 1, I need to show that these strategies are supported by a belief profile that satisfies Kreps and Wilson's (1982) consistency condition. This condition stipulates that there be a sequence of perturbed games (i.e., games in which every information set is reached with positive probability), such that the equilibrium belief profile is the limit of the sequence of belief profiles defined by Bayes' rule in each perturbed game. The full argument is presented in the Appendix.

3. Comparative statics of skepticism

■ Let us now turn to the comparative statics of the equilibrium described in Theorem 1 with respect to shifts in the probabilities θ_i. Since these are the probabilities with which the manager receives signals on the value of the firm, they can be seen as a measure of the severity of adverse selection in the market. What is of interest to us is that shifts in these probabilities will be met with commensurate shifts in the attitudes of the shareholders to the manager's announcements, so that in equilibrium, these probabilities will serve as a measure of the degree of skepticism in the market. A number of empirical predictions arise from such shifts of attitude, and it is these that we pursue here.

Denote by θ the vector $(\theta_2, \theta_3, \ldots, \theta_N)$. By the structure of the game, the set of admissible values of θ is the interior of the unit cube in $\mathbb{R}^{N-1}$. A limiting case of θ that is of particular interest as a benchmark is the case in which θ tends to the vector $(1, 1, \ldots, 1)$. In this limiting case, the manager knows the true state at the announcement stage, and prices are those associated with the "skeptical equilibrium" of Milgrom and Roberts (1986).

To see this, consider the conditional probability of state s_k on the event A_i according to the prior π. Denote this probability as

$$\pi(k \mid A_i). \tag{19}$$

This probability is positive if and only if $k \geq i$, and from the definition of π in (8),

$$\pi(k \mid A_i) = \frac{p_i}{p_i + p_{i+1}(1 - \theta_{i+1}) + \ldots + p_N \prod_{i+1}^{N} (1 - \theta_\ell)} \quad \text{if } k = i$$

$$= \frac{p_k \prod_{i+1}^{k} (1 - \theta_\ell)}{p_i + p_{i+1}(1 - \theta_{i+1}) + \ldots + p_N \prod_{i+1}^{N} (1 - \theta_\ell)} \quad \text{if } k > i. \tag{20}$$

As θ tends to $(1, 1, \ldots, 1)$, $\pi(k \mid A_i)$ tends to one if $k = i$ and tends to zero if $k > i$. Thus, in the limit, the equilibrium value of the firm following the announcement A_i is x_i, which is the worst outcome for the firm consistent with the announcement A_i.

In contrast, the limiting case in which θ tends to zero is the polar opposite to the skeptical equilibrium. From (20), prices in this limiting case are those obtained by naive updating by Bayes' rule on the *ex ante* probabilities p. In effect, the adverse selection problem disappears in the limit as θ tends to zero.

In general, if we define the vector inequality $\tilde{\theta} \leq \theta$ as $\tilde{\theta}_i < \theta_i$ for each component i, then the ordering of the set of admissible values of θ defined by the vector inequality can be shown to mirror the degree of skepticism in the market in the following sense. Denote by $\tilde{\pi}$ the pricing distribution associated with $\tilde{\theta}$.

Theorem 2. If $\tilde{\theta} \leq \theta$, then $E_{\tilde{\pi}}(x \mid A) \geq E_\pi(x \mid A)$ for any interval A announced in equilibrium.

That is, for a given announcement A by the manager, the market's skepticism will be more pronounced for parameter θ than for $\tilde{\theta}$, in the sense that the same disclosure leads to a lower valuation of the firm. The proof of this theorem appeals to first-degree stochastic dominance of the distribution $\tilde{\pi}$ over π. Define the complementary cumulative distribution function

$$G(k \mid A) \equiv \sum_{j \geq k} \pi(j \mid A), \tag{21}$$

and denote by $\tilde{G}(k \mid A)$ the corresponding expression for $\tilde{\pi}$. The argument from first-degree stochastic dominance rests on the following property of $G(k \mid A)$ and $\tilde{G}(k \mid A)$.

Lemma 1. If $\tilde{\theta} \leq \theta$, then $\tilde{G}(k \mid A) \geq G(k \mid A)$ for all k.

Proof. Let A be the interval $\{s_k \mid i \leq k \leq N\}$. Since $G(k \mid A) = \tilde{G}(k \mid A) = 1$ for $k \leq i$, we shall confine our attention to $k > i$. From the definition of π and $\tilde{\pi}$ in (8), $\tilde{\pi}(j \mid A)/\tilde{\pi}(k \mid A) \geq \pi(j \mid A)/\pi(k \mid A)$ for $j \geq k > i$. Summing over $j > k$,

$$\tilde{G}(k + 1 \mid A)/\tilde{\pi}(k \mid A) \geq G(k + 1 \mid A)/\pi(k \mid A). \tag{22}$$

Taking reciprocals, adding one to both sides, and taking reciprocals again, we have

$$\tilde{G}(k + 1 \mid A)/\tilde{G}(k \mid A) \geq G(k + 1 \mid A)/G(k \mid A). \tag{23}$$

The lemma then follows by induction. Given $\tilde{G}(k \mid A) \geq G(k \mid A)$, (23) implies

$$\frac{\tilde{G}(k+1 \mid A)}{G(k \mid A)} \geq \frac{\tilde{G}(k+1 \mid A)}{\tilde{G}(k \mid A)} \geq \frac{G(k+1 \mid A)}{G(k \mid A)}, \tag{24}$$

so that $\tilde{G}(k+1 \mid A) \geq G(k+1 \mid A)$. *Q.E.D.*

Proof of Theorem 2. The conditional expectation $E_\pi(x \mid A) = \sum_k \pi(k \mid A)x_k$ can be written as

$$\sum_{k=1}^{N-1} [G(k \mid A) - G(k+1 \mid A)]x_k + G(N \mid A)x_N. \tag{25}$$

If we define $\Delta(k \mid A)$ as $\Delta(k \mid A) \equiv \tilde{G}(k \mid A) - G(k \mid A)$, then

$$E_{\tilde{\pi}}(x \mid A) - E_\pi(x \mid A) = \sum_{k=1}^{N=1} [\Delta(k \mid A) - \Delta(k+1 \mid A)]x_k + \Delta(N \mid A)x_N$$

$$= \sum_{k=1}^{N-1} [\Delta(k \mid A)x_k + \Delta(k+1 \mid A)(x_{k+1} - x_k) - \Delta(k+1 \mid A)x_{k+1}] + \Delta(N \mid A)x_N.$$

Since $\Delta(1 \mid A) = 0$, this simplifies to

$$\sum_{k=1}^{N-1} \Delta(k+1 \mid A)(x_{k+1} - x_k). \tag{26}$$

As x_k is increasing in k, and as $\Delta(k \mid A)$ is nonnegative by Lemma 1, this expression is nonnegative. *Q.E.D.*

Theorem 2 points to one consequence of shifts in the vector θ in terms of the traded value of the firm. Indeed, the expression (26) gives us the explicit formula for this difference in the value of the firm. Another important consequence of shifts in θ is on the probabilities attached to each of the manager's announcements. The probability distribution over the set of announcements (considered as a function of θ) can be seen as a characteristic "signature" of θ. This opens up the prospect of drawing inferences on θ from the shape of the distribution of announcements.

Denote by λ_i the probability with which the manager announces the event A_i in equilibrium. We can derive an expression for λ_i from (13) by multiplying by p_j and summing over j. This yields

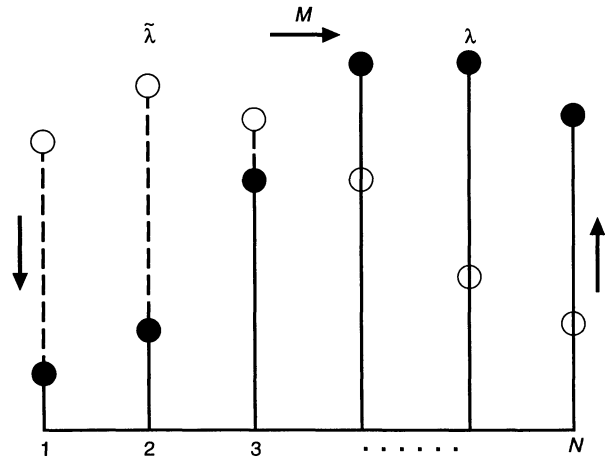
$$\lambda_i = F(i)\left[p_i + \sum_{k=i+1}^{N} p_k \prod_{\ell=i+1}^{k} (1 - \theta_\ell)\right], \tag{27}$$

where $F(i)$ is the function defined in (14). Denote by λ the (row) vector of these probabilities, and call this the manager's *disclosure profile*. We shall be particularly interested in the association between λ and the value of the firm, since this association provides an empirically verifiable consequence of shifts in the vector θ. This intuition forms the basis of the empirical hypothesis to be reported in the next section. If we denote by $\tilde{\lambda}$ the disclosure profile associated with the vector $\tilde{\theta}$, we can state the following theorem on the comparative statics of disclosures.

Theorem 3. If $\tilde{\theta} \leq \theta$, then $\lambda = \tilde{\lambda}M$ for some upper triangular Markov matrix M.

The effect of the transformation embodied in an upper triangular Markov matrix is to transfer some of the probability mass of each state to states with *higher* indices. One way of visualizing this transformation is to see M as a transition matrix that allows transitions to states with higher indices but blocks any transition to states with lower indices.

FIGURE 1



Thus, the overall effect of such a transformation is to shift the distribution to the right and increase the degree of (positive) skewness of the distribution. Figure 1 illustrates such a shift.

I prove Theorem 3 by constructing the upper triangular matrix in question. For our starting point, consider the $N \times N$ matrix whose (i, k)th entry is given by $\pi(k \mid A_i)$. Denote this matrix by Π. By construction, Π is upper triangular, with all nonzero entries being positive. Π is nonsingular, and its inverse Π^{-1} plays an important role. It is useful to have an explicit expression for the entries in Π^{-1}. For this purpose, define the matrix D whose (i, k)th entry is given by

$$d(i, k) \equiv \begin{cases} 1 & \text{if } \pi(k \mid A_i) > 0 \\ 0 & \text{otherwise}. \end{cases} \tag{28}$$

D is an upper triangular matrix for which every entry on or above the leading diagonal is one. D is nonsingular, and its inverse is given by

$$D^{-1} = \begin{bmatrix} 1 & -1 & & & \\ & 1 & -1 & & \\ & & 1 & \ddots & \\ & & & \ddots & -1 \\ & & & & 1 \end{bmatrix}$$

That is, the (i, k)th entry of D^{-1} is given by

$$d^{(-1)}(i, k) = \begin{cases} 1 & \text{if } k = i \\ -1 & \text{if } k = i + 1 \\ 0 & \text{otherwise}. \end{cases} \tag{29}$$

If we denote by $G(k)$ the sum $\sum_{j \geq k} \pi_j$, then we can verify by multiplication that the (i, k)th entry of Π^{-1} is given by

$$\pi^{(-1)}(i, k) = d^{(-1)}(i, k) \frac{G(k)}{\pi_i}. \tag{30}$$

We can then prove the following lemma.

Lemma 2. $\lambda = p\Pi^{-1}$.

Proof. Consider the product $p\Pi^{-1}$, which is a row vector with N components. From (30), the ith component of $p\Pi^{-1}$ is given by

$$\sum_k p_k \pi^{(-1)}(k, i) = \begin{cases} p_1/\pi_1 & \text{if } i = 1 \\ G(i)[(p_i/\pi_i) - (p_{i-1}/\pi_{i-1})] & \text{if } i \geq 2. \end{cases} \tag{31}$$

This simplifies to

$$F(i)\left[p_i + \sum_{k=i+1}^{N} p_k \sum_{\ell=i+1}^{k} (1 - \theta_\ell)\right], \tag{32}$$

which is identical to the expression for λ_i in (27). *Q.E.D.*

Lemma 2 has a natural interpretation. It states that $p = \lambda\Pi$, so if we denote by x the column vector whose ith component is x_i, we have $px = \lambda\Pi x$. In other words, the *ex ante* value of the firm is the convex combination of the possible values of the firm at the trading stage, where the weights are the probabilities attached to each announcement by the manager.

Let us denote by $\tilde{\Pi}$ the pricing matrix associated with $\tilde{\pi}$. Then, Lemma 2 implies that $p = \tilde{\lambda}\tilde{\Pi} = \lambda\Pi$, so that

$$\lambda = \tilde{\lambda}\tilde{\Pi}\Pi^{-1}. \tag{33}$$

To prove Theorem 3, it remains to show that $\tilde{\Pi}\Pi^{-1}$ is an upper triangular Markov matrix. To see that $\tilde{\Pi}\Pi^{-1}$ is upper triangular, note from (29) and (30) that Π^{-1} is upper triangular, and recall that the product of two triangular matrices is itself triangular.

To show that all nonzero entries of $\tilde{\Pi}\Pi^{-1}$ are positive, consider the (i, k)th entry, where $k \geq i$. By (30), it is given by

$$\sum_j d(i, j) \frac{\tilde{\pi}_j}{\tilde{G}(i)} d^{(-1)}(j, k) \frac{G(k)}{\pi_j} = \frac{G(k)}{\tilde{G}(i)} \left[\frac{\tilde{\pi}_k}{\pi_k} - \frac{\tilde{\pi}_{k-1}}{\pi_{k-1}}\right]. \tag{34}$$

However, from (9), the expression in the square brackets is nonnegative. Since $\tilde{\Pi}\Pi^{-1}$ is triangular, every nonzero entry is positive.

Finally, we need to show that each row of $\tilde{\Pi}\Pi^{-1}$ sums to one. For this, it suffices to show that each row of Π^{-1} sums to one, since then, the ith row of $\tilde{\Pi}\Pi^{-1}$ sums to $\sum_j\sum_k \tilde{\pi}(i, k)\pi(k, j) = \sum_k \tilde{\pi}(i, k)\sum_j \pi(k, j) = 1$. From (30), the ith row of Π^{-1} sums to

$$\sum_k d^{(-1)}(i, k) \frac{G(k)}{\pi_i} = \frac{G(i)}{\pi_i} - \frac{G(i+1)}{\pi_i} = 1. \tag{35}$$

This concludes the proof of Theorem 3.

4. Empirical hypothesis

■ One area in which my theory has potentially fruitful applications is in the explanation of divergent price/earnings ratios for otherwise similar firms. The earnings reports of firms are notorious for their malleability. Within the bounds of "generally accepted accounting practices," firms have considerable discretion in managing their earnings figures. Elton and Gruber (1987) cite an example in which a given balance sheet gives rise to a range of earnings figures from \$1.79 to \$0.80 as the treatment of items such as depreciation, inventories, and R&D expenditures are treated in different ways, all within accepted accounting rules. I have already alluded to the spate of recent, highly publicized corporate

collapses in the United Kingdom, and the response of the accounting regulators to these events.

The main empirical hypothesis suggested by Theorems 2 and 3 is that the disclosure profiles of firms provide information on the degree of credence placed on the firm's reports. In particular, we have seen that a firm with a *lower* level of credence has a disclosure profile with a greater degree of skewness to the right—toward the good states for the firm. At the same time, we know from Theorem 2 that for a given news event, a firm with lower credence is priced at a lower level than an equivalent firm with higher credence. The empirical counterpart to this would be that a firm with lower credence has a lower price/earnings ratio. I thus propose the following empirical hypothesis.

Hypothesis. For a set of firms that are otherwise identical, low price/earnings ratios are associated with a greater degree of positive skewness in the distribution of earnings reports.

Actual empirical tests of this hypothesis will be feasible once we have settled on a procedure for standardizing the earnings figures between firms in order to make the comparisons across firms meaningful. Many issues of interest would seem to arise in implementing this empirical project, and would be worthy of a systematic empirical investigation.

Appendix

■ In this appendix, I complete the proof of Theorem 1 by exhibiting a belief profile that satisfies the consistency condition of Kreps and Wilson (1982). In our game, every information set belonging to the manager is reached with positive probability, so we need only consider beliefs at the shareholders' information sets. I shall define a sequence of perturbed games based on our persuasion game in which the manager "trembles," and announces every interval of the state space with positive probability. I then note the shareholders' beliefs implied by Bayes' rule, and show that the limiting belief profile supports the strategies described in Theorem 1.

In the perturbed game, the manager makes small mistakes in implementing the sanitization strategy. The mistake takes the form of the manager revealing a "bad" realization of a signal with some probability $\eta > 0$. However, the manager does not make any mistakes in revealing all the good realizations. In other words, the trembles in the perturbed game modify (6) to

$$\sigma_i(s^*) = 1 \quad \Rightarrow \sigma_i \in R$$

$$\sigma_i(s^*) = 0 \quad \Rightarrow \begin{cases} \sigma_i \in R \text{ with probability } \eta \\ \sigma_i \notin R \text{ with probability } 1 - \eta. \end{cases} \tag{A1}$$

Assume that these trembles are independent of each other and of nature's decisions in stage 1. Let us denote by $A_{i\ell}$ the interval $\{s_k \mid i \leq k \leq \ell\}$, and denote by

$$\bar{\mu}(A_{i\ell} \mid j) \tag{A2}$$

the probability in the perturbed game of the manager announcing the interval $A_{i\ell}$ given that s_j is the true state. If we define

$$H(j, \ell) \equiv \begin{cases} \prod_{j+1}^{\ell} (1 - \eta\theta_m) & \text{if } \ell > j \\ 1 & \text{if } \ell = j \\ 0 & \text{if } \ell < j \end{cases} \tag{A3}$$

and

$$I(\ell) \equiv \begin{cases} \eta\theta_\ell & \text{if } \ell < N \\ 1 & \text{if } \ell = N\,, \end{cases}$$

then similar reasoning to that used in the derivation of (13) yields the following expression for $\bar{\mu}(A_{i\ell} \mid j)$:

$$\bar{\mu}(A_{i\ell} \mid j) = F(i)G(i, j)H(j, \ell)I(\ell). \tag{A4}$$

It can be verified that $\sum_\ell \bar{\mu}(A_{i\ell} \mid j) = \mu(A_i \mid j)$, so that the sum of probabilities over all possible announcements is $\sum_i \sum_\ell \bar{\mu}(A_{i\ell} \mid j) = \sum_i \mu(A_i \mid j) = 1$. Moreover, (A4) implies that for any integers $i \leq j < k \leq \ell$,

$$\frac{\bar{\mu}(A_{i\ell} \mid k)}{\bar{\mu}(A_{i\ell} \mid j)} = \prod_{j+1}^{k} (1 - \theta_m) \Big/ \prod_{j+1}^{k} (1 - \eta\theta_m). \tag{A5}$$

So, by Bayes' rule,

$$\frac{\bar{\mu}(k \mid A_{i\ell})}{\bar{\mu}(j \mid A_{i\ell})} = p_k \prod_{j+1}^{k} (1 - \theta_m) \Big/ p_j \prod_{j+1}^{k} (1 - \eta\theta_m). \tag{A6}$$

As η tends to zero, this expression tends to π_k/π_j. Thus, together with the fact that announcements are truthful (i.e., $\bar{\mu}(j \mid A_{i\ell}) > 0$ only if $i \leq j \leq \ell$), we can conclude that $\bar{\mu}(j \mid A_{i\ell})$ converges to the conditional probability of state s_j on the event $A_i\ell$ according to the prior distribution π. Hence, shareholder 1's strategy of setting $V = E_\pi(x \mid A)$ is supported by a belief profile that is obtained as the limit of a sequence of belief profiles of perturbed games, so that the consistency condition for sequential equilibrium is satisfied. This completes the proof of Theorem 1.

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