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Imprecision in Accounting Measurement: Can It Be Value Enhancing?

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ABSTRACT

Accounting measurements of firms' investments are usually imprecise. We study the economic consequences of such imprecision when it interacts with information asymmetry regarding an investment project's ex ante profitability, known only by the firm's managers. Absent agency and risk-sharing considerations, we find that some degree of accounting imprecision could actually be value enhancing. We characterize the optimal degree of imprecision and identify its key determinants. The greater the information asymmetry regarding the project's profitability, the greater is the imprecision that should be tolerated in the measurement of the firm's investment.

1. *Introduction*

Most accounting measurements are imprecise and provide, at best, a noisy representation of a firm's operations and underlying events. Intuitively, it may seem that such imprecision is always undesirable and should be eliminated to the extent possible. This intuition is consistent with many well-known results. More information is always preferred to less when a single

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decision maker interacts with the forces of nature. In moral hazard settings, contracts between a principal and an agent are more efficient when measured outcomes convey more information about the agent's hidden actions. When capital markets are viewed as purely trading institutions with firms' cash flows described by fixed exogenous distributions, more information about these cash flows is preferred to less because information reduces assessed risk and decreases the risk premium imbedded in capital market values.

In this article we analyze a capital market setting, which is not uncommon, and develop the counterintuitive result that accounting imprecision is not necessarily harmful and, in fact, can be value enhancing. Moreover, we find that there is an optimal degree of imprecision, and that level is not zero. Thus, an emphasis on eliminating imprecision in public disclosure to capital markets may actually destroy wealth. We identify a key factor that determines how much accounting imprecision should be tolerated.

We study the investment problem of a value-maximizing firm, given that its manager has private information regarding the profitability of that investment. The firm's investment cannot be directly observed by capital market participants. Instead, accounting measurements of investment are observed, and these measurements are imprecise. These assumptions seem realistic and representative of many situations. Accounting measurements of investment depend on many subjective judgments, estimates, and simplistic conventions that are necessitated by the inherent difficulty of separating investments (tangible and intangible) from operating cash flows. These judgments and conventions introduce random error into accounting measurements. Additionally, it seems realistic that, at the time they choose the firm's investment, managers have superior information relative to the capital market regarding the profitability of that investment. Given that, in practice, managers expend enormous amounts of time and resources to collect and analyze information about alternative investment projects, much of which is sensitive and unverifiable, it seems reasonable to assume that such information cannot be immediately shared with the capital market. We assume there are no agency conflicts and the firm's manager chooses investment to maximize the expected payoff to the firm's current shareholders. We call imperfect accounting measurement "imprecision" and the market's lack of information about the investment's profitability "ignorance."

The firm's future cash flows depend on both the size of its investment and the profitability of its operations. However, the firm's incentive to invest depends critically upon how the capital market perceives and prices those investments and, therefore, on the information available to the capital market. Thus, when investment is not measured at all, or measured imprecisely, the firm's incentives to invest are different from settings where the capital market perfectly observes the firm's investments. Additionally, the capital market's response to accounting measurements is affected by the awareness that the firm's management possesses private information about the ex ante profitability of investment. A priori, there are two unknowns in the capital market: the magnitude of the firm's investment and the ex ante profitability of the investment. These two variables are economically related; therefore, measurement of one leads to inferences about the other. These measurements and inferences combine to determine the pricing of the firm in the capital market and, consequently, the firm's incentives for investment. Our investigation of accounting imprecision takes into account the interaction between measurements and inferences and their economic consequences.

The ideal situation, leading to first-best investment and market prices, is one where neither ignorance of profitability nor measurement imprecision of the firm's investment exists. In this sense, both ignorance and imprecision are undesirable. Therefore, it is tempting to conclude that the presence of either condition makes the elimination or minimization of the other more desirable. However, we show that exactly the opposite is true. The presence of either one (ignorance or imprecision) without the other has disastrous consequences, whereas together they work reasonably well. If managers possess superior information about a project's profitability, some degree of imprecision in the measurement of the firm's investment is value enhancing. Conversely, if measurement of investment is imprecise, some degree of capital market ignorance of the project's profitability is value enhancing. There is an optimal balance of measurement imprecision and ignorance for the capital market.

The intuition for these results is developed by initially studying the effect of each information asymmetry in the absence of the other. In the first setting, we pose the hypothetical question: What are the economic consequences of providing perfect information about the project's profitability but imprecise measurement of its size, that is, imprecision without ignorance? In the second regime we pose the complementary question: What are the consequences if the measurement of investment were made infinitely precise without changing the information asymmetry about project profitability, that is, ignorance but no imprecision?

In the first informational regime—imprecision without ignorance—we find that *any* imprecision in measurement, no matter how small, makes the measurement completely uninformative and is ignored by the capital market. This happens because the market rationally believes it can perfectly anticipate the firm's investment from its knowledge of the project's profitability. Thus, when the accountant's measurement of investment does not coincide with the market's prior anticipation, the difference is attributed solely to measurement error and is ignored. In this situation, the equilibrium price in the capital market becomes insensitive to reductions in the firm's true investment. Firms rationally respond to this situation by underinvesting; the market anticipates the underinvestment and prices the firm accordingly. Thus, imprecision without ignorance results in a bad equilibrium, with significant underinvestment and destruction of value.

In the second informational regime, we find that eliminating imprecision but not ignorance induces firms to overinvest and thus destroy value. Perfect measurement imparts an informational value to the firm's investment because the market seeks to infer the manager's private knowledge of project profitability from observation of its investment. In equilibrium, higher investment leads to inferences of higher profitability, resulting in a classical Spence-type [1974] signaling equilibrium with overinvestment.

These two extremes provide the intuition of why the existence of both imprecision and ignorance are value enhancing. Information asymmetry about the project's profitability (ignorance) prevents perfect anticipation of the firm's investment and allows imprecise measurements to have information content, thus alleviating the underinvestment problem. Imprecision in the measurement of investment counteracts the firm's overinvestment incentive that is associated with fully revealing signaling equilibria. As the noise in the accounting measurement increases, the signal value of the firm's investment decreases. Given both ignorance and imprecision, market inferences consist of assessed distributions of the project's profitability and the magnitude of its investment, resulting in noisy signaling equilibria.

We find conditions under which an appropriate balance of imprecision and ignorance actually restores the first-best investment schedule and achieves the first-best expected payoff to the firm. In this case, we are able to precisely characterize the optimal degree of imprecision as a function of exogenous parameters. We obtain the surprising result that the greater the information asymmetry between the manager and the market regarding the project's profitability, the *less* precise should be the accounting measurement of investment. Conversely, given some exogenous level of imprecision in the accounting measurement, there is an optimal degree of ignorance for the capital market; the greater the imprecision in accounting measurement, the greater should be the information asymmetry about the project's profitability.

Our results indicate that there is an externality between two noisy signals when one signal reveals direct information about the firm's type and the other signal reveals direct information about the firm's decision. As the first signal becomes more informative, the value of the second signal declines because the market relies more on its prior beliefs regarding the firm's decisions. In the limit, when the firm's type is perfectly known, the accounting signal becomes worthless. Conversely, when the market obtains precise information about the firm's decision, the market tends to ignore the signal on type, relying more on indirect inferences made from observation of the firm's decision. Unless both signals can be made infinitely precise, there is an optimal balance in the precision of the two signals.

The Financial Accounting Standards Board's (FASB) Statement of Financial Accounting Concepts No. 2 [1980] states that "reliability" (freedom from error and bias) and "representational faithfulness" are key desirable characteristics of accounting statements. Using these criteria, the inability to measure with sufficient reliability is sometimes cited as a reason for nonmeasurement and nondisclosure. Our findings caution against an excessive insistence on reliability. Rather than being harmful, some imprecision in accounting measurements could actually be value enhancing. At the same time, it is not easy to determine just how much imprecision should be tolerated. Our results indicate that the degree of information asymmetry about related variables, which are inferred from the variables that accountants measure, is a crucial factor that should affect this judgment.

In the academic literature, there are at least three other well-known instances where perfect measurements and full disclosure of information are undesirable. In the first, more information through public disclosure destroys risk-sharing opportunities (Hirshleifer [1971], Diamond [1985], Verrecchia [1982]). In the second, disclosure imposes proprietary costs on a firm by informing competitors' actions (Dye [1986], Gigler [1994], Verrecchia [1983]). In the third, agents' payoffs depend both on economic fundamentals and the similarity of their actions, and increasing the transparency of public information may coordinate agents' beliefs in such a way that the incidence of inefficient social outcomes is increased (Morris and Shin [2002, 2004]). Our analysis shows that there is an additional compelling demand for measurement imprecision that does not arise from risk-sharing, competitive, or coordination considerations but arises solely from a valuation perspective. The result that noisy signals of endogenous actions have no information content is closely related to the results in Bebchuk and Stole [1993], Bagwell [1995], and Kanodia and Mukherji [1996].[1] Bagwell establishes that in a leader-follower oligopoly, the leader's first-mover advantage is destroyed if observation of the leader's output is noisy. Maggi [1999] extends Bagwell's analysis by showing that if the leader's output is based on private information, noisy signals on that output are indeed informative. Maggi further establishes that in some cases there is a critical degree of noise that would fully restore the first-mover advantage of the leader. This result is similar to our result on the optimality of noise in the measurement of a firm's investment. However, Maggi explicitly avoids signaling considerations by having the leader privately observe a parameter that is not directly relevant to the follower. Noisy signaling lies at the heart of our analysis because the private information on which the firm's investment is based is essential to the pricing of the firm in the capital market.

The literature on noisy signaling is sparse. Methodologically, the study closest to our work is Matthews and Mirman's [1983] study of entry deterrence with limit pricing. In Matthews and Mirman, an incumbent producer, with private knowledge of an industry demand parameter, chooses an output level that stochastically affects the equilibrium price in the commodity market. A potential entrant extracts information from the observed price and decides whether to enter. Unlike our work, their analysis is considerably simplified by the binary nature of the entrant's decision and they do not provide any insights into an optimal degree of noise.

The remainder of this article is organized as follows. In section 2, we describe three benchmark models of the firm's investment decision under different information structures. Section 2.1 studies the case of full

[1] See also Dutta and Reichelstein [2003], Fudenberg and Tirole [1986], Narayanan [1985], and Stein [1989].

information. Section 2.2 studies accounting imprecision when there is no other information asymmetry between the market and the firm's manager. In section 2.3, we introduce asymmetric information (ignorance) about the profitability of investment and examine the consequences of perfect measurement of investment. Section 3 characterizes noisy signaling equilibria when both ignorance and imprecision are present. In sections 4 and 5 we develop the optimality of accounting imprecision under two alternative representations of noise. Section 6 concludes. Proofs of all propositions are contained in the appendix.

2. *Benchmark Models of the Firm's Investment Decision*

Consider the investment problem of a firm that is traded in a capital market. The firm's investment yields short- and long-term returns. Short-term returns are consumed directly and privately by the firm's current shareholders, but long-term returns are consumed through the pricing of the firm in the capital market. Investment of k units yields a short-term return of $\theta k - c(k)$, where the parameter θ is a summary statistic representing the profitability of the project in which the firm invests, and $c(k)$ is the cost of investment, which is assumed to be increasing and strictly convex. The profitability parameter is drawn from a distribution with density function $h(\theta)$. The firm's manager observes the parameter θ before choosing the firm's investment. There are no agency conflicts between the firm's manager and its current shareholders, and all investors in the economy are risk neutral.

2.1 INVESTMENT WITH COMPLETE INFORMATION

Suppose that the capital market has full information; that is, θ is common knowledge and the firm's investment is perfectly and directly observed. The firm's chosen investment k and its profitability θ also affect (perhaps stochastically) the long-term returns generated by the firm. Hence, its price in the capital market, net of the short-term return, is described by some function $v(k, \theta)$. This pricing rule for the complete information setting is exogenous. We assume that $v(k, \theta)$ satisfies $v_k > 0$, $v_{kk} \leq 0$, $v_\theta > 0$, and $v_{k\theta} \geq 0$. Given the absence of agency conflicts, the firm invests to maximize the expected payoff to its current shareholders. Thus, the firm's problem is described by

$$\max_k \theta k - c(k) + v(k, \theta).$$

The firm's optimal investment schedule is described by the first-order condition,

$$c'(k) = \theta + v_k(k, \theta). \tag{1}$$

Let $k_{FB}(\theta)$ be the solution to (1), where the subscript *FB* denotes first best.

The investment model captures, in a simple way, the two-way interaction between a firm's investment and its capital market price. Not only does the firm's investment affect its capital market value, as described by $v(k, \theta)$, but also the market's price response affects the firm's choice of investment, as indicated in (1). The firm's investment policy characterized in (1) is consistent with the net present value rule, which formally requires the firm to discount its expected future cash flows at an appropriate cost of capital. The market value $v(k, \theta)$ is, *in fact*, the present value obtained from the distribution of future cash flows and appropriately reflects both the market's assessment of this distribution and the time preferences of investors in the capital market. The firm's investment policy described in (1) indicates that the firm invests up to the point where the marginal cost of investment equals the sum of the marginal short-term return to investment and the marginal effect of investment on the value assigned by the capital market to the distribution of long-term returns. Modeling the firm's investment problem in this fashion allows us to study how accounting measurements and disclosures affect the firm's investment choices through its interaction with capital markets.

In principle, the valuation rule, $v(k, \theta)$, which is exogenous to our analysis, derives from a complex intertemporal equilibrium (see Kanodia [1980]). In such a model the profitability parameter θ could evolve stochastically over time and the firm could have opportunities for new investment at every point in time. If the firm's current profitability θ affects the distribution of future profitability, and if either the firm's current investment directly affects the distribution of future cash flows or indirectly affects that distribution by constraining future investment opportunities, the current value of the firm would indeed be a function of current investment k and current profitability θ. The assumptions we specify for this exogenous value $v(k, \theta)$ would likely be satisfied in such an intertemporal model where $v(k, \theta)$ is derived endogenously.

In this benchmark model of a firm's investment decision, there are two *potential* sources of information asymmetry between the firm's management and the capital market. First, contrary to the assumption in the full-information benchmark model, realistically the firm's investment is not directly and perfectly observed by the capital market. Instead, information on a firm's investment is conveyed by accounting measurements and reports, which are necessarily imprecise. Second, managers are likely to possess superior information on parameters, such as θ, that describe the distribution of future cash flows to new investments. We study how these two information asymmetries (imprecision and ignorance) interact to determine simultaneously the firm's investment and its pricing in the capital market. However, to gain insight into the relative role of each, we first examine pricing and investment in two additional benchmark cases where one information asymmetry is present and the other is not.

2.2 INVESTMENT UNDER IMPRECISION AND NO IGNORANCE

Here, we examine the consequences of imprecision in accounting measurements of investment when the profitability parameter θ is common knowledge. Let $\tilde{s}$ denote the accounting report on the firm's investment. Because the accounting measurement is stochastically related to the firm's actual investment, we model $\tilde{s}$ as a drawing from a distribution $F(s \mid k)$ parameterized by the *true* level of investment. At this point in the analysis, we assume only that F has density $f(s \mid k)$ and fixed support $[\underline{s}, \bar{s}]$.

The pricing rule in the capital market can depend only on observable variables: the known parameter, θ, and the imprecise accounting measurement, s. Therefore the equilibrium price in the capital market is some function $\varphi(s, \theta)$.

DEFINITION. *A pure strategy equilibrium consists of two schedules, an investment schedule, $k_M(\theta)$, and a pricing schedule, $\varphi(s, \theta)$, satisfying:*

(i) Given $\varphi(s, \theta)$, $k_M(\theta)$ is optimal for the firm; that is, for each θ, $k_M(\theta)$ solves

$$\max_k \theta k - c(k) + \int_{\underline{s}}^{\bar{s}} \varphi(s, \theta) f(s \mid k)\, ds. \tag{2}$$

(ii) $\varphi(s, \theta) = E[v(k_M(\theta), \theta) \mid s]$.

Condition (ii) is the rational expectations requirement that market prices incorporate beliefs that are consistent with the equilibrium investment schedule $k_M(\theta)$ and the firm's intrinsic value $v(k, \theta)$. Such an equilibrium is characterized in Proposition 1.

PROPOSITION 1. *When the firm's investment is measured imprecisely and its profitability parameter θ is common knowledge, the firm's equilibrium investment is described by $c'(k_M(\theta)) = \theta$ and the equilibrium price in the capital market is $v(k_M(\theta), \theta), \forall\, s$.*

Proposition 1 indicates that the accounting signal is completely ignored by the capital market even though there is a well-defined statistical relationship between the signal and the firm's investment. The firm underinvests and behaves myopically to maximize only its short-term return of $\theta k - c(k)$. It follows that if the marginal, long-term return to investment is high, the magnitude of underinvestment could be substantial. The intuition underlying Proposition 1 is as follows: The equilibrium price in the capital market is based on an *anticipated* level of investment rather than the firm's actual investment. When the market observes a signal realization different from the anticipated investment, the market attributes the difference to measurement error and therefore has no reason to revise its beliefs. Therefore, if the firm departs from the market's anticipation of its investment, there is no change in the equilibrium market price, even though the distribution of the accounting signal does change. The firm responds to this situation by choosing its investment to maximize only its short-term return, which is directly consumed by its current shareholders. Because such myopic investment is optimal for the firm regardless of what anticipated investment is incorporated in the equilibrium market price, the *only* sustainable anticipation by the market is that the firm will indeed invest myopically. The equilibrium market price reflects this rational anticipation.

Proposition 1 is a stark, but unrealistic, result. In the real world, imprecision in accounting measurements of investment is pervasive. It is difficult to believe that such measurements have no information content and are ignored by the capital market. In addition, it seems unlikely that real investment in the economy exhibits the extreme myopia characterized in the preceding equilibrium. The fundamental reason for this myopia is that investors in the capital market can step into the manager's shoes and solve the manager's investment problem. We think this perfect anticipation of the firm's investment is unrealistic and indefensible. The market has been assumed to know too much!

It seems realistic that corporate managers possess superior information about the profitability of new investment projects—at least at the time they are initiated. Screening alternative projects, assessing the future demand for new products, making cost and revenue projections, anticipating the retaliatory moves of competitors, and making judgments about future technological innovations are all tasks that have deliberately been delegated by shareholders to corporate managers presumably for informational reasons. If managers possess such firm-specific information that is not directly available to the capital market, perfect anticipation of the firm's investment is no longer possible. In such asymmetric information environments, noisy measurements of the firm's investment will have information content and will affect equilibrium capital market prices. In fact, given that the lack of information asymmetry results in unrealistic myopia, it is difficult to justify the study of accounting imprecision in settings without asymmetric information.

2.3 INVESTMENT UNDER IGNORANCE AND PERFECT MEASUREMENT

Having studied imprecision without ignorance, we now study how *perfect* measurements of investment will affect the equilibrium when the firm's manager privately knows the value of θ. Because the market does not a priori know the firm's profitability parameter θ, the equilibrium price in the capital market is described by some function $\varphi(k)$. But because the manager chooses investment using private information, the market would seek to make inferences about project profitability from the perfectly measured investment. These inferences are embedded in the equilibrium pricing schedule $\varphi(k)$. Because measurement is perfect, there is the possibility that the market's inference of θ is also perfect, resulting in a fully revealing signaling equilibrium similar to Spence [1974] and others. Such a fully revealing signaling equilibrium is constructed next.

DEFINITION. *A fully revealing signaling equilibrium is a triple* $\{k(\theta), \varphi(k), I(k)\}$ *that satisfy:*

(i) $k(\theta) = \arg\max_k \theta k - c(k) + \varphi(k)$,
(ii) $\varphi(k) = v(k, I(k))$, *and*
(iii) $I(k(\theta)) = \theta, \forall \theta$.

Condition (i) requires that the equilibrium investment schedule maximizes the firm's payoff, given the pricing rule in the capital market. Condition (ii) requires that each possible investment that could be chosen by the firm is priced consistent with the market's point inference, $I(k)$, of the project's profitability. Thus, when the accounting system reports an investment of k and the market infers that the value of θ must be $I(k)$, the equilibrium price that must prevail in the market is $v(k, I(k))$. Condition (iii) requires that, in equilibrium, the market's inference of θ from each observed investment coincides with the value of θ that gave rise to that investment. Such an equilibrium is characterized in Proposition 2.

PROPOSITION 2. *In a setting where the firm's manager privately observes θ before choosing the investment, and investment is perfectly measured and reported by the accounting system, any fully revealing equilibrium investment schedule must satisfy the monotonicity condition $k'(\theta) > 0$, and the first-order differential equation,*

$$k'(\theta)[c'(k(\theta)) - \theta - v_k] = v_\theta. \tag{3}$$

The firm overinvests at each $\theta > \underline{\theta}$.

Riley [1979] shows that differential equations of this nature have a one-parameter family of solutions and that the exogenous parameter can be chosen so that the worst type invests the first-best quantity, in which case $k'(\theta) > 0$. Given $k'(\theta) > 0$ and $v_\theta > 0$, (3) can only be satisfied if $\theta + v_k - c'(k(\theta)) < 0$, which implies that the firm overinvests because the first-best investment satisfies $\theta + v_k - c'(k(\theta)) = 0$. The greater the value of v_θ, the greater is the degree of overinvestment.[2]

The reason for overinvestment is that investment acquires an informational value. To make inferences about the profitability parameter θ from the firm's observed investment, market participants must form beliefs about the firm's investment policy. How each observed investment is priced in the capital market depends strongly on these beliefs and inferences. Inferences based on the first-best investment schedule cannot be sustained because such inferences lead to market prices that increase too rapidly in observed investment. Given such pricing, high levels of investment become so much more attractive—relative to low levels of investment—that low θ types choose investment levels that the market believes only high types would choose. Market participants would be systematically deceived and lose money, thereby inducing a revision in their beliefs. In equilibrium, beliefs shift in such a way that the market is no longer deceived, and equilibrium

[2] In the special case where $v_\theta \equiv 0$, investment is first best as implied by (3). This case would occur when the long-term return to the firm's investment is independent of the current profitability parameter θ, which implies that the market does not need to make any inferences about θ from observed investment. Because market inferences are moot there is no distortion to the firm's investment. However, the myopia results obtained when investment is measured with noise and θ is public information continue to hold.

market prices are consistent with both the observed investment and its underlying profitability. However, the shift in beliefs that occurs because of the *possibility* of deception induces firms to overinvest, and the cost of this overinvestment is borne entirely by the firm's current shareholders.

Once again, the economy is trapped in a bad equilibrium. Now the firm is induced to overinvest, whereas previously it was optimal to underinvest. We now investigate the more realistic setting where there is both ignorance and imprecision; that is, the manager is better informed than the capital market and the accounting measurement is imprecise. We show that together ignorance and imprecision can sustain more efficient equilibria.

3. *Imprecision and Ignorance: Noisy Signaling Equilibria*

Now, assume the manager privately observes the profitability parameter θ before choosing the firm's investment and that investment is measured imprecisely by the accounting system (i.e., there is both ignorance and imprecision). The market observes only the accounting measurement of investment and knows that θ is a drawing from the density $h(\theta)$ with support Θ. As before, the accountant's imprecise measurement system is represented by a probability density function $f(s \mid k)$, where s is the accounting signal and k is the firm's true investment. Now, the price in the capital market can be a function only of s, say $\varphi(s)$. Embedded in this pricing rule are the market's inferences about the firm's investment and its profitability from observation of the accounting measure.

We show that when the market perfectly observes the firm's investment it can make a perfect inference of profitability, and when the market directly observes profitability it can make a perfect inference of the firm's investment. However, when both θ and k are unobservable, the market's inference can no longer be perfect. Market inferences must take the form of a Bayesian posterior distribution on feasible values of (k, θ) conditional on s. This posterior reduces to a distribution on Θ conditional on s because, in equilibrium, the market knows the firm's investment policy. If the market believes that the firm's investment schedule is $\hat{k}(\theta)$, the assessed posterior distribution on Θ conditional on s must satisfy

$$g(\theta \mid s) = \frac{f(s \mid \hat{k}(\theta))\, h(\theta)}{\int_{\Theta} f(s \mid \hat{k}(t))\, h(t)\, dt}.$$

In this equation, $f(s \mid \hat{k}(\theta))$ is the appropriate density at s conditional on θ because the market believes that at θ the firm chooses investment of $\hat{k}(\theta)$.

DEFINITION. *An equilibrium is a triple* $\{k(\theta), g(\theta \mid s), \varphi(s)\}$ *such that:*

(i) Given $\varphi(s)$, $k(\theta)$ *is optimal for the firm; that is,* $\forall\, \theta$, $k(\theta)$ *solves*

$$\max \theta k - c(k) + \int_{s} \varphi(s)\, f(s \mid k)\, ds.$$

(ii) The market's beliefs are consistent with the equilibrium investment schedule of the firm; that is,

$$g(\theta \mid s) = \frac{f(s \mid k(\theta))\, h(\theta)}{\int_{\Theta} f(s \mid k(t))\, h(t)\, dt}. \tag{4}$$

(iii) $\varphi(s)$ is sequentially rational; that is,

$$\varphi(s) = \int_{\Theta} v(k(\theta), \theta)\, g(\theta \mid s)\, d\theta. \tag{5}$$

This definition describes a noisy signaling equilibrium in the sense of Matthews and Mirman [1983]. The firm's investment affects the *distribution* of a signal, which is then priced in the market in accordance with the rational, but noisy, inferences made by the market. Unlike the perfect measurement case, (5) indicates that the equilibrium price in the market incorporates a pooling of types. However, unlike traditional notions of pooling where the weight on each type is defined by the prior distribution $h(\theta)$, here the weights are *equilibrium* weights that depend on (1) the equilibrium investment schedule, (2) the accounting measurement system, and (3) the prior distribution of types. In a fully revealing signaling equilibrium, the prior distribution on types is irrelevant. Here, the prior distribution affects the firm's investment through its effect on equilibrium capital market prices.

In general, in a noisy signaling environment the equilibrium investment schedule is characterized by an integral equation of the form described in Proposition 3.

PROPOSITION 3. *In a setting where the firm's manager privately observes θ before choosing an investment, and the investment is measured imprecisely (in accordance with the probability density function $f(s \mid k)$), any equilibrium investment schedule $k(\theta)$ must satisfy the integral equation,*

$$\int_S \left\{ \int_{\Theta} v(k(t), t) \frac{f(s \mid k(t))\, h(t)}{\int_{\Theta} f(s \mid k(\tau))\, h(\tau)\, d\tau}\, dt \right\} f_k(s \mid k(\theta))\, ds = c'(k(\theta)) - \theta, \tag{6}$$

and $k(\theta)$ must be increasing in θ.

The preceding equilibria studied in sections 2.2 and 2.3 are special cases of the more general equilibrium described in (6). Myopia occurs when the value of θ is publicly observed. Let θ^0 be the observed value of θ. Then, (interpreting $g(\cdot)$ as a probability), $\forall\, s$, $g(\theta \mid s) = 1$ if $\theta = \theta^0$, and $g(\theta \mid s) = 0$ if $\theta \neq \theta^0$. Given that all of the probability mass is on θ^0, (5) implies that $\varphi(s) = v(k(\theta^0), \theta^0)$, $\forall\, s$. Thus (6) becomes

$$\int_S v(k(\theta^0), \theta^0)\, f_k(s \mid k(\theta^0))\, ds = c'(k(\theta^0)) - \theta^0.$$

Because $\int_S f_k(s \mid k)\, ds = 0, \forall\, k$, the equation collapses to $c'(k(\theta^0)) - \theta^0 = 0$ or myopic investment. Perfect measurement is equivalent to $s \equiv k$. Let $K(\theta)$ be the equilibrium perfect-measurement investment schedule, and let k denote an observed level of investment. Then, the posterior density $g(\theta \mid k)$ is described by $g(\theta \mid k) = 1$ if $\theta = K^{-1}(k)$ and $g(\theta \mid k) = 0$ for all other values of θ. Then, (5) becomes

$$\varphi(k) = \int_\Theta v(K(\theta), \theta) g(\theta \mid k)\, d\theta = v(k, K^{-1}(k)).$$

In this case (6) is equivalent to

$$\frac{d}{dk}\{v(k, K^{-1}(k))\} = c'(K(\theta)) - \theta,$$

which is equivalent to (3).

It seems intuitive that imprecise accounting measurements of investment would have the property that on average the accounting measure is higher when the firm's investment is higher. In turn, this property is implied by the first-order stochastic dominance (FSD) condition that higher investment shifts the distribution of the accounting measure to the right. The corollary shows that the standard regularity condition that guarantees the preceding two properties of imprecise accounting measurements is sufficient to ensure that such measurements have value.

COROLLARY TO PROPOSITION 3. *If $f(s \mid k)$ satisfies the monotone likelihood ratio property (MLRP), any solution to the integral equation (6) must have the property that at each $\theta > 0$, the firm's equilibrium investment is greater than the myopic amount.*

Although the corollary shows that noisy measurements of investment alleviate myopia, it is not a priori obvious whether such measurements yield an improvement over perfect measurements of investment. The answer must depend on the extent of imprecision in the accounting measure.[3] To study the effect of imprecision on the firm's equilibrium investment, we need to study how the solution to the integral equation (6) changes in response to variations in the precision of the accounting system $f(s \mid k)$. We model imprecision of the accounting measure two ways. In the first model (imprecision as normally distributed noise), the accounting measure is the firm's true investment perturbed by a normally distributed error term. The greater the variance of the noise term, the more imprecise is the accounting measure. In the second model (imprecision as a mixture of distributions), the accounting system *perfectly* measures the firm's investment with probability $(1 - \epsilon)$ and provides an uninformative signal with probability ϵ. In this

[3] As discussed in footnote 2, when $v_\theta \equiv 0$ the signal s is used solely to assess a posterior distribution on investment. Therefore, noisy measurements of investment would only decrease the sensitivity of the price φ to s, which in turn would decrease the firm's incentive to invest. In this case, noisy measurements are necessarily inferior to perfect measurement.

latter model, the parameter ϵ can be thought of as the degree of imprecision with higher values of ϵ representing greater imprecision. The second model requires no additional specification of the prior distribution of the profitability parameter θ or of the valuation $v(k, \theta)$, whereas the first model does. In both models, we fully characterize the equilibrium as a function of the degree of imprecision and establish that a small amount of imprecision yields a strict improvement over perfect measurement of the firm's investment.

4. *Optimality of Imprecision with Normally Distributed Noise*

Here we assume:

A1) $\tilde{s} = k + \tilde{\epsilon}$, $\tilde{\epsilon}$ is distributed normally with $E(\tilde{\epsilon}) = 0$, $var(\tilde{\epsilon}) = \sigma_\epsilon^2$.

For tractability reasons we specialize the exogenous features of the environment so that linear investment schedules can be supported as equilibria. Specifically, we make the following additional assumptions:

A2) The prior distribution of $\tilde{\theta}$ is normal with $E(\tilde{\theta}) = \mu$, $var(\tilde{\theta}) = \sigma_\theta^2$,

A3) $v(k, \theta) = \gamma\theta k + m\theta^2$, where $\gamma > 0$ and $m \geq 0$ are known constants, and

A4) $c(k) = \frac{1}{2}ck^2$.

Assumption A1 requires accounting measurement rules to be unbiased and errors to be normally distributed. Larger values of σ_ϵ^2 correspond to less precise accounting measurement rules. Assumption A2 implies that optimal investments could become negative when the profitability parameter θ is sufficiently negative. We allow such negative investments to avoid truncating the distribution of θ, though the interpretation could be problematic.[4] Varying the parameter σ_θ^2 allows us to make the prior information about θ more or less precise, and increases in μ make prior beliefs more optimistic. Assumption A3 says that in a complete information economy, where θ and k are directly observed, the equilibrium valuation rule in the capital market has two components. The first component, $\gamma\theta k$, represents the persistence in expected returns from the firm's current investment, where the parameter γ could be interpreted as an earnings multiple or as the number of years of useful life of the project or as a present value factor. The second component, $m\theta^2$, which does not depend on current investment, captures the effect of current profitability on the expected returns from anticipated *future* investment.[5]

[4] This issue is additionally discussed in footnote 6.

[5] We use the square of θ to reflect the assumption that negative investment is feasible thus making negative values of θ similar to positive values of θ.

4.1 CHARACTERIZATION OF NOISY SIGNALING EQUILIBRIA

Given assumption A3, the first-best investment schedule $k_{FB}(\theta)$ is $(\frac{1+\gamma}{c})\theta$ and the myopic investment schedule is $k_M(\theta) = \frac{\theta}{c}$. Using (3), the fully revealing investment schedule, which obtains when $\sigma_\epsilon^2 = 0$, must satisfy $k'(\theta)[ck(\theta) - (1+\gamma)\theta] = \gamma k(\theta) + 2m\theta$. This differential equation admits the linear solution,

$$k_{PM}(\theta) = \left[\frac{1 + 2\gamma + \sqrt{(1+2\gamma)^2 + 8mc}}{2c}\right]\theta. \tag{7}$$

Because all three of the investment schedules are linear in θ, we investigate the family of linear investment schedules as candidates for noisy signaling equilibria. Consider investment schedules of the form $k(\theta) = a + b\theta$, where a and b are endogenously determined. Given the linear investment schedule, the joint distribution of $(\tilde{s}, \tilde{\theta})$ and the conditional density $g(\theta \mid s)$ are also normal. This allows us to characterize the equilibrium investment schedule, in closed form, in Lemma 1.

LEMMA 1. *Linear investment schedules, $k(\theta) = a + b\theta$, that are sustainable as equilibria are those that satisfy*

$$b \in \mathcal{B} \equiv \left\{ b \in R^+ \,\middle|\, \frac{1}{c} \le b \le \frac{1 + 2\gamma + \sqrt{(1+2\gamma)^2 + 8mc}}{2c} \right\}, \tag{8}$$

with the intercept a given by

$$a = \frac{2(b\gamma + m)\beta(1-\beta)\mu}{1 - \beta\left[\gamma - 2(b\gamma + m)\dfrac{\beta}{b}\right]}, \tag{9}$$

where β satisfies $\beta^2 = \frac{(bc-1)b}{2(b\gamma+m)}$. The noise σ_ϵ^2 that is needed to sustain any $b \in \mathcal{B}$ is characterized by

$$\sigma_\epsilon^2 = b^2\sigma_\theta^2\left[\sqrt{\frac{2(b\gamma + m)}{(bc-1)b}} - 1\right]. \tag{10}$$

To construct a sustainable linear investment schedule, first choose a sustainable value of b (as specified in (8)), then solve for β from the expression specified in the lemma, then solve (9) for the value of a that is implied by these values of b and β, and then determine σ_ϵ^2 from (10). The procedure described earlier for construction of a sustainable investment schedule yields unique values for a, β, and σ_ϵ^2 as functions of b. Unfortunately, the relationship between b and σ_ϵ^2 is not one to one. For values of σ_ϵ^2 that correspond to multiple values of b, there are multiple equilibria.

4.2 COMPLEMENTARITY OF IMPRECISION AND IGNORANCE

We characterize the linear investment schedules that can be supported as equilibria for every degree of imprecision in accounting measurements.

Is there an *optimal* extent of imprecision? How much of an improvement do imprecise accounting measurements provide relative to the perfect measurement equilibrium? How does the optimal degree of imprecision in accounting measurements depend on the initial degree of information asymmetry between the firm's manager and the capital market? In this section we study these issues and provide some surprising answers.

We study the optimality of accounting imprecision from an ex ante perspective; that is, we assume that accounting measurement rules are chosen before the manager has observed the firm's profitability θ. An optimal measurement rule is one that maximizes the expected payoff to the firm's current shareholders, $E_\theta[\theta k(\theta) - \frac{ck^2(\theta)}{2} + E_s(\varphi(s)\,|\,\theta)]$. The last term can be simplified by using the law of iterative expectations:

$$\begin{aligned} E_\theta[E_s(\varphi(s)\,|\,\theta)] &= E_s(\varphi(s)) \\ &= E_s[E_\theta\{v(k(\theta),\theta)\,|\,s\}] \\ &= E_\theta[v(k(\theta),\theta)]. \end{aligned} \tag{11}$$

An accounting policy maker choosing among alternative measurement rules should be concerned with how these measurement rules affect the *expected* price in the capital market rather than the price response to specific realizations of s and θ. The derivation in (11) establishes that measurement rules affect the equilibrium expected price *only* through its effect on the equilibrium investment schedule of the firm. Because the first-best investment schedule maximizes $\theta k(\theta) - \frac{ck^2(\theta)}{2} + v(k(\theta),\theta)$ at each θ, it follows from the preceding analysis that if there is an accounting measurement rule that sustains the first-best investment schedule in equilibrium, that measurement rule is optimal even when perfect measurement is an option. The next proposition characterizes such measurement rules given assumptions A1 through A4.

PROPOSITION 4. *The first-best investment schedule is sustainable if and only if* $\mu \equiv E(\theta) = 0$. *In this case, the optimal degree of imprecision in accounting measurement is characterized by*

$$\sigma_\epsilon^2 = \sigma_\theta^2\left(\frac{1+\gamma}{c}\right)^2\left[\sqrt{2\left(1+\frac{mc}{\gamma(1+\gamma)}\right)} - 1\right]. \tag{12}$$

The intuition underlying Proposition 4 is as follows. The firm's incentive to invest depends on the sensitivity of its capital market price $\varphi(s)$ to the accounting measure. The more rapidly $\varphi(s)$ increases, the more the firm would want to invest at any θ. In the proof of Proposition 4 we show that the equilibrium market price has the quadratic form $\varphi(s) = \alpha_0 + \alpha_1 s + \alpha_2 s^2$. Thus, the sensitivity of the price to the accounting measure is described by $\varphi'(s) = \alpha_1 + 2\alpha_2 s$. The component α_1 that is independent of the accounting measurement provides a common incentive for investment, and therefore the value of α_1 determines the firm's choice of a in the linear investment schedule $k(\theta) = a + b\theta$. In turn, the sensitivity of the market price to the accounting measurement depends on the assessed posterior density $g(\theta \mid s)$. Holding the imprecision σ_ϵ^2 fixed, this posterior density depends entirely on the market's conjecture, $\hat{a} + \hat{b}\theta$, of the firm's investment schedule. As shown in the appendix (see (A9)) higher values of $\hat{a}$ shift the density to the left, decreasing $\varphi'(s)$ at every s, thereby decreasing the firm's incentive to invest. Now, consider the case $\mu > 0$. Suppose the market's conjecture has $\hat{a} = 0$, and the price schedule $\varphi(s)$ is based on this conjecture. The value of the coefficient α_1 embedded in this price schedule is, $\alpha_1 = \frac{2(b\gamma + m)}{b}\beta(1-\beta)\mu > 0$, as calculated from (A14). The firm would respond to such a pricing rule by choosing $a > 0$, as derived in (A16), disconfirming the market's conjecture. This implies that the posterior distribution $g(\theta \mid s)$ that is assessed by the market lies to the right of the equilibrium distribution and the conjecture $\hat{a} = 0$ cannot be sustained. This is why the first-best investment schedule cannot be sustained when $\mu > 0$.

Because the firm's manager observes θ before choosing the firm's investment, whereas the capital market does not, the parameter σ_θ^2 can be interpreted as the degree of information asymmetry between the manager and the market (or the extent of the market's ignorance) regarding the profitability of the firm's investment. It might appear that the greater this information asymmetry, the more precision one would like in the accounting measurement (if feasible). However, Proposition 4 implies the surprising result that the opposite is true.[6] We formalize this result in the following corollary.

COROLLARY TO PROPOSITION 4. *When first-best investment is sustainable, the greater is the information asymmetry between the firm's manager and the capital market regarding the profitability of the firm's investment, the lower should be the precision with which the firm's investment is measured.*

The intuition for this result is as follows. The market uses the accounting signal to update its beliefs about the firm's profitability. If the prior information is precise (i.e., σ_θ^2 is small) the market's revision of beliefs is not very sensitive to the accounting signal. This induces the firm to invest myopically. An increase in the precision of the accounting measurement (i.e., reducing σ_ϵ^2) increases the sensitivity of the market price to the accounting signal, thus moving the firm away from myopia. Conversely, when the market's prior information about project profitability is imprecise, the weight assigned to the accounting signal is large, making the market price sensitive to the accounting signal, which in turn induces the firm to overinvest. In

[6] Equation (12) depends on the assumption that μ, the prior mean of θ, is zero. This has the unfortunate implication that the firm's investment is negative with probability one-half. This concern is mitigated by our result in Proposition 5, where we establish that some degree of imprecision is optimal regardless of the value of μ. Additionally, we show through numerical analysis that the relationship between σ_ϵ^2 and σ_θ^2 derived in equation (12) continues to hold for values of μ large enough such that the probability of negative investment is negligible.

this case, it is desirable to decrease the precision of the accounting signal, thus inducing the firm to reduce its investment.

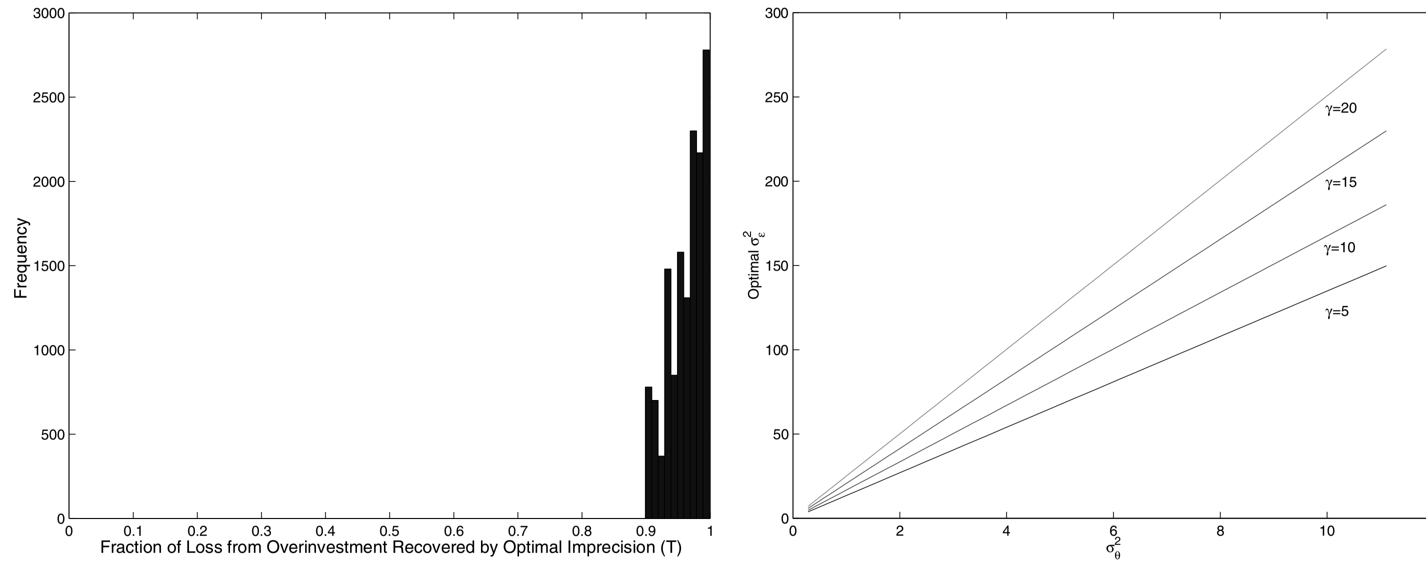
The preceding result seems to be qualitatively consistent with current accounting practice. A firm's financial statements, under generally accepted accounting principles (GAAP), convey more precise information about the firm's investment in property, plant and equipment than its investment in R&D. Even though a firm's total expenditures on R&D are reported, no attempt is made to distinguish between productive R&D investment and worthless R&D expenses, resulting in imprecise estimates by individual investors attempting to disentangle these two components. At the same time, it is likely that there is more information asymmetry between the market and the firm's managers regarding the future returns, or market value of productive R&D, than of property, plant and equipment.

The result in (12) also has implications for a firm's disclosure policy regarding the profitability of its investment. Suppose the imprecision in reporting the firm's investment is beyond the manager's control, and the manager has the opportunity to commit ex ante to a reporting policy that credibly reveals information about the profitability parameter θ. Our results indicate that the manager should not commit to fully reveal his or her information. If σ_θ^2 is interpreted as the *posterior* variance of the market's assessed distribution of θ conditional on the information released by the manager, (12) indicates that the more noise there is in measurement of investment, the less information the manager should reveal about the profitability of investment. For any amount of measurement imprecision, there is a unique, optimal level of ignorance about profitability.

It is difficult to characterize the optimal imprecision when the first-best investment schedule cannot be sustained (i.e., when $\mu \neq 0$). However, we show that some degree of imprecision in accounting measurement is still desirable.

PROPOSITION 5. *When the firm's manager has an information advantage over the capital market regarding the profitability of investment, some degree of imprecision in accounting measurements of investment is optimal, irrespective of the value of* μ.

Proposition 5 establishes that *some* degree of imprecision is always value enhancing, but it does not characterize the optimal level of imprecision. To obtain insights into the relationship between the optimal level of imprecision and our exogenous variables, we analyze the problem numerically and illustrate the extent of improvement over perfect measurement. The results are surprising. We solve for the optimal level of imprecision for 14,320 combinations of the parameters, μ, σ_θ^2, γ, m, and c and summarize the results in figure 1. On average, with perfect measurement of investment only 16% of the first-best expected payoff is attained. The optimal level of imprecision recoups between 90.1% and 99.9% (with an average of 96.2% and a median of 97.04%) of the efficiency loss associated with perfect measurement. The relationship between the optimal level of imprecision and the



(a) Improvement in Payoffs

(b) Optimal Measurement Imprecision

FIG. 1.—**Numerical analysis of the benefits from measurement.** Here we numerically solve for the optimal imprecision for the following parameter values: $\mu \in \{2, 4, \ldots, 18, 20\}$, $\sigma_\theta = \{\frac{1}{18}\mu, \frac{2}{18}\mu, \ldots, \frac{6}{18}\mu\}$, $\gamma \in \{2, 4, \ldots, 18, 20\}$, $m \in \{0, 2, \ldots, 10\}$, and $c \in \{1, 2, \ldots, 5\}$. There are 15,000 combinations of parameters. For each combination we calculated the optimal value of b, and the corresponding value of a and σ_ϵ^2 using the Nelder-Mead type simplex search method to maximize the firm's expected payoff. Of these 15,000 the algorithm did not converge for 680 combinations and was consequently dropped. Given the optimal values of b, we calculated the following statistic: $T = \frac{U(b^*) - U(b_{PM})}{U_{FB} - U(b_{PM})}$, where $U(b^*)$ is the firm's expected payoff (40) calculated under the optimal level of imprecision, $U(b_{PM})$ is the expected payoff under perfect measurement, and U_{FB} is the first best expected payoff. The statistic $T \in [0, 1]$ measures the fraction of the loss in expected payoff due to perfect measurement that is recovered by optimal imprecise measurement. $T = 0$ indicates that perfect measurement is optimal and $T = 1$ indicates that b^* implements the first best. We plot the histogram for T in panel 1a. In panel 1b, we illustrate the effect of an increase in the level of information asymmetry regarding the profitability parameter θ on the optimal imprecision of the accounting measure. The parameter values used are $c = 2$, $\mu = 10$, and $m = 10$. Consistent with Proposition 4, the optimal imprecision increases linearly in the level of information asymmetry. Panel 1b also indicates a positive relationship between the optimal imprecision and the capitalization factor γ.

information asymmetry between the manager and the market, characterized in Proposition 4, continues to hold even when $\mu > 0$; the higher the level of information asymmetry, the higher is the optimal level of imprecision.

5. *Optimality of Imprecision with a Mixture of Two Distributions*

In section 4, we assume normally distributed noise and a specific form for the valuation rule $v(k, \theta)$. To demonstrate the desirability of imprecise measurements more generally, we analyze here a different form of noise without imposing specific valuation rules and distributions.

Assume now that when the firm's true investment is k the accounting measurement provides a signal s that equals k with probability $(1 - \epsilon)$ and a random drawing from an uninformative distribution $l(s)$ with probability ϵ. We assume that the market does not know a priori whether it observes the true investment or a drawing from $l(s)$. Thus, ϵ can be interpreted as accounting imprecision—the higher the value of ϵ, the less precise is the accounting measurement of investment.

Conjecture an equilibrium with a strictly increasing investment schedule $k(\theta)$ and let $I(k)$ be its inverse. Given an observation s, the market assesses

$$Prob[k = s] = \frac{(1 - \epsilon)h(I(s))}{(1 - \epsilon)h(I(s)) + \epsilon l(s)}. \tag{13}$$

In the (13), $h(I(s))$ is the probability density of the accounting signal when investment is measured perfectly because $I(s)$ is the value of θ that would produce an investment of s and $h(\cdot)$ is the prior probability density of θ. We make the further assumption that the uninformative signal is a random drawing from the *equilibrium* ex ante distribution of the firm's true investment, that is, $l(s) = h(I(s))$ for $s \in [k(\underline{\theta}), k(\bar{\theta})]$. This assumption implies that realizations of the accounting signal s provide no information about which of the two densities $h(I(s))$ or $l(s)$ produced that signal. Inserting $l(s) = h(I(s))$ into (13) yields $Prob[k = s] = 1 - \epsilon, \forall s$. Hence, the market price $\varphi(s)$ is

$$\varphi(s) = (1 - \epsilon)v(s, I(s)) + \epsilon\bar{v}, \tag{14}$$

where $\bar{v} \equiv \int v(k(\theta), \theta)h(\theta)\, d\theta$ is the value of the firm given an uninformative signal and the prior conjecture that the firm's equilibrium schedule is $k(\theta)$. The firm chooses its investment k with the knowledge that the accounting signal will report $s = k$ with probability $(1 - \epsilon)$, and with probability ϵ the accounting signal will be a drawing from the uninformative density $h(I(s))$. Thus type θ's investment is a solution to the maximization problem:

$$\max_k -c(k) + \theta k + (1 - \epsilon)\varphi(k) + \epsilon \int \varphi(s)h(I(s))\, ds. \tag{15}$$

Using (14), $\int \varphi(s)h(I(s))\, ds = \int [(1-\epsilon)v(s, I(s)) + \epsilon\bar{v}]h(I(s))\, ds = \bar{v}$. Inserting this into (15) yields the equivalent objective function,

$$\max_k -c(k) + \theta k + (1-\epsilon)\varphi(k) + \epsilon\bar{v}. \tag{16}$$

The properties of the equilibrium investment schedule are described by the following proposition.

PROPOSITION 6. *In a setting where the firm's manager privately observes θ before choosing investment and the accounting signal s equals its chosen investment with probability $(1-\epsilon)$ and is a random drawing from an uninformative distribution with probability ϵ, the equilibrium investment schedule $k(\theta)$ is characterized by,*

(i) $k(\theta)$ is increasing in θ and

$$k'(\theta) = \frac{(1-\epsilon)^2 v_\theta(k(\theta), \theta)}{c'(k(\theta)) - \theta - (1-\epsilon)^2 v_k(k(\theta), \theta)}, \tag{17}$$

(ii) $k(\underline{\theta})$ satisfies

$$-c'(k) + \underline{\theta} + (1-\varepsilon)v_k(k, \underline{\theta}) = 0. \tag{18}$$

For every $\epsilon > 0$, the equilibrium investment schedule $k(\theta)$ is strictly below the fully revealing equilibrium schedule $k_{PM}(\theta)$, which would prevail in the absence of any measurement imprecision. Moreover, an increase in ϵ reduces the equilibrium investment and decreases the slope of the investment schedule at every θ.

The equilibrium constructed here is similar to the fully revealing equilibrium that prevails when accounting measurements are perfect. The main difference in the current setting is that the firm affects its market value through its investment choice only with probability $(1-\epsilon)$. This is what decreases the firm's incentives to overinvest as in the perfect measurement case. Is such a weakening of incentives value enhancing?

In the case where the noise in accounting measurements consists of normally distributed errors, we establish that the firm is strictly better off, in an ex ante sense, relative to perfect measurement if the noise is sufficiently small. In the following proposition, we establish a similar result in the current setting.

PROPOSITION 7. *In a setting where the firm's manager privately observes θ before choosing investment and the accounting signal s equals its chosen investment with probability $(1-\epsilon)$ and is a random drawing from an uninformative distribution with probability ϵ, the firm's ex ante expected payoff is strictly higher relative to perfect measurement if ϵ is small enough.*

The intuition for this result is illustrated in figure 2. Increases in ϵ not only reduce the firm's incentives for investment at every θ but also flatten the entire investment schedule relative to perfect measurement. With perfect measurement, the equilibrium investment schedule is close to first best at low values of θ but deviates significantly from first best at high values of θ.

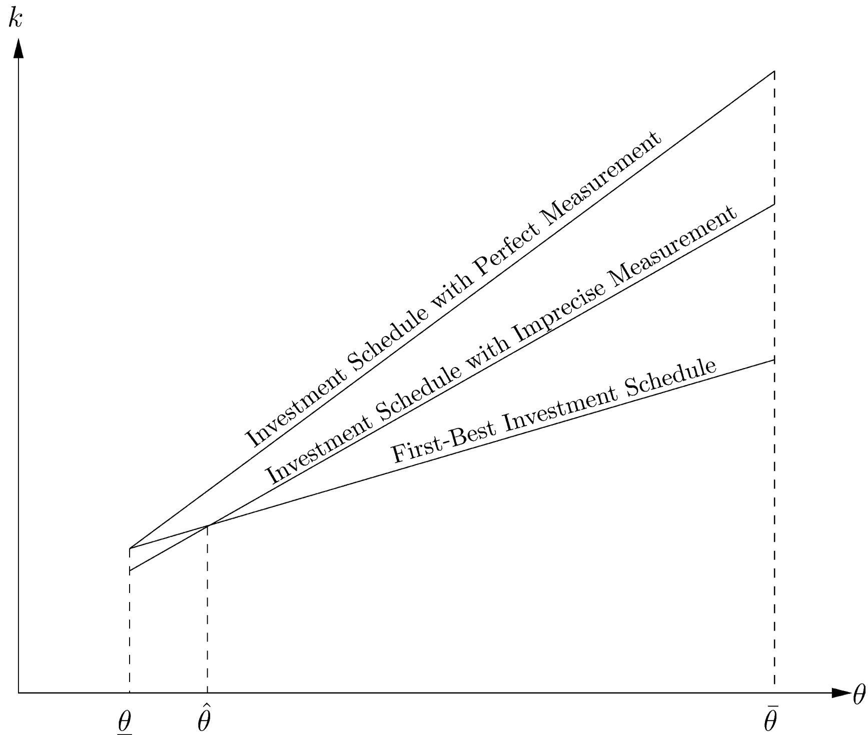


FIG. 2.—Effect of imprecision on the equilibrium investment schedule when the accounting signal is drawn from a mixture of two distributions.

Thus, a downward shift in the investment schedule results in a gain in the firm's expected payoff for high values of θ (i.e., $\theta > \hat{\theta}$ in figure 2) and a loss at low values of θ (i.e., $\theta < \hat{\theta}$). However, as seen in figure 2, the gain at higher values of θ is big and the losses at low values of θ are relatively small. For ϵ sufficiently close to zero, the gain more than offsets the loss.

6. *Conclusion*

Our results contradict the conventional wisdom that imprecision in accounting measurement should be eliminated to the extent possible. We study a plausible market setting where (1) firms' managers have information superior to the market regarding the environment in which managerial decisions are made, and (2) firms' managers are concerned about how their decisions are priced in the capital market. We show that in such settings precise accounting measurements actually destroy value and reduce shareholder wealth. Some degree of imprecision in accounting measurements induces more efficient equilibria. There is an optimal degree of imprecision that is strictly increasing in the information advantage that the manager has over the capital market regarding the project's profitability.

Conversely, given imprecision in accounting measurement, it is desirable that managers retain some information superiority over the capital market regarding the firm's profitability. In this sense, ignorance supports imprecision and imprecision supports ignorance. An appropriate mix of ignorance and imprecision produces outcomes that are reasonably close to first best.

Our findings should be tempered by some of our assumptions that may not hold in real-world settings. We assume that capital market participants have no opportunity to augment accounting information through private information search. If such search opportunities exist but are unequal across individuals, imprecision in accounting measurements would allow the privileged few to gain an informational advantage over the average investor. This may be socially undesirable. We additionally assume that the support of the accounting signal is independent of the true investment of the firm. Perhaps, real-world measurements exhibit moving support. We do not investigate such moving support cases because they give rise to difficult issues concerning off-equilibrium beliefs. Finally, we assume aggregate risk neutrality for the capital market. If, instead, there is aggregate risk aversion in the capital market, imprecision in accounting reports would increase the risk premium in the equilibrium capital market price, decreasing the benefits to imprecision. Investigation of these issues would enrich the understanding of the costs and benefits of imprecision in accounting measurements.

APPENDIX

Proof of Proposition 1. First, we establish that the equilibrium market price cannot depend on the accounting measure $\tilde{s}$. Any assumption that it does leads to a contradiction. Consider any pricing rule $\varphi(s, \theta)$, that depends nontrivially on s. Given any such pricing rule, the firm's objective function specified in (2) implies some investment policy that is a function only of θ, say $\hat{k}(\theta)$. Now, as required by condition (ii) of the equilibrium, rationality of beliefs implies that $\varphi(s, \theta)$ must satisfy

$$\varphi(s, \theta) = E[v(\hat{k}(\theta), \theta) \mid s]. \tag{A1}$$

Because the capital market understands the structure of the firm's problem—that is, the market knows that the firm chooses investment to maximize (2)—the market can calculate the firm's investment policy as a function of θ. Given that the market additionally knows the parameter θ, the market believes it knows exactly how much the firm has invested, even though this investment is not directly observed by the market. Thus, the conditional expectation in (A1) is vacuous, implying that

$$\varphi(s, \theta) = v(\hat{k}(\theta), \theta), \forall\, s. \tag{A2}$$

This implies that the equilibrium price in the market does not depend on s and is described by some function of θ alone, say $\hat{\varphi}(\theta)$ that incorporates the anticipated investment $\hat{k}(\theta)$.

Given that the market prices the firm in this manner, the firm's objective function described in (2) becomes

$$\max_{k} \theta k - c(k) + \hat{\varphi}(\theta),$$

which implies the myopic investment characterized by the first order condition $c'(k) = \theta$. This leads to the equilibrium described in Proposition 1.

QED.

Proof of Proposition 2. We use the mechanism-design methodology to characterize investment schedules that are consistent with fully revealing signaling equilibria.[7] If $k(\theta)$ is an equilibrium investment schedule, it must be that for any two types θ and $\hat{\theta}$, type θ prefers $k(\theta)$ to $k(\hat{\theta})$ and type $\hat{\theta}$ prefers $k(\hat{\theta})$ to $k(\theta)$. If additionally, $k(\cdot)$ is a *fully revealing* equilibrium investment schedule, it must satisfy the following incentive compatibility conditions:

$$\theta k(\theta) - c(k(\theta)) + v(k(\theta), \theta) \geq \theta k(\hat{\theta}) - c(k(\hat{\theta})) + v(k(\hat{\theta}), \hat{\theta}), \forall \theta, \hat{\theta}. \quad \text{(A3)}$$

Conditions (ii) and (iii) of equilibrium are embedded in (A3). Denote the left-hand side of (A3) by $\Omega(\theta)$ so that the incentive compatibility conditions can be expressed as

$$\Omega(\theta) \geq \Omega(\hat{\theta}) - k(\hat{\theta})[\hat{\theta} - \theta], \forall \theta, \hat{\theta}. \quad \text{(A4)}$$

Analysis of (A4), using techniques that are standard in the adverse selection literature yields the result: an investment schedule $k(\theta)$ is incentive compatible if and only if (i) $\Omega'(\theta) = k(\theta), \forall \theta$, and (ii) $k(\theta)$ is increasing.

These necessary and sufficient conditions for incentive compatibility are used to characterize fully revealing equilibrium investment schedules in the form of a differential equation. Let the interval $\Theta \equiv [\underline{\theta}, \bar{\theta}]$ be the support of the distribution of θ. Then, from (i), it follows that

$$\int_{\underline{\theta}}^{\theta} \Omega'(t)\, dt = \int_{\underline{\theta}}^{\theta} k(t)\, dt,$$

which implies that

$$\Omega(\theta) = \int_{\underline{\theta}}^{\theta} k(t)\, dt + \Omega(\underline{\theta}).$$

Using this with the definition of $\Omega(\cdot)$ implies that an equilibrium investment schedule must satisfy

$$c(k(\theta)) - \theta k(\theta) + \int_{\underline{\theta}}^{\theta} k(t)\, dt + \Omega(\underline{\theta}) = v(k(\theta), \theta). \quad \text{(A5)}$$

Equation (A5) should not be interpreted as a constraint on the market's pricing rule $v(\cdot)$, which is required to be sequentially rational and market

[7] The link between the Spence-Riley methodology of constructing signaling equilibria and the mechanism design approach used here is formalized in Kanodia and Lee [1998].

clearing, but rather as a condition on the equilibrium investment schedule. Differentiating (A5) with respect to θ yields $k'(\theta)[c'(k(\theta)) - \theta - v_k] = v_\theta$, as specified in Proposition 2. *QED.*

Proof of Proposition 3. As in the perfect measurement case, the equilibrium investment schedule is characterized by the mechanism-design approach. Given a pricing rule $\varphi(s)$, if $k(\theta)$ is an optimal investment schedule it must satisfy the incentive-compatibility conditions:

$$\theta k(\theta) - \frac{c}{2}k^2(\theta) + \int_s \varphi(s)\, f(s \mid k(\theta))\, ds$$
$$\geq \theta k(\hat{\theta}) - \frac{c}{2}k^2(\hat{\theta}) + \int_s \varphi(s)\, f(s \mid k(\hat{\theta}))\, ds \quad \forall \theta, \hat{\theta}. \tag{A6}$$

Denoting the left-hand side of (A6) by $\Lambda(\theta)$, the preceding inequalities are equivalent to

$$\Lambda(\theta) \geq \Lambda(\hat{\theta}) - k(\hat{\theta})(\hat{\theta} - \theta). \tag{A7}$$

Inequalities (A6) and (A7) are identical to (A3) and (A4) except that the pricing rule $v(k(\theta), \theta)$ is replaced by $\int_s \varphi(s)\, f(s \mid k(\theta))\, ds$. Hence, a result similar to Proposition 2 holds; that is, the investment schedule $k(\theta)$ satisfies (A7) if and only if

(i) $\Lambda'(\theta) = k(\theta), \forall \theta$ and
(ii) $k(\theta)$ is increasing.

Using the preceding results in exactly the same way as in Proposition 2, we find that the investment schedule must satisfy

$$\int_S \varphi(s)\, f(s \mid k(\theta))\, ds = \frac{ck^2(\theta)}{2} - \theta k(\theta) + \int_{\underline{\theta}}^{\theta} k(t)\, dt + \Lambda(\underline{\theta}).$$

Differentiating with respect to θ and canceling common terms yields the equivalent of the first-order condition to the firm's optimization program:

$$\int_S \varphi(s)\, f_k(s \mid k(\theta))\, ds = ck(\theta) - \theta. \tag{A8}$$

In equilibrium, the pricing rule in the capital market must be consistent with the investment schedule that is incentive compatible relative to that pricing rule. Inserting (4) and (5) into (A8) yields the desired result. *QED.*

Proof of Corollary to Proposition 3. For any given investment schedule $k(\theta)$ satisfying $k'(\theta) > 0$, let $n(s \mid \theta) \equiv f(s \mid k(\theta))$. Then,

$$\frac{n_\theta(s \mid \theta)}{n(s \mid \theta)} = \frac{f_k(s \mid k(\theta))k'(\theta)}{f(s \mid k(\theta))}.$$

Thus, $\frac{f_k(s \mid k)}{f(s \mid k)}$ strictly increasing in s (from MLRP) and $k'(\theta) > 0$ implies that $n(s \mid \theta)$ inherits MLRP. Milgrom [1981] establishes that if $n(s \mid \theta)$ satisfies MLRP, the induced posterior distribution on θ conditional on the signal s satisfies FSD for *every* nondegenerate prior distribution on θ. Thus, the *equilibrium* posterior density $g(\theta \mid s)$ satisfies FSD. Now, because $\varphi(s) = \int v(k(\theta), \theta) g(\theta \mid s)\, d\theta$ and v is strictly increasing in θ, $\varphi(s)$ is strictly increasing. In turn, this implies that $\int \varphi(s) f_k(s \mid k)\, ds = -\int \varphi'(s) F_k(s \mid k)\, ds > 0$. Using this fact together with the firm's first-order condition for a maximum implies that any solution to the integral equation (6) must have the property that at each $\theta > 0$, the firm's equilibrium investment is greater than the myopic amount. *QED.*

Proof of Lemma 1. Given the conjectured equilibrium investment schedule $k = a + b\theta$, the accounting measure $\tilde{s}$ is equivalent to $a + b\tilde{\theta} + \tilde{\epsilon}$. Hence, when $b \neq 0$, the joint distribution of $(\tilde{s}, \tilde{\theta})$ is normal, and the conditional density $g(\theta \mid s)$ is also normal, with parameters:

$$E(\tilde{\theta} \mid s) = (1 - \beta)\mu + \beta\left(\frac{s - a}{b}\right), \tag{A9}$$

$$\operatorname{var}(\tilde{\theta} \mid s) = (1 - \beta)\sigma_\theta^2, \tag{A10}$$

where $\beta \equiv \frac{b^2\sigma_\theta^2}{b^2\sigma_\theta^2 + \sigma_\epsilon^2}$. Now, a closed-form expression for the left-hand side of (6) is obtained as follows. Inserting $k(\theta) = a + b\theta$, and $v(k, \theta) = \gamma\theta k + m\theta^2$ into the expression $\varphi(s) = E[(v(k(\theta), \theta) \mid s]$ gives

$$\varphi(s) = [a\gamma]E(\theta \mid s) + [b\gamma + m]E(\theta^2 \mid s). \tag{A11}$$

Replacing $E(\theta \mid s)$ by (A9) and using $E(\theta^2 \mid s) = \operatorname{var}(\theta \mid s) + [E(\theta \mid s)]^2$, where $\operatorname{var}(\theta \mid s)$ is given by (A10), yields the following quadratic expression for $\varphi(s)$:

$$\varphi(s) = \alpha_0 + \alpha_1 s + \alpha_2 s^2, \tag{A12}$$

where

$$\begin{aligned}\alpha_0 = {} & a\gamma\left[(1 - \beta)\mu - \frac{\beta a}{b}\right] \\ & + (b\gamma + m)\left[(1 - \beta)\sigma_\theta^2 + \left\{(1 - \beta)\mu - \frac{\beta a}{b}\right\}^2\right],\end{aligned} \tag{A13}$$

$$\alpha_1 = \frac{\beta}{b}\left[a\gamma + 2(b\gamma + m)\left\{(1 - \beta)\mu - \frac{\beta a}{b}\right\}\right], \tag{A14}$$

$$\alpha_2 = \frac{\beta^2(b\gamma + m)}{b^2}. \tag{A15}$$

Using (A12) the left-hand side of (6) becomes

$$\int_S \varphi(s) f_k(s \mid k(\theta))\, ds = \alpha_1 \int_S s f_k(s \mid k(\theta))\, ds + \alpha_2 \int_S s^2 f_k(s \mid k(\theta))\, ds.$$

For the normal density, we have $f_k(s \mid k) = f(s \mid k)[\frac{s-k}{\sigma_\epsilon^2}]$. Therefore, for any k,

$$\begin{aligned}\int_S \varphi(s)\, f_k(s \mid k)\, ds &= \frac{\alpha_1}{\sigma_\epsilon^2}[E(s^2 \mid k) - kE(s \mid k)] + \frac{\alpha_2}{\sigma_\epsilon^2}[E(s^3 \mid k) - kE(s^2 \mid k)] \\ &= \frac{\alpha_1}{\sigma_\epsilon^2}\left[\sigma_\epsilon^2 + k^2 - k^2\right] + \frac{\alpha_2}{\sigma_\epsilon^2}\left[k^3 + 3k\sigma_\epsilon^2 - k^3 - k\sigma_\epsilon^2\right] \\ &= \alpha_1 + 2\alpha_2 k.\end{aligned}$$

The integral equation, described in (6), reduces to $\alpha_1 + 2\alpha_2 k = ck - \theta$. Thus, consistent with our conjecture, $k(\theta)$ has a linear form,

$$k(\theta) = \frac{\alpha_1}{c - 2\alpha_2} + \frac{1}{c - 2\alpha_2}\theta. \tag{A16}$$

The second-order condition for a maximum is satisfied if $c - 2\alpha_2 > 0$. Matching coefficients, sustainable conjectures must satisfy:

$$b = \frac{1}{c - 2\alpha_2}, \tag{A17}$$

$$a = \frac{\alpha_1}{c - 2\alpha_2} = b\alpha_1, \tag{A18}$$

where α_1 and α_2 are functions of a and b, as specified in (A14) and (A15). Equation (A17) indicates that the second-order condition $c - 2\alpha_2 > 0$ is equivalent to $b > 0$ (i.e., $k'(\theta) > 0$, as required by incentive compatibility). This requirement that $b > 0$ is satisfied for each $b \in \mathcal{B}$. For every sustainable value of b, the values of β, a, and σ_ϵ^2 can be expressed as functions of b. Solving for β from (A15) and (A17) yields

$$\beta^2 = \frac{(bc - 1)b}{2(b\gamma + m)}. \tag{A19}$$

Similarly solving for a from (A14) and (A18) yields

$$a = \frac{2(b\gamma + m)\beta(1 - \beta)\mu}{1 - \beta\left[\gamma - 2(b\gamma + m)\dfrac{\beta}{b}\right]}. \tag{A20}$$

Now, substituting $\beta = \frac{b^2\sigma_\theta^2}{b^2\sigma_\theta^2 + \sigma_\epsilon^2}$ into equation (A19) and solving for σ_ϵ^2 yields

$$\sigma_\epsilon^2 = b^2\sigma_\theta^2\left[\sqrt{\frac{2(b\gamma + m)}{(bc - 1)b}} - 1\right]. \tag{A21}$$

The sustainable values of b are determined from the requirement that $\sigma_\epsilon^2 \geq 0$. As $b \to \frac{1}{c}$, $\beta \to 0$, $\sigma_\epsilon^2 \to \infty$, and $a \to 0$. This corresponds to the myopic investment schedule. As $b \to b_{PM} \equiv \frac{1 + 2\gamma + \sqrt{(1 + 2\gamma)^2 + 8mc}}{2c}$, it can be verified that $2(b\gamma + m) \to (bc - 1)b$, implying that σ_ϵ^2 as characterized in

(A21) converges to zero. This corresponds to perfect measurement, yielding the fully revealing signaling equilibrium characterized in (7). Given $b > 0$ and the requirement that $\sigma_\epsilon^2 \geq 0$, (A21) implies that $bc - 1 > 0$, yielding the lower bound on b. Using the non-negativity of σ_ϵ^2 and (A21) implies that sustainable values of b must satisfy

$$\sqrt{\frac{2(b\gamma + m)}{(bc-1)b}} - 1 \geq 0,$$

which is equivalent to

$$N(b) \equiv b^2 c - (1+2\gamma)b - 2m \leq 0.$$

At $b = \frac{1}{c}$, $N(b) = -\frac{2(\gamma + mc)}{c} < 0$. Because $N'(b) = 2bc - (1+2\gamma)$, $N(b)$ is strictly decreasing at every $b < \frac{1+2\gamma}{2c}$ and strictly increasing at every $b > \frac{1+2\gamma}{2c}$. Therefore, $N(b) < 0$ over the interval $[\frac{1}{c}, \frac{1+2\gamma}{2c}]$. Now, $N(b_{PM}) = 0$ and $b_{PM} > \frac{1+2\gamma}{2c}$. Therefore $N(b) > 0$, $\forall\, b > b_{PM}$, implying that any $b > b_{PM}$ cannot be sustained by any feasible choice of σ_ϵ^2. This completes the proof. *QED.*

Proof of Proposition 4. The first-best investment schedule is $k_{FB}(\theta) = a_{FB} + b_{FB}\theta$, where $a_{FB} = 0$, $b_{FB} = (\frac{1+\gamma}{c})$. The linear investment schedules $k(\theta) = a + b\theta$ that can be sustained in equilibrium are characterized in Lemma 1. From (A20) it is clear that $a = a_{FB} = 0$ if and only if $\mu = 0$ or $\beta = 0$ or $\beta = 1$. But as shown earlier, $\beta = 0$ results in myopic investment and $\beta = 1$ results in the fully revealing investment schedule. Therefore, $\mu = 0$ is necessary to sustain the first-best investment schedule. With $\mu = 0$, $a = 0$ is the only self-fulfilling conjecture by the market regardless of the value of σ_ϵ^2. Therefore, σ_ϵ^2 can be chosen solely to optimize the slope b of the investment schedule. Equation (A21) characterizes the value of σ_ϵ^2 that sustains feasible values of b. Inserting $b = b_{FB}$ in (A21) and solving for σ_ϵ^2 gives the desired result. *QED.*

Proof of Proposition 5. The problem of finding the optimal precision of accounting measurement is equivalent to searching over all sustainable (linear) investment schedules of the form $k(\theta) = a + b\theta$ to maximize the firm's expected payoff. Having found the optimal pair $\{a, b\}$ from the sustainable set, one can calculate the corresponding value of σ_ϵ^2 from (A21). In Lemma 1, we characterize the sustainable set of $\{a, b\}$ pairs as those that satisfy $b \in \mathcal{B}$ and $a = a(b)$, where $a(b)$ is defined by (A19) and (A20). Therefore, the optimal sustainable investment schedule, $a^* + b^*\theta$, is characterized by

$$\begin{aligned} b^* &\in \arg\max_{b\in\mathcal{B}} U(b) \\ &\equiv E_\theta\left[\theta(a(b) + b\theta) - \frac{c}{2}(a(b)+b\theta)^2 + \gamma\theta(a(b)+b\theta) + m\theta^2\right]. \end{aligned} \tag{A22}$$

Evaluating the expectation with respect to θ and collecting terms yields

$$U(b) = \left[(1+\gamma)b - \frac{c}{2}b^2 + m\right]\left(\sigma_\theta^2 + \mu^2\right) + (1+\gamma-bc)a\mu - \frac{c}{2}a^2. \quad \text{(A23)}$$

Differentiating with respect to b,

$$U'(b) = (1+\gamma-bc)\left[\sigma_\theta^2 + \mu^2 + \mu\frac{\partial a}{\partial b}\right] - ca\left(\mu + \frac{\partial a}{\partial b}\right). \quad \text{(A24)}$$

We show that as $b \to b_{PM}$, $\lim\ U'(b)$ is negative. Now, as $b \longrightarrow b_{PM}$, $a \longrightarrow 0$ and $\beta \longrightarrow 1$. Therefore,

$$\lim U'(b) = (1+\gamma-b_{PM}c)\left[\sigma_\theta^2 + \mu^2 + \mu \lim\left\{\frac{\partial a}{\partial b}\right\}\right]. \quad \text{(A25)}$$

To investigate $\frac{\partial a}{\partial b}$, from (A20),

$$a = I(b, \beta(b)) \equiv \frac{2(m+\gamma b)\beta\mu(1-\beta)}{1-\left(\gamma - 2(m+\gamma b)\frac{\beta}{b}\right)\beta},$$

where $\beta(b)$ is defined by,

$$\beta^2 = \frac{b^2c - b}{2(b\gamma + m)}. \quad \text{(A26)}$$

Then, $\frac{\partial a}{\partial b} = \frac{\partial I}{\partial b} + \frac{\partial I}{\partial \beta}\frac{\partial \beta}{\partial b}$. Now, $\frac{\partial I}{\partial b} = 0$ when evaluated at $\beta = 1$, and

$$\frac{\partial I}{\partial \beta} = \frac{-2(m+\gamma b_{PM})\mu b_{PM}}{b_{PM}(1+\gamma) + 2m}.$$

Totally differentiating (A26) yields

$$2\beta\frac{\partial \beta}{\partial b} = \frac{2(m+\gamma b)(2bc-1) - 2\gamma(bc-1)b}{4(m+\gamma b)^2}.$$

Simplifying and evaluating at $\beta = 1$, $b = b_{PM}$ gives

$$\frac{\partial \beta}{\partial b} = \frac{(2b_{PM}c - 1)m + b_{PM}^2 c\gamma}{4(m+\gamma b_{PM})^2}.$$

Therefore,

$$\lim\frac{\partial a}{\partial b} = \frac{-\mu b_{PM}}{b_{PM}(1+\gamma) + 2m}\,\frac{(2b_{PM}c - 1)m + b_{PM}^2 c\gamma}{2(m+\gamma b_{PM})}.$$

Inserting this expression in (A25) gives

$$\lim U'(b) = (1+\gamma-b_{PM}c)\left[\sigma_\theta^2 + \mu^2\left\{1 - \frac{b_{PM}}{b_{PM}(1+\gamma)+2m}\right.\right.$$
$$\left.\left.\times\frac{(2b_{PM}c - 1)m + b_{PM}^2 c\gamma}{2(m+\gamma b_{PM})}\right\}\right].$$

Because $1+\gamma-b_{PM}c<0$, a sufficient condition for $\lim U'(b)<0$ is

$$2(b_{PM}(1+\gamma)+2m)(m+\gamma b_{PM})-b_{PM}\left(2b_{PM}cm-m+b_{PM}^2c\gamma\right)>0.$$

Inserting $b_{PM}=\frac{1+2\gamma+\sqrt{(1+2\gamma)^2+8mc}}{2c}$, the preceding inequality reduces (after considerable simplification) to

$$\frac{1}{2c^2}\Big(\sqrt{(1+4\gamma+4\gamma^2+8cm)}(\gamma+2\gamma^2+cm)$$
$$+\gamma+4\gamma^2+4\gamma^3+6cm\gamma+cm\Big)>0,$$

which is obviously satisfied. Because $b=b_{PM}$ is sustainable if and only if $\sigma_\epsilon^2=0$, every lower value of b requires corresponding positive imprecision, that is, $\sigma_\epsilon^2>0$. This completes the proof. *QED.*

Proof of Proposition 6. The optimal investment of firm type θ is characterized by the first order condition:

$$\theta-c'(k)+(1-\epsilon)\frac{\partial}{\partial k}\varphi(k)=0. \tag{A27}$$

Substituting (14) for $\varphi(k)$ in (A27) and collecting terms we obtain

$$\theta-c'(k)+(1-\epsilon)^2v_k(k,I(k))+(1-\epsilon)^2v_\theta(k,I(k))I'(k)=0. \tag{A28}$$

In equilibrium, the investment schedule conjectured by the market coincides with the actual investments chosen by the firm. Inserting this requirement into the firm's first-order condition (A28) and using $I'(k)=\frac{1}{k'(\theta)}$ yields the following first-order differential equation characterizing the equilibrium investment schedule:

$$k'(\theta)=\frac{(1-\epsilon)^2v_\theta(k(\theta),\theta)}{c'(k(\theta))-\theta-(1-\epsilon)^2v_k(k(\theta),\theta)}. \tag{A29}$$

The investment schedule $k(\theta)$ is strictly increasing if the denominator in (A29) is strictly positive. If we think of the new first best problem of the firm as a maximization of the strictly concave objective function $-c(k)+k\theta+(1-\epsilon)^2v(k,\theta)$, the requirement on the denominator of (A29) is equivalent to overinvestment relative to this first best. As in standard fully revealing signaling equilibria, this condition is guaranteed to hold so long as the first-best objective function has the single-crossing-property. Our assumption $v_{k\theta}>0$ ensures that, in fact, the single-crossing property is satisfied.

As in Spence-type [1974] fully revealing equilibria, the solution to (A29) is a one-parameter family of investment schedules, which can be refined to a specific equilibrium schedule by establishing the investment of the lowest type $\underline{\theta}$. To address the incentives of type $\underline{\theta}$, we need to specify the off-equilibrium beliefs of the market when an accounting measure $s<k(\underline{\theta})$ is observed. Given that the support of $l(\cdot)$ is $[k(\underline{\theta}),k(\bar{\theta})]$, any such signal must communicate *for certain* that the firm chose an investment lower than $k(\underline{\theta})$ and that the observed signal coincides with the firm's true investment.

Thus, for any off-equilibrium accounting signal $s < k(\underline{\theta}), \varphi(s) = v(s, \underline{\theta})$. Given this pricing rule, type $\underline{\theta}$'s expected payoff from any $k \leq k(\underline{\theta})$ is $-c(k) + k\underline{\theta} + (1-\epsilon)v(k, \underline{\theta}) + \epsilon\bar{v}$. Because $k(\underline{\theta})$ as specified in Proposition 6 maximizes the previous expression, type $\underline{\theta}$ has no incentive to deviate to an investment below $k(\underline{\theta})$. The equilibrium condition specified in (17) insures that deviations to higher investment levels are unprofitable.

To establish that higher values of ϵ reduce investment at every θ, we differentiate the first-order condition with respect to ϵ. Let $\Lambda(k, \epsilon, \theta) \equiv \theta - c'(k) + (1-\epsilon)^2 v_k(k, I(k)) + (1-\epsilon)^2 v_\theta(k, I(k))\ I'(k)$, where k should be thought of as a function of θ and ϵ. Because $\Lambda(k, \epsilon, \theta) \equiv 0$,

$$\frac{d}{d\epsilon}\Lambda(k(\theta,\epsilon),\epsilon,\theta) = \Lambda_k(k,\epsilon,\theta)k_\epsilon(\theta,\epsilon) + \Lambda_\epsilon(k,\epsilon,\theta) = 0$$

$$\Longrightarrow k_\epsilon(\theta,\epsilon) = -\frac{\Lambda_\epsilon(k,\epsilon,\theta)}{\Lambda_k(k,\epsilon,\theta)}.$$

Because the second-order condition requires $\Lambda_k(k, \epsilon, \theta) < 0$, $\text{sign}[k_\epsilon(\theta, \epsilon)] = \text{sign}[\Lambda_\epsilon(k, \epsilon, \theta)]$. But $\Lambda_\epsilon(k, \epsilon, \theta) = -2(1-\epsilon)\ [(v_k(k, I(k)) + v_\theta(k, I(k))\ I'(k))] < 0\ \forall \epsilon \geq 0$ because $v_k > 0$, $v_\theta > 0$, and $I'(k) > 0$. This establishes that $k(\cdot)$ is strictly decreasing in ϵ at every θ. Comparing (3) with (17), it is clear that at $\epsilon = 0$, $k(\theta)$ coincides with $k_{PM}(\theta)$. This observation together with $k_\epsilon < 0$ implies that $k(\theta) < K_{PM}(\theta)$ at every $\epsilon > 0$.

To establish that $k'(\theta)$ is decreasing in ϵ at every θ, examine (A29). The numerator of (A29) is decreasing in ϵ because k decreases in ϵ at every θ and by assumption $v_{k\theta} > 0$. The denominator is increasing in ϵ because by assumption, $v_{kk} < 0$. *QED.*

Proof of Proposition 7. The ex ante expected payoff of the firm is

$$\int_\theta (-c(k(\theta,\epsilon)) + \theta k(\theta,\epsilon) + E_s[\varphi(s) \mid k(\theta,\epsilon)])h(\theta)\,d\theta. \tag{A30}$$

As derived in (11) the preceding expression reduces to:

$$W(\epsilon) \equiv \int_\theta (-c(k(\theta,\epsilon)) + \theta k(\theta,\epsilon) + v(k(\theta,\epsilon),\theta))h(\theta)\,d\theta.$$

Differentiating with respect to ϵ we obtain

$$\begin{aligned}\frac{dW(\epsilon)}{d\epsilon} &= \int_\theta [-c'(k(\theta,\epsilon))k_\epsilon(\theta,\epsilon) + \theta k_\epsilon(\theta,\epsilon) + k_\epsilon(\theta,\epsilon)v_k(k(\theta,\epsilon),\theta)]h(\theta)\,d\theta \\ &= \int_\theta k_\epsilon(\theta,\epsilon)[-c'(k(\theta,\epsilon)) + \theta + v_k(k(\theta,\epsilon),\theta)]h(\theta)\,d\theta.\end{aligned}$$

Adding and subtracting $(1-\epsilon)^2 v_k(k(\theta, \epsilon), \theta)$, the preceding reduces to

$$\begin{aligned}\frac{dW(\epsilon)}{d\epsilon} = \int_\theta k_\epsilon(\theta,\epsilon)\big[\big\{-c'(k(\theta,\epsilon)) + \theta + (1-\varepsilon)^2 v_k(k(\theta,\epsilon),\theta)\big\} \\ + \epsilon(2-\epsilon)v_k(k(\theta,\epsilon),\theta)\big]h(\theta)\,d\theta.\end{aligned}$$

Using (17) to substitute for $\{-c'(k(\theta,\epsilon)) + \theta + (1-\epsilon)^2 v_k(k(\theta,\epsilon),\theta)\}$, we get

$$\frac{dW(\epsilon)}{d\epsilon} = \int_\theta k_\epsilon(\theta,\epsilon)\bigg[-\frac{(1-\epsilon)^2 v_\theta(k(\theta,\epsilon),\theta)}{k_\theta(\theta,\epsilon)} + \epsilon(2-\epsilon)v_k(k(\theta,\epsilon),\theta)\bigg]h(\theta)\,d\theta.$$

Thus,

$$\lim_{\epsilon\to 0}\frac{dW(\epsilon)}{d\epsilon} = \int_\theta -k_\epsilon(\theta,\epsilon)\frac{v_\theta(k(\theta,\epsilon),\theta)}{k_\theta(\theta,\epsilon)}h(\theta)\,d\theta.$$

Because, as shown in the proof of Proposition 6, $k_\epsilon(\theta,\epsilon) < 0\ \forall\,\epsilon$, $k_\theta(\theta,\epsilon) > 0$ and by assumption $v_\theta(k(\theta,\epsilon),\theta) > 0$; therefore, $\lim_{\epsilon\to 0}\frac{dW(\epsilon)}{d\epsilon} > 0$. *QED.*

REFERENCES

BAGWELL, K. “Commitment and Observability in Games.” *Games and Economic Behavior* 8 (1995): 271–80.

BEBCHUK, L. A., AND L. A. STOLE. “Do Short-Term Objectives Lead to Under- or Overinvestment in Long-Term Projects?” *Journal of Finance* 48 (1993): 719–29.

DIAMOND, D. “Optimal Release of Information by Firms.” *Journal of Finance* 40 (1985): 1071–94.

DUTTA, S., AND S. REICHELSTEIN. “Stock Price, Earnings and Book Value in Managerial Performance Measures.” Working paper, Stanford Univeriesty, 2003.

DYE, R. A. “Proprietary and Nonproprietary Disclosures.” *Journal of Business* 59 (1986): 331–66.

Financial Accounting Standards Board (FASB). “Qualitative Characteristics of Accounting Information.” Statement of Financial Accounting Concepts No. 2. Norwalk, CT: FASB, 1980.

FUDENBERG, D., AND J. TIROLE. “A Signal-Jamming Theory of Predation.” *Rand Journal of Economics* 17 (1986): 366–76.

GIGLER, F. “Self-Enforcing Voluntary Disclosures.” *Journal of Accounting Research* 40 (1994): 1071–94.

HIRSHLEIFER, J. “The Private and Social Value of Information and the Reward to Inventive Activity.” *American Economic Review* 61 (1971): 561–74.

KANODIA, C. “Effects of Shareholder Information on Corporate Decisions and Capital Market Equilibrium.” *Econometrica* 48 (1980): 923–53.

KANODIA, C., AND D. LEE. “Invetment and Disclosure: The Disciplinary Role of Periodic Performance Reports.” *Journal of Accounting Research* 3 (1998): 33–55.

KANODIA, C., AND A. MUKHERJI. “Real Effects of Separating Investment and Operating Cash Flows.” *Review of Accounting Studies* 1 (1996): 51–72.

MAGGI, G. “The Value of Commitment with Imperfect Observability and Private Information.” *Rand Journal of Economics* 4 (1999): 555–74.

MATTHEWS, S. A., AND L. J. MIRMAN. “Equilibrium Limit Pricing: The Effects of Private Information and Stochastic Demand.” *Econometrica* 51 (1983): 981–96.

MILGROM, P. R. “Good News and Bad News: Representation Theorems and Applications.” *Bell Journal of Economics* 12 (1981): 380–91.

MORRIS, S., AND H. S. SHIN. “Social Value of Public Information.” *American Economic Review* 92 (2002): 1521–34.

MORRIS, S., AND H. S. SHIN. “Coordination Risk and the Price of Debt.” *European Economic Review* 48 (2004): 133–53.

NARAYANAN, M. “Managerial Incentives for Short-Term Results.” *Journal of Finance* 40 (1985): 1469–84.

RILEY, J. G. “Informational Equilibrium.” *Econometrica* 47 (1979): 331–59.

SPENCE, A. M. "Competitive and Optimal Responses to Signals: An Analysis of Efficiency and Distribution." *Journal of Economic Theory* 7 (1974): 296–32.

STEIN, J. "Efficient Capital Markets, Inefficient Firms: A Model of Myopic Corporate Behavior." *Quarterly Journal of Economics* 104 (1989): 655–69.

VERRECCHIA, R. E. "The Use of Mathematical Models in Financial Accounting." *Journal of Accounting Research* 20 (Supplement) (1982): 1–55.

VERRECCHIA, R. E. "Discretionary Disclosure." *Journal of Accounting and Economics* 5 (1983): 179–94.