Ph.D. Seminar in Corporate Finance
Lecture 4

An Incomplete Contracts Approach to Financial Contracting

(Aghion-Bolton, *Review of Economic Studies*, 1982)

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The paper analyzes capital structure from the perspective of *control rights* - who owns the asset controls it - in a framework of incomplete contracts with wealth constraints. The existence of wealth constraints is a key difference from the original Grossman-Hart (1986) paper. Note that these approaches always lead to debt, never equity. For some recent work on equity financing, see Myers, *Journal of Finance,* 1998.

The Model Setup

In this bilateral contracting setting, the two parties are:

- the entrepreneur/manager: needs setup costs K for a project, has zero wealth, and has all the bargaining power (i.e. he can make a take-it-or-leave-it offer);
- the wealthy investor: needs an expected payoff of K.

The timing of the game is as follows: at time $t = 0$ investment K is taken, at time $t = 1$ the state θ is realized and the signal s is observed, then an action is taken, and finally at time $t = 2$, the returns r are realized.

Both the entrepreneur (manager) and the investor are risk-neutral, with utility functions

$$\begin{aligned} U_E(r, a, \theta) &= r + l(a, \theta) \\ U_I(r, a, \theta) &= r \end{aligned}$$

where r is the realization of the returns and $l(a, \theta)$ is a non-monetary payoff to the entrepreneur.

Note that in a complete contracts framework, the contract specifies a mapping $\alpha : \Theta \rightarrow A$, so that $\alpha(\theta) \rightarrow a$, where $\theta \in \Theta$ and $a \in A$. Then this needs to be enforced.

Assumption 1: ***The state of nature θ cannot be contracted on. Ex-post, θ is known, but it is not contractible.***

Assumption 2: ***There exists a publicly verifiable signal s that is imperfectly correlated with θ.***

Assumption 3: ***All monetary payoffs are verifiable and contractible (i.e. r).***

Other assumptions:

- There are two states of nature, so that $\Theta = \{\theta_g, \theta_b\}$;

- There are two actions in the action set: $A = \{a_g, a_b\}$. These actions are the state-dependent optimal actions, i.e. $a_g = a^*(\theta_g)$ and $a_b = a^*(\theta_b)$;

- The signal $s \in \{0,1\}$. Letting $\beta^\theta = P[s = 1|\theta]$, we assume that

$$\begin{aligned}\beta^g &= \Pr[s = 1|\theta = \theta_g] > \frac{1}{2} \\ \beta^b &= \Pr[s = 1|\theta = \theta_b] < \frac{1}{2}\end{aligned}$$

  Given β^g and β^b defined above, let $d(\tilde{\beta}, (1,0)) = [|1 - \beta^g| + |0 - \beta^b|]$. Then d measures the degree of "contract incompleteness". If $d = 0$ there is perfect correlation between the state of nature θ and the signal s. If $d = 1$ there is no correlation.

- The return $r \in \{0,1\}$

Let y^i_j denote the expected final-period return in state θ_i when action a_j is chosen, and l^i_j the private benefit of the entrepreneur in state θ_i when action a_j is chosen. It follows that

$$y^i_j = E(r|\theta = \theta_i, a = a_j) = \Pr[r = 1|\theta = \theta_i, a = a_j]$$

The First Best

$$\begin{aligned}qy^g_g + (1-q)y^b_b &> K && \text{(feasibility)} \\ y^g_g + l^g_g &> y^g_b + l^g_b && (a_g \text{ when } \theta_g) \\ y^b_b + l^b_b &> y^b_g + l^b_g && (a_b \text{ when } \theta_b)\end{aligned}$$

1 where $q = \Pr[\theta = \theta_g]$.

The Contract

2 A contract consists of:

(i) Compensation schedule for the entrepreneur/manager

$$\begin{aligned}t(s,r) &\geq 0 \\ t(a,s,r) &\geq 0 \quad \text{if actions are verifiable}\end{aligned}$$

(ii) Control allocation rule

- Individual control: $(\alpha_0, \alpha_1) \in [0,1]^2$ where $\alpha_s \equiv$ probability that the entrepreneur gets the right to decide what action to choose when $s = 1$ or $s = 0$, and $(1 - \alpha_s) \equiv$ probability that the investor gets control
- Joint control: $(\mu_0^I, \mu_1^I) \in (0,1)^2$ and $(\mu_0^E, \mu_1^E) \in (0,1)^2$ where $\mu_s^I + \mu_s^E > 1$ for some $s \in \{0, 1\}$.

In the case of joint control, both parties have control in some state of the world, and they have to agree on an action, otherwise they both get zero payoffs. It is assumed that the entrepreneur has all the bargaining power and thus makes a take-it-or-leave-it offer. If the investor accepts, the proposed action is taken; if he rejects, the payoff vector is $(0, 0)$.

Non-verifiable Actions

Three types of control allocation rules are considered: unilateral control, contingent control and joint control. The latter is shown to be (weakly) dominated.

Entrepreneur Control

Since there are only two possible returns ($r \in \{0, 1\}$), we can always write:

$$
\begin{aligned}
t(s,r) &= (t(s,1) - t(s,0))r + t(s,0) \\
&= t_s r + t'_s \\
&\therefore \text{ the contract is affine with only two states}
\end{aligned}
$$

The entrepreneur chooses an action $a^E(\theta_i, s)$ such that:

$$a^E(\theta_i, s) = \arg\max_{a_j \in A}\{t_s y_j^i + t_s' + l_j^i\}$$

Renegotiation

Suppose a contract that specifies (t_s, t_s') results in entrepreneur's decision that for any θ

$$\begin{aligned} a_g \text{ when } s &= 1 \\ a_b \text{ when } s &= 0 \end{aligned}$$

Consider the case $(\theta_b, s = 1)$. In this case, the optimal action is a_b. If the contract is not renegotiated, the investor gets:

$$\begin{aligned} U_I &= y_g^b(1 - t_1) - t_1' \\ &= y_g^b - (t_1 y_g^b + t_1') \end{aligned}$$

If the contract is renegotiated, the entrepreneur offers the investor $y_g^b(1 - t_1) - t_1'$ but takes action a_b and gets:

$$\begin{aligned} & (y_b^b + l_b^b) - y_g^b(1 - t_1) + t_1' \\ > \;& (y_g^b + l_g^b) - y_g^b(1 - t_1) + t_1' = t_1 y_g^b + t_1' + l_g^b \end{aligned}$$

A similar argument applies to the case $(\theta_g, s = 0)$.

Implication 1 **If the entrepreneur has full control, ex-post renegotiation guarantees achieving the first best.**

Implication 2 **This is optimal if the investor's ex-ante individual rationality constraint can be met.**

3 **Proposition 1** ***If private benefits l are comonotonic with $(y + l)$, i.e. if $l_g^g > l_b^g$ and $l_b^b > l_g^b$, entrepreneur control is always feasible.***

4 **Proof.** **Choose $t(s,r) = t \quad \forall s, r \quad \Rightarrow$ the entrepreneur chooses first best because**

$$\begin{aligned} t + l_g^g &> t + l_b^g \\ t + l_b^b &> t + l_g^b \end{aligned}$$

5 **Now choose t such that $[q y_g^g + (1-q) y_b^b] - t = K$. Feasibility ensures such a t exists.■**

6 **Suppose now that $l_b^b < l_g^b$, i.e. comonotonicity is lost. Define:**

$$\begin{aligned} \Delta^b &\equiv (y_b^b + l_b^b) - (y_g^b + l_g^b) > 0 \\ \Delta_y^b &\equiv y_b^b - y_g^b > 0 \\ \pi_1 &\equiv [q y_g^g + (1-q) y_b^b] \frac{\Delta^b}{\Delta_y^b} \\ \pi_2 &\equiv q y_g^g + (1-q) y_g^b \\ \pi_3 &\equiv q[\beta^g y_g^g + (1-\beta^g) y_g^g \frac{\Delta^b}{\Delta_y^b}] + (1-q)[\beta^b y_g^b + (1-\beta^b) y_b^b \frac{\Delta^b}{\Delta_y^b}] \end{aligned}$$

7 Proposition 2 ***Entrepreneur control is feasible and implements the first best solution if and only if*** $\max(\pi_1, \pi_2, \pi_3) \geq K$. ***If*** $K \in [\max(\pi_1, \pi_2, \pi_3), qy_g^g + (1-q)y_b^b]$ ***entrepreneur control is not feasible.***

Example **Let** $\Pr[\theta = \theta_g] = \frac{1}{2}$, $y_g^g = 100$, $l_g^g = 150$, $y_b^g = 200$, $l_b^g = 0$, $y_b^b = 50$, $l_b^b = 0$, $y_g^b = 0$, $l_g^b = 49$. **Restrict attention to contracts of the form** $t_s' = 0, t_s \leq 1$. **We can do this because if** $t_s' > 0$, **then we can lower** t_s', **so that the investor's IC is relaxed while incentives are not affected. Then we can reallocate in a way not to affect incentives** $\therefore$ **consider** $t(s, r) = t_s r$ **where** $t_s \leq 1$.

7.1 No Renegotiation

Note the investor always chooses a_g **in state** θ_g. **In state** θ_b, **we need** t_s **big enough so**

$$\begin{aligned} t_s y_b^b + l_b^b &\geq t_s y_g^b + l_g^b \\ t_s 50 &\geq 49 \\ t_s &\geq \frac{49}{50} \end{aligned}$$

So for $t_s = \frac{49}{50}$ **no renegotiation occurs. Investor's maximum payoff is:**

$$\begin{aligned} \pi_1 &= \frac{1}{2}\frac{1}{50}100 + \frac{1}{2}\frac{1}{50}500 = \frac{3}{2} \\ &\therefore \ K < \frac{3}{2} \end{aligned}$$

7.2 Full Renegotiation in state θ_b

7.3 In this case the contract is such that

$$\begin{aligned} t_s y_b^b + l_b^b &< t_s y_g^b + l_g^b \\ t_s y_b^b &< l_g^b \\ t_s &< \frac{49}{50} \end{aligned}$$

7.4 Since $t_s < \frac{49}{50}$, the entrepreneur chooses a_g without renegotiation, yielding a payoff of zero for the investor $\Longrightarrow$ on renegotiation, the investor gets zero as well. The ex-ante expected payoffs from the contract for the two parties are:

$$\begin{aligned} \text{Entrepreneur} \quad &: \quad (\frac{1}{2}t_s 100 + \frac{1}{2}50) + \frac{1}{2}50 \\ \text{Investor} \quad &: \quad \frac{1}{2}(1 - t_s)100 \end{aligned}$$

7.5 Therefore, the investor expected payoff is $\pi_2 = \frac{1}{2}\frac{1}{50}100 = 1$ if $t_s = \frac{49}{50}$, and $\pi_2 = \frac{1}{2}100 = 50$ if $t_s = 0 \Longrightarrow$ the range of investments that can be supported in this case is $K \in (1, 50)$. If $K \in (1, \frac{3}{2})$, full renegotiation yields the same outcome in terms of efficiency as no renegotiation (i.e. the first best is implemented).

7.6 Partial renegotiation in state θ_b

7.7 In this case, we have $t_1 \neq t_0$. By setting $t_1 < \frac{49}{50}$, renegotiation occurs in state θ_b only when $s = 1$, so that a_b is implemented after renegotiation. By setting $t_0 > \frac{49}{50}$, the action a_b is implemented in θ_b when $s = 0$. With these constraints, the values for t_s that maximize investor's payoff are: $t_1 = 0, t_0 = \frac{49}{50}$. The investor's expected payoff is:

$$\begin{aligned} \pi_3 &= \frac{1}{2}[\beta^g 100 + (1 - \beta^g)(1 - \frac{49}{50})100] + \frac{1}{2}[\beta^g 0 + (1 - \beta^g)(1 - \frac{49}{50})50] \\ &= 50\beta^g + 1 - \beta^g + \frac{1 - \beta^b}{2} \end{aligned}$$

7.8 **As $\beta^g \to 1$ and $\beta^b \to 0$, π_3 gets larger and we get $\pi_3 \longrightarrow 50.5 > 50$. For $K \in (50, 50.5)$ only partial renegotiation can occur.**

Investor Control

8 **Proposition 3** ***When monetary benefits are comonotonic with total revenues ($y_g^g > y_b^g$ and $y_b^b > y_g^b$) the first best can be achieved under investor control.***

9 **Proof:** **Let $t(s,r) = t \cdot r$. Then:**

$$\begin{aligned} (1-t)y_g^g &> (1-t)y_b^g \Rightarrow \\ \pi^g(a_g) &> \pi^g(a_b) \quad \text{q.e.d.} \end{aligned}$$

10 **Set t such that the investor's participation constraint holds.■**

11 **Without loss of generality, consider now the case when $y_g^g < y_b^g$, so that comonotonicity fails. Set $t_s' = 0$. Since $(1-t_s)y_g^g < (1-t_s)y_b^g$, the investor does not take the first-best action a_g in state θ_g.**

12 **When $t_s < 1$, we need to renegotiate (i.e. agree on a payment $\hat{t}_s$)so that**

$$\begin{aligned} (1-\hat{t}_s)y_g^g &\geq (1-t_s)y_b^g \\ \text{or } \hat{t}_s &\leq 1-(1-t_s)\frac{y_b^g}{y_g^g} \end{aligned}$$

13 So if $\frac{y_b^g}{y_g^g}$ is large or t_s is small, the entrepreneur's wealth constraint may be violated. We need to ensure that

$$\begin{aligned} \hat{t}_s &\geq 0 \\ \text{or} \quad (1-t_s)\frac{y_b^g}{y_g^g} &\leq 1 \\ \text{thus} \quad t_s &\geq 1-\frac{y_b^g}{y_g^g} \end{aligned}$$

14 Proposition 4 ***If monetary benefits are not comonotonic with total returns, a necessary and sufficient condition for implementing the first-best action plan under investor control is:***

$$\pi_4 = [qy_g^g + (1-q)y_b^b]\frac{y_b^g}{y_g^g} \geq K$$

15 Example $y_b^b = 50; \quad y_g^g = 100 < y_b^g = 200$ $\Rightarrow$ **Investor control is first-best efficient if** $\pi_4 = (\frac{1}{2}200 + \frac{1}{2}50)\frac{1}{2} = 62.5 > K.$

Contingent Control - Debt

16 **To analyze this type of control, we assume that neither monetary nor private benefits are comonotonic with total benefits: $y_g^g < y_b^g$ and $l_b^b < l_g^b$. Consequently, in state θ_b, the investor chooses the first-best, while in state θ_g the entrepreneur chooses the first-best. However, control cannot be made contingent on the state θ, but only on the signal s. If s is highly corrleated with θ, and setting $t(s,r) = 0$, the optimal contract will specify:** $\begin{cases} s = 1 \Rightarrow \text{ Entrepreneur control} \\ s = 0 \Rightarrow \text{ Investor control} \end{cases}$

17 **With no renegotiation of contract and contingent control on s, we have:**

17.1 **In state θ_g,** $a = \begin{cases} a_g \text{ if } s = 1 \text{ - entrepreneur (efficient)} \\ a_b \text{ if } s = 0 \text{ - investor (inefficient)} \end{cases}$

17.2 **In state θ_b,** $a = \begin{cases} a_g \text{ if } s = 1 \text{ - entrepreneur (inefficient)} \\ a_b \text{ if } s = 0 \text{ - investor (efficient)} \end{cases}$

18 **The investor's expected payoff is:**

$$\pi_C = q[\beta^g y_g^g + (1-\beta^g) y_b^g] + (1-q)[\beta^b y_g^b + (1-\beta^b) y_b^b]$$

19 **As $\beta^g \longrightarrow 1$ and $\beta^b \longrightarrow 0$, π_C approaches the first-best monetary payoff $\Rightarrow \pi_C > \pi_1, \pi_C > \pi_2$.**

20 **As $\beta^g \longrightarrow 1$ and $\beta^b \longrightarrow 0$, π_3 approaches the first best monetary payoff as well, but**

$$\pi_C - \pi_3 = q(1-\beta^g)(y_g^b - y_g^g \frac{\Delta^b}{\Delta_y^b}) + (1-q)(1-\beta^b)(1 - \frac{\Delta^b}{\Delta_y^b}) y_b^b$$

21 Now $\frac{\Delta^b}{\Delta^b_y} < 1$ since $l^b_b < l^b_g$ and $y^g_g < y^g_b$, so $\pi_C > \pi_3 \Rightarrow$ as $\beta^g \longrightarrow 1$ and $\beta^b \longrightarrow 0$ there exist values for K for which contingent control is feasible but entrepreneurial control is not. Suppose $\pi_4 < K$ (so the condition for Proposition 4 does not hold). Then investor control achieves a_b in state θ_g, not the first-best. Under contingent control, as $\beta^g \longrightarrow 1$ and $\beta^b \longrightarrow 0$ the first-best is implemented.

22 Proposition 5 ***When neither monetary nor private benefits are comonotonic with total benefits, there exist values of K such that:***

22.1 ***(i) entrepreneur control is not feasible***

22.2 ***(ii) investor control is not first-best efficient***

22.3 ***(iii) contingent control ($\alpha_0 = 0, \alpha_1 = 1$) dominates when $(\beta^g, \beta^b) \longrightarrow (1, 0)$.***

23 The alternative of joint control/ownership exhibits the ex-post hold-up problem as either party has veto power. Examples include joint ventures and alliances. The intruments that implement the control structures discussed in the paper are:

Entrepreneur control	$\longrightarrow$	Preffered stock
Investor control	$\longrightarrow$	Equity voting
Contingent control	$\longrightarrow$	Debt
Joint control	$\longrightarrow$	Partnership

24 Venture Capital applications seem to be consistent.