Collateral and Capital Structure*

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Abstract

We develop a dynamic model of investment, capital structure, leasing, and risk management based on the firm's need to collateralize promises to pay with tangible assets. Both financing and risk management involve promises to pay that are limited by collateral constraints. Leasing is costly financing that allows higher leverage due to its greater collateralization. More constrained firms engage in less risk management and lease more, both cross-sectionally and dynamically. Even mature firms that suffer adverse cash flow shocks may cut risk management and sell and lease back assets. Persistence of firms' productivity reduces the benefits to hedging low cash flow states and, for plausible levels of persistence, firms may not hedge at all.

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1 Introduction

We argue that collateral determines the capital structure and develop a dynamic agency based model of firm financing based on the need to collateralize promises to pay with tangible assets. We maintain that the enforcement of payments is a critical determinant of both firm financing and whether asset ownership resides with the user or the financier, that is, whether firms purchase or lease assets. We study a dynamic neoclassical model of the firm in which financing is subject to collateral constraints derived from limited enforcement and firms choose between purchasing and renting assets. Our theory of optimal investment, capital structure, leasing, and risk management enables the first dynamic analysis of the financing vs. risk management trade-off and of firm financing when firms can rent capital.

In the frictionless neoclassical model asset ownership is indeterminate and firms are assumed to rent all capital. The recent dynamic agency models of firm financing ignore the possibility that firms rent capital. Of course, a frictionless rental market for capital would obviate financial constraints. We explicitly consider firms' dynamic lease vs. buy decision, modeling leasing as highly collateralized albeit costly financing. When capital is leased, the financier retains ownership which facilitates repossession and strengthens the collateralization of the financier's claim. Leasing is costly since the lessor incurs monitoring costs to avoid agency problems due to the separation of ownership and control.

We provide a definition of the user cost of capital in our model of investment with financial constraints that is similar in spirit to Jorgenson's (1963) definition of the user cost of capital in the frictionless neoclassical model. The user cost of capital in our model is effectively the sum of Jorgenson's user cost and a term which captures the additional cost due to the scarcity of internal funds; this user cost of capital can also be written in a "weighted average cost of capital" way. In our model, firms require both tangible and intangible capital, but the enforcement constraints imply that only tangible capital can be pledged as collateral and borrowed against; tangibility restricts leverage.

There is a fundamental connection between the optimal financing and risk management policy that has not been previously recognized. Both financing and risk management involve promises to pay by the firm, leading to a trade off when such promises are limited by collateral constraints. Indeed, firms with sufficiently low net worth do not engage in risk management at all because the need to finance investment overrides the hedging concerns. This result is in contrast to the extant theory, such as Froot, Scharfstein, and Stein (1993), and is consistent with the evidence that more constrained firms hedge less provided by Rampini, Sufi, and Viswanathan (2011) and the literature cited therein.

With constant investment opportunities, risk management depends only on firms' net worth and incomplete hedging is optimal. That is, firms do not hedge to the point where the marginal value of net worth is equated across all states. In fact, firms abstain from risk management with positive probability under the stationary distribution. Thus, even mature firms that suffer a sequence of adverse cash flow shocks may see their net worth decline to the point where they find it optimal to discontinue risk management.

With stochastic investment opportunities, risk management depends not only on firms' net worth but also on their productivity. If productivity is persistent, the overall level of risk management is reduced, because cash flows and investment opportunities are positively correlated due to the positive correlation between current productivity and future expected productivity. There is less benefit to hedging low cash flow states. Moreover, risk management is lower when current productivity is high, as higher expected productivity implies higher investment and raises the opportunity cost of risk management. With sufficient but empirically plausible levels of persistence, the firm abstains from risk management altogether, providing an additional reason why risk management is so limited in practice. Furthermore, when the persistence of productivity is strong, firms hedge investment opportunities, that is, states with high productivity, as the financing needs for increased investment rise more than cash flows.

Leasing tangible assets requires less net worth per unit of capital and hence allows firms to borrow more. Financially constrained firms, that is, firms with low net worth, lease capital because they value the higher debt capacity; indeed, severely constrained firms lease all their tangible capital. Over time, as firms accumulate net worth, they grow in size and start to buy capital. Thus, the model predicts that small firms and young firms lease capital. We show that the ability to lease capital enables firms to grow faster. Dynamically, mature firms that are hit by a sequence of low cash flows may sell assets and lease them back, that is, sale-leaseback transactions may occur under the stationary distribution. Moreover, leasing has interesting implications for risk management: leasing enables high implicit leverage; this may lead firms to engage in risk management to reduce the volatility of net worth that such high leverage would otherwise imply.

In the data, we show that tangible assets are a key determinant of firm leverage. Leverage varies by a factor 3 from the lowest to the highest tangibility quartile for Compustat firms. Moreover, tangible assets are an important explanation for the "low leverage puzzle" in the sense that firms with low leverage are largely firms with few tangible assets. We also take firms' ability to deploy tangible assets by renting or leasing such assets into account. We show that accounting for leased assets in the measurement of leverage and tangibility reduces the fraction of low leverage firms drastically and that firms with low lease adjusted leverage are firms with low lease adjusted tangible assets. Finally, we show that accounting for leased capital has a striking effect on the relation between leverage and size in the cross section of Compustat firms. This relation is essentially flat when leased capital is taken into account. In contrast, when leased capital is ignored, as is done in the literature, leverage increases in size, that is, small firms seem less levered than large firms. Thus, basic stylized facts about the capital structure need to be revisited. Importantly, the lease adjustments to the capital structure we propose based on our theory are common in practice, and accounting rule changes are currently being considered by the US and international accounting boards that would result in the implementation of lease adjustments similar to ours throughout financial accounting. Our model and empirical evidence together suggest a collateral view of the capital structure.

Our paper is part of a recent and growing literature which considers dynamic incentive problems as the main determinant of the capital structure. The incentive problem in our model is limited enforcement of claims. Most closely related to our work are Albuquerque and Hopenhayn (2004), Lorenzoni and Walentin (2007), and Rampini and Viswanathan (2010). Albuquerque and Hopenhayn (2004) study dynamic firm financing with limited enforcement. In their setting, the value of default is exogenous, albeit with fairly general properties, whereas in our model the value of default is endogenous as firms are not excluded from markets following default. Moreover, they do not consider the standard neoclassical investment problem.[1] Lorenzoni and Walentin (2007) consider limits on enforcement similar to ours in a model with constant returns to scale and adjustment costs on aggregate investment which implies that all firms are equally constrained at any given time. However, they assume that all enforcement constraints always bind, which is not the case in our model, and focus on the relation between investment and Tobin's q rather than the capital structure. Rampini and Viswanathan (2010) consider a two period model in a similar setting with heterogeneity in firm productivities and focus on the distributional implications of limited risk management. While they consider the comparative statics with respect to exogenously given initial net worth, net worth is endogenously determined in our model, and our model moreover allows the analysis of the dynamics of risk management, risk management by mature firms in the long run, and the effect of persistence on the extent of risk management.

The rationale for risk management in our model is related to the one in Froot, Scharfstein, and Stein (1993) who show that firms subject to financial constraints are effectively risk averse and hence engage in risk management. Holmström and Tirole (2000) recognize that financial constraints may limit ex ante risk management. Neither of these papers provides a dynamic analysis of the financing vs. risk management trade-off.

Capital structure and investment dynamics determined by incentive problems due to

[1]The aggregate implications of firm financing with limited enforcement are studied by Cooley, Marimon, and Quadrini (2004) and Jermann and Quadrini (2007). Schmid (2008) considers the quantitative implications for the dynamics of firm financing.

private information about cash flows or moral hazard are studied by Quadrini (2004), Clementi and Hopenhayn (2006), DeMarzo and Fishman (2007a), DeMarzo, Fishman, He, and Wang (2011), and Biais, Mariotti, Rochet, and Villeneuve (2010). Capital structure dynamics subject to similar incentive problems but abstracting from investment decisions are analyzed by DeMarzo and Fishman (2007b), DeMarzo and Sannikov (2006), and Biais, Mariotti, Plantin, and Rochet (2007).[2] In these models, collateral plays no role.

Moreover, none of these models consider intangible capital or the option to lease capital. An exception is Eisfeldt and Rampini (2009) who argue that leasing amounts to a particularly strong form of collateralization due to the relative ease with which leased capital can be repossessed, albeit in a static model. We are the first, to the best of our knowledge, to consider the role of leased capital in a dynamic model of firm financing and provide a dynamic theory of sale-and-leaseback transactions.

In Section 2 we report some stylized empirical facts about collateralized financing, tangibility, and leverage, taking leased capital into account. Section 3 describes the model, defines the user cost of tangible, intangible, and leased capital, and characterizes the optimal payout policy. Section 4 analyzes optimal risk management, Section 5 characterizes the optimal leasing and capital structure policy, and Section 6 concludes. All proofs are in Appendix B.

2 Stylized facts

This section provides some aggregate and cross-sectional evidence that highlights the first order importance of tangible assets as a determinant of the capital structure in the data. We first take an aggregate perspective and then document the relation between tangible assets and leverage across firms. We take leased capital into account explicitly and show that it has quantitatively and qualitatively large effects on basic stylized facts about the capital structure, such as the relation between leverage and size. Tangibility also turns out to be one of the few robust factors explaining firm leverage in the extensive empirical literature on capital structure, but we do not attempt to summarize this literature here.

[2] Relatedly, Gromb (1995) analyzes a multi-period version of Bolton and Scharfstein (1990)'s two period dynamic firm financing problem with privately observed cash flows. Gertler (1992) considers the aggregate implications of a multi-period firm financing problem with privately observed cash flows. Atkeson and Cole (2008) consider a two period firm financing problem with costly monitoring of cash flows.

2.1 Collateralized financing: the aggregate perspective

From the aggregate point of view, the importance of tangible assets is striking. Consider the balance sheet data from the Flow of Funds Accounts of the U.S. for (nonfinancial) corporate businesses, (nonfinancial) noncorporate businesses, and households reported in Table 1 for the years 1999 to 2008 (detailed definitions of variables are in the caption of the table). For businesses, tangible assets include real estate, equipment and software, and inventories, and for households mainly real estate and consumer durables.

Panel A documents that from an aggregate perspective, the liabilities of corporate and noncorporate businesses and households are less than their tangible assets and indeed typically considerably less, and in this sense all liabilities are collateralized. For corporate businesses, debt in terms of credit market instruments is 48.5% of tangible assets. Even total liabilities, which include also miscellaneous liabilities and trade payables, are only 83.0% of tangible assets. For noncorporate businesses and households, liabilities vary between 37.8% and 54.9% of tangible assets and are remarkably similar for the two sectors.

Note that we do not consider whether liabilities are explicitly collateralized or only implicitly in the sense that firms have tangible assets exceeding their liabilities. Our reasoning is that even if liabilities are not explicitly collateralized, they are implicitly collateralized since restrictions on further investment, asset sales, and additional borrowing through covenants and the ability not to refinance debt allow lenders to effectively limit borrowing to the value of collateral in the form of tangible assets. That said, households' liabilities are largely explicitly collateralized. Households' mortgages, which make up the bulk of households' liabilities, account for 41.2% of the value of real estate, while consumer credit amounts to 56.1% of the value of households' consumer durables.

Finally, aggregating across all balance sheets and ignoring the rest of the world implies that tangible assets make up 79.2% of the net worth of U.S. households, with real estate making up 60.2%, equipment and software 8.3%, and consumer durables 7.6% (see Panel B). While this provides a coarse picture of collateral, it highlights the quantitative importance of tangible assets as well as the relation between tangible assets and liabilities in the aggregate.

2.2 Tangibility and leverage

To document the relation between tangibility and leverage, we analyze data for a cross section of Compustat firms. We sort firms into quartiles by tangibility measured as the value of property, plant, and equipment divided by the market value of assets. The results are reported in Panel A of Table 2, which also provides a detailed description of the construction of the variables. We measure leverage as long term debt to the market value of assets.

The first observation that we want to stress is that across tangibility quartiles, (median) leverage varies from 7.4% for low tangibility firms (that is, firms in the lowest quartile) to 22.6% for high tangibility firms (that is, firms in the highest quartile), that is, by a factor 3.[3] Tangibility also varies substantially across quartiles; the cut-off value for the first quartile is 6.3% and for the fourth quartile is 32.2%.

To assess the role of tangibility as an explanation for the observation that some firms have very low leverage (the so-called "low leverage puzzle"), we compute the fraction of firms in each tangibility quartile which have low leverage, specifically leverage less than 10%.[4] The fraction of firms with low leverage decreases from 58.3% in the low tangibility quartile to 14.9% in the high tangibility quartile. Thus, low leverage firms are largely firms with relatively few tangible assets.

2.3 Leased capital and leverage

Thus far, we have ignored leased capital which is the conventional approach in the literature. To account for leased (or rented) capital, we simply capitalize the rental expense (Compustat item #47).[5] This allows us to impute capital deployed via operating leases, which are the bulk of leasing in practice.[6] To capitalize the rental expense, recall that Jorgenson's (1963) user cost of capital is $u \equiv r + \delta$, that is, the user cost is the sum of the interest cost and the depreciation rate. Thus, the frictionless rental expense for an amount of capital k is

$$\text{Rent} = (r + \delta)k.$$

Given data on rental payments, we can hence infer the amount of capital rented by capitalizing the rental expense using the factor $1/(r + \delta)$. For simplicity, we capitalize the rental expense by a factor 10. We adjust firms' assets, tangible assets, and liabilities by adding 10 times rental expense to obtain measures of lease adjusted assets, lease adjusted tangible assets, and lease adjusted leverage.

We proceed as before and sort firms into quartiles by lease adjusted tangibility. The results are reported in Panel B of Table 2. Lease adjusted debt leverage is somewhat lower

[3] Mean leverage varies somewhat less, by a factor 2.2 from 10.8% to 24.2%.

[4] We do not think that our results change if lower cutoff values are considered.

[5] In accounting this approach to capitalization is known as constructive capitalization and is frequently used in practice, with "8 x rent" being the most commonly used. For example, *Moody's* rating methodology uses multiples of 5x, 6x, 8x, and 10x current rent expense, depending on the industry.

[6] Note that capital leases are already accounted for as they are capitalized on the balance sheet for accounting purposes. For a description of the specifics of leasing in terms of the law, accounting, and taxation see Eisfeldt and Rampini (2009) and the references cited therein.

as we divide by lease adjusted assets here. There is a strong relation between lease adjusted tangibility and lease adjusted leverage (as before), with the median lease adjusted debt leverage varying again by a factor of about 3. Rental leverage also increases with lease adjusted tangibility by about a factor 2 for the median and more than 3 for the mean. Similarly, lease adjusted leverage, which we define as the sum of debt leverage and rental leverage, also increases with tangibility by a factor 3.

Taking rental leverage into account reduces the fraction of firms with low leverage drastically, in particular for firms outside the low tangibility quartile. Lease adjusted tangibility is an even more important explanation for the "low leverage puzzle." Indeed, less than 4% of firms with high tangibility have low lease adjusted leverage.

It is also worth noting that the median rental leverage is on the order of half of debt leverage or more, and is hence quantitatively important. Overall, we conclude that tangibility, when adjusted for leased capital, emerges as a key determinant of leverage and the fraction of firms with low leverage.

2.4 Leverage and size revisited

Considering leased capital changes basic cross-sectional properties of the capital structure. Here we document the relationship between firm size and leverage (see Table 3 and Figure 1). We sort Compustat firms into deciles by size. We measure size by lease adjusted assets here, although using unadjusted assets makes our results even more stark. Debt leverage is increasing in size, in particular for small firms, when leased capital is ignored. Rental leverage, by contrast, decreases in size, in particular for small firms.[7] Indeed, rental leverage is substantially larger than debt leverage for small firms. Lease adjusted leverage, that is, the sum of debt and rental leverage, is roughly constant across Compustat size deciles. In our view, this evidence provides a strong case that leased capital cannot be ignored if one wants to understand the capital structure.

3 Model

This section provides a dynamic agency based model to understand the first order importance of tangible assets and rented assets for firm financing and the capital structure documented above. Dynamic financing is subject to collateral constraints due to limited enforcement. We consider both tangible and intangible capital as well as firms' ability

[7]Eisfeldt and Rampini (2009) show that this is even more dramatically the case in Census data, which includes firms that are not in Compustat and hence much smaller, and argue that for such firms renting capital may be the most important source of external finance.

to lease capital. We define the user cost of tangible, intangible, and leased capital, and provide a weighted average cost of capital type representation of the user cost of capital. Finally, we characterize the dividend policy and show how tangibility and collateralizability of assets affect the capital structure in the special case without leasing.

3.1 Environment

A risk neutral firm, who is subject to limited liability and discounts the future at rate $\beta \in (0, 1)$, requires financing for investment. The investment problem has an infinite horizon and we write the problem recursively. The firm starts the period with net worth w and has access to a standard neoclassical production function with decreasing returns to scale.

There are two types of capital, tangible capital and intangible capital. Tangible capital can be either purchased (k_p) or leased (k_l), while intangible capital (k_i) can only be purchased. The total amount of capital is $k \equiv k_i + k_p + k_l$ and we refer to total capital k often simply as capital. For simplicity, we assume that tangible and intangible capital are required in fixed proportions and denote the fraction of tangible capital required by φ.[8] Both tangible and intangible capital can be purchased at a price normalized to 1 and depreciate at the same rate δ.[9] There are no adjustment costs. An amount of invested capital k yields stochastic cash flow $A(s')f(k)$ next period, where $A(s')$ is the realized total factor productivity of the technology in state s', which we assume follows a Markov process described by the transition function $\Pi(s, s')$ on $s' \in S$.

Tangible capital which the firm owns can be used as collateral for state-contingent one period debt up to a fraction $\theta \in (0, 1)$ of its resale value. These collateral constraints are motivated by limited enforcement. We assume that enforcement is limited in that firms can abscond with all cash flows, all intangible capital, and $1 - \theta$ of purchased tangible capital k_p. Further we assume that firms cannot abscond with leased capital k_l, that is, leased capital enjoys a repossession advantage. It is easier for a lessor, who retains ownership of the asset, to repossess it, than for a secured lender, who only has a security

[8] We are implicitly using the Leontief aggregator of tangible and intangible capital $\min\{(k_p + k_l)/\varphi, k_i/(1-\varphi)\}$ which yields $k_i = (1-\varphi)k$, $k_p + k_l = \varphi k$, and $k = k_i + k_p + k_l$ as above, simplifying the firm's investment problem to the choice of capital k and leased capital k_l only. If tangible and intangible capital were not used in fixed proportions and had a constant elasticity of substitution $\gamma > -\infty$, that is, the aggregator of tangible and intangible capital was $[\sigma(k_p + k_l)^\gamma + (1-\sigma)k_i^\gamma]^{1/\gamma}$ with $\gamma \leq 1$, then the composition of capital would vary with firms' financial condition, with more financially constrained firms using a lower fraction of intangible capital.

[9] We can normalize both prices to 1 by rescaling the units of capital and adjusting the parameters of the production function and capital aggregator accordingly.

interest, to recover the collateral backing the loan.[10] Importantly, we assume that firms who abscond cannot be excluded from any market: the market for intangible capital, tangible capital, loans, and rented capital. As we show in Appendix A, these dynamic enforcement constraints imply the above collateral constraints, which are similar to the ones in Kiyotaki and Moore (1997), albeit state contingent, and are described in more detail below.[11] We emphasize that any long-term contract that satisfies the enforcement constraints can be implemented with such one-period ahead state contingent debt subject to the above collateral constraints and hence long-term contracts are not ruled out. The motivation for our assumption about the lack of exclusion is two-fold. First, it allows us to provide a tractable model of dynamic collateralized firm financing. The equivalence of the two problems (with limited enforcement and collateral constraints, respectively) enables us to work directly with the problem with collateral constraints and use net worth as the state variable. In contrast, the outside options considered in the literature result in continuation utility being the appropriate state variable, which typically makes the dual problem easier to work with (see, e.g., Albuquerque and Hopenhayn (2004)). Second, a model based on this assumption has implications which are empirically plausible, in particular by putting the focus squarely on tangibility.

We assume that intangible capital can neither be collateralized nor leased. The idea is that intangible capital cannot be repossessed due to its lack of tangibility and can be deployed in production only by the owner, since the agency problems involved in separating ownership from control are too severe.[12]

Our model considers the role of leased capital in a dynamic model of firm financing subject to limited enforcement. The assumption that firms cannot abscond with leased capital captures the fact that leased capital can be repossessed more easily. Leased capital involves monitoring costs m per unit of capital incurred by the lessor at the end of the period, which are reflected in the user cost of leased capital u_l. Leasing separates ownership

[10]Leasing enjoys such a repossession advantage under U.S. law and, we believe, in most legal systems.

[11]These collateral constraints are derived from a explictly dynamic model of limited enforcement similar to the one considered by Kehoe and Levine (1993). The main difference to their limits on enforcement is that we assume that firms who abscond cannot be excluded from future borrowing whereas they assume that borrowers are in fact excluded from intertemporal trade after default. Similar constraints have been considered by Lustig (2007) in an endowment economy, by Lorenzoni and Walentin (2007) in a production economy with constant returns to scale, and by Rampini and Viswanathan (2010) in a production economy with a finite horizon. Krueger and Uhlig (2006) find that similar limits on enforcement in an endowment economy without collateral imply short-sale constraints, which would be true in our model in the special case where $\theta = 0$.

[12]The assumption that intangible capital cannot be collateralized or leased *at all* simplifies the analysis, but is not required for the main results. Assuming that intangible capital is less collateralizable and more costly to lease would suffice.

and control and the lessor must pay the cost m to ensure that the lessee uses and maintains the asset appropriately.[13] A competitive lessor with a cost of capital $R \equiv 1 + r$ charges a user cost of

$$u_l \equiv r + \delta + m$$

per unit of capital. Equivalently, we could assume that leased capital depreciates faster due to the agency problem at rate $\delta_l > \delta$ and set $m = \delta_l - \delta$. Due to the constraints on enforcement, the user cost of leased capital is charged at the beginning of the period and hence the firm pays $R^{-1}u_l$ per unit of leased capital up front. Specifically, since the lessor recovers the leased capital at the end of the period, no additional payments at the end of the period can be enforced and the leasing fee must hence be charged up front. Recall that in the frictionless neoclassical model, the rental cost of capital is Jorgenson (1963)'s user cost $u \equiv r + \delta$. Thus the only difference to the rental cost in our model is the positive monitoring cost m (or, equivalently, the costs due to faster depreciation $\delta_l - \delta$). Note that as in Jorgenson's definition, we define the user cost of capital in terms of value at the end of the period.[14]

We assume that the firm has access to lenders who have deep pockets in all dates and states and discount the future at rate $R^{-1} \in (\beta, 1)$. These lenders are thus willing to lend in a state-contingent way at an expected return R. The assumption that firms are less patient than lenders, which is quite common,[15] implies that firms' financing policy matters even in the long run, that is, even for mature firms, and that the financing policy is uniquely determined. Moreover, firms are never completely unconstrained and firms which currently pay dividends that are hit by a sequence of low cash flow shocks may eventually stop dividend payments, cut risk management, and switch back to leasing capital, implications that are empirically plausible.[16]

[13] In practice, there may be a link between the lessor's monitoring and the repossession advantage of leasing. In order to monitor the use and maintenance of the asset, the lessor needs to keep track of the asset which makes it harder for the lessee to abscond with it.

[14] To impute the amount of capital rented from rental payments, we should hence capitalize rental payments by $1/(r+\delta+m)$. In documenting the stylized facts, we assumed that this factor takes a value of 10. The implicit debt associated with rented capital is $R^{-1}(1-\delta)$ times the amount of capital rented, so in adjusting liabilities, we should adjust by $R^{-1}(1-\delta)$ times 10 to be precise. In documenting the stylized facts, we ignored the correction $R^{-1}(1-\delta)$, implicitly assuming that it is approximately equal to 1.

[15] For example, this assumption is made by DeMarzo and Sannikov (2006), Lorenzoni and Walentin (2007), Biais, Mariotti, Plantin, and Rochet (2007), Biais, Mariotti, Rochet, and Villeneuve (2010), and DeMarzo, Fishman, He, and Wang (2011); DeMarzo and Fishman (2007a, 2007b) consider $\beta \leq R^{-1}$; in contrast, firms and lenders are assumed to be equally patient in Albuquerque and Hopenhayn (2004), Quadrini (2004), Clementi and Hopenhayn (2006), and Rampini and Viswanathan (2010).

[16] While we do not explicitly consider taxes here, our assumption about discount rates can also be interpreted as a reduced form way of taking into account the tax-deductibility of interest, which effectively

3.2 Firm's problem

The firm's problem can be written recursively as the problem of maximizing the discounted expected value of future dividends by choosing the current dividend d, capital k, leased capital k_l, net worth $w(s')$ in state s' next period, and state-contingent debt $b(s')$ given current net worth w and state s:

$$V(w,s) \equiv \max_{\{d,k,k_l,w(s'),b(s')\}\in\mathbb{R}_+^{3+S}\times\mathbb{R}^S} d+\beta\sum_{s'\in S}\Pi(s,s')V(w(s'),s') \tag{1}$$

subject to the budget constraints for the current and next period

$$w+\sum_{s'\in S}\Pi(s,s')b(s') \geq d+k-(1-R^{-1}u_l)k_l \tag{2}$$

$$A(s')f(k)+(k-k_l)(1-\delta) \geq w(s')+Rb(s'),\ \forall s'\in S, \tag{3}$$

the collateral constraints

$$\theta(\varphi k-k_l)(1-\delta)\geq Rb(s'), \qquad \forall s'\in S, \tag{4}$$

and the constraint that only tangible capital can be leased

$$\varphi k\geq k_l. \tag{5}$$

The program in (1)-(5) requires that dividends d and net worth $w(s')$ are non-negative which is due to limited liability. Furthermore, capital k and leased capital k_l have to be non-negative as well. We write the budget constraints as inequality constraints, despite the fact that they bind at an optimal contract, since this makes the constraint set convex as shown below. There are only two state variables in this recursive formulation, net worth w and the state of productivity s. This is due to our assumption that there are no adjustment costs of any kind and greatly simplifies the analysis. Net worth in state s' next period $w(s')=A(s')f(k)+(k-k_l)(1-\delta)-Rb(s')$, that is, equals cash flow plus the depreciated resale value of owned capital minus the amount to be repaid on state s' contingent debt. Borrowing against state s' next period by issuing state s' contingent debt $b(s')$ reduces net worth $w(s')$ in that state. In other words, borrowing less than the maximum amount which satisfies the collateral constraint (4) against state s' amounts to conserving net worth for that state and allows the firm to hedge the available net worth in that state.

We make the following assumptions about the stochastic process describing productivity and the production function:

lowers the cost of debt finance.

Assumption 1 *For all* $\hat{s}, s \in S$ *such that* $\hat{s} > s$, **(i)** $A(\hat{s}) > A(s)$ *and* **(ii)** $A(s) > 0$.

Assumption 2 f *is strictly increasing, strictly concave,* $f(0) = 0$, $\lim_{k\to 0} f_k(k) = +\infty$, *and* $\lim_{k\to +\infty} f_k(k) = 0$.

We first show that the firm financing problem is a well-behaved dynamic programming problem and that there exists a unique value function V which solves the problem. To simplify notation, we introduce the shorthand for the choice variables x, where $x \equiv [d, k, k_l, w(s'), b(s')]'$, and the shorthand for the constraint set $\Gamma(w, s)$ given the state variables w and s, defined as the set of $x \in \mathbb{R}_+^{3+S} \times \mathbb{R}^S$ such that (2)-(5) are satisfied. Define operator T as

$$(Tg)(w, s) = \max_{x\in\Gamma(w,s)} d + \beta \sum_{s'\in S} \Pi(s, s')g(w(s'), s').$$

We prove the following result about the firm financing problem in (1)-(5):

Proposition 1 **(i)** $\Gamma(w, s)$ *is convex, given* (w, s)*, and convex in* w *and monotone in the sense that* $w \leq \hat{w}$ *implies* $\Gamma(w, s) \subseteq \Gamma(\hat{w}, s)$. **(ii)** *The operator* T *satisfies Blackwell's sufficient conditions for a contraction and has a unique fixed point* V. **(iii)** V *is continuous, strictly increasing, and concave in* w. **(iv)** *Without leasing,* $V(w, s)$ *is strictly concave in* w *for* $w \in int\{w : d(w, s) = 0\}$. **(v)** *Assuming that for all* $\hat{s}, s \in S$ *such that* $\hat{s} > s$, $\Pi(\hat{s}, s')$ *strictly first order stochastically dominates* $\Pi(s, s')$, V *is strictly increasing in* s.

The proofs of part (i)-(iii) of the proposition are relatively standard. Part (iii) however only states that the value function is concave, not strictly concave. The value function is linear in net worth when dividends are paid. The value function may also be linear in net worth on some intervals where no dividends are paid, due to the substitution between leased and owned capital. All our proofs below hence rely on weak concavity only. Nevertheless we can show that without leasing, the value function is strictly concave where no dividends are paid (see part (iv) of the proposition).

Consider the first order conditions of the firm financing problem in equations (1)-(5). Denote the multipliers on the constraints (2), (3), (4), and (5) by μ, $\Pi(s, s')\beta\mu(s')$, $\Pi(s, s')\beta\lambda(s')$, and $\bar{\nu}_l$.[17] Let ν_d and $\underline{\nu}_l$ be the multipliers on the constraint $d \geq 0$ and $k_l \geq 0$.

[17]Note that we scale some of the multipliers by $\Pi(s, s')\beta$ to simplify the notation.

The first order conditions are

$$\mu = 1 + \nu_d \quad (6)$$

$$\mu = \sum_{s' \in S} \Pi(s,s')\beta \left\{\mu(s')\left[A(s')f_k(k) + (1-\delta)\right] + \lambda(s')\theta\varphi(1-\delta)\right\} + \bar{\nu}_l \varphi \quad (7)$$

$$(1 - R^{-1}u_l)\mu = \sum_{s' \in S} \Pi(s,s')\beta \left\{\mu(s')(1-\delta) + \lambda(s')\theta(1-\delta)\right\} + \bar{\nu}_l - \underline{\nu}_l \quad (8)$$

$$\mu(s') = V_w(w(s'),s'), \qquad \forall s' \in S, \quad (9)$$

$$\mu = \beta\mu(s')R + \beta\lambda(s')R, \qquad \forall s' \in S, \quad (10)$$

where we use the fact that the constraints $k \geq 0$ and $w(s') \geq 0$, $\forall s' \in S$, are slack as Lemma 6 in Appendix B shows.[18] The envelope condition is $V_w(w,s) = \mu$; the marginal value of (current) net worth is μ. Similarly, the marginal value of net worth in state s' next period is $\mu(s')$.

Using equations (7) and (10), we obtain the investment Euler equation,

$$1 \geq \sum_{s' \in S} \Pi(s,s')\beta \frac{\mu(s')}{\mu} \frac{A(s')f_k(k) + (1-\theta\varphi)(1-\delta)}{1 - R^{-1}\theta\varphi(1-\delta)}, \quad (11)$$

which holds with equality if the firm does not lease all its tangible assets (that is, $\bar{\nu}_l = 0$). Notice that $\beta\mu(s')/\mu$ is the firm's stochastic discount factor; collateral constraints render the firm as if risk averse and hence provide a rationale for risk management.

3.3 User cost of capital

This section defines the user cost of (purchased) tangible and intangible capital, extending Jorgenson's (1963) definition to our model with collateral constraints.[19] The definitions clarify the main economic intuition behind our results and allow a simple characterization of the leasing decision as we show in Section 5.

Let $\rho(w,s)$ denote the premium on internal funds for a firm with net worth w in state s and define it implicitly using the firm's stochastic discount factor as $1/(1+r+\rho(w,s)) \equiv \sum_{s' \in S} \Pi(s,s')\beta\mu(s')/\mu$. Our definition of the user cost of tangible capital which is purchased $u_p(w,s)$ for a firm with net worth w in state s is

$$u_p(w,s) \equiv r + \delta + \frac{\rho(w,s)}{R + \rho(w,s)}(1-\theta)(1-\delta)$$

[18] Since the marginal product of capital is unbounded as capital goes to zero by Assumption 2, the amount of capital is strictly positive. Because the firm's ability to issue promises against capital is limited, this in turn implies that the firm's net worth is positive in all states in the next period.

[19] Lucas and Prescott (1971), Abel (1983), and Abel and Eberly (1996) define the user cost of capital for models with adjustment costs.

where $\rho(w,s)/(R+\rho(w,s)) = \sum_{s'\in S}\Pi(s,s')R\beta\lambda(s')/\mu$. To emphasize the dependence of the premium on internal funds and the user cost of purchased tangible (and intangible) capital on the firm's net worth w and state s, we make it explicit (in contrast to the dependence of the choice variables and multipliers on w and s which is suppressed throughout). Note that $\rho(w,s) > 0$ as long as the multiplier on the state s' collateral constraint $\lambda(s') > 0$, for some $s' \in S$. The user cost of purchased tangible capital has two components. The first component is simply the Jorgensonian user cost of capital. The second component captures the additional cost of internal funds, which command a premium $\rho(w,s)$ due to the collateral constraints. Indeed, $(1-\theta)(1-\delta)$ is the fraction of the resale value of capital recovered the next period that the firm cannot credibly pledge to lenders and hence is financed internally. Similarly, we define the user cost of intangible capital as $u_i(w,s) \equiv r + \delta + \rho(w,s)/(R+\rho(w,s))(1-\delta)$. The only difference is that intangible capital needs to be financed entirely with internal funds and hence the second term involves fraction $1-\delta$ rather than only a fraction $1-\theta$ of that amount.

Using our definitions of the user cost of capital and equations (8) and (10), we can rewrite the first order condition for capital, equation (7), as

$$\varphi \min\{u_p(w,s), u_l\} + (1-\varphi)u_i(w,s) = \sum_{s'\in S}\Pi(s,s')R\beta\frac{\mu(s')}{\mu}A(s')f_k(k).$$

Optimal investment equates the weighted average of the user cost of tangible and intangible capital with the expected marginal product of capital, where the applicable user cost of tangible capital is the minimum of the user cost of purchased and leased tangible capital.

The user cost of tangible capital can be rearranged in a *weighted average (user) cost of capital* form as

$$u_p(w,s) = \frac{R}{R+\rho(w,s)}\left((r+\rho(w,s))\left(1-R^{-1}\theta(1-\delta)\right) + r\left(R^{-1}\theta(1-\delta)\right) + \delta\right),$$

where the fraction of tangible capital that can be financed with external funds, $R^{-1}\theta(1-\delta)$, is charged a cost of capital r, while the fraction of tangible capital that has to be financed with internal funds, $1-R^{-1}\theta(1-\delta)$, is charged a cost of capital $r+\rho(w,s)$.

3.4 Dividend payout policy

We start by characterizing the firm's payout policy. The firm's dividend policy is very intuitive: there is a state-contingent cutoff level of net worth $\bar{w}(s)$, $\forall s \in S$, above which the firm pays dividends. Moreover, whenever the firm has net worth w exceeding the cutoff $\bar{w}(s)$, paying dividends in the amount $w - \bar{w}(s)$ is optimal. All firms with net worth w exceeding the cutoff $\bar{w}(s)$ in a given state s, choose the same level of capital. Finally, the investment policy is unique in terms of the choice of capital k. The following proposition summarizes the characterization of firms' payout policy:

Proposition 2 (Dividend policy) *There is a state-contingent cutoff level of net worth, above which the marginal value of net worth is one and the firm pays dividends:* **(i)** $\forall s \in S$, $\exists \bar{w}(s)$ *such that,* $\forall w \geq \bar{w}(s)$, $\mu(w,s) = 1$. **(ii)** *For* $\forall w \geq \bar{w}(s)$,

$$[d_o(w,s), k_o(w,s), k_{l,o}(w,s), w_o(s'), b_o(s')] = [w - \bar{w}(s), \bar{k}_o(s), \bar{k}_{l,o}(s), \bar{w}_o(s'), \bar{b}_o(s')]$$

where $\bar{x}_o \equiv [0, \bar{k}_o(s), \bar{k}_{l,o}(s), \bar{w}_o(s'), \bar{b}_o(s')]$ *attains* $V(\bar{w}(s), s)$. *Indeed,* $k_o(w,s)$ *is unique for all* w *and* s. **(iii)** *Without leasing, the optimal policy* x_o *is unique.*

3.5 Effect of tangibility and collateralizability without leasing

We distinguish between the fraction of tangible assets required for production, φ, and the fraction of tangible assets θ that the borrower cannot abscond with and that is hence collateralizable. This distinction is important to understand differences in the capital structure across industries, as the fraction of tangible assets required for production varies considerably at the industry level whereas the fraction of tangible assets that is collateralizable primarily depends on the type of capital, such as structures versus equipment (which we do not distinguish here). Thus, industry variation in φ needs to be taken into account in empirical work. That said, in the special case without leasing, higher tangibility φ and higher collateralizability θ are equivalent in our model. This result is immediate as without leasing, φ and θ affect only (4) and only the product of the two matters. Thus, firms that operate in industries that require more intangible capital are more constrained, all else equal. Furthermore, the fact that firms can only borrow against a fraction $\varphi\theta$ of total capital is quantitatively relevant as the model predicts much lower, and empirically plausible, leverage ratios.

4 Risk management and the capital structure

Our model allows an explicit analysis of dynamic risk management since firms have access to complete markets subject to the collateral constraints. We first provide a general result about the optimal absence of risk management for firms with sufficiently low net worth. We then show how to interpret the state-contingent debt in our model in terms of risk management. Next, we prove the optimality of incomplete hedging with constant investment opportunities, that is, when productivity shocks are independently and identically distributed. Indeed, we show that firms abstain from risk management with positive probability under the stationary distribution, implying that even mature firms who experience a sequence of low cash flows eliminate risk management. Moreover, we study the effect of stochastic investment opportunities on optimal risk management and show that persistent shocks further reduce risk management and may result in a complete absence of risk management for empirically plausible levels of persistence. Strong persistence of productivity may result in firms hedging higher productivity states, because financing needs for increased investment rise more than cash flows.

4.1 Optimal absence of risk management

Severely constrained firms optimally abstain from risk management altogether:

Proposition 3 (Optimal absence of risk management) *Firms with sufficiently low net worth do not engage in risk management, that is,* $\forall s \in S$, $\exists \underline{w}_h(s) > 0$, *such that* $\forall w \leq \underline{w}_h(s)$, *all collateral constraints bind,* $\lambda(s') > 0$, $\forall s' \in S$.

Collateral constraints imply that there is an opportunity cost to issuing promises to pay in high net worth states next period to hedge low net worth states next period, as such promises can also be used to finance current investment.[20] The proposition shows that when net worth is sufficiently low, the opportunity cost of risk management due to the financing needs must exceed the benefit. Hence, the firm optimally does not hedge at all.[21] Notice that the result obtains for a general Markov process for productivity. The result is consistent with the evidence that firms with low net worth hedge less, and is in contrast to the conclusions from static models in the extant literature, such as Froot, Scharfstein, and Stein (1993). The key difference is that our model explicitly considers dynamic financing needs for investment as well as the limits on firms' ability to promise to pay.

In order to characterize risk management and corporate hedging policy, define risk management in terms of financial slack for state s' as

$$h(s') \equiv \theta(\varphi k - k_l)(1-\delta) - Rb(s'). \tag{12}$$

The collateral constraints (4) can then be rewritten as

$$h(s') \geq 0, \qquad \forall s' \in S, \tag{13}$$

[20] Since firms in our model can replicate forward contracts or futures that do not involve payments up front, our proposition holds for forward contracts and futures, too. Such contracts involve promises to pay in some states next period which count against the collateral constraints in those states. There is an opportunity cost for such promises, because promises against these states could alternatively be used to finance current investment. Hence, Proposition 3 implies that firms with sufficiently low net worth do not hedge with forward contracts or futures.

[21] Rampini and Viswanathan (2010) derive a special case of this result in a two period model with uncertainty in the first period only.

implying that financial slack has to be non-negative. Our model with state-contingent debt $b(s')$ thus is equivalent to a model in which firms borrow as much as they can against each unit of tangible capital which they purchase, that is, borrow $R^{-1}\theta(1-\delta)$ per unit of capital, and keep financial slack by purchasing Arrow securities with a payoff of $h(s')$ for state s'. Under this interpretation, firms' debt is not state-contingent, since we assume that the price of capital is constant for all states. Our model with state-contingent borrowing is hence a model of financing and risk management. The proposition above states that all collateral constraints bind, which means that the firm does not purchase any Arrow securities at all. In this sense, the firm does not engage in risk management. In the numerical examples in Subsections 4.2 and 4.4, we show that the extent to which firms hedge low states is in fact increasing in net worth.

In our model, we do not take a stand on whether the productivity shocks are firm specific or aggregate. Since all states are observable, as the only friction considered is limited enforcement, our analysis applies either way. Hedging can hence be interpreted either as using loan commitments, for example, to hedge idiosyncratic shocks to firms' net worth or as using traded assets to hedge aggregate shocks which affect firms' cash flows.

4.2 Risk management with constant investment opportunities

With independent productivity shocks, risk management only depends on the firm's net worth, because the expected productivity of capital is independent of the current state s, that is, investment opportunities are constant. More generally, both cash flows and investment opportunities vary, and the correlation between the two affects the desirability of hedging, as we show in Subsection 4.4 below.

With constant investment opportunities, the marginal value of net worth is higher in states with low cash flows due to low realizations of productivity and complete hedging is never optimal:

Proposition 4 (Optimality of incomplete hedging) *Suppose that* $\Pi(s,s') = \Pi(s')$, $\forall s, s' \in S$. **(i)** *The marginal value of net worth next period* $\mu(s') = V_w(w(s'))$ *is (weakly) decreasing in the state* s', *and the multipliers on the collateral constraints are (weakly) increasing in the state* s', *that is,* $\forall s', s'_+ \in S$ *such that* $s'_+ > s'$, $\mu(s'_+) \leq \mu(s')$ *and* $\lambda(s'_+) \geq \lambda(s')$. **(ii)** *Incomplete hedging is optimal, that is,* $\exists s' \in S$, *such that* $\lambda(s') > 0$. *Indeed,* $\exists s', \hat{s}' \in S$, *such that* $w(s') \neq w(\hat{s}')$. *Moreover, the firm never hedges the highest state, that is, is always borrowing constrained against the highest state,* $\lambda(\bar{s}') > 0$ *where* $\bar{s}' = \max\{s' : s' \in S\}$. *The firm hedges a lower interval of states,* $[\underline{s}', \ldots, s'_h]$, *where* $\underline{s}' = \min\{s' : s' \in S\}$, *if at all.*

Complete hedging would imply that all collateral constraints are slack and consequently the marginal value of net worth is equalized across all states next period. But hedging involves conserving net worth in a state-contingent way at a return R. Given the firm's relative impatience, it can never be optimal to save in this state-contingent way for all states next period. Thus, incomplete hedging is optimal. Further, since the marginal value of net worth is higher in states with low cash flow realizations, it is optimal to hedge the net worth in these states, if it is optimal to hedge at all. Firms' optimal hedging policy implicitly ensures a minimum level of net worth in all states next period.

We emphasize that our explicit dynamic model of collateral constraints due to limited enforcement is essential for this result. If the firm's ability to pledge were not limited, then the firm would always want to pledge more against high net worth states next period to equate net worth across all states. However, in our model the ability to credibly pledge to pay is limited and there is an opportunity cost to pledging to pay in high net worth states next period, since such pledges are also required for financing current investment. This opportunity cost implies that the firm never chooses to fully hedge net worth shocks.

To illustrate the interaction between financing needs and risk management, we compute a numerical example. We assume that productivity is independent and, for simplicity, that productivity takes on two values only, $A(s_1) < A(s_2)$, and that there is no leasing. The results and details of the parameterization are reported in Figure 2.

Investment as a function of net worth is shown in Panel A, which illustrates Proposition 2. Above a threshold $\bar{w}$, firms pay dividends and investment is constant. Below the threshold, investment is increasing in net worth and dividends are zero.

The dependence of the risk management policy on net worth is illustrated in Panel B. With independent shocks, Proposition 4 implies that the firm never hedges the high state, that is, $h(s_2') = 0$, where $h(s')$ is defined as in equation (12). Panel B thus displays the extent to which the firm hedges the low state only, that is, the payoff of the Arrow securities that the firm purchases to hedge the low state, $h(s_1')$. Most importantly, note that the hedging policy is increasing in firm net worth, that is, better capitalized firms hedge more. This illustrates the main conclusion from our model for risk management. Above the threshold $\bar{w}$, risk management is constant (as Proposition 2 shows). Below the threshold, hedging is increasing, and for sufficiently low values of net worth w, the firm does not hedge at all, as Proposition 3 shows more generally. Note that hedging is zero until net worth reaches a value of around 0.1 in the example, that is, for a sizable range of values of net worth, then increases, and then is constant above $\bar{w}$.

The values of net worth next period are displayed in Panel C and illustrate the endogenous dynamics of net worth. The figure illustrates the optimality of incomplete hedging from Proposition 4. Net worth next period is higher in state s_2' than in state s_1' despite the fact that firms have access to complete markets (except for collateral constraints). The figure moreover plots the 45 degree line (dotted), that is, the locus where $w = w'$, to facilitate the characterization of the stationary distribution. The ergodic set of net worth must be bounded below by the intersection of $w(s_1')$ and the 45 degree line, which we denote $\underline{w}(s_1)$, and above by the intersection of $w(s_2')$ and the 45 degree line, which we denote $\bar{w}(s_2)$. The support of the stationary distribution is a subset of the interval $[\underline{w}(s_1), \bar{w}(s_2)]$. Levels of net worth below $\underline{w}(s_1)$ and above $\bar{w}(s_2)$ are transient. Indeed, firms with net worth above $\bar{w}(s_2)$ will pay out the extra net worth and start the next period within the ergodic set. Moreover, evaluating the first order condition (10) for $b(s')$ at $\underline{w}(s_1)$ and $s' = s_1$, we have $V_w(\underline{w}(s_1)) = R\beta V_w(\underline{w}(s_1)) + R\beta\lambda(\underline{w}(s_1))$ and thus $\lambda(\underline{w}(s_1)) > 0$. This means that the firm abstains from risk management altogether at $\underline{w}(s_1)$. By continuity, the absence of risk management is optimal for sufficiently low values of net worth in the stationary distribution. This is, in fact, a general result, as Proposition 5 below shows.

The multipliers on the collateral constraints $\beta\lambda(s')$ are shown in Panel D. Recall that the first order conditions (10) for $b(s')$ imply that $\mu(s_1') + \lambda(s_1') = \mu(s_2') + \lambda(s_2')$. The firm thus does not simply equate the marginal value of net worth across states, but the sum of the marginal value of net worth and the multiplier on the collateral constraint. From Proposition 4 we know that $\lambda(s_2') > 0$ for all w and that $\lambda(s_2') \geq \lambda(s_1')$ as the figure shows. Moreover, for levels of w at which the firm (partially) hedges the low state, the multiplier on the collateral constraint for the low state $\lambda(s_1') = 0$. For lower levels of net worth w the firm abstains from risk management and $\lambda(s_1') > 0$. Collateral constraints result in a trade off between financing and risk management.

4.3 Risk management under the stationary distribution

We now show that firms abstain from risk management at the lower bound of the stationary distribution as we observed in the example above. Indeed, there is a unique stationary distribution and firms abstain from risk management with positive probability under the stationary distribution.

Proposition 5 (Absence of risk management under the stationary distribution) *Suppose* $\Pi(s, s') = \Pi(s')$, $\forall s, s' \in S$, *and* $m = +\infty$ *(no leasing).* **(i)** *For the lowest state* $\underline{s}'$, *the wealth level* $\underline{w}$ *for which* $w(\underline{s}') = \underline{w}$ *is unique and the firm abstains from risk management at* $\underline{w}$. **(ii)** *There exists a unique stationary distribution of net worth and the firm abstains from risk management with positive probability under the stationary distribution.*

Proposition 5 implies that even if a firm is currently relatively well capitalized and paying dividends, a sufficiently long sequence of low cash flows will leave the firm so constrained that it chooses to discontinue risk management.[22] Thus, it is not only young firms with very low net worth that abstain from risk management, but also mature firms that suffer adverse cash flow shocks. Consistently, Rampini, Sufi, and Viswanathan (2011) show that airlines that hit financial distress dramatically cut their fuel price risk management.

4.4 Risk management with stochastic investment opportunities

With stochastic investment opportunities, risk management depends not only on net worth but also on the firm's productivity, since the conditional expectation of future productivity varies with current productivity when productivity is persistent. Positive autocorrelation reduces the benefit to hedging and may eliminate the need to hedge completely.

As a first step, we show that incomplete hedging is optimal even when investment opportunities are stochastic:

Proposition 6 (Optimality of incomplete hedging with persistence) *Optimal risk management is incomplete with positive probability, that is, $\exists s$ such that for all w, $\lambda(s') > 0$, for some s'.*

This proposition shows that firms engage in incomplete risk management for any Markov process of productivity, generalizing the results from Proposition 4.

With a general Markov process for productivity, as in the case without persistence, firms engage in risk management to transfer funds into states in which the marginal value of net worth is highest (if at all). With persistence, however, the marginal value of net worth depends not only on net worth but also on investment opportunities going forward. With positive autocorrelation, which is typical in practice, investment opportunities are worse when productivity is low, reducing the benefits of hedging such states; similarly, investment opportunities are higher when productivity is high, increasing the marginal value of net worth in such states. This reduces the benefit to hedging and thus firms hedge less or not at all. Indeed, if this effect is strong enough, firms may hedge states with high productivity, despite the fact that cash flows are high then, too. Current investment opportunities also affect the benefits to investing and the opportunity cost of hedging, and thus the extent of risk management depends on current productivity as well.

To further characterize the effect of stochastic investment opportunities, we reconsider the example with a two state symmetric Markov process for productivity from the previous subsection and study the effect of persistence. The results are reported in Figure 3. We

[22] Equation (10) implies that there is an upper bound on the extent to which the marginal value of net worth can increase as $V_w(w(s'), s')/V_w(w, s) \leq (\beta R)^{-1}$; however, the marginal value of net worth could increase, for example, when cash flows are sufficiently low.

increase the transition probabilities $\Pi(s_1, s_1) = \Pi(s_2, s_2) \equiv \pi$, which are 0.5 when investment opportunities are constant, progressively to 0.55, 0.60, 0.75, and 0.90. Note that the autocorrelation of a symmetric two state Markov process is $2\pi - 1$; the autocorrelation is thus progressively raised from 0 to 0.1, 0.2, 0.5, and 0.8. Given the symmetry, the stationary distribution of the productivity process over the two states is 0.5 and 0.5 across all our examples, and the unconditional expected productivity is hence the same as well. Panels A and B display the investment and hedging policies in the case without persistence from the previous subsection (see also Panels A and B of Figure 2). With constant investment opportunities, the policy functions are independent of the state of productivity s and hence there is only one function for each policy. Panels C and D show the investment and hedging policies with some positive autocorrelation. The solid lines denote the policies when current productivity is low (s_1) and the dashed lines the ones when current productivity is high (s_2). Panel C shows that, for given net worth w, investment is higher when current productivity is higher, because the conditional expected productivity is higher. Panel D shows that hedging (for the low state) decreases relative to the case of constant investment opportunities, but more so when current productivity is high. Most notably, for given net worth, the firm keeps less financial slack (for the low state) when current productivity is high than when it is low. The economic intuition has two aspects. First, hedging decreases relative to the case of constant investment opportunities because persistent shocks reduce the marginal value of net worth when productivity is low (since the conditional expected productivity is low then too) while raising the marginal value of net worth when productivity is high. Thus, there is less reason to hedge. Second, with persistence, high current productivity implies high expected productivity (as well as higher opportunity cost of risk management) and higher investment, which raises net worth next period, further reducing the hedging need when current productivity is high. In contrast, when current productivity is low (and expected productivity and the opportunity cost of risk management are low), investment and hence net worth next period is reduced, which would raise the need for risk management all else equal. However, the first effect goes the other way and dominates the second effect in our example.

When persistence is raised further, as illustrated in Panels E and F, the benefit to hedging is still lower and the firm abstains from hedging completely when current productivity is high and only hedges (the low state) when current productivity is low. Persistent productivity shocks thus further reduce the optimal amount of risk management. Indeed, when persistence is still higher, the benefit to hedging is so low that the firm does not hedge any state (see Panels G and H). Note this level of persistence, implying an autocorrelation of productivity of 0.5, is not unreasonable; for example, Gomes (2001) uses an autocorrelation of 0.62 in his calibrations. This suggests that, for empirically plausible parameterizations, even firms with high net worth may engage in only limited risk management or none at all.

For the persistence levels considered thus far, firms hedge the low cash flow state when they engage in risk management. With severe persistence (see Panels I and J), the difference in investment is very large across the two productivity states (e.g., a factor of more than 10 for dividend paying firms). Consequently, there is an incentive to hedge the high state due to its substantially greater investment opportunities when current productivity is low. Indeed, the dash-dotted line in Panel J denotes the hedging of the high state next period when the current state is low. But notice that even the hedging of investment opportunities is increasing in net worth.

This example illustrates the dynamic trade-off between financing current investment and risk management. First, current expected productivity affects the benefits to investing and hence the opportunity cost of risk management. Second, expected productivity next period affects the benefits to hedging and which states the firm hedges; for plausible levels of persistence even less constrained firms may abstain from risk management. However, whatever the persistence, firms do not hedge at all when they are severely constrained.

5 Leasing and the capital structure

This section analyzes the dynamic leasing decision. We first prove a general result about the optimality of leasing for firms with sufficiently low net worth. We then analyze the dynamic choice between leasing and secured financing in the deterministic case, which facilitates explicit characterization because the collateral constraint binds throughout, to highlight the economic intuition; leasing allows firms to grow faster. In the stochastic case, leasing and risk management are jointly determined and firms that lease may engage in risk management because leasing enables higher leverage but reduces net worth in low cash flow states. Mature firms may engage in sale-and-leaseback transactions when they experience adverse cash flow shocks. Moreover, the model's implications for the capital structure are consistent with the empirical facts documented in Section 2.

5.1 Optimality of leasing

Using the definitions of the user cost of tangible capital and (10), the first order condition with respect to leased capital, (8), simplifies to

$$u_p(w, s) = u_l + \frac{R}{\mu}(\bar{\nu}_l - \underline{\nu}_l). \tag{14}$$

The decision between purchasing capital and leasing reduces to a straight comparison of the user costs. If the user cost of leasing exceeds the user cost of purchased capital, which depends on the firm's net worth w and state s, $\underline{\nu}_l > 0$ and the firm purchases all capital. If the reverse is true, $\bar{\nu}_l > 0$ and all capital is leased. When $u_p(w,s) = u_l$, the firm is indifferent between leasing and purchasing capital at the margin.

Using equation (8) and (10) we moreover obtain an Euler equation for the substitution of purchased capital for leased capital

$$1 = \sum_{s' \in S} \Pi(s,s')\beta \frac{\mu(s')}{\mu} \frac{(1-\theta)(1-\delta)}{(1-R^{-1}\theta(1-\delta)) - R^{-1}u_l} + \frac{\bar{\nu}_l - \underline{\nu}_l}{(1-R^{-1}\theta(1-\delta)) - R^{-1}u_l}. \quad (15)$$

Purchasing a unit of tangible capital instead of leasing it requires an additional outlay of $(1-R^{-1}\theta(1-\delta)) - R^{-1}u_l$, which is the difference between the minimal down payment required to purchase a unit of capital and the leasing fee, and results in an incremental payoff of $(1-\theta)(1-\delta)$, which is the amount that the firm cannot pledge to lenders. The return on buying instead of leasing is thus constant, whereas the return on investment is decreasing as one can see from the investment Euler equation (11). The basic tradeoff is that when the firm's net worth is low, investment is low and hence the marginal return on investment is high whereas the return on purchasing instead of leasing is constant, and hence the firm does not purchase any of its tangible capital. In contrast, when the firm's net worth is high, investment is high and the marginal return on investment low, and the firm may purchase some or all of its tangible capital.

The following assumption ensures that the monitoring cost are such that leasing is neither dominated nor dominating, which rules out the uninteresting special cases in which firms never lease or always lease tangible assets:

Assumption 3 *Leasing is neither dominated nor dominating, that is,*

$$(1-\theta)(1-\delta) > m > (1-R\beta)(1-\theta)(1-\delta).$$

We maintain this assumption throughout. The left most expression and the right most expression are the opportunity costs of the additional down payment requirement when purchasing capital, which depend on the firm's discount rate. The amount due to the additional down payment requirement recovered the next period is $(1-\theta)(1-\delta)$. If the firm is very constrained, the recovered funds are not valued at all, which yields the expression on the left. The left inequality moreover ensures that the additional outlay for purchasing capital instead of leasing it in equation (15) above is strictly positive. If the firm is least constrained, the recovered funds are valued at β, the discount factor of the firm, and the opportunity cost is only the wedge between the funds discounted at the lenders' discount rate and the firm's discount rate, hence, the term $R\beta$.

We can now prove that severely constrained firms lease all their tangible assets:

Proposition 7 (Optimality of leasing) *Firms with sufficiently low net worth lease all tangible capital, that is,* $\forall s \in S$, $\exists \underline{w}_l(s) > 0$, *such that* $\forall w \leq \underline{w}_l(s)$, $k_l = \varphi k$.

The proposition holds for any Markov process for productivity, and hence cash flows, and does not require any further assumptions. The intuition is that when net worth is sufficiently low, the firm's investment must be very low and hence its marginal product very high. But then the firm's financing need must be so severe, that it must find the higher debt capacity of leasing worthwhile.

5.2 Dynamic deterministic choice between leasing and financing

We consider the capital structure dynamics in the deterministic case next. To start, consider the deterministic dynamics of firm financing without leasing. As long as net worth is below a cutoff $\bar{w}$, firms pay no dividends and accumulate net worth over time which allows them to increase the amount of capital they deploy. Once net worth reaches $\bar{w}$, dividends are positive and firms no longer grow.

When leasing is an option, firms have to choose a leasing policy in addition to the investment, financing and payout policy. In this case, the financing dynamics are as follows: when firms have low net worth, they lease all the tangible capital and purchase only the intangible capital. Over time, firms accumulate net worth and increase their total capital. When they reach a certain net worth threshold, they start to substitute owned capital for leased capital, continuing to accumulate net worth. Once firms own all their tangible and intangible capital, they further accumulate net worth and increase the capital stock until they start to pay dividends. At that point, capital stays constant. The following proposition summarizes the deterministic dynamics:

Proposition 8 (Deterministic capital structure dynamics) **(i)** *Suppose* $m = +\infty$ *(no leasing). For* $w \leq \bar{w}$*, no dividends are paid and capital is strictly increasing in* w *and over time. For* $w > \bar{w}$*, dividends are strictly positive and capital is constant at a level* $\bar{k}$*.* **(ii)** *Suppose* m *satisfies Assumption 3. For* $w \leq \bar{w}$*, no dividends are paid and capital is increasing in* w *and over time. For* $w > \bar{w}$*, dividends are strictly positive and capital is constant at a level* $\bar{k}$*. There exist* $\underline{w}_l < \bar{w}_l < \bar{w}$*, such that for* $w \leq \underline{w}_l$ *all tangible capital is leased and for* $w < \bar{w}_l$ *some capital is leased with the fraction of capital leased linearly decreasing in* w *between* $\underline{w}_l$ *and* $\bar{w}_l$*.*

Leasing allows constrained firms to grow faster. To see this note that the minimum amount of internal funds required to purchase one unit of capital is $1 - R^{-1}\theta\varphi(1-\delta)$, since the firm can borrow against fraction θ of the resale value of tangible capital, which is fraction φ of capital. The minimum amount of internal funds required when tangible capital is leased is $1 - \varphi + R^{-1}u_l\varphi$, since the firm has to finance all intangible capital with internal funds $(1 - \varphi)$ and pay the leasing fee on tangible capital up front $(R^{-1}u_l\varphi)$. Per unit of internal funds, the firm can hence buy capital in the amount of one over these minimum amounts of internal funds. Under Assumption 3, leasing allows higher leverage, that is, $1/(1 - \varphi + R^{-1}u_l\varphi) > 1/(1 - R^{-1}\theta\varphi(1 - \delta))$. Thus, leasing allows firms to deploy more capital and hence to grow faster.

Corollary 1 (Leasing and firm growth) *Leasing enables firms to grow faster.*

The same economic intuition carries over to the stochastic case analyzed below. However, the high leverage that leasing entails can change firms' risk management policy considerably.

5.3 Leasing, leverage, and risk management

To study the leasing, financing, and risk management policy jointly, we consider the stochastic example with constant investment opportunities analyzed in Subsection 4.2 and introduce leasing. The parameters are mostly as before, with details in the caption of Figure 4 which displays the results. Panel A displays the investment and leasing policy, which is very similar to the one in the deterministic case; more constrained firms lease more, if not all, of their tangible capital; further, as net worth increases, firms substitute owning and borrowing for leasing and eventually stop leasing altogether.

Particularly noteworthy are the implications for risk management in Panel B. For high values of net worth the figure shows the by now familiar pattern for risk management, with risk management increasing in net worth until the dividend paying region is reached and constant from there on. However, for lower values of net worth at which the firm leases a substantial amount of capital, risk management first increases and then drops quite dramatically and in fact drops back to zero. To understand this result, recall that leasing allows higher leverage and hence firms which are very constrained choose to lease to be able to lever up more. But the high implicit leverage reduces firms' net worth in the low state, and, if this effect is sufficiently strong, firms undo some of it by keeping some financial slack for the low cash flow state next period. Effectively, firms use leasing to borrow more from the high state next period while at least partially undoing the effects of higher leverage for the low state next period via risk management. Panel D shows that the multiplier on the collateral constraint for the low state next period $(\lambda(s_1))$ is non-monotone. It is positive for low values of net worth, consistent with Proposition 7 (and Proposition 3). It is zero for an interval in which firms engage in risk management when leasing a substantial amount of capital. It then is positive again when firms purchases enough of their capital and finally goes back to zero for sufficiently well capitalized firms.

The transition function of net worth is displayed in Panel C and is reminiscent of Panel C of Figure 2. What is remarkable however is that the solid line which denotes $w(s_1')$ crosses the 45 degree line below the point where firms lease the maximal amount. Recall that this intersection bounds the support of the stationary distribution from below. This implies that even mature firms that are hit by a sequence of low productivity shocks eventually will return to leasing capital, that is, firms engage in sale-leaseback transactions under the stationary distribution. Sale-leaseback transactions by financially constrained firms are relatively common in practice. In our model, such transactions enable firms that are hit by adverse shocks to free up net worth and cut investment by less than they would have to otherwise.

Panel E illustrates the implication that rental leverage is decreasing in net worth while debt leverage is increasing and total leverage is approximately constant, which is consistent with the empirical leverage size relation shown in Figure 1 and Table 3. Thus, our model matches the basic cross sectional facts on leasing and the capital structure documented in Section 2.

6 Conclusion

We argue that collateral determines the capital structure. We provide a dynamic agency based model of the capital structure of a firm with a standard neoclassical production function subject to collateral constraints due to limited enforcement. In the model firms require both tangible and intangible capital, and the fraction of tangible assets required is a key determinant of leverage and the dynamics of firm financing.

There is a fundamental connection between firms' financing and risk management policy, since both involve promises to pay by firms, and financing needs can override hedging concerns. In fact, poorly capitalized firms optimally do not engage in risk management and firms abstain from risk management with positive probability under the stationary distribution. Our dynamic model which allows explicit analysis of the financing needs for investment and the limits on firms' ability to promise to pay is critical for this result. Moreover, we prove the optimality of incomplete hedging. It is not optimal for the firm to hedge to the point that the marginal value of internal funds is equal across all states. Our dynamic analysis of risk management shows that for plausible levels of the autocorrelation of productivity, firms may not hedge at all and that even dividend-paying firms that are hit by a sequence of adverse shocks eventually become so constrained that they cut risk management.

Firms' ability to lease capital is explicitly taken into account in contrast to previous dynamic models of firm financing and investment with financial constraints. The extent to which firms lease is determined by firms' financial condition, and more constrained firms lease more. Indeed, severely constrained firms find it optimal to lease all their tangible capital. Sale-leaseback transactions free up net worth and can be an optimal response to adverse cash flow shocks. Moreover, leasing enables firms to grow faster. Leased capital is an important mode of financing, in particular for constrained firms, and should be taken into account not only in corporate finance, but also in studies of the effect of financing on development and growth. Indeed, changes to financial accounting standards are currently being considered by accounting boards that would result in adjustments to firms' capital structure similar to the ones suggested here.

We also provide stylized empirical facts that highlight the importance of tangibility as a determinant of the capital structure in the data. Firm leverage changes substantially with the fraction of assets which is tangible. Moreover, the lack of tangible assets largely explains why some firms have low leverage, and hence addresses the "low leverage puzzle." Leased capital is quantitatively important and further reduces the fraction of firms with low leverage.

We conclude that the tangibility of assets and firms' ability to lease capital are critical determinants of the capital structure. The simple form of the optimal contract in our dynamic agency based capital structure model should facilitate the calibration and empirical implementation, which has remained a challenge for other such agency based models. Moreover, due to its simplicity, our model may also prove to be a useful framework to address other theoretical questions in dynamic corporate finance and financial macroeconomics.

Appendix A: Enforcement versus collateral constraints

In this appendix we prove the equivalence of enforcement constraints and collateral constraints. For simplicity, we abstract from the option to lease capital, but the proof can be extended in a straightforward way by recognizing that the firm cannot abscond with leased capital. The firm's problem with limited enforcement at any time $\tau \geq 0$, denoted $P_\tau(w(s^\tau))$, is the problem of maximizing the discounted expected value of future dividends by choosing the sequence of dividends, capital levels, and net payments to the lender $\{x(s^t)\}_{t\geq\tau}$, where $x(s^t) = \{d(s^t), k(s^t), p(s^t)\}$ and $s^t \equiv \{s_0, s_1, \ldots, s_t\}$, given current net worth $w(s^\tau)$ and history s^τ to maximize

$$E_\tau \left[\sum_{t=\tau}^{\infty} \beta^{(t-\tau)} d_t \right] \tag{16}$$

subject to the budget constraints

$$\begin{aligned} w(s^\tau) &\geq d(s^\tau) + k(s^\tau) + p(s^\tau), && (17) \\ A(s^t) f(k(s^{t-1})) + k(s^{t-1})(1-\delta) &\geq d(s^t) + k(s^t) + p(s^t), \qquad \forall t > \tau, && (18) \end{aligned}$$

the lender's participation constraint,

$$E_\tau \left[\sum_{t=\tau}^{\infty} R^{-(t-\tau)} p_t \right] \geq 0 \tag{19}$$

and the enforcement constraints

$$E_{\tau'} \left[\sum_{t=\tau'}^{\infty} \beta^{(t-\tau')} d_t \right] \geq E_{\tau'} \left[\sum_{t=\tau'}^{\infty} \beta^{(t-\tau')} \hat{d}_t \right], \quad \forall \tau' \geq \tau, \text{ and } \forall \{\hat{d}(s^t)\}_{t=\tau'}^{\infty}, \tag{20}$$

where $\{\hat{d}(s^t)\}_{t=\tau'}^{\infty}$ together with $\{\hat{k}(s^t)\}_{t=\tau'}^{\infty}$ and $\{\hat{p}(s^t)\}_{t=\tau'}^{\infty}$ solve $P_{\tau'}(w(s^{\tau'}))$ given net worth $w(s^{\tau'}) = A(s^{\tau'}) f(k(s^{\tau'-1})) + (1-\theta\varphi) k(s^{\tau'-1})(1-\delta)$, that is, the same problem with a different level of net worth. We say a sequence of net payments is implementable if it satisfies the lender's participation constraint and the enforcement constraints.

Proposition 9 (Equivalence of enforcement and collateral constraints) **(i)** *Any sequence of net payments $\{p(s^t)\}_{t=\tau}^{\infty}$ to the lender is implementable in problem $P_\tau(w(s^\tau))$ iff*

$$\theta\varphi k(s^{\tau'-1})(1-\delta) \geq E_{\tau'} \left[\sum_{t=\tau'}^{\infty} R^{-(t-\tau')} p_t \right], \qquad \forall \tau' \geq \tau, \tag{21}$$

that is, the present value of the remaining net payments never exceeds the current collateral value. **(ii)** *Moreover, the set of sequences of net payments that satisfy (21) is equivalent to the set of sequences of one period state contingent claims $\{b(s^t)\}_{t=\tau}^{\infty}$ which satisfy*

$$\theta\varphi k(s^{t-1})(1-\delta) \geq R b(s^t), \qquad \forall t > \tau, \tag{22}$$

Proof of Proposition 9. Part (i): Suppose the sequence $\{p(s^t)\}_{t=\tau}^{\infty}$ is such that (21) is violated for some $s^{\tau'}$, $\tau' > \tau$, that is,

$$\theta\varphi k(s^{\tau'-1})(1-\delta) < E_{\tau'}\left[\sum_{t=\tau'}^{\infty} R^{-(t-\tau')} p_t\right].$$

Without loss of generality, assume $\tau' = \tau + 1$. Suppose the firm defaults in state $s^{\tau+1}$ at time $\tau + 1$ and issues a new sequence of net payments $\{\hat{p}(s^t)\}_{t=\tau+1}^{\infty}$ such that

$$E_{\tau+1}\left[\sum_{t=\tau+1}^{\infty} R^{-(t-(\tau+1))}\hat{p}_t\right] = 0$$

with $\hat{p}(s^t) = p(s^t)$, $\forall t > \tau + 1$, and $\hat{p}(s^{\tau+1}) = -E_{\tau+1}\left[\sum_{t=\tau+2}^{\infty} R^{-(t-(\tau+1))}\hat{p}_t\right]$ (which has zero net present value by construction), keeping the dividend and investment policies the same, except for the dividend in state $s^{\tau+1}$ at time $\tau + 1$. This dividend increases since the firm makes payment $\hat{p}(s^{\tau+1})$ instead of $p(s^{\tau+1})$ while buying back the tangible assets which have been seized, that is, $\theta\varphi k(s^{\tau})(1-\delta)$, and thus

$$\begin{aligned}\hat{d}(s^{\tau+1}) &= d(s^{\tau+1}) + \big(p(s^{\tau+1}) - \hat{p}(s^{\tau+1}) - \theta\varphi k(s^{\tau})(1-\delta)\big) \\ &= d(s^{\tau+1}) + \left(E_{\tau+1}\left[\sum_{t=\tau+1}^{\infty} R^{-(t-(\tau+1))}p_t\right] - \theta\varphi k(s^{\tau})(1-\delta)\right) > d(s^{\tau+1}).\end{aligned}$$

Such a deviation would hence constitute an improvement, a contradiction. Conversely, if (21) is satisfied $\forall \tau' \geq \tau$, then defaulting cannot make the firm better off.

Part (ii): Take any sequence of net payments $\{p(s^t)\}_{t=\tau}^{\infty}$ and define

$$Rb(s^{\tau'}) \equiv E_{\tau'}\left[\sum_{t=\tau'}^{\infty} R^{-(t-\tau')}p_t\right] \leq \theta\varphi k(s^{\tau'-1})(1-\delta), \qquad \forall \tau' > \tau.$$

Then $Rb(s^{\tau'}) = p(s^{\tau'}) + R^{-1}E_{\tau'}\left[Rb(s^{\tau'+1})\right]$ and thus $p(s^{\tau'}) = Rb(s^{\tau'}) - E_{\tau'}\left[b(s^{\tau'+1})\right]$ and equation (18) can be rewritten as

$$A(s^t)f(k(s^{t-1})) + k(s^{t-1})(1-\delta) + E_t\left[b(s^{t+1})\right] \geq d(s^t) + k(s^t) + Rb(s^t), \qquad \forall t > \tau. \tag{23}$$

Similarly, setting $b(s^{\tau}) = 0$ yields $p(s^{\tau}) = -E_{\tau}\left[b(s^{\tau+1})\right]$. Thus, any sequence of net payments satisfying (21) can be implemented with a sequence of one period contingent claims satisfying (22).

Conversely, take any sequence $\{b(s^t)\}_{t=\tau}^{\infty}$ satisfying (22) and define $p(s^t) = Rb(s^t) - E_t\left[b(s^{t+1})\right]$, $\forall t \geq \tau$. Then, $\forall \tau' > \tau$,

$$E_{\tau'}\left[\sum_{t=\tau'}^{\infty} R^{-(t-\tau')}p_t\right] = E_{\tau'}\left[\sum_{t=\tau'}^{\infty} R^{-(t-\tau')}(Rb^t - b^{t+1})\right] = Rb(s^{\tau'}) \leq \theta\varphi k(s^{\tau'-1})(1-\delta),$$

that is, the sequence of one period contingent claims satisfying (22) can be implemented with a sequence of net payments satisfying (21). □

Given Proposition 9, the sequence problem P_τ in equations (16)-(20) is equivalent to the problem of maximizing (16) subject to $w(s^\tau) \geq d(s^\tau) + k(s^\tau) - E_\tau\left[b(s^{\tau+1})\right]$, (22), and (23). Defining the net worth after repayment of the one period claims issued the previous period as $w(s^t) \equiv A(s^t)f(k(s^{t-1})) + k(s^{t-1})(1-\delta) - Rb(s^t)$, $\forall t > \tau$, the problem can be written recursively as in equations (1)-(5).

Appendix B: Proofs

Proof of Proposition 1. The proposition is proved in Lemma 1-5 below.

Lemma 1 $\Gamma(w,s)$ *is convex, given* (w,s)*, and convex in* w *and monotone in the sense that* $w \leq \hat{w}$ *implies* $\Gamma(w,s) \subseteq \Gamma(\hat{w},s)$.

Proof of Lemma 1. Suppose $x, \hat{x} \in \Gamma(w,s)$. For $\phi \in (0,1)$, let $x_\phi \equiv \phi x + (1-\phi)\hat{x}$. Then $x_\phi \in \Gamma(w,s)$ since equations (2), (4), and (5), as well as the right hand side of equation (3) are linear and, since f is concave,

$$\begin{aligned} A(s')f(k_\phi) + (k_\phi - k_{l,\phi})(1-\delta) \quad \geq \quad & \phi[A(s')f(k) + (k-k_l)(1-\delta)] \\ & +(1-\phi)[A(s')f(\hat{k}) + (\hat{k}-\hat{k}_l)(1-\delta)]. \end{aligned}$$

Let $x \in \Gamma(w,s)$ and $\hat{x} \in \Gamma(\hat{w},s)$. For $\phi \in (0,1)$, let $x_\phi \equiv \phi x + (1-\phi)\hat{x}$. Since equations (3), (4), and (5) do not involve w and $\hat{w}$, respectively, and $\Gamma(w,s)$ is convex given w, x_ϕ satisfies these equations. Moreover, since x and $\hat{x}$ satisfy equation (2) at w and $\hat{w}$, respectively, and equation (2) is linear in x and w, x_ϕ satisfies the equation at w_ϕ. Thus, $x_\phi \in \Gamma(\phi w + (1-\phi)\hat{w}, s)$. In this sense, $\Gamma(w,s)$ is convex in w.

If $w \leq \hat{w}$, then $\Gamma(w,s) \subseteq \Gamma(\hat{w},s)$. □

Lemma 2 *The operator* T *satisfies Blackwell's sufficient conditions for a contraction and has a unique fixed point* V.

Proof of Lemma 2. Suppose $\hat{g}(w,s) \geq g(w,s)$, for all $(w,s) \in \mathbf{R}_+ \times S$. Then, for any $x \in \Gamma(w,s)$,

$$(T\hat{g})(w,s) \geq d + \beta \sum_{s' \in S} \Pi(s,s')\hat{g}(w(s'),s') \geq d + \beta \sum_{s' \in S} \Pi(s,s')g(w(s'),s').$$

Hence,

$$(T\hat{g})(w,s) \geq \max_{x \in \Gamma(w,s)} d + \beta \sum_{s' \in S} \Pi(s,s')g(w(s'),s') = (Tg)(w,s)$$

for all $(w,s) \in \mathbb{R}_+ \times S$. Thus, T satisfies monotonicity.

Operator T satisfies discounting since

$$T(g+a)(w,s) \leq \max_{x \in \Gamma(w,s)} d + \beta \sum_{s' \in S} \Pi(s,s')(g+a)(w(s'),s') = (Tg)(w,s) + \beta a.$$

Therefore, operator T is a contraction and, by the contraction mapping theorem, has a unique fixed point V. □

Lemma 3 *V is strictly increasing and concave in w.*

Proof of Lemma 3. Let $x_o \in \Gamma(w,s)$ and $\hat{x}_o \in \Gamma(\hat{w},s)$ attain $(Tg)(w,s)$ and $(Tg)(\hat{w},s)$, respectively. Suppose g is increasing in w and suppose $w \leq \hat{w}$. Then,

$$(Tg)(\hat{w},s) = \hat{d}_o + \beta \sum_{s' \in S} \Pi(s,s') g(\hat{w}_o(s'),s') \geq d + \beta \sum_{s' \in S} \Pi(s,s') g(w(s'),s').$$

Hence,

$$(Tg)(\hat{w},s) \geq \max_{x \in \Gamma(w,s)} d + \beta \sum_{s' \in S} \Pi(s,s') g(w(s'),s') = (Tg)(w,s),$$

that is, Tg is increasing in w. Moreover, suppose $w < \hat{w}$, then

$$(Tg)(\hat{w},s) \geq (\hat{w} - w) + d_o + \beta \sum_{s' \in S} \Pi(s,s') g(w_o(s'),s') > (Tg)(w,s),$$

implying that Tg is strictly increasing. Hence, T maps increasing functions into strictly increasing functions, which implies that V is strictly increasing.

Suppose g is concave. Then, for $\phi \in (0,1)$, let $x_{o,\phi} \equiv \phi x_o + (1-\phi)\hat{x}_o$ and $w_\phi \equiv \phi w + (1-\phi)\hat{w}$, we have

$$\begin{aligned}
(Tg)(w_\phi,s) &\geq d_{o,\phi} + \beta \sum_{s' \in S} \Pi(s,s') g(w_{o,\phi}(s'),s') \\
&\geq \phi \left[d_o + \beta \sum_{s' \in S} \Pi(s,s') g(w_o(s'),s') \right] + (1-\phi) \left[\hat{d}_o + \beta \sum_{s' \in S} \Pi(s,s') g(\hat{w}_o(s'),s') \right] \\
&= \phi (Tg)(w,s) + (1-\phi)(Tg)(\hat{w},s).
\end{aligned}$$

Thus, Tg is concave, and T maps concave functions into concave functions, which implies that V is concave. Since V is increasing and concave in w, it must be continuous in w. □

Lemma 4 *Without leasing, $V(w,s)$ is strictly concave in w for $w \in int\{w : d(w,s) = 0\}$.*

Proof of Lemma 4. Without leasing, k_l is set to zero throughout and all the prior results continue to hold. Suppose $w, \hat{w} \in int\{w : d(w,s) = 0\}$, $\hat{w} > w$. There must exists some state s_*^t, where $s^t = \{s_0, s_1, \ldots, s_t\}$, which has strictly positive probability and in which the capital stock choice at $\hat{w}$ is different from the choice at w, i.e., $\hat{k}(s_*^t) \neq k(s_*^t)$. Suppose instead that $\hat{k}(s^t) = k(s^t)$, $\forall s^t \in S^t, t = 0, 1, \ldots$. Then there must exist some state s_{**}^t with strictly positive probability in which $\hat{d}_o(s_{**}^t) > d_o(s_{**}^t)$ and for which borrowing is not constrained along the path of s_{**}^t. Reducing $\hat{d}_o(s_{**}^t)$ by η and paying out the present value at time 0 instead changes the objective by $(R^{-t} - \beta^t)(\hat{d}_o(s^t) - d_o(s^t)) > 0$, contradicting the optimality of $d(\hat{w},s) = 0$.

Assume, without loss of generality, that $\hat{k}_o(s_*') \neq k_o(s_*')$, for some $s_*' \in S$. Rewrite the Bellman equation as

$$V(w,s) = \max_{\substack{x \in \Gamma(w,s), \\ x(s') \in \Gamma(w(s'),s'), \forall s' \in S}} d + \beta \sum_{s' \in S} \Pi(s,s') \left\{ d(s') + \beta \sum_{s'' \in S} \Pi(s',s'') V(w(s''),s'') \right\}$$

and note the convexity of the constraint set. Using the fact that $\hat{k}_o(s'_*) \neq k_o(s'_*)$, that V is concave and strictly increasing, and that $f(k)$ is strictly concave, we have, for $\phi \in (0,1)$, and denoting $x_{o,\phi} = \phi x_o + (1-\phi)\hat{x}_o$ and analogously for other variables,

$$
\begin{aligned}
V(w_\phi, s) &> d_{o,\phi} + \beta \sum_{s' \in S} \Pi(s,s') \left\{ d_{o,\phi}(s') + \beta \sum_{s'' \in S} \Pi(s', s'') V(w_{o,\phi}(s''), s'') \right\} \\
&\geq \phi V(w,s) + (1-\phi) V(\hat{w}, s).
\end{aligned}
$$

The first (strict) inequality is due to the fact that for s'' following s'_* equation (3) is slack and hence a net worth $w(s'') > w_{o,\phi}(s'')$ is feasible. The second inequality is due to concavity of V. □

Lemma 5 *Assuming that for all $\hat{s}, s \in S$ such that $\hat{s} > s$, $\Pi(\hat{s}, s')$ strictly first order stochastically dominates $\Pi(s,s')$, V is strictly increasing in s.*

Proof of Lemma 5. Let $S = \{s_1, \ldots, s_n\}$, with $s_{i-1} < s_i$, $\forall i = 2, \ldots, n$ and $N = \{1, \ldots, n\}$. Define the step function on the unit interval $b : [0,1] \to \mathbb{R}$ as $b(\omega) = \sum_{i=1}^n b(s'_i)\mathbf{1}_{B_i}(\omega)$, $\forall \omega \in [0,1]$, where $\mathbf{1}$ is the indicator function, $B_1 = [0, \Pi(s, s'_1)]$, and

$$
B_i = \left(\sum_{j=1}^{i-1} \Pi(s, s'_j), \sum_{j=1}^{i} \Pi(s, s'_j) \right], \qquad i = 2, \ldots, n.
$$

For $\hat{s}$, define $\hat{B}_i$, $\forall i \in N$, analogously. Let $B_{ij} = B_i \cap \hat{B}_j$, $\forall i,j \in N$, of which at most $2n-1$ are non-empty. Then, we can define the step function $\hat{b} : [0,1] \to \mathbb{R}$ as

$$
\hat{b}(\omega) = \sum_{j=1}^{n} \sum_{i=1}^{n} b(s'_i) \mathbf{1}_{B_{ij}}(\omega), \qquad \forall \omega \in [0,1].
$$

We can then define the stochastic debt policy for $\hat{B}_j$, $\forall j \in N$, with positive Lebesgue measure ($\lambda(\hat{B}_j) > 0$), as $\hat{b}(s'_i|s'_j) = b(s'_i)$ with conditional probability $\pi(s'_i|s'_j) = \lambda(B_{ij})/\lambda(\hat{B}_j)$. Moreover, this implies a stochastic net worth

$$
\begin{aligned}
\hat{w}(s'_i|s'_j) &= A(s'_j)f(k) + (k - k_l)(1-\delta) - R\hat{b}(s'_i|s'_j) \\
&\geq A(s'_i)f(k) + (k - k_l)(1-\delta) - R\hat{b}(s'_i) = w(s'_i), \qquad a.e.,
\end{aligned}
$$

with strict inequality when $i < j$, since under the assumption in the statement of the lemma, $\lambda(B_{ij}) = 0$ whenever $i > j$. Moreover, $\hat{w}(s'_i|s'_j) > w(s'_i)$ with positive probability given that assumption.

Now suppose $\hat{s} > s$ and $g(w, \hat{s}) \geq g(w,s)$, $\forall w \in \mathbb{R}_+$. Let x_o attain the $(Tg)(w,s)$. Then

$$
\begin{aligned}
(Tg)(w, \hat{s}) &\geq d_o + \beta \sum_{\hat{s}' \in S} \Pi(\hat{s}, \hat{s}') \sum_{s' \in S} \pi(s'|\hat{s}') g(\hat{w}_o(s'|\hat{s}'), \hat{s}') \\
&> d_o + \beta \sum_{s' \in S} \Pi(s,s') g(w_o(s'), s') = (Tg)(w,s).
\end{aligned}
$$

Thus, T maps increasing functions into strictly increasing functions, implying that V is strictly increasing in s. □

Lemma 6 *Under Assumption 2, capital and net worth in all states are strictly positive, $k > 0$ and $w(s') > 0$, $\forall s' \in S$.*

Proof of Lemma 6. We first show that if $k > 0$, then $w(s') > 0$, $\forall s' \in S$. Note that (3) holds with equality. Using (4) we conclude

$$w(s') = A(s')f(k) + (k - k_l)(1-\delta) - Rb(s') \geq A(s')f(k) + ((k - k_l) - \theta(\varphi k - k_l))(1-\delta) > 0.$$

To show that $k > 0$, note that (7) and (10) imply that

$$\mu(1 - R^{-1}\theta\varphi(1-\delta)) \geq \sum_{s' \in S} \Pi(s,s')\beta\mu(s')\left[A(s')f_k(k) + (1-\theta\varphi)(1-\delta)\right]. \tag{24}$$

Suppose that $\mu = 1$. Then $k > 0$ since $\mu(s') = V_w(w(s'), s') \geq 1$ and hence the right hand side goes to $+\infty$ as $k \to 0$, a contradiction. Suppose instead that $\mu > 1$ and hence $d = 0$. For k sufficiently small, $\exists \hat{s}' \in S$, such that $\mu(\hat{s}') = (R\beta)^{-1}\mu$. But then

$$\begin{aligned} 0 \quad \geq \quad & \sum_{s' \in S \setminus \hat{s}} \Pi(s,s')\beta\mu(s')\left[A(s')f_k(k) + (1-\theta\varphi)(1-\delta)\right] \\ & + \left\{\Pi(s,\hat{s}')R^{-1}[A(\hat{s}')f_k(k) + (1-\theta\varphi)(1-\delta)] - (1 - R^{-1}\theta\varphi(1-\delta))\right\}\mu. \end{aligned}$$

where the first term is positive and the second term goes to $+\infty$ as $k \to 0$, a contradiction.
□

Proof of Proposition 2. Part (i): By the envelope condition, $\mu(w,s) = V_w(w,s)$. By Lemma 3, V is concave in w and hence $\mu(w,s)$ is decreasing in w. The first order condition (6) implies that $\mu(w,s) \geq 1$. If $d(\hat{w},s) > 0$, then $\mu(\hat{w},s) = 1$ and $\mu(w,s) = 1$ for all $w \geq \hat{w}$. Let $\bar{w}(s) = \inf\{w : d(w,s) > 0\}$.

Part (ii): Suppose $w > \hat{w} \geq \bar{w}(s)$ and let $\hat{x}_o$ attain $V(\hat{w},s)$. Since $V_w(w,s) = 1$ for $w \geq \bar{w}(s)$, $V(w,s) = V(\hat{w},s) + \int_{\hat{w}}^{w} dv$. The choice $x_o = [w - \hat{w} + \hat{d}_o, \hat{k}_o, \hat{k}_{l,o}, \hat{w}_o(s'), \hat{b}_o(s')]$ attains $V(w,s)$ and thus is weakly optimal.

The optimal choice $\hat{x}_o$ is unique in terms of the capital stock $\hat{k}_o$. To see this, suppose instead that $\hat{x}_o$ and $\tilde{x}_o$ both attain $V(\hat{w},s)$, but $\hat{k}_o \neq \tilde{k}_o$. Recalling that $\Gamma(\hat{w},s)$ is convex and noting that

$$\begin{aligned} A(s')f(k_{o,\phi}) + (k_{o,\phi} - k_{l,o,\phi})(1-\delta) \quad > \quad & \phi[A(s')f(\hat{k}_o) + (\hat{k}_o - \hat{k}_{l,o})(1-\delta)] \\ & +(1-\phi)[A(s')f(\tilde{k}_o) + (\tilde{k}_o - \tilde{k}_{l,o})(1-\delta)], \end{aligned}$$

where $x_{o,\phi}$ is defined as usual, we conclude that at $x_{o,\phi}$, (3) is slack, and hence there exists a feasible choice that attains a strictly higher value, a contradiction. Indeed, $x_o(w,s)$ is unique in terms of $k_o(w,s)$, for all w and s.

Now take $w > \hat{w}$ and let x_o attain $V(w,s)$. By part (i) of Proposition 1, $x_{o,\phi} \in \Gamma(w_\phi, s)$. Moreover, if $k_o \neq \hat{k}_o$, then there exists a feasible choice such that $V(w_\phi) > \phi V(w,s) + (1-\phi)V(\hat{w},s)$ contradicting the linearity of V. Thus, $k_o(w,s) = \bar{k}_o(s)$, $\forall w \geq \bar{w}(s)$.

Part (iii): We now show that without leasing the optimal policy is unique also in terms of state-contingent net worth, state-contingent borrowing, and the dividend. Define

$\hat{S}^0 = \{s' : \hat{w}_o(s') < \bar{w}(s')\}$ and $\hat{S}^+ = S \setminus \hat{S}^0$. Then $\forall s' \in \hat{S}^0$, $\hat{w}_o(s')$ is unique. To see this suppose instead that there is a $\tilde{x}_o$ with $\tilde{w}_o(s') \neq \hat{w}_o(s')$ for some $s' \in \hat{S}^0$ that also attains $V(\hat{w}, s)$. Then a convex combination $x_{o,\phi}$ is feasible and attains a strictly higher value due to strict concavity of $V(w, s')$ for $w < \bar{w}(s')$ (part (iv) of Proposition 1). For the alternative optimal policy $\tilde{x}_o$ define $\tilde{S}^0$ and $\tilde{S}^+$ analogously to $\hat{S}^0$ and $\hat{S}^+$. By above, $\tilde{S}^0 \supseteq \hat{S}^0$. For any $s' \in \hat{S}^+$, $\tilde{w}_o(s') \geq \bar{w}(s')$. For suppose instead that $\tilde{w}_o(s') < \bar{w}(s')$, then by strict concavity of V for $w < \bar{w}(s')$ a convex combination would again constitute a feasible improvement. Thus, $\tilde{S}^+ \supseteq \hat{S}^+$ and as a consequence $S^+ \equiv \tilde{S}^+ = \hat{S}^+$ and $S^0 \equiv \tilde{S}^0 = \hat{S}^0$. For $s' \in S^0$, $b_o(s')$ is uniquely determined by (3). For $s' \in S^+$, equation (4) holds with equality and determines $b_o(s')$ uniquely, and $w_o(s')$ is then uniquely determined by (3). Hence, the optimal policy is unique. Moreover, the policy determined by part (ii) (with $k_{l,o}(w, s)$ set to 0) is the unique optimal policy for $w > \bar{w}(s)$. $\square$

Proof of Proposition 3. Using the first order conditions for investment (7) and substituting for $\lambda(s')$ using (10) we have

$$\begin{aligned} 1 &\geq \sum_{s' \in S} \Pi(s, s')\beta \frac{\mu(s')}{\mu} \frac{[A(s')f_k(k) + (1-\theta\varphi)(1-\delta)]}{1 - R^{-1}\theta\varphi(1-\delta)} \\ &\geq \Pi(s, s')\beta \frac{\mu(s')}{\mu} \frac{A(s')f_k(k)}{1 - R^{-1}\theta\varphi(1-\delta)}. \end{aligned} \tag{25}$$

Using the budget constraint (2) and the collateral constraints (4), we have

$$w \geq (1-\varphi)k + (\varphi k - k_l)(1 - R^{-1}\theta(1-\delta)) + R^{-1}u_l k_l,$$

and thus as $w \to 0$, investment $k \to 0$. But then the marginal product of capital $f_k(k) \to +\infty$, which implies by (25) that $\mu(s')/\mu \to 0$, and using (10), that $\lambda(s')/\mu = (R\beta)^{-1} - \mu(s')/\mu \to (R\beta)^{-1} > 0$, $\forall s' \in S$. Therefore, by continuity, $\forall s \in S$, $\exists \underline{w}_h(s) > 0$, such that $\forall w \leq \underline{w}_h(s)$, $\lambda(s') > 0$, $\forall s' \in S$. $\square$

Proof of Proposition 4. Part (i): If $w(s') \leq w(s'_+)$, then $\mu(s') \geq \mu(s'_+)$ by concavity. Moreover, $\mu(s') + \lambda(s') = \mu(s'_+) + \lambda(s'_+)$, so $\lambda(s') \leq \lambda(s'_+)$. Suppose instead that $w(s') > w(s'_+)$. Then $\lambda(s') = 0$ since otherwise net worth in state s' could not be larger than in state s'_+. But then $\mu(s') = \mu(s'_+) + \lambda(s'_+)$, implying $\mu(s'_+) \leq \mu(s')$. If $\mu(s'_+) = \mu(s')$, then $\lambda(s'_+) = \lambda(s')$ and the assertion is true. If instead $\mu(s'_+) < \mu(s')$, then by concavity $w(s'_+) \geq w(s')$, a contradiction.

Part (ii): Suppose that $\lambda(s') = 0$, $\forall s' \in S$. Then (9), (10), and the envelope condition imply that $V_w(w) = \mu = \beta\mu(s')R = R\beta V_w(w(s')) < V_w(w(s'))$ and, by concavity, $w > w(s')$, $\forall s' \in S$. If $d = 0$, then saving the entire net worth w at R would imply net worth $Rw > w(s')$ in all states next period and hence attain a higher value of the objective, contradicting optimality. Suppose $d > 0$ and hence $w > \bar{w}$ as defined in Proposition 2. That proposition also implies that $V(w)$ can be attained by the same optimal policy as at $\bar{w}$ except that $d = w - \bar{w}$. Since $V_w(w(s')) > 1$, we conclude that $w(s') < \bar{w}$. But then paying out $d = w - \bar{w}$ as before and saving $\bar{w}$ at R raises net worth in all states next period and hence improves the value of the objective, a contradiction.

Hence, $\exists s' \in S$ such that $\lambda(s') > 0$, and, since $\lambda(s')$ is increasing in s' by part (i), $\lambda(\bar{s}') > 0$ where $\bar{s}' = \max\{s' : s' \in S\}$. If $\lambda(s') > 0$, $\forall s'$, then $w(s') = A(s')f(k) + k(1 - \theta\varphi)(1-\delta) - k_l(1-\theta)(1-\delta)$ and hence $w(s') \neq w(\hat{s}')$, $s \neq \hat{s}'$. If $\lambda(\hat{s}') = 0$ for some $\hat{s}'$, then $\mu(\hat{s}') = \mu(\bar{s}') + \lambda(\bar{s}') > \mu(\bar{s}')$ and $w(\hat{s}') < w(\bar{s}')$.

Suppose $\lambda(s') = 0$ for some $s' \in S$. For any $s'_- < s'$, $\mu(s'_-) \geq \mu(s')$ by part (i), and $\mu(s'_-) \leq \mu(s'_-) + \lambda(s'_-) = \mu(s')$, implying $\mu(s'_-) = \mu(s')$. Thus, the firm hedges all states below $s'_h = \max\{s' : \lambda(s') = 0\}$. Note that the set may be empty, that is, the firm may not hedge at all. □

Lemma 7 (Net worth transition dynamics) *Suppose* $\Pi(s, s') = \Pi(s')$, $\forall s, s' \in S$, *and* $m = +\infty$ *(no leasing).* **(i)** $\forall s', s'_+ \in S$, *such that* $s'_+ > s'$, $w(s'_+) \geq w(s')$, *with strict inequality iff* $s'_+ > s'_h$ *where* s'_h *is defined in Proposition 4.* **(ii)** $w(s')$ *is increasing in* w, $\forall s' \in S$; *for* w *sufficiently small,* $w(s') > w$, $\forall s' \in S$; *and for* w *sufficiently large,* $w(s') < w$, $\forall s' \in S$. **(iii)** $\forall s' \in S$, $\exists w$ *dependent on* s' *such that* $w(s') = w$.

Proof of Lemma 7. Part (i): By part (iv) of Proposition 1 $V(w)$ is strictly concave unless $w > \bar{w}$. By Proposition 4, the firm hedges a lower set of states $[\underline{s}', \ldots, s'_h]$ if at all. If $s'_+ \leq s'_h$, then $\mu(s') = \mu(s'_+) > 1$ and hence $w(s') = w(s'_+) < \bar{w}$. If $s'_+ > s'_h$, then either $\lambda(s') = 0$ and $\mu(s') > \mu(s'_+)$, implying $w(s'_+) > w(s')$, or $\lambda(s') > 0$, which together with (3) and (4) at equality implies $w(s'_+) > w(s')$.

Part (ii): If $d > 0$, then $w(s')$ is constant by Proposition 2 and hence (weakly) increasing. If $d = 0$, then $w(s')$ is strictly increasing in w for $\{s'|\lambda(s') = 0\}$ using strict concavity of V and the fact that $V_w(w) = R\beta V_w(w(s'))$. For $\{s'|\lambda(s') > 0\}$, (3) and (4) hold with equality and hence $w(s')$ is increasing in w if k is. We now show that k is strictly increasing in w for $w \leq \bar{w}$. If $\lambda(s') > 0$, $\forall s \in S$, then $k = w/(1 - R^{-1}\theta\varphi(1-\delta)$ and k is hence strictly increasing. Suppose $\lambda(s') = 0$, some $s \in S$. Then using the first order conditions for investment (7) and substituting for $\lambda(s')$ using (10) we have

$$
\begin{aligned}
1 &= \sum_{s' \in S} \Pi(s')\beta \frac{\mu(s')}{\mu} \frac{[A(s')f_k(k) + (1-\theta\varphi)(1-\delta)]}{1 - R^{-1}\theta\varphi(1-\delta)} \\
&= \sum_{\{s'|\lambda(s')>0\}} \Pi(s')\beta \frac{\mu(s')}{\mu} \frac{[A(s')f_k(k) + (1-\theta\varphi)(1-\delta)]}{1 - R^{-1}\theta\varphi(1-\delta)} \\
&\quad + \sum_{\{s'|\lambda(s')=0\}} \Pi(s')\beta \frac{\mu(s')}{\mu} \frac{[A(s')f_k(k) + (1-\theta\varphi)(1-\delta)]}{1 - R^{-1}\theta\varphi(1-\delta)}. \qquad (26)
\end{aligned}
$$

Take $w^+ > w$ and suppose that $k^+ \leq k$ with the usual abuse of notation. Then $f_k(k^+) \geq f_k(k)$. Moreover, for $\{s'|\lambda(s') = 0\}$, $\mu(s')/\mu = (R\beta)^{-1}$. Since $\mu^+ < \mu$, (26) implies that $\exists s'$ such that $\mu^+(s') < \mu(s')$. But $k^+ \leq k$ implies that for $\{s'|\lambda(s') > 0\}$, $w^+(s') \leq w(s')$ and hence $\mu^+(s') \geq \mu(s')$, a contradiction. Hence, k and $w(s')$ are strictly increasing in w for $w \leq \bar{w}$.

To show that $\exists w$ such that $w(s') > w$, $\forall s' \in S$, note that Proposition 3 implies that for w sufficiently small, $\lambda(s') > 0$, $\forall s' \in S$, and thus $w = k/(1 - R^{-1}\theta\varphi(1-\delta))$ and $w(s') = A(s')f(k) + k(1-\theta\varphi)(1-\delta)$. But then

$$
\frac{dw(s')}{dw} = \frac{A(s')f_k(k) + (1-\theta\varphi)(1-\delta)}{1 - R^{-1}\theta\varphi(1-\delta)} = \frac{A(s')f_k(k)k + k(1-\theta\varphi)(1-\delta)}{k(1 - R^{-1}\theta\varphi(1-\delta))} < \frac{w(s')}{w},
$$

where the inequality uses the strict concavity of $f(\cdot)$. Moreover, as $w \to 0$, $f_k(k) \to +\infty$ and thus $dw(s')/dw \to +\infty$ and $w(s')/w > 1$ for w sufficiently low.

To show that $\exists w$ such that $w(s') < w, \forall s' \in S$, it is sufficient to show that such a w exists for $w(\bar{s}')$ given part (i). By Proposition 2, $\forall w \geq \bar{w}$, the optimal policy x_o is independent of w (except for the current dividend), and thus $w_o(\bar{s}') = A(\bar{s}')f(\bar{k}_o) + \bar{k}_o(1-\delta) - R\bar{b}_o(\bar{s}') < +\infty$, and hence for $w > w_o(\bar{s}')$ the assertion holds.

Part (iii): By the theorem of the maximum $w(s')$ is continuous in w, and the intermediate value theorem and part (ii) hence imply the result. □

Proof of Proposition 5. Part (i): Denoting the wealth level as defined in part (iii) of Lemma 7 for the lowest state $\underline{s}'$ by $\underline{w}$ and using $w(\underline{s}') = \underline{w}$, (9), (10), and the envelope condition, we have $V_w(\underline{w}) = R\beta V_w(\underline{w}) + R\beta\lambda(\underline{s}')$ and thus $\lambda(\underline{s}') > 0$, and by Proposition 4 the firm abstains from risk management altogether at $\underline{w}$. The level of net worth $\underline{w}$ is unique, since either $d > 0$ at $\underline{w}$, and hence $w(\underline{s}')$ is constant, or $d = 0$ and then using $\lambda(s') > 0$, $\forall s \in S$, and evaluating $dw(\underline{s}')/dw$ as in part (ii) at $\underline{w}$

$$\left.\frac{dw(\underline{s}')}{dw}\right|_{w=\underline{w}} = \left.\frac{A(\underline{s}')f_k(k)k + k(1-\theta\varphi)(1-\delta)}{k(1-R^{-1}\theta\varphi(1-\delta))}\right|_{w=\underline{w}} < \frac{w(\underline{s}')}{\underline{w}} = 1.$$

Thus the locus of $w(\underline{s}')$ crosses the 45 degree line from above, that is, at most once. Moreover, $w(\underline{s}')/w < 1$, $\forall w > \underline{w}$.

Part (ii): We adapt Theorem 12.12 from Stokey, Lucas, and Prescott (1989). Let $\varepsilon_w > 0$ and $w_{\text{bnd}} = A(\bar{s}')f(k_{\text{bnd}}) + k_{\text{bnd}}(1-\delta)$ where k_{bnd} such that $A(\bar{s}')f_k(k_{\text{bnd}}) = r + \delta$. Define the induced state space $W = [\varepsilon_w, w_{\text{bnd}}] \subset \mathbb{R}$ with its Borel subsets $\mathcal{W}$. Take P to be the induced transition function on $(W, \mathcal{W})$, with the associated operator on bounded continuous functions $T : B(W, \mathcal{W}) \to B(W, \mathcal{W})$ and the associated operator on probability measures $T^* : \mathcal{P}(W, \mathcal{W}) \to \mathcal{P}(W, \mathcal{W})$.

We need to show that P is monotone (that is, for any bounded, increasing function g, the function Tg defined by $(Tg)(w) = \int g(w')P(w, dw')$, $\forall w$, is also increasing), has the Feller property (that is, for any bounded, continuous function g, the function Tg is also continuous), and $\exists w^o \in W$, $\varepsilon > 0$, and $N \geq 1$, such that $P^N(\varepsilon_w, [w^o, w_{\text{bnd}}]) \geq \varepsilon$ and $P^N(w_{\text{bnd}}, [\varepsilon_w, w^o]) \geq \varepsilon$.

Take any bounded, increasing function g. Then $(Tg)(w) = \sum_{s' \in s} \Pi(s')g(w(s')(w))$ is increasing since $w(s')(w)$ is increasing by part (ii) of Lemma 7. For any bounded, continuous function g , $(Tg)(w)$ is moreover continuous as $w(s')(w)$ is continuous by the theorem of the maximum.

From Lemma 7 and part (i) we know that levels of net worth below $\underline{w}$ and above $w_o(\bar{s}')$ are transient. We now provide an explicit characterization of the stationary solution when $\bar{w} \leq \underline{w}$ and then show that otherwise w^o can be set to $w^o = \bar{w}$, where $\bar{w}$ is the level of net worth above which the firm pays dividends (see Proposition 2).

If $\bar{w} \leq \underline{w}$, then the stationary distribution is a subset of the dividend paying set and the solution is quasi-deterministic, in the sense that capital $\bar{k}_o$ is constant under the stationary distribution. In this case, $\bar{k}_o$ solves

$$1 = \beta\frac{\sum_{s' \in S} \Pi(s')A(s')f_k(\bar{k}_o) + (1-\theta\varphi)(1-\delta)}{1 - R^{-1}\theta\varphi(1-\delta)}.$$

Then $\bar{w} = \bar{k}_o(1 - R^{-1}\theta\varphi(1-\delta))$ and

$$\begin{aligned}\underline{w} &= w(\underline{s}') = A(\underline{s}')f(\bar{k}_o) + \bar{k}_o(1-\theta\varphi)(1-\delta) \\ &\geq \bar{w} = \bar{k}_o(1 - R^{-1}\theta\varphi(1-\delta)) = \beta\left(\sum_{s'\in S}\Pi(s')A(s')f_k(\bar{k}_o)\bar{k}_o + \bar{k}_o(1-\theta\varphi)(1-\delta)\right).\end{aligned}$$

The condition for $\bar{w} \leq \underline{w}$ is thus

$$(1-\beta)(1-\theta\varphi)(1-\delta) \geq \beta\sum_{s'\in S}\Pi(s')A(s')f_k(\bar{k}_o) - A(\underline{s}')\frac{f(\bar{k}_o)}{\bar{k}_o}. \tag{27}$$

Concavity implies that $f(\bar{k}_o)/\bar{k}_o > f_k(\bar{k}_o)$ and thus a sufficient condition is that $A(\underline{s}') \geq \beta\sum_{s'\in S}\Pi(s')A(s')$. If $f(k) = k^\alpha$, a sufficient condition is $A(\underline{s}') \geq \alpha\beta\sum_{s'\in S}\Pi(s')A(s')$. Note that in the quasi-deterministic case the firm abstains from risk management with probability 1 under the stationary distribution.

If (27) is violated, then $\underline{w} < \bar{w}$. Moreover, the firm cannot be paying dividends in the lowest state next period if it currently is paying dividends, for if it paid dividends, it would choose $\bar{k}_o$ as determined above and

$$\begin{aligned}w(\underline{s}') &= A(\underline{s}')f(\bar{k}_o) + \bar{k}_o(1-\theta\varphi)(1-\delta) \\ &< \beta\left(\sum_{s'\in S}\Pi(s')A(s')f_k(\bar{k}_o)\bar{k}_o + \bar{k}_o(1-\theta\varphi)(1-\delta)\right) = \bar{k}_o(1 - R^{-1}\theta\varphi(1-\delta)) = \bar{w},\end{aligned}$$

a contradiction. Thus, if $w^o = \bar{w}$, then $P(w_{\text{bnd}}, [\varepsilon_w, \bar{w}]) \geq \Pi(\underline{s}')$ and $N_1 = 1$ and $\exists\varepsilon_1 > 0$ such that $\Pi(\underline{s}') > \varepsilon_1 > 0$.

Now given $w = \varepsilon_w$, and since the net worth could be paid out, the objective has to exceed the value of net worth,

$$0 < \varepsilon_w = w \leq E\left[\sum_{t=0}^{\infty}\beta^t d_t\right] = E\left[\sum_{t=0}^{N}\beta^t d_t\right] + E\left[\sum_{t=N+1}^{\infty}\beta^t d_t\right].$$

Note that $d_t \leq d_o(\bar{s}') = w_o(\bar{s}') - \bar{w}$ and the last expectation above is thus bounded by $\beta^{N+1}d_o(\bar{s}')/(1-\beta)$. For any $\varepsilon_d > 0$ such that $\varepsilon_w > \varepsilon_d$, $\exists N_2 < \infty$ such that the last expectation is less than ε_d. But then $P^{N_2}(\varepsilon_w, [\bar{w}, w_{\text{bnd}}]) \geq (\varepsilon_w - \varepsilon_d)/d_o(\bar{s}') > 0$. Let $\varepsilon_2 \equiv (\varepsilon_w - \varepsilon_d)/d_o(\bar{s}')$, $\varepsilon = \min\{\varepsilon_1, \varepsilon_2\}$, and $N = \max\{N_1, N_2\}$. Finally, when $\underline{w} < \bar{w}$, $dw(\underline{s}')/dw < 1$ at $\underline{w}$ and $w(\underline{s}') < w$ for all $w \geq \underline{w}$, and thus a sufficiently long sequence of the lowest productivity realization results in a net worth in a neighborhood of $\underline{w}$ and hence the firm abstains from risk management with positive probability. □

Proof of Proposition 6. We first show that, for each s, $k \leq k_{fb}(s)$ where $k_{fb}(s)$ solves $r + \delta = \sum_{s'\in S}\Pi(s,s')A(s')f_k(k_{fb}(s))$, that is, $k_{fb}(s)$ is the capital level in the frictionless case when cash flows are discounted at r. Using the first order conditions (7) and (10), the investment Euler equation implies

$$\begin{aligned}1 &= \sum_{s'\in S}\Pi(s,s')\beta\frac{\mu(s')}{\mu}\frac{[A(s')f_k(k)+(1-\theta\varphi)(1-\delta)]}{1-R^{-1}\theta\varphi(1-\delta)} \\ &\leq R^{-1}\frac{\sum_{s'\in S}\Pi(s,s')A(s')f_k(k)+(1-\theta\varphi)(1-\delta)}{1-R^{-1}\theta\varphi(1-\delta)}.\end{aligned} \tag{28}$$

Since the term on the right of the inequality equals 1 at $k_{fb}(s)$ and decreases in k the result is immediate.

Second, we show that, given s, k is weakly increasing in w. We focus on the non-dividend paying region (as k is constant otherwise). Let $w^+ > w$. By the concavity of the value function (Proposition 1, Part (iii)), $\mu^+ \leq \mu$. Suppose $k^+ < k$. If $\lambda(s') = 0$, $\forall s' \in S$, at w and w^+, then $k = k_{fb}(s)$ at both levels of wealth, a contradiction. Thus, suppose $\exists s' \in S$, such that $\lambda(s') > 0$ at w and assume that $\{s' : \lambda(s') > 0\}$ is the same at w and w^+. Using (28) we have

$$\begin{aligned}1 &= \sum_{\{s':\lambda(s')>0\}}\Pi(s,s')\beta\frac{\mu(s')}{\mu}\frac{[A(s')f_k(k)+(1-\theta\varphi)(1-\delta)]}{1-R^{-1}\theta\varphi(1-\delta)} \\ &+ \sum_{\{s':\lambda(s')=0\}}\Pi(s,s')R^{-1}\frac{[A(s')f_k(k)+(1-\theta\varphi)(1-\delta)]}{1-R^{-1}\theta\varphi(1-\delta)}.\end{aligned}$$

The second summation must be higher at w^+ and hence $\exists s' \in \{s' : \lambda(s') > 0\}$, such that $\mu^+(s') < \mu(s')$. But for such an s', $w^+(s') < w(s')$ and hence $\mu^+(s') \geq \mu(s')$, a contradiction. Therefore, $k^+ \geq k$, that is, investment is weakly increasing in w.

Finally, we show that $\exists s$, such that $\forall w$, there is an s' for which $\lambda(s') > 0$, that is, incomplete hedging in state s is optimal for all wealth levels. To see this note that there must be a current state (w, s) and state next period s' such that $d = 0$ at (w, s) and $d(s') > 0$, implying that $\lambda(s') > 0$ at (w, s). (Otherwise, the firm would either never pay a dividend, which is not possible, or would always pay a dividend in which case $\lambda(s') > 0$, $\forall s'$, in any state (w, s).) Since the firm pays a dividend in state s' next period, $w(s') > \bar{w}(s')$. Take $w^+ > w$. Since k is weakly increasing in w, $w^+(s') \geq w(s') > \bar{w}(s')$, that is, the firm still pays dividends in s' and is hence constrained against s' at (w^+, s), too. Picking $w^+ \geq \bar{w}(s)$ implies that $\lambda(s') > 0$ and thus $\bar{k}(s) < k_{fb}(s)$ arguing as in the first step of the proof. But then, given s, $k \leq \bar{k}(s) < k_{fb}(s)$, which implies that $\exists s'$ such that $\lambda(s') > 0$, $\forall w$ (using the argument in the first step of the proof again). Note however that the state s' against which the firm is constrained in state s may not be the same for wealth levels below the w we started with. Since state s has positive probability, hedging is incomplete with positive probability. (We can moreover conclude that k is strictly increasing in w for $w \leq \bar{w}(s)$ in state s. To see this use the fact that the value function is strictly concave (Proposition 1, Part (iv)) and the fact that $\{s' : \lambda(s') > 0\} \neq \emptyset$ and argue as in the second step of the proof.) □

Proof of Proposition 7. Proceeding as in the proof of Proposition 3, we conclude that as $w \to 0$, investment $k \to 0$ and $\lambda(s')/\mu \to (R\beta)^{-1} > 0$, $\forall s' \in S$. Therefore, given Assumption 3, the user cost of owned tangible capital exceeds the user cost of leased capital in the limit as $u_p(w,s) \equiv r + \delta + \sum_{s' \in S} \Pi(s,s') R\beta\lambda(s')/\mu(1-\theta)(1-\delta)$ goes to $r + \delta + (1-\theta)(1-\delta) > r + \delta + m = u_l$. Consequently, using (14) we obtain $R/\mu(\bar{\nu}_l - \underline{\nu}_l) = u_p(w,s) - u_l > 0$, so $\bar{\nu}_l > 0$ (and $\underline{\nu}_l = 0$); all tangible capital is leased in the limit. By continuity, $\forall s \in S$, $\exists \underline{w}_l(s) > 0$, such that $\forall w \leq \underline{w}_l(s)$, $u_p(w,s) > u_l$ and all tangible capital is leased. □

Proof of Proposition 8. Part (i): Denote with a prime variables which in the stochastic case were a function of the state tomorrow, that is, w', b', μ', and λ'. We first characterize a steady state. From (9) and the envelope condition we have $\mu' = \mu$. Then (10) implies $\lambda' = ((R\beta)^{-1} - 1)\mu > 0$, that is, the firm is constrained in the steady state, and (7) can be written as $1 - [R^{-1}\theta\varphi + \beta(1-\theta\varphi)](1-\delta) = \beta A' f_k(k)$, which implicitly defines the steady state value of capital $\bar{k}$. Denoting steady state variables with a bar, using (4) and (3) at equality, we have $\bar{b} = R^{-1}\theta\varphi\bar{k}(1-\delta)$ and the cum-dividend net worth in the steady state $\bar{w}_{cum} = A' f(\bar{k}) + \bar{k}(1-\theta\varphi)(1-\delta)$. Dividends in the steady state are

$$\begin{aligned}
\bar{d} &= A' f(\bar{k}) - \bar{k}(1 - [R^{-1}\theta\varphi + (1-\theta\varphi)](1-\delta)) \\
&> A' f(\bar{k}) - \beta^{-1}\bar{k}(1 - [R^{-1}\theta\varphi + \beta(1-\theta\varphi)](1-\delta)) \\
&= \int_0^{\bar{k}} \left\{ A' f_k(k) - \beta^{-1}(1 - [R^{-1}\theta\varphi + \beta(1-\theta\varphi)](1-\delta)) \right\} dk > 0
\end{aligned}$$

and hence $\bar{\mu} = 1$. The lowest level of net worth for which $\bar{k}$ is feasible is $\bar{w} \equiv \bar{w}_{cum} - \bar{d}$, and $\bar{w}$ is the ex-dividend net worth in the steady state. Thus, for $w < \bar{w}$, $k < \bar{k}$. Using the first order conditions and the envelope condition we have

$$\frac{V_w(w)}{V_w(w')} = \frac{\mu}{\mu'} = \beta \frac{A' f_k(k) + (1-\theta\varphi)(1-\delta)}{1 - R^{-1}\theta\varphi(1-\delta)}.$$

Note that the right hand side equals 1 at $\bar{k}$ and is decreasing in k. Thus, if $k < (>)\, \bar{k}$, $V_w(w) > (<)\, V_w(w')$ and $w < (>)\, w'$. Since $k < \bar{k}$ for $w < \bar{w}$, $w < w'$ and w increases over time. If $w > \bar{w}$, then either $d > 0$ (and $V_w(w) = 1$) or $d = 0$ and $k > \bar{k}$. In the first case, concavity and the fact that $V_w(w') \geq 1$ imply $V_w(w') = 1$ and hence $k = \bar{k}$. In the second case, $w > w'$, but simply saving w at R would result in higher net worth and hence $k > \bar{k}$ cannot be optimal.

Part (ii): Consider the optimal policy without leasing from part (i). The user cost of tangible capital at $\bar{w}$ is $\bar{u}_p = r + \delta + (1-R\beta)(1-\theta)(1-\delta) < u_l$ under Assumption 3. Thus, there is no leasing at $\bar{w}$ and the solution is as before as long as w is sufficiently high. Recall that as w decreases μ'/μ decreases and hence λ'/μ increases. Note also that under Assumption 2, as w goes to zero, k and μ'/μ go to zero and hence λ'/μ goes to $(R\beta)^{-1}$ and $u_p(w)$ goes to $r + \delta + (1-\theta)(1-\delta) > u_l$ given Assumption 3. When $\lambda'/\mu = (R\beta)^{-1}m/((1-\theta)(1-\delta))$, $u_l = u_p(w)$ and (7) simplifies to

$$1 - R^{-1}\theta\varphi(1-\delta) = R^{-1}\left(1 - \frac{m}{(1-\theta)(1-\delta)}\right)[Af_k(k) + (1-\theta\varphi)(1-\delta)],$$

which defines $\underline{k}$. At $\bar{w}_l \equiv (1 - R^{-1}\theta\varphi(1-\delta))\underline{k}$ all the tangible capital is owned and at $\underline{w}_l \equiv (1 - \varphi + R^{-1}u_l\varphi)\underline{k}$ all the tangible capital is leased. For $w \in [\underline{w}_l, \bar{w}_l]$, leased capital is

$$k_l = \frac{(1 - R^{-1}\theta\varphi(1-\delta))\underline{k} - w}{1 - R^{-1}\theta(1-\delta) - R^{-1}u_l}$$

which is linear and decreasing in w. Moreover, w' is linearly decreasing in k_l and hence linearly increasing in w. For $w < \underline{w}_l$, $k = w/(1-\varphi+R^{-1}u_l\varphi)$ and $w' = A'f(k)+k(1-\varphi)(1-\delta)$. □

References

Abel, Andrew B., 1983, Optimal investment under uncertainty, *American Economic Review* 73, 228-233.

Abel, Andrew B., and Janice C. Eberly, 1996, Optimal investment with costly reversibility, *Review of Economic Studies* 63, 581-593.

Albuquerque, Rui, and Hugo A. Hopenhayn, 2004, Optimal dynamic lending contracts with imperfect enforceability, *Review of Economic Studies* 71, 285-315.

Atkeson, Andrew, and Harold Cole, 2008, A dynamic theory of optimal capital structure and executive compensation,Working paper, UCLA and University of Pennsylvania.

Biais, Bruno, Thomas Mariotti, Guillaume Plantin, and Jean-Charles Rochet, 2007, Dynamic security design: Convergence to continuous time and asset pricing implications, *Review of Economic Studies* 74, 345-390.

Biais, Bruno, Thomas Mariotti, Jean-Charles Rochet, and Stéphane Villeneuve, 2010, Large risks, limited liability and dynamic moral hazard, *Econometrica* 78, 73-118.

Bolton, Patrick, and David S. Scharfstein, 1990, A theory of predation based on agency problems in financial contracting, *American Economic Review* 80, 93-106.

Clementi, Gian Luca, and Hugo A. Hopenhayn, 2006, A theory of financing constraints and firm dynamics, *Quarterly Journal of Economics* 121, 229-265.

Cooley, Thomas, Ramon Marimon, and Vincenzo Quadrini, 2004, Aggregate consequences of limited contract enforceability, *Journal of Political Economy* 112, 817-847.

DeMarzo, Peter M., and Michael J. Fishman, 2007a, Agency and optimal investment dynamics, *Review of Financial Studies* 20, 151-188.

DeMarzo, Peter M., and Michael J. Fishman, 2007b, Optimal long-term financial contracting, *Review of Financial Studies* 20, 2079-2128.

DeMarzo, Peter M., Michael J. Fishman, Zhiguo He, and Neng Wang, 2011, Dynamic agency and the q theory of investment, *Journal of Finance*, forthcoming.

DeMarzo, Peter M., and Yuliy Sannikov, 2006, Optimal security design and dynamic capital structure in a continuous-time agency model, *Journal of Finance* 61, 2681-2724.

Eisfeldt, Andrea L., and Adriano A. Rampini, 2009, Leasing, ability to repossess, and debt capacity, *Review of Financial Studies* 22, 1621-1657.

Froot, Kenneth A., David S. Scharfstein, and Jeremy C. Stein, 1993, Risk management: Coordinating corporate investment and financing policies, *Journal of Finance* 48, 1629-1658.

Gertler, Mark, 1992, Financial capacity and output fluctuations in an economy with multi-period financial relationships, *Review of Economic Studies* 59, 455-472.

Gomes, Joao F., 2001, Financing investment, *American Economic Review* 91, 1263-1285.

Gromb, Denis, 1995, Renegotiation in debt contracts, Working paper, MIT.

Holmström, Bengt, and Jean Tirole, 2000, Liquidity and risk management, *Journal of Money, Credit and Banking* 32, 295-319.

Jermann, Urban J., and Vincenzo Quadrini, 2007, Stock market boom and the productivity gains of the 1990s, *Journal of Monetary Economics* 54, 413-432.

Jorgenson, Dale W., 1963, Capital theory and investment behavior, *American Economic Review (Papers and Proceedings)* 53, 247-259

Kehoe, Timothy J., and David K. Levine, 1993, Debt-constrained asset markets, *Review of Economic Studies* 60, 865-888.

Kiyotaki, Nobuhiro, and John Moore, 1997, Credit cycles, *Journal of Political Economy* 105, 211-248.

Krueger, Dirk, and Harald Uhlig, 2006, Competitive risk sharing contracts with one-sided commitment, *Journal of Monetary Economics* 53, 1661-1691.

Lorenzoni, Guido, and Karl Walentin, 2007, Financial frictions, investment, and Tobin's q, Working paper, MIT and Sveriges Riksbank.

Lucas, Robert E., Jr., and Edward C. Prescott, 1971, Investment under uncertainty, *Econometrica* 39, 659-681.

Lustig, Hanno, 2007, The market price of aggregate risk and the wealth distribution, Working paper, UCLA.

Rampini, Adriano A., A. Sufi, and S. Viswanathan, 2011, Dynamic risk management, Working paper, Duke University and University of Chicago.

Rampini, Adriano A., and S. Viswanathan, 2010, Collateral, risk management, and the distribution of debt capacity, *Journal of Finance* 65, 2293-2322.

Schmid, Lukas, 2008, A quantitative dynamic agency model of financing constraints, Working paper, Duke University.

Stokey, Nancy L., Robert E. Lucas, and Edward C. Prescott, 1989, *Recursive Methods in Economic Dynamics,* Harvard University Press, Cambridge, MA.

Quadrini, Vincenzo, 2004, Investment and liquidation in renegotiation-proof contracts with moral hazard, *Journal of Monetary Economics* 51, 713-751.

Table 1: Tangible assets and liabilities

This table reports balance sheet data on tangible assets and liabilities from the Flow of Funds Accounts of the United States for the 10 years from 1999 to 2008 [Federal Reserve Statistical Release Z.1], Tables B.100, B.102, B.103, and L.229. Panel A measures liabilities two ways. Debt is *Credit Market Instruments* which for (nonfinancial) businesses are primarily corporate bonds, other loans, and mortgages and for households are primarily home mortgages and consumer credit. Total liabilities are *Liabilities,* which, in addition to debt as defined before, include for (nonfinancial) businesses primarily miscellaneous liabilities and trade payables and for households primarily the trade payables (of nonprofit organizations) and security credit. For (nonfinancial) businesses, we subtract *Foreign Direct Investment in the U.S.* from Table L.229 from reported miscellaneous liabilities as Table F.229 suggests that these claims are largely equity. For households, real estate is mortgage debt divided by the value of real estate, and consumer durables is consumer credit divided by the value of consumer durables. Panel B reports the total tangible assets of households and noncorporate and corporate businesses relative to the total net worth of households. The main types of tangible assets, real estate, consumer durables, equipment and software, and inventories are also separately aggregated across the three sectors.

Panel A: Liabilities (% of tangible assets)

Sector	Debt (% of tangible assets)	Total liabilities (% of tangible assets)
(Nonfinancial) corporate businesses	48.5%	83.0%
(Nonfinancial) noncorporate businesses	37.8%	54.9%
Households and nonprofit organizations		
Total tangible assets	45.2%	47.1%
Real estate	41.2%	
Consumer durables	56.1%	

Panel B: Tangible assets (% of household net worth)

Assets by type	Tangible assets (% of household net worth)
Total tangible assets	79.2%
Real estate	60.2%
Equipment and software	8.3%
Consumer durables	7.6%
Inventories	3.1%

Table 2: Tangible assets and debt, rental, and lease adjusted leverage

Panel A displays the relation between tangibility and (debt) leverage and Panel B displays the relation between tangibility and leverage adjusted for rented assets. Annual firm level Compustat data for 2007 are used excluding financial firms.

Panel A: Tangible assets and debt leverage

Tangibility: *Property, Plant, and Equipment–Total (Net)* (Item #8) divided by Assets; Assets: *Assets–Total* (Item #6) plus *Price–Close* (Item #24) times *Common Shares Outstanding* (Item #25) minus *Common Equity–Total* (Item #60) minus *Deferred Taxes* (Item #74); Leverage: *Long-Term Debt–Total* (Item #9) divided by Assets.

Tangibility quartile	Quartile cutoff (%)	Leverage (%) median	Leverage (%) mean	Low leverage firms (%) (leverage $\leq$ 10%)
1	6.3	7.4	10.8	58.3
2	14.3	9.8	14.0	50.4
3	32.2	12.4	15.5	40.6
4	n.a.	22.6	24.2	14.9

Panel B: Tangible assets and debt, rental, and lease adjusted leverage

Lease Adjusted Tangibility: *Property, Plant, and Equipment–Total (Net)* plus 10 times *Rental Expense* (Item #47) divided by Lease Adjusted Assets; Lease Adjusted Assets: Assets (as above) plus 10 times *Rental Expense*; Debt Leverage: *Long-Term Debt–Total* divided by Lease Adjusted Assets; Rental Leverage: 10 times *Rental Expense* divided by Lease Adjusted Assets; Lease Adjusted Leverage: Debt Leverage plus Rental Leverage.

Lease adjusted tangibility quartile	Quartile cutoff (%)	Leverage (%)						Low leverage firms (%) (leverage $\leq$ 10%)		
		Debt		Rental		Lease adjusted		Debt	Rental	Lease adjusted
		median	mean	median	mean	median	mean			
1	13.2	6.5	10.4	3.7	4.2	11.4	14.6	61.7	97.7	46.0
2	24.1	9.8	12.9	6.9	8.1	18.4	21.0	50.1	68.2	16.1
3	40.1	13.1	14.8	8.0	10.5	24.2	25.3	41.7	60.6	12.0
4	n.a.	18.4	20.4	7.2	13.8	32.3	34.2	24.4	57.3	3.7

Table 3: Leverage and size revisited

This table displays debt and rental leverage across size deciles (measured by lease adjusted book assets). Lease Adjusted Book Assets: Assets – Total plus 10 times Rental Expense; Debt Leverage: Long-Term Debt – Total divided by Lease Adjusted Book Assets; Rental Leverage: 10 times Rental Expense divided by Lease Adjusted Book Assets; Lease Adjusted Leverage: Debt Leverage plus Rental Leverage. For details of data and variables used see caption of Table 2.

	Size deciles									
Median leverage	1	2	3	4	5	6	7	8	9	10
Debt	6.0	7.3	7.4	14.1	19.5	22.6	20.6	20.2	21.6	17.8
Rental	21.8	14.6	10.8	11.1	11.2	9.1	9.7	9.1	7.8	7.3
Lease adjusted	30.6	24.2	21.0	28.8	36.4	37.7	33.4	36.6	31.7	26.3

Figure 1: Leverage versus size revisited

Lease adjusted leverage (solid), debt leverage (dashed), and rental leverage (dash dotted) across size decicles for Compustat firms. For details see caption of Table 3.

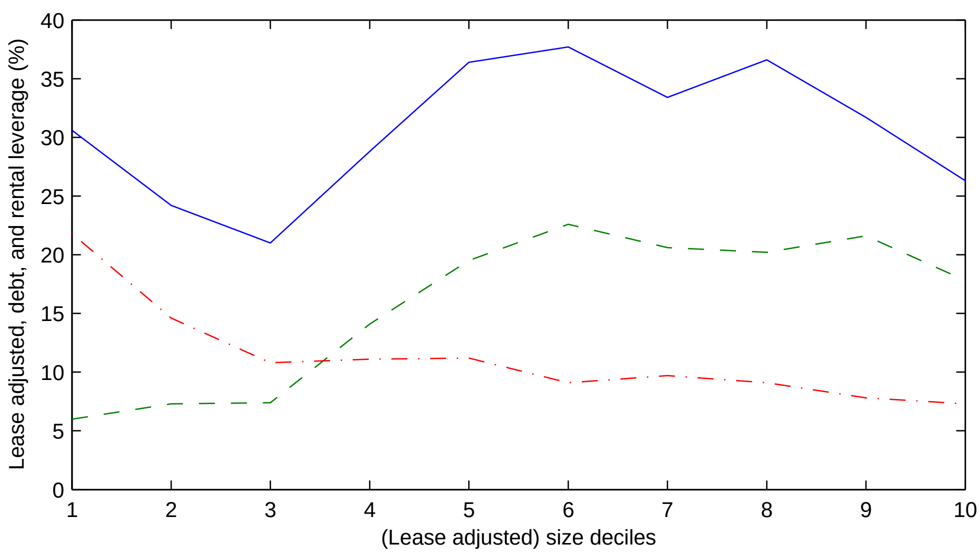

Figure 2: Investment and risk management

Panel A shows investment k; Panel B shows risk management for the low state $h(s_1')$; Panel C shows net worth in low state next period $w(s_1')$ (solid) and in high state next period $w(s_2')$ (dashed); Panel D shows scaled multipliers on the collateral constraint for the low state next period $\beta\lambda(s_1')/\mu$ (solid) and for the high state next period $\beta\lambda(s_2')/\mu$ (dashed); all as a function of current net worth w. The parameter values are: $\beta = 0.93$, $r = 0.05$, $\delta = 0.10$, $m = +\infty$, $\theta = 0.80$, $\varphi = 1$, $A(s_2) = 0.6$, $A(s_1) = 0.05$, and $\Pi(s, s') = 0.5$, $\forall s, s' \in S$, and $f(k) = k^{\alpha}$ with $\alpha = 0.333$.

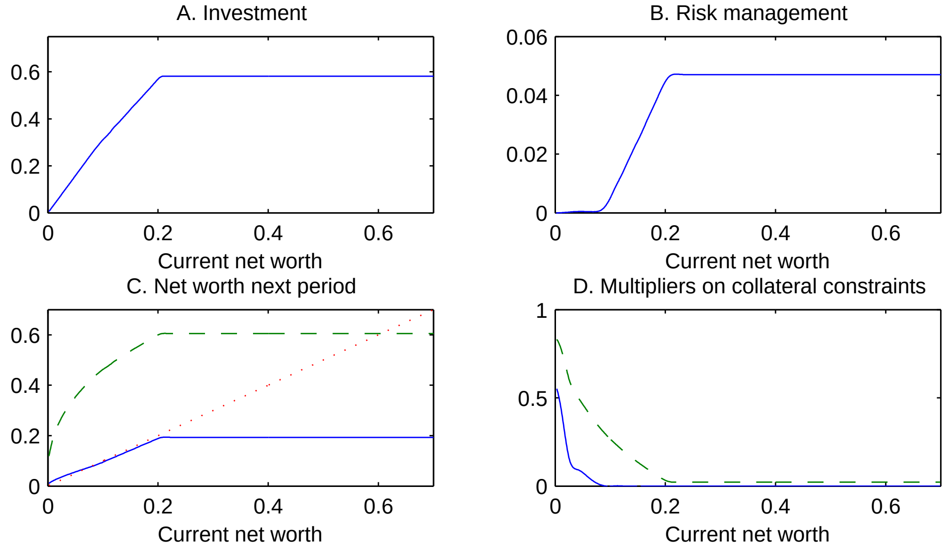

Figure 3: Risk management with stochastic investment opportunities

Panels A through I: Investment (k) and risk management for the low state ($h_1(s')$) as a function of current net worth w for low current productivity (s_1) (solid) and high current productivity (s_2) (dashed). Panel J: Risk management for the high state ($h_2(s')$) as a function of current net worth w for low current productivity (s_1) (dash dotted). Persistence measured by $\Pi(s_1, s_1) = \Pi(s_2, s_2) \equiv \pi$ is 0.50, 0.55, 0.60, 0.75, and 0.90 in Panels A/B (no persistence), Panels C/D (some persistence), Panels E/F (more persistence), Panels G/H (high persistence), and Panels I/J (severe persistence). For other parameter values see the caption of Figure 2.

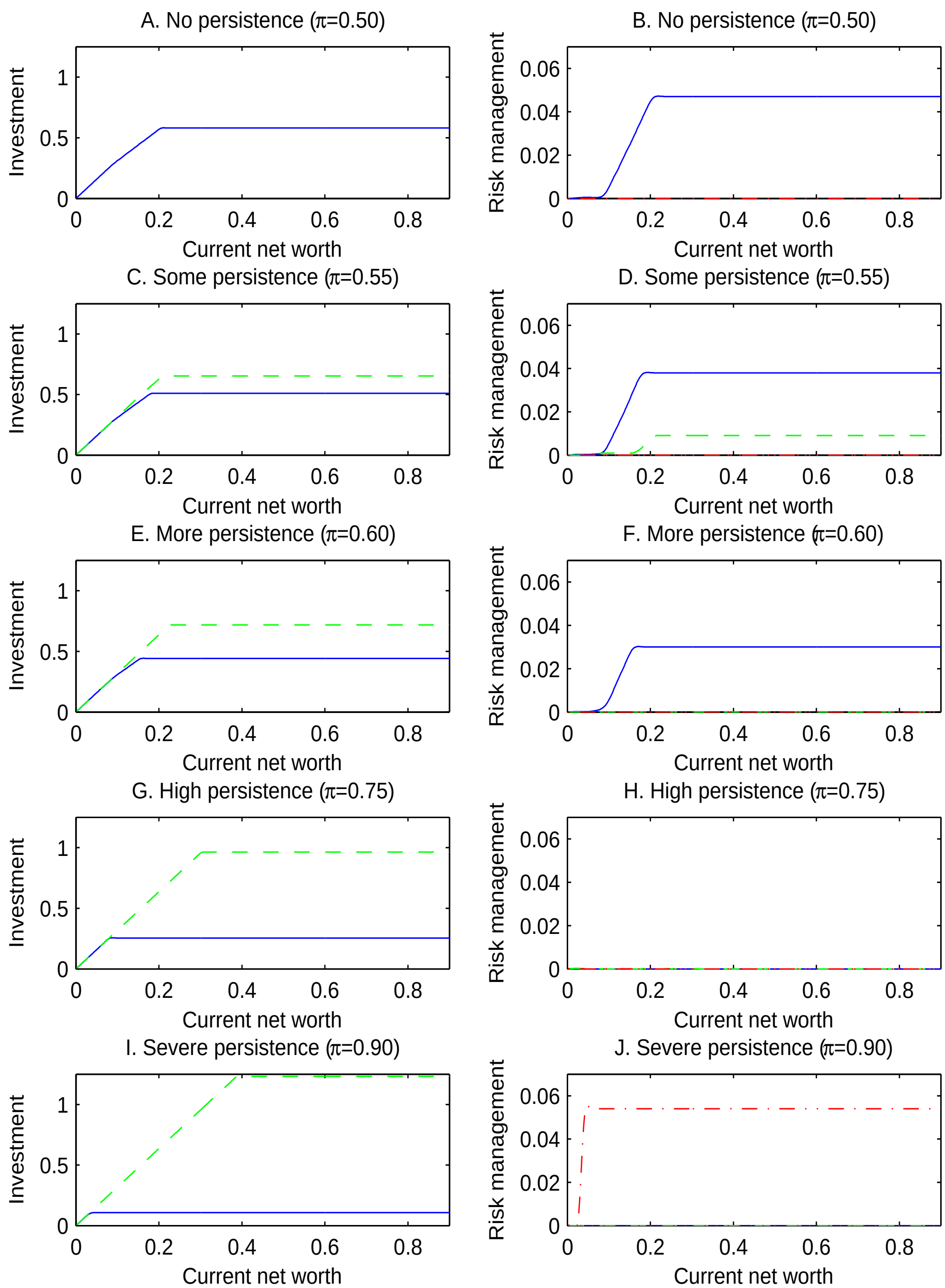

Figure 4: Leasing, leverage, and risk management

Panel A shows investment (k) (solid) and leasing (k_l) (dashed); Panel B shows risk management for the low state ($h_1(s')$); Panel C shows net worth next period in the low state next period ($w(s'_1)$) (solid) and in the high state next period ($w(s'_2)$) (dashed); Panel D shows the multipliers on the collateral constraints for the low state ($\beta\lambda(s'_1)$) (solid) and for the high state ($\beta\lambda(s'_2)$) (dashed); and Panel E shows total leverage $(\theta(\varphi k - k_l) + k_l)/k$ (solid), debt leverage $\theta(\varphi k - k_l)/k$ (dashed), and rental leverage k_l/k (dash dotted); all as a function of current net worth w. For other parameter values see the caption of Figure 2 except that $m = 0.01$ and $\varphi = 0.8$.