

Cointegration and Long-Run Asset Allocation

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We show that economic restrictions of cointegration between asset cash flows and aggregate consumption have important implications for return dynamics and optimal portfolio rules, particularly at long investment horizons. When cash flows and consumption share a common stochastic trend (i.e., are cointegrated), temporary deviations between their levels forecast long-horizon dividend growth rates and returns, and consequently, alter the term profile of risks and expected returns. We show that the optimal asset allocation based on the error-correction vector autoregression (EC-VAR) specification can be quite different relative to a traditional VAR that ignores the cointegrating relation. Unlike the EC-VAR, the commonly used VAR approach to model expected returns focuses on short-run forecasts and can considerably miss on long-horizon return dynamics, and hence, the optimal portfolio mix in the presence of cointegration. We develop and implement methods to account for parameter uncertainty in the EC-VAR setup and highlight the importance of the error-correction channel for optimal portfolio decisions at various investment horizons.

KEY WORDS: Asset allocation; Cointegration; Long-run risks.

1. INTRODUCTION

Risks facing a short-run and long-run investor can be quite different. While at very short horizons, the contribution of cash-flow news to the variance of return may be small, as the investment horizon increases, cash-flow fluctuations become the dominant source of return variability. Hence, understanding and modeling the behavior of asset returns, especially at long horizons, depend critically on understanding and modeling the dynamics of their cash flows. In this article we argue that deviations between cash-flow levels and aggregate consumption (the error-correction term) contain important information about means and variances of future cash-flow growth rates, and consequently, returns. Incorporating this cointegration restriction in return dynamics yields interesting implications for the term-structure of expected returns and risks, and hence, asset allocations at various investment horizons. In particular, we show that the error-correction mechanism significantly alters the risk-return tradeoff and the shape of optimal portfolio rules implied by models where the long-run adjustment of cash flows is ignored.

Our motivation for including the error-correction mechanism is based on the ideas of long-run risks developed in [Bansal and Yaron \(2004\)](#), [Hansen, Heaton, and Li \(2005\)](#), [Bansal, Dittmar, and Lundblad \(2005\)](#), and [Bansal, Dittmar, and Kiku \(2009\)](#). These articles, both theoretically and empirically, highlight the importance of the long-run relation between cash flows and aggregate consumption for understanding the magnitude of the risk premium and its cross-sectional variation. Built on this evidence, our article aims to explore the effect of long-run properties of asset cash flows on the optimal portfolio mix at various investment horizons. Intuitively, if the long-run dynamics of asset dividends are described by a cointegrating relation with aggregate consumption, then current deviations between their levels should forecast future dividend growth rates (see [Engle and Granger 1987](#)). Further, as risks in long-horizon returns are

dominated by cash-flow news, the predictability of asset dividends emanating from the error-correction mechanism may significantly alter the future dynamics of multihorizon returns and their volatilities. This suggests that the error-correction channel may be very important for determining the optimal asset allocation at intermediate and long investment horizons. Earlier portfolio choice literature, including [Kandel and Stambaugh \(1996\)](#), [Barberis \(2000\)](#), [Chan, Campbell, and Viceira \(2003\)](#), and [Jurek and Viceira \(2005\)](#), model asset returns via a standard vector-autoregression, and hence, ignore the consequences of the long-run dividend dynamics for the risk-return tradeoff and allocation decisions.

We measure the long-run relation between asset dividends and aggregate consumption via a stochastic cointegration. Based on the implications of the cointegrating relation, we model dividend growth rates, price-dividend ratios, and returns using an error-correction specification of a vector autoregression (EC-VAR) model. Our time-series specification allows us to compute the term profile of conditional and unconditional means and the variance-covariance structure of asset returns, which we subsequently use to derive the optimal conditional and unconditional portfolio rules. To highlight the importance of the error-correction dynamics in dividends, we compare the resulting allocations with those implied by a standard VAR model, which excludes the error-correction variable from investors' information set.

We solve the portfolio choice problem for buy-and-hold mean-variance investors with different investment horizons, ranging from 1 to 15 years, and different levels of risk aversion. To emphasize the implications of long-run cash-flow dynamics for the risk-return tradeoff and optimal portfolio mix, we focus on equities that are known to display large dispersion in average returns and opposite long-run (cointegrating)

characteristics. In particular, we consider investors who allocate their wealth across value and growth (i.e., high and low book-to-market) stocks and the one-year Treasury bond. We distinguish between conditional and unconditional portfolio choice problems and highlight the differences between the two.

To keep the analysis simple and transparent, we abstract from any types of dynamic rebalancing and focus on the first-order effect of the error-correction mechanism captured by the solution to the mean-variance problem. As shown in [Jurek and Viceira \(2005\)](#), regardless of investors' risk aversion, the intertemporal hedging demand contributes a very small portion (less than 5%) to the variation of the overall portfolio weights across time. Thus, the volatility of the optimal portfolio is largely dominated by its myopic component, which our approach captures. Considering reasonable alternative preference specifications, while straightforward, is unlikely to materially alter our evidence.

We establish several interesting results. Consistent with [Bansal, Dittmar, and Lundblad \(2001\)](#), [Hansen, Heaton, and Li \(2005\)](#), and [Bansal, Dittmar, and Kiku \(2009\)](#), we find that value and growth stocks significantly differ in their exposures to long-run consumption risks. While cash flows of value firms respond positively to low-frequency consumption fluctuations, growth firms display a negative response in the long run. Importantly, we find that current deviations in the dividend-consumption pair (the cointegrating residual) contain distinct information about future dynamics of both cash flows and multihorizon returns, which is missing in the VAR setup. In particular, if the error-correction dynamics are ignored and returns are modeled via the standard VAR, one is able to account for about 11% and 52% of the variation in growth and value returns at the 10-year horizon. With the cointegration-based specification, the long-run predictability of growth and value returns rises to striking 42% and 65%, respectively.

The forecasting ability of the error-correction term significantly alters variances (and covariances) of asset returns relative to the growth rates-based VAR, especially at intermediate and long horizons. As expected, the EC-VAR model generates a declining pattern in the term structure of unconditional volatilities of both value and growth stocks. The standard deviation of value returns in the VAR specification, on the other hand, is slightly increasing with the horizon. Hence, the EC-VAR model potentially is able to capture much larger benefits of time-diversification relative to the traditional VAR approach. Indeed, if the error-correction channel is ignored, the unconditional allocation to value stocks steadily declines: the VAR investors reduce their holdings of value stocks from 66% to about 52% as the investment horizon changes from 1 to 10 years. This pattern is consistent with the VAR-based evidence of [Jurek and Viceira \(2005\)](#). In contrast, relying on the cointegration-based specification, investors tend to allocate a much larger fraction of their wealth to value stocks as the investment horizon lengthens. In particular, the holding of value firms increases from 76% at the one-year horizon to about 96% for the 10-year investment. Thus, optimal portfolio prescriptions based on the standard VAR and EC-VAR models can be very different—these differences are a reflection of the error-correction mechanism between asset cash flows and aggregate consumption and the ensuing time-diversification effect. Given a strong economic

appeal of cointegration in dividend-consumption relation, our evidence suggests that investors should rely on the optimal portfolio mix based on the EC-VAR model.

It is well recognized in the literature that asset allocation decisions may be quite sensitive to parameter uncertainty. To ensure that our results are robust to estimation errors, we supplement our evidence by deriving optimal allocations of a Bayesian-type investor who recognizes and accounts for uncertainty about model parameters. The impact of parameter uncertainty in a standard VAR framework was earlier analyzed in [Kandel and Stambaugh \(1996\)](#) and [Barberis \(2000\)](#). We extend their approach and develop a method that allows us to handle parameter uncertainty in the cointegration setup. We find the Bayesian-based evidence to be qualitatively similar to the no-uncertainty case. To be specific, even after accounting for uncertainty in model parameters, the EC-VAR and VAR specifications deliver quite different portfolio rules, particularly in the intermediate and long run. As the horizon increases, the allocation to value continues to rise within the EC-VAR specification (from about 47% to 64% at the horizon extends from 1 to 10 years) and keeps on falling in the growth rates VAR framework (from 43% to 24%, respectively). Further, similar to [Barberis \(2000\)](#), we find that investors that doubt reliability of the estimated model parameters tend to shift their wealth away from equities toward safer securities. Depending on the horizon, the allocation to the Treasury bond increases by 20% to 40% compared to the no-uncertainty case. Taken together, our evidence suggests that parameter uncertainty affects the scale but not the shape of optimal asset allocations.

The rest of the article is structured as follows. Section 2 describes the evolution of risks across different investment horizons and points toward the importance of long-run dynamics of dividend growth rates for optimal decision rules. Section 3 outlines the portfolio choice problem, highlights implications of cointegration, and describes the dynamic model for asset returns. Our empirical results and their discussion are presented in Section 4. Finally, Section 5 concludes.

2. SOURCES OF RISKS AT DIFFERENT HORIZONS

Our motivation for incorporating long-run (cointegration) restrictions is the changing nature of risks across investment horizons. Although short and long-horizon investors are confronted by both risks in dividend growth rates and risks in price-dividend ratios, their concerns about the two are likely to be quite different since the relative contribution of dividend and price news to the overall return variation changes considerably with the horizon. While at short horizons, price risks are very important, their impact gradually diminishes due to stationarity of the price-dividend ratio. Consequently, at long investment horizons, variation in returns is dominated by risks in dividends.

To formalize this intuition, we perform a variance decomposition of returns using a first-order VAR model for dividend growth rates and log price-dividend ratios. Specifically, we project dividend growth of an asset on its own lag and regress the price-dividend ratio on one lag of the dividend growth, as well as its own lag. To provide a clean interpretation of the role of price shocks versus dividend shocks we orthogonalize the VAR innovations by assuming that dividend news leads to price

Table 1. Data summary

	Returns		Growth rates		log(P/D)	
	Mean	StdDev	Mean	StdDev	Mean	StdDev
Growth portfolio	8.45	19.56	1.31	9.07	3.91	0.40
Value portfolio	14.09	22.13	3.23	16.41	3.23	0.43
1-yr Treasury bond	2.25	2.04				
Consumption			2.16	1.13		

NOTE: This table presents descriptive statistics for returns, dividend growth rates and logarithms of price/dividend ratios of value and growth firms, the return on the one-year Treasury bond and consumption growth. Value firms represent companies in the highest book-to-market quintile of all NYSE, AMEX, and NASDAQ firms. Growth firms correspond to the lowest book-to-market quintile. Portfolios are constructed as in Fama and French (1993). Returns are value-weighted, price/dividend ratios are constructed by dividing the end-of-year price by the annual per-share dividend, growth rates are constructed by taking the first difference of the logarithm of per-share dividend series. Time-series for the Treasury bond are taken from the CRSP Fama-Bliss Discount Bonds files. Data on the per-capita consumption of nondurables and services come from the NIPA tables available from the Bureau of Economic Analysis. All data are sampled on at the annual frequency, converted to real using personal consumption deflator, and cover the period from 1954 to 2003.

movements, but price innovations do not lead to contemporaneous responses in dividends. We implement variance decomposition for two equity portfolios: growth and value stocks that we subsequently use in our asset allocation analysis. Growth and value stocks represent the lowest and the highest quintile of book-to-market sorted portfolios, respectively. The construction of portfolios and their dynamics are presented in Table 1.

We find that the contribution of dividends and price-dividend ratios to return variation changes significantly with the horizon. In the short run, price risks dominate—for both portfolios, about 98% of return variation over the one-year horizon is attributed to price news. However, as the holding interval increases, risks in returns shift toward risks in dividends. By the 10-year horizon, more than half of return variation is due to dividend shocks. As the horizon reaches 20 years, dividend growth risks account for 75% of variation in growth returns and more than 90% of risks in value returns. This evidence suggests that asset allocations at long investment horizons are mostly about managing dividend risks. Thus, understanding low-frequency dynamics of asset dividends and integrating them into a model for the risk-return tradeoff is critical in designing optimal allocations for long-horizon investors. In this article, we model the dynamics of asset dividends via a cointegrating relation with aggregate consumption and show that the ensuing error-correction channel has important implications for optimal portfolio rules at intermediate and long investment horizons. In the next section, we set out the portfolio choice problem and describe the dynamics of asset returns that account for long-run consumption risks in dividends.

3. ASSET ALLOCATION FRAMEWORK

3.1 Portfolio Choice Problem

We consider investors with constant relative risk aversion preferences (CRRA) preferences who follow a buy-and-hold strategy over different holding horizons. At time t , an investor chooses an allocation that maximizes her expected end-of-period utility and is locked into the chosen portfolio till the end of her investment horizon. Specifically, the s -period investor solves

$$\max_{\alpha_{s,t}} E_t[U_{t+s}] = \max_{\alpha_{s,t}} E_t \left[\frac{W_{t+s}^{1-\gamma}}{1-\gamma} \right], \quad (1)$$

where $\alpha_{s,t}$ is the vector of portfolio weights, W_{t+s} is the terminal wealth, and γ is the coefficient of risk aversion (RA). Letting $R_{t+1 \rightarrow t+s}^p$ denote the (gross) return on the portfolio held by the investor,

$$R_{t+1 \rightarrow t+s}^p = \alpha'_{s,t} \mathbf{R}_{t+1 \rightarrow t+s}, \quad (2)$$

where $\mathbf{R}_{t+1 \rightarrow t+s}$ is the vector of compounded asset returns, the evolution of wealth is described by

$$W_{t+s} = W_t * R_{t+1 \rightarrow t+s}^p. \quad (3)$$

We distinguish between the conditional and unconditional stock allocation problems. The conditional problem is stated above and uses the conditional distribution of future returns. The unconditional asset allocation relies on the unconditional distribution of asset returns to maximize expected utility.

To make the problem tractable, we will assume throughout that gross asset returns are lognormally distributed. As shown in Campbell and Viceira (2002), the investor's objective function in this case can be written as

$$\max_{\alpha_{s,t}} \left\{ \left[E_t[r_{t+1 \rightarrow t+s}^p] + \frac{1}{2} \text{Var}_t(r_{t+1 \rightarrow t+s}^p) \right] - \frac{\gamma}{2} \text{Var}_t(r_{t+1 \rightarrow t+s}^p) \right\}, \quad (4)$$

where $r_{t+1 \rightarrow t+s}^p$ is the log return on a portfolio bought at time t and held up to $t+s$. The unconditional problem can be restated analogously by dropping the time subscripts in the expression above. We will refer to $E_t[r_{t+1 \rightarrow t+s}^p]$ as the expected log return and $E_t[r_{t+1 \rightarrow t+s}^p] + \frac{1}{2} \text{Var}_t(r_{t+1 \rightarrow t+s}^p)$ as the arithmetic mean return. In the empirical section, the reported mean returns correspond to arithmetic means. To enhance the comparison across different holding periods, we measure and express all asset return moments per unit of time, that is, we scale both means and variances by the investment horizon.

There are three assets available to investors: in addition to the one-year Treasury bond, they allocate their wealth between growth and value stocks. We focus on stocks with opposite book-to-market characteristics that, historically, are known to display large dispersion in average returns (as shown in Table 1). The data employed in our empirical work are sampled on the annual frequency, thus, a single investment period corresponds to one year. We set risk aversion at five in our benchmark case; in addition, we highlight the implications of investors' preferences by entertaining a higher risk aversion level of 10.

3.2 Modeling Asset Returns

3.2.1 Cointegration Specification. We describe the long-run dynamics of dividends and consumption via a cointegrating relation,

$$d_t = \tau_0 + \tau_1 t + \delta c_t + \epsilon_{d,t}, \tag{5}$$

where d_t is the log level of an asset's dividend, c_t is the log level of aggregate consumption, and $\epsilon_{d,t} \sim I(0)$ is the cointegrating residual or the error-correction term. It follows from Equation (5) that dividend growth evolves as $\Delta d_t \equiv \tau_1 + \delta \Delta c_t + \Delta \epsilon_{d,t}$. Hence, a time-series model for $\epsilon_{d,t}$ and Δc_t is sufficient to model the dynamics of cash-flow growth rates.

Our specification implies that dividends and consumption share a common stochastic trend. The two, however, may exhibit different exposures to the underlying long-run risks as we do not impose a unit restriction on the cointegration parameter, δ . In addition, by including the time-trend in Equation (5), we allow for differences in deterministic trends in asset dividends and aggregate consumption. As we argue, imposing restrictions on either τ or δ may not be appropriate for the dividend series we rely on.

Following the existing asset pricing literature, we focus on dividends constructed on the per-share basis. These dividends correspond to a trading strategy of holding one share of a firm's stock at each point in time. An investor following the one-share strategy will consume all the dividends and reinvests only capital gains. Consider, alternatively, an investor who plows a portion of the received cash back into the firm. If the amount of reinvested income matches the net share issuance, such an investor will hold a claim to the total equity capital of the firm. Consequently, payout series associated with this alternative investment, which we refer to as aggregate dividends, are proportional to the firm's market capitalization. Notice the difference between the two measures—while the per-share series account for the growth of the share price, aggregate dividends reflect the appreciation of the firm's equity capital.

It may be theoretically appealing to omit the time trend and restrict the cointegration parameter of aggregate dividends on the market (or a particular sector of the economy) to one, as it will yield balanced growth paths of aggregate payouts and aggregate consumption. There is, however, no economic rationale for such restrictions for dividends per share. In order to illustrate and reinforce this important point, Figure 1 plots the log of the stock market dividend to consumption ratio for the two dividend measures. While the ratio of aggregate dividends to consumption appear to be stationary, the ratio of per-share dividends to consumption displays a dramatic decline over time. The reason per-share dividends fail to catch up with the level of aggregate consumption is due to the fact that per-share series, by construction, do not account for capital inflow in equity markets. [Bansal and Yaron \(2006\)](#) provided further discussion of the difference between the two trading strategies and implied dividend series.

From an econometric perspective, the distinction between aggregate and per-share dividends has important implications for modeling the dynamics of asset returns. As Figure 1 shows, per-share dividends and aggregate consumption tend to drift apart over time and the cointegrating relation between the two

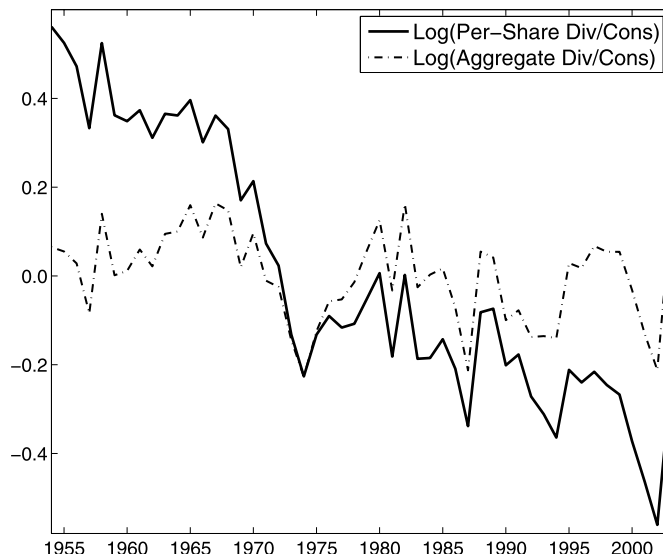


Figure 1. Dividend to consumption ratio. This figure plots the logarithm of per-share dividends to aggregate consumption (solid line), and the log ratio of aggregate dividends to consumption (dash-dotted line). Dividend series represent cash flows of the aggregate stock market portfolio. Per-share dividend series are constructed as $D_{t+1} = Y_{t+1}P_t$, where Y and P are the dividend yield and the price index respectively; the latter evolves according to $P_{t+1} = H_{t+1}P_t$, P_0 is normalized to 1, and H is the price gain. Aggregate dividend series are constructed as $D_{t+1}^{agg} = Y_{t+1}K_t$, where K is the market capitalization. The data are real and span the period from 1954 to 2003.

cannot be established under the $\{\delta = 1 \ \& \ \tau = 0\}$ restriction as confirmed by the augmented Dickey–Fuller test. Hence, omitting the time trend and imposing the $\{\delta = 1 \ \& \ \tau = 0\}$ restriction in the data leads to explosive/nonstationary return dynamics. This issue is also discussed in [Bansal, Dittmar, and Kiku \(2009\)](#).

The asset pricing literature typically focuses on the per-share dividends as their present value corresponds to the price of the asset, which is not true for aggregate dividend series. We follow this tradition and use series constructed on the per-share basis. Given the earlier discussion, we do not impose any restrictions on parameters that govern their long-run dynamics, letting the data decide on the underlying cointegrating relation between per-share dividends and aggregate consumption.

3.2.2 Return Dynamics. To describe the distribution of asset returns at various investment horizons, we model the dynamics of single-period returns and state variables jointly via the following EC-VAR,

$$\begin{pmatrix} b_{t+1} \\ \Delta c_{t+1} \\ \epsilon_{d,t+1} \\ z_{t+1} \\ r_{t+1} \end{pmatrix} = \begin{pmatrix} a_b \\ a_c \\ a_\epsilon \\ a_z \\ a_r \end{pmatrix} + \begin{pmatrix} \bullet & 0 & 0 & 0 & 0 \\ 0 & \bullet & 0 & 0 & 0 \\ 0 & \bullet & \bullet & \bullet & 0 \\ 0 & \bullet & \bullet & \bullet & 0 \\ 0 & \bullet & \bullet & \bullet & 0 \end{pmatrix} \begin{pmatrix} b_t \\ \Delta c_t \\ \epsilon_{d,t} \\ z_t \\ r_t \end{pmatrix} + \begin{pmatrix} u_{b,t+1} \\ u_{c,t+1} \\ u_{\epsilon,t+1} \\ u_{z,t+1} \\ u_{r,t+1} \end{pmatrix}. \tag{6}$$

That is, we project log bond return, b_t , and consumption growth, Δc_t , on their own lags, and regress the cointegrat-

ing residual, $\epsilon_{d,t}$, log price-dividend ratio, z_t , and log return, r_t , on their lags (excluding lagged return) and past consumption growth. Denoting $\mathbf{X}'_t = (b_t \ \Delta c_t \ \epsilon_{d,t} \ z_t \ r_t)$, we can rewrite the EC-VAR in a compact matrix form,

$$\mathbf{X}_{t+1} = \mathbf{a} + \mathbf{A}\mathbf{X}_t + \mathbf{u}_{t+1}, \quad (7)$$

where \mathbf{a} is the vector of intercepts, the matrix \mathbf{A} is defined earlier, and \mathbf{u} is a (5×1) -matrix of shocks that follow a normal distribution with zero mean and variance-covariance matrix Σ_u . For expositional purposes, we focus of the first-order EC-VAR. It is easy to allow for higher-order dynamics as they always can be mapped into the first-order representation.

The error-correction specification is the key dimension that differentiates our article from the existing portfolio choice literature. The latter typically models asset returns via a simple VAR that incorporates information on the price-dividend ratio (see Kandel and Stambaugh 1996; Barberis 2000; Chan, Campbell, and Viceira 2003; and Jurek and Viceira 2005 among others). In contrast to the traditional VAR approach, we describe the dynamics of asset returns using the error-correction framework that exploits the implications of long-run relation between dividends and consumption.

The conceptual difference between a standard VAR and the EC-VAR specifications is summarized by the error-correction variable, $\epsilon_{d,t}$. To see its implications for returns consider a Taylor series approximation of log returns (as in Campbell and Shiller 1988),

$$r_{t+1} = \kappa_0 + \Delta d_{t+1} + \kappa_1 z_{t+1} - z_t, \quad (8)$$

where κ 's are constants of linearization. As long-horizon returns can be computed via summing up both sides of Equation (8), multiperiod returns will depend on the dynamics of long-horizon dividend growth rates. Thus, long-run predictable variation in dividend growth via the cointegrating residual (Engle and Granger 1987) might alter predictability, and hence, distribution of multiperiod returns. In fact, cointegration between dividends and consumption has potentially the same economic consequences for returns as the unit cointegration restriction between prices and dividends. While price-dividend ratios are commonly used to forecast long-horizon returns, we argue that the error-correction residual, $\epsilon_{d,t}$, may be equally (if not more) important for predicting future returns; as such, it may significantly affect volatilities and correlations of multiperiod returns. Consequently, including the error-correction variable in the return dynamics may alter our views of the optimal long-run allocations.

To highlight the importance of the error-correction mechanism in cash flows for the risk-return tradeoff and optimal portfolio decisions, we will compare the implications of the cointegration-based EC-VAR to those implied by the traditional VAR specification. In the VAR setup, the error correction variable in Equation (6), $\epsilon_{d,t}$, is simply replaced by dividend growth, Δd_t .

Notice that instead of estimating the dynamics of asset returns directly as in Equation (6), one can infer them from the joint dynamics of price-dividend ratio and dividend growth according to the log-linear approximation in Equation (8). As shown in Campbell and Shiller (1988), the approximation works well at relatively short investment horizons. However,

once compounded, the approximation error can lead to sizable distortions in multiperiod return moments, and hence, long-horizon asset allocations. Quantitatively, we find that at horizons of 10 to 15 years, the volatility of approximate returns is likely to over or understate the true return volatility by 2% to 4%, or 15% to 20% in relative terms (see the Appendix for details). Although our empirical evidence will not materially change if we rely on log-linearization of asset returns, by modeling the dynamics of asset returns explicitly we are able to purge the effect of log-linearization and derive allocations that are not subject to the approximation error.

3.3 Term Structure of Expected Returns and Risks

The solution to the portfolio choice problem in Equation (4) hinges on the distribution of multiperiod returns, in particular, its first two moments. The required term-profile of expected returns and risks can be easily computed by exploiting the recursive structure of the EC-VAR as we outline in the following.

3.3.1 Unconditional Analysis. The solution to the unconditional problem is derived by fixing expected log returns on individual assets at their sample means, i.e.,

$$E[r_{t+1 \rightarrow t+s}] = \frac{1}{s} \sum_{j=1}^s \bar{r} = \bar{r}. \quad (9)$$

To compute the unconditional variance of asset returns at various investment horizons, we exploit the stationarity property of EC-VAR variables and present the original specification as an infinite-order moving average,

$$\mathbf{X}_{t+1} = (\mathbf{I} - \mathbf{A}L)^{-1} \mathbf{u}_{t+1} = \sum_{j=0}^{\infty} \mathbf{A}^j \mathbf{u}_{t+1-j}. \quad (10)$$

It follows, then, that the unconditional variance of \mathbf{X}_t is

$$\mathbf{\Omega}_0^* = \sum_{j=0}^{\infty} \mathbf{A}^j \Sigma_u \mathbf{A}^{j'}, \quad (11)$$

and the variance of the sum of s consecutive \mathbf{X} 's is given by

$$\mathbf{\Omega}_s^* = s\mathbf{\Omega}_0 + \sum_{j=1}^{s-1} (s-j)[\mathbf{V}_j + \mathbf{V}_j'], \quad (12)$$

where \mathbf{V}_j is the j -order autocovariance of \mathbf{X}_t defined as $\mathbf{V}_j = \mathbf{A}^j \mathbf{\Omega}_0$. Scaling $\mathbf{\Omega}^*$ by horizon, $\mathbf{\Omega}_s \equiv \frac{\mathbf{\Omega}_s^*}{s}$, the unconditional variance of multiperiod returns (expressed per-unit time) can be extracted via,

$$\text{Var}(r_{t+1 \rightarrow t+s}) = \iota_r' \mathbf{\Omega}_s \iota_r, \quad (13)$$

where ι_r is a (5×1) -indicator vector with the last element (corresponding to return) set equal to 1. As pointed out earlier, while the expected log returns $E[r_{t+1 \rightarrow t+s}]$ are constant across horizons, the unconditional variances may change with the horizon. Thus, although the unconditional problem does not accommodate market timing, it does exploit return predictability via horizon-dependent variances and correlations.

3.3.2 *Conditional Analysis.* In the conditional problem, we rely on the above EC-VAR to measure both expected values and variance–covariance structure of asset returns. Specifically, the mean of the continuously compounded return is computed as

$$E_t[r_{t+1 \rightarrow t+s}] = \frac{1}{s} \sum_{j=1}^s (\mathbf{G}_j \mathbf{a} + \mathbf{A}^j \mathbf{X}_t), \tag{14}$$

where $\mathbf{G}_j = \mathbf{G}_{j-1} + \mathbf{A}^{j-1}$ and $\mathbf{G}_0 = \mathbf{0}$, for $j = 1, \dots, s$. Further, for a given horizon $s \geq 1$, the innovation in the sum of s consecutive \mathbf{X} 's can be extracted as follows:

$$\sum_{j=1}^s \mathbf{X}_{t+j} - E_t \left[\sum_{j=1}^s \mathbf{X}_{t+j} \right] \equiv \boldsymbol{\zeta}_{t,t+s},$$

where $\boldsymbol{\zeta}_{t,t+s}$ is

$$\boldsymbol{\zeta}_{t,t+s} = \sum_{j=1}^s \mathbf{G}_j \mathbf{u}_{t+1+s-j}.$$

Exploiting the fact that errors are identically distributed and serially uncorrelated, the covariance matrix of $\boldsymbol{\zeta}_{t,t+s}$ for any given horizon s is

$$\boldsymbol{\Sigma}_s^* = \mathbf{G}_s \boldsymbol{\Sigma}_u \mathbf{G}_s' + \boldsymbol{\Sigma}_{s-1}^*, \tag{15}$$

where $\boldsymbol{\Sigma}_0^* = \mathbf{G}_0 \boldsymbol{\Sigma}_u \mathbf{G}_0' = \mathbf{0}$. As s increases, $\boldsymbol{\Sigma}_s^*$ grows without bound; hence we consider $\boldsymbol{\Sigma}_s \equiv \frac{\boldsymbol{\Sigma}_s^*}{s}$, that is, the covariance matrix of $\boldsymbol{\zeta}_{t,t+s}$ scaled by the horizon. Given $\boldsymbol{\Sigma}_u$ and \mathbf{G}_s , the evolution of $\boldsymbol{\Sigma}_s$ is given by

$$\boldsymbol{\Sigma}_s = \frac{1}{s} \mathbf{G}_s \boldsymbol{\Sigma}_u \mathbf{G}_s' + \left(1 - \frac{1}{s}\right) \boldsymbol{\Sigma}_{s-1}. \tag{16}$$

The term structure of risks in returns can now be extracted by taking the corresponding element of the s -horizon matrix,

$$\text{Var}_t(r_{t+1 \rightarrow t+s}) = \mathbf{t}_r' \boldsymbol{\Sigma}_s \mathbf{t}_r. \tag{17}$$

The arithmetic mean return can be constructed by adding half the variance to the expected log return given by Equation (14). The covariance between assets returns is calculated by stacking individual EC-VAR models and applying the same recursive procedure to the augmented system.

The solution to the conditional problem incorporates both horizon and time dimensions, allowing us to trace the impact of time-diversification as well as time-varying economic conditions on optimal allocations. Investors, in this case, are said to time the market by choosing their portfolios according to the current level of state variables.

3.4 Incorporating Parameter Uncertainty

Despite the growing evidence of time-variation in expected returns, it is well recognized that the true predictability of asset returns is highly uncertain. Furthermore, the predictive power of popular forecasting instruments, such as dividend yields, price–earning ratios or interest rates, is highly unstable across sample periods and sampling frequencies (Stambaugh 1999; Goyal and Welch 2003). This may raise concerns as to what

extent investors incorporate the data evidence on return predictability in their investment decisions. We address this issue in a Bayesian framework similar to Kandel and Stambaugh (1996) and Barberis (2000).

The difference between a frequentist and a Bayesian approaches lies in the probability distribution of asset returns that they rely on. In the former case, the term-structure of the risk–return relation is measured using the distribution conditioned on both the data and the point estimates. The Bayesian analysis, on the other hand, relies on the so-called predictive distribution of future returns conditioned only on the observed sample. To integrate out parameter uncertainty we use the standard Bayesian technique summarized in the Appendix.

Our analysis differs from Kandel and Stambaugh (1996) and Barberis (2000) as they designed optimal allocations using the standard VAR approach, and thus, do not entertain parameter uncertainty emanating from estimating cointegration parameters. Incorporating uncertainty in our EC-VAR setup leads to two layers of estimation risk. The first is induced by the uncertainty in the cointegrating relation, the second arises from the uncertainty about the EC-VAR parameters. Following the literature, we impose a flat prior on the EC-VAR model parameters, but consider an informative prior on the cointegrating relation between asset dividends and aggregate consumption. In particular, we assume that the prior distribution of the cointegration parameter is normal and centered at 1. To highlight the sensitivity of optimal asset allocations to prior uncertainty about the cointegration parameter, we allow for various degrees of investors' confidence. In the first case, which we refer to as the “tight” prior, we assume that 95% of the probability mass of the distribution of the cointegration parameter lies in the 0.5 to 1.5 range. In the second, “loose” prior case, we expand the confidence interval from -1 to 3 . In the case of the standard VAR we assume a noninformative prior.

4. EMPIRICAL RESULTS

4.1 Return Dynamics

In this section we discuss the dynamics of the state variables and returns across various investment horizons implied by our EC-VAR specification. Our benchmark results are based on parsimonious first-order dynamics. We subsequently highlight the robustness of our evidence to the inclusion of higher-order terms. We start by presenting empirical evidence on cointegration and analyzing the ability of the error-correction variable to predict future dividend growth rates and returns. Our consumption and financial data are standard and described in the footnote to Table 1.

4.1.1 *Cointegration Evidence.* We estimate cointegration parameters via ordinary least squares (OLS) by regressing log dividends on log consumption and a time trend. For both growth and value stocks, the sample autocorrelations of the resulting cointegrating residuals exhibit a rapid decline, from about 0.8 at the first lag to around -0.2 at the fifth lag. Formally, the augmented Dickey–Fuller test rejects the unit root null in the error-correction term at the 5% level for growth stocks and about 10% to 15% for value portfolio. This supports our assumption that the dynamics of the portfolios' dividends and aggregate consumption are tied together in the long run.

Long-run risk properties of value and growth firms, however, are very different. While cash flows of value firms respond positively to persistent shocks in aggregate consumption, growth firms' dividends exhibit an opposite, negative exposure to low-frequency consumption fluctuations. In particular, the parameter of cointegration is estimated at 1.94 (SE = 2.30) for high book-to-market firms and -4.84 (SE = 0.97) for low book-to-market firms. Similar estimates are obtained in the dynamic OLS framework of [Stock and Watson \(1993\)](#). The dynamic ordinary least squares (DOLS) estimate of the long-run exposure of asset dividends to consumption is equal to 1.58 (SE = 2.11) and -5.41 (SE = 1.01) for value and growth stocks, respectively. Our evidence is consistent with the cross-sectional pattern in long-run dividend betas documented in [Bansal, Dittmar, and Lundblad \(2001\)](#), [Hansen, Heaton, and Li \(2005\)](#), and [Bansal, Dittmar, and Kiku \(2009\)](#).

The implications of cointegration for future growth rates can intuitively be explained via the error-correction mechanism. Assume that dividends are unusually high today. Since the cointegrating residual is stationary, dividend growth rates are expected to fall since dividends have to adjust back to their long-run equilibrium with consumption. Thus, variation in future dividend growth rates should be accounted for by variation in the error-correction variable. Further, the slower the adjustment of dividends to the consumption level, the longer the effect of the cointegrating residual on future growth rates. Given that dividend growth is a key input in thinking about multihorizon returns, predictability of dividends emanating from the cointegrating relation may have important consequences for predictability and volatility of future returns.

4.1.2 Predictability Evidence. In this section, we examine the ability of the cointegrating residual to forecast asset returns at various horizons. To highlight the importance of the cointegrating relation, we compare R^2 's for return projections implied by the EC-VAR model with the corresponding R^2 's from the growth-rates-based VAR. The predictive state variables in the EC-VAR are consumption growth, price-dividend ratio, and cointegrating residual of the asset. In the VAR specification, we replace the asset's error-correction term ($\epsilon_{d,t}$) with dividend growth (Δd_t). Hence, in both cases we have three variables that forecast future equity returns. Note that we do not incorporate past prices and dividends of one asset when describing the dynamics of the other asset return as those bring virtually no additional predictive information. Statistically, once asset's own lagged attributes are included, the other asset does not improve the predictive capacity of the forecasting regressions.

Table 2 presents R^2 's implied by the EC-VAR and the alternative, growth rates model. As in [Hodrick \(1992\)](#), long-horizons R^2 's are calculated as one minus the ratio of the innovation variance in the return compounded over a given horizon s to the total variance of s -period returns, i.e.,

$$R_s^2 = 1 - \frac{t_r' \Sigma_s t_r}{t_r' \Omega_s t_r}, \quad (18)$$

where Σ_s and Ω_s are defined previously. The numbers reported in parentheses are the 2.5% and 97.5% percentiles of the corresponding bootstrap distributions. First, notice that return predictability implied by our EC-VAR specification improves considerably with the horizon. As Panel A shows, the EC-VAR

Table 2. Return predictability

Horizon	Growth	Value
Panel A: Error-correction VAR		
1	0.12 (0.07, 0.33)	0.17 (0.10, 0.39)
2	0.16 (0.11, 0.39)	0.26 (0.17, 0.45)
5	0.28 (0.17, 0.52)	0.46 (0.31, 0.62)
10	0.42 (0.22, 0.58)	0.65 (0.43, 0.73)
15	0.48 (0.23, 0.61)	0.73 (0.43, 0.79)
Panel B: Growth-rates VAR		
1	0.11 (0.07, 0.43)	0.18 (0.11, 0.44)
2	0.12 (0.07, 0.46)	0.27 (0.15, 0.57)
5	0.12 (0.05, 0.51)	0.45 (0.19, 0.75)
10	0.11 (0.03, 0.51)	0.52 (0.13, 0.70)
15	0.10 (0.02, 0.50)	0.47 (0.10, 0.60)

NOTE: This table presents return R^2 's implied by the EC-VAR specification (Panel A) and the alternative growth rates-based VAR model (Panel B). The latter ignores the implications of cointegration between asset cash flows and consumption. The entries are reported for value and growth portfolios across various holding horizons. Numbers in parentheses are the lower and upper bounds of the corresponding 95% bootstrap confidence intervals.

model accounts for only about 12% to 17% of the one-period return variation. However, by the 10-year horizon its predictive ability increases to striking 42% and 65% for growth and value stocks, respectively. Second, while none of the models seems to outperform the other in the short run, the growth rates VAR is noticeably dominated by the error-correction specification at longer horizons. This evidence suggests that the cointegrating residual incorporated in the EC-VAR specification contains distinct and important information about return dynamics, especially in the long run. To ensure robustness, we also considered direct projections of multiperiod returns on the EC-VAR and VAR predictive variables. The R^2 's from these regressions are very similar to those reported in Table 2.

As discussed earlier, dividend risks are an important component of return variation, especially at long horizons. Thus, predictability of multihorizon returns is largely driven by predictability of cash-flow growth rates. In the presence of cointegration, the error-correction specification should forecast dividend risks much better than the standard VAR. Indeed, we find a sizable improvement in predicting long-horizon dividend growth rates with the EC-VAR model. For example, in the VAR specification, the adjusted R^2 for predicting dividend growth rates at the five year horizon is about 22% for growth, and only 4% for value firms. In the EC-VAR, the corresponding numbers increase to 39% for growth and 18% for value stocks.

4.1.3 Term Profile of Means and Variances. Our predictability evidence suggests that the error-correction mechanism may have an important bearing on the evolution of the expected return-risk relation across investment horizons, which we now explore in details. The profile of arithmetic means and unconditional volatilities of asset returns is presented in Table 3. To emphasize the differences between the EC-VAR and the alternative VAR setup we display return moments for both models. As Panel A shows, the term structure of arithmetic mean returns on low and high book-to-market firms is declining with the horizon for the EC-VAR specification. In contrast, there is almost no decline in mean returns of value stocks in the VAR

Table 3. Term structure of expected returns and risks

Horizon	Expected return			Volatility		
	Growth	Value	Bond	Growth	Value	Bond
Panel A: Error-correction VAR						
1	0.082 (0.014)	0.133 (0.019)	0.022 (0.000)	0.186 (0.019)	0.203 (0.021)	0.000 (0.000)
2	0.081 (0.014)	0.131 (0.019)	0.022 (0.005)	0.181 (0.017)	0.193 (0.019)	0.022 (0.004)
5	0.078 (0.014)	0.128 (0.019)	0.023 (0.006)	0.168 (0.018)	0.175 (0.023)	0.030 (0.006)
10	0.076 (0.014)	0.126 (0.020)	0.023 (0.006)	0.153 (0.020)	0.161 (0.030)	0.034 (0.007)
15	0.074 (0.014)	0.125 (0.021)	0.023 (0.006)	0.141 (0.021)	0.155 (0.035)	0.036 (0.008)
Panel B: Growth-rates VAR						
1	0.080 (0.023)	0.132 (0.029)	0.022 (0.000)	0.179 (0.017)	0.198 (0.021)	0.000 (0.000)
2	0.080 (0.023)	0.132 (0.029)	0.022 (0.005)	0.176 (0.017)	0.195 (0.020)	0.022 (0.004)
5	0.078 (0.024)	0.132 (0.031)	0.023 (0.006)	0.168 (0.021)	0.195 (0.026)	0.030 (0.006)
10	0.077 (0.025)	0.133 (0.033)	0.023 (0.006)	0.159 (0.027)	0.199 (0.037)	0.034 (0.007)
15	0.076 (0.026)	0.133 (0.034)	0.023 (0.006)	0.153 (0.031)	0.203 (0.042)	0.036 (0.008)

NOTE: This table reports the profile of mean returns and volatilities by horizon. Expected returns and risks are presented for the EC-VAR specification (Panel A) and the alternative growth rates-based VAR model (Panel B). The latter ignores the implications of cointegration between asset cash flows and consumption. Bootstrap standard errors are reported in parentheses.

specification (see Panel B). Recall that in the unconditional case, the arithmetic mean return for a given horizon is defined as the mean log return plus one-half of the scaled variance of the multihorizon return. Clearly, the first component is the same in both the EC-VAR and VAR specifications, as it is simply determined by the historical average of log asset returns. The variance component, however, depends on the time-series dynamics and predictability of long-horizon returns, and consequently, may significantly differ between the competing models.

Indeed, we find that volatilities of asset returns at various horizons are quite different across the two specifications. As expected, the standard deviation of long-run returns is reduced within the error-correction framework for both value and growth stocks. The volatility of returns implied by the alternative VAR specification similarly decreases for growth firms, but displays a generally flat (slightly increasing) pattern for value firms. In particular, the standard deviation of value returns declines from 20% at the one-year horizon to about 15% at the long horizon for the EC-VAR specification, but stays at the initial 20% for the VAR model. Thus, the EC-VAR specification captures considerable time-diversification benefits in value returns that are overlooked by the VAR model.

Differences between the error-correction and growth rates specifications are pronounced not only for volatilities but also correlations of asset returns. In particular, in the EC-VAR the correlations of returns are much higher than in the VAR model. In the latter setup, the correlations decline from 0.63 to about 0.30 from the one to 15-year horizon. In contrast, for the EC-VAR, the correlation starts at 0.75 and gradually decreases to about 0.52. We should emphasize that these differences are not solely driven by the differences in returns' variances across the two models—the EC-VAR and VAR-implied covariances likewise significantly deviate from each other. This evidence implies that from the VAR perspective, diversification across assets can be quite important—growth asset can be valuable at long horizons despite its lower mean for purposes of reducing the overall volatility of the optimal portfolio. The cross-sectional diversification seems to be less valuable from the EC-VAR perspective.

To investigate the impact of the estimation error on return moments, in Table 4 we report return volatilities after accounting for parameter uncertainty. Mean returns are not reported for brevity—across all horizons, they are about 1% lower than their counterparts in the case when parameter uncertainty is ignored. Not surprisingly, the volatility of asset returns is higher when estimation errors are taken into account. However, the general pattern is similar to the case when parameter uncertainty is ignored.

To summarize, the empirical evidence presented in this section underscores the importance of the cointegration specification for risks and expected returns. Temporary deviations of cash flows from the permanent component in consumption contain important information about future dynamics of asset returns, and consequently, the term structure of the risk-return tradeoff. Furthermore, cointegration alters risk diversification properties of value and growth assets relative to the standard VAR model that, we expect, may significantly affect wealth allocation across the two stocks.

Table 4. Term structure of return volatilities with parameter uncertainty

Horizon	Growth	Value	Bond
Panel A: Error-correction VAR			
1	0.214	0.239	0.022
2	0.206	0.227	0.027
5	0.191	0.204	0.033
10	0.173	0.184	0.036
15	0.158	0.173	0.037
Panel B: Growth-rates VAR			
1	0.210	0.234	0.024
2	0.208	0.235	0.030
5	0.205	0.248	0.039
10	0.203	0.270	0.045
15	0.202	0.288	0.047

NOTE: This table reports standard deviations of the predictive distribution of multi-period returns. The term profile of asset risks is presented for the EC-VAR specification (Panel A) and the alternative growth rates-based VAR model (Panel B). Panel A is constructed imposing "tight" prior on the cointegration vector, centered around one.

4.2 Asset Allocation Decisions

Using the profile of constructed return moments, we solve for the optimal allocations of investors with different holding intervals. For brevity, in the benchmark case of no-parameter uncertainty, we report allocations for the risk aversion level of 5. The impact of investors' preferences is illustrated later on, for the case that incorporates estimation uncertainty in model parameters. In there we consider two values of risk aversion, 5 and 10, and two levels of prior confidence, "loose" and "tight," defined earlier. For simplicity, we ignore short selling constraints; the essential message is similar if one were to impose these restrictions.

4.2.1 Unconditional Analysis. Asset allocations of the EC-VAR-based investors are reported in Panel A of Table 5. We find that their investment strategy is considerably tilted toward value stocks at both short and long horizons. In particular, the allocation to value firms starts at about 76% at the one-year horizon, increasing to 95% at the 15-year horizon. The allocation to growth increases as well but it starts with a negative position. In addition, the level of growth investment is significantly lower than the allocation to value stocks at any horizon.

The horizon effect has an opposite pattern for the alternative VAR specification, as suggested by the entries in Panel B. At the very short horizon, the VAR-based investors still allocate more to value than to growth stocks. Their preferences toward the two assets, however, reverse as the holding period lengthens—as the horizon increases, the allocation to value declines and that to growth increases. This is consistent with the evidence in Jurek and Viceira (2005). In a similar VAR setup, they also found that long-run investors gradually shift their wealth away from value stocks.

The documented differences in the optimal portfolio mix across the two models arise due to different patterns in return volatilities and their correlations. The VAR-based investors perceive value stocks as quite risky, especially in the long run, thus, steadily reducing their allocations to high book-to-market firms. In contrast, the EC-VAR investors recognize that, via the error-correction mechanism, the relative riskiness of value investment shrinks over time. At short horizons, transitory risks in

dividends and prices make value stocks look quite risky. In the long run, transitory fluctuations are washed away and all risks in value returns come from permanent risks in their dividends. Importantly, the adjustment of value dividends, and thus, value returns to their long-run equilibrium relation with aggregate consumption is strongly predicted by the error-correction variable. This long-run predictability reduces the volatility of multihorizon returns making value firms more attractive for long-horizon investments. Further, while in the VAR, the cross-sectional diversification via growth returns increases, its benefits are significantly reduced in the cointegration-based framework. Thus, the EC-VAR-based investors do not view growth stocks as good substitutes for value.

We find that across all horizons, investors are better off by following the cointegration-based allocation strategy rather than that prescribed by the standard VAR. Utility gain associated with the EC-VAR specification is increasing with the horizon when predictability coming from the error-correction mechanism takes stronger effect. For example, at the one-year investment horizon, the EC-VAR and VAR specifications yield about 0.050 and 0.049 utilities, respectively. By the five-year horizon, the difference in utilities is about 20%. At the 15-year horizon, the error-correction implied utility is about 0.065, while the VAR-based allocation guarantees only about 0.050, which amounts to more than 30% difference.

Our evidence is quite robust with respect to the order of the EC-VAR and VAR dynamics. We find that across various plausible specifications, allocation to value stocks is always increasing with the horizon in the error-correction case and always declining in the growth rates-based VAR. For example, the EC-VAR(2)-based investor will choose to increase her holdings of value stocks from 0.70 to about 1.04 as the horizon increases from one to five years. Investors that follow VAR(2) strategy, on the other hand, will choose to allocate about 53% of their wealth to value stocks at the one-year horizon, and about 48% at the five-year horizon. Higher-order specifications, as well, do not change the documented patterns in asset allocations and do not alter the magnitudes of portfolio weights in any economically meaningful way.

We now turn the discussion to the portfolio choice in the presence of parameter uncertainty, which is reported in Table 6.

Table 5. Optimal allocation strategy

Horizon	Growth	Value	Bond
Panel A: Error-correction VAR			
1	-0.30 (-0.94, 0.10)	0.76 (0.15, 1.27)	0.54 (0.42, 0.93)
2	-0.30 (-0.93, 0.15)	0.82 (0.19, 1.29)	0.48 (0.38, 0.91)
5	-0.26 (-0.93, 0.21)	0.91 (0.17, 1.51)	0.35 (0.15, 0.87)
10	-0.14 (-0.92, 0.36)	0.96 (0.16, 1.93)	0.18 (-0.35, 0.82)
15	-0.00 (-0.88, 0.49)	0.95 (0.18, 2.32)	0.05 (-0.94, 0.79)
Panel B: Growth-rates VAR			
1	-0.13 (-0.66, 0.55)	0.66 (-0.21, 1.11)	0.47 (0.14, 0.98)
2	-0.07 (-0.64, 0.58)	0.64 (-0.18, 1.06)	0.43 (0.17, 0.95)
5	0.08 (-0.50, 0.64)	0.57 (-0.12, 1.10)	0.35 (0.00, 0.92)
10	0.23 (-0.39, 0.73)	0.52 (-0.10, 1.28)	0.25 (-0.25, 0.91)
15	0.33 (-0.35, 0.80)	0.51 (-0.09, 1.38)	0.16 (-0.41, 0.91)

NOTE: This table reports the optimal allocation across different investment horizons. Portfolio weights are presented for the EC-VAR specification (Panel A) and the alternative growth rates-based VAR model (Panel B). The latter ignores the implications of cointegration between asset cash flows and consumption. Numbers in parentheses are the lower and upper bounds of the corresponding 95% bootstrap confidence intervals.

Table 6. Optimal allocation strategy with parameter uncertainty

Horizon	Growth	Value	Bond
Panel A: Error-correction VAR			
1	-0.23	0.47	0.75
2	-0.21	0.50	0.71
5	-0.19	0.57	0.62
10	-0.14	0.64	0.50
15	-0.10	0.69	0.41
Panel B: Growth-rates VAR			
1	-0.09	0.43	0.66
2	-0.05	0.37	0.66
5	0.06	0.30	0.63
10	0.15	0.24	0.62
15	0.18	0.21	0.61

NOTE: This table reports the optimal allocation of Bayesian-type investors with different holding periods. Portfolio weights are presented for the EC-VAR specification (Panel A) and the alternative growth rates-based VAR model (Panel B). Panel A is constructed imposing “tight” prior on the cointegration vector, centered around one.

We maintain risk aversion of 5 and in the cointegration specification we use the “tight” prior centered at 1. First notice that incorporating parameter uncertainty significantly lowers positions in risky assets, which is similar to the findings of Barberis (2000). In the VAR specification, investors continue to cut down their allocations to value stocks as the horizon increases—starting at 43%, investment in value firms declines to 21% at the 15-year horizon. Similar to the no-parameter uncertainty case, the allocation to growth increases with the horizon due to the cross-sectional diversification effect. With the EC-VAR specification, the allocation to value increases with the investment horizon: from 47% at the one-year horizon to 69% under the 15-year buy-and-hold strategy. Consequently, for both specifications, parameter uncertainty affects the scale of the position, but does not affect the horizon pattern in allocations.

Table 7 highlights the effect of parameter uncertainty under various assumptions of the prior confidence about the cointegration parameter. The optimal portfolio weights are reported for four configurations based on high and low risk aversion,

Table 7. Optimal allocation strategy with parameter uncertainty under various assumptions

Horizon	Growth	Value	Bond	Growth	Value	Bond
RA = 5, tight prior			RA = 5, loose prior			
1	-0.23	0.47	0.75	-0.14	0.46	0.68
2	-0.21	0.50	0.71	-0.11	0.47	0.64
5	-0.19	0.57	0.62	-0.05	0.49	0.56
10	-0.14	0.64	0.50	0.04	0.49	0.47
15	-0.10	0.69	0.41	0.11	0.50	0.39
RA = 10, tight prior			RA = 10, loose prior			
1	-0.11	0.22	0.88	-0.08	0.23	0.86
2	-0.10	0.25	0.85	-0.07	0.24	0.83
5	-0.09	0.32	0.77	-0.05	0.25	0.79
10	-0.06	0.37	0.69	0.00	0.25	0.75
15	-0.01	0.37	0.64	0.03	0.24	0.73

NOTE: This table reports the optimal allocation of investors who rely on the EC-VAR specification and incorporate parameter uncertainty. Four panels present portfolio weights for different levels of investors’ risk aversion and their confidence that value and growth firms’ dividends exhibit a unit cointegration with aggregate consumption.

and “tight” and “loose” priors about the cointegration parameter. As expected, for a given prior, increasing the risk aversion shifts the allocation away from risky assets toward the T-bond. At the same time, both high and low risk aversion investors continue to hold on to value stocks. Although the allocation to growth increases with the horizon, it fails to keep up with the value investment. Further, for a given risk aversion, different prior beliefs do not dramatically affect portfolio weights at short horizons. The prior uncertainty, however, seems to matter for longer holding periods. With lax beliefs about the cointegration parameter, the long-horizon allocation to value stocks scales down relative to a tighter prior.

The key message of the evidence presented above is that the EC-VAR view of return dynamics significantly alters asset allocations, particularly at long horizons. Specifically, in the long run, value stocks seem to be far more attractive relative to the traditional VAR model for returns. Moreover, this result continues to hold even after accounting for uncertainty about model parameters. In the next section, we discuss the effect of the error-correction channel within the conditional framework.

4.2.2 Conditional Analysis. The optimal allocation of a conditional-type investor with the 10-year horizon is presented in Figure 2. We report the evidence for the case where parameter uncertainty about cointegration is incorporated using “loose” prior around unity and risk aversion of 10. The portfolio choice problem is solved for each date using the predictive density and the current value of the predictive state variables that are observable to the investor. Portfolio allocation problems that exploit market timing in somewhat different settings are also considered in Lynch and Balduzzi (2000), Ferson and Siegel (2001), Brandt and Ait-Sahalia (2001), Ang and Bekaert (2002), Bansal, Dahlquist, and Harvey (2004), and Brandt and Santa-Clara (2006) among others. The importance of conditioning information and predictability is highlighted in the earlier work of Hansen and Richard (1987).

Figure 2 suggests a number of interesting observations. The first-order difference between the EC-VAR and the VAR specifications is the difference in level allocations to value and growth firms. Similar to the unconditional analysis, the allocation to value is much higher under the EC-VAR specification. Second, a visual inspection of EC-VAR weights reveals an interesting business-cycle pattern in investment decisions of long-horizon investors. The holding of growth stocks tends to significantly decline at the outset of economic troughs. For example, the 10-year allocation to growth falls right before the recessions of 1970, 1973, 1990–1991, and 2001. A lag between a decrease in growth weight and an up-coming downturn measured by the National Bureau for Economic Research (NBER) business cycle indicator is about two to three years. Value holdings, on the other hand, strengthen during recessions. Table 8 summarizes business-cycle properties of conditional allocations by averaging portfolio weights across the NBER-dated expansions and recessions. As the table suggests, the optimal allocation to value stocks tends to be higher during recessions than booms. For example, the mean of the one-year allocation to value is about 50% when the economy is in a low state and only 24% in a high state. This difference, although somewhat less pronounced, is still noticeable at longer hori-

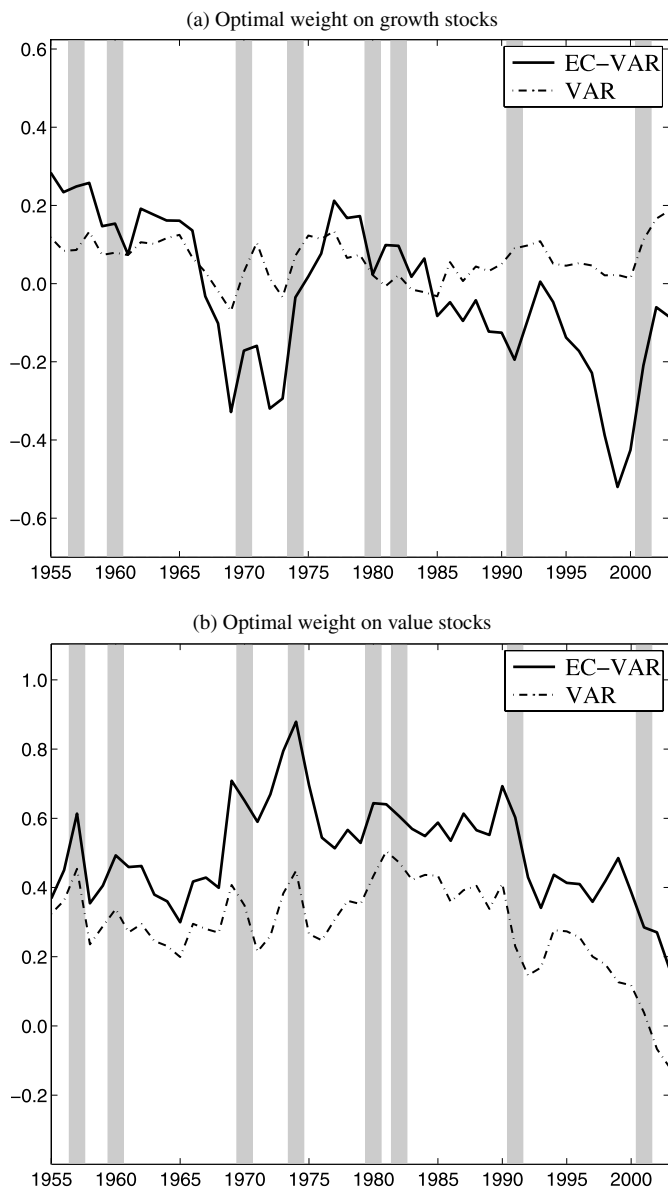


Figure 2. Portfolio weights. This figure displays time-series of optimal portfolio holdings of growth (a) and value (b) stocks for buy-and-hold investors with the 10-year investment horizons. Thick line corresponds to weights implied by the EC-VAR model, dash-dotted line is derived from the alternative growth rates-based VAR specification. Both allocations incorporate parameter uncertainty and assume risk aversion of 10. The prior of the cointegration parameter in the EC-VAR specification is “loose” and centered around 1.

zons. Notice also that bond holdings are highly pro-cyclical, increasing during expansions (i.e., when expected returns on equity are low) and declining when the economy moves down (i.e., when risk compensations are expected to rise). The market timing dimension is much less pronounced in the VAR setup. In particular, the correlation between the 10-year allocation to value and consumption growth is -17% in the EC-VAR specification but only -6% in the case of the VAR specification. The corresponding numbers for the five-year horizon are -30% and -15% for the EC-VAR and the standard VAR, respectively.

Table 8. Optimal allocations of a Bayesian, conditional-type EC-VAR investor

Horizon	Growth	Value	Bond
Panel A: Average across NBER recessions			
1	-0.08	0.50	0.58
5	-0.10	0.54	0.55
10	-0.01	0.60	0.41
Panel B: Average across NBER expansions			
1	-0.12	0.24	0.88
5	-0.10	0.39	0.71
10	-0.03	0.48	0.55

NOTE: This table reports optimal allocations of a conditional-type investor who relies on the EC-VAR specification and incorporates parameter uncertainty. Panels A and B report average allocations across economic recessions and expansions as defined by the NBER business cycle indicator over the period from 1954 to 2003. The prior of the cointegration parameter is “loose” and centered around 1, the level of risk aversion is set at 10.

5. CONCLUSION

In this article, we argue that the long-run equilibrium relation measured via a stochastic cointegration between aggregate consumption and dividends has significant implications for dividend growth rates and return dynamics. The error-correction mechanism between dividends and consumption and the ensuing EC-VAR provide an alternative specification for describing the dynamics of equity returns relative to the traditional VAR that ignores the implications of the long-run equilibrium. The recent long-run risks literature (including Bansal and Yaron 2004; Bansal, Dittmar, and Lundblad 2005; Hansen, Heaton, and Li 2005; and Bansal, Dittmar, and Kiku 2009) argued that the long-run relation between cash flows and consumption contains important information about asset risk premia. Our approach explores the implications of these ideas for optimal portfolio decisions across different investment horizons.

We show that the error-correction channel, incorporated by the EC-VAR representation, significantly alters return forecasts and the variance-covariance matrix of asset returns and shifts the optimal portfolio mix relative to the traditional VAR. The standard VAR specification captures the short-run return dynamics, but in the presence of cointegration, considerably misses the long-horizon dynamics of asset returns. In contrast, the EC-VAR specification successfully accounts for both short and long-horizon return dynamics. Consequently, as we show, the EC-VAR model is able to capture much larger benefits of time-diversification than the standard VAR framework does.

Incorporating parameter uncertainty in the cointegration-based specification, we highlight the effect of estimation errors on long-horizon asset allocations. We show that significant differences in the optimal portfolio decisions between the EC-VAR and VAR specifications persist even after accounting for parameter uncertainty. In sum, our evidence suggests that optimal allocations derived from the standard VAR may be quite suboptimal for investors with intermediate and long investment horizons.

APPENDIX A: EVALUATING THE ACCURACY OF LOG-LINEARIZATION

To assess the effect of log-linearization (Equation (8)) on the dynamics of multihorizon returns, we perform the following

simulation exercise. Using the point estimates of the EC-VAR specification, we simulate price and dividend levels and construct exact asset returns. Innovations are drawn from a normal distribution using the residual variance–covariance matrix. The length of the simulated sample is set at 50 years as in the actual data. Using the simulated sample, we estimate two error-correction specifications: one that directly incorporates the return projection and another one that excludes the return equation. In the latter case, we use the log-linear approximation to infer the dynamics of asset returns. We iterate on each specification to compute means and variances of multiperiod returns. Notice that any differences in return moments between the two specifications are driven entirely by the approximation error. Consistent with the evaluation in Campbell and Shiller (1988), we find that differences between exact and approximate returns are tiny at short horizons. However, as the horizon increases, the approximation error is magnified due to compounding and may lead to differences in standard deviations as high as 2% to 4%. For example, at the 15-year horizon, it is quite likely that the volatility of the approximate return on value stocks will over or understate the true return volatility by 3% or 4.2%, respectively. For growth stocks, the magnitude of the approximation error can be as large as 2% to 3%. In relative terms, the approximation error may lead to up to 15% to 20% distortion in return volatilities at long horizons, which can be significant from an asset allocation perspective. Similar results hold for the VAR specification.

APPENDIX B: INCORPORATING UNCERTAINTY IN THE VAR FRAMEWORK

To obtain the posterior distribution of the VAR parameters, we follow Bauwens, Lubrano, and Richard (1999) and re-write the model in the form of seemingly unrelated regressions. Specifically, we present the dynamics of each variable i ($i = 1, \dots, n$) as

$$\mathbf{X}_i = \mathbf{Z}_i \boldsymbol{\beta}_i + \mathbf{U}_i, \quad (\text{B.1})$$

where \mathbf{X}_i is $((T-1) \times 1)$ -vector of observations on the i th variable, \mathbf{Z}_i is the $((T-1) \times k_i)$ -matrix of relevant predictors, $\boldsymbol{\beta}_i$ is the $(k_i \times 1)$ -vector of regression coefficients (including the intercept), and \mathbf{U}_i is the $((T-1) \times 1)$ -vector of shocks. Stacking all the equations together, we can express Equation (B.1) compactly as

$$\mathbf{x} = \mathbf{z}\boldsymbol{\beta} + \mathbf{u}, \quad (\text{B.2})$$

where $\mathbf{x} = (\mathbf{X}'_1, \dots, \mathbf{X}'_n)$, $\boldsymbol{\beta} = (\boldsymbol{\beta}'_1, \dots, \boldsymbol{\beta}'_n)$, $\mathbf{u} = (\mathbf{U}'_1, \dots, \mathbf{U}'_n)$, and $\mathbf{z} = \text{diag}(\mathbf{Z}_1, \dots, \mathbf{Z}_n)$. Alternatively, the EC-VAR model can be cast in the following matrix form:

$$\mathbf{X} = \mathbf{Z}\mathbf{B} + \mathbf{U}, \quad (\text{B.3})$$

where $\mathbf{X} = (\mathbf{X}_1, \dots, \mathbf{X}_n)$, $\mathbf{Z} = (\mathbf{Z}_1, \dots, \mathbf{Z}_n)$, $\mathbf{U} = (\mathbf{U}_1, \dots, \mathbf{U}_n)$, and $\mathbf{B} = \text{diag}(\boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_n)$. To derive the posterior distribution, we assume that investors have no well-defined beliefs about the model parameters, and therefore, use a noninformative prior

$$\varphi(\boldsymbol{\beta}, \boldsymbol{\Sigma}_u) \propto |\boldsymbol{\Sigma}_u|^{-(n+1)/2}. \quad (\text{B.4})$$

As shown in Bauwens, Lubrano, and Richard (1999), the posterior density of the parameters can be factorized as

$$\begin{aligned} \boldsymbol{\beta} | \boldsymbol{\Sigma}_u &\sim N(\hat{\boldsymbol{\beta}}, [\mathbf{z}'(\boldsymbol{\Sigma}_u^{-1} \otimes \mathbf{I}_{T-1})\mathbf{z}]^{-1}), \\ \boldsymbol{\Sigma}_u | \boldsymbol{\beta} &\sim \text{IW}(\mathbf{Q}, T-1), \end{aligned} \quad (\text{B.5})$$

where

$$\begin{aligned} \hat{\boldsymbol{\beta}} &= [\mathbf{z}'(\boldsymbol{\Sigma}_u^{-1} \otimes \mathbf{I}_{T-1})\mathbf{z}]^{-1} \mathbf{z}'(\boldsymbol{\Sigma}_u^{-1} \otimes \mathbf{I}_{T-1})\mathbf{x}, \\ \mathbf{Q} &= (\mathbf{X} - \mathbf{Z}\mathbf{B})'(\mathbf{X} - \mathbf{Z}\mathbf{B}). \end{aligned}$$

Although marginal posterior densities of $\boldsymbol{\beta}$ and $\boldsymbol{\Sigma}_u$ are not available, the posterior analysis can be easily implemented by applying block-Gibbs sampling algorithm to the above conditional densities. Specifically, at the j th simulation, we draw $\boldsymbol{\beta}^j$ conditional on the previous $\boldsymbol{\Sigma}_u^{j-1}$, and close the loop by sampling $\boldsymbol{\Sigma}_u^j | \boldsymbol{\beta}^j$ from the inverse Wishart distribution. The chain is initialized using point estimates of the model parameters. We make two adjustments to the described sampling procedure. We discard the first 500 draws in order to eliminate the influence of the starting values. In addition, to ensure stationarity, we remove draws if matrix \mathbf{B} has any eigenvalues larger than 0.98.

Our final sample consists of 20,000 draws of parameter values from the posterior, $\{\boldsymbol{\beta}^j, \boldsymbol{\Sigma}_u^j\}$. We cast them back into the original VAR representation and calculate the means and variance–covariance matrices of multihorizon returns. The corresponding moments of the predictive distribution of asset returns are obtained by taking averages of the constructed quantities. This procedure delivers the term-structure of expected returns and risks conditioned only on the observed data, but not the VAR estimates.

APPENDIX C: INCORPORATING UNCERTAINTY IN THE EC-VAR FRAMEWORK

The approach of incorporating uncertainty in the cointegration vector along with the EC-VAR parameters is similar to the one discussed in Appendix B. After observing the sample, investors update their prior to form the posterior distribution of the model parameters. Letting $f_0(\boldsymbol{\alpha})$ denote the prior density of the elements of the cointegration vector $\boldsymbol{\alpha} \equiv (\tau_0, \tau_1, \delta)'$ and maintaining the noninformative prior for the EC-VAR parameters, we can summarize investors' ex ante beliefs in a composite form,

$$\varphi(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\Sigma}_u) \propto f_0(\boldsymbol{\alpha}) |\boldsymbol{\Sigma}_u|^{-(n+1)/2}. \quad (\text{C.1})$$

The conditional posterior densities are then given by

$$\begin{aligned} \boldsymbol{\beta} | \boldsymbol{\Sigma}_u, \boldsymbol{\alpha} &\sim N(\hat{\boldsymbol{\beta}}, [\mathbf{z}'(\boldsymbol{\Sigma}_u^{-1} \otimes \mathbf{I}_{T-1})\mathbf{z}]^{-1}), \\ \boldsymbol{\Sigma}_u | \boldsymbol{\beta}, \boldsymbol{\alpha} &\sim \text{IW}(\mathbf{Q}, T-1), \end{aligned} \quad (\text{C.2})$$

$$f(\boldsymbol{\alpha}) \propto f_0(\boldsymbol{\alpha}) \frac{|\boldsymbol{\alpha}'\mathbf{W}_0\boldsymbol{\alpha}|^{l_0}}{|\boldsymbol{\alpha}'\mathbf{W}_1\boldsymbol{\alpha}|^{l_1}},$$

where $\hat{\boldsymbol{\beta}}$ and \mathbf{Q} are defined as previously and matrices \mathbf{W}_0 , \mathbf{W}_1 and constants l_0 , l_1 depend on the observed sample (for detailed formulas see Bauwens and Lubrano 1996). Notice that the analytical expression for the conditional density of the cointegration parameter is not available. As suggested in Bauwens, Lubrano, and Richard (1999), $f(\boldsymbol{\alpha})$, in this case, can be simulated

by applying the Griddy–Gibbs sampling technique. The algorithm is implemented by evaluating the kernel over a grid of points and approximating the cumulative distribution using numerical integration. Making a normalization to obtain a proper distribution function, we sample the cointegration vector by inverting the constructed cdf. Conditionally on a given draw α^j , we generate the EC-VAR parameters applying the two-block Gibbs sampler to the conditional densities of β and Σ_u as discussed previously. Discarding the initial draws and rejecting draws that fall on the edge or outside the stationary region, we end up with 20,000 values from the posterior distribution and use them to construct the moments of the predictive distribution of asset returns. The resulting distribution and the optimal allocation it implies are conditioned only on the actual data and do not depend on any estimates of the predictive model for returns.

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