A Long-Run Risks Explanation of Predictability Puzzles in Bond and Currency Markets

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Abstract

We provide a long-run risks based explanation for the predictability puzzles in bond and currency markets. In our model, time-varying macroeconomic (i.e., consumption) volatility carries a separate risk compensation and ensures that the risk premium in asset markets is time-varying. In particular, periods of high macroeconomic volatility are associated with (i) an increase in nominal bond risk premium and a steeper slope of the nominal yield curve, and (ii) expected depreciation of the currency and a low domestic minus foreign interest rate differential, thus providing the economic channels for explaining the violations of expectations hypothesis. We show that the model qualitatively matches the violations of the expectations hypothesis seen in the data, while at the same time accounting for the level and volatilities of nominal yields and equity returns. In all, we argue that the long-run risks model provides a coherent explanation of the key puzzles in bond and currency markets.

1 Introduction

A prominent puzzle in financial economics is the violation of the expectations hypotheses and the ensuing predictability of returns in bond and currency markets. In this paper we show that Bansal and Yaron (2004) type long-run risks framework provides an explanation for these puzzles. The long-run risks framework we consider features a preference for early resolution of uncertainty, low-frequency movements in consumption, and time-varying short and long-run consumption volatility — this permits significant time-variation in risk premia which quantitatively explains the violations of expectations hypotheses in bond and currency markets. We also provide direct empirical evidence that the covariation between consumption volatility in the data with the bond and currency prices across countries agrees with the theoretical predictions from our model.

Empirical evidence presented in Campbell and Shiller (1991) and Dai and Singleton (2002) shows that typically there is a drop in long rates following periods of high long-short yield differential, which stands in sharp contrast to the implications of the expectations hypothesis of the yield curve. Additional evidence on the predictability of bond returns is presented more recently in Cochrane and Piazzesi (2005), who show that a single bond factor constructed from a linear combination of several forward rates can sharply forecast future bond returns. In all, this is consistent with time-varying risk-premia in bond markets. Similarly, in currency markets, Fama (1984), Bansal (1997), and Backus, Foresi, and Telmer (2001), show that the interest rate differential across countries forecasts future exchange rate changes — in particular, a rise in the domestic nominal rate forecasts an appreciation of the domestic currency. What economic mechanisms can account for these puzzles along with the level and volatility of bond yields? We argue that the economic channels of the long-run risks model can successfully account for the risk and returns in financial markets, and jointly explain the salient features of bond and currency markets.

In our model-setup a preference for early resolution of uncertainty ensures that volatility shocks carry a risk premium. Time-varying consumption uncertainty generates time-variation in expected excess returns. The expected growth component magnifies the role of the volatility channel and magnifies the risk-premia variation which enables the model to quantitatively explain the violations of the expectations hypotheses in the data. In earlier work, Bansal and Yaron (2004) highlight the implications of this model for the slope of the real yield curve. Eraker (2006), Piazzesi and Schneider (2005) consider the long-run risks model as well, and show that the inflation risk induces an upward sloping nominal yield curve in this model. These papers do not attempt to explain the violations of the expectations hypothesis, as in their specifications, consumption volatility and hence all the risk premia are constant. A similar two-country version of the model considered in Colacito and Croce (2005) also has constant risk premia and consequently cannot account for the currency market violations of the expectations hypothesis.

In terms of the model intuition, when agents prefer early resolution of uncertainty (i.e., when the risk aversion exceeds the reciprocal of the IES) a positive shock to long-run consumption volatility moves the expected excess bond returns and the long-short yield spread in the same direction. Therefore, the slope of the term structure forecasts positively future excess returns on bonds, which can quantitatively account for the violations of expectations hypothesis in the data. Similarly, in foreign exchange markets, in response to a positive shock to domestic short-run consumption volatility agents demand higher expected excess returns in foreign bonds, forecast appreciation of the foreign currency and at the same time push the yield on domestic risk-free assets down. This can quantitatively account for the violations of the expectations hypothesis in currency markets.

To highlight the quantitative implications of the model, we calibrate the preference and expected consumption growth parameters to the usual values in the long-run risks literature. Instead of the single volatility process used in Bansal and Yaron (2004), we model a short-run (fast decay) and a long-run (slow decay) time-varying consumption volatility process to capture fluctuations in the uncertainty in the economy; the short-run volatility is a local country variable, while the long-run volatility and long-run expected growth are similar across countries. This captures the intuition that the long-run growth prospects across countries look very similar, while in the short run there may be differences in the economic environment[1]. To derive implications for nominal yields, we also model the inflation process. We carefully document that our

[1]An earlier version of the paper shows that a specification with a single volatility process, less than perfectly correlated across countries, captures the bond-market puzzles almost equally well as the two-volatility specification; however, the latter one does better in explaining the violation of the expectation hypothesis in currency markets

assumed processes match the observed consumption and inflation dynamics in the data. With this specification we show that the model can reproduce an upward-sloping term structure of nominal yields, while the volatilities of yields decrease with maturity, as in the data. The real yield curve is downward sloping, which as discussed in earlier work by Bansal and Yaron (2004) and Piazzesi and Schneider (2005), is consistent with the long-sample UK evidence for real bonds documented in Evans (1998). More importantly, the model qualitatively and quantitatively captures the negative regression slope coefficients that characterize the violations of expectation hypothesis documented in the data. In the model, the negative slope coefficients increase with bond maturity, in absolute value, as in the data. We also find violations of expectations hypothesis for real bonds as well. Remarkably, the small-sample results in the model are consistent with the evidence for UK real bonds presented in Evans (1998), who documents that the slope coefficients in the real bond regressions are positive but less than one. Further, the model can capture the single-factor type predictability evidence documented in Cochrane and Piazzesi (2005). All this evidence indicates that the model can successfully explain the predictability of returns in bond markets. At the same parameter configuration, we show that the model can also account for the nature and scope of the expectations hypothesis violations in foreign exchange markets.

There is considerable support for time-varying consumption volatility in the data; see Kandel and Stambaugh (1990) and Stock and Watson (2002). Further, we document that in the data, the domestic minus foreign volatility of consumption growth correlates negatively both with foreign exchange rates and with the interest rate differential between domestic and foreign countries. This evidence is consistent with the predictions of the model, and provides empirical support for the economic channels highlighted in the paper.

There is considerable earlier work on the violation of expectations hypothesis using alternative approaches. Using the Campbell and Cochrane (1999) habits based model, Wachter (2006) explores the implications for yields and the violations of expectations hypothesis in bond markets, while Verdelhan (2005) explores its implications for currency markets. Using the rare disasters framework of Rietz (1988) and Barro (2006), Farhi and Gabaix (2008) explore the implications of time-variation in disaster risk for violations of expectation hypothesis in currency markets. Bekaert and Marshall

(2001) use a peso-problem argument to address the violations of the expectations hypothesis and explain the term structure of interest rates in United States, United Kingdom and Germany. Alvarez, Atkeson, and Kehoe (2006) argue that a model of limited and changing participation of agents in financial markets could account for forward premium anomaly at short and long horizons. Burnside, Eichenbaum, and Rebelo (2007) highlight the importance of asymmetric information and microstructure frictions to address the currency market puzzle. Lustig and Verdelhan (2007) construct portfolios of foreign currency returns sorted on the basis of foreign interest rates and test whether a consumption-based model can empirically explain their expected returns. Earlier equilibrium-model based efforts to explain some of the puzzles in currency markets include Bekaert (1996) and Backus, Gregory, and Telmer (1993).[2]

The rest of the paper is organized as follows. In the next section we document the violations of the expectations hypothesis in bond and currency markets. In Section 3 we setup the long-run risks model. We present the solution to the model and discuss its theoretical implications for domestic asset markets in Section 4. In Section 5 we describe an extension of the model to capture foreign country dimension and discuss the implications of the model for the foreign exchange markets. We describe the data and calibration of the economy and preference parameters in Section 6. Model implications for bond and currency markets values are addressed in Section 7. Conclusion follows.

2 Predictability Puzzles and Evidence

A standard benchmark for the analysis of returns on bonds is provided by the expectations hypothesis. It states that in domestic bond markets, a high long-short yield spread today is offset by an anticipated loss on long maturity bonds in the future, and therefore should forecast an increase in the long rates. In foreign exchange context, low risk-free rates at home are compensated by the future appreciation of dollar and therefore should predict expected depreciation of the foreign currency. These con-

[2]Recent works in no-arbitrage and statistical literature include affine models of Dai and Singleton (2002), regime-switching models of Bansal and Zhou (2002) and macro-finance specifications of the term structure by Ang and Piazzesi (2003), Ang, Dong, and Piazzesi (2005), Rudebusch and Wu (2004) and Bikbov and Chernov (2006).

clusions can be formally obtained in structural models in which the expected excess returns are constant, e.g., when investors are risk-neutral or economic uncertainty is constant.

As discussed below, none of the implications of the expectations hypothesis are supported by the data; in fact, the signs in predictability regressions are exactly the opposite. Reduced-form empirical projections imply that a high yield spread forecasts a drop in future long rates, and the regression coefficients become more negative with maturity. Likewise, low forward premium predicts appreciation of the foreign currency, though, the violations are less severe at longer horizon. Therefore, the forecasts of the change in future bond and currency prices based on expectations hypothesis or empirical projections will be radically different, both in terms of their magnitude and sign. The violations of these predictions in the data pose a challenge to the economic understanding of asset markets, and seriously question the constant (zero) expected excess return assumptions used to justify the expectations hypothesis model.

The economic principle of no-arbitrage across bond, currency and equity markets implies that the expected return in all these markets should be explained by common economic risk channels. In the context of long-run risks model, we show that these channels can successfully account for the predictability puzzles in bond and currency markets.

In the next two sections we establish the notation and document the key empirical findings on predictability of domestic and foreign bond returns.

2.1 Bond Market Puzzles

Denote $y_{t,n}$ the yield on the real discount bond and $f_{t,n}$ the real forward rate with n months to maturity. We will use variables with a dollar superscript to refer to nominal quantities, e.g., nominal one-month yield $y^{\$}_{t,1}$. To avoid clustering of superscripts, we lay out subsequent discussion using the notation for real variables.

Denote $rx_{t+m,n}$ the excess log return on buying an n month bond at time t and selling it at time $t+m$ as an $n-m$ period bond as

$$rx_{t+m,n} = ny_{t,n} - (n-m)y_{t+m,n-m} - my_{t,m}. \tag{2.1}$$

Under the expectations hypothesis, the expected excess bond returns are constant. This implies that the slope coefficient $\beta_{n,m}$ in bond regressions

$$y_{t+m,n-m} - y_{t,n} = const + \beta_{n,m}\frac{m}{n-m}(y_{t,n} - y_{t,m}) + error \tag{2.2}$$

should be equal to one at all maturities n and time steps m. Indeed, with rational expectations, the population value for the slope coefficient can be computed as

$$\beta_{n,m} = 1 - \frac{Cov(E_t rx_{t+m,n}, y_{t,n} - y_{t,m})}{mVar(y_{t,n} - y_{t,m})}. \tag{2.3}$$

If the term-spread $y_{t,n} - y_{t,m}$ contains no information about future excess bond returns, e.g. expected excess returns are constant under the expectations hypothesis, then the slope is equal to unity. Alternatively, a high long-short spread should predict a proportional decline in future bond prices, which eliminates the yield advantage to long-term bonds by expected capital loss.

In the data, however, the expectations hypothesis is strongly rejected, as the slope coefficients in the projections are well below one, as shown in Campbell and Shiller (1991) and Dai and Singleton (2002). In the second panel of Table 2, we tabulate the projection coefficients for nominal bond yields in the US and UK. Consistent with previous studies, the slope coefficients in the expectations hypothesis regressions in the data are negative for the two countries in our sample, and are increasing in absolute value with maturity for the US. The standard errors for these slope coefficients is quite large, the two standard error range includes a wide range of slope coefficients. Further, the violations of expectations hypothesis are considerably weaker in the UK, relative to US. Using UK data, Evans (1998) provides evidence against the expectations hypothesis for the real bonds; the slope coefficients are less than one, though not negative, so the violations seem less pronounced for the real bonds. All this evidence suggests that the expected bond returns are time-varying and predictable by the yield variables. In particular, for nominal yields, a high long-short yield spread forecasts a decrease in future bond yields, in sharp contrast to the implications of the expectations hypothesis.

Using multiple forecasting variables, Cochrane and Piazzesi (2005) provide stronger evidence for the time-variation in bond risk premia. Following their approach, we regress the average of 1-year nominal excess bond returns of 2 to 5 years to maturity on the forward rates of 1, 3 and 5 years to maturity:

$$\frac{1}{4}\sum_{n=2}^{5} rx_{t+12,12n} = \gamma_0 + \gamma_1 f_{t,12} + \gamma_2 f_{t,36} + \gamma_3 f_{t,60} + error. \tag{2.4}$$

We extract a single bond factor $\widehat{rx}_{t,m} = \hat{\gamma}_0 + \hat{\gamma}_1 f_{t,12} + \hat{\gamma}_2 f_{t,36} + \hat{\gamma}_3 f_{t,60}$ from this regression, which is subsequently used to forecast excess bond returns at each maturity n from 2 to 5 years:

$$rx_{t+m,n} = const + b_{m,n}\widehat{rx}_{t,m} + error. \tag{2.5}$$

Cochrane and Piazzesi (2005) show that the estimates $b_{m,n}$ are positive and increasing with horizon, and a single factor projection captures $20-30\%$ of the variation in bond returns. We document these results for US and UK bond markets in the last panel of Table 2. The levels of the slope coefficients and the amount of predictability in these regressions provide strong evidence for a substantial time-variation in the risk premium in bond markets. This evidence is also quite comparable across the US and UK and in this sense is a more robust finding. Our target is to qualitatively and quantitatively account for the bond market predictability evidence using the long-run risks model.

2.2 Currency Market Puzzles

Let s_t stand for a real spot exchange rate, in logs, per unit of foreign currency (dollars spot price of one pound), and denote by f_t^{FX} the logarithm of the foreign exchange forward rate, i.e. current dollar price of a contract to deliver one pound tomorrow. Superscript * will denote the corresponding variable in foreign country, e.g. $y_{t,1}^*$ stands for the foreign risk-free rate. To avoid clustering of superscripts, we present the discussion in real terms.

A one-period excess dollar return in foreign bonds is given by

$$rx_{t+1}^{FX} = s_{t+1} - s_t + y_{t,1}^{*} - y_{t,1}. \tag{2.6}$$

This corresponds to an excess return on buying foreign currency today, investing the money into the foreign risk-free asset and converting the proceeds back using the spot rate next period.

Under the expectations hypothesis in currency markets, the excess returns are constant. Therefore, the slope coefficient in the projection

$$s_{t+1} - s_t = const + \beta^{UIP}(y_{t,1} - y_{t,1}^{*}) + error. \tag{2.7}$$

should be equal to one. Indeed, with rational expectations, the population value for the regression coefficient can be written as,

$$\beta^{UIP} = 1 + \frac{Cov(E_t rx_{t+1}^{FX}, y_{t,1} - y_{t,1}^{*})}{Var(y_{t,1} - y_{t,1}^{*})}. \tag{2.8}$$

Therefore, if the forward premium $y_{t,1} - y_{t,1}^{*}$ contains no information about the foreign bond risk premium $E_t rx_{t+1}^{FX}$, e.g. the latter is constant under the expectations hypothesis, the projection coefficient is one. Alternatively, if the uncovered interest rate parity condition holds, high interest rate bearing countries are expected to experience a proportional depreciation of their currency.

Fama (1984), Hodrick (1987), Bansal (1997), Backus et al. (2001) show that at short maturities, the regression coefficient in foreign exchange projection (2.7) is negative and statistically significant. In the second panel in Table 3, we document these findings for UK, Germany and Japan for an investment horizon of 1 month. These findings suggest that the uncovered interest rate parity condition is violated in the data. Hence, the expected excess currency returns are time-varying and co-vary negatively with the yield spread between the two countries. An additional dimension of the data, documented in Alexius (2001), is that the negative slope coefficients become less pronounced at long horizons, though, the standard errors on these estimates are quite large. Our target is to quantitatively explain the nature and magnitude of the time-variation of the foreign exchange risk premium using the long-run risks model.

3 Model Specification

3.1 Preferences

We consider a discrete-time real endowment economy developed in Bansal and Yaron (2004). The investors preferences over the uncertain consumption stream C_t can be described by the Kreps-Porteus, Epstein-Zin recursive utility function, (see Epstein and Zin, 1989; Kreps and Porteus, 1978):

$$U_t = \left[(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta(E_t U_{t+1}^{1-\gamma})^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}}, \qquad (3.1)$$

where δ is the time discount factor, $\gamma \geq 0$ is the risk aversion parameter, and $\psi \geq 0$ is the intertemporal elasticity of substitution (IES). Parameter θ is defined $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$. Its sign is determined by the magnitudes of the risk aversion and the elasticity of substitution, so that if $\psi > 1$ and $\gamma > 1$, then θ will be negative. Note that when $\theta = 1$, that is, $\gamma = 1/\psi$, the above recursive preferences collapse to the standard case of expected utility. As is pointed out in Epstein and Zin (1989), in this case the agent is indifferent to the timing of the resolution of uncertainty of the consumption path. When risk aversion exceeds (is less than) the reciprocal of IES the agent prefers early (late) resolution of uncertainty of consumption path. Hence, these preferences allow for agent's preference for the timing of the resolution of uncertainty. In the long-run risk model agents prefer early resolution of uncertainty of the consumption path.

As shown in Epstein and Zin (1989), the logarithm of the Intertemporal Marginal Rate of Substitution (IMRS) for these preferences is given by

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1)r_{c,t+1}, \qquad (3.2)$$

where $\Delta c_{t+1} = \log(C_{t+1}/C_t)$ is the log growth rate of aggregate consumption and $r_{c,t+1}$ is the log of the return (i.e., continuous return) on an asset which delivers aggregate consumption as its dividends each time period. This return is not observable in the data. It is different from the observed return on the market portfolio as the levels of market dividends and consumption are not equal: aggregate consumption is much larger than aggregate dividends. Therefore, we assume an exogenous process for consumption growth and use a standard asset-pricing restriction

$$E_t[\exp(m_{t+1} + r_{t+1})] = 1 \tag{3.3}$$

which holds for any continuous return $r_{t+1} = \log(R_{t+1})$, including the one on the wealth portfolio, to solve for the unobserved wealth-to-consumption ratio in the model.

3.2 Real Economy

We adopt the long-run risks model of Bansal and Yaron (2004) and assume that the real consumption growth contains a small and persistent long-run expected growth component. We further allow for separate volatilities of immediate consumption innovations and shocks to expected growth, so that the consumption dynamics is the following:

$$\Delta c_{t+1} = \mu_g + x_t + \sigma_{gt}\eta_{t+1}, \tag{3.4}$$
$$x_{t+1} = \rho x_t + \sigma_{xt} e_{t+1}, \tag{3.5}$$
$$\sigma^2_{g,t+1} = \nu_g \sigma^2_{gt} + w_{g,t+1}, \tag{3.6}$$
$$\sigma^2_{x,t+1} = \nu_x \sigma^2_{xt} + w_{x,t+1}. \tag{3.7}$$

The consumption and expected growth innovations η_{t+1} and e_{t+1} are standard Normal. To ensure the positivity of volatility process, we pursue the approach of Barndorff-Nielsen and Shephard (2001) and assume that the innovations in volatility processes $w_{g,t+1}$ and $w_{x,t+1}$ follow Gamma distribution[3]. The Gamma distribution is

[3] While in the standard long-run risks model volatility shocks are Gaussian, our specification of the model is equally tractable, and guarantees that the variances always stay positive. Further, in the earlier draft of the paper, we assumed that volatility shocks are Gaussian, and our results were very similar.

characterized by two parameters, so we specify the mean and volatility of the volatility shocks as

$$\begin{aligned} E(w_{g,t+1}) &= \sigma_g^2(1-\nu_g), \quad Var(w_{g,t+1}) = \sigma_{gw}^2, \\ E(w_{x,t+1}) &= \sigma_x^2(1-\nu_x), \quad Var(w_{x,t+1}) = \sigma_{xw}^2. \end{aligned} \tag{3.8}$$

This structure implies that the unconditional mean of the time-varying variance of consumption growth is σ_g^2, while the unconditional level of the expected growth variance is σ_x^2. The parameters ρ, ν_g and ν_x control the persistence of shocks to expected growth, consumption and expected growth volatility, respectively. For analytical tractability, we assume that all the innovations are independent from each other.

Hansen, Heaton, and Li (2006) and Bansal, Kiku, and Yaron (2007) present empirical evidence in favor of the low-frequency expected growth factor x_t. In terms of time-varying volatility of consumption growth, Stock and Watson (2002) and Kandel and Stambaugh (1990) present strong evidence for the variation in conditional variance of consumption and other related macroeconomic series. Further, Bansal, Khatchatrian, and Yaron (2005) and Lettau, Ludvigson, and Wachter (2006) document a significant co-movement between macroeconomic volatility and asset prices using financial markets data. All this evidence highlights the importance of time-varying consumption volatility for asset prices. In our two-volatility model, the variance σ_{xt}^2 captures the persistent fluctuations in expected growth volatility. Its movements broadly correspond to business-cycle frequencies. The volatility of short-run consumption shocks, σ_{gt}^2, is not very persistent and will translate to short run risk-premia movements which will play a role primarily in currency markets. In the earlier draft of the paper we showed that one-volatility specification, as used in Bansal and Yaron (2004), can also account for many of the asset-market features. However, the two-volatility specification adds flexibility to quantitatively match bond and currency markets puzzles simultaneously.

3.3 Inflation Process

We write down inflation process to derive implications for nominal yields. This is important, as real bond data are not observed. Our approach to directly model inflation is similar to that pursued by Wachter (2006) and Piazzesi and Schneider (2005). In particular, we assume that the inflation process follows

$$\pi_{t+1} \;=\; \mu_\pi + z_t + \varphi_{\pi g}\sigma_{gt}\eta_{t+1} + \varphi_{\pi x}\sigma_{xt}e_{t+1} + \sigma_\pi\xi_{t+1}, \tag{3.9}$$

where the expected inflation state variable z_t is given by

$$z_{t+1} \;=\; \alpha_z z_t + \alpha_x x_t + \varphi_{zg}\sigma_{gt}\eta_{t+1} + \varphi_{zx}\sigma_{xt}e_{t+1} + \sigma_z\xi_{z,t+1}. \tag{3.10}$$

Note that the inflation process does not feed back into consumption, but we allow the shocks to real economy to affect the realized and expected inflation – in fact, this channel is going to be important to generate the inflation risk premium in the economy. Parameters $\varphi_{\pi g}, \varphi_{zg}$ and $\varphi_{\pi x}, \varphi_{zx}$ measure the sensitivity ("beta") of realized and expected inflation innovations to short and long-run consumption news.

To maintain parsimony, we assume that the inflation shocks ξ_{t+1} and $\xi_{z,t+1}$ are standard Normal and homoscedastic. This specification implies that the conditional variation in realized and expected inflation is driven by the short-run and long-run volatility processes σ_{gt}^2 and σ_{xt}^2 only:

$$\begin{aligned} Var_t\pi_{t+1} &= \varphi_{\pi g}^2\sigma_{gt}^2 + \varphi_{\pi x}^2\sigma_{xt}^2, \\ Var_t z_{t+1} &= \varphi_{zg}^2\sigma_{gt}^2 + \varphi_{zx}^2\sigma_{xt}^2. \end{aligned} \tag{3.11}$$

It is straightforward to extend inflation specification to include inflation-specific time-varying volatility, though, for parsimony we do not pursue this additional complication. In addition, in our main calibration for parsimony we zero out the inflation loadings on short-run news ($\varphi_{\pi g} = \varphi_{zg} = 0$), as they do not play a materially important role in our analysis.

4 Asset Markets

4.1 Discount Factor

The key ideas of the model rely on solutions which are derived using the standard log-linearization of returns. In particular, the log-linearized return on consumption claim is given by

$$r_{c,t+1} = \kappa_0 + \kappa_1 pc_{t+1} - pc_t + \Delta c_{t+1}, \tag{4.1}$$

where pc_t is the log price-to-consumption ratio. Parameters κ_0 and κ_1 are approximating constants which are based on the endogenous average price-to-consumption ratio in the economy. We follow Bansal et al. (2007) to solve for the endogenous constants κ_0 and κ_1 in equation (4.1). In Appendix B we show that the log-linearized solution to the model is very close to the solution of the model based on numerical methods.

The approximate solution for the price-consumption ratio is linear in states,

$$pc_t = A_0 + A_x x_t + A_{gs}\sigma^2_{gt} + A_{xs}\sigma^2_{xt}. \tag{4.2}$$

Using the Euler condition (3.2) and the assumed dynamics of consumption growth, we derive the solutions coefficients A_x, A_{gs} and A_{xs} :

$$A_x = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho}, \tag{4.3}$$

$$A_{gs} = \frac{(1-\gamma)(1-1/\psi)}{2(1-\kappa_1 \nu_g)}, \tag{4.4}$$

$$A_{xs} = \frac{(1-\gamma)(1-1/\psi)}{2(1-\kappa_1 \nu_x)} \left(\frac{\kappa_1}{1-\kappa_1 \rho} \right)^2. \tag{4.5}$$

The details for the model solution and the expressions for the endogenous log-linearization coefficients are provided in Appendix A.

It follows that A_x is positive if the IES, ψ, is greater than one. In this case the intertemporal substitution effect dominates the wealth effect. In response to higher expected growth, agents buy more assets, and consequently the wealth to consumption ratio rises. In the standard power utility model with risk aversion larger than one, the IES is less than one, and hence A_x is negative — a rise in expected growth potentially lowers asset valuations. That is, the wealth effect dominates the substitution effect.

Coefficients A_{gs} and A_{xs} measure the sensitivity of price-consumption ratio to volatility fluctuations. If the IES and risk aversion are larger than one, then these loadings are negative. In this case a rise in consumption or expected growth volatility lowers asset valuations and increases the risk premia on all assets. An increase in the permanence of volatility shocks magnifies the effects of volatility shocks on valuation ratios as changes in macroeconomic volatility are perceived by investors as being long lasting. A_{xs} also increases in absolute value with the persistence of expected consumption growth ρ, as the effects of expected volatility shocks are magnified as they feed in through the expected growth channel x_t.

Using the equilibrium solutions for the price-consumption ratio, we can provide an analytical expression for the marginal rate of substitution in (3.2):

$$\begin{aligned} m_{t+1} &= m_0 + m_x x_t + m_{gs}\sigma_{gt}^2 + m_{xs}\sigma_{xt}^2 \\ &\quad - \lambda_\eta \sigma_{gt}\eta_{t+1} - \lambda_e \sigma_{xt} e_{t+1} - \lambda_{gw} w_{g,t+1} - \lambda_{xw} w_{x,t+1}. \end{aligned} \tag{4.6}$$

In particular, the conditional mean of the IMRS is affine in expected growth and two volatility factors, where the loadings m_0, m_x, m_{gs} and m_{xs} depend on model and preference parameters, and are provided in the Appendix A.

The innovations in the IMRS are very important for thinking about risk compensation (risk premia) in various markets. The magnitudes of the risk compensation depend on the market prices of short-run, long-run and volatility risks λ_η, λ_e, and λ_{gw} and λ_{xw}. The market prices of systematic risks, including the compensation for stochastic volatility risk, can be expressed in terms of the underlying preferences and parameters that govern the evolution of consumption growth:

$$\lambda_\eta = \gamma, \tag{4.7}$$

$$\lambda_e = \left(\gamma - \frac{1}{\psi}\right)\frac{\kappa_1}{1-\kappa_1\rho}, \tag{4.8}$$

$$\lambda_{gw} = -\left(\gamma - \frac{1}{\psi}\right)(\gamma-1)\frac{\kappa_1}{2(1-\kappa_1\nu_g)}, \tag{4.9}$$

$$\lambda_{xw} = -\left(\gamma - \frac{1}{\psi}\right)(\gamma-1)\frac{\kappa_1}{2(1-\kappa_1\nu_x)}\left(\frac{\kappa_1}{1-\kappa_1\rho}\right)^2. \tag{4.10}$$

The compensation for the short-run consumption risks is standard and given by the risk-aversion coefficient γ. In the special case of power utility, $\gamma = \frac{1}{\psi}$, the risk compensation parameters λ_e, λ_{gw} and λ_{xw} are zero, and the IMRS collapses to the standard power utility specification,

$$m_{t+1}^{CRRA} = \log\delta - \gamma\Delta c_{t+1}. \tag{4.11}$$

With power utility there is no separate risk compensation for long-run growth rate risks and volatility risks — with generalized preferences these risks are priced. The pricing of long-run and volatility risks is an important feature of the long-run risks model.

The discount factor used to price nominal payoffs is given by,

$$m_{t+1}^{\$} = m_{t+1} - \pi_{t+1}. \tag{4.12}$$

The solution to the nominal discount factor is affine in the state variables, and nominal market prices of risks depend on the real prices of risks and inflation sensitivity to short and long-run consumption news. We can calculate one-period inflation risk premium as the covariance of the real discount factor with inflation:

$$Cov_t(m_{t+1},\pi_{t+1}) = -\varphi_{\pi g}\lambda_\eta\sigma_{gt}^2 - \varphi_{\pi x}\lambda_e\sigma_{xt}^2. \tag{4.13}$$

The inflation risk premium is determined by the inflation sensitivity to consumption news, and market prices of short and long run consumption risks. In particular, if inflation increases in bad times when consumption or expected consumption is low ($\varphi_{\pi g}, \varphi_{\pi x} < 0$), then nominal claims are particularly risky as their real payoff is low when marginal utility is high. In this case, the inflation risk premium is positive. In general, time-variation in the inflation risk premium is driven by short and long-run consumption volatilities. It is important to note that with power utility the price of long-run consumption risks λ_e is zero, so in this case the inflation risk premium varies only with the short-run volatility.

The dynamics of the multi-period discount factor takes into account the time-varying expected inflation, and the sensitivity of expected inflation to consumption shocks. Hence, the multi-period inflation risk premium also depends on the sensitivity of the expected inflation to consumption innovations. In particular, if expected inflation falls in bad times, the inflation risk premium is positive.

4.2 Bond Prices

The equilibrium real and nominal yields are affine in the state variables. Indeed, in Appendix A we show that real and nominal yields satisfy

$$y_{t,n} = \frac{1}{n}\left(B_{0,n} + B_{x,n}x_t + B_{gs,n}\sigma^2_{gt} + B_{xs,n}\sigma^2_{xt},\right), \tag{4.14}$$

$$y^{\$}_{t,n} = \frac{1}{n}\left(B^{\$}_{0,n} + B^{\$}_{x,n}x_t + B^{\$}_{gs,n}\sigma^2_{gt} + B^{\$}_{xs,n}\sigma^2_{xt} + B^{\$}_{z,n}z_t\right). \tag{4.15}$$

The bond coefficients, which measure the sensitivity of bond prices to the aggregate risks in the economy, are pinned down by the preference and model parameters – the expressions for the loadings are presented in Appendix A. For typical parameter values, real yields respond positively to expected growth shocks, $B_x > 0$, and negatively to volatility states $B_{gs} < 0, B_{xs} < 0$. While this is also true with power utility ($\gamma = \frac{1}{\psi}$), the full model with recursive preferences separates the risk aversion from the IES which allows for a greater flexibility in modeling and explaining the yield curve.

The solutions for nominal bond yields take into account inflation risks in the economy. For example, one-period nominal yield satisfies Fisher-type equation

$$y^{\$}_{t,1} = y_{t,1} + E_t\pi_{t+1} - \frac{1}{2}Var_t\pi_{t+1} + Cov_t(m_{t+1}, \pi_{t+1}), \tag{4.16}$$

where the expressions for the inflation variance and inflation risk premium are provided in (3.11) and (4.13). In particular, if inflation increases in bad times, the inflation risk premium $Cov_t(m_{t+1}, \pi_{t+1})$ is positive, which increases nominal yields. Further, in calibrations we show that the inflation risk premium is increasing with maturity, which allows us to match an upward sloping term-structure of nominal bonds.

As discussed in Section 2.1, the evidence on violations of expectations hypothesis and predictability of bond returns in the data suggests that expected excess bond returns are time-varying and co-vary positively with long-short yield slope. The long-run risks model can explain the time-variation in bond premia and positive covariation between expected excess bond return and the yield slope through a long-run volatility channel. Indeed, one-period expected excess return on a real bond with n months to maturity can be written in the following form:

$$\begin{aligned} E_t(rx_{t+1,n}) + \frac{1}{2}Var_t(rx_{t+1,n}) &= -Cov_t(m_{t+1}, rx_{t+1,n}) \\ &= -B_{xs,n-1}\lambda_{xw}\sigma^2_{xw} - B_{gs,n-1}\lambda_{gw}\sigma^2_{gw} - B_{x,n-1}\lambda_e\sigma^2_{xt}. \end{aligned} \tag{4.17}$$

The expected excess real bond returns are time-varying and driven only by long-run volatility σ^2_{xt}. Hence, future bond returns are predictable by the yield variables, which also depend on the long-run volatility, as shown in (4.14). To determine the direction of predictability, note that real bonds provide a hedge against expected consumption risks: as $B_{x,n-1} > 0$, real bond yields decrease and their prices increase when expected consumption is low. Hence, the risk premium on real bonds is negative, and is low when long-run consumption volatility is high. In the model, real yields decline when the volatility of expected growth is high; further, long yields are more sensitive to volatility σ^2_{xt} than short yields, which causes long minus short yield slope to decline as well when σ^2_{xt} is high. This leads to a positive correlation of the real term-spread and bond risk premia, as required to explain the violations of the expectations hypothesis in the data for real bonds. The actual magnitudes of the slopes coefficients in bond regressions (2.2) depend on the amount of persistence and variation in the bond risk premium generated by the model.

A similar discussion of the bond risk premia holds in nominal terms. Nominal yields and nominal risk premium now also include inflation risk components:

$$\begin{aligned} E_t(rx^{\$}_{t+1,n}) + \frac{1}{2}Var_t(rx^{\$}_{t+1,n}) &= -Cov_t(m^{\$}_{t+1}, rx^{\$}_{t+1,n}) \\ &= -B^{\$}_{xs,n-1}\lambda_{xw}\sigma^2_{xw} - B^{\$}_{gs,n-1}\lambda_{gw}\sigma^2_{gw} \\ &\quad - B^{\$}_{z,n-1}\varphi_{zg}(\lambda_\eta + \varphi_{\pi g})\sigma^2_{gt} - (B^{\$}_{x,n-1} + B^{\$}_{z,n-1}\varphi_{zx})(\lambda_e + \varphi_{\pi x})\sigma^2_{xt}. \end{aligned} \tag{4.18}$$

The nominal bond risk premium is time-varying and in general driven by the short and long-run consumption volatilities; hence, nominal bond returns are predictable by the yield variables as well. Let us focus on the time-varying component in the nominal bond premium driven by the long-run volatility σ^2_{xt}. When expected inflation increases when expected consumption is low ($\varphi_{zx} < 0$ and $\alpha_z < 0$), nominal bonds become additionally risky due to the inflation risk. This additional risk compensation makes nominal bond premium positive and high when the long-run volatility is high. Further, the long minus short nominal yield spread also increases with the long-run variance σ^2_{xt}. This enables the model to account for the violations of the expectations hypothesis and the negative slope coefficient in the expectations hypothesis regressions.

It is important to note that all the three ingredients of the long-run risks model – preference for early resolution of uncertainty, time-variation in expected consumption growth, and fluctuations in consumption volatility – are critical to explain the predictability of bond returns and violations of the expectations hypothesis in the data. Indeed, as can be seen from expression (4.17), the risk premium on holding period bond returns reflects the compensation for long-run and volatility risks. With power utility, these sources of risks are not priced, as λ_{gw}, λ_{xw} and λ_e are all zero, so up to Jensen's adjustment term, the one-period bond risk premium is zero. Further, the fluctuations in bond risk premium come from the volatility of short-run and long-run consumption news, so that if the expected consumption growth is constant or the volatilities are constant, the bond risk premium is constant as well, and the expectation hypothesis holds.

5 Currency Markets

5.1 Two-Country Setup

We extend our model to a two-country setup — a similar specification, without the time-varying volatility in consumption, is also entertained in Colacito and Croce (2005). We index foreign country variables by a superscript *. For tractability, we impose complete symmetry and assume that all the model parameters are identical across the two countries. Further, we impose a restriction that long-run expected growth and long-run volatility are identical across countries. This captures the intuition that the long run growth prospects across the two countries are nearly identical (the long-run means and volatilities are the same). However, there are short-run differences between countries, which are mirrored in the differences in the short-run consumption shocks and short-run volatility across two countries.

Given these assumptions, the endowment dynamics in the foreign country is given by,

$$\Delta c^*_{t+1} = \mu_g + x_t + \sigma^*_{gt}\eta^*_{t+1}, \tag{5.1}$$

$$x_{t+1} = \rho x_t + \sigma_{xt} e_{t+1}, \tag{5.2}$$

$$\sigma^{*2}_{g,t+1} = \nu_g \sigma^{*2}_{gt} + w^*_{g,t+1}, \tag{5.3}$$

$$\sigma^2_{x,t+1} = \nu_x \sigma^2_{xt} + w_{x,t+1}. \tag{5.4}$$

where η_{t+1} and η^*_{t+1}, and $w_{g,t+1}$ and $w^*_{g,t+1}$ are independent. We write down an inflation process for the foreign country same as our specification for US in (3.9)-(3.10). For parsimony, we assume that the individual inflation shocks are independent across countries, and the co-movement in inflation rates is obtained through their sensitivity to a common expected growth shocks.

5.2 Foreign Exchange Rate

The discount factor used to price assets denominated in the foreign currency is given by

$$m^*_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c^*_{t+1} + (\theta - 1) r^*_{c,t+1}, \tag{5.5}$$

where Δc^*_{t+1} is the log growth rate of foreign endowment growth, $r^*_{c,t+1}$ is the log return on foreign consumption portfolio, and δ, γ and ψ are the preference parameters of the representative agents which are assumed to be the same at home and abroad.

As discussed in Backus et al. (2001), with frictionless markets, the exchange rate is equal to the difference between the logarithms of the discount factors in the two countries:

$$s_{t+1} - s_t = m^*_{t+1} - m_{t+1}, \tag{5.6}$$

and a similar expression holds for nominal exchange rates using the nominal stochastic discount factor $m^{\$}$.

Therefore, given the equilibrium solution to the model, we can write down the solution to the interest rate differential across countries in the following way:

$$y_{t,1} - y^*_{t,1} = -\frac{1}{2} \left(\gamma + \frac{1}{\psi} (\gamma - 1) \right) (\sigma^2_{gt} - \sigma^{*2}_{gt}), \tag{5.7}$$

while the solution to the expected excess return on foreign bonds is given by,

$$\begin{aligned} E_t rx^{FX}_{t+1} &= E_t \left(s_{t+1} - s_t + y^*_{t,1} - y_{t,1} \right) \\ &= \frac{1}{2} \gamma^2 (\sigma^2_{gt} - \sigma^{*2}_{gt}). \end{aligned} \tag{5.8}$$

Notably, as the expected growth components and the long-run volatilities are assumed to be perfectly correlated across the two countries, they do not drive the variation in the yield differential and foreign exchange risk premium.

As can be seen from the above expressions, in the full long-run risks specification with time-varying consumption volatility, the expected excess return on foreign bonds unambiguously increases when the consumption volatility at home is high. In response to a rise in domestic uncertainty, the equilibrium dollar price of the foreign currency s_t drops immediately, so that relative to the new level today, the foreign currency is expected to appreciate tomorrow, and the dollar return on investments abroad is expected to be high (expression (5.8)). At the same time, when $\gamma > 1$, an increase in domestic consumption volatility lowers the yields on domestic bonds and the yield spread across countries (equation (5.9)). All together, in response to a positive shock to consumption uncertainty, agents demand higher expected excess returns in foreign bonds, forecast appreciation of the foreign currency, and at the same time push the yield on domestic risk-free assets down. This can qualitatively account for the violations of the uncovered interest rate parity condition in the data.

The magnitude of the model-implied slope coefficients in foreign exchange projections depend on the calibration of preference and consumption growth parameters. As evident from equation (5.8), if the short-run consumption volatility σ_{gt}^2 is constant, the expected excess returns on foreign bonds are constant as well, so that the expectations hypothesis holds and the projection coefficient in foreign exchange regressions (2.7) should be equal to one. On the other hand, when investors have power utility, the expected currency depreciation is constant, as the variations in foreign exchange risk premium are exactly offset by the interest rate differential between the two countries. In this case, the slope coefficient in foreign exchange regressions is zero.

The discussion for nominal variables is very similar. The expressions for the nominal interest rate differential is now augmented to include the gap between the two expected inflation rate across the countries and the inflation rate sensitivity to consumption risks:

$$y_{t,1}^{\$} - y_{t,1}^{\$*} = (z_t - z_t^*) - \frac{1}{2}\left(\gamma + \frac{1}{\psi}(\gamma - 1) + \varphi_{\pi g}(\varphi_{\pi g} + 2\gamma)\right)(\sigma_{gt}^2 - \sigma_{gt}^{*2}), \tag{5.9}$$

and the nominal foreign exchange risk premium is modified to account for the inflation risk:

$$\begin{aligned} E_t rx_{t+1}^{FX\$} &= E_t\left(s_{t+1}^{\$} - s_t^{\$} + y_{t,1}^{\$*} - y_{t,1}^{\$}\right) \\ &= \frac{1}{2}(\gamma + \varphi_{\pi g})^2(\sigma_{gt}^2 - \sigma_{gt}^{*2}). \end{aligned} \tag{5.10}$$

The expected inflation gap increases the volatility of the interest rate differential and somewhat dampens the violations of the expectations hypothesis in foreign exchange markets relative to the real bonds. In the calibrations we show that the slope coefficients in nominal UIP projections are negative at short maturities, and the violations become less severe with horizon, as in the data.

6 Data and Calibration

6.1 Financial Markets

We choose four countries for our empirical analysis, the United Kingdom, Germany, Japan and United States (domestic country). The financial data for the foreign countries is taken from Datastream, and include spot and forward exchange rates, 1 month nominal rates and MSCI returns for the period of January 1976 (July 1978 for Japan) to November 2005. Additional data on 1 to 5 year nominal discount bonds in US and UK come from CRSP and Bank of England, respectively. Market returns in US are calculated for a broad value-weighted portfolio from CRSP. The consumption and CPI measures for foreign countries are taken from the IMF's International Financial Statistics, while the US consumption data come from BEA tables of real expenditures on non-durable goods and services.

The first panel in Table 1 tabulates summary statistics for excess market returns and inflation-adjusted rates across the four countries[4]. One-month interest rates vary from 1.9% in Japan to 3.61% in UK, while their standard deviations range between 2.1% and 3.1% for the sample period. The average equity premium in the countries in our sample is about 5%, and the volatility of the market return is 15%.

In Table 2 we report summary statistics the for nominal bonds in the US and UK. The first panel of the Table shows that the nominal term structure is upward-sloping in both countries. The nominal yields in the US increase from 5.56% at one year horizon to 6.16% at 5 years, while the bond yield volatilities decrease from 2.91% to 2.72%. The evidence in the UK is similar, but the levels of nominal yields in the

[4] Inflation adjustment of interest rates is based on AR(2) filtered inflation, while realized inflation is used to adjust changes in foreign exchange rates.

UK are about 2% higher than in the US. The second and third panel of the Table 2 present the evidence for the violations of expectations hypothesis and predictability of bonds returns, as discussed in Section 2.1.

The first panel of Table 3 reports summary statistics for the nominal and real foreign exchange rates across the countries. Foreign exchange rates have a zero autoregressive coefficient, and the volatility of the exchange rates range between $11\% - 12\%$. The second panel of the Table reports the evidence for the violations of uncovered interest rate parity condition in the foreign exchange markets, as discussed in Section 2.2.

In the next Section we discuss the key features of the macroeconomic data and our calibration of the model.

6.2 Calibration of Consumption and Inflation

We calibrate our model for consumption and inflation outlined in (3.4)-(3.7) and (3.9)-(3.10) at a monthly frequency and time-aggregate the output from monthly simulations to match the key aspects of quarterly consumption growth and inflation rate in US from 1952 to 2006.

The baseline calibration parameter values for consumption dynamics, reported in the first panel of Table 4, are very similar to the ones used in Bansal and Yaron (2004) and Bansal et al. (2007). Specifically, the persistence in the expected consumption growth, on an annual basis, (i.e. ρ^{12}) is 0.897. The annualized volatility of monthly consumption growth, $\sigma_c \times \sqrt{12}$, is set at 1.45%, while the annualized long-run volatility is 0.06%. The calibrated annual persistence of the long-run volatility is 0.785, which implies that long-run volatility movements broadly correspond to business-cycle frequencies with a half-life of under 3 years. We calibrate the annual auto-correlation of the short-run volatility to 0.069, so that the short-run volatility shocks are very short-lived with a half-life of about 1 quarter.

We report the calibration output of our model, which is based on 1,000 simulations of 55 years of monthly data aggregated to quarterly horizon, in Table 5. As shown in the Table, the model can match very well the salient features of the consumption data. In particular, the volatility of consumption growth in the data and in the model are close to 1%, and the model delivers a low persistence of consumption growth of 0.35 versus 0.33 in the data. We further examine long-horizon properties of consumption series and compute its autoregressive coefficients and variance ratios at different lags. In the data, variance ratios increase from 1.3 at 2-quarter horizon to 2.4 at 10 quarters, and the model matches these dimensions very well, as all the estimates are well within the $5\% - 95\%$ confidence band. In the Table 5 we report that the model-implied correlation of the consumption growth rates between the two countries is equal to 0.18, which matches very well the historical estimate of 0.13 computed for the US and UK data from 1957 to 2006.

The baseline parameter values for our calibration of inflation dynamics are reported in the second panel of Table 4. We set the mean of inflation rate to 3.3%, and calibrate the volatilities of individual inflation and expected inflation shocks at 1.07% and 0.04%, annually. The expected inflation process is calibrated to match the key features of inflation rate in the data, as well as the joint dynamics of consumption and inflation series in our sample. In particular, the expected inflation loads negatively on the expected consumption growth, $\alpha_x = -0.34$, and its own annualized autoregressive coefficient is α_z^{12} is 0.188. To maintain parsimony, we zero out inflation and expected inflation betas to immediate consumption news, $\varphi_{\pi g} = \varphi_{zg} = 0$, and set their sensitivity to long-run risks to be negative, $\varphi_{\pi x} = -2$ and $\varphi_{\pi z} = -1$.

Table 6 shows the calibration output for the inflation process, which is based on 1,000 simulations of 55 years of monthly data aggregated to a quarterly horizon. The model can match very well the key dimensions of the inflation data, as all the estimates in the data are very close to their counterparts in the model. In particular, the model-implied volatility of inflation rate is 1.8% versus 1.6% in the data. The model matches the first-order persistence of the inflation rate of 0.7, as well as the long-horizon properties of the series. The variance ratios increase from 1.7 at 2-quarter horizon to 6.7 at 10 quarters, mirrored by 1.6 to 6.3 increase in the model. We further verify that the model can account for the correlation of the inflation rate and consumption growth (0.24 in the data versus -0.34 in the model), as well as the correlation of the inflation rates across the countries (0.58 based on the US and UK sample versus 0.64 in the model).

One of the central ingredients in our inflation calibration is a negative inflation sensitivity to expected consumption news, modeled by parameters $\varphi_{\pi x} = -2, \varphi_{zx} = -1$ as well as $\alpha_z = -0.34$. This calibration is consistent with Piazzesi and Schneider (2005), who document that consumption and inflation are negatively correlated contemporaneously, and inflation forecasts future consumption with a negative sign at long-horizons. To focus on the inflation risk dimension of the data, we fit a VAR(1) to the consumption and inflation data, and use the estimated model to compute a $k-$period inflation beta defined as $\widehat{Cov}_t(\sum_{i=1}^{k} \pi_{t+i}, \sum_{i=1}^{k} \Delta c_{t+i})/\widehat{Var}_t(\sum_{i=1}^{k} \Delta c_{t+i})$. A solid line in Figure 1 plots the estimated inflation beta up to 20-year horizon. Consistent with a long-run negative correlation of consumption and inflation rates in the data, the inflation beta is negative and stabilizes at -1.2 at long horizons. We perform identical computations in the simulations of the model based on the time-aggregated quarterly output, and plot the model-implied inflation beta as a dotted line on Figure 1. The model-implied inflation beta is -1.3 at 20-year horizon, and can match the historical estimate very well both at short and long end. Hence, our calibrated model quite successfully matches the univariate properties of the consumption and inflation series, as well as the joint behavior of the inflation and consumption growth rate.

6.3 Preference Parameters

We calibrate the subjective discount factor $\delta = 0.998$. The risk-aversion coefficient is set at $\gamma = 10$. Mehra and Prescott (1985) and Bansal and Yaron (2004) do not entertain risk aversion values larger than 10.

There is a debate in the literature about the magnitude of the IES. As in Bansal and Yaron (2004), we focus on an IES of 1.5 — an IES value larger than one is important for our quantitative results. Bansal et al. (2005) document that the asset valuations fall when consumption volatility is high, which is consistent only with $\psi > 1$.

7 Model Implications

7.1 Bond Markets

As shown in the first panel of Table 7, at the calibrated parameter values the model-implied term structure of nominal bond yields is upward sloping. The one-year nominal yield is 5.60%, and it increases to 7.43% at 5 years. The volatilities of the yields fall uniformly from 2.92% at 1 year to 2.53% at 5 year horizon. The model-implied values for levels and volatilities of nominal yields are consistent with US historical estimates reported in Table 2. The term-structure of real rates, reported in Table 7, is downward sloping: the model-implied real rate is 1.93% at one year horizon and 0.12% at 5 years. The volatilities of real yields decline from 1.1% for one-year real yields to 0.96% for 5-year yields. Our magnitudes for the real and nominal yields are comparable to Piazzesi and Schneider (2005). Long-term real bonds hedge expected consumption risks and thus demand a negative risk premium, which leads to a negative slope of the real term structure as seen in the data. The nominal yield curve is upward sloping as inflation risk premia rises with maturity. While the level of the inflation risk premia is important for the level of the nominal yields, the time-variation in the inflation risk premium, however, contributes very little to the variability of the nominal yields — at a 5-year horizon it contributes only 10% to the variance of nominal yield and at shorter maturities even less[5].

The Table 8 reports model-implied slope coefficients in expectations hypothesis projections (2.2). These regressions are done using simulated monthly yields with annual time step and bond maturities of 2 to 5 years, so they are directly comparable to the estimates in the data reported in Table 2. The population slope coefficients in nominal regressions are all negative and decreasing with maturity from −0.17 at 1-year horizon to −0.54 at 5 years. We additionally report small-sample slope coefficients based on 55 years of simulated monthly data, equal to the size of our historical sample. The small-sample slope coefficients are negative and decrease from −0.02 at

[5] We also computed the level, slope and curvature factors (see Litterman and Scheinkman, 1991). We find that the real level is primarily driven by expected growth (73%) and long-run volatility; the slope factor is driven by expected growth and long-run volatility (65% and 23%, respectively), while the curvature factor is driven mostly (59%) by short-run volatility. The discussion of nominal factors is very similar; the level factor, in addition to expected growth, is also driven by expected inflation.

1 year to -0.36 at 5 year maturities. The magnitudes of the slope coefficients and the regression R^2 in the model are comparable to the estimates in US and UK bond markets documented in Table 2, after accounting for standard errors. Comparing the population slope coefficients with their finite-sample counterparts in Table 2 reveals that there is an upward bias in finite samples.

In the second panel of the Table 8 we report the model-implied slope coefficients in expectations hypothesis projections using the real bond data. It is important to note that the population slope coefficients in real regressions are also negative and comparable to their nominal counterparts. Hence, the violations of expectations hypothesis are a real-economy phenomenon; indeed as discussed in Section 4.2 our model can generate positive correlation between the bond risk premium and the long-short term slope, required to explain violations of expectations hypothesis, both for real and nominal bonds. Interestingly, the model-implied small-sample regression coefficients are weakly positive for real bonds and center around 0.1-0.2. This model implication is remarkably consistent with the evidence on the expectation hypothesis violations for UK real bonds presented in Evans (1998), who documents that the slope coefficients in the real bond regressions are positive but less than one.

In Table 9 we report the model implications for single factor projections of Cochrane and Piazzesi (2005). As in the data, we use 3 forward rates to construct a single bond factor, which is used to predict future excess bond returns 2 to 5 years ahead. As evident from the Table, the slope coefficients in the second-stage regressions increase from 0.4 at 2-year maturity to 1.6 at 5 years, which matches very well the estimates of 0.4 and 1.5, respectively, in US and UK data reported in second panel of Table 2. The R^2 in these regressions reach $20\% - 30\%$ in the data, though, the standard errors on the R^2 are quite high and are just under 10%. The model-implied R^2 in the projections is about 15%, which after accounting for the standard errors matches the evidence in the data. We also report the small-sample regression results, which are nearly identical to the population ones.

The single factor projections for the real bonds deliver a very similar pattern for the second-stage slope coefficients, which increase from 0.4 at 1-year to 1.5 at 5 year maturities. The R^2s in real regressions are quite substantial, though, they somewhat decrease relative to their nominal counterparts to about 10%. Hence, the model implies that predictability of bond returns is intrinsically a feature of the real economy.

As discussed, the long-run risks framework, and a specification with early resolution for uncertainty ($\gamma > 1/\psi$), ensures that the yield spread and the bond risk premium respond in the same direction to shocks to economic uncertainty (see Section 4.2) – this channel is central to account for the expectations hypothesis violations in the data. Indeed, a the power utility specification ($\gamma = 1/\psi$), fails to account for the expectations hypothesis violations. This underscores the importance of the joint economic restrictions on the real rates and the market prices of risks for various risk sources to explain these violations.

Using a very different, empirically driven multi-factor affine model, Dai and Singleton (2002) document that a specification where market prices of risks are driven by three Gaussian factors also reproduces the expectations hypothesis violations, whereas a specification which incorporates stochastic volatility in the factors, at the MLE estimates, does not. Our approach differs from the Dai-Singleton approach in many important respects. First, while our long-run risks model falls within the affine class, it does not directly correspond to any of the specific versions entertained in Dai and Singleton (2002)[6]. Second, the reduced-form affine model specification does not impose any of the long-run risks based joint economic restrictions on the risk-free rate and the market prices of risks for the various shocks, which are critical to explain the expectations hypothesis violations. Third, we do not use MLE-based parameter estimates for our model, which in addition to the yield levels and risk-premia could be driven by additional data features, such as conditional second moments of yields. Instead, we target a different set of macroeconomic and financial data features, which include moments of consumption growth and inflation rate and various aspects of bond, currency and equity markets to set our parameters. It is quite possible that forcing the model to match additional data features in the MLE approach may potentially make it harder for the model to account for the expectations hypothesis violations. If so, tractable adaptations of the long-run risks model, such as regime

[6]For example, the stochastic volatility processes in Dai and Singleton (2002) follow a square-root specification, while in our model the variances of short and long-run consumption volatilities are constant.

shifts and/or incorporating confidence jumps (see Bansal and Shaliastovich (2008)), open up new channels to confront additional data features.

7.2 Currency Markets

An important aspect of our model is that the expected growth components across two countries and the long-run volatilities are perfectly correlated, while the short-run consumption shocks and short-run volatilities are uncorrelated. This captures the intuition that in the long-run, the distributions of the endowments in the two countries are nearly identical, while in the short-run they can be quite different. This implies that the interest rate differentials and the foreign exchange risk premium are driven by the short-run components of the economy – the long-run components cancel out across the two countries.

We report the foreign exchange output from the model in Table 10. The nominal slope coefficient in foreign exchange projections is equal to -1.07 at one month horizon, and it increases to 0.20 at 1 year. In the small-sample, we show that the slope coefficients are biased upward, but remain negative at 1 month to maturity and increases to positive values at 1 year and beyond. The value of the nominal projection coefficient matches well the empirical estimates shown in Table 3. The model implication that the violations of the expectations hypothesis are less pronounced at longer maturities is consistent with the empirical evidence reported in Alexius (2001). We also note that the estimation uncertainty around the foreign exchange coefficient is quite high in the model, as in the data.

Further, consistent with the data, our model delivers that the foreign exchanges rates are virtually unpredictable – the R^2s in foreign exchange projections and the persistence in changes in spot prices of foreign currencies are all very close to zero. In the model we also verify that the slope coefficients in real regressions are negative as well, and increase from about -4.79 at 1 month horizon to -3.23 at 1 year. These findings are broadly consistent with Hollifield and Yaron (2003), who argue that risks from the real side of the economy are potentially important to capture the violations of the uncovered interest rate parity condition.

We can provide a direct evidence for the volatility channel in the model using consumption and asset prices data across countries. In Figures 2 - 3 we plot inflation-adjusted spot prices and forward premia against the difference in consumption volatility across countries. We follow Bansal et al. (2005) and construct consumption volatility measures as a 4.5 year sum of absolute residuals from AR(3) projections of consumption growth rates. Inflation adjustment for the interest rates is based on the fitted values from AR(2) model for the inflation rate. Consistent with the theoretical predictions in Section 5, consumption volatility differential co-moves negatively with dollar prices of foreign currency and forward premia. Indeed, the correlation coefficients for the consumption volatility differential with the spot exchange prices range between -0.1 and -0.5 and are equal to about -0.3 for the forward premia for all the countries in the sample. This evidence provides a direct support for the volatility channel highlighted in our model.

7.3 Equity Return

We note that the model, at the calibrated values for preferences and consumption above, also matches the equity data. This is not surprising, given the earlier work on equity markets in Bansal and Yaron (2004). Following this work, we consider a dividend process of the form,

$$\Delta d_{t+1} = \mu_d + \phi x_t + \varphi_d \sigma_{gt} \eta_{d,t+1}. \tag{7.1}$$

and calibrate it similar to that in Bansal and Yaron (2004). We set the dividend mean equal to average consumption growth, $\mu_d = \mu$, and set the exposure of the corporate sector to long-run risks to $\phi = 1.25$. We choose $\varphi_d = 10$ to target volatility of annual dividend growth of 12%. For parsimony, we assume the idiosyncratic dividend shock $\eta_{d,t+1}$ is independent from consumption shocks. The model-implied dynamics of the dividend growth series matches dividend data very well, and is omitted for the interest of space.

The third panel of Table 7 shows summary statistics for excess return on market portfolio. The model generates a sizable equity premium of 5%, and the volatility of market returns of 15%, which matches well the historical estimates in the data (see Table 1).

Conclusion

We show that the long-run risks type model can explain the violations of expectation hypothesis and predictability puzzles in bond and currency markets. The key ingredients of the model include long-run growth fluctuations, time-varying short and long-run consumption volatility and preference for early resolution of uncertainty. This model-structure ensures that shocks to volatility carry a separate risk premium. The risk channels featured in the paper generate a significant variation in risk premia, driven by the consumption volatility, which can quantitatively account for the negative coefficients in the tests of expectations hypothesis and for the level of return predictability in bond and currency markets. Using consumption and asset markets data, we provide direct empirical evidence to support the key economic channels highlighted in the paper.

The model captures the intuition that a positive shock to long-run consumption volatility moves the expected excess bond returns and the yield-spread in the same direction, which explains negative slope coefficients in the expectations hypothesis regressions in bond markets. At the same time, following an increase in short-run volatility, forward premium decreases and the domestic currency is expected to depreciate, which accounts for the violations of the uncovered interest rate parity condition in currency markets. In numerical calibrations, we show that our model can quantitatively match the key dimensions of bond and currency markets in the data.

In future work it would be interesting to consider adaptations of the long-run risks framework which include regime shifts and/or confidence jumps (Bansal and Shaliastovich (2008)). This broader framework opens up new channels for risk premium variation, and also permits the model to confront additional data dimensions such as jumps and regime-shifts in yields and exchange rates. We leave this exploration for future research.

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Tables and Figures

Table 1: **Descriptive Statistics Across Countries**

		UK	Germany	Japan	US	US47
Inflation-Adjusted	Mean	3.61	2.73	1.91	2.59	0.82
Interest Rate:	Std. Dev.	3.06	2.06	2.80	3.07	2.76
	AR(1)	0.74	0.90	0.91	0.84	0.78
Excess Market Return:	Mean	5.46	4.86	2.83	5.50	7.42
	Std. Dev.	17.30	20.78	18.56	14.74	14.60
	AR(1)	0.00	0.02	0.05	0.03	0.06
	Sharpe Ratio	0.32	0.23	0.15	0.37	0.51
Nominal	Mean	9.15	5.27	3.57	6.82	4.59
Interest Rate:	Std. Dev.	3.91	2.53	3.26	3.76	2.90
	AR(1)	0.97	0.99	0.98	0.98	0.97
Inflation Rate:	Mean	5.52	2.55	1.84	4.24	3.76
	Std. Dev.	1.58	0.72	1.24	1.02	1.20
	AR(1)	0.63	0.44	0.21	0.71	0.57

Descriptive statistics for interest rates, inflation and equity returns across countries. Inflation-adjusted interest rate corresponds to 1 month nominal interest rate adjusted for expected inflation. Excess market return is the return on Morgan Stanley International Index (CRSP portfolio for US) over the one month interest rate. Nominal interest rate is Euro-Currency Middle Rate of 1 month to maturity (CRSP risk-free rate for US47). Monthly observations from Jan 1976 (July 1978 for Japanese interest rate) to Nov 2005, and Feb 1947 to Nov 2005 for US47. Means and standard deviations are annualized.

Table 2: **Bond Market Data**

		1y	2y	3y	4y	5y
Nominal Yield:						
US	Mean	5.56	5.77	5.94	6.07	6.16
	Std. Dev.	2.91	2.86	2.79	2.75	2.72
UK	Mean	7.33	7.35	7.39	7.43	7.46
	Std. Dev.	2.82	2.57	2.45	2.40	2.37
EH Projection:						
US	Slope		-0.68	-1.01	-1.39	-1.38
			(0.43)	(0.51)	(0.57)	(0.64)
	R^2		0.02	0.03	0.06	0.05
			(0.03)	(0.04)	(0.05)	(0.04)
UK	Slope		-0.14	-0.12	-0.10	-0.09
			(0.57)	(0.64)	(0.72)	(0.82)
	R^2		0.01	0.01	0.01	0.01
			(0.01)	(0.01)	(0.01)	(0.01)
Single Factor Projection:						
US	Slope		0.44	0.87	1.24	1.45
			(0.03)	(0.02)	(0.02)	(0.04)
	R^2		0.19	0.23	0.24	0.22
			(0.08)	(0.08)	(0.08)	(0.08)
UK	Slope		0.47	0.88	1.20	1.45
			(0.04)	(0.03)	(0.02)	(0.05)
	R^2		0.28	0.30	0.30	0.29
			(0.09)	(0.08)	(0.08)	(0.08)

Nominal term structure and tests of expectations hypothesis and return predictability in US and UK bond markets. Monthly observations of 1-5 year yields on US and UK discount bonds for June 1952 to Dec 2005 and Jan 1985 to Dec 2005, respectively. EH Projection reports slope coefficient $\beta^{\$}_{n,m}$ in regression $y^{\$}_{t+m,n-m} - y^{\$}_{t,n} = const + \beta^{\$}_{n,m}\frac{m}{n-m}(y^{\$}_{t,n} - y^{\$}_{t,m}) + error$, where time step m is set at 12 months and bond maturities n run from 2 to 5 years. Single Factor Projection report the slope coefficient $b^{\$}_{m,n}$ and R^2 in single latent factor regression $rx^{\$}_{t+m,n} = const + b^{\$}_{m,n}\widehat{rx}^{\$}_{t,m} + error$, where $rx^{\$}_{t+m,n}$ is an m-months excess return on $n-$period nominal bond, and $\widehat{rx}^{\$}_{t,m}$ corresponds to a single bond factor obtained from a first-stage projection of average bond returns on three forward rates. Standard errors are Newey-West adjusted with 10 lags, computed using GMM approach.

Table 3: **Currency Market Data**

		UK	Germany	Japan
Foreign Exchange Rate:				
Nominal	Mean	-0.53	1.47	3.10
	Std. Dev.	10.69	11.18	12.09
Real	Mean	0.72	-0.25	0.65
	Std. Dev.	10.80	11.17	12.18
UIP Projection:				
	Slope	-1.72	-0.85	-2.83
Nominal		(0.95)	(0.77)	(0.66)
	R^2	0.02	0.01	0.03
		(0.02)	(0.01)	(0.01)

Foreign exchange rate and tests of expectations hypothesis in currency markets. Monthly observations of changes in log spot foreign exchange rates from Jan 1976 to Nov 2005 for Germany and UK and from July 1978 to Nov 2005 for Japan. UIP Projection reports the slope coefficient $\beta^{UIP\$}$ and R^2 in regression $s^{\$}_{t+1} - s^{\$}_t = const + \beta^{UIP\$}(y^{\$}_{t,1} - y^{*\$}_{t,1}) + error$, where $y^{\$}_{t,1}$ and $y^{*\$}_{t,1}$ are US and foreign nominal interest rate, respectively, and $s^{\$}_t$ is the nominal exchange rate. Standard errors are Newey-West adjusted with 10 lags.

Table 4: **Model Parameter Values**

Parameter		Value
Consumption Dynamics:		
Mean of consumption growth	μ_g	1.92
Expected growth persistence	ρ	0.897
Short-run volatility level	σ_g	1.45
Short-run volatility persistence	ν_g	0.069
Short-run volatility of volatility	σ_{gw}	5.47e-03
Long-run volatility level	σ_x	0.058
Long-run volatility persistence	ν_x	0.784
Long-run volatility of volatility	σ_{xw}	1.96e-06
Inflation Dynamics:		
Mean of inflation rate	μ_π	3.30
Inflation sensitivity to short-run news	$\varphi_{\pi g}$	0.0
Inflation sensitivity to long-run news	$\varphi_{\pi x}$	-2.0
Inflation shock volatility	σ_π	1.07
Expected inflation persistence	α_z	0.188
Expected inflation leverage on long-run news	α_x	-0.34
Expected inflation sensitivity to short-run news	φ_{zg}	0.0
Expected inflation sensitivity on long-run news	φ_{zx}	-1.0
Expected inflation shock volatility	σ_z	0.058
Preference Parameters:		
Subjective discount factor	δ	0.9978
Intertemporal elasticity of substitution	ψ	1.5
Risk aversion coefficient	γ	10

Calibrated parameter values for the baseline model. The model is calibrated at monthly frequency. We report annualized parameter values where we multiply consumption and inflation mean μ and μ_π by 12×100, raise persistence parameters $\rho, \nu_g, \nu_x, \alpha_x$ to power 12, and multiply volatility parameters $\sigma_g, \sigma_{gw}, \sigma_x, \sigma_{xw}, \sigma_\pi, \sigma_z$ by $\sqrt{12} \times 100$.

Table 5: **Consumption Growth Dynamics**

	Data		Model		
Variable	Estimate	S.E.	Median	5%	95%
$\sigma(\Delta c)$	0.91	(0.06)	1.35	1.12	1.64
AR(1)	0.33	(0.06)	0.35	0.17	0.54
AR(2)	0.20	(0.06)	0.17	-0.02	0.41
VR(2)	1.33	(0.06)	1.35	1.17	1.54
VR(5)	2.01	(0.19)	1.93	1.33	2.80
VR(10)	2.40	(0.44)	2.65	1.42	4.65
$Corr(\Delta c, \Delta c^*)$	0.13	(0.07)	0.18	0.03	0.41

Calibration of consumption growth. Quarterly observations of US real consumption growth from 1952 to 2006. Cross-country correlation is computed for US and UK series from 1957 to 2006. Standard errors are Newey-West corrected using 10 lags. Model output is based on 1000 simulations of 55 years of monthly data aggregated to quarterly horizon.

Table 6: **Inflation Dynamics**

	Data		Model		
Variable	Estimate	S.E.	Median	5%	95%
$\sigma(\pi)$	1.58	(0.23)	1.82	1.36	2.74
AR(1)	0.69	(0.10)	0.65	0.33	0.85
AR(2)	0.71	(0.09)	0.63	0.34	0.84
VR(2)	1.68	(0.10)	1.64	1.34	1.84
VR(5)	3.77	(0.42)	3.50	2.32	4.32
VR(10)	6.71	(1.33)	6.27	3.58	8.26
$Corr(\pi, \Delta c)$	-0.24	(0.09)	-0.34	-0.59	-0.12
$Corr(\pi, \pi^*)$	0.58	(0.04)	0.64	0.34	0.84

Calibration of inflation rate. Quarterly observations of US inflation rate from 1952 to 2006. Cross-country correlations are computed for US and UK series from 1957 to 2006. Standard errors are Newey-West corrected using 10 lags. Model output is based on 1000 simulations of 55 years of monthly data aggregated to quarterly horizon.

Table 7: **Model Implications: Bond and Equity Markets**

	1y	2y	3y	4y	5y
Nominal Term Structure:					
Mean:	5.60	5.85	6.28	6.82	7.43
Std. Dev.	2.92	2.81	2.71	2.61	2.53
Real Term Structure:					
Mean:	1.93	1.53	1.09	0.62	0.12
Std. Dev.	1.07	1.04	1.01	0.99	0.96
Market Return:					
Equity premium		5.01			
Std. Dev.		15.21			

Model-implied nominal and real term structure, and mean equity premium and volatility of market return. Population values, annualized in percent.

Table 8: **Model Implications: Expectations Hypothesis Tests**

	1y	2y	3y	4y	5y
Nominal EH Projection:					
Slope:					
Population		-0.17	-0.39	-0.49	-0.54
Small Sample		-0.02	-0.16	-0.29	-0.36
SE		(0.70)	(0.84)	(0.93)	(1.00)
R^2:					
Population		0.01	0.01	0.01	0.01
Small Sample		0.01	0.02	0.02	0.02
SE		(0.04)	(0.04)	(0.04)	(0.04)
Real EH Projection:					
Slope:					
Population		-0.23	-0.38	-0.42	-0.38
Small Sample		0.15	0.06	0.07	0.13
SE		(1.14)	(1.34)	(1.50)	(1.62)
R^2:					
Population		0.01	0.01	0.01	0.01
Small Sample		0.01	0.01	0.01	0.01
SE		(0.03)	(0.03)	(0.03)	(0.03)

Model-implied slope coefficient and R^2 in tests of expectations hypothesis. Population values, and small sample statistics based on 1000 simulations of 55 years of monthly data.

Table 9: **Model Implications: Single Factor Projection**

	1y	2y	3y	4y	5y
Nominal Regression:					
Slope:					
Population		0.39	0.82	1.22	1.57
Small Sample		0.38	0.81	1.22	1.60
SE		(0.03)	(0.03)	(0.01)	(0.05)
R^2:					
Population		0.15	0.15	0.15	0.15
Small Sample		0.15	0.16	0.16	0.16
SE		(0.12)	(0.12)	(0.11)	(0.11)
Real Regression:					
Slope:					
Population		0.44	0.84	1.20	1.52
Small Sample		0.42	0.83	1.20	1.55
SE		(0.03)	(0.02)	(0.01)	(0.04)
R^2:					
Population		0.11	0.10	0.10	0.10
Small Sample		0.15	0.14	0.14	0.14
SE		(0.09)	(0.08)	(0.08)	(0.08)

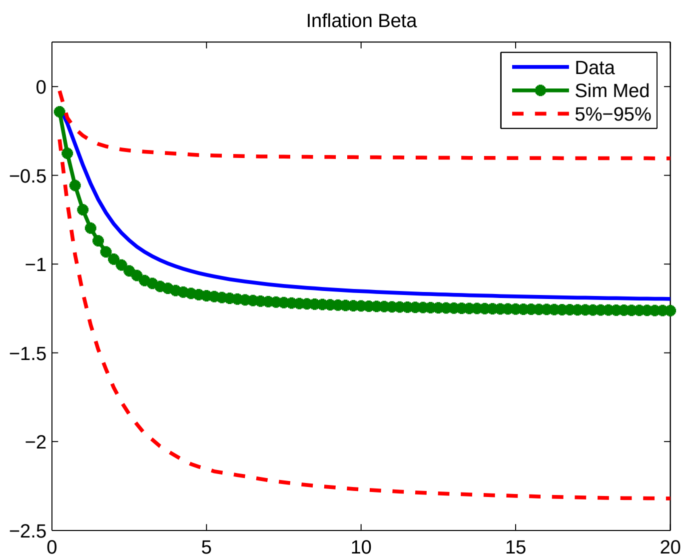
Model-implied slope coefficient and R^2 in single factor projections. Population values and small sample statistics based on 1000 simulations of 55 years of monthly data.

Table 10: **Model Implications: Currency Markets**

	1m	3m	1y
UIP Projections:			
Nominal Slope:			
Population:	-1.07	-0.75	0.20
Small Sample:	-0.95	0.16	2.26
SE:	(7.86)	(7.50)	(8.99)
Real Slope:			
Population:	-4.79	-4.59	-3.23
Nominal FX Rate:			
Std. Dev.	19.49		
AR(1)	0.00		

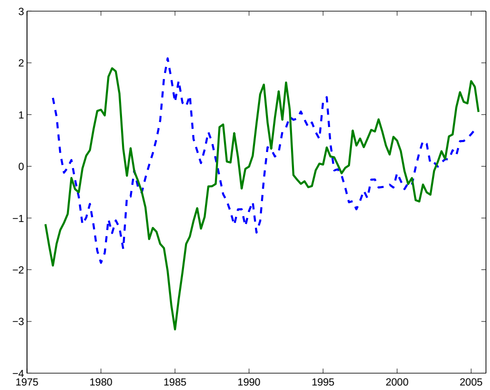
Model-implied foreign exchange rate and tests of uncovered interest rate parity condition. Population values and small-sample statistics based on 1000 simulations of 30 years of monthly data.

Figure 1: **Inflation Beta**

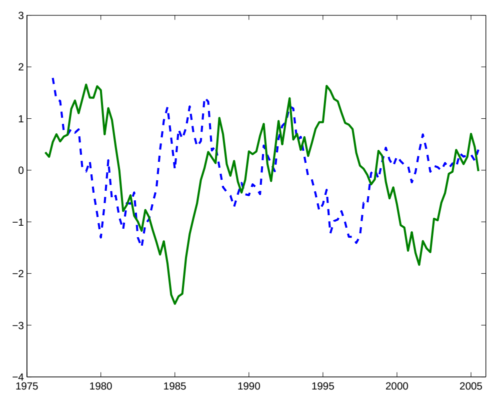


Long-run inflation beta to consumption risks. Computations are based on VAR(1) fit of consumption growth and inflation rate up to 20 year horizon. Data are quarterly observations of US real consumption growth and inflation from 1952 to 2005. Model output is based on 1,000 simulations of 55 months aggregated to quarterly horizon.

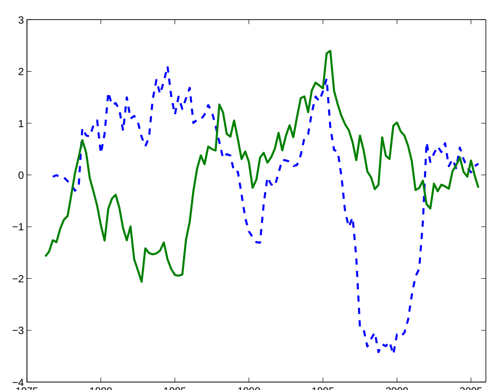
Figure 2: **Real Exchange Rate and Consumption Volatility**



(a) UK



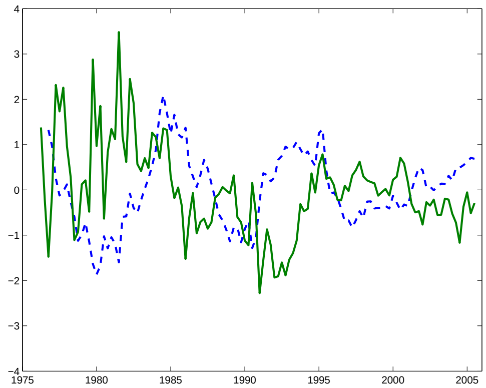
(b) Germany



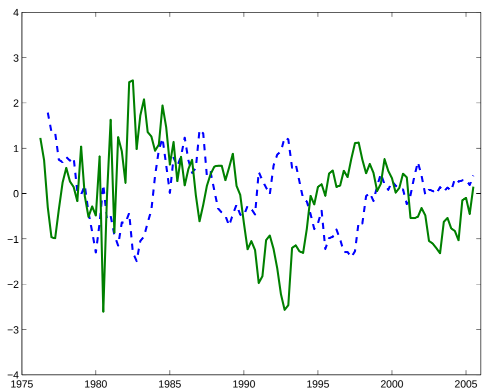
(c) Japan

Real exchange rate (solid line) and domestic minus foreign consumption volatility (dashed line).

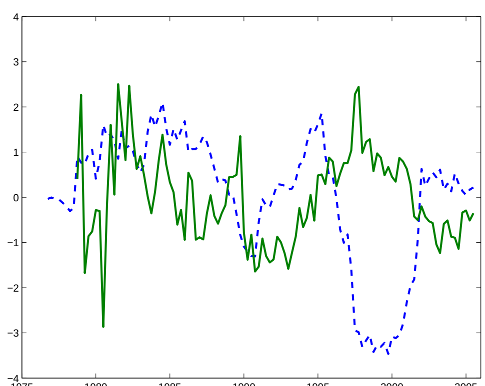
Figure 3: **Forward Premium and Consumption Volatility**



(a) UK



(b) Germany



(c) Japan

Forward premium (solid line) and domestic minus foreign consumption volatility (dashed line).

A Model Solution

Denote $\Psi_c(u)$ and $\Psi_c(u)$ the moment-generating functions for the Gamma-distributed shocks in volatility states:

$$\Psi_c(u) = Ee^{uw_{g,t+1}}, \quad \Psi_x(u) = Ee^{uw_{x,t+1}}. \tag{A.1}$$

For our parametrization of Gamma distribution, the expressions for the moment-generating functions are given by,

$$\Psi_c(u) = \left(1 - \frac{\sigma_{gw}^2}{\sigma_g^2(1-\nu_g)}u\right)^{-\left(\frac{\sigma_g^2(1-\nu_g)}{\sigma_{gw}}\right)^2}, \quad \text{for } u < \frac{\sigma_g^2(1-\nu_g)}{\sigma_{gw}^2}, \tag{A.2}$$

and similar for $\Psi_x(u)$.

It is important to note that even though the volatility shocks are non-Gaussian, our model specification belongs to the exponentially affine class. Indeed, the expectations of the exponential of the state variables is exponentially linear in the current states, which greatly facilitates the solution of the model. In particular,

$$\begin{aligned} &E_t e^{F_0+F_g\Delta c_{t+1}+F_x x_{t+1}+F_c\sigma_{g,t+1}^2+F_s\sigma_{x,t+1}^2} \\ &= e^{F_0+F_g\mu_g+(F_g+\rho F_x)x_t+(F_c\nu_g+\frac{1}{2}F_g^2)\sigma_{gt}^2+(F_s\nu_x+\frac{1}{2}F_x^2)\sigma_{xt}^2+\log\Psi_c(F_c)+\log\Psi_x(F_s)}. \end{aligned} \tag{A.3}$$

Using the Euler equation for the consumption asset, we obtain that the equilibrium log price-to-consumption ratio pc_t is linear in the states of the economy:

$$pc_t = A_0 + A_x x_t + A_{gs}\sigma_{gt}^2 + A_{xs}\sigma_{xt}^2. \tag{A.4}$$

The price-to-consumption loadings satisfy the equations given in (4.3)-(4.5), while the solution to the log-linearization coefficient κ_1 is given by

$$\begin{aligned} \log\kappa_1 &= \log\delta + \left(1-\frac{1}{\psi}\right)\mu + A_{gs}(1-\kappa_1)\sigma_g^2 + A_{xs}(1-\kappa_1)\sigma_x^2 \\ &\quad + \frac{1}{\theta}\left(\log\Psi_c(\theta\kappa_1 A_{gs}) + \log\Psi_x(\theta\kappa_1 A_{xs})\right). \end{aligned} \tag{A.5}$$

The solution to the price-to-consumption intercept A_0 can be found using the following condition:

$$E(pc_t) = A_0 + A_{gs}\sigma_g^2 + A_{xs}\sigma_x^2 = \log\left(\frac{\kappa_1}{1-\kappa_1}\right). \tag{A.6}$$

Using the equilibrium solution to the price-consumption ratio, we can write down the expression for the real discount factor in the following way:

$$\begin{aligned} m_{t+1} &= m_0 + m_x x_t + m_{gs}\sigma_{gt}^2 + m_{xs}\sigma_{xt}^2 \\ &\quad - \lambda_\eta \sigma_{gt}\eta_{t+1} - \lambda_e \sigma_{xt} e_{t+1} - \lambda_{gw} w_{g,t+1} - \lambda_{xw} w_{x,t+1}. \end{aligned} \tag{A.7}$$

The solution to the discount factor loadings are given by

$$\begin{aligned} &m_x = -\frac{1}{\psi}, \qquad m_{gs} = -\frac{1}{2}(\gamma - \frac{1}{\psi})(\gamma - 1), \qquad m_{xs} = -\frac{1}{2}(\gamma - \frac{1}{\psi})(\gamma - 1)\left(\frac{\kappa_1}{1-\kappa_1\rho}\right)^2, \\ &m_0 = \theta\log\delta + (1-\theta)\log\kappa_1 - \gamma\mu + (\theta-1)(1-\kappa_1)(A_{gs}\sigma_g^2 + A_{xs}\sigma_x^2), \end{aligned} \tag{A.8}$$

and the expressions for the market prices of risks are provided in equations (4.7).

Denote $q_{t,n}$ and $q_{t,n}^{\$}$ the equilibrium solution to the real and nominal n-period bond prices, respectively. Using the Euler equation, we find that the equilibrium solutions to the real bond prices are affine in the state variables:

$$q_{t,n} = -B_{0,n} - B_{x,n}x_t - B_{gs,n}\sigma_{gt}^2 - B_{xs,n}\sigma_{xt}^2, \tag{A.9}$$

where the loadings satisfy the recursions

$$\begin{aligned} B_{x,n} &= \rho B_{x,n-1} - m_x, \\ B_{gs,n} &= \nu_g B_{gs,n-1} - m_{gs} - \frac{1}{2}\gamma^2, \\ B_{xs,n} &= \nu_x B_{xs,n-1} - m_{xs} - \frac{1}{2}(\lambda_e + B_{x,n-1})^2, \\ B_{0,n} &= B_{0,n-1} - m_0 - \log\Psi_c(-\lambda_{gw} - B_{gs,n-1}) - \log\Psi_x(-\lambda_{xw} - B_{xs,n-1}). \end{aligned} \tag{A.10}$$

Similarly, using the equilibrium solution to the nominal discount factor and the Euler equation, we find that the nominal bond prices are affine in the state variables:

$$q_{t,n}^{\$} = -B_{0,n}^{\$} - B_{x,n}^{\$}x_t - B_{gs,n}^{\$}\sigma_{gt}^2 - B_{xs,n}^{\$}\sigma_{xt}^2 - B_{z,n}^{\$}z_t, \tag{A.11}$$

where the nominal bond loadings satisfy the recursions

$$\begin{aligned}
B^{\$}_{x,n} &= \rho B^{\$}_{x,n-1} + \alpha_x B^{\$}_{z,n-1} - m_x, \\
B^{\$}_{z,n} &= \alpha_z B^{\$}_{z,n-1} + 1, \\
B^{\$}_{gs,n} &= \nu_g B^{\$}_{gs,n-1} - m_{gs} - \frac{1}{2}\left(\varphi_{\pi g} + \gamma + \varphi_{zg} B^{\$}_{z,n-1}\right)^2, \\
B^{\$}_{xs,n} &= \nu_x B^{\$}_{xs,n-1} - m_{xs} - \frac{1}{2}\left(\varphi_{\pi x} + \lambda_e + B^{\$}_{x,n-1} + \varphi_{zx} B^{\$}_{z,n-1}\right)^2, \\
B^{\$}_{0,n} &= B^{\$}_{0,n-1} - m_0 + \mu_\pi - \frac{1}{2}\left(\sigma_\pi^2 + (B^{\$}_{z,n-1}\sigma_z)^2\right) \\
&\quad - \log\Psi_c(-\lambda_{cw} - B^{\$}_{gs,n-1}) - \log\Psi_x(-\lambda_{xw} - B^{\$}_{xs,n-1}).
\end{aligned} \tag{A.12}$$

B Numerical Solution of the Model

The exact solution to the log of consumption asset return is given by,

$$r_{c,t+1} = \Delta c_{t+1} - pc_t + \log\left(e^{pc_{t+1}} + 1\right), \tag{B.1}$$

where pc_t stands for the log of price-to-consumption ratio. We plug in this expression for $r_{c,t+1}$ into the solution for the discount factor in equation (3.2) to obtain,

$$m_{t+1} = \theta\log\delta - \gamma\Delta c_{t+1} - (\theta-1)pc_t + (\theta-1)\log\left(e^{pc_{t+1}} + 1\right). \tag{B.2}$$

Using the Euler condition for the consumption asset, and the fact that short-run consumption shocks are conditionally independent from other sources of risks, we obtain the following expression to the price-consumption ratio:

$$pc_t = \log\delta + (1 - \frac{1}{\psi})\left(\mu + x_t + \frac{1}{2}(1-\gamma)\sigma_{gt}^2\right) + \frac{1}{\theta}\log E_t\left(1 + e^{pc_{t+1}}\right)^\theta. \tag{B.3}$$

The solution to the risk-free rate is derived in a similar way and is given by

$$rf_t = -\theta\log\delta + \gamma\left(\mu + x_t - \frac{1}{2}\gamma\sigma_{gt}^2\right) - (1-\theta)pc_t - \frac{1}{\theta}\log E_t\left(1 + e^{pc_{t+1}}\right)^{(\theta-1)}. \tag{B.4}$$

We solve the two equations above numerically by discretizing the grid for the three states x_t, σ_{gt}^2 and σ_{xt}^2. In particular, we use 15 grid points for expected growth state x_t and 12 grid points for long-run consumption volatility σ_{xt}^2 to capture the persistence of these series, and

2 grid points for short-run volatility σ_{gt}^2 as this process is very short-lived. In the first step, we solve price-to-consumption ratio as a fixed point in the equation (B.3), and then use the solution to price-to-consumption to characterize the discount factor, consumption asset return and the risk-free rate. We provide the output for the numerical and log-linearized solution to the model in Table 11 below. As can be seen from the Table, the moments of price-to-consumption ratio, risk-free rate, consumption asset return and discount factor based on the numerical solution of the model are very close to their counterparts using the log-linearized solution to the model. Hence, the approximation errors using the log-linearized solution are expected to be small. As log-linearization of the model is accurate and provides closed-form expressions for the asset prices, we rely on it in our main discussion in the paper.

Table 11: **Numerical Model Solution**

	Mean	Vol
Price-Consumption Ratio		
Numerical	6.02	0.03
Log-linearized	6.05	0.04
One-month risk-free rate		
Numerical	2.29	0.95
Log-linearized	2.25	1.15
Return on consumption asset		
Numerical	4.87	2.67
Log-linearized	4.79	2.54
Discount Factor		
Numerical	-0.28	0.63
Log-linearized	-0.26	0.59

Numerical solution is based on discretization of expected growth, long-run volatility and short-run volatility states. Mean and volatility of consumption asset return and discount factor are annualized.