A Note on the Economics and Statistics of Predictability: A Long Run Risks Perspective*

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Abstract

Asset return and cash flow predictability is of considerable interest in financial economics. In this note, we show that the magnitude of this predictability in the data is quite small and is consistent with the implications of the long-run risks model.

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1 Introduction

Predictability of asset returns and cash flows is a topic of considerable interest for financial economists. The source and magnitude of predictability in these components determine asset price fluctuations and impose restrictions on economic models that help evaluate asset pricing models. We use the long-run risks model of Bansal and Yaron (2004) to evaluate the economic and statistical plausibility of predictability of returns and cash flows. That is, we ask how much predictability is plausible in the data, both from a statistical and the long-run risks model perspective.

The evidence on predictability is voluminous and contentious (see for example, Keim and Stambaugh (1986), Campbell and Shiller (1988), Fama and French (1988), Hodrick (1992), Stambaugh (1999), Goyal and Welch (2003), Valkanov (2003), Lewellen (2004), and Boudoukh, Richardson, and Whitelaw (2006)). One view, (see Campbell and Cochrane (1999) and Cochrane (2006)) is that returns are sharply predictable while consumption and cash flow growth rates are not. This view, therefore, associates movements in asset prices to discount rate variation rather than time varying cash flow growth. However, on statistical grounds, Ang and Bekaert (2007), Boudoukh, Richardson, and Whitelaw (2006) question the magnitude of return predictability in the data and argue that returns do not have significant predictability. An alternative view is that cash flow growth rates are predictable in ways that have important implications for asset prices (see Bansal and Yaron (2006), Lettau and Ludvigson (2005), and Hansen, Heaton, and Li (2006)). Hence, the magnitude of predictability of returns and cashflows in the data is a source of considerable debate and discussion.

The main focus in this paper is about magnitudes: what is a plausible magnitude of predictability from the statistical perspective and from the perspective of an economic model – the long-run risks model. The economic model, which is broadly consistent with a wide-range of asset market facts, provides a framework to evaluate the plausibility of predictability in the data. We confine our attention to the standard excess return and consumption growth rate predictability. Our evidence shows that based on dividend-price ratios returns are modestly predictable, though this predictability is quite fragile. For example, when we use dividend-price ratios adjusted by the risk-free rate, we get a more stationary and better behaved predictor variable, however, the level of return predictability declines considerably and is close to zero.[1] The magnitude of predictability of consumption growth rate in the data is also quite small. For both returns and consumption growth, the finite sample distribution of the coefficients and adjusted R^2's are quite wide.

We calibrate a version of the long-run risks model of Bansal and Yaron (2004) and use an improved model solution based on approximate analytical method from Bansal, Kiku, and Yaron (2007) to show that the model can generate finite sample properties that are consistent with the aforementioned empirical findings. Excess return predictability in the model is due to the time variation of risk premia, induced by the presence of time varying volatility of consumption and cash flows. Consumption growth in the model is driven by a small, persistent component that, in equilibrium, governs the dynamics of asset prices. Thus, current asset valuations should contain important information about future consumption growth. However, price-dividend ratios in the model move not only on news about future economic growth but also on news about future economic uncertainty (or discount-rate news). Price fluctuations emanating from time-variation in discount rates may significantly diminish the informational content of asset valuations about future growth and, consequently, limit their ability to forecast future dynamics of consumption growth. Indeed, we show, that consistent with the data evidence, the model-implied predictability of consumption growth by the market dividend-price ratio is quite small.

Overall our results support the view that there is a small time-varying component in returns and in cash flows. The evidence in this paper shows that the long-run risks model can quantitatively explain the level of predictability of returns and consumption growth consistent with that observed in the data.

The paper continues as follows: Section 2 discusses the data and provides the results of our empirical analysis. Section 3 presents the model and provides the corresponding predictability results. Section 4 provides concluding comments.

[1] This difference in the magnitude of the R^2 between dividend-price and risk-free rate adjusted dividend-price ratio is most likely due to the very high persistence in the dividend yield. For this issue also see Hodrick (1992).

2 Empirical Findings

We use annual data on consumption and asset prices for the time period from 1930 till 2006. The annual data provides the longest available sample and is arguably the least susceptible to measurement errors. Consumption data are based on seasonally adjusted per-capita series on real consumption from the NIPA tables available on the Bureau of Economic Analysis website. Aggregate consumption is defined as consumer expenditures on non-durables and services. Growth rates are constructed by taking the first difference of the corresponding log series. Our asset menu comprises the aggregate stock market portfolio on the value weighted return of the NYSE/AMEX/NASDAQ from CRSP and a proxy of a risk-less asset. The real interest rate is constructed by subtracting realized annual inflation from the annualized yield on the 3-month Treasury bill taken from the CRSP treasury files.

Table I presents descriptive statistics for consumption growth, the return and dividend yield of the aggregate stock market and the risk-free rate. All entries are expressed in real percentage terms. Standard errors are based on the Newey and West (1987) estimator with 8 lags. This particular sample results in the standard and well known features of the data such as a low risk free rate, a large equity premium and a relatively low consumption volatility.

Table II provides the results of consumption growth predictability using the log of the dividend-price ratio as a regressor. The table presents estimates of slope coefficients ($\hat{\beta}$), robust t-statistics and R^2s from projecting 1-, 3- and 5-year consumption growth onto lagged log dividend-price ratio of the aggregate stock market portfolio. The point estimates are insignificantly different from zero and the R^2s are less than 2%. In addition, the right columns display bootstrap distributions of the reported statistics. Empirical percentiles are constructed by resampling the data 10,000 times in blocks of 8 years with replacement. At the 5-year horizon, the median R^2 is 4 percent while the 90 percentile includes an R^2 as high as 18%. This evidence suggests that the level of the consumption predictability in the data includes a wide range of predictability estimates and R^2s.

It is very important to note that the above predictability evidence is solely based on using the dividend-price ratio as a predictive variable. Bansal, Kiku, and Yaron (2007) provide evidence that when additional predictive variables are used, the consumption predictability is considerably higher. For example, if the risk-free rate is included as an additional predictive variable, the R^2 for the one-year horizon rises to 17% and at the two-year horizon is about

12%. Clearly other forecasting variables, such as earnings to consumption ratio used in Hansen, Heaton, and Li (2006), would further increase short- and long-run predictability of consumption. Expanding the information set beyond financial ratios to forecast future growth is motivated by economic considerations as discussed in Bansal, Kiku, and Yaron (2007).

Table III provides evidence on predictability of multi-period excess returns. In panel A the log of dividend-price ratio is used to forecast returns. Consistent with evidence is earlier papers, the R^2s rise with maturity from 4.5% at the 1-year to 29% at the 5-year horizon. Note that the slope coefficient estimates are only marginally significant for all three horizons. The bootstrap t-statistics and R^2s have a wide distribution and range from 0.2 to 3 for the t-statistics and from 0 to 40% for the R^2. This evidence of predictability is highly fragile. Panel B of Table III runs the same regressions save for the fact the regressor is now the log dividend-price ratio minus the risk free rate. We do so to ensure that the predictive variable is well behaved — adjusting the dividend-price ratio for the risk free rate lowers the high persistence in the predictive variable. The results of return predictability are now much weaker. In particular, at all horizons, the slope coefficients are insignificant. The R^2s are now below 4.5% for all horizons. The range for the bootstrap t-statistics and R^2s is now tighter and covers 0.23 to 2.8 for the t-statistic, and 0 to 21% for the R^2. This is consistent with a view that the actual magnitude for return predictability is quite small. The difference in predictability between Panel A and Panel B also clearly suggests that much of the ability of the dividend-yield to predict future returns might be spurious and simply due to its very persistent nature for this particular sample. The fragility of the return predictability evidence is one of the reasons for the ongoing debate about the presence and magnitude of return predictability discussed in the introduction.

3 Model

In this section we specify a model based on Bansal and Yaron (2004). The underlying environment is one with complete markets and the representative agent has Epstein and Zin (1989) type recursive preferences in which she maximizes her life-time utility,

$$V_t = \left[(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta\Big(E_t\big[V_{t+1}^{1-\gamma}\big]\Big)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}}, \tag{1}$$

where C_t is consumption at time t, $0 < \delta < 1$ reflects the agent's time preferences, γ is the coefficient of risk aversion, $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$, and ψ is the elasticity of intertemporal substitution (IES). Utility maximization is subject to the budget constraint,

$$W_{t+1} = (W_t - C_t)R_{c,t+1} \ , \tag{2}$$

where W_t is the wealth of the agent, and $R_{c,t}$ is the return on all invested wealth.

Consumption and dividends have the following joint dynamics:

$$\begin{aligned}
\Delta c_{t+1} &= \mu_c + x_t + \sigma_t \eta_{t+1} \\
x_{t+1} &= \rho x_t + \varphi_e \sigma_t e_{t+1} \\
\sigma^2_{t+1} &= \bar{\sigma}^2 + \nu(\sigma^2_t - \bar{\sigma}^2) + \sigma_w w_{t+1}, \\
\Delta d_{t+1} &= \mu_d + \phi x_t + \pi \sigma_t \eta_{t+1} + \varphi \sigma_t u_{d,t+1}
\end{aligned} \tag{3}$$

where Δc_{t+1}, and Δd_{t+1} are the growth rate of consumption and dividends respectively. In addition, we assume that all shocks are $i.i.d$ normal and are orthogonal to each other. As in the long-run risks model of Bansal and Yaron (2004), $\mu_c + x_t$ is the conditional expectation of consumption growth, and x_t is a small but persistent component that captures long-run risks in consumption growth. For parsimony, as in Bansal and Yaron (2004), we have a common time-varying volatility in consumption and dividends, which, as shown in their paper, leads to time-varying risk premia. Dividends have a levered exposure to the persistent component in consumption, x_t, which is captured by the parameter ϕ. In addition, we allow the $i.i.d$ consumption shock η_{t+1} to influence the dividend process, and thus serve as an additional source of risk premia. The magnitude of this influence is governed by the parameter π.[2] Save for this addition, the dynamics are similar to those in Bansal and Yaron (2004).

As in Epstein and Zin (1989), it is easily shown that, for any asset j, the first order condition yields the following asset pricing Euler condition,

$$E_t \left[\exp\left(m_{t+1} + r_{j,t+1} \right) \right] = 1, \tag{4}$$

where m_{t+1} is the log of the intertemporal marginal rate of substitution and $r_{j,t+1}$ is the

[2] Note that equivalently we could have specified the correlation between η_{t+1} and $u_{d,t+1}$ to be non-zero, and set $\pi = 0$.

log of the gross return on asset j. Further, the log of the Intertemporal Marginal Rate of Substitution (IMRS), m_{t+1}, is

$$m_{t+1} \;=\; \theta \log \delta \;-\; \frac{\theta}{\psi}\Delta c_{t+1} \;+\; (\theta - 1) r_{c,t+1}, \tag{5}$$

where $r_{c,t+1}$ is the continuous return on the consumption asset. To solve for the return on wealth (the return on the consumption asset), we use the log-linear approximation for the continuous return on the wealth portfolio, namely,

$$r_{c,t+1} = \kappa_0 + \kappa_1 z_{t+1} + \Delta c_{t+1} - z_t \;, \tag{6}$$

where $z_t = \log(P_t/C_t)$ is log price to consumption ratio (the valuation ratio corresponding to a claim that pays consumption) and the κ's are log linearization constants which are discussed in more detail below.

To solve for asset prices we provide a simpler and more efficient way to solve the Bansal and Yaron (2004) long-run risks model. We use approximate analytical solutions (instead of the polynomial-based numerical approximation in the original paper), which, we find, provide a more accurate solution to the model.[3] This easier-to-implement solution and a refined configuration leads to similar economic magnitudes but allows us to better address certain predictability dimensions. Specifically, we conjecture the price to consumption ratio follows,

$$z_t = A_0 + A_1 x_t + A_2 \sigma_t^2 \tag{7}$$

and solve for the A's using the Euler equation (4), the return equation (6) and the conjectured equation (7) for the price-consumption ratio. The solution for the A's depends on all the preference and technology parameters and is derived in Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2007), and for completeness is reproduced in the Appendix. In solving for the price-consumption ratio we impose model consistency between the average price consumption ratio $\bar{z}$ and the approximation κ's, which themselves depend on the average price-consumption ratio. This is important to impose, as any change in the model parameters will alter $\bar{z}$ and hence the approximation κ's. The model-based endogenous solution to $\bar{z}$ can

[3]Bansal, Kiku, and Yaron (2007) evaluate the various approaches and find the approximate-analytical solution to be the most accurate and easy to implement.

be obtained by solving the equation,

$$\bar{z} = A_0(\bar{z}) + A_2(\bar{z})\bar{\sigma}^2 \ , \tag{8}$$

recognizing that $\kappa_0 = \log(1 + \exp(\bar{z})) - \kappa_1 \bar{z}$ and $\kappa_1 = \frac{\exp(\bar{z})}{1+\exp(\bar{z})}$. Implementing equation (8) to solve for $\bar{z}$ is quite easy in practice. The endogeneity of $\bar{z}$ has also been emphasized in Campbell and Koo (1997).

Given the solution for z_t, the innovation to the return to wealth can be derived, which in turn allows us to specify the innovations to the IMRS and thus facilitate computing risk premia for various assets. In particular, it immediately follows that the risk premium on the market portfolio (that is, the return on the dividend paying asset) carries three sources of risks. That is

$$E_t[r_{m,t+1} - r_{f,t} + 0.5\sigma^2_{t,r_m}] = \beta_{\eta,m}\lambda_\eta\sigma^2_t + \beta_{e,m}\lambda_e\sigma^2_t + \beta_{w,m}\lambda_w\sigma^2_w \tag{9}$$

where $\beta_{m,j}, \ j = \{\eta, e, w\}$ are respectively the betas of the market return with respect to the "short run" risk, η_t, the long-run risk innovation, e_t, and the economic uncertainty (volatility) risk, w_t. The λ's represent the corresponding market prices of risks. The appendix and Bansal, Kiku, and Yaron (2007) provide the solution for the market price of risks and the market return's betas in terms of the underlying preference and technology parameters.

Table IV provides the parameter configuration we use to calibrate the model — these are chosen to match several key statistics of consumption data, dividend data, and asset returns. Table V presents moments of simulated annualized consumption and dividend growth rates along with asset pricing implications of the model. Reported statistics are based on 10,000 simulated samples with 77×12 monthly observations that match the length of the actual data. The entries represent the median, 5^{th} and 95^{th} percentiles of the monte-carlo distributions of the corresponding statistics. These results show that the model distribution for the mean, standard deviation and first autocorrelation of consumption and dividend growth and their correlation are consistent with the data. Moreover, the model generates a distribution of asset returns that captures the key features of the data. In particular, the model's median equity premium is just under 7% and the volatility of the market return is just about that of the data at 19%. The model further generates a low risk free rate level and volatility and a plausible level and volatility of the price-dividend ratio.

Table VI provides the model-implied consumption growth predictability by the log of the dividend-price ratio. The model's median estimate indicates a significant negative slope coefficient for predicting the one- to five-year ahead consumption growth. Although the median R^2s are somewhat large relative to their data counterpart, the data t-statistics and R^2s are within the 95% confidence interval generated by the model. It is important to note that, to maintain parsimony and keep the number of calibrated parameters small, we have assumed that all consumption shocks are orthogonal to each other. We have, however, explored the sensitivity of the model implications to the innovation correlation structure. We find that relaxing a zero-correlation restriction between long-run and volatility risks allows the model to even better capture the low ability of the dividend-price ratio to forecast future consumption growth, as observed in the data. Under reasonable correlation parameterizations, the model is able to diminish short- and long-horizon consumption growth predictability to about 5-6% without altering other asset pricing predictions (this evidence is available upon request).[4]

Panel A of Table VII provides the analogous results for model-implied statistics for return predictability by the log dividend-price ratio. As in the data, the regression coefficients and R^2s rise with the horizon. The median return predictability coefficients are not significant at conventional levels and the median estimate for R^2 is 5% at the 5-year horizon. The data's estimates across horizons are all well within their corresponding 90% model-based confidence intervals. Panel B provides the analogous projections when the log dividend price ratio is adjusted for the risk free rate. The model-based results are very close to those in the data reported in Panel B of Table III — both the level of the slope coefficient and the R^2 are a close match. In all this evidence implies that the model can match the return predictability observed in the data. Note that in the model the level of predictability is not sensitive to using the dividend-price ratio or the adjusted dividend-price ratio; this is because the information in the predictive variable, consistent with theory, should not change when one adjusts the price-dividend ratio for the risk free rate. This model-based evidence, along with the sharp differences in the data across Panels A and B of Table III, indicates that the actual return predictability is close to what one finds using the risk-free adjusted dividend-price

[4]Using the parameter configuration of Bansal and Yaron (2004) may lead to somewhat higher predictability of consumption growth as pointed out in Bui (2007). However, the somewhat different configuration we rely on does equally well in reproducing key asset pricing features as well as replicating low predictability of consumption as shown in Table VI. As discussed above, the model-implied predictability of consumption growth is even lower when one allows for a non-zero correlation in shocks in the consumption growth dynamics.

ratio. The model, as documented above, can completely match this data feature.

4 Conclusions

The debate regarding return and cash flow predictability has been at center stage in finance for several decades. From the statistical point of view, both returns and consumption growth are predictable by the dividend-price ratios only to a limited extent. We show that the implications of the long-run risks model are consistent with the view that the data contains a small predictive component in both returns and consumption growth.

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5 Appendix

The solutions for **A**s are given by,

$$
\begin{aligned}
A_0 &= \frac{1}{1-\kappa_1}\left[\log\delta + \kappa_0 + \Big(1-\frac{1}{\psi}\Big)\mu_c + \kappa_1 A_2(1-\nu)\bar{\sigma}^2 + \frac{\theta}{2}\Big(\kappa_1 A_2 \sigma_w\Big)^2\right] \\
A_1 &= \frac{1-\frac{1}{\psi}}{1-\kappa_1\rho} \\
A_2 &= -\frac{(\gamma-1)(1-\frac{1}{\psi})}{2\,(1-\kappa_1\nu)}\left[1+\Big(\frac{\kappa_1\varphi_e}{1-\kappa_1\rho}\Big)^2\right]
\end{aligned} \tag{10}
$$

As shown in Bansal and Yaron (2004) the solution for the market price of risks,

$$
\begin{aligned}
\lambda_\eta &= \gamma \\
\lambda_e &= (1-\theta)\kappa_1 A_1 \varphi_e = (\gamma - \frac{1}{\psi})\frac{\kappa_1\varphi_e}{1-\kappa_1\rho} \\
\lambda_w &= (1-\theta)\kappa_1 A_2 = -(\gamma-1)(\gamma-\frac{1}{\psi})\frac{\kappa_1}{2\,(1-\kappa_1\nu)}\big[1+(\frac{\kappa_1\varphi_e}{1-\kappa_1\rho})^2\big]
\end{aligned} \tag{11}
$$

where these respectively represent the market prices of transient (η_{t+1}), long-run (e_{t+1}) and volatility (w_{t+1}) risks respectively.

The price-dividend ratio for the market claim to dividends, $z_{m,t} = A_{0,m} + A_{1,m}x_t + A_{2,m}\sigma_t^2$, where

$$
\begin{aligned}
A_{0,m} &= \frac{1}{1-\kappa_{1,m}}\left[\Gamma_0 + \kappa_{0,m} + \mu_d + \kappa_{1,m}A_{2,m}(1-\nu)\bar{\sigma}^2 + \frac{1}{2}\Big(\kappa_{1,m}A_{2,m} - \lambda_w\Big)^2\sigma_w^2\right] \\
A_{1,m} &= \frac{\phi - \frac{1}{\psi}}{1-\kappa_{1,m}\rho} \\
A_{2,m} &= \frac{1}{1-\kappa_{1,m}\nu}\left[\Gamma_2 + \frac{1}{2}\Big(\varphi^2 + (\pi-\lambda_\eta)^2 + (\kappa_{1,m}A_{1,m}\varphi_e - \lambda_e)^2\Big)\right]
\end{aligned} \tag{12}
$$

where $\Gamma_0 = \log\delta - \frac{1}{\psi}\mu_c - (\theta-1)\left[A_2(1-\nu)\bar{\sigma}^2 + \frac{\theta}{2}\Big(\kappa_1 A_2\sigma_w\Big)^2\right]$ and $\Gamma_2 = (\theta-1)(\kappa_1\nu-1)A_2$

The risk premium is determined by the covariation of the return innovation with the innovation into the pricing kernel. Thus, the risk premium for $r_{m,t+1}$ is equal to the asset's exposures to systematic risks multiplied by the corresponding risk prices,

$$
\begin{aligned}
E_t(r_{m,t+1} - r_{f,t}) + 0.5\sigma^2_{t,r_m} &= -Cov_t\Big(m_{t+1} - E_t(m_{t+1}), r_{m,t+1} - E_t(r_{m,t+1})\Big) \\
&= \lambda_\eta \sigma_t^2 \beta_{\eta,m} + \lambda_e \sigma_t^2 \beta_{e,m} + \lambda_w \sigma_w^2 \beta_{w,m}
\end{aligned}
$$

where the asset's βs are defined as,

$$
\begin{aligned}
\beta_{\eta,m} &= \pi \\
\beta_{e,m} &= \kappa_{1,m} A_{1,m} \varphi_e \\
\beta_{w,m} &= \kappa_{1,m} A_{2,m}
\end{aligned}
$$

Table I
Summary Statistics

	Mean		Volatility	
	Estimate	SE	Estimate	SE
Cons. Growth (Δc)	1.95	0.31	2.13	0.44
Market Return (R)	8.51	1.69	19.65	2.14
Div. Yield (D/P)	3.97	0.43	1.52	0.23
Risk-free Rate (R^f)	0.95	0.91	3.95	0.75

Table I presents descriptive statistics for consumption growth, return and dividend yield of the aggregate stock market, and the risk-free rate. All entries are expressed in percentage terms. Standard errors are based on the Newey and West (1987) estimator with 8 lags. The data are real, sampled on an annual frequency and cover the period from 1930 to 2006.

Table II

Predictability of Consumption Growth

Horizon (yr)		Estimate	5%	10%	50%	90%	95%
	$\hat{\beta}$	-0.007	-0.02	-0.01	-0.00	0.00	0.00
1	t-stat	-1.286	-1.97	-1.64	-0.59	0.72	1.12
	R^2	0.022	0.00	0.00	0.01	0.05	0.06
	$\hat{\beta}$	-0.004	-0.02	-0.01	0.01	0.03	0.04
3	t-stat	-0.363	-1.31	-0.86	0.70	1.85	2.12
	R^2	0.001	0.00	0.00	0.02	0.08	0.11
	$\hat{\beta}$	0.005	-0.03	-0.02	0.02	0.07	0.09
5	t-stat	0.294	-1.29	-0.81	1.04	2.38	2.71
	R^2	0.002	0.00	0.00	0.04	0.18	0.23

Table II presents estimates of slope coefficients ($\hat{\beta}$), robust t-statistics and R^2s from projecting 1-, 3- and 5-year consumption growth onto lagged dividend-price ratio of the aggregate stock market portfolio. Robust t-statistics are computed using Hodrick (1992)-adjusted standard errors. The right columns display bootstrap distributions of the reported statistics. Empirical percentiles are constructed by resampling the data 10,000 times in blocks of 8 years with replacement. The data employed in estimation are real, compounded continuously, sampled on an annual frequency and cover the period from 1930 to 2006.

Table III

Predictability of Excess Returns

Panel A: Predictability by Dividend-Price Ratio

Horizon (yr)		Estimate	5%	10%	50%	90%	95%
1	$\hat{\beta}$	0.094	0.03	0.05	0.10	0.20	0.25
	t-stat	1.779	0.58	0.90	1.91	2.83	3.10
	R^2	0.045	0.00	0.01	0.05	0.13	0.15
3	$\hat{\beta}$	0.276	0.08	0.13	0.29	0.56	0.67
	t-stat	1.755	0.54	0.90	1.96	2.87	3.10
	R^2	0.175	0.01	0.03	0.15	0.32	0.38
5	$\hat{\beta}$	0.455	0.05	0.12	0.38	0.76	0.87
	t-stat	1.857	0.21	0.56	1.67	2.58	2.81
	R^2	0.294	0.01	0.02	0.17	0.40	0.46

Panel B: Predictability by Dividend-Price Ratio Adjusted for Risk-free Rate

Horizon (yr)		Estimate	5%	10%	50%	90%	95%
1	$\hat{\beta}$	0.009	0.00	0.00	0.01	0.01	0.02
	t-stat	1.418	0.23	0.48	1.40	2.48	2.80
	R^2	0.034	0.00	0.00	0.03	0.08	0.10
3	$\hat{\beta}$	0.012	-0.02	-0.01	0.01	0.03	0.04
	t-stat	0.839	-0.83	-0.46	0.93	2.24	2.54
	R^2	0.033	0.00	0.00	0.03	0.15	0.20
5	$\hat{\beta}$	0.017	-0.02	-0.01	0.01	0.04	0.05
	t-stat	1.025	-1.08	-0.70	0.76	2.03	2.33
	R^2	0.045	0.00	0.00	0.03	0.16	0.21

Panel A of Table III presents estimates of slope coefficients ($\hat{\beta}$), robust t-statistics and R^2s from projecting 1-, 3- and 5-year excess returns onto lagged dividend-price ratio of the aggregate stock market portfolio. Evidence on predictability of multi-period excess returns by the dividend-price ratio adjusted for the risk-free rate is reported in Panel B. Robust t-statistics are computed using Hodrick (1992)-adjusted standard errors. The right columns display bootstrap distributions of the reported statistics. Empirical percentiles are constructed by resampling the data 10,000 times in blocks of 8 years with replacement. The data employed in estimation are real, compounded continuously, sampled on an annual frequency and cover the period from 1930 to 2006.

Table IV

Configuration of Model Parameters

Preferences	δ	γ	ψ			
	0.9989	10	1.5			
Consumption	μ	ρ	ϕ_x	$\bar{\sigma}$	ν	σ_w
	0.0015	0.975	0.038	0.0072	0.999	0.0000028
Dividends	μ_d	ϕ	φ_d	π		
	0.0015	2.5	5.96	2.6		

Table IV reports configuration of investors' preferences and time-series parameters that describe dynamics of consumption and dividend growth rates. The model is calibrated on a monthly decision interval.

Table V

Model-Implied Dynamics of Growth Rates and Prices

Moments	Median	5%	95%
Consumption:			
$E[\Delta c]$	1.80	0.88	2.69
$\sigma(\Delta c)$	2.46	1.58	3.62
$AC(1)$	0.39	0.18	0.57
Dividends:			
$E[\Delta d]$	1.81	-2.02	5.63
$\sigma(\Delta d)$	13.87	8.95	20.37
$Corr(\Delta d, \Delta c)$	0.46	0.25	0.62
Market:			
$E[R]$	8.16	4.38	13.73
$\sigma(R)$	20.39	13.28	30.62
$E[D/P]$	4.42	3.71	6.07
$\sigma[D/P]$	0.83	0.43	1.73
Risk-free Rate:			
$E[R^f]$	1.23	0.28	1.81
$\sigma(R^f)$	0.95	0.58	1.48

Table V presents moments of simulated annualized consumption and dividend growth rates along with asset pricing implications of the model. Reported statistics are based on 10,000 simulated samples with 77×12 monthly observations that match the length of the actual data. The entries represent the median, 5^{th} and 95^{th} percentiles of the monte-carlo distributions of the corresponding statistics.

Table VI

Model-Implied Predictability of Consumption Growth

Horizon (yr)		Median	5%	10%	90%	95%
	$\hat{\beta}$	-0.05	-0.09	-0.08	-0.02	-0.01
1	t-stat	-2.88	-4.22	-3.96	-1.40	-0.83
	R^2	0.15	0.01	0.03	0.31	0.36
	$\hat{\beta}$	-0.10	-0.20	-0.18	-0.02	0.01
3	t-stat	-2.35	-4.02	-3.72	-0.44	0.22
	R^2	0.11	0.00	0.01	0.31	0.37
	$\hat{\beta}$	-0.12	-0.29	-0.25	0.01	0.05
5	t-stat	-1.94	-3.89	-3.50	0.17	0.85
	R^2	0.09	0.00	0.00	0.31	0.38

Table VI reports implications of the Long-Run Risks model for consumption growth predictability. The entries represent estimates of slope coefficients ($\hat{\beta}$), robust t-statistics and R^2s from projecting 1-, 3- and 5-year consumption growth onto lagged dividend-price ratio of the aggregate stock market portfolio. Robust t-statistics are computed using Hodrick (1992)-adjusted standard errors. The entries present distributions of the corresponding moments across 10,000 simulated samples.

Table VII
Model-Implied Predictability of Excess Returns

Panel A: Predictability by Dividend-Price Ratio

Horizon (yr)		Median	5%	10%	90%	95%
	$\hat{\beta}$	0.09	-0.11	-0.06	0.27	0.33
1	t-stat	0.76	-0.95	-0.57	1.98	2.30
	R^2	0.01	0.00	0.00	0.06	0.08
	$\hat{\beta}$	0.27	-0.24	-0.14	0.69	0.82
3	t-stat	0.87	-0.83	-0.47	2.08	2.40
	R^2	0.03	0.00	0.00	0.14	0.18
	$\hat{\beta}$	0.42	-0.38	-0.20	1.04	1.21
5	t-stat	0.90	-0.86	-0.45	2.11	2.44
	R^2	0.05	0.00	0.00	0.20	0.25

Panel B: Predictability by Dividend-Price Ratio Adjusted for Risk-free Rate

Horizon (yr)		Median	5%	10%	90%	95%
	$\hat{\beta}$	0.01	-0.02	-0.02	0.04	0.05
1	t-stat	0.52	-1.17	-0.81	1.74	2.08
	R^2	0.01	0.00	0.00	0.05	0.07
	$\hat{\beta}$	0.03	-0.06	-0.04	0.11	0.13
3	t-stat	0.52	-1.15	-0.80	1.77	2.10
	R^2	0.02	0.00	0.00	0.12	0.17
	$\hat{\beta}$	0.04	-0.10	-0.07	0.16	0.19
5	t-stat	0.54	-1.19	-0.82	1.82	2.14
	R^2	0.04	0.00	0.00	0.18	0.23

Table VII reports predictability evidence for excess returns implied by the Long-Run Risks model. Panel A presents estimates of slope coefficients ($\hat{\beta}$), robust t-statistics and R^2s from projecting 1-, 3- and 5-year excess returns onto lagged dividend-price ratio of the aggregate stock market portfolio. Evidence on predictability of multi-period excess returns by the dividend-price ratio adjusted for the risk-free rate is reported in Panel B. Robust t-statistics are computed using Hodrick (1992)-adjusted standard errors. The entries present distributions of the corresponding moments across 10,000 simulated samples.