Decisive Entrepreneurs and Cautious Investors*

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Very Preliminary and Incomplete
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Abstract

Entrepreneurs may undertake investments when returns cannot be precisely evaluated: they are decisive. Lenders may require high compensation to place funds in those ventures: they are cautious. We analyze optimal financing contracts between entrepreneurs and lenders. Decisiveness and cautiousness are characterized using Bewley's (1986) incomplete preferences framework. Preferences are represented by a utility function and a set of probability distributions. Suppose a project's expected net return is positive for some distributions and negative for others. Then, invest and leave money in the bank are non comparable alternatives. Undertaking the venture is a decisive action: entrepreneurship makes it attractive. Cautiousness corresponds to leaving money in the bank unless there is an alternative with higher expected return for all distributions. Given the optimal contract, the entrepreneur can invest only if she owns some assets to use as collateral. Cautious lenders require too high a remuneration; collateral has a safe value and lowers their demands. All available assets are used as collateral: self financing is the preferred source of funds. When there are two lenders with different levels of cautiousness, the optimal contract gives the less cautious a stake in the project, and the more cautious a safer contract.

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1 Introduction

Recently, the question of what defines an entrepreneur has received some attention in both academic circles and the popular press. While there is no agreement on the answer, many suggests entrepreneurship may be a personality trait, an attitude towards new business ventures. Some individuals have entrepreneurial attitude, and they are different from the rest of us. Because of this attitude, they undertake projects we might not. In this paper, we encompass entrepreneurs' unique drive towards new ventures, and analyze their interactions with individuals who lack the same spirit. We focus on the issue of financing the new venture when the entrepreneur's attitude towards the project differs from that of potential providers of capital.

An important feature of our analysis is that the entrepreneur is neither more nor less informed than other individuals and, crucially, she is as rational as they are. Instead, she is willing to embark on projects when others would not. We follow Bewley [1989], and define the entrepreneur as an individual who might take a project even without precisely knowing the probabilities of different outcomes. In this setting, the project's returns are uncertain in the sense of Knight [1921] but the entrepreneur wants to start the venture regardless.[1] In a sense, the entrepreneur relies on factors beyond pure rationality to decide on a course of action. This idea is reminiscent of both Knight [1921] description of entrepreneurs as individuals willing to initiate uncertain investments, as well as entrepreneurial animal spirits a là Keynes [1936]. In both views, entrepreneurs are radically different from other economic agents because of the way they tackle uncertainty. We account for this difference by assuming non entrepreneurial individuals might not be willing to invest in an uncertain project.

We study the following model. Suppose an investment opportunity is characterized by Knightian uncertainty: its expected returns cannot be evaluated according to a unique probability measure. In this case, the net present value of the project cannot be uniquely computed. It is different across different probabilities. If, despite this, the net present value is always positive, everybody would agree this is a profitable investment. If, on the contrary, the net present value is positive for some probabilities and negative for others, nobody knows whether this is a profitable investment or not. In this framework, entrepreneurs are assumed to be decisive. They think the venture is worth pursuing and they want to undertake it. At the other extreme, investors are assumed to be cautious. They think the only projects worth pursuing are those which have positive net present value for all probability distributions.

We find the optimal contract for a decisive entrepreneur whose project is financed by cautious investors. Since only the entrepreneur is willing to embark on the venture, there seems to be little hope she can get money to finance it. This intuition is only partially correct. Entrepreneurs who own some assets that can be used as collateral are able to loan funds. Intuitively, cautious investors require a very high lending interest rate; so high that the returns left to the entrepreneur make her unwilling to proceed with the venture. Collateral, on the other hand, makes the loan safer and reduces the lending rate. Moreover, the entrepreneur uses all the assets she owns as collateral to finance it. Loosely speaking, she puts her money where her mouth is. Having done this, there is an entire class of financial contracts which are optimal. A Modigliani-Miller type result

[1] In Knight's language, projects are risky if possible outcomes have known probabilities, and uncertain if they do not.

applies: some specifics of the contract are undetermined. Whether debt-like contracts or stock-like contracts are used makes no difference to the value of the firm.

We show that the implicit interest rate required by investors increases with the degree of uncertainty associated with the project, measured by how many probability distributions are considered. Informally, this is a measure of the degree of cautiousness of financiers. One is naturally led to ask what happens when the entrepreneur can obtain funds from investors who differ in their cautiousness. She always chooses to deal with the less cautious individuals. Therefore, we study the case in which the less cautious investor is a *professional* who imposes a cost on the entrepreneur when providing funds. If this cost is not too high, and if the professional cannot commit enough funds to be the only financier, the entrepreneur uses both sources of funds. The financial contract with the professional investor resembles a stock, while the contract with a cautious investor resembles a bond. Intuitively, professionals have greater similarity to the entrepreneur; hence, she prefers having them as partners.

Our approach has many advantages. The interaction between entrepreneurial and non entrepreneurial attitudes is explicitly taken into account in optimal financial contracting. It results in optimal contracts which are consistent with some stylized facts about financing of new ventures.[2] Self financing is optimal, and different sources of funds correspond to different financial contracts. The framework in which these results are obtained is technically simple. In our opinion, the model provides a reasonable description of the entrepreneur's behavior. For example, there are no incentive problems. This is in stark contrast with some recent work on optimal financial contracts based on moral hazard models. In that approach, without appropriate incentives, a firm's owner (or manager) opts for personal benefits over the venture's success.[3] When it comes to small entrepreneurial firms, this description seems at odds with reality. Anecdotal evidence suggest entrepreneurs are 'driven to succeed'. A venture's success is their ultimate objective, so the moral hazard paradigm might be inappropriate. The approach in this paper constitutes a viable and simple alternative.

Recently, a few papers have investigated the relationship between entrepreneurship and financing of new ventures. Among these are De Meza and Webb [1996], Manove [1996], and Manove and Padilla [1997]. They are all based on the idea that entrepreneurs are unreasonably optimistic about the stream of profits generated by new ventures. The focus of the analysis, then, is on how they interact with potential lenders who are able to correctly asses these profits. In a sense, entrepreneurs are not rational because they could learn from lenders a project's true probability of success. Unlike these authors, we do not assume any irrationality. Our entrepreneur is rational; she makes the best use of all her information. Her preferences are such that she invests in projects others deem too uncertain to be worthwhile. But conditional on this feature of her preferences, she maximizes her expected profits.

2 The Setup

In this section, we summarize Bewley's [1986] model of decision making under uncertainty. Then, we show how it can be used to characterize the behavior of entrepreneurial

[2] See De Meza and Webb [1996] and Holmstrom [1993].

[3] For a summary of this literature, see Hart [1995].

and non entrepreneurial economic agents faced with an uncertain business venture. The main idea is to distinguish these two groups according to their decisiveness.

2.1 Incomplete Preferences and Imprecise Beliefs

In Bewley [1986], individuals's preferences are not necessarily complete and, as a result, they are represented by a Von Neumann-Morgenstern utility function and a *set* of probability distributions. In other words, without completeness the individual uses many probabilities to evaluate outcomes. She associates many expected utilities to each uncertain payoff. Incompleteness is reflected by multiplicity of beliefs.

More precisely, if a preference ordering satisfies the completeness axiom, a decision maker can compare any two random payoffs. She decides which one is better based on their respective expected utilities. If a preference ordering does not satisfy completeness, a decision maker is not necessarily able to compare any two payoffs. As a consequence, she cannot compute a unique expected utility. Let Δ be a set of subjective probability distributions, and let x and y be monetary payoffs defined over the same state space. When preferences are not complete, we say

$$x \text{ is preferred to } y \text{ if and only if } E_\delta[u(x)] > E_\delta[u(y)] \text{ for all } \delta \text{ in } \Delta \tag{2.1}$$

where E_δ indicates an expectation taken with respect to the probability distribution δ, and $u(\cdot)$ is the Von Neumann-Morgenstern utility derived from the outcomes associated with the two alternatives.[4] If Δ has only one member, the preference ordering is complete, and the usual representation obtains.

Bewley notes this expression is a way to model Knight's [1921] distinction between risk and uncertainty. A payoff is risky when the probabilities of different outcomes are known; if they are unknown, the payoff is uncertain. Hence, payoffs are risky when Δ has only one element and uncertain otherwise. Roughly speaking, one can also gauge the amount of uncertainty the individual perceives by how large the set Δ is.

In our paper, the relevant state space has only two elements and individuals are risk neutral. Then, the probability of each state covers an interval and we can describe Δ using the upper and lower bound of this interval for one of the two states. Formally, (2.1) x is preferred to y if and only if

$$\delta x_1 + (1-\delta)\, x_2 > \delta y_1 + (1-\delta)\, y_2 \qquad \text{for all } \delta \text{ in } \left[\underline{\delta}, \bar{\delta}\right]$$

where δ is the probability of state 1. In this framework, $\bar{\delta}$ and $\underline{\delta}$ are referred to as upper and lower probability of state 1 respectively.

2.2 Decisiveness, Cautiousness, and the Decision to Invest

We first describe the entrepreneur's decision whether to undertake a project or not, and then compare her decision to what other potential investors might do.

Let x_1 and x_0 be the returns to some investment opportunity which costs I, with $x_1 > x_0$. Suppose this project's income stream displays Knightian uncertainty as defined in the previous section. That is, the probabilities of the two outcomes are not

[4] A description of the axioms needed for this representation result and its proof can be found in Bewley [1986].

uniquely determined. According to the model of the previous section, we let p denote the probability of success and assume it belongs to the interval $\left[\underline{p}, \overline{p}\right]$.

One needs to compute the project's net present value to decide wether to go ahead with it. Since the returns are uncertain, there are many net present values; one for each element of the interval $\left[\underline{p}, \overline{p}\right]$. Formally, if we let r be the interest rate on a safe investment opportunity (money in the bank), these net present values are given by:

$$NPV\left(p\right) \equiv px_1 + \left(1-p\right)x_0 - \left(1+r\right)I,$$

where $p \in \left[\underline{p}, \overline{p}\right]$.

Clearly, if $NPV\left(p\right) \geq 0$ for all p the new venture dominates keeping money in the bank. What if $NPV\left(p'\right) > 0$ for some $p' \in \left[\underline{p}, \overline{p}\right]$ and $NPV\left(p''\right) < 0$ for some other $p'' \in \left[\underline{p}, \overline{p}\right]$? In this case, the alternatives 'start the project' or 'invest in the safe asset' are not comparable. For some probabilities of success the venture seems profitable, for others it does not.[5]

The framework of the previous section does not help resolve this decision problem. At the same time, it makes possible to to consider two opposite reactions to this situation. Some individuals are *decisive*. The new venture attracts them even though they know full well that, on purely rational grounds, it does not dominate keeping money in the bank. Other individuals are *cautious*. The new venture scares them exactly because it does not dominate keeping money in the bank. The first group we call entrepreneurs; the second, for reasons which will be apparent shortly, we call investors.

A decisive entrepreneur wants to start the project. Informally, we may think she has some *intuition* about the project that tells her to go ahead with it. Formally, this intuition can be represented by some probability $\hat{p}$ for which $NPV\left(\hat{p}\right) > 0$. In other words, the entrepreneur's intuition resolves the inability to decide. This is so because the project's 'intuitive' net present value is positive. While the question of what exactly determines $\hat{p}$ is very interesting, no conclusions in our model should depend on some particular value for it. By definition, one's intuition is not observable. All we observe, is that entrepreneurs start new ventures and other individuals do not. In principle, $\hat{p}$ does not necessarily need to be unique. That is, it could be that $\hat{p} \in \left[\underline{p}', \overline{p}'\right] \subset \left[\underline{p}, \overline{p}\right]$; this would not change the analysis that follows.

A cautious investor only start new projects whose net present value is positive for any probability of success. His behavior conforms to what Bewley [1986] states as *the inertia assumption*. Only alternatives which are preferred to some status quo are chosen. In our framework, the status quo is naturally represented by the safe alternative which yields interest rate r. Since $x_1 > x_0$, individuals obeying the inertia assumption start the project only if $NPV\left(\underline{p}\right) \geq 0$; their decision is 'triggered' by the lowest probability of success.

Summarizing, we divide individuals in two possible groups: entrepreneurs and non-entepreneurs. In a framework where there is Knightian uncertainty about a project's returns and investing or not are non comparable choices, we assume only entrepreneurs want to invest. Therefore entrepreneurs are characterized on purely behavioral grounds.

[5] Notice that if 'start the project' and 'keep money in the bank' are not comparable, we must have $NPV\left(\overline{p}\right) > 0$ and $NPV\left(\underline{p}\right) < 0$ (since $x_1 > x_0$).

What makes them different is not superior information nor optimism, but their drive to do "their own thing".

The remainder of the paper deals with the following question. If entrepreneurs are behaviorally different, what happens when, in their running the business, they face individuals who react to uncertainty differently? We focus on one particular situation: an entrepreneur willing to start a new venture does not have sufficient funds to cover the initial outlay. Therefore, she is decisive but has to borrow money from cautious investors.

2.3 The Financing Problem

The entrepreneur owns an investment opportunity with uncertain returns. This investment costs I and has two possible outcomes, R_1 and R_0. We assume $R_1 > R_0$. Therefore, we will refer to R_1 as success and to R_0 as failure. The project's income stream is uncertain. That is, the probabilities of the two outcomes are not uniquely determined. According to the model of the previous section, we let p denote the probability of success and assume it belongs to the interval $\left[\underline{p}, \overline{p}\right]$. The project's net present value(s) are given by

$$NPV\left(p\right) \equiv pR_1 + \left(1-p\right) R_0 - \left(1+r\right) I,$$

where $p \in \left[\underline{p}, \overline{p}\right]$. Two situations are possible, either $NPV\left(p\right)$ is positive for all p or $NPV\left(p\right)$ is negative for some p. In principle, we do not rule out either. Furthermore, we do not assume anything about the relationship between the project's revenues and its costs. We want to allow for the possibility that revenues are sufficient to cover $\left(1+r\right) I$ in both states; if this is the case, $NPV\left(p\right)$ is necessarily positive for all p. If $R_0 < \left(1+r\right) I$, $NPV\left(p\right)$ may be negative for some p.

In any case, the entrepreneur is decisive and would like to undertake the project. Her willingness to start the venture is expressed formally as $NPV\left(\hat{p}\right) > 0$ where, following the notation of the previous section, $\hat{p}$ is the entrepreneur's intuition about the project. We assume $\hat{p} \in \left(\underline{p}, \overline{p}\right)$ to rule out the case in which entrepreneur and investors wildly disagree on the project.

Notice that, even when the projects is valuable for cautious investors, the entrepreneur should still be the one running it. This simply follows from the observation that $NPV\left(\hat{p}\right) > NPV\left(\underline{p}\right)$. The entrepreneur's behavior is triggered by probability of success $\hat{p}$; invetsors behavior is triggered by $\underline{p}$, which is smaller than $\hat{p}$.

The entrepreneur owns some (perfectly liquid) personal wealth W which is smaller than the cost of the project, $W < I$.[6] Therefore, she needs to raise funds to start the venture. The amount she has to borrow depends on how much wealth she is willing to put in the venture (instead of the bank). If the entrepreneur decides to go ahead with this project, a firm is created. This firm is subject to limited liability.

The amount of self-financing, s, is given by all assets the entrepreneur formally makes part of the firm. Assets not used for self-financing, $W - s$, are invested in the safe opportunity and contribute $\left(W - s\right)\left(1+r\right)$ to the entrepreneur's expected wealth. Self-financing has in turn two possible uses. It may reduce the amount that must be borrowed, or it may be invested (by the firm) in the safe opportunity. In the latter case, the proceeds yield extra funds available as repayment to creditors, in addition to the project's returns.

[6] We could also assume the assets W are not perfectly liquid without altering the intuition or the results that follow.

We let w be self-financing towards reduced borrowing; $I - w$ then represents what must be borrowed; and $(s-w)(1+r)$ represents the sum available to pay creditors in excess of the project's returns.

The entrepreneur raises funds by means of a financial contract. This contract specifies amount borrowed and future payments to lenders. These depend on the outcome of the project and on the self-financing decisions of the entrepreneur. Formally, the entrepreneur borrows D and repays d_1 in case of success or d_0 in case of failure. Repayments cannot exceed the amount of money the project generates plus the proceed from the safe asset which are available to creditors.

The Knightian uncertainty associated with the project is thus transfered onto the contract's returns. Lenders, not being enrepreneurs, are cautious toward any uncertain investments, including the contract the entrepreneur offers them. Therefore, they accept this contract only if it yield no less than the safe investment for all probabilities of success. Formally, lenders accept the contract only if

$$pd_1 + (1-p)\,d_0 \geq (1+r)\,D \qquad \text{for all } p \text{ in } \left[\underline{p}, \overline{p}\right]$$

In Bewley's [1986] language, investors lend money to the entrepreneur only if the corresponding contract is preferred to their status quo, represented by the possibility to keep money in the bank and earn interest rate r.

3 Optimal Contracts between a Decisive Entrepreneur and Cautious Lenders

In this section, we look for the contract that maximizes the entrepreneur's expected wealth, given she and lenders have different attitudes towards the venture's uncertain returns. To illustrate some of the optimal contract's features, we first solve the case in which the entrepreneurs' wealth is zero, and then look at the more general case. Before that, we set up the general problem solved by optimal contracts.

The entrepreneur has to borrow money since she cannot sustain the venture's cost. Therefore, she subtracts the cost of obtaining funds from the project's net present value and then considers whether investing in the venture is still better than investing in the safe asset. This last option would yield $W(1+r)$.

Let $V(\hat{p})$ denote the entrepreneur's expected wealth when she undertakes the new venture; this is given by

$$\begin{aligned} V(\hat{p}) &\equiv \hat{p}R_1 + (1-\hat{p})\,R_0 - \{\hat{p}\,[d_1 - (s-w)(1+r)] + (1-\hat{p})\,[d_0 - (s-w)(1+r)]\} + (W-s)(1+r) \\ &= \hat{p}R_1 + (1-\hat{p})\,R_0 - [\hat{p}d_1 + (1-\hat{p})\,d_0] + (W-w)(1+r)\,. \end{aligned}$$

We write explicitly the dependence of expected wealth on the entrepreneur's intuition to remind ourselves that results may depend on its value. In the following, we will prove that the optimal contract between a decisive entrepreneur and cautious lenders does not depend on the entrepreneur's intuition.

An optimal contract maximizes expected wealth. Having solved for it, the entrepreneur considers wether starting up the firm is still better than keeping money in the bank. That is, she considers whether the project's income net of financing cost still justifies going ahead with the venture. This condition can be expressed formally as

$V(\hat{p}) \geq W(1+r)$. Notice that this calculation uses the project's intuitive net present value defined previously.

An optimal contract solves the problem

$$\max_{s,w,d_1,d_0} V(\hat{p})$$

subject to

$$d_1 \leq R_1 + (s-w)(1+r) \quad (3.2)$$

$$d_0 \leq R_0 + (s-w)(1+r) \quad (3.3)$$

$$D(1+r) \leq pd_1 + (1-p)d_0 \qquad \text{for all } p \text{ in } \left[\underline{p}, \overline{p}\right] \quad (3.4)$$

$$0 \leq s \leq W$$

$$0 \leq w \leq s$$

$$D = I - w$$

Constraints (3.2) and (3.3) represent how much can the entrepreneur promise to pay in case the projects succeeds or fails respectively; constraint (3.4) represents the lenders' participation constraint. If we let the four-tuple (s^*, w^*, d_1^*, d_0^*) denote a solution to this problem, and the corresponding expected wealth be $V^*(\hat{p})$, the entrepreneur prefers the venture to the safe opportunity if and only if $V^*(\hat{p}) \geq W(1+r)$.

3.1 Decisive but Poor Entrepreneurs

Here, we assume the entrepreneurs owns no assets. In this case, the problem the entrepreneur faces is much simpler since $W = s = w = 0$. An optimal contract then solves

$$\max_{d_1,d_0} V(\hat{p}) = \hat{p}R_1 + (1-\hat{p})R_0 - [\hat{p}d_1 + (1-\hat{p})d_0]$$

subject to

$$d_1 \leq R_1$$

$$d_0 \leq R_0$$

$$I(1+r) \leq pd_1 + (1-p)d_0 \qquad \text{for all } p \text{ in } \left[\underline{p}, \overline{p}\right]$$

The entrepreneur then invests in the venture if her expected wealth is positive. That is, she invests if the condition $V^*(\hat{p}) \geq 0$ is satisfied. First, we characterize the solution to the optimal contracting problem. Then, we look at the decision to start the venture, after considering the optimal financial contract that insues.

The optimal contract is easy to characterize. A payment equal to the minimum between failure revenues and $I(1+r)$ is made when the project fails. When the project succeeds, lenders receive the smallest amount required to keep them in the contract. Formally, these statements are summarized as follows.

Proposition 3.1 *If an entrepreneur who owns no assets decides to invest, the unique optimal contract is given by*

$$d_0^* = \min\{R_0, I(1+r)\},$$

$$d_1^* = \begin{cases} \frac{1}{\underline{p}} I(1+r) & if \quad R_0 < I(1+r) \\ I(1+r) & if \quad R_0 \geq I(1+r) \end{cases}$$

Proof.

At an optimum, the lenders' participation constraint computed according to the probability that yields the payments' lowest expected value must bind. If not, the entrepreneur is being unnecessarily generous. Formally, let $\tilde{d}_1$ and $\tilde{d}_0$ be a candidate optimal contract; then, we have

$$I\left(1+r\right)=\tilde{p}\tilde{d}_1+\left(1-\tilde{p}\right)\tilde{d}_0, \tag{3.5}$$

where $\tilde{p}$ is given by

$$\tilde{p}\in\arg\min_{p\in\left[\underline{p},\overline{p}\right]}p\tilde{d}_1+\left(1-p\right)\tilde{d}_0.$$

Note that $\tilde{p}=\underline{p}$ if $\tilde{d}_0<\tilde{d}_1$, and $\tilde{p}=\overline{p}$ if $\tilde{d}_0>\tilde{d}_1$. The probability corresponding to a contract's lowest expected payment is determined by having the largest weight on the smallest payment. This probability is crucial in the eyes of the entrepreneur since it triggers lenders behavior.

Rewriting equation (3.5) as

$$\tilde{d}_1=\frac{1}{\tilde{p}}\left[I\left(1+r\right)-\tilde{d}_0\right]+\tilde{d}_0 \tag{3.6}$$

and substituting in the objective function, one has

$$\begin{aligned}V\left(\hat{p}\right) &= \hat{p}R_1+\left(1-\hat{p}\right)R_0-\left[\frac{\hat{p}}{\tilde{p}}\left[I\left(1+r\right)-\tilde{d}_0\right]+\hat{p}\tilde{d}_0+\left(1-\hat{p}\right)d_0\right]\\ &= \hat{p}R_1+\left(1-\hat{p}\right)R_0-\frac{\hat{p}}{\tilde{p}}I\left(1+r\right)-\left[1-\frac{\hat{p}}{\tilde{p}}\right]\tilde{d}_0. \end{aligned} \tag{3.7}$$

The sign of the term in square bracket depends on the relation between $\tilde{d}_0$ and $\tilde{d}_1$. If $\tilde{d}_0>\tilde{d}_1$, $\tilde{p}=\overline{p}$ and $\frac{\hat{p}}{\tilde{p}}<1$; in this case the entrepreneur can profit by reducing $\tilde{d}_0$. Therefore, at an optimum $\tilde{d}_0\leq\tilde{d}_1$ must hold. If $\tilde{d}_0<\tilde{d}_1$, $\tilde{p}=\underline{p}$ and $\frac{\hat{p}}{\tilde{p}}>1$; in this case the entrepreneur can profit by increasing $\tilde{d}_0$. We conclude that if the safe contract is feasible, it is also optimal. If the safe contract is not feasible, it must be because $R_0<I\left(1+r\right)$ since we assumed $R_0<R_1$. In this case, the payment when the project fails is as large as possible. Summarizing,

$$d_0^*=\min\left\{R_0,I\left(1+r\right)\right\}$$

as claimed. The corresponding payment when the project succeds, d_1^*, is then obtained using equation (3.6). ■

Proposition 3.1 says that when starting the new venture is profitable for the entrpreneur, the corresponding unique optimal contract is debt-like; furthermore safe debt is optimal whenever it is feasible.

This follows because lenders' behavior when facing uncertainty is different from the entrepreneur's. As noted previously, lenders' attitude towards the contract is triggered by their lowest expected value of payments. The trigger is given by $\tilde{p}\in\arg\min_{p\in\left[\underline{p},\overline{p}\right]}pd_1+\left(1-p\right)d_0$. The entrepreneur's perception of the cost of this contract, on the other hand, is determined by her intuition $\hat{p}$. Any contract with $d_1\neq d_0$ induces a wedge between these two probabilities. This wedge is positive when lenders receive more in the failure state and negative in the opposite case. In the first case, the entrepreneur's intuition tells her the failure state is less likely than what the lenders trigger probability implies. Therefore, if a contract does not pays as much as possible in that state it is more costly than necessary. In the second case, the opposite reasoning applies; a contract which does not pay as much as possible in the success state is more costly than necessary.

These observations can be made formally. For any borrowed amount D, an optimal contract satisfies

$$D(1+r) = \tilde{p}d_1 + (1-\tilde{p})\, d_0.$$

The expected cost of financing is then given by

$$\begin{aligned}
\hat{p}d_1 + (1-\hat{p})\, d_0 &= \frac{\hat{p}}{\tilde{p}}\left[D(1+r) - (1-\tilde{p})\, d_0\right] + (1-\hat{p})\, d_0 \\
&= \frac{\hat{p}}{\tilde{p}} D(1+r) - \left(\frac{\hat{p}}{\tilde{p}} - \hat{p}\right) d_0 + (1-\hat{p})\, d_0 \\
&= \frac{\hat{p}}{\tilde{p}}\left[D(1+r) - d_0\right] + d_0.
\end{aligned}$$

If $d_1 < D(1+r) < d_0$, $\tilde{p} = \overline{p}$ and the expected cost of the contract is increasing in d_1. If $d_1 > D(1+r) > d_0$, $\tilde{p} = \underline{p}$ and this cost is decreasing in d_0.[7] Therefore, if feasible, a safe contract is always optimal. Intuitively, the entrepreneur looks for the 'flatter' (less diversified) payment scheme available. Diversification is unnecessarily costly in this environment because cautious lenders require an uncertainty premium to provide funds.

This result is very different from what one obtains in a framework where there is no uncertainty and the entrepreneur is 'optimistic' about the project's outcome. In that case, she would assess a larger probability of success and try to maximize the payment to lenders in the failure state. The unpleasant implication of this story is that we should never observe safe contracts, even if they are feasible. In this situation, our model uniquely predicts safe debt as the contract of choice. This issue is relevant since feasibility of the safe contract implies that the project's net present value is positive for all probabilities of success. According to either model, entrepreneurs are the ones valuing the project most and so should be the ones carrying it out. Then the conclusion that safe debt is never observed in an optimistic entrepreneur model is disturbing, particularly if optimism is thought of as a character trait of entrepreneurs. Unless, of course, we believe another character trait is that entrepreneurs never invest in projects which have a positive net present value for them as well as lenders.

After examining the optimal contract between a decisive enrtepreneurs and cautious lenders, we are now ready to check if, given this contract, the entrepreneur is willing to start the new venture. In a nutshell, the issue is whether the cost of financing is low enough to keep the venture profitable. The result is that the entrepreneur starts the new venture only if its net present value is positive for all probabilities of success.

[7] Notice that in the first case, the cost of financing depends positively on $\overline{p}$, while in the second it depends positively on $\underline{p}$. In a loose sense, the cost of funds increases with the length of the interval $\left[\underline{p}, \overline{p}\right]$. This length is a rough measure of the degree of cautiousness of lenders. Higher cautiousness implies higher financing costs. This result will play an important role when we let the entrepreneur choose between lenders with different cautiousness (section ????).

Proposition 3.2 *Given the optimal contract derived in proposition 3.1, the entrepreneur starts the new venture only if its lowest net present value according to cautious lenders is positive. That is, she starts the venture only if* $NPV\left(\underline{p}\right) \geq 0$.

Proof.

Substituting the optimal contract in equation (3.7), we compute the entrepreneur's expected wealth if she undertakes the venture. This is

$$V^*(\hat{p}) = \hat{p}R_1 + (1-\hat{p})R_0 - \frac{\hat{p}}{\tilde{p}}I(1+r) - \left[1 - \frac{\hat{p}}{\tilde{p}}\right]\min\{R_0, I(1+r)\};$$

notice that, since $d_0^* \leq d_1^*$, we can simplify this expression by letting $\tilde{p} = \underline{p}$:

$$V^*(\hat{p}) = \hat{p}R_1 + (1-\hat{p})R_0 - \frac{\hat{p}}{\underline{p}}I(1+r) - \left[1 - \frac{\hat{p}}{\underline{p}}\right]\min\{R_0, I(1+r)\}.$$

Therefore, we have

$$V^*(\hat{p}) = \begin{cases} \frac{\hat{p}}{\underline{p}}\left[\underline{p}R_1 - \left(1-\underline{p}\right)R_0 - I(1+r)\right] & if \quad R_0 \leq I(1+r) \\ \hat{p}R_1 + (1-\hat{p})R_0 - I(1+r) & if \quad R_0 > I(1+r) \end{cases}$$

which is more simply written as

$$V^*(\hat{p}) = \begin{cases} \frac{\hat{p}}{\underline{p}}NPV\left(\underline{p}\right) & if \quad R_0 \leq I(1+r) \\ NPV(\hat{p}) & if \quad R_0 > I(1+r) \end{cases}$$

In the first case, the sign of $V^*(\hat{p})$ is equal to the sign of $NPV\left(\underline{p}\right)$. Hence, the entrepreneur loses money in expectation whenever $NPV\left(\underline{p}\right) < 0$. In the second case, $V^*(\hat{p})$ is positive and so is $NPV\left(\underline{p}\right)$ since $NPV(p) \geq 0$ for all p in this case. This proves that the entrepreneur starts the new venture if and only if $NPV\left(\underline{p}\right) \geq 0$. ■

The conclusion of Proposition 3.2 is negative. Decisive entrepreneurs who are poor do not start a new business unless cautious lenders agree the endeavor is worth it. Cautiousness implies such a high financing cost that the entrepreneur does not have enough left for herself. Once the cost of financing is taken into account, the venture is not worth it anymore. Entrepreneurial attitude is not enough. This leads us to analyze the case in which the entrepreneurs assets are positive.

3.2 Decisive and Rich Entrepreneurs

We now return to the more general case in which the entrepreneur's wealth can be used to partially self finance the venture. An optimal contract solves the problem

$$\max_{s,w,d_1,d_0} V(\hat{p}) = \hat{p}R_1 + (1-\hat{p})R_0 - [\hat{p}d_1 + (1-\hat{p})d_0] + (W-w)(1+r)$$

subject to

$$\begin{aligned} d_1 &\leq R_1 + (s-w)(1+r) \\ d_0 &\leq R_0 + (s-w)(1+r) \\ (I-w)(1+r) &\leq pd_1 + (1-p)d_0 \qquad \text{for all } p \text{ in } \left[\underline{p}, \overline{p}\right] \\ 0 &\leq s \leq W \\ 0 &\leq w \leq s \end{aligned}$$

Given the solution to this problem, the entrepreneur starts the new venture if her expected wealth is larger than $W(1+r)$.

At the optimum, the entrepreneur uses all her wealth as self financing. Having done this, reducing borrowing or increasing future payments makes no difference to her. Because of this, the optimal contract is partially unspecified. It makes the largest possible payment when the project fails if a safe contract is not feasible.

When all self financing goes into reducing borrowing, the optimal contract resembles a share. If some self financing goes into future payments, the optimal contract resembles a bond, since it makes the largest possible payment when the project fails. In any case, if the entrepreneur's wealth is large enough, she undertakes the venture even if its lowest net presented value is negative. The following proposition makes these points formally.

Proposition 3.3 *The optimal contract is given by*

$$d_0^* = \min\left\{R_0 + s^*(1+r), I(1+r)\right\} - w(1+r)$$

$$d_1^* =$$

$$d_1^* = \begin{cases} \frac{1}{\underline{p}}(I-W)(1+r) + (W-w^*)(1+r) & if \quad R_0 + W(1+r) \leq I(1+r) \\ (I-w^*)(1+r) & otherwise \end{cases}$$

$$0 \leq w^* \leq s^*$$

$$s^* = W \quad if \quad R_0 + W(1+r) \leq I(1+r), \quad and \quad I - R_0(1+r)^{-1} \leq s^* \leq W \quad otherwise$$

Proof.

First, we obtain the optimal contract assuming the entrepreneur decided to start the project; then, we describe when doing so is optimal, considering the optimal contract.

The lenders' participation constraint binds when computed according to the probability that yields the lowest expected value of payments. If not, the entrepreneur is being unnecessarily generous. Formally, if we let $\tilde{w}, \tilde{s}, \tilde{d}_1, \tilde{d}_0$ be a candidate optimal contract, we have

$$(I-\tilde{w})(1+r) = \tilde{p}\tilde{d}_1 + (1-\tilde{p})\tilde{d}_0, \tag{3.8}$$

where $\tilde{p}$ is given by

$$\tilde{p} = \arg\min_{p\in[\underline{p},\overline{p}]} p\tilde{d}_1 + (1-p)\tilde{d}_0.$$

This is the probability that triggers lenders behavior. Note that $\tilde{p} = \underline{p}$ if $\tilde{d}_0 < \tilde{d}_1$, and $\tilde{p} = \overline{p}$ if $\tilde{d}_0 > \tilde{d}_1$.

Rewriting equation (3.8) as

$$\tilde{d}_1 = \frac{1}{\tilde{p}}\left[(I-\tilde{w})(1+r) - \tilde{d}_0\right] + \tilde{d}_0 \tag{3.9}$$

and substituting in the objective function, one has

$$\begin{aligned} V(\hat{p}) &= \hat{p}R_1 + (1-\hat{p})R_0 - \left[\frac{\hat{p}}{\tilde{p}}\left[(I-\tilde{w})(1+r) - \tilde{d}_0\right] + \hat{p}\tilde{d}_0 + (1-\hat{p})\tilde{d}_0\right] + (W-\tilde{w})(1+r) \\ &= \hat{p}R_1 + (1-\hat{p})R_0 - \left[\frac{\hat{p}}{\tilde{p}}(I-\tilde{w})(1+r) + \left(1-\frac{\hat{p}}{\tilde{p}}\right)\tilde{d}_0\right] + (W-\tilde{w})(1+r). \end{aligned}$$

The sign of the term in square bracket depends on the relation between $\tilde{d}_0$ and $\tilde{d}_1$. If $\tilde{d}_0 > \tilde{d}_1$, $\tilde{p} = \overline{p}$ and $\frac{\hat{p}}{\tilde{p}} < 1$; in this case the entrepreneur can profit by reducing $\tilde{d}_0$. Therefore, at an optimum $\tilde{d}_0 \leq \tilde{d}_1$ must hold. If $\tilde{d}_0 < \tilde{d}_1$, $\tilde{p} = \underline{p}$ and $\frac{\hat{p}}{\tilde{p}} > 1$; in this case the entrepreneur can profit by increasing $\tilde{d}_0$. We conclude that if the safe contract is feasible, it is also optimal. If the safe contract is not feasible, it must be because $R_0 + (\tilde{s} - \tilde{w})(1+r) < (I - \tilde{w})(1+r)$, since we assumed $R_0 < R_1$. In this case, the payment when the project fails is as large as possible. Summarizing,

$$\tilde{d}_0 = \min\{R_0 + (\tilde{s} - \tilde{w})(1+r), (I - \tilde{w})(1+r)\} = \min\{R_0 + \tilde{s}(1+r), I(1+r)\} - \tilde{w}(1+r); \tag{3.10}$$

since $\tilde{d}_0 \leq \tilde{d}_1$, one can substitute $\tilde{p}$ with $\underline{p}$ and obtain the following expression for expected wealth:

$$\begin{aligned} V(\hat{p}) &= \hat{p}R_1 + (1-\hat{p})R_0 - \left[\frac{\hat{p}}{\underline{p}}(I - \tilde{w})(1+r) + \left(1 - \frac{\hat{p}}{\underline{p}}\right)[\min\{R_0 + \tilde{s}(1+r), I(1+r)\} - \tilde{w}(1+r)]\right] + \\ &= \hat{p}R_1 + (1-\hat{p})R_0 - \frac{\hat{p}}{\underline{p}}I(1+r) + \left(\frac{\hat{p}}{\underline{p}} - 1\right)\min\{R_0 + \tilde{s}(1+r), I(1+r)\} + W(1+r). \end{aligned}$$

From this, we see that $\tilde{w}$ disappears and thus its value does not affect the entrepreneur's expected wealth. A second conclusion deals with $\tilde{s}$. The coefficient in front of $\min\{\cdot\}$ is positive; hence, increasing $\tilde{s}$ benefits the entrepreneur, as longs as $R_0 + \tilde{s}(1+r) \leq I(1+r)$. In the opposite case, $\tilde{s}$ drops out. Therefore, we conclude

$$\begin{array}{ll} \tilde{s} = W & if \quad W(1+r) + R_0 \leq I(1+r) \\ I - R_0(1+r)^{-1} \leq \tilde{s} \leq W & otherwise \end{array} \tag{3.12}$$

The first case says the entrepreneur uses all her wealth to self-finance the project. The second says that after using enough wealth as self-finance to make the contract safe, what the entrepreneur does with the rest of it has no effect; whether she keeps it inside or outside the firm, it can only be invested in the safe asset. The payments are then computed using (3.12) in equations (3.9) and (3.10). ■

Proposition 3.3 shows that maximum self-financing is optimal. An example clarifies the intuition of this result. Keeping the amount borrowed $I - w$ fixed, consider the effect of an extra dollar 'inside' the firm. It increases the expected cost of the contract by $\left(1 - \frac{1}{\underline{p}}\right)(1+r)$ in the success state and $(1+r)$ in the failure state. The overall effect, therefore, is a cost reduction equal to $\frac{\hat{p}}{\underline{p}}(1+r)$. If this dollar is left 'outside' the firm, it earns only $(1+r)$. Since $\frac{\hat{p}}{\underline{p}} > 1$, the entrepreneur uses all her assets to self finance the project. Quite obviously, this effect disappears if a safe contract is feasible. Essentially, when the contract is not safe, assets used as self-finance can be used to make the contract flatter, and we have seen that a flatter contract is less costlty.

The optimal contract is unaffected by the trade-off between reducing in borrowing and increasing ability to repay. This can be seen by looking at its expected cost, given by

$$\hat{p}d_1^* + (1-\hat{p})d_0^* = \hat{p}\left[\frac{(I-w)(1+r)}{\underline{p}} + \left(1 - \frac{1}{\underline{p}}\right)(s-w)(1+r)\right] + (1-\hat{p})(s-w)(1+r).$$

Keeping self financing s fixed, suppose borrowing is reduced by one dollar by increasing w. On one hand, there is a cash in the bank effect; the firm loses $(1+r)$ in both states. On the other hand, there is a reduced payments effect; the firm gains $\frac{1}{\underline{p}}(1+r)+\left(1-\frac{1}{\underline{p}}\right)(1+r)$ in case of success and $(1+r)$ in case of failure. Overall, these two effects exactly compensate each other. This equivalence result depends on the assumption that the entrepreneur's assets are like cash: perfectly liquid and irrelevant to the project's net present value. If these assets are essential to the project, their liquidation may negatively affect this value. Then, reducing borrowing is dominated by increasing future payments. One example is the case in which W represents a building the entrepreneur uses to develop the venture. Having to move to a smaller space, might reduce the revenues generated by the project in case of success.

After examining the optimal contract between a decisive enrtepreneurs and cautious lenders, we are now ask if, given this contract, the entrepreneur is willing to start the new venture. As before, the issue is whether the cost of financing is low enough to keep the venture profitable. The entrepreneur starts the new venture only if its her wealth is above some value which depends on lenders lowest net present value. When this is positive, the wwealth requiement is not necessary.

Proposition 3.4 *Given the optimal contract derived in proposition 3.3, the entrepreneur starts the new venture only if*

$$W \geq -\frac{\hat{p}}{\hat{p}-\underline{p}}\left[NPV\left(\underline{p}\right)\right](1+r)^{-1}$$

Proof.

We can compute the entrepreneur's expected wealth when she starts the project substituting (3.12) in (3.11):

$$\begin{aligned} V^*(\hat{p}) &= \hat{p}R_1+(1-\hat{p})R_0-\frac{\hat{p}}{\underline{p}}I(1+r)+\left(\frac{\hat{p}}{\underline{p}}-1\right)\min\left\{R_0+s^*(1+r),I(1+r)\right\}+W(1+r) \\ &= \begin{cases} \frac{\hat{p}}{\underline{p}}\left[\underline{p}R_1+\left(1-\underline{p}\right)R_0-I(1+r)+W(1+r)\right] & if \quad W(1+r)+R_0\leq I(1+r) \\ \hat{p}R_1+(1-\hat{p})R_0-I(1+r)+W(1+r) & otherwise \end{cases} \end{aligned}$$

which is more simply written as

$$V^*(\hat{p})=\begin{cases} \frac{\hat{p}}{\underline{p}}\left[NPV\left(\underline{p}\right)+W(1+r)\right] & if \quad W(1+r)+R_0\leq I(1+r) \\ NPV(\hat{p})+W(1+r) & otherwise \end{cases}$$

She starts the venture only if doing so is better than the safe opportunity,

$$V^*(\hat{p})\geq W(1+r).$$

In our case, this condition is reduces to:

$$\begin{array}{ll} \frac{\hat{p}}{\underline{p}}\left[NPV\left(\underline{p}\right)+W(1+r)\right]\geq W(1+r) & if \quad W(1+r)+R_0\leq I(1+r) \\ NPV(\hat{p})+W(1+r)\geq W(1+r) & otherwise \end{array}$$

or

$$\begin{array}{ll} W \geq -\frac{\hat{p}}{\hat{p}-\underline{p}} NPV\left(\underline{p}\right)(1+r)^{-1} & if \quad W(1+r)+R_0 \leq I(1+r) \\ NPV(\hat{p}) \geq 0 & otherwise \end{array}$$

The second condition is always satisfied. The first one, on the other hand, establishes the lower bound on wealth which makes undertaking the venture profitable. ■

Proposition 3.4 shows that an entrepreneur who owns enough wealth can profitably finance a new venture. In particular, given the project's return when successful, and the lowest probability of success, we can define the minimum amount of wealth necessary to undertake the project as

$$\underline{W} \equiv -\frac{\hat{p}}{\hat{p}-\underline{p}} NPV\left(\underline{p}\right)(1+r)^{-1}$$

$\underline{W}$ is negative if the lowest net present value of the project is positive. Intuitively, if lenders see the project as profitable, the firm does not need collateral. $\underline{W}$ is positive if the lowest net present value of the project is negative. If lenders do not see the project as profitable, the entrepreneur needs collateral to make the loan safer. In this case, entrepreneurs without assets are forced to pass up an opportunity they deem profitable. $\underline{W}$ depends negatively on the lower probability of success. The entrepreneur needs less wealth when investors are more optimistic about the project's success.

4 Different Sources of Funds

In the previous section, we analyzed optimal financing decisions of an entrepreneur faced by cautious lenders. In this section, we ask how would an entrepreneur choose among fund providers with different degree of cautiousness. We assume less cautious lenders can impose an additional cost of funds. One can think of them as professional investors, experts in providing funds. The entrepreneur has two possible sources of funds, professional investors and lenders. She can choose to use both, or only one of them.

We assume professional investors face less Knightian uncertainty than lenders. This might follow from their expertise in evaluating firms' investment opportunities. From the entrepreneur's point of view, this expertise has two effects. It is advantageous because, as showed previously, the expected cost of a contract depends positively on lenders' cautiousness. On the other hand, it is harmful because professionals require a higher rate of return on investment. The trade-off between these two forces determines the entrepreneur's decision to obtain funds from professionals or from lenders.

The main result of this section states that, if both sources of funds are used, they receive contracts with a radically different payment structure. In particular, when the project fails lenders receive all the entrepreneur's revenues while professionals receive nothing. The contract for professionals looks like a partnership in which they get a share of the proceeds of the venture. The contract for lenders looks like a standard debt contract. The entrepreneur prefers professionals as partners because they require a smaller share of the revenues in case of success. This is because, professionals are more similar to her, and are more confident than lenders of the outcome of the venture.

Let D' be the amount of money the entrepreneur borrows from a professional, and let b_1 and b_0 be the payment scheme used to repay the loan. This investor must be compensated at a premium over the safe interest rate. To keep things simple, I assume this premium is fixed independently of the borrowed amount. The contract payments display Knightian uncertainty and professionals are cautious. Formally, professional investors accept the contract only if

$$pb_1 + (1-p)\,b_0 \geq (1+r)\,B + C \qquad \text{for all } p \text{ in } \left[\underline{p}', \overline{p}'\right]$$

Professional investors are faced with Knightian uncertainty as lenders, but to a smaller extent. In other words, we assume they are less cautious in deciding whether to finance a project whose returns are uncertain. We can model this situation with the following.

Assumption 4.1 *The probability interval of professional investors is smaller than the probability interval of lenders, and it contains the entrepreneur's intuition; that is:*

$$\underline{p} < \underline{p}' < \hat{p} < \overline{p}' < \overline{p}$$

4.1 Optimal Contracts with Different Sources of Funds

Except for the presence of the professional, the model is the same as in the previous section. Now, though, the entrepreneur has one more option. She may choose whether to get funds from the lenders we model in last section, or from a professional investor.

The set up is as follows. The entrepreneur's financing needs can be covered in different ways: self-financing as debt reduction w, borrowing from the lenders D, borrowing from a professional B. We assume he has limited wealth, denoted $\overline{B}$, and assume $\overline{B} < I - W$. The combined wealth of the entrepreneur and the professional is not sufficient to cover the project's cost.

The entrepreneur needs to borrow to complete the project. She chooses contracts with lenders and a professional investor to maximize her expected wealth, given with each party's participation constraint and the limited liability constraints. Formally, entrepreneur's the expected wealth is given by:

$$V(\hat{p}) = \hat{p}R_1 + (1-\hat{p})\,R_0 - \left[\hat{p}\,(d_1 + b_1) + (1-\hat{p})\,(d_0 + b_0)\right] + (W - w)\,(1+r)$$

The optimal contract solves:

$$\begin{aligned}
&\max_{s,w,D,B,d_1,b_1,d_0,b_0} V(\hat{p}) \\
&\text{subject to} \\
&d_0 + b_0 \leq R_0 + (s-w)\,(1+r) \\
&d_1 + b_1 \leq R_1 + (s-w)\,(1+r) \\
&D\,(1+r) \leq pd_1 + (1-p)\,d_0 \qquad \text{for all } p \text{ in } \left[\underline{p}, \overline{p}\right] \\
&B\,(1+r) \leq pb_1 + (1-p)\,b_0 \qquad \text{for all } p \text{ in } \left[\underline{p}', \overline{p}'\right] \\
&D + B = I - w \\
&0 \leq s \leq W \\
&0 \leq w \leq s \\
&0 \leq B \leq \overline{B}
\end{aligned}$$

A full characterization of the solution to this problem is not easy. In the following, we summarize its main features. Similarly to the previous section, self financing is optimal, while using it to reduce borrowing or increase possible payments is irrelevant. If the premium charged by the professional is too high, he is not used as a source of funds and the optimal contract is the one described in Proposition 3.3. If the premium charged by the professional is low and he has enough wealth, he is used as the only source of funds. If the premium charged by the professional is low and he does not have enough wealth, both sources of funds are used. In this case, the entrepreneur borrows as much as possible from the professional before using the lenders. The payment to the two professional when the project fails is in this case zero. Formally, the solution to the entrepreneur's problem is characterized as follows.

Proposition 4.2 *(B): If $\overline{B}+W<I$ and $\overline{B}\geq\frac{\underline{p}}{\underline{p}'-\underline{p}}C\left(1+r\right)^{-1}$, the optimal contract is:*

$$\begin{aligned}
s^* &= W \\
B^* &= \overline{B} \\
W &\geq w^* \geq 0 \\
D^* &= I-s-\overline{B} \\
b_0^* &= 0 \\
b_1^* &= \frac{\hat{p}}{\underline{p}'}\overline{B}\left(1+r\right)+\frac{\hat{p}}{\underline{p}'}C \\
d_0^* &= \left(W-w^*\right)\left(1+r\right) \\
d_1^* &= \frac{1}{\underline{p}}\left(I-w^*-\overline{B}\right)\left(1+r\right)+\left(1-\frac{1}{\underline{p}}\right)\left(W-w^*\right)\left(1+r\right)
\end{aligned}$$

and the entrepreneur's expected wealth is:

$$V^B=\hat{p}R-\frac{\hat{p}}{\underline{p}}\left(I-W\right)\left(1+r\right)-\left(\frac{\hat{p}}{\underline{p}'}-\frac{\hat{p}}{\underline{p}}\right)\overline{B}\left(1+r\right)-\frac{\hat{p}}{\underline{p}'}C$$

The proof of this result is rather involved, and is relegated to the Appendix. Here, we prefer to focus on the case in which the firm must use lenders and the professional to gain some intuition about the contract in (B). The details of the two contracts are not entirely specified, but they have a distinctive feature. All that is available in the failure state goes to lenders. The professional always makes as much as the entrepreneur in that state. One can conclude the professional is always treated like a partner. He gets a share of the profits the venture makes, after deducting the cost of funds provided by lenders. The reason for this result lies in the intuition about the nature of the optimal contract between the entrepreneur and lenders described in the previous section. There, we argued that the entrepreneur is betting with fund providers on the likelihood of success. Here, there are two types of providers. Both underestimate the likelihood of success, but lenders do it more than professionals. Therefore, the entrepreneur pays them the smallest possible amount in that state; this corresponds to paying them the largest possible amount in the failure state. Relative to lenders, professionals agree more with the entrepreneur on the project's probability of success. Hence, they receive a contracts whose payment stream resemble the entrepreneur's revenues.

5 Conclusions

We presented a model of investment financing that explicitly captures some ideas about entrepreneurship. In particular, an entrepreneur's favorable attitude towards new ventures is explicitly captured by assuming she is willing to invest when nobody else would. If this is true, the entrepreneur is also different from possible providers of funds. Knightian uncertainty, allowing a distinction between cautiousness and decisiveness, enables us to account for this difference in a natural way. The entrepreneur acts decisively, while fund providers act cautiously.

The main conclusion is that attitude is not enough to make a project viable. Some cash is also needed. When this is available, the entrepreneur uses all of it as means of self financing. In this setting, we also derived some familiar corporate finance results like the equivalence between reduction in borrowing and increase in future payments possibilities. We also considered the case in which the entrepreneur has access to fund providers with differing attitudes about the project. In some instances, she uses all providers, but strikes very different deals with them. Less cautious lenders receive a contract that resembles a share of the project. More cautious lenders receive a contract resembling debt.

Among the possible extensions of our model, a more sophisticated way of modelling this case seems the most pressing one. Our assumptions about the interaction between a professional less cautious investor and an ordinary lender are quite simplistic. For example, can the professional investor solicit funds directly from the lenders? In our model, this cannot have any impact; among other reasons, this also depends on the fixed premium the professional charges the entrepreneur. One can imagine a professional's cost of fund dependent on possible deposits, and thus the professional attempting to be a fully fledged intermediary.

The results we derived are important not only because they mirror some observed patterns of corporate financing, but also because of the simplicity of the model used to generate them. In a sense, the fact the model can deliver results similar to the standard literature is, we believe, among its strength. The standard paradigm in modern corporate finance is based on the idea of conflict of interests. A firm's manager is not interested in maximizing the probability a project succeeds unless some incentives are provided for him to do so. This story does not seem appropriate for entrepreneurs. Their desire to succeed is often quoted as a defining characteristics of entrepreneurial behavior. If this is the case, the standard paradigm does not apply. Our paper is an attempt to complement it with a description of entrepreneurial behavior more similar to reality.

Appendix

Proof of Proposition 4.2. To find the optimal contract, we proceed as follows. First, we write the optimal payments to lenders and to the professional when the project succeeds as a function of the payment to the professional when the project fails. With this, we derive a formula for the expected cost of the contract where the latter is the only payment considered. Second, we use this formula to obtain the optimal value of the payment when the project fails as a function of the amount borrowed from the various sources and the self-financing choices. Third, we consider the optimal choice of these variables to characterize the optimal contract. Fourth, we establish conditions for the entrepreneur to prefer this contract to the one without the professional's funds. The first two steps are provided as lemmas.

At the optimum, the participation constraints for the professional and the lenders bind for the lowest expected value of the received payments. Using this result, one can write the expected cost of the contract as a function of the different amounts borrowed and the payment to the professional in state 0. The optimal value of this payment is then derived as a function of the entrepreneur's other choice variables. These results are summarized by the following lemmas.

Lemma 5.1 *Assume $B > 0$. Then, at an optimum, the payment the firm has to make to the public and to the professional in state 1 can be written as*

$$\begin{aligned} d_0 &= \min\left\{(s-w) - b_0\,(1+r)^{-1}, D\right\}(1+r) \\ d_1 &= \tfrac{1}{\underline{p}}D\,(1+r) + \left(1-\tfrac{1}{\underline{p}}\right)\min\left\{(s-w) - b_0\,(1+r)^{-1}, D\right\}(1+r) \end{aligned} \tag{5.13}$$

$$b_1 = -\frac{1}{\tilde{p}'}\,(1-\tilde{p}')\,b_0 + \frac{1}{\tilde{p}'}B^i\,(1+r) + \frac{1}{\tilde{p}'}C \tag{5.14}$$

Hence, the expected cost of an optimal financial contract

$$EC \equiv \hat{p}\,(d_1+b_1) + (1-\hat{p})\,(d_0+b_0)$$

is given by

$$\begin{aligned} EC &= \hat{p}\left(\frac{1}{\underline{p}}\,(I-w)\,(1+r) + \left(\frac{1}{\tilde{p}'} - \frac{1}{\underline{p}}\right)B\,(1+r) + \frac{1}{\tilde{p}'}C\right) + \left(1-\frac{\hat{p}}{\tilde{p}'}\right)b_0 \\ &\quad + \left(1-\frac{\hat{p}}{\underline{p}}\right)\min\left\{(s-w) - b_0\,(1+r)^{-1}, (I-w-B)\right\}(1+r) \end{aligned}$$

Proof.

At the optimum, the participation constraint for the lenders binds for the lowest expected value of the received payments. If not, the firm is being unnecessarily generous. Let $\tilde{p}$ be the probability of success that yields this expectation. Hence:

$$\begin{aligned} D\,(1+r) &= \tilde{p}d_1 + (1-\tilde{p})\,d_0 \\ d_1 &= \frac{1}{\tilde{p}}D\,(1+r) - \frac{1}{\tilde{p}}d_0 + d_0 \end{aligned}$$

and the expected cost to the firm of the contract with the lenders is given by:

$$\hat{p}d_1 + (1-\hat{p})\, d_0 = \frac{\hat{p}}{\tilde{p}} D\,(1+r) + \left(1 - \frac{\hat{p}}{\tilde{p}}\right) d_0$$

Depending on the relative magnitude of the payments the professional makes, one needs to distinguish three cases: $d_0 > d_1$, $d_0 < d_1$, and $d_0 = d_1$. We claim that choosing any $d_0 > d_1$ is not optimal. In that case, $\tilde{p} = \overline{p}$ and the coefficient multiplying d_0 is positive. Hence, reducing d_0 reduces the expected cost of the contract. For any $d_0 < d_1$, the limited liability constraint in state 0 binds at the optimum because $\tilde{p} = \underline{p}$ and the coefficient multiplying d_0 is negative. If $d_0 = d_1$, the lenders' participation constraint implies they are equal to $D\,(1+r)$. This discussion can be summarized as follows. At the optimum, the following must hold:

$$\begin{aligned} d_0 &= \min\left\{(s-w) - b_0\,(1+r)^{-1}, D\right\}(1+r) \\ d_1 &= \frac{1}{\underline{p}} D\,(1+r) + \left(1 - \frac{1}{\underline{p}}\right) \min\left\{(s-w) - b_0\,(1+r)^{-1}, D\right\}(1+r) \end{aligned}$$

At the optimum, the participation constraint for the investor binds for the lowest expected value of F. If not, the firm is being needlessly generous. Let $\tilde{p}'$ be the probability of success that yields this expectation. This implies:

$$\begin{aligned} \tilde{p}' b_1 + (1-\tilde{p}')\, b_0 + \left(\overline{B} - B\right)(1+r) - C &= \overline{B}\,(1+r) \\ \tilde{p}' b_1 + (1-\tilde{p}')\, b_0 - B\,(1+r) - C &= 0 \end{aligned}$$

which can be rewritten as

$$\tilde{p}' b_1 = -\,(1-\tilde{p}')\, b_0 + B\,(1+r) + C$$

Solving this for b_1, one obtains:

$$b_1 = -\frac{1}{\tilde{p}'}\,(1-\tilde{p}')\, b_0 + \frac{1}{\tilde{p}'} B\,(1+r) + \frac{1}{\tilde{p}'} C$$

The expected cost to the firm of the contracts with the professional and the lenders is given by:

$$\begin{aligned} EC &= \hat{p}\,(d_1 + b_1) + (1-\hat{p})\,(d_0 + b_0) \\ &= \hat{p} \left(\begin{array}{c} -\frac{1}{\tilde{p}'}\,(1-\tilde{p}')\, b_0 + \frac{1}{\tilde{p}'} B\,(1+r) + \frac{1}{\tilde{p}'} C \\ \frac{1}{\underline{p}} D\,(1+r) + \left(1 - \frac{1}{\underline{p}}\right) \min\left\{(s-w) - b_0\,(1+r)^{-1}, D\right\}(1+r) \end{array} \right) \\ &\quad + (1-\hat{p}) \left(\min\left\{(s-w) - b_0\,(1+r)^{-1}, D\right\}(1+r) + b_0 \right) \end{aligned}$$

which can be rearranged as:

$$\begin{aligned} EC &= \hat{p} \left(\frac{1}{\underline{p}} D\,(1+r) + \frac{1}{\tilde{p}'} B\,(1+r) + \frac{1}{\tilde{p}'} C \right) + \left(1 - \frac{\hat{p}}{\tilde{p}'}\right) b_0 \\ &\quad + \left(1 - \frac{\hat{p}}{\underline{p}}\right) \min\left\{(s-w) - b_0\,(1+r)^{-1}, D\right\}(1+r) \end{aligned}$$

Finally, using constraint (vii), one obtains:

$$\begin{aligned} EC &= \hat{p}\left(\frac{1}{\underline{p}}(I-w)(1+r)+\left(\frac{1}{\tilde{p}'}-\frac{1}{\underline{p}}\right)B(1+r)+\frac{1}{\tilde{p}'}C\right)+\left(1-\frac{\hat{p}}{\tilde{p}'}\right)b_0 \\ &+\left(1-\frac{\hat{p}}{\underline{p}}\right)\min\left\{(s-w)-b_0(1+r)^{-1},(I-w-B)\right\}(1+r) \end{aligned}$$

This conclude the proof. ■

Lemma 5.2 *When Lemma 5.1 holds, at the optimum, the payment to the professional in state* 0 *is given by the following:*

$$b_0=\begin{cases}(s-I+B)(1+r) & \text{if } (s-I+B)\geq 0 \\ 0 & \text{otherwise}\end{cases}$$

Moreover, the payment the professional receives in state 1 *is larger than* b_0.

Proof.

Lemma 5.1 gives a formula for the expected cost of the optimal contract. That formula is the starting point of this proof. I distinguish two cases, according to whether the contract with the lenders is risk-free or not.

CASE 1: Assume $\min\left\{(s-w)-b_0(1+r)^{-1},(I-w-B)\right\}=(s-w)-b_0(1+r)^{-1}$

Substituting this expression in the expected cost of the financial contracts, one has:

$$\begin{aligned} EC &= \hat{p}\left(\frac{1}{\underline{p}}(I-w)(1+r)+\left(\frac{1}{\tilde{p}'}-\frac{1}{\underline{p}}\right)B(1+r)+\frac{1}{\tilde{p}'}C\right) \\ &+\left(1-\frac{\hat{p}}{\underline{p}}\right)(s-w)(1+r)+\left(\frac{\hat{p}}{\underline{p}}-\frac{\hat{p}}{\tilde{p}'}\right)b_0 \end{aligned}$$

By assumption, $\left(\frac{\hat{p}}{\underline{p}}-\frac{\hat{p}}{\tilde{p}'}\right)>0$. Thus, the firm chooses the smallest possible b_0. This is given by solving

$$(s-w)-b_0(1+r)^{-1}=(I-w-B)$$

when $(s-I+B)>0$ and zero when $(s-I+B)<0$. This concludes the proof for CASE 1.

CASE 2 $\min\left\{(s-w)-b_0(1+r)^{-1},(I-w-B)\right\}=(I-w-B)$

Substituting this expression in the expected cost of the financial contracts, one has:

$$\begin{aligned} EC &= \hat{p}\left(\frac{1}{\underline{p}}(I-w)(1+r)+\frac{1}{\tilde{p}'}C\right)+\left(\frac{\hat{p}}{\tilde{p}'}-1\right)B(1+r) \\ &+\left(1-\frac{\hat{p}}{\underline{p}}\right)(I-w)(1+r)+\left(1-\frac{\hat{p}}{\tilde{p}'}\right)b_0 \end{aligned}$$

I claim that no $b_0\geq d_1^i$ can be optimal. Suppose not. Then, $\tilde{p}'=\overline{p}'$ and $\left(1-\frac{\underline{p}}{\overline{p}'}\right)>0$. The firm optimally reduces b_1 until $b_1=b_0=B(1+r)+C$. Thus, we need to have:

$$\min\left\{(s-w)-B-C(1+r)^{-1},(I-w-B)\right\}=(I-w-B)$$

which implies

$$s \geq I + C(1+r)^{-1}$$

This is impossible because the firm's wealth W is smaller than I.

Thus, we restrict our attention to the case in which $b_0 < b_1$. The lowest expectation that the professional computes is the given by $\underline{p}'$. The coefficient of the payment in the bad state is then $\left(1 - \frac{\hat{p}}{\underline{p}'}\right) < 0$. Thus, at the optimum the firm chooses the highest possible non-negative b_0. This is given by

$$(s - I + B)(1+r) = b_0$$

when $(s - I + B) > 0$ and zero when $(s - I + B) < 0$. This is the same conclusion we arrived at for CASE 1.

We established that, at the optimum, $s < I + C(1+r)^{-1}$. Thus

$$(s - I + B) < I + C(1+r)^{-1} - I + B = B + C(1+r)^{-1}$$

Hence, the payment to the professional in state 1 must be larger than the one in state 0. This concludes the proof of this lemma. ■

Because the professional receives more in state 1, the lowest expectation she can compute is the one according to $\underline{p}'$. Then, the payment the professional receives in state 0 is positive only if $D + s \geq I$, otherwise it is zero. This is possible only if $\overline{B} + W \geq I$. I analyze this case, and derive the corresponding optimal contract. This corresponds to (A) in the statement of Proposition 4.2.

CASE (A): Assume $\overline{B} + W \geq I$

Using the previous Lemmas, the expected cost of the contract is:

$$\begin{aligned} EC &= \frac{\hat{p}}{\underline{p}}(I - w)(1+r) + \left(\frac{\hat{p}}{\underline{p}'} - \frac{\hat{p}}{\underline{p}}\right) B(1+r) + \frac{\hat{p}}{\underline{p}'}C \\ &\quad + \left(1 - \frac{\hat{p}}{\underline{p}}\right)(I - w - B)(1+r) + \left(1 - \frac{\hat{p}}{\underline{p}'}\right)(s - I + B)(1+r) \\ &= (I - w)(1+r) + \frac{\hat{p}}{\underline{p}'}C + \left(1 - \frac{\hat{p}}{\underline{p}'}\right)(s - I)(1+r) \end{aligned}$$

Then, the entrepreneur's expected wealth is given by:

$$\begin{aligned} V &= \hat{p}R - EC + (W - w)(1+r) \\ &= \hat{p}R + W(1+r) \\ &\quad -I(1+r) - \left(1 - \frac{\hat{p}}{\underline{p}'}\right)(s - I)(1+r) - \frac{\hat{p}}{\underline{p}'}C \end{aligned}$$

Because $\left(1 - \frac{\hat{p}}{\underline{p}'}\right) < 0$, the optimal s is equal to W. The entrepreneur's expected wealth does not depend on the amount of funding the professional provides, as long as $B + W \geq I$. Thus, any $B \geq I - W$ is a possible solution.

Summarizing, if $\overline{B} + W \geq I$, the optimal contract is: $s = W$, $B \geq I - W$, $D = I - w - B$, and $b_0 = (W - I + B)(1+r)$. Equations (5.13) and (5.14) can then be used to obtain the other payments the firm has to make. The entrepreneur's expected wealth is

$$\hat{V}^A = pR + \frac{\hat{p}}{\underline{p}'}(W - I)(1 + r) - \frac{\hat{p}}{\underline{p}'}C$$

This concludes the proof of (A).

CASE B: Assume $\overline{B} + W < I$

Using the previous Lemmas, the payment the professional receives in state 0 is zero. Thus, the expected cost of the contract is:

$$\begin{aligned} EC &= \hat{p}\left(\frac{1}{\underline{p}}(I - w)(1 + r) + \left(\frac{1}{\underline{p}'} - \frac{1}{\underline{p}}\right)B(1 + r) + \frac{1}{\underline{p}'}C\right) \\ &+ \left(1 - \frac{\hat{p}}{\underline{p}}\right)\min\{(s - w), (I - w - B)\}(1 + r) \\ &= \frac{\hat{p}}{\underline{p}}(I - w)(1 + r) + \left(\frac{\hat{p}}{\underline{p}'} - \frac{\hat{p}}{\underline{p}}\right)B(1 + r) + \frac{\hat{p}}{\underline{p}'}C \\ &+ \left(1 - \frac{\hat{p}}{\underline{p}}\right)(s - w)(1 + r) \end{aligned}$$

The last equality follows because, if $\overline{B} + W < I$, $\min\{(s - w), (I - w - B)\} = (s - w)$ as shown below:

$$s - w \leq W - w < I - w - \overline{B} \leq I - w - B$$

By assumption, $\left(1 - \frac{\hat{p}}{\underline{p}}\right) < 0$ and $\left(\frac{1}{\underline{p}'} - \frac{1}{\underline{p}}\right) < 0$. Hence, at the optimum, both s and B are as large as possible. That is, $s = W$ and $B = \overline{B}$. Using this, the entrepreneur's expected wealth is:

$$\begin{aligned} \hat{V} &= \hat{p}R + (W - w)(1 + r) - EC \\ &= \hat{p}R - \frac{\hat{p}}{\underline{p}}I(1 + r) - \left(\frac{\hat{p}}{\underline{p}'} - \frac{\hat{p}}{\underline{p}}\right)\overline{B}(1 + r) - \frac{\hat{p}}{\underline{p}'}C + \frac{\hat{p}}{\underline{p}}W(1 + r) \end{aligned}$$

which does not depend on w.

Summarizing, if $\overline{B} + W < I$, the optimal contract is: $s = W$, $b_0 = 0$, $B = \overline{B}$, and $D = I - w - \overline{B} > 0$. Equations (5.13) and (5.14) can be used to obtain the other payments the firm has to make. This establishes (B).

To conclude the proof, I need to show that the contract with $B = 0$ is optimal when $\overline{B} < \frac{\underline{p}}{\underline{p}' - \underline{p}}C(1 + r)^{-1}$.

If $B = 0$, the optimal contract has been already derived, and the corresponding entrepreneur's expected wealth is:

$$V^C = \hat{p}R - \frac{\hat{p}}{\underline{p}}(I - W)(1 + r)$$

Thus, I need to show this is higher than the entrepreneur's expected wealth corresponding to the optimal contracts in (A) and (B).

(A): If $\overline{B} + W \geq I$, the entrepreneur's expected wealth is:

$$\hat{V}^A = \hat{p}R - \frac{\hat{p}}{\underline{p}'}(I - W)(1 + r) - \frac{\hat{p}}{\underline{p}'}C$$

Thus,

$$\begin{aligned} V - \hat{V}^A &= \left(\frac{\hat{p}}{\underline{p}'} - \frac{\hat{p}}{\underline{p}}\right)(I - W)(1 + r) + \frac{\hat{p}}{\underline{p}'}C \\ &= \hat{p}(1 + r)\left[\frac{\underline{p} - \underline{p}'}{\underline{p}'}(I - W) + \frac{1}{\underline{p}'}C(1 + r)^{-1}\right] \\ &> \hat{p}(1 + r)\left[\frac{\underline{p} - \underline{p}'}{\underline{p}\underline{p}'}(I - W) + \frac{\underline{p}' - \underline{p}}{\underline{p}}\frac{1}{\underline{p}'}\overline{B}\right] \\ &= \hat{p}(1 + r)\frac{\underline{p}' - \underline{p}}{\underline{p}}\left(\overline{B} - I - W\right) > 0 \end{aligned}$$

(B): If $\overline{B} + W < I$, the entrepreneur's expected wealth is:

$$\hat{V}^B = pR - \frac{\hat{p}}{\underline{p}}(I - W)(1 + r) - \left(\frac{\hat{p}}{\underline{p}'} - \frac{\hat{p}}{\underline{p}}\right)\overline{B}(1 + r) - \frac{\hat{p}}{\underline{p}'}C$$

Thus,

$$\begin{aligned} V - \hat{V}^B &= \left(\frac{\hat{p}}{\underline{p}'} - \frac{\hat{p}}{\underline{p}}\right)\overline{B}(1 + r) + \frac{\hat{p}}{\underline{p}'}C \\ & \quad \hat{p}\frac{\underline{p} - \underline{p}'}{\underline{p}\underline{p}'}\overline{B}(1 + r) + \frac{\hat{p}}{\underline{p}'}C \\ &> \hat{p}\frac{\underline{p} - \underline{p}'}{\underline{p}\underline{p}'}\overline{B}(1 + r) + \frac{\underline{p}' - \underline{p}}{\underline{p}'\underline{p}}\overline{B}(1 + r) = 0 \end{aligned}$$

This proves (C), and concludes the proof of Proposition 4.2. ■.

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