EMPTY VOTING AND THE EFFICIENCY OF CORPORATE GOVERNANCE

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Abstract. We model corporate voting outcomes when an informed trader, such as a hedge fund, can establish separate positions in a firm's shares and votes ("empty voting"). The positions are separated by borrowing shares on the record date, hedging economic exposure, or trading between record and voting dates. We find that the trader's presence can improve efficiency overall despite the fact that it sometimes ends up selling short and then voting to decrease firm value. An efficiency improvement is likely if other shareholders' votes are not highly correlated with the correct decision *or* if it is relatively expensive to separate votes from shares on the record date. On the other hand, empty voting will tend to decrease efficiency if it is relatively inexpensive to separate votes from shares *and* other shareholders are likely to vote the right way. We also study how market depth affects efficiency.

Keywords: voting, trading, hedge funds, corporate governance

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1. Introduction

The impact of hedge funds on corporate governance has received considerable attention recently as the rise in popularity of hedge funds has coincided with an increased focus on governance in general. Much of the attention has been devoted to "activist" funds that take significant stakes in firms and then advocate for change.[1] However, other more subtle strategies undertaken by hedge funds or other strategic traders can also significantly affect the efficiency of the corporate governance system.

In particular, recent work has shown that some funds may use "empty voting" – a practice whereby they accumulate voting power in excess of their economic share ownership – to manipulate shareholder vote outcomes and generate trading gains. This practice is possible even when one share, one vote is the explicit rule. It can be accomplished, for example, by borrowing shares of stock on the record date or hedging economic exposure in the derivatives markets. Hu and Black (2006, 2007) provide a number of examples where such behavior seems to result in perverse voting incentives. In one case, a hedge fund acquired votes by borrowing shares, then voted against a buyout proposal and apparently profited from a short position when the share price dropped following the vote.[2] These authors suggest that some form of regulation, starting with additional disclosure requirements, may be necessary to curb the negative effects of such activities.

Regulators have expressed significant concern over empty voting, particularly given the boom in the hedge fund industry and the increasing number and importance of items requiring a shareholder vote. The Wall Street Journal (January 26, 2007, p. A1) quotes SEC chairman Christopher Cox as saying that the practice of empty voting "is almost certainly going to force further regulatory response to ensure that investors' interests are protected...This is already a serious issue and it is showing all signs of growing." Many large institutional

[1]See Kahan and Rock (2006), Clifford (2007), Klein and Zur (2006), and Brav, Jiang, Partnoy, and Thomas (2008).

[2]This incident involved a Hong Kong company named Henderson Land, which wanted to buy out a 25% minority interest in its publicly traded affiliate Henderson Investment.

shareholders are examining their share lending practices in response to these concerns. In addition, some companies have recently amended their bylaws to force additional disclosure of complex transactions in their securities due to concerns about corporate governance implications (The Wall Street Journal, July 14, 2008, p. B4).

On the other hand, Christoffersen, Geczy, Musto, and Reed (2007) argue that "vote trading" in the share lending market can increase efficiency because information about proposals can be costly to acquire. Uninformed shareholders who are not willing to pay the cost to become informed can sell their votes to informed parties in order to increase the efficiency of the voting outcome. Of course, this argument requires that the vote buyer and vote seller have coincident interests, which often seems to be violated in the examples cited by Hu and Black (2006, 2007). To date, there is no agreement on whether empty voting constitutes a significant problem that should be regulated. Importantly, the literature does not currently provide an integrated theoretical framework to help assess the tradeoff between increased information efficiency and the cost of possible manipulations via empty voting.

In this paper, we develop a theoretical model to explore this trade-off. We derive the optimal share and vote position of a strategic trader that has the ability to acquire unique information about the value of a management proposal and the ability to acquire votes separately from shares. We show that while the trader may sometimes reduce efficiency by shorting the stock and then "voting the wrong way" (from a firm value perspective), the cost of these possible manipulations can be offset by a greater probability that the trader will "do the right thing" and vote to maximize firm value. In other words, in equilibrium both the presence of the strategic trader and the ability to separate votes from economic ownership can increase overall efficiency by making the "right" outcome more likely. This occurs when *either* the establishment of an empty voting stake on the record date is relatively expensive *or* other shareholders' votes are not very highly correlated with the true state. However, we find that a negative efficiency effect is likely when separating votes from shares is relatively inexpensive *and* other shareholders are relatively likely to vote the right way.

Our analysis deals with deviations from the one share, one vote rule, on which there is a large existing literature dating back to at least Manne (1964). Much of the modern literature focuses on how the one share, one vote rule affects the efficiency of the market for corporate control (see, e.g., Harris and Raviv (1988), Grossman and Hart (1988), and Burkart and Lee (2007)), or how disparities between cash flow and voting rights held by *insiders* affects efficiency (e.g., DeAngelo and DeAngelo (1985), Gilson (1987)). These studies generally focus on long-term deviations from one share, one vote that are codified in the corporate charter. An important recent exception is Kalay and Pant (2008), which shows that the ability to separate economic and voting interests via derivatives markets can increase efficiency by allowing shareholders to extract more surplus in a control contest. Like Kalay and Pant (2008), we examine short-term deviations arising from activities in the derivatives or share lending markets. However, we focus on how these deviations affect the efficiency of voting by outsiders on day-to-day proposals (rather than control contests). We think of outsiders as parties who do not make proposals themselves, but face uncertainty over whether an insider's proposal is value-increasing or instead self-serving. There are many types of proposals other than proxy contests or takeover bids for control that can have important value implications for the firm. Examples include proposals for the purchase of another firm, a divestiture, or a change in the corporate charter (often involving a takeover defense).

In our model, the firm's management initially proposes an action that requires shareholder approval. The proposed action may be either good or bad (ie, its approval may either increase or decrease firm value), and its value is not observable at this stage. All shares are held either by atomistic shareholders or a single strategic trader. In the benchmark model, the strategic trader holds no shares prior to the announcement of the forthcoming vote. After the proposal is announced the strategic trader can buy or sell shares in a transparent market prior to the record date (ie, with no noise trading) and can also acquire "extra" votes in excess of its economic ownership by paying a convex cost. This cost represents, for example, increasing difficulty in finding shareholders from whom to borrow shares, or the increasing cost of finding counterparties to hedge a large economic interest.

On the record date, voting interests are set according to share or vote ownership on that day - all votes are held either by atomistic shareholders or the strategic trader. After the record date, there is a significant time lag before the actual date of the vote, so the strategic trader is able to both learn about the value of the proposal, and further adjust its economic ownership, but not its voting interest. At this intermediate trading stage, however, the market is not completely transparent, because there is noise trading by atomistic investors. Finally, on the voting date the strategic trader votes according to its economic incentives, while the voting of atomistic shareholders is effectively random. We do not explicitly model the atomistic holders' voting decisions; the important feature is that their behavior induces randomness in the final voting outcome.

Our assumptions are meant to reflect the realities of corporate governance in the United States. Christoffersen, Geczy, Musto, and Reed (2007) report that there is a significant time lag between the record date and the meeting date (a median of 54 calendar days in their sample) as opposed to the relatively short time between the announcement of the agenda and the record date. Thus, it seems reasonable that there would be little ability to trade strategically prior to the record date (which corresponds to our assumption of a transparent market at that stage), but a significant opportunity to gather information and trade less transparently between the record and voting dates.

It is important to note that we highlight two ways in which empty voting can occur in the U.S. corporate governance system. In addition to the lending and derivatives markets, there is also the time lag between record and voting dates. Even if voting and economic interests have to be aligned on the record date, it is possible to divorce the two prior to the voting date by trading in the stock market during the intervening period. Our model allows us to separate the two effects. As will become clear, we find that the ease with which votes and shares can be separated on the record date is of key importance with respect to whether empty voting helps or hurts efficiency.

We first solve a benchmark model with no ex ante ownership by the strategic trader. In this case, we find that the strategic trader optimally trades to a long economic position on the record date while simultaneously acquiring "extra" votes, both of which set the stage for the possibility of future trading gains. The number of extra votes acquired depends on the cost of the votes versus the value (in terms of larger expected trading profits) of the increased ability to affect the vote outcome. For the bulk of the analysis, we assume that the cost of separating votes from ownership is high enough that the trader will not acquire enough votes to singlehandedly determine the voting outcome.

The trader's *economic* position on the record date is driven by a separate tradeoff. On the positive side, a larger economic stake increases the trader's voting power. This is valuable when extra votes are costly. On the negative side, greater economic ownership reduces expected trading gains for two reasons. First, the "future self" of the trader will be concerned with protecting the value of its stake in addition to maximizing trading gains. This "commitment effect" of owning an economic stake on the record date thus reduces expected future trading profits. Second, the strategic trader's position reduces overall market depth. In equilibrium, the extent of the long position is determined by the expected amount of noise trading between the record and voting dates, and the ease with which votes can be acquired separately from shares on the record date.

After the record date, the strategic trader becomes informed[3] about whether the proposal is good or bad and then plays a mixed strategy; it either buys additional shares and votes to maximize firm value, or it sells to a short position and votes to minimize firm value. We find in our benchmark model that the presence of the strategic trader is good for efficiency overall when the other shareholders' votes are not highly correlated with the correct decision, or when the ability to separate shares and votes on the record date is highly restricted. Because of its long position on the record date, the strategic trader tends to "vote the right way" more often than not, increasing the probability of a correct decision in these situations. As market depth increases, the optimal long position on the record date increases, intensifying the positive effect. Thus, we find that allowing for trading gains by a strategic trader can

[3]The timing of when the trader becomes informed actually does not matter - it can occur either before or after the record date with no change in the analysis.

increase efficiency even though the trader sometimes engages in value-reducing strategies. Also, since the strategic trader would not acquire any votes or shares in the absence of possible trading gains, noise trading improves voting efficiency.

The positive efficiency effect we document for these cases is driven by the fact that the strategic trader has unique value-relevant information that other shareholders do not have. If the strategic trader brought no new information to the model, there would be no possibility of an efficiency improvement, but manipulation could still be possible. As such, our model provides a framework for determining whether and how an informed trader's unique information is ultimately reflected in the final voting outcome and thus firm value.

On the other hand, we go on to show that efficiency can be reduced by the strategic trader when other shareholders' votes are sufficiently biased toward the correct decision *and* it is not too expensive to separate votes from shares on the record date. Intuitively, when it is easy to separate votes from shares on the record date, the trader chooses a smaller long position and the commitment effect is attenuated. In these cases the trader's efficiency-reducing votes are *relatively* more likely than its efficiency-enhancing votes to change the actual decision if other shareholders are likely to vote the right way on their own. Thus, the negative effects can start to outweigh the positive ones. This occurs despite the fact that the trader brings value-relevant information to the table.

We also investigate how changes in the underlying parameters affect the trader's strategy as well as overall efficiency. We find that making it easier to separate votes from economic ownership on the record date tends to decrease the trader's economic ownership (in order to maximize trading gains by avoiding the commitment effect – as noted above). This increases the probability that the trader will go short and vote the wrong way later. However, the net effect on efficiency can be positive when votes are not too cheap, since the additional votes also increase the trader's ability to affect the voting outcome. This increased ability to affect the outcome can outweigh the higher probability of voting the wrong way. This result can be reversed, however, if votes are cheap enough and the correlation between others' votes and the correct decision is high enough.

Finally, we analyze how the model would change if the strategic trader had an ex ante long position in the stock. We find that the trader's optimal economic interest on the record date is increasing in its initial ownership. If initial ownership is high enough, the trader will actually find it optimal to forgo trading gains, instead buying enough shares and/or votes to ensure that the vote outcome always maximizes firm value. In such a case, an increase in market depth or a decrease in the price of votes separated from shares can reverse that decision and cause the trader to manipulate in order to generate trading gains, reducing overall efficiency. In this setting, empty voting and market liquidity can reduce the extent to which the trader's information positively affects firm value.

Our results may provide some guidance on the efficacy of proposed regulatory reforms designed to curb or eliminate the negative effects of empty voting. For example, Hu and Black (2006, 2007) advocate additional disclosure requirements as a reasonable starting point. In the framework of our model, disclosure of an "empty voting" position on the record date would have no effect, because we already assume that the market maker observes the strategic trader's actions at that stage. Disclosure of a change in economic position relative to voting rights between the record and voting dates would have the effect of reducing or eliminating any trading profits the strategic trader could otherwise generate. This would reduce the trader's willingness to gather information and vote. Thus, the efficiency effect of such a rule would depend case by case on whether the model predicted a positive or negative effect from the trader's presence. Overall, our results imply that regulators should consider the possibility that curbs to empty voting behavior could be costly in cases where there is significant uncertainty about the value of a proposal.

1.1. **Related Literature.** Our model obviously involves a form of stock price manipulation, but where the manipulation is accomplished by affecting the firm's real operations. Closely related are Kyle and Vila (1991), Maug (1998), and Kahn and Winton (1998). In all of these models, a strategic trader can directly take an action that will affect firm value, and its ability to trade in a noisy stock market affects its incentives to do so. The main difference between our analysis and theirs is that we endogenize the trader's ability to affect firm value by modeling the voting game and deriving the optimal ex ante share and vote position of the trader.

Another strand of literature studies incentives for manipulation by traders when managers make investment decisions partially based on information gleaned from stock prices. In particular, Khanna and Sonti (2004) and Khanna and Marietta-Westberg (2005) show that *informed* traders may trade against their private information if that will send a valuable signal to managers about investment prospects. On the other hand, Goldstein and Guembel (2007) show that *uninformed* traders may take advantage of feedback effects and sell shares to manipulate prices negatively and generate trading gains. Attari, Banerjee, and Noe (2006) show that informed investors may have incentives to dump shares and move prices to induce shareholder activism. Other models of manipulation involving real activities include Bagnoli and Lipman (1996) and Vila (1989), both of which study manipulation involving direct actions such as a takeover bid. Manipulation based on information alone has also been studied widely, such as by Allen and Gale (1992) and Chakraborty and Yilmaz (2004).

Many of these papers contribute to a more general literature on how large shareholders affect corporate governance. Other papers in this literature tend to focus either on blockholders who exercise "voice" by directly intervening in the firm's activities (Shleifer and Vishny (1986), Burkart, Gromb, and Panunzi (1997)), or those who use informed trading, also called "exit," to improve stock price efficiency and encourage correct actions by managers (Admati and Pfleiderer (2006), Edmans (2007), Edmans and Manso (2007)). Several recent empirical papers specifically study activism by hedge funds (Kahan and Rock (2006), Clifford (2007), Klein and Zur (2006), Brav, Jiang, Partnoy, and Thomas (2008)). Such strategies contrast sharply with the trading strategy we study this paper, where the fund optimally hides its information about the value of a proposal while strategically using it to manipulate real outcomes and generate trading gains. Olaru and Zachariadis (2008) study the governance implications of a similarly subtle strategy, wherein hedge funds may own both debt and equity in a distressed firm, which will affect their voting on a reorganization plan.

Our analysis is also closely related to the small but growing literature on vote buying. For example, Blair, Golbe, and Gerard (1989) and Neeman and Orosel (2007) show that allowing a contest for votes in addition to a contest for shares can have efficiency advantages. However, they do not model how stock trading interacts with vote buying.

Other authors have modeled trading and voting together, but without allowing for "empty voting" For example, Maug (1999) models a strategic voting game in which voting and trading both help aggregate dispersed information about the value of a proposal. Musto and Yilmaz (2003) study how the operation of a financial market affects political voting.

Our study is also related to papers studying the value of corporate votes. For example, Zwiebel (1995) models shareholders' incentives to form blocks and participate in voting coalitions. Barclay and Holderness (1989) study block trades and find that there is a significant premium paid for large minority blocks, which indicates that less than majority voting control can be valuable.

The paper proceeds as follows. In section 2 we describe the model. In section 3 we derive the equilibrium in the absence of initial shareholdings. In section 4 we allow the strategic trader to hold shares ex ante. We discuss implications in section 5, and conclude in section 6.

2. The Model

The model focuses on a firm with an upcoming shareholder vote. The firm has one perfectly divisible share outstanding. The players consist of the management of the firm, a strategic trader, a market maker, and atomistic shareholders. Management sets the agenda for the vote, but does not hold any shares and cannot vote. The market maker also does not vote.

At the beginning of the game, management proposes an action that can be either good or bad. The proposal's ultimate approval status determines firm value, which is either $\underline{v}$ or $\bar{v}$. In particular, with a good proposal firm value equals $\underline{v}$ if the proposal is defeated and

$\bar{v} = \underline{v} + \Delta v$ if the proposal is approved. With a bad proposal firm value is $\bar{v}$ if the proposal is defeated and $\underline{v}$ if it is approved.[4] We do not model the reason why management may make a bad proposal. As an example, it could be caused by an agency problem or a lack of ability or information on management's part.

All players are initially uninformed about whether the proposal is good or bad. We assume that H costlessly becomes informed about whether the proposal is good or bad at some point prior to the voting date (it makes no difference whether this occurs before or after the record date). The market maker and the atomistic shareholders do not become informed. The strategic trader, hereafter H, may also hold an ex ante long position in the firm's stock equal to α_h shares (or, equivalently, α_h percent of the shares).

After the proposal is announced, but before the record date, H can submit a market order to buy or sell shares in the firm. The order is filled by the market maker at a price equal to the expected value of the shares (ie, the market maker and H have the same information at this point, including any "extra" votes H may acquire, and H's trade is transparent to the market maker). For simplicity, we assume that the market maker holds no inventory at any stage of the model. So, for example, if it sells shares to H it is immediately able to purchase shares from atomistic holders at the same price. This simplifies the analysis because it implies that all shares will be held by either H or atomistic stockholders on the record date. The important feature of the assumption is that ownership of shares by H reduces ownership by atomistic shareholders, who are the only other parties allowed to vote, and who also determine later market liquidity. We denote H's final economic position on the record date by α_H.

The strategic trader may also be able to acquire votes in addition to those represented by its record date share ownership. It can do this at a cost, $c(\alpha_X)$, that is increasing and

[4]The assumption that firm value can take on only two values is made for tractability. An alternative specification where firm value equals one if any proposal is defeated and is increased by Δv if a good proposal is approved or decreased by Δv if a bad proposal is approved yields the same qualititative results but requires additional simplifying assumptions.

convex in the number of "extra" votes, α_X. This cost reflects any expense H incurs in separating its voting interest from its economic ownership. For example, the extra votes could be purchased on the share lending market.[5] When H approaches a given atomistic shareholder to borrow its shares, H may have all of the bargaining power, and thus be able to borrow the share at effectively zero cost – the shareholder does not believe its vote will be pivotal, and thus is willing to sell the vote for any nominal price. However, since the lending market is decentralized H must first find the shareholder. Our assumption then corresponds to a convex search cost function (it becomes harder to find the next shareholder the more you have already located). For simplicity, we assume that the cost function takes the form $c(\alpha_X) = max[0, (\alpha_X - \bar{\alpha_X})K]$, where $\bar{\alpha_X} \geq 0$ and K is very large. Thus, extra votes are free up to $\bar{\alpha_X}$ and then prohibitively expensive beyond that.[6] Consistent with these assumptions, Christoffersen, Geczy, Musto, and Reed (2007) find that the average vote sells for zero in the share lending market. Furthermore, Kolasinski, Reed, and Ringgenberg (2008) show that the share loan supply schedule is relatively flat at lower quantity levels but becomes very steep at higher levels, and that the share lending market exhibits features consistent with significant search frictions.

On the record date, all shares are held either by H or by atomistic shareholders. Their holdings on that date determine their final voting power. Next, some of the *original* atomistic shareholders are hit by a random liquidity shock that causes them to sell their interest in the firm. We assume that with probability $\frac{1}{2}$, a proportion α_Z of the atomistic shareholders who held shares prior to H's record date trading place market orders to sell their shares,

[5]We focus on buying votes through the share lending market for expositional simplicity. The analysis is equivalent if the empty voting position arises because H buys shares to gain votes and then hedges part of the economic exposure. In that case, the cost of extra votes becomes the cost of finding counterparties to hedge the economic exposure.

[6]All of the qualitative results of the paper are unchanged if a continuous, convex function is assumed for $c(\alpha_X)$, as long as the function is sufficiently "steep."

and that otherwise there is no trading by atomistic shareholders.[7] Since H owned α_h shares originally, and owns α_H shares at this point, this means that the total number of shares sold by atomistic holders when the liquidity shock hits is $\alpha_Z(1 - max[\alpha_h, \alpha_H])$. Thus, if H *buys* shares before the record date, this offsets market liquidity. If H *sells* shares before the record date, liquidity is not affected.[8] H can also place a market order at this time to buy or sell whatever quantity it wishes (without first observing whether atomistic shareholders sell). The market maker observes only the total net order flow. There are no short sale constraints.

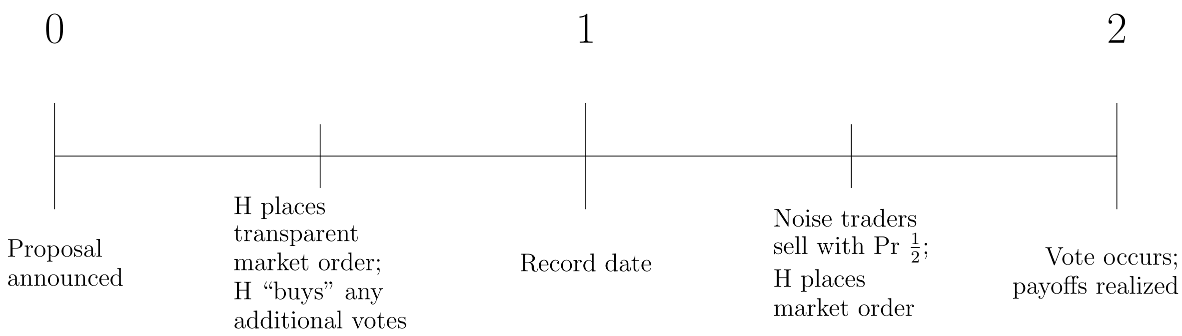
Finally, on the voting date, H votes its $max[0, \alpha_H]+\alpha_X$ votes according to its own economic incentives and information. If at least $\frac{1}{2}$ of the total votes are cast in favor, the proposal passes. For the $1 - (max[0, \alpha_H] + \alpha_X)$ votes held by atomistic stockholders, we assume that the *proportion* of "yes" votes cast, denoted by Y, is distributed on [0,1] according to the distribution function $F_G(\cdot)$ with associated density $f_G(\cdot) \in \mathcal{C}^2$ for a good proposal, and $F_B(\cdot)$ with associated density $f_B(\cdot) \in \mathcal{C}^2$ for a bad proposal. We assume different distributions for good versus bad proposals to allow for the possibility of correlation between atomistic stockholders' votes and the true state. For tractability, we assume that $F_G(\cdot)$ and $F_B(\cdot)$ are always symmetric, ie, that for any $\eta \in [0, \frac{1}{2}]$, $F_B(\frac{1}{2} - \eta) = 1 - F_G(\frac{1}{2} + \eta)$. This implies that if $F_G(\cdot) = F_B(\cdot)$ (ie, there is no correlation with the true state) then the common distribution must be symmetric around $\frac{1}{2}$, and thus the expected ex-ante probability of approval if H has no votes equals $\frac{1}{2}$ in this case. Note that the atomistic holders' total number of "yes" votes equals $Y(1 - max[0, \alpha_H] - \alpha_X)$. After the proposal passes or fails, the resulting value of the firm is realized and immediately reflected in the share price.

Figure 1 illustrates the timeline of the game.

[7]This is equivalent to assuming that the atomistic traders buy $\frac{\alpha_Z}{2}$ shares with probability $\frac{1}{2}$ and sell $\frac{\alpha_Z}{2}$ shares with probability $\frac{1}{2}$.

[8]It seems reasonable to assume that a new shareholder buying just prior to the record date is less likely to face a liquidity shock in the short run than are pre-existing, long-term shareholders.

Figure 1. Time Line



3. Equilibrium With No Ex Ante Holdings

We first solve the model assuming no ex ante shareholdings by H ($\alpha_h = 0$). We derive a subgame perfect Nash equilibrium using backward induction. We assume throughout that $\alpha_Z < \frac{1}{2}$, K is sufficiently large to deter any $\alpha_X > \bar{\alpha_X}$, and $\bar{\alpha_X}$ is low enough relative to α_Z that H will never optimally take full control of the vote by setting $\alpha_H \geq \frac{1}{2} - \bar{\alpha_X}$.[9]

Behavior at the voting date is straightforward. As noted above, a random proportion Y of the atomistic shareholders vote in favor of the proposal according to $F_G(\cdot)$ or $F_B(\cdot)$. H's vote depends on its economic position in the shares. Since there is no information asymmetry after the vote, the shares will trade at their true value of $\underline{v}$ or $\bar{v}$. Thus, if H is long in the stock it maximizes the value of its stake by voting in the direction that makes the $\bar{v}$ outcome more likely, i.e., in favor of a good proposal and against a bad proposal. If it is short it

[9]Proposition 2 below shows that given $\alpha_Z < \frac{1}{2}$, the optimal α_H can never exceed $\frac{1}{5}$, so this last restriction is quite easy to satisfy.

maximizes the value of its stake by voting in the opposite direction, making the $\underline{v}$ outcome more likely.

We now derive the expected value of the firm depending on whether H is long or short on the record date. First consider a good proposal. If H votes in favor of the proposal the probability of acceptance is

$$Pr\left[max[0,\alpha_H]+\alpha_X+Y(1-max[0,\alpha_H]-\alpha_X)>\frac{1}{2}\right]=Pr\left[Y>\frac{\frac{1}{2}-max[0,\alpha_H]-\alpha_X}{1-max[0,\alpha_H]-\alpha_X}\right]$$
$$=1-F_G\left(\frac{\frac{1}{2}-max[0,\alpha_H]-\alpha_X}{1-max[0,\alpha_H]-\alpha_X}\right).$$

Similarly, if H votes against, the probability of acceptance is $1-F_G\left(\frac{\frac{1}{2}}{1-max[0,\alpha_H]-\alpha_X}\right)$. To economize on notation, let $\gamma\equiv\frac{\frac{1}{2}}{1-max[0,\alpha_H]-\alpha_X}$ and $\delta\equiv\frac{\frac{1}{2}-max[0,\alpha_H]-\alpha_X}{1-max[0,\alpha_H]-\alpha_X}$. Then expected firm value conditional on a good proposal and H being long is

$$V_B\equiv\underline{v}+\Delta v\left[1-F_G\left(\delta\right)\right].$$

Similarly, expected firm value conditional on a good proposal and H being short is

$$V_S\equiv\underline{v}+\Delta v\left[1-F_G\left(\gamma\right)\right].$$

With a bad proposal, the probability of approval given that H is long and votes against is $1-F_B\left(\gamma\right)$, which implies that the probability of achieving the value $\bar{v}$ is $F_B\left(\gamma\right)$. From the symmetry of $F_G(\cdot)$ and $F_B(\cdot)$, we know that

$$F_B\left(\gamma\right)=1-F_G\left(\delta\right),$$

which implies that expected firm value if H is long equals V_B as derived above whether the proposal is good or bad. Using a similar argument it is easy to see that the expected value if H is short will always equal V_S as derived above.

Now consider H's trading decision between the record and voting dates given its economic position in the stock on the record date, α_H, and its voting power, $max[0,\alpha_H]+\alpha_X$. This stage of the model can be viewed as a modified version of Maug (1998)'s model of a large shareholder's trading/intervention decision. The main difference here is that H can only probabilistically affect the value of the firm by voting its shares ex post. In equilibrium, H plays a mixed strategy. With some probability q H sells α_S shares to arrive at an overall short position and then votes to minimize firm value, while with probability $(1-q)$ it buys α_B additional shares to arrive at a long position and votes to maximize firm value. Departing from Maug (1998), we assume that the trading quantities and mixing probability are chosen simultaneously by H. We first derive the equilibrium assuming this behavior, then prove that it is the unique equilibrium of the subgame in Lemma 1 below.

In order for the mixed trading strategy to be profitable, H must set its buy and sell quantities such that the market maker cannot always see what H is doing. In particular, the market maker must be unable to distinguish between cases where atomistic holders sell while H buys, and cases where H sells while atomistic holders do not sell. This provides the following constraint on H's buy/sell quantities:

$$\alpha_Z(1 - max[0, \alpha_H]) - \alpha_B = \alpha_S. \tag{1}$$

We must now determine the optimal mixing probability, q, and optimal buying quantity, α_B. Given the expressions for expected firm value derived above, H's expected profit from buying α_B shares and voting to maximize firm value is

$$\alpha_B \left(V_B - \left[\frac{1}{2} V_B + \frac{1}{2} [qV_S + (1-q)V_B] \right] \right) + \alpha_H V_B. \tag{2}$$

The first term in the equation equals H's expected trading profits. The expected price at which H buys shares, $\left[\frac{1}{2}V_B + \frac{1}{2}[qV_S + (1-q)V_B]\right]$ reflects the fact that half of the time, no atomistic holders will sell, and its strategy will be transparent to the market maker. The rest of the time it successfully hides among the noise, and the market maker sets the price at the "uninformed" expected value. The second term in the equation reflects the value of H's existing stake in the firm.

Similarly, H's expected profit from selling and voting to minimize firm value equals

$$(\alpha_B - \alpha_Z(1 - max[0, \alpha_H])) \left(V_S - \left[\frac{1}{2} V_S + \frac{1}{2} [qV_S + (1-q)V_B] \right] \right) + \alpha_H V_S, \tag{3}$$

where the term $(\alpha_B - \alpha_Z(1 - max[0, \alpha_H]))$ reflects the constraint given by equation (1). Equation (2) must equal (3) in order for H to be indifferent and willing to mix. In order to solve for the optimal q and α_B, we set the two equations equal and first solve for the q that makes H indifferent for a given α_B. We then maximize H's overall expected profits to solve for the optimal α_B (we show in Lemma 1 that this is the unique equilibrium).

Setting (2) equal to (3) and solving for q yields

$$q = 1 - \frac{\alpha_B + 2\alpha_H}{\alpha_Z(1 - max[0, \alpha_H])}. \tag{4}$$

H's overall expected profit equals $(1-q)$ times (2) plus q times (3). Plugging the expression above for q into this overall profit function yields a concave function in α_B that is easily shown to have a maximum at

$$\alpha_B^* = \frac{\alpha_Z(1 - max[0, \alpha_H])}{2} - \alpha_H.$$

Plugging this into the expression for q above implies an optimal mixing quantity of

$$q^* = \frac{1}{2} - \frac{\alpha_H}{\alpha_Z(1 - max[0, \alpha_H])}.$$

We have the following result (all proofs are provided in the appendix).

Lemma 1. In the unique subgame perfect Nash equilibrium of the post-record date subgame, H plays a mixed strategy in which it sells $\alpha_Z(1 - max[0, \alpha_H]) - \alpha_B^*$ shares with probability q^* and buys α_B^* shares with probability $(1 - q^*)$.

Using these optimal quantities, we can rewrite H's overall expected payoff for this stage as

$$\frac{(V_B - V_S)(\alpha_Z^2(1 - max[0, \alpha_H])^2 - 4\alpha_H^2)}{8\alpha_Z(1 - max[0, \alpha_H])} + \alpha_H \left(q^* V_S + (1 - q^*)V_B\right).$$

The first term represents H's expected trading profits, while the second is simply the expected value of its stake. Note from above that the "value wedge," $V_B - V_S$, that H can generate by randomizing its trade (and therefore its vote) can be expressed as

$$V_B - V_S = \Delta v \left[F_G(\gamma) - F_G(\delta)\right] \geq 0, \tag{5}$$

where the inequality follows from $\gamma \geq \delta$.

Given the optimal trading strategy derived above, we can proceed to solve for H's optimal share and vote trading prior to the record date. H's expected profits viewed from this stage of the game include its expected future trading profits plus the expected value of the stake it acquires today, less the price it pays for the stake and the cost of any "excess" votes, $c(\alpha_X)$. From above, H's expected future trading profits are $\frac{(V_B - V_S)(\alpha_Z^2(1-max[0,\alpha_H])^2 - 4\alpha_H^2)}{8\alpha_Z(1-max[0,\alpha_H])}$. Given our assumption that the shares H buys at this stage are priced at their true expected value, the price of the stake exactly offsets its expected value, so H chooses its economic and voting stakes solely to maximize expected future trading profits less the cost of extra votes. We thus have the objective function

$$(6) \qquad \max_{\alpha_H, \alpha_X} \frac{(V_B - V_S)(\alpha_Z^2(1 - max[0, \alpha_H])^2 - 4\alpha_H^2)}{8\alpha_Z(1 - max[0, \alpha_H])} - max[0, (\alpha_X - \bar{\alpha_X})K],$$

where $V_B - V_S$ is given by (5) above.

The basic tension in H's stake purchase decision can be seen in these equations. For a given value wedge $(V_B - V_S)$, trading profits are maximized by choosing a stake of $\alpha_H = 0$. This is reflected in the term $\frac{(\alpha_Z^2(1-max[0,\alpha_H])^2 - 4\alpha_H^2)}{8\alpha_Z(1-max[0,\alpha_H])}$, which decreases as α_H rises or falls from zero. Intuitively, the larger the stake H holds (in absolute value) at the later trading stage, the more it will worry at that point about protecting the value of that stake rather than generating trading gains. Furthermore, positive economic ownership by H offsets ownership by atomistic shareholders, and thus reduces market depth and the potential for profitable trading. On the other hand, the greater the (positive) stake, the more voting power H has, so the larger is the value wedge it can generate. This is reflected in the expression given in (5) for $V_B - V_S$, in which γ is increasing in α_H and δ is decreasing in α_H. The optimal stake trades off these effects. The optimal amount of extra votes, α_X, affects trading profits only indirectly, through the increased value wedge. Thus, this positive effect is weighed against the direct cost $max[0, (\alpha_X - \bar{\alpha_X})K]$.

Analyzing (6) yields the following result.

Proposition 2. The strategic trader's optimal record date share and vote position is characterized by a long economic position, $0 \leq \alpha_H < \frac{\alpha_Z}{2+\alpha_Z}$, and an optimal number of "extra" votes $\alpha_X^* = \bar{\alpha_X}$.

The solution for H's optimal quantity of extra votes, $\alpha_X^* = \bar{\alpha_X}$, is trivial given the votes' usefulness in generating a value wedge together with the assumed cost function. The economic stake, α_H, is determined by the tradeoff between voting power and trading gains. Going short is never optimal since it confers no votes and has no effect on liquidity, but reduces future trading gains by biasing H toward value destruction. A long position reduces trading gains both by reducing liquidity and by biasing H toward value creation (the commitment effect). Indeed, if the stake gets too large (larger than $\frac{\alpha_Z}{2+\alpha_Z}$), the desire to protect the value of the stake will completely overcome any incentive to profit by trading (q^* goes to zero). Thus, when choosing an ex ante stake purchased at its expected value, H will significantly limit the size of the stake purchase. However the stake is generally positive since a long position increases H's voting power and enhances its ability to create a value wedge. Note that without further assumptions on the other shareholders' vote distributions we cannot show that there is a unique optimal α_H, just that it is weakly positive and within the specified range. Also, the only instances where H does not buy a strictly positive stake are when $\bar{\alpha_X}$ is sufficiently large and/or the distribution of Y is sufficiently concentrated that the extra votes are essentially worthless.

Now that we have the solution to H's share and vote purchase strategy, it remains to determine how H's actions will affect efficiency overall. We measure efficiency by the probability with which the correct value-maximizing decision is made on the voting date. H votes to minimize firm value with probability q, so the ex ante probability of the correct decision is

$$q\left[1 - F_G(\gamma)\right] + (1-q)\left[1 - F_G(\delta)\right]. \tag{7}$$

Here we again use the symmetry of $F_G(\cdot)$ and $F_B(\cdot)$ to express the probability in terms of $F_G(\cdot)$ alone regardless of the true state. Plugging in the equilibrium probability q^* from above and analyzing (7) yields the following result.

Proposition 3. Whenever $F_G(\cdot) = F_B(\cdot)$, the presence of the strategic trader (weakly) increases the ex ante probability of a correct decision.

Thus, despite the fact that H will seek to generate trading profits by sometimes voting the wrong way and manipulating the firm's decisions to decrease value, its presence overall is actually beneficial to the firm from an ex ante perspective whenever other shareholders' votes are uncorrelated with the true state. (The only times this is not true are in the exceptional cases discussed above where $\alpha_H^* = 0$, in which case H's presence does not affect efficiency). This is because the positive economic position H takes in order to increase voting power and help generate a value wedge causes it to vote the right way more often than not. Thus, the more rare cases where H manipulates negatively can be seen as the "price to be paid" for greater overall voting efficiency when H's information is particularly valuable. Note that if there were no liquidity and thus no possibility of trading gains ($\alpha_Z = 0$), H would have no incentive to purchase shares or votes, or to try to learn the value of the proposal. It is the possibility of trading gains introduced by noise trade that induces the information gathering and thereby increases efficiency.

The key to the unambiguous nature of Proposition 3 is that the "noise" induced by the random votes is centered around the threshold when $F_G(\cdot) = F_B(\cdot)$. In other words, the expected proportion of yes votes by atomistic stockholders is the same as the acceptance threshold, $\frac{1}{2}$. When that is not the case, it is possible for H's presence to reduce overall efficiency. Thus, a correct interpretation of Proposition 3 is that finding a negative overall efficiency effect will *require* that atomistic shareholders' votes be correlated with the actual value of the proposal. We now proceed to investigate these issues in greater depth. In order to do so we must make further assumptions on the distributions $F_G(\cdot)$ and $F_B(\cdot)$.

For the remainder of this section we assume that the underlying probability density functions, $f_G(\cdot)$ and $f_B(\cdot)$, are linear on $[0,1]$. Further, we assume that $f_G(\cdot)$ is (weakly) positively sloped while $f_G(\cdot)$ is symmetrically (weakly) negatively sloped. Thus, for good projects we have $f_G(y) = 1 + a(y - \frac{1}{2})$, where $a \in [0,2]$ is the slope parameter, and symmetrically $f_B(y) = 1 - a(y - \frac{1}{2})$. When $a = 0$ the distributions are uniform, and as a increases the probability of acceptance increases for a good proposal and decreases for a bad proposal. Using these properties, we first derive the following comparative statics for the optimal stake size.

Proposition 4. Assume $f_G(\cdot)$ and $f_B(\cdot)$ are linear on $[0,1]$. Then there is a unique optimal stake size $\alpha_H^* > 0$. The optimal stake size is increasing in α_Z, decreasing in $\bar{\alpha_X}$, and unaffected by Δv and a.

The greater is market depth (α_Z), the greater is the potential for trading profits, and the more equity H will purchase, in general, in order to take advantage. On the other hand, as votes become easier to purchase directly, i.e., $\bar{\alpha_X}$ increases, control can be achieved by other means, so a smaller economic stake is taken to reduce the commitment effect and increase future trading gains. The importance of the proposal, Δv, does not affect H's stake purchase decision. The magnitude of Δv will certainly affect the magnitude of trading profits H is able to generate, but it does not affect the trade-off between trading profits and the ability to generate a value wedge, which determines the optimal stake size. Finally, the fact that changes in a, which measures the correlation between other shareholders' votes and the true state, do not affect $\alpha_H^* > 0$ is a function of the linear distributional assumption.

Next we consider how efficiency is affected. First we derive results on how the probability of a correct decision depends on H's economic and voting positions, *without* taking into account the equilibrium choices of α_H and α_X.

Proposition 5. Assume $f_G(\cdot)$ and $f_B(\cdot)$ are linear on $[0,1]$. Then the probability of a correct decision is increasing in α_H. When $a > 0$ the probability of a correct decision is increasing in α_X for sufficiently large α_H, and decreasing in α_X for sufficiently small α_H.

An increase in α_H increases both H's voting power and the probability that H will vote to maximize firm value, tending to reinforce a positive efficiency effect. On the other hand, an increase in α_X increases only H's voting power, so its effect on overall efficiency depends on H's propensity for voting the right way, measured by the extent of its long position α_H. These findings lead directly to the following important result.

Corollary 1. *Assume $f_G(\cdot)$ and $f_B(\cdot)$ are linear on $[0,1]$. Then if $\bar{\alpha_X}$ is sufficiently low, H's presence always (weakly) increases efficiency.*

When H's voting power is closely tied to its economic stake on the record date, we see that allowing H to "play games" by sometimes selling short and voting to destroy value always increases efficiency. In other words, allowing for empty voting stakes generated between the record and voting dates always improves efficiency as long as the record date stakes are close enough to being equal, implying that the commitment effect is large. Similar to Proposition 3, the negative outcomes are the price to be paid for inducing H to participate and contribute its information to the voting process.

Next we consider how efficiency is affected as other shareholders' votes become more correlated with the true state.

Proposition 6. Assume $f_G(\cdot)$ and $f_B(\cdot)$ are linear on $[0,1]$. Then the probability of a correct decision is increasing in a, but the rate of increase decreases in both α_H and α_X.

As expected, as others' votes become more "informed," efficiency improves. However, this is attenuated when H controls more votes. As shown in Proposition 4, changes in a do not affect α_H^* in equilibrium, so Proposition 6 provides the *equilibrium* result that efficiency will not rise as quickly with a when H is present versus when there is no strategic trader.

Putting Propositions 5 and 6 together with Corollary 1 leads to the following conclusion.

Corollary 2. *Assume $f_G(\cdot)$ and $f_B(\cdot)$ are linear on $[0,1]$. Then H's presence is more likely to decrease efficiency the larger is a, and a negative efficiency effect requires a sufficiently large $\bar{\alpha_X}$.*

Not surprisingly, H is more likely to reduce efficiency as other shareholders are more likely to arrive at the correct decision on their own. Furthermore, as noted above, a negative efficiency effect requires that a significant empty voting stake be created on the record date.

Finally, we relax the assumption that H never finds it optimal to take full control of the vote, and look at the extreme case where separating votes from ownership at the record date is essentially unlimited ($\bar{\alpha_X} \geq \frac{1}{2}$). The equilibrium is as follows.

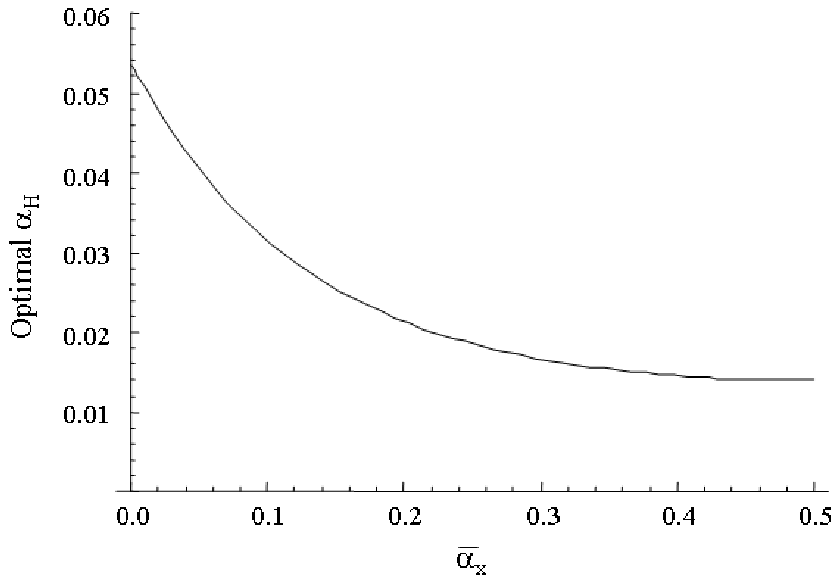
Proposition 7. Assume $\bar{\alpha_X} \geq \frac{1}{2}$ and $f_G(\cdot)$ and $f_B(\cdot)$ are linear on $[0, 1]$. Then H will not trade in the stock prior to the record date (i.e., $\alpha_H^* = 0$), but will accumulate sufficient votes to determine the election outcome (i.e., $\alpha_X^* \geq \frac{1}{2}$). The probability of H selling short and voting to minimize firm value will be $\frac{1}{2}$ in equilibrium, and H's presence will not affect ex ante efficiency if $F_G(\cdot) = F_B(\cdot)$, but will decrease efficiency if $F_G(\cdot) \neq F_B(\cdot)$.

This result reflects the fact that H's trading gains are maximized when its stake is zero. Since buying enough votes to swing the election maximizes the value wedge, $V_B - V_S$, there is no longer any reason for H to take a position in the stock on the record date. It maximizes its trading profits by buying/selling with equal probability. It is interesting to note that this is an extreme version of the model in which H's ability to manipulate firm value is maximized, yet the overall effect is neutral for ex ante firm value in the absence of correlated voting by others.

We now provide a numerical example to illustrate the above results. Continuing with the case of linear probability density functions, we consider the cases $a = 0$ (uniform), $a = 1$ (the probability of a correct decision in the absence of H is $\frac{5}{8}$), and $a = 2$ (the probability of a correct decision in the absence of H is $\frac{3}{4}$). We also set $\alpha_Z = 0.2$.

We first consider how H's optimal record-date economic ownership, α_H^*, varies as votes become easier to acquire in the lending market, i.e., as $\bar{\alpha_X}$ increases. Figure 2 below graphs this relationship. As noted in Proposition 4, α_H^* does not depend on a so this graph has a single line.

Figure 2

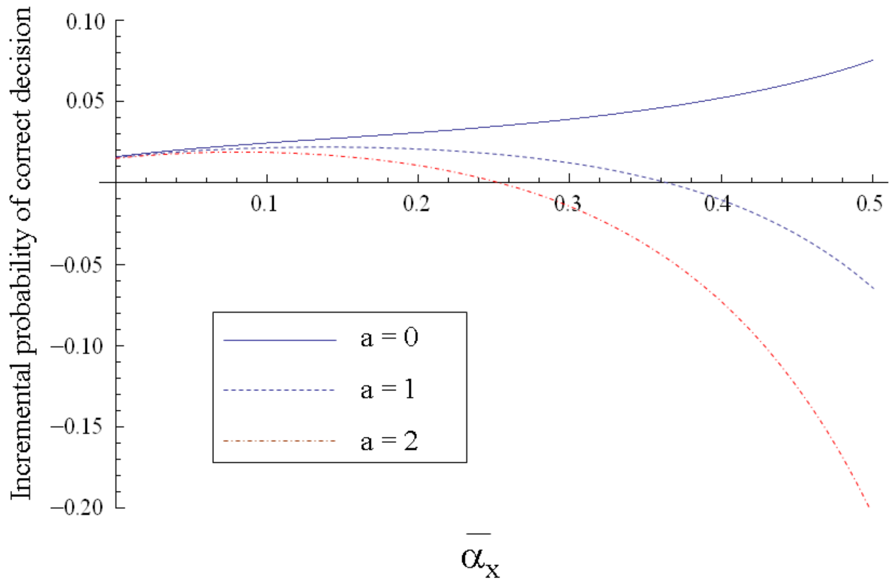


As indicated in Proposition 4, the extent of H's long position on the record date is clearly decreasing in the number of extra votes it can buy in the share lending market. In this case it falls from a maximum of about 25% of α_Z to about 8% of α_Z as α_X approaches its maximum value.

Next we consider how the ease of purchasing extra votes affects efficiency. Figure 3 graphs the relationship between the number of extra votes available, $\bar{\alpha_X}$, and the difference in the probability of a correct decision caused by the presence of H. In particular, the graph plots the probability when H plays its optimal strategy minus the probability when H is not present in the model. Here, the results depend critically on the correlation between others' votes and the true state.

When atomistic holders' votes are uniformly distributed ($a = 0$), making more votes available to H *always* improves efficiency as long as $\bar{\alpha_X} < \frac{1}{2}$. This occurs despite the fact

Figure 3



that extra votes result in a smaller economic stake (as in Proposition 4 and Figure 2 above) and therefore a smaller commitment effect and a greater probability that H will go short and vote to minimize firm value. The offsetting force is the greater voting power H has in equilibrium despite his reduced economic stake – ie, the direct increase in voting power via $\bar{\alpha_X}$ is greater in equilibrium than the decrease in α_H. This direct positive effect (with H still voting the right way more often than not) outweighs the negative effect of a greater probability of going short.

With the correlated distributions ($a = 1$, $a = 2$), H's presence still improves efficiency when α_X is sufficiently low, as shown in Corollary 1. However, as indicated by Proposition 5, the extra votes at some point start to reduce efficiency as α_H is reduced and H's increased voting power together with the reduced commitment effect works against the other shareholders, who are now fairly likely to arrive at the correct decision on their own. Ultimately, H's presence reduces efficiency overall when $\bar{\alpha_X}$ is sufficiently large, as indicated by Corollary 2.

4. Ex Ante Shareholdings

In this section, we consider how the equilibrium changes if the strategic trader holds a long position in the stock, $0 < \alpha_h < \frac{1}{2}$, prior to the announcement of a proposal. In other words, if H is already a long-term shareholder. For tractability, we assume for now that buying votes separately from shares at the record date is impossible, so $\bar{\alpha_X} = 0$. In other words, empty voting is only possible by trading away from the record date position before the voting date. We also assume throughout this section that Y is uniformly distributed on $[0, 1]$ regardless of whether the proposal is good or bad. Finally, since H still has no reason to trade to a short position on the record date, the equations below reflect $\alpha_H \geq 0$.

In this case, H takes into account the value of its existing stake when deciding how to trade prior to the record date. However, following the record date the model is solved exactly as before, with one caveat. In some cases H ends up owning more than $\frac{\alpha_Z}{2+\alpha_Z}$ shares, in which event H always votes to maximize firm value (since manipulating to produce trading profits is not optimal - i.e. the optimal q is zero when $\alpha_H \geq \frac{\alpha_Z}{2+\alpha_Z}$).

Given this, H's objective function for pre-record date trading, assuming $\alpha_H < \frac{\alpha_Z}{2+\alpha_Z}$, becomes

$$(8) \qquad \max_{\alpha_H} \frac{\left(\frac{\Delta v(\alpha_H)}{1-\alpha_H}\right)(\alpha_Z^2(1 - max[\alpha_h, \alpha_H])^2 - 4\alpha_H^2)}{8\alpha_Z(1 - max[\alpha_h, \alpha_H])} + E[\alpha_h(q^* V_S^* + (1 - q^*)V_B^*)]$$

where $\frac{\Delta v(\alpha_H)}{1-\alpha_H} = V_B - V_S$ given the properties of the uniform distribution. We use the superscript * in the second term to denote the fact that the quantities are being evaluated at their equilibrium values (derived above) given α_H. Intuitively, H is again maximizing its future trading gains, represented by the first term, but is also taking into account the effect of its future actions on the value of its initial stake, represented by the second term.

In the model with no ex ante ownership, it was never optimal for H to accumulate a stake as large as $\frac{\alpha_Z}{2+\alpha_Z}$, because in the future trading round it would then never have an incentive to manipulate and generate trading gains. However, when H has an ex ante long position, this can change. It is easy to show that if H is always going to vote to maximize firm value, then it is optimal for H to acquire enough shares to have full control over the vote outcome ($\alpha_H^* \geq \frac{1}{2}$) and ensure the right decision. This is because the shares are bought at their expected "cash flow" value, but the value of the votes conferred on the holder is not reflected in the price. Since these votes will have positive value to H, it is willing to acquire the shares.

Using this, and analyzing (8) yields the following result.

Proposition 8. There exists a threshold level of initial holdings, $\hat{\alpha_h} < \frac{\alpha_Z}{2+\alpha_Z}$, such that:
a) for all $\alpha_h < \hat{\alpha_h}$, H purchases additional shares to reach a record date position $\alpha_h < \alpha_H^* < \frac{\alpha_Z}{2+\alpha_Z}$ and plays the mixed strategy derived in Lemma 1; and
(b) for all $\alpha_h \geq \hat{\alpha_h}$, H trades to reach a record date position of $\alpha_H = \frac{1}{2}$, always votes to maximize firm value, and thus maximizes overall efficiency.

When H has initial shareholdings, it is willing to sacrifice some trading profits in order to protect the value of its existing stake, thus it favors a larger stake than in the benchmark model. If the initial stake is large enough, it gives up on future trading profits entirely, and makes sure that it has sufficient votes to guarantee the right voting outcome.

We can also derive the following comparative static for the extended model.

Proposition 9. The threshold level of initial holdings, $\hat{\alpha_h}$, is increasing in α_Z.

Just as in the benchmark model, greater market depth induces greater trading gains, which means that a larger initial stake is required to induce H to give up those gains. Together with the prior result, we can now say something about how an increase in market depth may affect overall efficiency.

Corollary 10. Consider an increase in α_Z. Then for all α_h such that $\alpha_h \geq \hat{\alpha_h}$ before the increase, but $\alpha_h < \hat{\alpha_h}$ afterwards, the increase in market depth reduces ex ante efficiency.

Increased market depth can be bad for efficiency because it might convince a large shareholder who would otherwise have given up on trading gains (and thus maximized firm value) to instead try to generate some trading gains by threatening to short sell and vote to minimize firm value.

Finally, consider what happens if H is allowed to buy votes separately from shares on the record date. For tractability, consider the extreme case where $\bar{\alpha_X} \geq \frac{1}{2}$. In this case, H can swing the election no matter its record date holding. The equilibrium is as follows.

Proposition 11. Assume $\bar{\alpha_X} \geq \frac{1}{2}$. Then H does not trade in the stock, but accumulates sufficient votes to determine the election outcome (i.e., $\alpha_h + \alpha_X^* \geq \frac{1}{2}$), and:
(a) if $\alpha_h < \frac{\alpha_Z}{2+\alpha_Z}$, H subsequently plays the mixed strategy described in Lemma 1 with $\alpha_H^* = \alpha_h$; or
(b) if $\alpha_h \geq \frac{\alpha_Z}{2+\alpha_Z}$, H always votes ex post to maximize firm value.

Comparing this result to Propostion 8, it is clear that allowing H to buy votes separately from economic ownership at the record date can reduce efficiency even when $F_G(\cdot) = F_B(\cdot)$. In Proposition 11, H ends up owning fewer than $\frac{\alpha_Z}{2+\alpha_Z}$ shares anytime $\alpha_h \leq \frac{\alpha_Z}{2+\alpha_Z}$, so it short sells and votes to minimize firm value with some probability in the future. On the other hand, in Proposition 8, in the range $\alpha_h \in [\hat{\alpha_h}, \frac{\alpha_Z}{2+\alpha_Z}]$, full efficiency is achieved since H finds it optimal to take voting control and maximize firm value. So for at least all α_h in that range, allowing the separation of votes and ownership at the record date decreases efficiency.

5. Implications

Our model is stylized, but it nevertheless provides a number of useful empirical implications. Most obviously, it implies that empty voting behavior should result in both positive and negative outcomes from an efficiency perspective. Thus, the negative anecdotes described by Hu and Black (2006, 2007) should be only one side of the story. However, it is likely that direct, large-scale evidence of favorable (or unfavorable) empty voting behavior would be difficult or impossible to gather.

Our model can also guide less direct empirical investigations. For instance, with slight adjustments to the model, we could predict which types of firms and proposals are more likely to be targeted by strategic empty voters. In particular, if we add an information cost that the strategic trader must pay after the record date to become informed about the value of the proposal, the model would predict that such traders are likely to target firms where the strategy is most profitable. This would tend to be firms with:
(a) high liquidity (high α_Z), which could be measured by trading volume or a statistic summarizing the dispersion of share ownership;
(b) potentially important proposals pending (high Δv), which could be measured either directly by looking at types of proposals, or indirectly using a proxy for the quality of corporate governance;
(c) greater availability of "extra" votes (high $\bar{\alpha_X}$), which could be measured by volume and specialness in the lending market, dispersion of ownership, or the availability of derivatives to hedge economic exposure; and
(d) voting outcomes that are uncertain (ie, the vote is fairly likely to go either way) but where relatively few votes are needed to swing the result (which would correspond to a tighter distribution of non-strategic votes).
This last point could again be related to the quality of corporate governance (outcomes are likely to be less certain when bad proposals are more likely). Another example could be a vote with a supermajority rule where a high percentage is needed for approval of the proposal. The distribution of share ownership between institutions and individuals could also be important if different types of shareholders have different information or voting incentives.

Our model would therefore predict that more empty voting activity should occur in these types of settings. Direct evidence could be sought by looking at share lending volumes around record dates and/or share trading by certain types of investors (such as hedge funds) before and after record dates. The comparative statics in Proposition 4 could also be tested in this context. Furthermore, the model provides predictions about when the actual decisions are likely to be more efficient when empty voting occurs. Thus, an indirect test of the model could focus on ex post measures of how voting outcomes affect firm value.

Finally, to the extent that firms have existing blockholders who could implement the strategy, the results of section 4 could be tested using data on those holders' trading patterns and how their behavior varies with measures of liquidity and the ease of buying extra votes.

As noted in the introduction, our results may also provide some guidance on the efficacy of proposed regulatory reforms designed to curb or eliminate the negative effects of empty voting. For example, Hu and Black (2006, 2007) advocate additional disclosure requirements as a reasonable starting point. In the framework of our model, disclosure of an empty voting position on the record date would have no effect, because we already assume that the market maker observes the strategic trader's actions at that stage. The effect of a rule requiring disclosure of a change in economic position relative to voting rights between the record and voting dates depends on how the rule is implemented. If the rule made it more difficult for the trader to hide its trades from the market maker, this would have the effect of reducing or eliminating any trading profits the strategic trader could otherwise generate. Thus, the rule could reduce efficiency if it causes the trader not to accumulate votes in the first place (which is likely if there is a cost to gathering information about the quality of the proposal) and either separating shares from votes is not very expensive *or* other shareholders' votes are not too highly correlated with the correct decision. Otherwise it is likely to improve efficiency. In the model with initial holdings by the strategic trader, the reduction or elimination of trading gains after the record date could cause the trader to forego the mixed strategy and therefore either not trade and always vote the right way, or accumulate additional votes in order to ensure the correct outcome. In either of these cases, the rule is likely to enhance efficiency.

6. Conclusion

We provide a model of empty voting in the U.S. corporate governance system. We find that allowing empty voting can improve the efficiency of corporate governance despite the fact that it may cause a strategic trader to sometimes "vote the wrong way" in order to generate trading gains. However, the practice can reduce efficiency if it confers too much control too quickly or motivates manipulation by a long-term shareholder that would not otherwise occur.

While our model provides a coherent framework for addressing the efficiency consequences of empty voting, there are a number of issues that remain unexplored. For example, it would be interesting to study how the results would change if there were multiple strategic traders who compete to generate trading gains. It would also be interesting to study the interaction between a non-shareholder and an existing large shareholder who could both act to influence the vote outcome. Finally, we would like to more closely investigate specific mechanisms by which shares and votes can be separated, such as the share lending market. For example, if the uninformed shareholders were not all atomistic, how would they analyze the decision of whether to lend their shares on the record date? How would this affect pricing in the lending market? Our framework should provide a platform for exploring these issues in the future.

Appendix

Proof of Lemma 1: Any realization of total net order flow, say n, (which is all the market maker observes) could occur in two possible ways: atomistic holders could have placed no orders while H placed an order of n, or atomistic holders could have placed an order to sell $\alpha_Z(1 - max[0, \alpha_H])$ while H placed an order equal to $n + \alpha_Z(1 - max[0, \alpha_H])$. For all n, we call each pair of possible trading quantities for H, $(n, n + \alpha_Z(1 - max[0, \alpha_H]))$ a "quantity pair," indexed by n.

In any subgame perfect equilibrium, upon observing net order flow of n the market maker must set a price that is optimal given H's equilibrium (mixed) trading strategy. Now recall that if H ends up with a long position, it will always vote to maximize firm value, while if it ends up with a short position it will always vote to minimize firm value. Because of this, if H places any mixing weight on either element of a quantity pair such that H would end up long with both orders or short with both orders in the quantity pair (ie, quantity pairs such that $n > -\alpha_H$ so that H would end up long with either quantity, or pairs such that $n + \alpha_Z(1 - max[0, \alpha_H]) < -\alpha_H$ so that H would end up short with either quantity), the optimal price for the associated order flow n must be either V_B or V_S since there is no uncertainty over H's voting behavior. Thus, H cannot generate any trading gains by placing such an order, and no such order can be part of an equilibrium strategy for H.

This implies that H will only consider trading quantities that are elements of quantity pairs such that $n + \alpha_Z(1 - max[0, \alpha_H]) > -\alpha_H$ and $n < -\alpha_H$ (hereafter "feasible pairs"), so that H would end up long after the former order and short after the latter. Next note that if H puts mixing weight on one element of a feasible pair, it must also put weight on the other element. If it did not, then in equilibrium the market maker, upon observing the associated n, would be able to infer H's final trading position, and would price the stock accordingly, so that no trading gains are possible.

Next we prove that H puts weight only on the elements of one feasible pair, corresponding to the strategy in the lemma. First note that for any given feasible pair, there is a unique price corresponding to the associated n that will make H indifferent between the two quantities in that pair. The prices will be different for different feasible pairs, and must correspond to the prices derived by plugging the mixing quantity q from equation (4) into the pricing equation $qV_S + (1-q)V_B$. Thus, the equilibrium price p_n for a given n must satisfy $p_n = (\frac{n+2\alpha_H}{\alpha_Z(1-max[0,\alpha_H])})V_S + (1 - \frac{n+2\alpha_H}{\alpha_Z(1-max[0,\alpha_H])})V_B$ (this is derived by simply plugging $\alpha_B = n + \alpha_Z(1-max[0,\alpha_H])$ into equation (4) and using this in $p = qV_S + (1-q)V_B$). This is clearly decreasing in n since $V_S < V_B$.

But from the text, we know that at these prices, the feasible pair with $\alpha_B = \alpha_B^*$ (which corresponds to $n = -(\alpha_Z(1-max[0,\alpha_H]) - \alpha_B^*)$, and which we denote hereafter as n^*) results in greater expected trading profits for each trading quantity within the pair than any trading quantity in any other feasible pair. Thus, if the pair n^* receives positive mixing probability in equilibrium, no other feasible pair can also receive positive probability. Thus, it remains to prove that the equilibrium must involve the feasible pair n^*.

We prove the result by contradiction. Suppose that there is an equilibrium in which the pair n^* is not used, but some other feasible pair, say corresponding to $n = \hat{n}$, receives positive mixing probability in equilibrium. The price when the market maker observes $n = \hat{n}$ must then equal $p_{\hat{n}}$ as derived above. Now consider whether there are prices that can be set when the market maker observes the off-equilibrium order flow n^* that will prevent any deviation by H to one of the quantities in the n^* pair. If the price p_{n^*} is set, H will deviate since the n^* node offers the greatest profits given this pricing. If a price $p < p_{n^*}$ is set, the profit to the "buy" element in the n^* pair will be higher than if p_{n^*} were the price, so deviation to that element must be optimal. Similarly, if a price $p > p_{n^*}$ is set, the profit to the "sell" element in the n^* pair will be higher, so deviation to that element must be profitable. QED

Proof of Proposition 2: The result for α_X^* is immediate since $V_B - V_S$ is clearly increasing in α_X. Next, note that when $\alpha_H < 0$, $V_B - V_S$ does not vary with α_H. Given this, it is obvious that (6) is increasing in α_H for all $\alpha_H < 0$, so $\alpha_H^* \geq 0$ follows.

Now consider $\alpha_H \geq 0$. First note that the objective function (6) will be negative for any feasible $\alpha_H > \frac{\alpha_Z}{2+\alpha_Z}$, but is positive at $\alpha_H = 0$, which proves $\alpha_H^* < \frac{\alpha_Z}{2+\alpha_Z}$. Taking the first derivative of (6) with respect to α_H yields

(9)
$$\frac{\Delta v(f_G(\delta)+f_G(\gamma))}{2(1-\alpha_H-\alpha_X)^2}\left(\frac{\alpha_Z^2(1-\alpha_H)^2-4\alpha_H^2}{8\alpha_Z(1-\alpha_H)}\right)+\Delta v(F_G(\gamma)-F_G(\delta))\left(\frac{\alpha_H(\alpha_H-2)}{2\alpha_Z(\alpha_H-1)^2}-\frac{\alpha_Z}{8}\right).$$

It is easy to show that this is positive when $\alpha_H = 0$ and $\alpha_X = 0$, so α_H^* will be positive at least for sufficiently small α_X. QED

Proof of Proposition 3: If $F_G(\cdot) = F_B(\cdot)$ then, as noted in the text, $F_G(\cdot)$ must be symmetric around $\frac{1}{2}$. This implies both that the probability of approval without H is $\frac{1}{2}$ and that $F_G(\delta) = 1 - F_G(\gamma)$. Thus, if $\alpha_H = 0$, the optimal mixing probability is $q^* = \frac{1}{2}$, and the probability of a correct decision becomes $\frac{1}{2}F_G(\delta) + \frac{1}{2}(1 - F_G(\delta)) = \frac{1}{2}$, so H does not affect efficiency regardless of α_X. Thus, it suffices to show that (7) is increasing in α_H for any feasible α_X. Taking the derivative of (7) with respect to α_H, noting that $f_G(\gamma) = f_G(\delta)$ when $F_G(\cdot)$ is symmetric around $\frac{1}{2}$, and rearranging yields

$$\frac{F_G(\gamma)-F_G(\delta)}{\alpha_Z(\alpha_H-1)^2}+\frac{f(\gamma)(1-2q^*)}{2(1-\alpha_H-\alpha_X)^2}>0, \tag{10}$$

where the inequality follows from $q^* < \frac{1}{2}$. QED

Proof of Proposition 4: To prove $\alpha_H^* > 0$ we evaluate (9) using the properties of the linear distribution. Setting α_H to zero and simplifying, (9) becomes

$$\Delta v\frac{\alpha_Z}{8}\left(\frac{1-\alpha_X(1-\alpha_X)}{(1-\alpha_X)^2}\right)>0,$$

where the inequality follows from $\alpha_X < \frac{1}{2}$. To prove there is a unique maximum, it suffices to show that (6) is concave in the relevant range, $\alpha_H \in [0, \frac{\alpha_Z}{2+\alpha_Z}]$. This is straightforward (if algebraically tedious) to show by taking the derivative of (9) with respect to α_H and simplifying using the formula for the linear density function.

Next note that with the linear density function, $V_B - V_S$ simplifies to $\frac{\alpha_H+\alpha_X}{1-\alpha_H-\alpha_X}$. Thus, it is independent of a, so α_H^* will also be independent of a. This proves the last comparative static result.

For the first two comparative static results, it suffices to sign the appropriate cross partial derivative. Taking the derivative of (9) with respect to α_Z yields the cross-partial

(11)
$$\frac{1}{8}\left[\frac{\Delta v(f_G(\delta)+f_G(\gamma))}{2(1-\alpha_H-\alpha_X)^2}\left(1-\alpha_H+\frac{4\alpha_H}{\alpha_Z^2(1-\alpha_H)}\right)+\Delta v(F_G(\gamma)-F_G(\delta))\left(-1+\frac{4\alpha_H(2-\alpha_H)}{\alpha_Z^2(1-\alpha_H)^2}\right)\right]>0.$$

The inequality is derived as follows. It is straightforward to show that $\frac{\Delta v(f_G(\delta)+f_G(\gamma))}{2(1-\alpha_H-\alpha_X)^2} \geq 4\Delta v(F_G(\gamma)-F_G(\delta))$ given a linear density function and $\alpha_H+\alpha_X<\frac{1}{2}$, which we have assumed. The result then follows from the fact that $\left(-1+\frac{4\alpha_H(2-\alpha_H)}{\alpha_Z^2(1-\alpha_H)^2}\right)\geq -1$ clearly always holds, while $\left(1-\alpha_H+\frac{4\alpha_H}{\alpha_Z^2(1-\alpha_H)}\right)\geq\frac{4}{5}$ must hold given that $\alpha_H<\frac{1}{5}$ in equilibrium according to Proposition 2 given our assumption that $\alpha_Z<\frac{1}{2}$.

Next, taking the derivative of (9) with respect to α_X and using the properties of the linear density yields the cross-partial

(12)
$$\frac{\Delta v}{(1-\alpha_H-\alpha_X)^2}\left[\frac{2}{(1-\alpha_H-\alpha_X)}\left(\frac{\alpha_Z^2(1-\alpha_H)^2-4\alpha_H^2}{8\alpha_Z(1-\alpha_H)}\right)+\left(\frac{\alpha_H(\alpha_H-2)}{2\alpha_Z(\alpha_H-1)^2}-\frac{\alpha_Z}{8}\right)\right]<0.$$

The inequality is derived as follows. Consider the terms in the square brackets. The last term is clearly negative, while the first is positive. Now note that in the first order condition $[(9)=0]$, $\left(\frac{\alpha_H(\alpha_H-2)}{2\alpha_Z(\alpha_H-1)^2}-\frac{\alpha_Z}{8}\right)$ multiplies $\Delta v(F_G(\gamma)-F_G(\delta))$, while $\left(\frac{\alpha_Z^2(1-\alpha_H)^2-4\alpha_H^2}{8\alpha_Z(1-\alpha_H)}\right)$ multiplies $\frac{\Delta v(f_G(\delta)+f_G(\gamma))}{2(1-\alpha_H-\alpha_X)^2}$. As above, we have $\frac{\Delta v(f_G(\delta)+f_G(\gamma))}{2(1-\alpha_H-\alpha_X)^2}\geq 4\Delta v(F_G(\gamma)-F_G(\delta))$, so, from the first order condition we must have $\left[-\left(\frac{\alpha_H(\alpha_H-2)}{2\alpha_Z(\alpha_H-1)^2}-\frac{\alpha_Z}{8}\right)\right]\geq 4\left(\frac{\alpha_Z^2(1-\alpha_H)^2-4\alpha_H^2}{8\alpha_Z(1-\alpha_H)}\right)$ in equilibrium. This proves the result since $\frac{2}{(1-\alpha_H-\alpha_X)}\leq 4$.

For the third comparative static result, note that when setting (9) equal to zero and solving for α_H, Δv will drop out of the solution. QED

Proof of Proposition 5: Taking the derivative of (7) with respect to α_H using the linear density function yields

$$\frac{a(\alpha_H+\alpha_X)(-1+\alpha_H)^2\alpha_Z-4(-1+\alpha_H+\alpha_X)(2\alpha_H^2+2\alpha_H(-1+\alpha_X)+(-1+\alpha_X)\alpha_X)}{4(-1+\alpha_H+\alpha_X)^3(-1+\alpha_H)^2\alpha_Z}>0.$$

The inequality is derived as follows. The denominator is clearly negative. The numerator is maximized at $a=2$. It is straightforward (though algebraically tedious) to show that the numerator is negative at $a = 2$ given $\alpha_X + \alpha_H < \frac{1}{2}$ and $\alpha_H < \frac{1}{5}$, so it must be negative for all a, which proves the result with respect to α_H.

Taking the derivative of (7) with respect to α_X yields

$$(1-q^*)\left(\frac{f_G(\delta)}{2(1-\alpha_H-\alpha_X)^2}\right) - q^*\left(\frac{f_G(\gamma)}{2(1-\alpha_H-\alpha_X)^2}\right).$$

Since $f_G(\cdot)$ is linear and increasing, we have $f_G(\delta) \leq f_G(\gamma)$. Also, as α_H falls from $\frac{\alpha_Z}{2+\alpha_Z}$ to zero q^* rises from zero to $\frac{1}{2}$. The result for α_X follows. QED

Proof of Proposition 6: Taking the derivative of (7) with respect to a using the linear density yields $\frac{1-2(\alpha_H+\alpha_X)}{8(1-\alpha_H-\alpha_X)^2} > 0$. The derivative of this with respect to α_H or α_X equals $-\frac{\alpha_H+\alpha_X}{4(1-\alpha_H-\alpha_X)^3} < 0$. QED

Proof of Proposition 7: With $\bar{\alpha_X} \geq \frac{1}{2}$, votes are free up to the point where H takes complete control of the vote. It is easy to see that the value wedge H can create, $V_B - V_S$, is maximized when H can singlehandedly determine the outcome. Furthermore, its maximized value must equal Δv. Thus, H will acquire at least $\frac{1}{2}$ of the votes, and the value wedge will no longer depend on α_H. H's maximization problem with respect to α_H is therefore given by (note that there is still no incentive to go short, as the analysis in the proof of Proposition 2 above is valid here):

$$\max_{\alpha_H} \frac{(\Delta v)(\alpha_Z^2(1-\alpha_H)^2 - 4\alpha_H^2)}{8\alpha_Z(1-\alpha_H)}. \tag{13}$$

It is easy to show that this is decreasing in α_H. Finally, with respect to the efficiency results, note that the probability of approval is $\frac{1}{2}$ with or without H's presence when $F_G(\cdot) = F_B(\cdot)$. On the other hand, when $F_G(\cdot) \neq F_B(\cdot)$, Proposition 5 implies that the probability of a correct decision is decreasing in α_X when $\alpha_H = 0$. QED

Proof of Proposition 8: We need to treat the cases $\alpha_H > \alpha_h$ and $\alpha_H < \alpha_h$ separately. First consider the latter case (where H is selling). In this case, the derivative of the objective function with respect to α_H equals:

$$\Delta v(4\alpha_H[(4\alpha_h - 3\alpha_H) + 2\alpha_H(\alpha_H - \alpha_h)] + \alpha_Z^2(1-\alpha_H)^2) \over 8\alpha_Z(1-\alpha_h)(1-\alpha_H)^2} > 0. \tag{14}$$

Now consider the former case. Assume H's problem has an optimum at some $\alpha_h < \alpha_H^* < \frac{\alpha_Z}{2+\alpha_Z}$. Evaluating the objective function (8) assuming $\alpha_H > \alpha_h$ and taking the derivative with respect to α_H yields

$$\frac{\Delta v\left[(\alpha_H - 1)^4\alpha_Z^2 - 4\alpha_H(\alpha_H - 1)(4\alpha_h + \alpha_H(\alpha_H - 3))\right]}{8\alpha_Z(\alpha_H - 1)^4}. \tag{15}$$

Taking the second derivative with respect to α_H yields

$$\frac{\Delta v(2\alpha_h + \alpha_H(4\alpha_h - 3))}{\alpha_Z(\alpha_H - 1)^4}. \tag{16}$$

This will be positive for all $\alpha_H \in [0, \frac{2\alpha_h}{3-4\alpha_h}]$ but negative for higher α_H. Thus, the objective function is either always convex or convex then concave as α_H rises from zero over the relevant range.

Evaluating (15) at $\alpha_H = \alpha_h$ yields

$$\frac{\Delta v(\alpha_Z^2(\alpha_h - 1)^3 - 4\alpha_h^2(1 + \alpha_h))}{8\alpha_Z(\alpha_h - 1)^3} > 0, \tag{17}$$

so we must have $\alpha_H^* > \alpha_h$.

Now consider the possibility that the optimal solution has $\alpha_H \geq \frac{\alpha_Z}{2+\alpha_Z}$. In this case, H will always find it optimal to vote in favor of maximizing firm value (the probabiliy of going short and voting to minimize value goes to zero as α_H approaches $\frac{\alpha_Z}{2+\alpha_Z}$). But since shares prior to the record date sell at their expected value, if H expects to always vote the right way, it must be optimal to acquire enough shares so that H's voting power is at least $\frac{1}{2}$. The expected payoff to this strategy, the "always vote to maximize value" strategy, equals the expected increase in the value of H's initial stake, α_h, due to the higher probability of a correct decision.

Now consider the overall equilibrium. From the previous analysis, we know that when $\alpha_h = 0$ there is a strictly interior maximum with $\alpha_H^* < \frac{\alpha_Z}{2+\alpha_Z}$. Thus, for small enough α_h this will still be true. As α_h increases, the optimal α_H must increase (the cross partial derivative of the objective function with respect to α_H and α_h equals $-\frac{2\Delta v \alpha_H}{\alpha_Z(\alpha_H-1)^3} > 0$). Thus, at some $\alpha_h < \frac{\alpha_Z}{2+\alpha_Z}$, say α_h^*, we will have $\alpha_H^* = \frac{\alpha_Z}{2+\alpha_Z}$. Now note that at that point, H will always vote to maximize firm value, but does not have enough voting power to swing the election, thus the "always vote to maximize firm value" strategy where it instead acquires enough shares to reach α_H must be better. Since the payoff to this strategy is zero at $\alpha_h = 0$ and increasing in α_h, there must be some $\hat{\alpha_h} \in [0, \alpha_h^*]$ where the profit of the "always vote to maximize value" strategy equals the profit of the interior trading strategy, which proves the result. QED

Proof of Proposition 9: The result follows directly from the fact that, for any $\alpha_h < \alpha_h^*$ and $\alpha_H < \frac{\alpha_Z}{2+\alpha_Z}$, the profitability of playing the mixed strategy increases in α_Z, while the profitability of the "always vote to maximize value" strategy is not affected by α_Z. QED

Proof of Proposition 11: As in Proposition 7, it is easy to show that when votes are free, H will always buy enough to have voting power of at least $\frac{1}{2}$. From the results above, it is immediate that if $\alpha_h \geq \frac{\alpha_Z}{2+\alpha_Z}$, H will find it optimal to simply vote to maximize the value of the firm and enjoy the gain in the expected value of its shares. Thus, there is no incentive for trading.

Now consider cases with $\alpha_h < \frac{\alpha_Z}{2+\alpha_Z}$. Here, H can swing the election, so if it chooses to play the mixed strategy, it can maximize the value wedge at $V_B - V_S = \Delta v$. Its objective function, assuming a stake $\alpha_H < \frac{\alpha_Z}{2+\alpha_Z}$, is then

$$\max_{\alpha_H} \frac{\Delta v(\alpha_Z^2(1 - max[0, \alpha_H])^2 - 4\alpha_H^2)}{8\alpha_Z(1 - max[0, \alpha_H])} + E[\alpha_h(q^* V_S^* + (1 - q^*)V_B^*)]. \tag{18}$$

First assume $\alpha_H < \alpha_h$. The derivative of (18) with respect to α_H is then

$$\frac{\Delta v(\alpha_h - \alpha_H)}{\alpha_Z(1 - \alpha_h)} > 0, \tag{19}$$

so we must have $\alpha_H^* \geq \alpha_h$ if the solution is interior.

Now assume $\alpha_H \geq \alpha_h$. It is easy to show that (18) is concave over the relevant range. The derivative of (18) with respect to α_H evaluated at $\alpha_H = \alpha_h$ equals

$$\frac{\alpha_h^2 \Delta v}{2\alpha_Z(1-\alpha_h)^2} - \frac{\Delta v \alpha_Z}{8}, \tag{20}$$

which is negative for all $\alpha_h < \frac{\alpha_Z}{2+\alpha_Z}$, so we must have $\alpha_H^* = \alpha_h$ if the solution is interior. The final step is to prove that the interior solution exceeds the payoff to always voting the right way – but this must be true since, in this case, the vote outcome will be determined by H either way, so there is no discrete difference in payoffs for setting $\alpha_H = \frac{\alpha_Z}{2+\alpha_Z}$ and always voting to maximize value versus setting $\alpha_H = \frac{1}{2}$ and always voting to maximize value. QED

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