

# Optimal Equity Stakes and Corporate Control

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I show that firms may optimally sell blocks of their own equity to other firms in anticipation of future corporate control activity. In the model, a target and one potential acquirer, who may also be an alliance partner, can negotiate before synergy values are learned. I find that equity implements an optimal mechanism, allowing the partners to extract surplus from outside bidders who may arrive later. The stake is limited by the outsiders' willingness to investigate. The results imply that corporate control may motivate an equity sale even when no takeover activity is apparent at the time or occurs *ex post*. (*JEL* D44, G32, G34)

Young firms often sell blocks of their own equity to other firms.<sup>1</sup> Generally, the firms involved either avoid discussing control issues altogether or claim that no further corporate control activity is planned. However, the issuing firm is often acquired some time later by the block purchaser or a third party.<sup>2</sup> This raises several interesting questions. Are these equity sales driven by the possibility of future corporate control activity? If so, when are such sales likely to be observed and how are stake sizes and prices determined?

In this article, I show that an equity sale between a target and potential bidder can be used to extract surplus from other bidders as efficiently as possible. The equity transaction creates an optimal toehold that shapes the stakeholder's behavior if there is a subsequent auction. While several prior studies have shown that a bidder with an exogenous toehold will overbid in a takeover contest (Burkart (1995), Singh (1998), and Bulow, et al. (1999)), I show that this effect can be exploited by a target firm in

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<sup>1</sup> Allen and Phillips (2000) find 227 instances of direct sales of significant blocks of equity to corporations in the period from 1980 to 1991. The average block size is 15% of the seller's equity. This figure does not include transactions involving private selling firms or transactions where the selling firm was taken over within 2 years.

<sup>2</sup> Allen and Phillips (2000) state that 84 of the 402 issuers in their sample of block purchases are acquired in the period beginning 2 years after the initial purchase, with 11 of those 84 involving acquisitions by the block purchaser (the initial purchases are not all direct sales, however). They do not provide information on how many issuing firms are acquired within 2 years of the block purchase; these transactions are excluded from their sample. Also, their sample does not include private issuers.

partnership with a potential acquirer to implement a jointly optimal selling mechanism.

The optimality of equity arises from the sequential nature of the target's relationship with its bidders in the model. In particular, one bidder arrives before any information can be learned about the value of a takeover. Other likely bidders can be identified only after the resolution of some uncertainty.

To understand the relevance of this sequentiality, it is important to note that many equity sales coincide with the formation of a strategic alliance or other operational relationship.<sup>3</sup> In a typical scenario, a young, innovative firm enters an alliance with an established firm and sells it a minority equity stake at the same time. For example, NCR Corp. formed a "joint development arrangement" with Teradata Corp. in March of 1990 and simultaneously purchased a 9% equity stake in Teradata. In December of 1991, NCR acquired the remainder of Teradata. At the time of the merger, the CEO of NCR stated that "This merger is a logical step due to the success of our existing joint development organization. . ." (PR Newswire, December 2, 1991). In this case, the seller was eventually acquired by its partner. In many other cases, the innovative firm remains independent or is acquired by a different firm. For example, Amazon.com took a 16.6% stake in Ashford.com in December of 1999 as part of a deal to sell luxury goods on Amazon.com (The Economic Times, December 2, 1999). In September of 2001, Ashford.com agreed to be acquired by Global Sports Inc. (Anderson (2001)).

These facts suggest a general scenario with a young, innovative target firm that is developing a new product or market with uncertain final characteristics. The value of the firm in a future takeover depends on these characteristics, and therefore cannot be discovered until the uncertainty is resolved. However, at an early stage the firm is able to identify an established firm that is a natural alliance partner, and, as a result, a likely future acquirer.

If the target firm in this scenario could bargain with *all* potential bidders prior to the resolution of uncertainty, it would generally like to do so. It is well known that a seller with the ability to commit to a sale process can extract all of the expected surplus in such a situation. For example, it can commit to run an efficient auction and require an up-front entry fee from each potential bidder equal to its *ex ante* expected payoff. However, there are several reasons why this could be difficult for an innovative firm. First, the identities of some potential bidders could be difficult to discern at an early stage because of product or market uncertainties. Second, if the established partner's participation in an alliance is important for the target's future value in a takeover, the partner could have the power

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<sup>3</sup> See Allen and Phillips (2000) and Pablo and Subramaniam (2002).

to force exclusive negotiation. Finally, it could be costly to attempt a negotiation with multiple parties, particularly if valuable proprietary information would be revealed. On the other hand, an early alliance partner is likely to be a natural equity partner since the two parties will take the possibility of future control activity into account when forming their alliance.

The model is designed to capture this scenario as succinctly as possible. I assume that each potential acquirer has an unknown, private synergy value for the target in a future takeover. The target can identify and approach one, and only one, of the potential acquirers, the “insider,” before synergy values can be learned. The target and insider may also have an operational relationship such as a strategic alliance, but that relationship is not explicitly modeled.

When the target and insider negotiate *ex ante*, they can commit to a selling mechanism that will govern any future trade in the target’s assets. I allow them to choose the mechanism from a very general class, and assume that they can bargain efficiently. Thus, they behave as a single entity, or coalition, in designing the selling mechanism and then split the resulting joint surplus with an up-front transfer payment.

As a coalition the partners perceive that they will either retain the firm’s assets and realize the insider’s synergy value, or sell the assets to one of several outsiders with random private values. The analysis shows that their optimal mechanism corresponds to running a second-price auction for the outside bidders, with an optimal reserve price that is set as a function of the insider’s final synergy value (which is privately known by the insider following the resolution of uncertainty). The aggressiveness of the reserve price is constrained by the outsiders’ willingness to engage in a costly investigation of the target. If no outsider meets the reserve, the target’s assets are transferred to the insider.

Next I show that the partners can generate the same outcome by transferring a block of the target’s equity to the insider up front and committing to use a second-price or equivalent auction later, where the later auction involves all willing bidders, including the insider. This arrangement allows the use of a standard auction format but does not require cooperation between the target and insider at the auction stage. Intuitively, the contingent reserve price from the optimal mechanism described above is implemented in practice via the insider’s bid in the second-price auction. The up-front equity transfer ensures that the insider’s bid is appropriately inflated relative to its value. Even though the insider’s synergy value is private and nonverifiable, and it behaves opportunistically in the auction, stake ownership binds it to behave optimally for the coalition. Thus, I show that the previously documented overbidding effect of a toehold can be mapped precisely into the coalition’s optimal mechanism, allowing them to attain their joint maximum in a realistic way.

It is also important to note that if the future takeover contest were exogenously specified as a second-price or English auction, as is generally assumed in the takeover literature, then the equity stake derived in this article would represent an optimal contract between the target and insider. The result is therefore applicable whether or not one believes that the bargainers can commit to a future auction format.

The model is consistent with existing evidence that abnormal returns around corporate equity purchases are closely related to future control events (see Mikkelson and Ruback (1985) and Choi (1991)).<sup>4</sup> It also implies that a stake sale can be motivated by the extraction of surplus in a future takeover even if it is not actually followed by a takeover, since the sale occurs before synergy values are known. In other words, *ex post* control outcomes do not imply *ex ante* motivations. Thus, the model is consistent with Allen and Phillips' (2000) finding that many firms remain independent after selling stakes to other firms.

The analysis also provides testable results for observed stake sizes. Since the outsiders must be induced to pay an investigation cost, the optimal stake size for the insider is defined by the point at which any additional equity would make investigation unprofitable for the outsiders. Thus, the stake size naturally decreases in the outsiders' investigation costs. This implies that observed stake sizes should be closely related to how difficult it is for an outsider to learn about the value of a takeover. On the other hand, the stake size increases in the probability that a takeover turns out to be profitable, as this increases the outsiders' expected payoff and offsets their investigation costs. This is consistent with existing evidence that firms entering alliances have larger abnormal returns when they simultaneously sell equity to their partner (Pablo and Subramaniam (2002)).

The optimal stake size is also sensitive to changes in the *ex ante* probability distributions of the potential acquirers' synergy values. In particular, I show that any shift of the insider's distribution to one that is stochastically dominant (in the first-order sense) will decrease the optimal stake size. Intuitively, as the insider's distribution "improves," the outsiders' expected payoffs decrease, causing their participation constraints to bind at a lower stake size. This implies that observed stake sizes should be larger when the stake purchaser is expected to be a relatively weak competitor.

Existing evidence shows that private placements are generally priced at a discount to the issuer's stock price, while trades of existing blocks are priced at a premium (see Hertzler, et al. (2002), Barclay and Holderness (1989) and Barclay, et al. (2001)). My model provides specific predictions relevant to the subset of private placements that involve strategic purchasers. Under the assumption that a standard second-price auction is held in case of

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<sup>4</sup> Note that these studies do not focus exclusively on stakes purchased directly from the issuing firm.

disagreement between the target and insider, I find that the stake will always sell for a discount to the *post-trade* market price. The discount compensates the insider for its provision of the “public good” of overbidding, which is *ex post* costly in states where it wins and has to pay more for the remaining equity than it is worth. The result can also be understood by noting that a third party buying equity *after* the insider acquires its stake would be willing to pay more per share because it can enjoy the benefits of the overbidding while suffering none of the costs. It is important to note, however, that a premium can be generated under different assumptions about what happens following disagreement.

## 1. Related Literature

There has been significant prior work on the importance of block ownership for corporate control transactions. In particular, several articles have investigated the impact of *exogenous* toeholds in takeover contests. Burkart (1995) and Singh (1998) show that a bidder with a toehold will optimally overbid in a private values takeover auction, which forms the basis for the optimality of equity in my setting. Similarly, Eckbo and Thornton (2005) show that a bank/bidder coalition will optimally overbid in a bankruptcy auction because of the bank’s status as residual claimant. Bulow, et al. (1999) show that toeholds will also cause aggressive bidding in common value auctions, but bidders without toeholds will optimally respond by bidding more conservatively to avoid the winner’s curse. Klemperer (2002), among others, suggests that toehold ownership can cause other potential bidders to avoid takeover contests. Burkart, et al. (2000) show that an incumbent blockholder and a potential acquirer of the firm may choose to trade the block rather than compete in a tender offer contest. Grossman and Hart (1980) and Shleifer and Vishny (1986) argue that block ownership can make a takeover attempt more likely if there are free rider problems among atomistic target shareholders.

Shleifer and Vishny (1986) also contribute to a line of research addressing the optimality of *open market* toehold purchases by prospective bidders. They show that if shareholdings are initially dispersed, it will not pay a potential blockholder to accumulate a block unless they can trade secretly. In a closely related article, Kyle and Vila (1991) show that noise trader activity can make open market purchases by a potential raider more profitable, and increase the probability of a takeover. Other studies show that open market purchases may not be profitable despite having to buy fewer shares if a takeover is successful (see Ravid and Spiegel (1999), Chowdry and Jegadeesh (1994), and Goldman and Qian (2005)).

A few other articles in the corporate control literature also endogenize the formation of an equity block, but in settings that are quite different from mine. Bulow, et al. (1999) (see above) discuss a target firm’s incentive

to “level the playing field” by selling a stake cheaply to a bidder without a toehold assuming another bidder already owns a toehold. Their analysis exogenously imposes an equity contract, and does not claim to explain stake sales apart from an immediate control contest. Zingales (1995) and Bebchuk and Zingales (2000) study an entrepreneur’s incentives to allocate control and cash flow rights between himself and dispersed outside investors in anticipation of a future transfer of control. Che and Lewis (2002) show that a target firm may sell a toehold directly to a potential bidder to encourage it to enter or initiate a takeover contest when there are significant investigation costs.

There are other explanations for corporate equity purchases that are not based on corporate control. Dasgupta and Tao (2000), Van den Steen (2002), and Harbaugh (2001) all construct models showing how equity stakes can solve traditional hold-up problems in cooperative relationships. Equity sales paired with strategic alliances and other cooperative relationships have received considerable attention recently because of their increasing popularity.<sup>5</sup> Empirical work by Pablo and Subramaniam (2002) and Allen and Phillips (2000) generally supports the hold-up theories, but is also consistent with my model (see the discussion in Sections 3 and 4 below). Related strands of literature have considered the role of corporate venture capital in the financing of start-up firms (Hellmann (2002) and Fulghieri and Sevilir (2003)) and the industrial organization implications of cross ownership among direct competitors (see, e.g. Davidson and Deneckere (1984), Reynolds and Snapp (1986), Farrell and Shapiro (1990), and Malueg (1992)).

Finally, the sequential mechanism derived in this article relates to recent work by Povel and Singh (2006). These authors show that when bidders are asymmetrically informed about a target’s value, the target will find it optimal to approach the bidders sequentially, first trying to strike a deal with the better informed bidder. They assume that the selling mechanism is chosen after all bidders have received their signals of value, and thus the sequential nature of their equilibrium is driven by an *ex post* disparity in the quality of information, as opposed to my model where the sequentiality of the mechanism arises from the early arrival of one bidder. They also rule out the possibility of a toehold.

The article proceeds as follows. Section 2 describes the model. The main results on the optimality of equity and a second-price auction are derived in Section 3. Section 4 provides comparative statics results for the optimal

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<sup>5</sup> Pablo and Subramaniam (2002) find that 12.8% of the 759 alliances in their sample involve a direct equity sale. In a sample of 402 purchases of blocks of equity by corporate acquirers, Allen and Phillips (2000) report that approximately 37% of the deals involve some form of product market cooperation between the firms. Andersen Consulting estimates that the total value of strategic alliances will approach \$25 to \$40 trillion by 2004. Robinson (2003) finds that the number of alliances and joint ventures has been growing at a rate of approximately 16% per year since 1985.

stake size and considers how the price paid for the equity stake compares to the market price of the target's shares. Section 5 concludes. All proofs are collected in the Appendix.

## 2. The Model

Consider a young, innovative firm, hereafter the "target" or  $T$ , that may be a valuable future takeover target for several other firms. Without loss of generality, its stand-alone value is normalized to zero. It is an all-equity firm and there is one perfectly divisible share of equity outstanding.

The target interacts with  $N + 1$  potential acquirers over three stages. Each potential acquirer has a random future synergy value for the target's assets that is unknown at stage 1.<sup>6</sup> The uncertainty over synergy values relates to product or market characteristics that will be determined by stage 2. Enjoyment of the synergies requires the acquisition of 100% of the target's assets in stage 3.

The joint distribution of future synergy values is characterized by two possible regimes. In the no synergy regime all firms have zero or negative synergies with the target. In the positive synergy regime some subset of the potential acquirers have independently distributed, nonnegative private synergy values.<sup>7</sup> The random variable  $Z \in \{z_-, z_+\}$  indexes the synergy regime, i.e.  $Z = z_+$  signifies the positive synergy regime and  $Z = z_-$  signifies the no synergy regime. It is distributed such that  $\Pr[Z = z_+] = s$  and  $\Pr[Z = z_-] = (1 - s)$ . The realization of  $Z$  becomes known in stage 2.

One potential bidder, the "insider" or  $I$ , always has positive synergies with the target in the positive synergy regime.<sup>8</sup> The target is able to approach and negotiate with the insider in stage 1. The  $N$  "outsiders" are denoted by  $O_j$ , where  $j \in \{1, 2, \dots, N\}$ , and can be one of two types, high and low. Only high types can have positive synergies with the target. Types are unknown in stage 1, at which point the outsiders are not available for negotiation. The outsiders can learn their types at the beginning of stage 2, before  $Z$  becomes known, at a cost  $c$ . This cost represents any resources expended in investigating the target, including the opportunity cost of management time.

The outsiders' types are distributed as follows. Let  $\Omega$  represent the set of all possible random vectors  $\omega = (\omega^1, \omega^2, \dots, \omega^N)$  where  $\omega^j = 1$  if outsider  $O_j$  is a high type and  $\omega^j = 0$  otherwise. I assume the vectors  $\omega$  are distributed such that each outsider faces a symmetric probability structure

<sup>6</sup> The source of any takeover synergies is intentionally left unspecified. The term is intended to represent any operational or financial benefit that may arise from control of the target's assets.

<sup>7</sup> This structure is used to introduce uncertainty over whether a takeover will occur in the simplest possible way without loss of insight. It is similar to the structure of uncertainty in Fishman (1988).

<sup>8</sup> The insider may also have an unmodelled strategic alliance or other operational relationship with the target, which could account for the fact that it is a more likely possible acquirer.

of the relevant possible events. In other words, each outsider faces the same *ex ante* probability of being a low type, of being the only high type, of being one of two high types, etc. For example, they could each have an equal and independent probability of being a high type.

Potential acquirer *i*'s synergy value is represented by the random variable  $V_i$ , where  $i \in \{I, O_1, O_2, \dots, O_N\}$ . Synergy realizations cannot be discovered until the end of stage 2, after  $Z$  becomes known. The insider's value  $V_I$  is independently distributed according to the continuous distribution function  $F_I(\cdot)$  conditional on the positive synergy regime ( $Z = z_+$ ), with corresponding density function  $f_I(\cdot)$  and support  $[0, \lambda_I]$ . All high type outsiders have i.i.d. synergy distributions  $F_O(\cdot)$  conditional on  $Z = z_+$  with density  $f_O(\cdot)$  and support  $[0, \lambda_O]$ .<sup>9</sup> Throughout the article I assume that the monotone hazard rate assumption holds for both distributions, i.e.  $H_i'(v) \geq 0$  for all  $v \in [0, \lambda_i]$  and all  $i \in \{I, O\}$ , where  $H_i(v) = \frac{f_i(v)}{1-F_i(v)}$  is the hazard function. This is a standard assumption that is satisfied by many common distributions, including the uniform, exponential, normal, logistic, extreme-value, chi-square, chi, and Laplace distributions, as well as any truncations thereof.

Conditional on the negative synergy regime ( $Z = z_-$ ), the insider and all high type outsiders have zero synergies with the target. I assume that low type outsiders have negative synergies regardless of the regime, and of sufficient magnitude to rule out the possibility of an outsider trying to acquire the target's assets without first learning its type.<sup>10</sup>

Managers and original shareholders are uniquely identified with their firm, so all references to actions by a firm refer to actions by its managers and all references to a firm's payoffs refer to the payoffs of its original shareholders. I also assume that all managers act in the interests of their current shareholders and that the structure of the game is common knowledge. All parties are risk-neutral, and risk-free interest rates are zero.

The game's timeline is as follows.

In **stage 1**,  $T$  discovers the identity of potential acquirer  $I$  and can approach it to negotiate an agreement detailing a "takeover mechanism" (defined below) that will be used to govern any transfer of the target's assets in stage 3.<sup>11</sup> They can also agree to an up-front transfer payment from the insider to the target, denoted as  $P$ . If the payment is positive, it is immediately distributed to the target's previously existing shareholders

<sup>9</sup> I assume that both density functions are continuous and twice continuously differentiable.

<sup>10</sup> For example, for a given outsider  $O_j$  it is sufficient to assume that its synergy as a low type conditional on  $Z = z_+$  is less than  $-\frac{\Pr[\omega^j=1|Z=z_+]\lambda_O}{\Pr[\omega^j=0|Z=z_+]}$ . In words, it would have a negative expected payoff even if it could acquire the target for a price of zero.

<sup>11</sup> If the insider and target are also engaged in a strategic alliance, this could explain the relative ease with which the insider is identified by the target.

following consummation of the agreement. The details of any agreement become public knowledge immediately.

The negotiation is modeled as generalized Nash bargaining (see Svejnar (1986)) with bargaining powers of  $\theta_T$  for the target and  $(1 - \theta_T)$  for the insider. The firms' disagreement payoffs, or threat points, are denoted generically as  $D_T$  for the target and  $D_I$  for the insider. Specific formulations of the disagreement payoffs will be considered in Section 4, where I analyze pricing issues; for now I simply assume that there is scope for agreement, i.e. the bargainers' maximized joint surplus exceeds the sum of their disagreement payoffs. I show later that this assumption will hold very generally, even when the target is allowed to choose from a general class of mechanisms following disagreement.

A takeover is modeled as a payment to the target firm in exchange for its assets, where the payment is then distributed proportionally to the existing equity holders as a terminating dividend. To define the space of allowable takeover mechanisms, I use a modified version of the standard mechanism design problem outlined in Chapter 5 of Krishna (2002). Let the vector  $\varpi = (\varpi^1, \varpi^2, \dots, \varpi^N)$  where  $\varpi^j = 1$  if outsider  $O_j$  is a high type and chooses to participate, and  $\varpi^j = 0$  otherwise. For a given  $\varpi$ , a "conditional mechanism"  $(\beta^\varpi, \Gamma^\varpi, \mu^\varpi)$  consists of the following: a set of possible "messages" or "bids"  $\beta_i^\varpi$  for each participating bidder (including the insider), an allocation rule  $\Gamma^\varpi : \beta^\varpi \rightarrow \Delta^\varpi$ , where  $\Delta^\varpi$  is the set of probability distributions over the target and the participating bidders, and a payment rule  $\mu^\varpi : \beta^\varpi \rightarrow \mathbb{R}^{n_\varpi+1}$ , where  $n_\varpi = \sum_j \varpi^j$ . The allocation rule gives the probability that each player will get the target's assets conditional on the set of messages, and the payment rule gives the expected payment to  $T$  by each bidder. A takeover mechanism is defined as a set of conditional mechanisms,  $\{(\beta^\varpi, \Gamma^\varpi, \mu^\varpi)\}$ . In effect, I assume that  $T$  and  $I$  can commit to a different selling mechanism for every possible combination of participating bidders.<sup>12</sup>

In **stage 2**, the outsiders can privately learn their types by paying  $c$ . Next, all players learn the realization of  $Z$ , and the insider and any high types that have paid  $c$  observe a private, unverifiable signal of  $v_i$ , the realization of their synergy value  $V_i$ . Note that the solution of the model is qualitatively unchanged if the outsiders can wait until after observing  $Z$  to investigate. Also note that I am making the implicit assumption that the outsiders' signals are just as precise as the insider's signal, which may be unrealistic if the insider's information is obtained through an alliance. However, since all parties are risk-neutral, adding random noise to the outsiders' signals will not affect the equilibrium.

<sup>12</sup> The solution would not be affected if the mechanism could be conditioned only on the number of participating outsiders.

In **stage 3**, if synergies are positive ( $Z = z_+$ ) and an agreement was reached in stage 1, the potential acquirers first choose whether to participate in the takeover mechanism and then the appropriate conditional mechanism is implemented. If synergies are negative ( $Z = z_-$ ), no transactions take place. Finally, any payments to the target firm are proportionally distributed to its shareholders and all values are realized.

Throughout the article, I assume that once the potential acquirers agree to participate in the takeover mechanism, they cannot refuse to abide by the outcome (i.e. a bidder cannot renege). I also assume that if a potential acquirer chooses not to participate, they have no recourse to acquire the target's assets. Trading by any party in any context other than the negotiation and takeover mechanism is ruled out.<sup>13</sup>

Note that all parties share common *ex ante* beliefs about the synergy value distributions and the distribution of the outsiders' types, but only the potential acquirers can observe their actual values. This does not mean that the model will only apply when firms have no private information prior to value discovery. In Section 5, I briefly discuss the relaxation of this assumption.

Another important assumption in the model is that the bargainers can commit to a takeover mechanism in stage 1. As will be discussed later, it turns out that the bargainers would arrive at an equivalent contracting solution if the stage 3 interaction were exogenously specified as a standard second-price or English ascending-bid auction, which is a common assumption in the literature.

### 3. Equity and the Optimal Mechanism

#### 3.1 A benchmark

The sequentiality of the model is key to the results, so I start with a short discussion of a benchmark model with no sequentiality. For this purpose, assume that the target is able to simultaneously negotiate with *all* of the potential acquirers in stage 1. For intuitive clarity assume that the target has all of the bargaining power, and so makes take-it-or-leave-it offers to all potential acquirers. Also assume the target can commit fully to any future action (such as refusing to sell its assets or provide information to a particular bidder), as is standard in mechanism design.

The solution to this model is well known. In order to maximize total surplus, the target offers to hold a fully efficient auction in stage 3, such as a standard second-price auction. It then demands up-front payments from the potential bidders equal to their total expected payoff net of any

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<sup>13</sup> Note that in addition to ruling out open market transactions in the target's securities, this assumption also rules out the possibility of the winning bidder selling the target to a losing bidder after the mechanism has been implemented. This is seldom observed in reality, and the assumption ensures the model's tractability.

investigation costs. In this way, the target extracts all available expected surplus. This represents a first best situation for the target firm.

If a simultaneous negotiation is not possible, the solution changes significantly. If the target must either negotiate with only one potential bidder *ex ante*, or must negotiate with them in sequence, full efficiency is no longer optimal since the target and its initial counterparty seek to maximize their bilateral surplus at the expense of other potential bidders. This also puts the target in a second best situation because it is no longer able to extract the full expected surplus.

### 3.2 The bargainers' problem

Moving on to the solution of the sequential model outlined in Section 2, I first set up the general problem faced by the bargainers in the first stage. Let  $\pi^i, i \in \{I, T\}$ , denote party  $i$ 's expected continuation payoff for the subgame equilibrium induced by the stage 1 agreement, excluding the initial transfer  $P$ . Generalized Nash bargaining between the target and insider in stage 1 corresponds to the maximization of the quantity

$$[\pi^T + P - D_T]^{\theta_T} [\pi^I - P - D_I]^{1-\theta_T}. \quad (1)$$

Since  $P$  represents a simple transfer from the insider to the target, this can be conceptualized as a two-step problem: in the first step the bargainers set the terms of the takeover mechanism to maximize their joint "coalition" continuation payoff,  $\pi^{Coalition} \equiv \pi^T + \pi^I$ , and in the second step they maximize (1), evaluated given the chosen mechanism, with respect to  $P$ .

For simplicity, from this point on I assume that the outsiders' investigation cost,  $c$ , is low enough that they would all choose to investigate if a standard second-price auction with no reserve were to be held in stage 3. Given this assumption and the fact that the coalition can never do worse than realizing the insider's value, the maximization of  $\pi^{Coalition}$  will clearly be constrained by ensuring the participation of at least some of the outsiders. As it turns out, the bargainers will find it optimal to ensure that all of the outsiders investigate (see Proposition 1). The terms of the agreement therefore have to satisfy the incentive compatibility (IC) and individual rationality (IR), or participation, constraints of the outsiders.

### 3.3 An optimal takeover mechanism

I solve the model by assuming that the bargainers must choose a takeover mechanism that consists of a set of *direct* conditional mechanisms,  $\{(\mathbf{Q}^\omega, \mathbf{M}^\omega)\}$ , in which the bidders are asked to directly report their values  $v_i$ . A direct conditional mechanism consists of a set of functions  $\mathbf{Q}^\omega: \chi^\omega \rightarrow \Delta$  and  $\mathbf{M}^\omega: \chi^\omega \rightarrow \mathbb{R}^{n_\omega+1}$ , where  $\chi^\omega \equiv [0, \lambda_I] \times [0, \lambda_O]^{n_\omega}$ ,  $Q_i^\omega(\mathbf{v})$  gives the probability that player  $i$  will get the target's assets with the vector of reported values  $\mathbf{v}$ , and correspondingly  $M_i^\omega(\mathbf{v})$  gives the expected

payment by  $i$ . The Revelation Principle applies, so there is no loss of generality in focusing on direct mechanisms.

The full proof of the solution is given in the Appendix. Here, I provide an intuitive discussion that highlights the economics underlying the optimal takeover mechanism. The key to this discussion is in thinking of the target and insider as a single coalition facing the outsiders. The coalition either retains the target's assets and realizes the insider's value  $v_I$  (which is unknown *ex ante*) or transfers them to an outsider. For a given number of participating outsiders  $n$  and given value  $v_I$  this problem is precisely analogous to that faced by a seller with a single good facing  $n$  symmetric buyers with independent, identically distributed random values.

When the bidders' value distributions satisfy the monotone hazard rate condition, it is well known that the seller in such a situation will find it optimal to design a selling mechanism that corresponds to a second-price auction with an optimal reserve price. To derive the optimal reserve price,  $b$ , first let  $v_{O_j} - \frac{1}{H_O(v_{O_j})}$  be defined as outsider  $O_j$ 's "virtual valuation." Now, following Bulow and Roberts (1989), think of the coalition as a seller facing  $n$  high type outsiders. You could say that it faces a "demand" curve for each outsider equal to  $1 - F_O(b)$ . You can then write the outsiders' inverse demand functions as  $b(q) \equiv F_O^{-1}(1 - q)$ , which translates into a "revenue" function for the coalition of  $qb(q) = qF_O^{-1}(1 - q)$ . Taking the derivative with respect to  $q$  and simplifying gives the coalition's marginal revenue as a function of the reserve price,  $MR(b) = b - \frac{1}{H_O(b)}$ .

Finding the optimal reserve price then amounts to setting marginal revenue equal to marginal cost. The marginal cost to the coalition of giving up the target's assets equals the insider's value,  $v_I$ . Thus, assuming  $v_I < \lambda_O$ , the optimal reserve price  $b$  for a given value  $v_I$  solves  $MR(b) = MC \Leftrightarrow b - \frac{1}{H_O(b)} = v_I$ . The firm's assets will thus be sold to outsider  $O_j$  whenever it has the highest value among the participating outsiders *and* its value is greater than the optimal reserve price, i.e.  $v_{O_j} > b$ , or, in terms of virtual valuations,  $v_{O_j} - \frac{1}{H_O(v_{O_j})} > b - \frac{1}{H_O(b)} = v_I$ . This allocation can be implemented in a direct mechanism by comparing the insider's value to the outsiders' virtual valuations and awarding the assets accordingly.

This analysis is sufficient to describe the optimal mechanism for a *given* synergy value for the insider and a *known* set of participating outsiders. There are three additional wrinkles here, however: the insider's value  $v_I$  is not yet known, the outsiders' types are random, and the outsiders must pay an investigation cost to determine their type. The fact that  $v_I$  is unknown just means that the optimal reserve price will vary with the eventual realization of  $v_I$ . The latter two factors imply that the coalition must choose a *set* of conditional mechanisms (i.e. a takeover mechanism),

and that set must be such that at least some of the outsiders find it optimal to discover their types.

Assume for now that the bargainers choose their takeover mechanism so that all of the outsiders will investigate. I show in the proof of Proposition 1 that this is optimal. The IR condition that must be satisfied for outsider  $O_j$  to find investigation profitable is

$$s \sum_{\{\omega: \omega^j=1\}} \Pr[\omega] \int_{\chi^\omega} \left[ Q_{O_j}^\omega(\mathbf{v}) v_{O_j} - M_{O_j}^\omega(\mathbf{v}) \right] f^\omega(\mathbf{v}) d\mathbf{v} \geq c, \quad (2)$$

where  $f^\omega(\mathbf{v})$  is the joint distribution of the vector of values of participating bidders, denoted by  $\mathbf{v}$ , conditional on  $\omega$  and  $Z = z_+$ . Since the standard seller's problem is generally solved assuming that the buyers already know their values, the coalition's aggressiveness in each conditional mechanism may have to be tempered relative to that of the standard seller in order to preserve the outsiders' participation. As Proposition 1 shows, the optimal takeover mechanism deals with the outsiders' participation constraints using a slight modification of their virtual valuations.

**Proposition 1.** *The following defines an optimal takeover mechanism: an allocation rule conditioned on  $\omega$*

$$Q_{O_j}^\omega(\mathbf{v}) = \begin{cases} 1 & \text{if } v_{O_j} - \frac{(1-\gamma)}{H_O(v_{O_j})} > \max \left[ v_I, \max_{\{i \neq j: \omega^i=1\}} (v_{O_i} - \frac{(1-\gamma)}{H_O(v_{O_i})}) \right] \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

$$Q_I^\omega(\mathbf{v}) = \begin{cases} 1 & \text{if } \max_{\{j: \omega^j=1\}} (v_{O_j} - \frac{(1-\gamma)}{H_O(v_{O_j})}) < v_I \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

and a payment rule conditioned on  $\omega$

$$M_{O_j}^\omega(\mathbf{v}) = \begin{cases} \max[b^*, \max_{\{i \neq j: \omega^i=1\}} v_{O_i}] & \text{if } Q_{O_j}^\omega(\mathbf{v}) = 1 \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

$$M_I^\omega(\mathbf{v}) = \begin{cases} \gamma \max_{\{j: \omega^j=1\}} v_{O_j} & \text{if } Q_I^\omega(\mathbf{v}) = 1 \\ (\gamma - 1) \max[b^*, \max_{\{j \neq i^*: \omega^j=1\}} v_{O_j}], \\ \quad \text{where } i^* = \arg \max_{\{i: \omega^i=1\}} v_{O_i}, & \text{if } Q_I^\omega(\mathbf{v}) = 0 \end{cases} \quad (6)$$

where  $b^*$  is implicitly defined by  $b^* - \frac{(1-\gamma)}{H_O(b^*)} = v_I$  and  $\gamma \in [0, 1]$  either is set so that the set of direct conditional mechanisms  $\{(Q^\omega, M^\omega)\}$  satisfies the participation constraint (2) for all  $N$  outsiders with equality, or equals zero and (2) is satisfied with inequality.

Except for the variable  $\gamma$ , this is directly analogous to the optimal mechanism for the standard symmetric bidder problem. With  $\gamma = 0$ , the conditional allocation rule  $Q^\omega$  awards the target's assets to the outsider

with the highest value whenever its virtual valuation is higher than the insider's *actual* value, and otherwise awards them to the insider. The payment rule requires no payment from an outsider unless it wins. When an outsider does win, it pays a price equal to the higher of the next highest valuation and the coalition's optimal reserve price,  $b^*$ . Finally,  $T$  gives all of the proceeds from any payments to  $I$ , and  $I$  makes no payment if it receives the assets. In other words, when  $\gamma = 0$  the takeover mechanism is equivalent to an up-front sale of 100% of the firm to  $I$ , who then sets an optimal reserve price equal to  $b^*$  in a subsequent second-price auction.

Now consider how  $\gamma$  differentiates an outsider's "modified" virtual valuation,  $v_O - \frac{(1-\gamma)}{H_O(v_O)}$ , from its virtual valuation,  $v_O - \frac{1}{H_O(v_O)}$ , and thus distinguishes the coalition's optimal reserve price  $b^*$  from the standard seller's. If some or all of the outsiders will not investigate when  $\gamma = 0$ , this means that the coalition cannot behave as aggressively as a standard seller. They must temper their behavior in order to induce full investigation. Now note that their "aggressiveness" declines as  $\gamma$  rises. When  $\gamma = 0$  the reserve price equals the coalition's unconstrained optimal reserve price. As  $\gamma$  rises the reserve price falls, until at  $\gamma = 1$  the coalition effectively sets a reserve price equal to its own value,  $v_I$ . So a higher cost of investigation for the outsiders forces the coalition to commit to less aggressive behavior via a higher  $\gamma$ .

It is interesting to consider at this point why the coalition doesn't just subsidize the outsiders' investigation costs by guaranteeing them a fixed payment. This would allow the coalition to set  $\gamma = 0$  while ensuring that all of the outsiders participate. However, this turns out to be suboptimal. While the coalition's payoff decreases as  $\gamma$  rises, the total expected surplus increases because there are fewer cases where the coalition inefficiently retains the assets when there is a higher-valuing outsider. In other words, an increase in  $\gamma$  helps the outsiders more than it hurts the coalition. So increasing  $\gamma$  allows the coalition to boost the outsiders' expected surplus and induce investigation at a lower cost to the coalition.

It is also instructive to compare Proposition 1 to the result of a standard auction design problem with asymmetric bidders, where the target designs a selling mechanism by itself *after* values are known. The solution to that problem allocates the target's assets to the bidder with the highest virtual valuation, provided it is positive (see Krishna (2002)). Proposition 1 shows that instead of comparing virtual valuations for all bidders, the optimal takeover mechanism for my case compares the participating outsiders' *modified* virtual valuations to the insider's *actual value*. The use of modified virtual valuations for the outsiders reflects the need to induce investigation. The use of the insider's actual value reflects the extra stage of efficient bargaining between the target and insider before values are known. By bargaining *ex ante*, they eliminate a layer of inefficiency relative to the standard problem. Note that in the takeover mechanism

derived above, the assets always go to the insider if no outsider meets the reserve, whereas in the standard problem the target sometimes retains its own assets as a result of its attempt to extract surplus from all bidders (including the insider). Finally, note that the bargainers also use their superior information about the insider's value, relative to an uninformed target acting alone, to reduce the outsiders' informational rents and extract more surplus.

As noted above, Proposition 1 implies that the coalition should ensure that all of the outsiders will investigate. Alternatively, they could choose a takeover mechanism that extracts more surplus from the participating outsiders, but induces less investigation. This is not optimal because each additional bidder brings more expected surplus to the table than it extracts for itself, and this extra surplus can be captured by the bargainers.

### **3.4 The optimality of equity**

I now show that an equity contract together with a second-price or equivalent auction defines a realistic and empirically relevant way for the firms to implement the outcome of the optimal takeover mechanism derived above.

To begin, note that the transfer of some of the target's equity to the insider in stage 1 combined with a standard second-price auction in stage 3 corresponds to a set of conditional mechanisms within the general class allowed by the model. The bids in the second-price auction are the acquirers' "messages," and the net effect of the payments required by the auction rules and the equity contract defines a payment rule. Thus, it remains to prove that this arrangement can result in the same outcome as the takeover mechanism derived above.

I start with the assumption that a standard second-price auction with no reserve is held in stage 3. I also assume that the insider owns a proportion  $\alpha$  of the target's total equity. The analysis of bidding behavior basically follows that of Burkart (1995), with the addition of more outside bidders. I assume throughout that all bids correspond to a valuation for 100% of the firm's assets, although the insider effectively pays only  $(1 - \alpha)$  times the corresponding sale price if it wins.

In the stage 3 auction, the outsiders have a weakly dominant strategy of bidding their value, just as in a standard second-price auction with private values. Given this strategy for the outsiders, the insider selects its bid,  $b_I$ , to maximize its expected payoff. Let  $V_O^1$  be the highest order statistic of the  $n$  participating outsiders' values, and  $V_O^2$  be the second highest. Then the insider's expected payoff in the auction can be written as

$$v_I F_O(b_I)^n + \alpha b_I [1 - F_O(b_I)^n - (1 - n F_O(b_I)^{n-1} + (n-1) F_O(b_I)^n)]$$

$$\begin{aligned}
 & + \alpha E[V_O^2 | b_I < v_O^2] \left[ 1 - nF_O(b_I)^{n-1} + (n-1)F_O(b_I)^n \right] \\
 & - (1 - \alpha) \int_0^{b_I} u n F_O(u)^{n-1} f_O(u) du.
 \end{aligned} \tag{7}$$

The first term is the expected synergy value  $I$  realizes from states where it wins the auction, the second term is the expected profit  $I$  gets from the buyout of its stake in states where it loses the auction but has the price-setting bid, the third term is the analogous expected profit from states where it loses the auction and does not have the price-setting bid, and the fourth term represents the expected payment by  $I$  to  $T$  net of the cash it will receive back from its equity claim on  $T$ . Note that if  $\lambda_I > \lambda_O$ , this objective function is valid only as long as  $v_I \leq \lambda_O$ . If  $v_I > \lambda_O$ ,  $I$  will be indifferent among all bids weakly greater than  $\lambda_O$ .

Maximizing the objective function (7) with respect to the insider's bid  $b_I$  shows<sup>14</sup> that the insider's optimal bid is implicitly defined by the first-order condition (FOC)

$$b_I = v_I + \frac{\alpha}{H_O(b_I)}. \tag{8}$$

The insider's bid always weakly exceeds its value, so equity ownership makes the insider a more aggressive bidder (as previously shown by Burkart (1995) and Singh (1998)). The overbidding occurs because the insider's bid represents both a bid for the remaining shares of  $T$  and an ask for its own block. Note that  $I$ 's bid is increasing in both  $\alpha$  and its own value,  $v_I$ , and it is independent of the number of outside bidders,  $n$ .

Now note the correspondence between this bidding behavior by  $I$  and the allocation rule of the takeover mechanism described in Proposition 1. Each conditional mechanism requires that for a given realization of the insider's value,  $v_I$ , the insider gets the target's assets if and only if  $v_I$  exceeds  $v_O^1 - \frac{(1-\gamma)}{H_O(v_O^1)}$ , where  $v_O^1$  is the highest realized outside value. Otherwise the outsider with the highest modified virtual valuation should win. Given the outsiders' symmetry, this means that the outsider with the highest actual value should win. This allocation can be accomplished in a second-price auction if the insider bids such that it wins the auction whenever  $v_I > v_O^1 - \frac{(1-\gamma)}{H_O(v_O^1)}$ . To implement this, the insider's bid  $b_I$  should be such that  $b_I = b^*$  as defined in Proposition 1. Thus the insider's bidding strategy should solve  $b_I - \frac{(1-\gamma)}{H_O(b_I)} = v_I$ . Finally, note that this bidding strategy is equivalent to that defined in (8) when  $\alpha = (1 - \gamma)$ .

This analysis implies that transferring new shares to  $I$  such that its stake is  $\alpha = (1 - \gamma)$  will result in an allocation that conforms to the takeover

<sup>14</sup> Concavity of the objective function is guaranteed by the monotone hazard rate condition.

mechanism in Proposition 1, assuming a second-price auction in stage 3. The bargainers should be able to use this correspondence to implement an optimal takeover mechanism as long as it is incentive compatible and individually rational for all players, and specifies equivalent payments. Proposition 2 confirms that this is the case.

**Proposition 2.** *The following stage 1 agreement is optimal for the bargainers:*

- (i) *Commit to hold a standard second-price or equivalent auction with no reserve for 100% of the target firm's assets in stage 3 if  $Z = z_+$ ;*
- (ii) *transfer  $\frac{(1-\gamma)}{\gamma}$  new shares of the target's equity to the insider in stage 1, where  $\gamma$  is as defined in Proposition 1; and*
- (iii) *exchange a transfer payment,  $P$ , that satisfies the maximization of the Nash bargaining objective function (1) conditional on the above.*

When the insider overbids, it effectively sets a reserve price for the outsiders. I noted above that the takeover mechanism from Proposition 1 corresponds to a second-price auction with a jointly optimal reserve price, where the reserve is contingent on the insider's value. Proposition 2 shows that exactly the same behavior can be implemented without cooperation at the bidding stage by giving the insider  $\frac{(1-\gamma)}{\gamma}$  shares of new equity in  $T$ , which corresponds to a proportional holding of  $\left[\frac{(1-\gamma)}{\gamma}\right] / \left[1 + \frac{(1-\gamma)}{\gamma}\right] = (1 - \gamma)$ . Holding the equity credibly commits the insider to behave optimally for the coalition even though it opportunistically chooses its bid in the stage 3 auction.

Proposition 2 also implies that equity is an optimal contract even if stage 3 is exogenously specified as a second-price or English ascending-bid auction.<sup>15</sup> Prior work on takeovers has generally assumed this form of contest due to the nature of existing laws that require boards to act as efficient auctioneers when their firms are put up for sale, as well as the public nature of tender offer battles that involve ascending bids by competing acquirers. Thus, even if the assumption of full commitment ability is invalid, the same equity stake will still constitute an optimal contract when the standard auction format is imposed.<sup>16</sup>

Also note that if the outsiders' participation constraints never bind, Proposition 2 implies that it is optimal to sell 100% of the target's equity

<sup>15</sup> Burkart (1995) proved the equivalence of the second-price and English ascending-bid auction formats with two bidders with or without equity stakes. It is easy to show that the necessary equivalence still holds in a setting with one stakeholder and  $n$  symmetric other bidders.

<sup>16</sup> Since the acquirers learn their values simultaneously, the model implicitly precludes the possibility of preemptive bidding as modeled by Fishman (1988).

to the insider in stage 1.<sup>17</sup> The intuition for this is as follows. When the bargainers can be as aggressive as they want in specifying the takeover mechanism, they will want their aggressiveness to match that of a single owner of the firm facing a set of symmetric bidders and setting an optimal reserve price. This can be implemented via an equity transfer only if 100% is transferred. Note that from the target's perspective this outcome is still inferior to the benchmark model with no sequentiality, in which the target extracts all available surplus.

I now briefly return to my original assumption that there is scope for agreement, i.e. that the bargainers' joint surplus from the equilibrium in Proposition 2 exceeds the sum of their disagreement payoffs. Since the equilibrium presented in Proposition 2 results from efficient bargaining and its outcome conforms to the optimal takeover mechanism from Proposition 1, this assumption will hold for any disagreement game that involves the implementation of another takeover mechanism within the same general class (which includes all standard auction formats). Specifically, it will hold if such a mechanism is exogenously imposed following disagreement or if the target is able to independently choose a mechanism. So choosing to bargain with the insider at stage 1 will be optimal for the target under any such assumption.

### **3.5 Empirical implications and relation to existing evidence**

The equilibrium characterized in Proposition 2 has a number of interesting empirical implications. First, if the market does not anticipate the equity transaction or later potential takeover, then the model predicts an increase in the target's market value at the announcement of the stake sale equal to its total expected payoff less its stand-alone value, or  $\pi^T + P$ . However, if no takeover materializes ( $Z = z_-$ ), the target's stock will exhibit negative abnormal returns over time as the market learns that the firm is not a viable takeover target. This is exactly the pattern following strategic stake purchases observed by Mikkelsen and Ruback (1985), Choi (1991) and Barclay and Holderness (1991), although their samples do not focus specifically on direct sales. To my knowledge, this hypothesis has not been tested on a large sample of direct sales.

Next, note that the model implies that some stake sales motivated by surplus extraction in a future takeover will not actually be followed by a takeover. In fact, the probability of a takeover ( $s$ ) may be quite low and still lead to an equity sale when the outsiders' investigation costs ( $c$ ) are low relative to their expected payoff in the auction. Thus, the evidence

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<sup>17</sup> To see this, note from the discussion following Proposition 1 that absent a binding participation constraint, the insiders will find it optimal to set  $\gamma$  equal to zero. But from Proposition 2 this implies that the target management should issue infinite new equity to the insider, effectively selling it 100% of the firm's post-deal equity.

provided by Allen and Phillips (2000) that many firms remain independent after selling stakes to other firms is actually consistent with the model.

Allen and Phillips (2000) also find that firms selling stakes experience higher abnormal returns if they also have an alliance with the purchaser. My model will be particularly applicable to alliance situations if alliances allow potential acquirers, and perhaps outsiders, to learn about the value of the target firm in a takeover. It therefore seems reasonable that stake sales concurrent with alliances may be more likely to signal a potentially profitable takeover (or a greater increase in the probability of one) than stake sales without alliances. This could imply higher abnormal returns, depending, of course, on the motivation for the other sales.

#### **4. Optimal Stake Sizes and Stake Pricing**

Following Proposition 2, let  $\alpha^* \equiv (1 - \gamma)$  denote the “optimal stake size.” Assuming an interior solution ( $\gamma \in (0, 1)$ ), the following comparative statics are derived.

**Proposition 3.** *The optimal stake size,  $\alpha^*$ , is increasing in the probability of a takeover,  $s$ , and decreasing in the outsiders’ cost of investigation,  $c$ .*

An increase in the probability of the positive synergy regime, and thus the probability of a takeover, increases the outsiders’ overall expected payoff, relaxing the participation constraint. This allows the coalition to behave more aggressively, which is accomplished via a larger stake size. An increase in the cost of investigation has precisely the opposite effect.

Since the optimal stake size is determined by the satisfaction of the outsiders’ participation constraints, it will also vary with changes in the potential acquirers’ synergy value distributions, since any such changes will affect the outsiders’ expected payoff. It is possible to characterize an intuitive analytical result for changes in the insider’s distribution based on the concept of first-order stochastic dominance. For this purpose, I define any transformation from one distribution, say  $F(v)$  to another distribution, say  $F'(v)$ , with the same support to be a *positive stochastic shift* in the distribution if  $F'(v) \leq F(v)$  for all  $v$  in their common support.

**Proposition 4.** *The optimal stake size,  $\alpha^*$ , decreases with any positive stochastic shift in the insider’s synergy value distribution.*

A shift to a more dominant distribution for the insider does not affect its bid for any given realization of its value, but it makes its higher values more likely, which directly reduces the outsiders’ expected payoff. Thus, the coalition must decrease the amount of equity transferred since the outsiders will otherwise not investigate. A shift in the outsiders’ distribution, on the other hand, will affect their expected payoff both directly, by making

their own higher values more likely, and indirectly, because the insider's optimal bid will increase for any realization of its value below  $\lambda_0$ . A similar analytical characterization based on stochastic orders is therefore impossible.

These results have several interesting empirical implications. First, note that Proposition 3 predicts that an equity stake is more likely to be sold the higher is the probability of a takeover (the higher is  $s$ ). Thus, if strategic alliances allow firms to learn about the value of further integration, a simultaneous stake sale could signal a higher likelihood of a profitable takeover. This is consistent with the finding of Pablo and Subramaniam (2002) that firms that sell equity when forming strategic alliances have larger announcement returns on average than those that form alliances but do not sell equity. Proposition 3 also implies that observed stake sizes should be closely related to the probability of a subsequent control contest and the difficulty of determining synergy values, which could be proxied by the degree of asymmetric information or uncertainty. Proposition 4 implies that a larger stake should be expected when the stake purchaser is expected to be a relatively weak competitor in a future control contest.

Next I derive results for the pricing of the equity stake relative to the target's market value. It is a stylized fact that private placements of equity are priced at a discount to the *pre-trade* stock price of the issuing firm on average.<sup>18</sup> However, Barclay and Holderness (1989) and Barclay, et al. (2001) find that sales of existing blocks are generally priced at a premium to the issuing firm's *post-trade* stock price. Allen and Phillips (2000) find that block purchases by corporations, including both private placements and trades of existing blocks, generally trade at a premium to the *pre-trade* price, although they do not have data for the equity alliances in their sample. My model provides predictions relevant to more focused investigations of private placements to corporate acquirers.

I assume that the transfer payment  $P$  represents the price paid for the equity. To avoid having to specify what, exactly, the market knows prior to the stake sale, I consider the sale price relative to the *post-trade* price of the target's stock, which is the market price immediately following the sale. In the context of the model, I define the post-trade price as the expected continuation payoff per target share at the end of stage 1 following the consummation of the sale and the distribution of the proceeds.

To analyze the equilibrium premium, I need a specification for the threat points,  $D_I$  and  $D_T$ . For simplicity and tractability, I assume a standard second-price auction with no reserve is held in stage 3 in case of disagreement. As noted earlier, this is a standard assumption in the literature on takeovers, and is consistent with an interpretation of the model in which the auction format is exogenous and equity arises as an optimal

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<sup>18</sup> See, e.g., Hertzfel, et al. (2002).

contract between the target and insider in stage 1. I also assume that all outsiders will choose to investigate if there is no agreement in stage 1.

**Proposition 5.** *Assume a standard second-price auction with no reserve would be held in stage 3 following disagreement in stage 1. Then the equity stake always sells for a discount to the target's post-trade price.*

The difference between the equilibrium outlined in Proposition 2 and an exogenously imposed second-price auction with no stake sale boils down to the insider's overbidding behavior. This behavior raises the partners' joint payoff, but the insider pays an *ex post* "cost" in that it sometimes wins the auction and pays more than the assets are worth. The discount effectively compensates the insider for that cost. It can also be thought of as compensation for providing the "public good" of overbidding. Thus, selling an equity stake to a potential acquirer for less than its third party value can sometimes be beneficial for a firm's shareholders. From their point of view, the strategic benefit of overbidding outweighs the cost of the discount.

## 5. Conclusion

This article shows how corporate control considerations can motivate firms to sell blocks of their own equity to other firms. I show that if there is some probability of a future control contest for the issuer and the value of the firm in a takeover is unknown, the sale of an equity block can implement an optimal mechanism that maximizes the combined expected payoff of the target and block purchaser. Ownership of the block commits the block owner to behave more aggressively in a control contest and allows the coalition of the target and block owner to extract surplus from outside bidders. In the model, this effect is traded off against the possibility that too much extraction will dissuade outsiders from investigating the target.

I assume throughout the article that the partners have symmetric information about potential synergy values at the time of the equity sale. However, in some cases it could be more realistic to assume that the insider has more information about its own synergy distribution. If this were incorporated into the analysis, the universal optimality of equity would not be guaranteed. Such an extension creates significant technical difficulties since bargaining over the stake sale is no longer fully efficient. However, I have derived results for a simple model where the insider has positive synergies with some probability that is privately known, and the target knows that this probability can take one of two values. The optimal mechanism in this case can often be implemented using equity. The full analysis is available upon request. Further exploration of the issue is left to future work.

An important implication of my analysis is that the *ex ante* motivation for a stake sale cannot be determined by simply observing whether or not

there is a subsequent corporate control event. While the model is consistent with the existing evidence, more work is needed to determine whether interfirm equity sales are generally motivated by corporate control as modeled here or by other factors that have been proposed in the literature, such as improved cooperation in alliances. Toward that end, it would be interesting to test the model's prediction that selling firms that are not subsequently acquired will experience long-term negative abnormal returns if no takeover materializes, which has not been done using a focused sample of direct sales. The comparative statics for the optimal stake size and the model's predictions for the stake sale premia may also be particularly fertile grounds for empirical study.

## Appendix

*Proof of Proposition 1.* I first prove the result assuming the takeover mechanism is designed so that all outsiders investigate. I then verify that this conjecture is true.

Turning to the outsiders' IC and IR constraints, I first define the function

$$q_{O_j}^\omega(u_{O_j}) = \int_{x_{-O_j}^\omega} Q_{O_j}^\omega(\mathbf{v}_{-O_j}, u_{O_j}) f_{-O_j}^\omega(\mathbf{v}_{-O_j}) d\mathbf{v}_{-O_j}, \quad (\text{A1})$$

which equals the probability under the conditional mechanism  $(\mathbf{Q}^\omega, \mathbf{M}^\omega)$  that outsider  $O_j$  will get control of the target's assets when it reports its value as  $u_{O_j}$ . I also define

$$m_{O_j}^\omega(u_{O_j}) = \int_{x_{-O_j}^\omega} M_{O_j}^\omega(\mathbf{v}_{-O_j}, u_{O_j}) f_{-O_j}^\omega(\mathbf{v}_{-O_j}) d\mathbf{v}_{-O_j}, \quad (\text{A2})$$

which equals outsider  $O_j$ 's expected payment under  $(\mathbf{Q}^\omega, \mathbf{M}^\omega)$  when it reports  $u_{O_j}$ . Following Krishna (2002) (see his Section 5.1.2), it is easy to show that IC for this outsider requires that its expected payment given  $v_{O_j}$  must satisfy

$$m_{O_j}^\omega(v_{O_j}) = m_{O_j}^\omega(0) + q_{O_j}^\omega(v_{O_j})v_{O_j} - \int_0^{v_{O_j}} q_{O_j}^\omega(t) dt \quad (\text{A3})$$

and that the function  $q_{O_j}^\omega(v_{O_j})$  must be nondecreasing. Note that this implies that any two incentive compatible conditional mechanisms with the same allocation rule must have the same payment rule up to a constant equal to  $m_{O_j}^\omega(0)$ . Also note that the same analysis applies to the insider for analogous functions  $q_I^\omega(v_I)$  and  $m_I^\omega(v_I)$ .

As noted in the text, IR for outsider  $O_j$  requires that its total *ex ante* expected surplus conditional on participating in the takeover mechanism whenever it is a high type equal or exceed its investigation cost, or

$$s \sum_{\{\omega: \omega^J=1\}} \Pr[\omega] \int_0^{\lambda O} [q_{O_j}^\omega(v_{O_j})v_{O_j} - m_{O_j}^\omega(v_{O_j})] f_O(v_{O_j}) dv_{O_j} \geq c, \quad (\text{A4})$$

which rewrites the participation constraint (2). Individual rationality for the outsiders also requires that for all realizations  $\omega$  and  $v_{O_j}$ , outsider  $O_j$ 's expected payoff be nonnegative (i.e.,  $m_{O_j}^\omega(0) \geq 0$ ). If this did not hold, outsider  $O_j$  would choose not to participate in the

takeover mechanism at the beginning of stage 3 when a value with a negative expected payoff was observed. If IR does hold, then choosing not to participate will be weakly dominated by participating.

With these constraints in place it is possible to characterize the problem faced by the bargainers,  $T$  and  $I$ , in stage 1 of the model. They design the set of conditional mechanisms to maximize their joint continuation payoff,  $\pi^{Coalition}$ , which equals the *ex ante* expected payments by the outsiders plus the *ex ante* expectation of the realization of the insider's synergy value. This can be written as

$$\pi^{Coalition} = s \sum_{\omega \in \Omega} \Pr[\omega] \left[ \sum_{\{j:\omega^j=1\}} \int_0^{\lambda_O} m_{O_j}^\omega(v_{O_j}) f_O(v_{O_j}) dv_{O_j} + \int_0^{\lambda_I} v_I q_I^\omega(v_I) f_I(v_I) dv_I \right] \quad (A5)$$

where  $q_I^\omega(v_I)$  is defined analogously to  $q_{O_j}^\omega(v_{O_j})$  as the probability conditional on  $\omega$  that the insider gets the target's assets if it has value  $v_I$ . Any *ex post* payments from  $I$  to  $T$  do not enter the objective function since they maximize their joint surplus.

The bargainers' problem is thus to maximize (A5) subject to the outsiders' IC and IR constraints. I show later that the solution is also incentive compatible and individually rational for the insider. Substituting the main IC constraint (A3) into the first term within the brackets in the objective function (A5) yields a new term

$$\sum_{\{j:\omega^j=1\}} \left[ m_{O_j}^\omega(0) + \int_0^{\lambda_O} q_{O_j}^\omega(v_{O_j}) v_{O_j} f_O(v_{O_j}) dv_{O_j} - \int_0^{\lambda_O} \int_0^{v_{O_j}} q_{O_j}^\omega(t) f_O(v_{O_j}) dt dv_{O_j} \right]. \quad (A6)$$

Changing the order of integration in the last term within the brackets in (A6), inserting the definition of  $q_{O_j}^\omega(v_{O_j})$ , and simplifying the entire expression yields

$$\sum_{\{j:\omega^j=1\}} \left[ m_{O_j}^\omega(0) + \int_{\mathcal{X}^\omega} \left( v_{O_j} - \frac{1 - F_O(v_{O_j})}{f_O(v_{O_j})} \right) Q_{O_j}^\omega(\mathbf{v}) f^\omega(\mathbf{v}) d\mathbf{v} \right]. \quad (A7)$$

Taking this into account in both the objective function (A5) and the IR/participation constraint (A4), the bargainers' problem can be written as

$$\max_{Q_{O_j}^\omega, m_{O_j}^\omega(0)} s \sum_{\omega \in \Omega} \Pr[\omega] \left[ \int_{\mathcal{X}^\omega} \left\{ \sum_{\{j:\omega^j=1\}} \left[ \left( v_{O_j} - \frac{1}{H_O(v_{O_j})} \right) Q_{O_j}^\omega(\mathbf{v}) \right] + v_I Q_I^\omega(\mathbf{v}) \right\} f^\omega(\mathbf{v}) d\mathbf{v} \right] + \sum_{\{j:\omega^j=1\}} m_{O_j}^\omega(0) \quad (A8)$$

subject to

$$c - s \sum_{\{\omega:\omega^j=1\}} \Pr[\omega] \left[ \int_{\mathcal{X}^\omega} \left( \frac{1}{H_O(v_{O_j})} \right) Q_{O_j}^\omega(\mathbf{v}) f(\mathbf{v}) d\mathbf{v} - m_{O_j}^\omega(0) \right] \leq 0 \text{ for all } j \quad (A9)$$

$$m_{O_j}^\omega(0) \leq 0 \text{ for all } \omega, j \quad (A10)$$

where  $Q_{O_j}^\omega(\mathbf{v})$ ,  $Q_I^\omega(\mathbf{v}) \in [0, 1]$  and  $\sum_{\{j:\omega^j=1\}} Q_{O_j}^\omega(\mathbf{v}) + Q_I^\omega(\mathbf{v}) \leq 1$  for all  $\omega$  and all  $\mathbf{v}$ , and  $q_{O_j}^\omega(v_{O_j})$  must be nondecreasing. Letting  $\{\gamma_j\}$  and  $\{\mu_{I_j}^\omega\}$  be the sets of nonnegative multipliers

on constraints (A9) and (A10), respectively, the Lagrangian can be written as

$$L = s \sum_{\omega \in \Omega} \Pr[\omega] \left[ \int_{\mathcal{X}^\omega} \left\{ \sum_{\{j:\omega^j=1\}} \left( v_{O_j} - \frac{(1-\gamma_j)}{H_O(v_{O_j})} \right) Q_{O_j}^\omega(\mathbf{v}) + v_I Q_I^\omega(\mathbf{v}) \right\} f^\omega(\mathbf{v}) d\mathbf{v} \right. \\ \left. + \sum_{\{j:\omega^j=1\}} \left[ (1-\gamma_j) - \frac{1}{s \Pr[\omega]} \mu_{1j}^\omega \right] m_{O_j}^\omega(0) \right] - \sum_j \gamma_j c \tag{A11}$$

First consider the expression inside the curly brackets. By the symmetry of the problem we can set  $\gamma_j = \gamma$  for all  $j$ , yielding

$$\left\{ \sum_{\{j:\omega^j=1\}} \left( v_{O_j} - \frac{(1-\gamma)}{H_O(v_{O_j})} \right) Q_{O_j}^\omega(\mathbf{v}) + v_I Q_I^\omega(\mathbf{v}) \right\}. \tag{A12}$$

Maximizing this over  $\mathbf{Q}^\omega$  given  $Q_{O_j}^\omega(\mathbf{v}), Q_I^\omega(\mathbf{v}) \in [0, 1]$  and  $\sum_{\{j:\omega^j=1\}} Q_{O_j}^\omega(\mathbf{v}) + Q_I^\omega(\mathbf{v}) \leq 1$ , and thus viewing  $Q_{O_j}^\omega(\mathbf{v})$  and  $Q_I^\omega(\mathbf{v})$  as weights on each  $\left( v_{O_j} - \frac{(1-\gamma)}{H_O(v_{O_j})} \right)$  and  $v_I$ , respectively, results in a FOC for a given  $\omega$  and  $\mathbf{v}$  equivalent to the allocation rule given in the text of the proposition (note: ties are safely ignored since they occur with zero probability). Since this maximizes the expression for any given  $\omega$  and  $\mathbf{v}$ , the associated system of FOCs for all possible vectors  $\omega$  and  $\mathbf{v}$  correspond to the FOC for the maximization of the integral. The FOCs for the remaining control variables are

$$\frac{\partial L}{\partial m_{O_j}^\omega(0)} = s \Pr[\omega] \left( (1-\gamma) - \frac{1}{s \Pr[\omega]} \mu_{1j}^\omega \right) = 0. \tag{A13}$$

Finally, we have the complementary slackness conditions

$$\gamma, \mu_{1j}^\omega \geq 0 \tag{A14}$$

for all  $\omega$  and  $j$ , where

$$\gamma, \mu_{1j}^\omega = 0 \tag{A15}$$

if the corresponding constraint (A9) or (A10) holds with inequality.

To see that the solution laid out in the text of the proposition solves this problem, first consider the free parameters of the outsiders' payment rule,  $m_{O_j}^\omega(0)$ . Assume for now that  $\gamma \in [0, 1]$  (this will be confirmed below). By complementary slackness, if  $m_{O_j}^\omega(0) < 0$ , then  $\mu_{1j}^\omega = 0$ , and (A13) cannot hold. Thus, we must have  $m_{O_j}^\omega(0) = 0$  and  $\mu_{1j}^\omega = s \Pr[\omega](1-\gamma)$ . This is clearly satisfied by the allocation and payment rules in Proposition 1.

Now note that complementary slackness implies that if the outsiders' IR constraints (A9) are never binding, we have  $\gamma = 0$ . It is also clear that the coalition's payoff (A8) is monotonically decreasing in  $\gamma$  under the allocation rule defined in the proposition. As  $\gamma$  increases, the target's assets are allocated to the outsiders in more states, and clearly each time this reallocation occurs for any state, this must reduce (A8), which would be maximized by allocating the firm according to the allocation rule in the text of the proposition with  $\gamma = 0$ . Also note that when  $\gamma = 1$ , the allocation defined in the proposition conforms to that in a standard second-price auction. This, together with my assumption that  $c$  is low enough that the outsiders would choose to investigate if the chosen mechanism were a standard second-price auction, is therefore sufficient to prove that  $\gamma \in [0, 1]$  either is set so that the resulting takeover mechanism satisfies (2) with equality, or equals zero and (2) is satisfied with inequality.

As noted above, given the insertion of the IC constraint (A3) into the objective function, each conditional mechanism will be incentive compatible for the outsiders as long as  $q_{O_j}^\omega(v_{O_j})$

is nondecreasing. This is guaranteed by the monotone hazard rate condition, which ensures that  $v_{O_j} - \frac{(1-\gamma)}{H_O(v_{O_j})}$  is nondecreasing in  $v_{O_j}$ . Finally, note that this procedure does not pin down the payment rule for the insider. With efficient bargaining all expected *ex post* payments can be offset with a change in the *ex ante* transfer. Thus, it must simply be verified that the payment rule for the insider has  $m_j^o(0) \leq 0$  and is incentive compatible. The former is easily verified for the rule given in the proposition. Incentive compatibility follows from the proof of Proposition 2 below. To see this, note from that proof that the takeover mechanism detailed in Proposition 2 has equivalent allocation and payment rules to that in Proposition 1. The direct calculation of bidding strategies proves IC for the payment rule of the mechanism in Proposition 2, which implies IC for the payment rule in Proposition 1.

The final task is to prove that it is optimal for the coalition to ensure that all outsiders investigate. To see this, first note that conditional on inducing a given number  $n$  of the outsiders to investigate, the coalition will set  $\gamma$  such that the outsiders get zero surplus in expectation. For a given  $n$  and its implied  $\gamma$ , the coalition could instead raise  $\gamma$  to the point where  $n + 1$  outsiders would investigate. Now assume that  $\gamma$  is raised to such a level, but no additional outsiders investigate. In this case it is easy to see that the total surplus generated by the takeover mechanism increases, since the target's assets are retained by the coalition in fewer cases where there is an outsider with a superior value. However, the aggregate expected payoff of the  $n$  outsiders rises by more than the increase in total surplus, and thus hurts the coalition (otherwise the old level of  $\gamma$  would not have been optimal conditional on  $n$  outsiders). Now consider what happens when the additional outsider investigates. Since the new level of  $\gamma$  is optimal for  $n + 1$  outsiders, all of the outsiders now get zero surplus in expectation, but the presence of the  $(n + 1)^{st}$  outsider increases total surplus again. Thus, all of the outsiders are back to zero expected surplus, but total expected surplus has risen, so the coalition must be better off by raising  $\gamma$  and inducing the participation of another outsider. ■

*Proof of Proposition 2.* The discussion in the text proves that the transfer of  $\frac{(1-\gamma)}{\gamma}$  shares of equity in  $T$  followed in stage 3 by a second-price or equivalent auction implements the allocation rule described in Proposition 1. That discussion also shows that the prescribed behavior in the auction will be incentive compatible for all potential acquirers once the equity is transferred since their optimal bidding strategies are explicitly derived, which implies IC for the implicit payment rules. Now consider the bidders' IR constraints. These will be satisfied if the payment rules under the takeover mechanism described in Proposition 2 are equivalent to those in Proposition 1. It is easy to verify that this is the case given than  $b_I = b^*$  in equilibrium. Thus, participation is weakly dominant for the insider and all high type outsiders at stage 3. Individual rationality and incentive compatibility for the purchase of equity at the price  $P$  by the insider is clearly satisfied since I have assumed efficient bargaining between the partners and that mutually beneficial agreement is possible (the coalition's maximized joint surplus,  $\pi^T + \pi^I$ , exceeds the sum of disagreement payoffs,  $D_T + D_I$ ). ■

*Proof of Proposition 3.* Follows directly from the fact that the outsiders' IR constraint (2) holds with equality in equilibrium (assuming it is not satisfied at  $\gamma = 0$ ), the left-hand side of the constraint is increasing in  $s$ , the right-hand side is increasing in  $c$ , and the left-hand side is increasing in  $\gamma$  given the allocation rule in Proposition 1. ■

*Proof of Proposition 4.* Using the results from the proof of Proposition 1, the outsiders' IR constraint (2) can be re-written as

$$s \sum_{\{\omega: \omega^j = 1\}} \Pr[\omega] \left[ \int_{\chi^\omega} \left( \frac{1}{H_O(v_{O_j})} \right) Q_{O_j}^\omega(\mathbf{v}) f^\omega(\mathbf{v}) d\mathbf{v} - m_{O_j}^\omega(0) \right] \geq c. \quad (A16)$$

Now note that a positive stochastic shift in  $F_I(\cdot)$  will affect (A16) only via the resulting change in  $q_{O_j}^\omega(v_{O_j})$  for all  $\omega$  such that  $\omega^j = 1$ . Given  $q_{O_j}^\omega(v_{O_j}) = \int_{\chi_{-O_j}^\omega} Q_{O_j}^\omega(\mathbf{v}) f^\omega_{-O_j}(\mathbf{v}_{-O_j}) d\mathbf{v}_{-O_j}$

and  $Q_{O_j}^\omega(\mathbf{v})$  as defined in Proposition 1, we can rewrite  $q_{O_j}^\omega(v_{O_j})$  as  $F_O(v_{O_j})^{n_\omega-1} + F_I(v_{O_j} - \frac{(1-\gamma)}{H_O(v_{O_j})})$ , which equals the probability that both the highest order statistic among the  $n_\omega - 1$  other outsiders' values (bids) and the insider's bid are lower than  $v_{O_j}$ , whenever  $\omega^j = 1$ . Thus, since a positive stochastic shift in  $F_I(\cdot)$  reduces  $F_I(\cdot)$  at every point,  $q_{O_j}^\omega(v_{O_j})$  must decrease with the shift at every  $v_{O_j}$ , which implies that the left-hand side of (A16) decreases. Therefore, (2) must hold with equality at a higher  $\gamma$ , which is equivalent to a smaller optimal stake size. ■

*Proof of Proposition 5.* Since I have assumed that there is one target share outstanding and that the target issues new shares to the insider, the post-trade price per share is simply the target's continuation payoff,  $\pi^T$ . With the definition of the optimal stake size  $\alpha^* \equiv (1 - \gamma)$ , the number of shares can also be written as  $\frac{\alpha^*}{1-\alpha^*}$ , and the price paid per share can be written as  $\frac{1-\alpha^*}{\alpha^*}P$ . I define the stake sale premium (discount) as the percentage premium (discount) paid over (under) the post-trade price, or

$$\frac{(1 - \alpha^*)P}{\alpha^* \pi^T} - 1. \tag{A17}$$

Given the chosen stake size  $\alpha^*$ , the bargainers maximize (1), conditional on a transfer of that stake followed by a second-price auction, with respect to  $P$ . The resulting price is

$$P = \theta_T(\pi^I - D_I) - (1 - \theta_T)(\pi^T - D_T). \tag{A18}$$

The premium is clearly increasing in  $\theta_T$ , so it suffices to prove the result for  $\theta_T = 1$ . We therefore require proof that

$$\frac{(1 - \alpha^*)}{\alpha^*}(\pi^I - D_I) < \pi^T. \tag{A19}$$

First note that  $\pi^T$  represents the expected payoff of one share held by an outside party. For any given realization of the vector of values  $\mathbf{v}$ , this equals  $(1 - \alpha^*)$  times the expected sales price of the target's assets, denoted by  $E[price]$ . Thus, we can rewrite (A19) as

$$(\pi^I - D_I) < \alpha^* E[price]. \tag{A20}$$

Since both sides of this equation are expectations taken over all possible realizations of  $\omega$  and  $\mathbf{v}$ , it suffices to prove that

$$(\pi^I - D_I) \leq \alpha^* E[price]. \tag{A21}$$

holds conditional on  $\omega$  and  $\mathbf{v}$  for every possible  $\omega$  and  $\mathbf{v}$ , and (A20) holds for some  $\omega$  and  $\mathbf{v}$ .

First fix  $\omega$  and let  $\pi^I|_{\mathbf{v}} - D_I|_{\mathbf{v}}$  represent the left-hand-side of (A20) and (A21) as measured conditional on  $\mathbf{v}$ . Since a second-price auction is held in stage 3 whether or not an agreement is reached in stage 1,  $\pi^I|_{\mathbf{v}} - D_I|_{\mathbf{v}}$  simply represents the difference in payoff for the insider in the second-price auction with a stake of size  $\alpha^*$  versus without a stake. On the other hand,  $\alpha^* E[price]$  represents a sure payoff equal to  $\alpha^*$  times the second highest bidder's bid for a given realization of  $\mathbf{v}$ . To prove the result, I break the realizations of  $\mathbf{v}$  into three sets: set  $W$  consists of all realizations such that  $I$  loses the auction no matter what (that is, whether or not an equity stake is transferred in stage 1); set  $X$  consists of all realizations such that  $I$  wins the auction no matter what; and set  $Y$  consists of all realizations such that  $I$  would lose the auction if there were no stake transfer, but win the auction if there were a stake transfer. Note that since  $I$  always overbids, the union of these three sets contains all possible realizations of  $\mathbf{v}$ .

Now consider the quantity  $\pi^I|_{\mathbf{v}} - D_I|_{\mathbf{v}}$  when  $\mathbf{v} \in W$ . If a stake transfer occurs,  $I$ 's total payoff in these states is  $\alpha^*$  times the second highest bid, because it gets its proportional

payout of the price paid by the winning outsider. On the other hand,  $I$ 's total payoff in these states with no stake transfer is zero. Thus,  $\pi^I|_v - D_I|_v$  equals  $\alpha^*$  times the second highest bid for all  $\mathbf{v} \in W$ , so for any  $\mathbf{v} \in W$  equation (A21) holds with equality conditional on  $\mathbf{v}$  for all  $\mathbf{v} \in W$ .

Next consider set  $X$ .  $I$ 's total payoff if a stake transfer occurs is  $v_I - (1 - \alpha^*)v_O^1$ , and if no stake transfer occurs it is  $v_I - v_O^1$ , so  $\pi^I|_v - D_I|_v = \alpha^*v_O^1$ , which equals  $\alpha^*$  times the second highest bid for any  $\mathbf{v} \in X$ . Thus, (A21) holds with equality conditional on  $\mathbf{v}$  for all  $\mathbf{v} \in X$ . Finally, consider set  $Y$ . With a stake transfer,  $I$ 's total payoff is  $v_I - (1 - \alpha^*)v_O^1$ , while with no transfer  $I$  gets a payoff of zero. Thus,  $\pi^I|_v - D_I|_v = v_I - v_O^1 + \alpha^*v_O^1$ . For (A20) to hold for  $\mathbf{v} \in Y$ , we must have  $v_I - v_O^1 + \alpha^*v_O^1 < \alpha^*v_O^1$ . But clearly, for  $\mathbf{v} \in Y$  to hold, it must be the case that  $v_I < v_O^1$ , because otherwise  $I$  would win the auction if there were no stake transfer. So (A20) holds conditional on  $\mathbf{v}$  for all  $\mathbf{v} \in Y$ . ■

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