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Strategic alliances, equity stakes, and entry deterrence ☆

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Abstract

I study how strategic alliances and their impact on future competitive incentives can motivate interfirm equity sales. In the model, an alliance between an entrepreneurial firm and an established firm improves efficiency for both. However, the requisite knowledge transfer heightens the established firm's incentive to enter one of its partner's markets. I show that equity can eliminate the entry incentive, but accommodation is sometimes chosen to encourage entrepreneurial effort on future growth options. I analyze stake sizes, block pricing, and welfare effects. The results have implications for equity alliances, corporate venture capital, and the organization of research activities.

JEL classification: D43; G32; G34; L13; L14

Keywords: Strategic alliances; Cooperation; Equity stakes; Entry deterrence; Corporate venture capital

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1. Introduction

Strategic alliances are increasingly common. Robinson (2003) finds that the number of alliance transactions has grown by approximately 16% per year in the US since 1985 and that the number of alliances rivaled the number of mergers and acquisitions between 1985 and 1999. Many alliances pair a small, entrepreneurial firm with a larger, more established partner. The smaller firm commonly sells a stake of its own equity to its partner when the relationship is formed.[1] These "equity alliance" transactions raise several interesting questions. Why is it optimal for one partner to hold another's equity? Is it simply a financing transaction or are there important strategic implications? How are transaction characteristics such as stake sizes and prices influenced by these factors?

In this paper I develop a theory of equity alliances based on the partners' conflicting opportunities for cooperation and competition. The theory begins with the fact that efficiency benefits often require transfers of knowledge from entrepreneurial firms to their established partners. I argue that such transfers could create or intensify incentives for established firms to enter and compete directly in their partners' markets. Using a simple model, I show that equity can help ensure cooperation when it is difficult or impossible to contract on entry and the use of transferred knowledge. This arises from the fact that a partial owner internalizes the effect of its decisions on its partner's profits. The partners can exploit this fact to structure an equity transaction that noncontractually eliminates the entry incentive. However, they sometimes limit the equity transfer and accommodate entry when the entrepreneur's innovative incentives would be overly diluted.

A number of empirical papers have established that research and development (R&D) or technology intensive alliances are more likely to involve equity transfers, which is fully consistent with my model.[2] In the past this has generally been interpreted in a transaction cost or property rights framework. A common argument is that the uncertainty inherent in these activities leaves room for ex post negotiation and creates a hold-up problem. Stake ownership, it is argued, could alleviate the problem. However, such a framework cannot capture the impact of entry incentives that are not subject to negotiation.

Several key factors highlight the importance of knowledge transfers and their effect on entry incentives. First, entrepreneurial firms in alliances with established firms do not, in general, simply hand over their technology or know-how in return for cash. Instead, they hope to gain access to their partners' resources in areas such as manufacturing, distribution, or marketing (Alvarez and Barney, 2001). This implies that the entrepreneurial firm, on the one hand, is directly engaged in some product market activity, and it seeks to gain efficiencies through the alliance. Established partners, on the other hand, gain access to knowledge that can improve their efficiency in other new or pre-existing markets. Second, established firms often enter new markets themselves in competition with entrepreneurial firms. Third, contracting over specific uses of knowledge, even well-defined intellectual property, is often difficult. Finally, legal restrictions or contracting imperfections can make it impossible to eliminate future competition using direct contracts.

[1]See Alvarez and Barney (2001), Pablo and Subramaniam (2002), and Hellmann (2002) for related evidence.
[2]See, e.g., Gulati and Singh (1998), Oxley (1997), Pisano (1989), and Robinson and Stuart (2003).

The management literature stresses that enabling a potential competitor is a major concern for entrepreneurial alliance partners. Alvarez and Barney (2001, p. 144–145) quote a software company executive:

> My biggest concern about technology exchange is that you potentially really hurt your own ability to market. If you've developed something of great value, to make it available to an alliance partner in anticipation of getting something in the future that will make your own firm successful is foolish. All that you've really done as a small company is enable a competitor who can outmarket you any time that they want. . . . [I]t's my sense that all you end up doing is enabling a competitor that can beat the tar out of you in the marketplace.

Similarly, Hitt et al. (1995, p.14) discuss the "painful lessons learned [by US companies] in the 1970s and 1980s, when they were deceived by partners who obtained technology and market knowledge only to use it to compete against them." Oxley (1997) and Gulati and Singh (1998) argue that fears about technology appropriation are a key concern in many alliances. Finally, Gans and Stern (2000) and Gans et al. (2003) argue that the ability of small firms to protect their intellectual property determines whether they form cooperative relationships with established firms or enter new markets independently.

I use a simple model based on Cournot competition to study the role of equity in these situations. In the model a small, entrepreneurial firm and a large, established firm operating in distinct (but related) markets can form an alliance. The alliance requires a transfer of the entrepreneurial firm's technological knowledge or know-how to the established firm. This transfer makes both firms more efficient in their existing markets, but it also makes the established partner a more effective entry threat for its partner's market. For simplicity, I assume initially that the firms would be equal competitors in that market following the alliance.

I further assume that the firms cannot contract directly on profits or entry. However, they can negotiate a transfer of equity and a monetary transfer when the alliance is formed. Under these assumptions, I show that the partners can always structure an equity transaction that ensures cooperation and deters the established firm from entering. When the established firm would otherwise enter, its stake in its partner's profits makes entry unprofitable. However, the equity transfer also dilutes the entrepreneur's incentives to innovate in new markets in the future. As a result, the partners sometimes accommodate entry by the established firm, settling for reduced surplus in the contested market to preserve the value of the entrepreneurial firm's growth options.

The analysis does not employ security design techniques. This is consistent with the idea that the partners use equity to partially hide their true intentions, which could be more difficult with less traditional funding contracts. Also, because of the model's static nature I assume that renegotiation is prohibitively costly. Finally, I assume that the entrepreneurial firm cannot purchase significant equity in the established firm, which reflects their disproportionate sizes. These assumptions maintain the model's tractability, and their reasonableness is discussed in the paper.

The analysis provides a novel perspective on existing empirical results and generates a number of new testable predictions. It shows that equity is more likely to be transferred, and the stake size required to deter entry is larger, the more attractive is entry. Entry is more attractive the lower is the cost of entry and the more the alliance improves efficiency in the contested market. The analysis also shows that equity is sometimes used to deter entry that would not occur in the absence of an alliance.

When deterring entry with an alliance is too costly to the small firm's future growth options, the partners sometimes form an alliance and sell a smaller stake to soften competition, and sometimes they avoid an alliance altogether. The optimal structure depends critically on the magnitude of the established firm's initial cost disadvantage in the contested market. When this disadvantage is large, the firms are more likely to avoid an alliance because it is easier to deter entry or soften competition without one.

I also consider the effect of changes in the degree of intellectual property protection. I find that increased protection reduces the stake size required to deter entry, given that it reduces the profitability of entry. However, it increases the optimal stake size under accommodation. This arises because equity's competitive softening effect is more valuable on the margin when there is a larger cost disparity. Increased protection also makes an alliance more likely in general. This is true when entry is deterred because the stake size required to deter entry falls. It is true when entry is accommodated because greater protection means the alliance causes a smaller increase in competitiveness in the contested market. I also find that the partners are more likely to form an alliance when they can control the amount of knowledge transferred.

The empirical literature on equity blocks shows that trades of existing blocks tend to occur at a premium to the post-trade price (Barclay and Holderness, 1989), while private placements tend to occur at a discount to the pre-trade price (see, e.g., Hertzel et al., 2002). However, Barclay et al. (2001) find that private placements to corporations with a nonfinancial relationship with the issuer are priced more like trades of existing blocks. I show that stakes transferred in the model can sell for a premium or discount to the post-trade price. A premium is more likely if the established firm would not enter in the absence of a deal, the greater is the entrepreneurial firm's bargaining power, and the more the alliance helps the large firm in its existing markets. A premium both compensates the entrepreneur for his private effort and provides him a share of the large firm's benefits.

1.1. Related theoretical literature

In application this study is most closely related to papers that focus on the strategic implications of equity investments or organizational choice in innovative activities. Hellmann (2002, 1998) models an entrepreneur's choice between a strategic investor and a traditional venture capitalist when the success of the start-up affects the value of the strategic investor's existing assets. One of the scenarios considered in Hellmann (1998) includes the possibility that the strategic investor could fund a competing internal project. However, in that model there is no possibility of product market cooperation between the firms, no transfer of knowledge, and no analysis of when varying stake sizes accomplish entry deterrence (a 100% stake is assumed). Arping and Troege (2002) analyze the polar opposite of my case; they study alliance and funding agreements between an entrenched incumbent and an entrepreneurial potential competitor. They allow for the established firm to purchase the smaller firm and shelve its product or alternatively provide funding and resource access in an alliance. In contrast with my paper, they focus on agency conflicts in the alliance and find that the main role for equity is to induce the large firm to provide the small firm with access to its resources despite the scope for competition.

Other papers focus on cash flow and control rights in cooperative relationships. In a seminal paper on alliance contracting, Aghion and Tirole (1994) study the optimal allocation of property rights, but they also show that partial ownership does not solve the traditional hold-up problem and thus has no role in their model.[3] However, they do not consider the possibility of ex post competition between the two firms. More recently, Dasgupta and Tao (2000), Harbaugh (2001), and Van den Steen (2002) all show that partial ownership can solve some variation of the hold-up problem in a specific setting. My model is distinct from these because I assume that ex post contracting to split joint surplus is not possible, so no traditional hold-up problem exists. Dessein (2005) focuses on the allocation of control rights as opposed to cash flow rights and finds that control could be given to the larger firm to signal the congruence of the entrepreneur's objectives. Such control right issues are not explicitly considered in this paper.

Gromb and Scharfstein (2002), Fulghieri and Sevilir (2003), and Ambec and Poitevin (2001) develop models of equilibrium organizational form when a new R&D project can be undertaken within an established firm or by a new start-up firm. In general, their models assume that the research firm and the established firm can be separated with no fear of future competition between them. Anton and Yao (1995) model a different situation in which an employee with a new invention must decide whether to reveal the discovery to his current employer or leave and begin a start-up. They allow for future competition, but not an alliance.

Other relevant papers deal with the competitive implications of either cross-ownership or strategic alliances in isolation. For example, Reynolds and Snapp (1986) were the first to show that partial ownership tends to soften competition, resulting in decreased output and increased price.[4] They also argue informally that partial ownership by a potential competitor can reduce entry incentives and affect the incumbent's use of traditional entry deterrence strategies such as predatory pricing. However, none of the existing papers formally analyze when stake ownership deters entry or model an efficiency-enhancing relationship. Another large literature focuses on the competitive effects of R&D alliances among current competitors (see, e.g., D'Aspremont and Jacquemin, 1988; Grossman and Shapiro, 1986; Kamien et al., 1992).

A few other papers deal with financial or alliance-based entry deterrence. Chen and Ross (2000) demonstrate how a "sharing alliance" can be used to deter entry. They show that an incumbent could offer to share its facilities with a potential entrant to deter larger scale entry, and they analyze the resulting welfare consequences. Clayton and Jorgensen (2005) show that allowing cross-ownership among potential competitors can deter entry, but in their model the firms trade in the open market and take short positions in each other's equity to commit to aggressive behavior. Finally, Cestone and White (2003) show that giving a risky equity claim to a financial investor can cause the investor to deny funding to other entrants when credit markets are imperfectly competitive.

[3]For traditional formulations of hold-up in a transaction cost context, see Williamson (1979, 1985) and Klein et al. (1978). For the related property rights framework, see Grossman and Hart (1986) and Hart and Moore (1990).

[4]In a repeated game setting, Malueg (1992) shows that cross-ownership could reduce the effectiveness of punishment strategies, which attenuates the softening effect. Davidson and Deneckere (1984) demonstrate a similar result in the context of mergers. Farrell and Shapiro (1990) show that the purchase of a competitor's equity increases joint profits unless the purchaser is much larger than the selling firm. Reitman (1994) endogenizes the formation of cross-ownership interests among current Cournot competitors in a conjectural variations model.

The remainder of the paper is organized as follows. Section 2 provides the details of the model. The major results on the formation of the alliance and the transfer of equity are derived in Section 3. Section 4 analyzes the pricing of the stake. Section 5 contains a welfare analysis. Various extensions are presented in Section 6, and the robustness of the model is discussed in Section 7. Section 8 concludes. Proofs not provided in the text are collected in Appendix A. Appendix B derives technical conditions required for the construction of one of the figures.

2. The model

The model has two firms, A and B, and three markets, 1, 2, and 3. Firm A is a large, established firm with a professional manager. It currently operates in Market 1, representing all of its (possibly numerous) existing markets, and prospectively operates in Market 2. Firm B is a small, innovative firm with an owner-manager referred to as the "entrepreneur." It currently operates as a monopolist in Market 2 and prospectively operates in Market 3. The latter represents a future growth opportunity for B that arises from its operation in Market 2. Note that A is a potential entrant against B in Market 2. This is the only market in which they can compete head to head.

Firm A's status quo expected payoff in Market 1 is V_1. Market 2 is characterized by a downward sloping inverse demand function $P(Q)$, where Q is total industry output.[5] Firm B has a constant marginal cost of production c in this market and has already sunk any required fixed costs. Firm A can pay a fixed fee, $F \geqslant 0$, to enter and compete with B with a homogeneous product. If it enters in the status quo, A is less efficient than B. Specifically, upon entry A has a constant marginal cost of production equal to $(1+\gamma)c$, where $\gamma \geqslant 0$.

Exploitation of Market 3 is contingent on B continuing to operate in Market 2 and generating a necessary innovation in its technology. Successful innovation depends on effort expended by the entrepreneur.[6] If innovation is successful, B is able to generate a net "success" payoff in Market 3 of $V_3 \geqslant 0$. Choosing effort $e \in [0,1]$ costs the entrepreneur $\frac{e^2}{2\beta}$ and induces probability of success e.

Prior to A's entry decision for Market 2, the entrepreneur's effort decision for Market 3, and any production decisions, the firms have the opportunity to cooperate in a strategic alliance. The alliance requires a transfer of knowledge from B to A and has three basic effects. First, it generates a net incremental benefit for A in Market 1 of $\Delta V_1 > 0$. Second, it increases the success payoff in Market 3 by $\Delta V_3 \geqslant 0$. Third, it reduces *both firms*' marginal cost in Market 2 to $(1-\delta)c$, where $\delta \in [0,1]$. In other words, the alliance improves the entrepreneurial firm's efficiency in its current market but also makes the established firm an equal competitor in that market.

The assumption of equal marginal costs following the alliance simplifies the analysis and represents a limiting case in which A can completely appropriate B's technology. In reality this will depend on the strength of intellectual property rights, the amount of knowledge transferred, and A's ability to assimilate that knowledge. In Section 6.1.1 I consider an extension with varying degrees of property rights protection, and in Section 6.1.2 I consider an extension in which the degree of knowledge transfer is endogenous.

[5]I assume that $P(\cdot)$ is continuous and twice continuously differentiable.

[6]For notational simplicity, I assume the innovation process has no costs other than the entrepreneur's effort cost, but such an assumption is not necessary.

The game takes place over three stages. In Stage 1, the two firms bargain over whether to have an alliance and a transfer of new shares of B's equity to A. The equity deal consists of a pair (α, T), where α measures the post-deal proportion of B's equity to be owned by A and T is a monetary transfer from A to B. If T is negative, this is effectively a transfer from B to A. I assume that there is one existing share of B's equity outstanding prior to the stake transfer, and it is held by the entrepreneur. This implies that the number of shares transferred with an agreed stake size of α is $\frac{\alpha}{1-\alpha}$.

The bargaining game is modeled as generalized Nash bargaining with bargaining powers of θ for the entrepreneur and $(1-\theta)$ for A (see Svejnar, 1986). If a deal is made, any monetary transfer is immediately distributed to the existing shareholders of the relevant firm and the game proceeds to Stage 2. If there is no deal, the game proceeds directly to Stage 2. For simplicity, I assume that renegotiation is prohibitively costly and that any stake owned by A cannot be sold to a third party. The reasonableness of these two assumptions is discussed in Sections 7.2 and 7.3.

In Stage 2, the entrepreneur chooses his effort level and A decides whether or not to enter Market 2. If A enters it immediately pays F. In Stage 3, the firms simultaneously choose their quantities for Market 2 if A has entered. If A has not entered, B produces its monopoly quantity. Finally, all payoffs are realized. Firm A receives its own profits from Markets 1 and 2 plus its proportional share (α) of B's profits from Markets 2 and 3, while the entrepreneur receives the remaining profits of B. There is no discounting.

2.1. Discussion of the model

The model restricts the partners' contracting ability in several important ways. Specifically, (1) the firms cannot contract separately on different activities within the same firm, (2) the only available security is equity, and (3) they can trade equity only in B. The first restriction is meant to reflect the fact that information on intrafirm activities can be much more difficult to verify than overall firm results. Contracting on separate activities can be especially difficult in innovative firms, where different activities are often intertwined and depend on interrelated technologies. The second restriction is addressed below. The third restriction is justified by the empirical regularity that the purchasing firm in an equity alliance is generally much larger than its partner. For example, in a sample of 97 equity alliances formed between 1989 and 1997, Pablo and Subramaniam (2002) find that the buying partner has an average (median) market value of more than \$26 billion (\$12 billion) versus less than \$800 million (\$150 million) for the selling partner. Similarly, Allen and Phillips (2000) find that the value of the selling firm's total assets in equity alliance deals is on average approximately 1.3% of the value of the buying firm's total assets. Furthermore, both studies exclude deals with private selling firms. Thus, it is very unlikely that the smaller partner could purchase a significant stake in the larger partner in most real-life situations. Even if such a purchase were possible, it would likely be prohibitively costly. See Section 7.4 for a more rigorous discussion of this issue.

The model also implicitly assumes that firms cannot contractually agree not to compete. Together with the assumption that equity is the only available security, this is intended to reflect two facts. First, explicit agreements in restraint of trade are illegal in the United States under Section 1 of the Sherman Act. Second, it could be difficult to structure an ex ante contractual agreement because of incomplete contracting problems arising from uncertainties not modeled here. However, an equity investment may not be considered per se illegal and requires no contractual agreement on competition. Also, if the firms are attempting to mask their competitive motives in what looks like a financing deal, it is likely that they will not stray too far from standard securities. See Section 7.1 for a more rigorous discussion of security design issues.

The fact that the entrepreneur's effort relates to a separate growth option market instead of the contested market allows for a clean separation between the costs and benefits of selling equity. It also makes the effort decision completely independent of A's entry decision. However, it does not allow for any interaction between the costs and benefits of equity and thus limits the model's applicability to situations in which the entrepreneurial firm's effort has an important impact on the contested market. An alternative specification would have the entrepreneur's effort affect efficiency in Market 2. However, this results in significant technical complications. In Section 6.2 I analyze a simplified version of such a specification to explore the important implications and show that the model's main insights are not peculiar to the assumption of separate markets.

The model as it stands cannot distinguish between an equity alliance and a merger that is followed by the provision of a bonus or profit sharing system for the entrepreneur. However, the difficulty in contracting over specific activities within the firm could explain why an equity alliance is often preferred. For example, assume that the ability to innovate and create value in Market 3 is inseparable from B's operation in Market 2, but the resulting cash flows from Market 3 are noncontractible. Then transferring the property right over all of B's activities to A could generate a hold-up problem. Specifically, once the entrepreneur has secured the necessary innovation to enter Market 3, A might be able to expropriate all of the surplus from that endeavor, causing the entrepreneur to exert no effort ex ante. Thus, keeping B as a stand-alone firm with full property rights over its activities could be important for preserving incentives. See Aghion and Tirole (1994) and Robinson (2003) for related analyses of alliance contracting.

Finally, the model does not address the possibility of royalty payments. Royalties are generally paid by an established firm to an entrepreneurial firm for use of its technology. In such cases the specific use of the technology must be contractible, which violates one of my major assumptions. If such royalties could be demanded by B on A's revenue in Market 2, the firms could assure no entry by A if they simply assign all this revenue to B. I assume this is not possible. However, this does not mean that the model is inconsistent with the use of royalties in alliance contracts. For example, there would be no effect on my results if the firms could write a royalty contract for payments related to A's use of B's technology in Market 1.

3. Equity stakes and entry deterrence

This section provides the major results on the formation of the alliance, the equilibrium equity transfer, and entry. A subgame perfect Nash equilibrium is derived using backward induction. For the remainder of the paper I use the term "status quo" to refer the case in which there is no alliance and no stake sale.

3.1. Analysis of stages 2 and 3

The various markets can be analyzed separately for the subgame starting at Stage 2 because equity guarantees proportional payouts of all Firm B cash flows in Stage 3.

3.1.1. Market 2

Throughout the analysis, I make the usual Cournot assumptions that all equilibria are stable, and that quantities must be non-negative.[7] Any monetary transfer in Stage 1 is sunk and does not affect A's entry decision in Stage 2 or the firms' quantity decisions. I also assume throughout the paper that A never enters when indifferent.

Four classes of possible subgame equilibria are relevant for Market 2. They are indexed by whether or not an alliance is formed and whether or not A enters. In each case A could own a proportion $\alpha \in [0, 1]$ of B's equity. The cases are as follows: (1) No Alliance with No Entry (N/N); (2) No Alliance with Entry (N/E); (3) Alliance with No Entry (SA/N); and (4) Alliance with Entry (SA/E).

In Case 1, N/N, B behaves as a monopolist with marginal cost c and profits are split according to ownership of B's equity. Let π_m denote the total profit from Market 2 in this case. Then the entrepreneur gets a total payoff from Market 2 of $(1-\alpha)\pi_m$ and A gets a total payoff from Market 2 of $\alpha\pi_m$.

In Case 2, N/E, A and B compete as Cournot duopolists with costs $(1+\gamma)c$ and c, respectively, and A gets a proportion α of B's profit. Let $\pi_A(\alpha)$ and $\pi_B(\alpha)$ denote the associated profits of the firms' facilities as a function of the stake size.[8] In other words, $\pi_A(\alpha)$ does not include A's share of B's profit, and $\pi_B(\alpha)$ equals all of B's profits in Market 2, not just the portion kept by the entrepreneur. In this case, the entrepreneur gets a total payoff from Market 2 of $(1-\alpha)\pi_B(\alpha)$ and A gets $\pi_A(\alpha)+\alpha\pi_B(\alpha)$.

In Case 3, SA/N, B acts as a monopolist with marginal cost $(1-\delta)c$ and profits are split according to ownership of its equity. Let π_m^{SA} denote the associated total profit in Market 2. Total payoffs are analogous to those in Case 1.

In Case 4, SA/E, A and B compete as Cournot duopolists with equal marginal costs of $(1-\delta)c$ and A gets a proportion α of B's profit. Let $\pi_A^{\mathrm{SA}}(\alpha)$ and $\pi_B^{\mathrm{SA}}(\alpha)$ denote the associated profits of the individual firms' facilities. Total payoffs are analogous to those in Case 2.

An increase in α reduces A's quantity, increases B's quantity, and reduces total output in Cases 2 and 4 (see the proof of Proposition 1). This is the basic result in Reynolds and Snapp (1986); competition is softened as a result of partial ownership by a rival. This together with constant marginal costs also implies that monopoly profits always exceed the sum of duopoly profits because total output strictly decreases to the level of monopoly output as α rises.

Entry by A results in competition according to Case 2 or 4, depending on whether there is an alliance. Thus, in each case there is an entry condition based on the level of the entry fee, F. For each case I define a threshold fixed fee such that if F is below the threshold, entry occurs. The two thresholds are

$$f(\alpha) = \pi_A(\alpha) + \alpha\pi_B(\alpha) - \alpha\pi_m \tag{1}$$

for the no alliance case and

$$f^{\mathrm{SA}}(\alpha) = \pi_A^{\mathrm{SA}}(\alpha) + \alpha\pi_B^{\mathrm{SA}}(\alpha) - \alpha\pi_m^{\mathrm{SA}} \tag{2}$$

for the alliance case. An analysis of these thresholds provides Proposition 1.

[7]Formally, stability requires that $qP'' + P' < 0$ and, for cases in which a stake is sold, $(q_B + \alpha q_A)P'' + P' < 0$.

[8]Whenever the simultaneous solution of the first-order conditions in any of the cases implies a negative quantity for A, I assume that A produces zero and B adjusts its quantity correspondingly.

Proposition 1. *The thresholds* $f(\alpha)$ *and* $f^{\mathrm{SA}}(\alpha)$ *decrease strictly as* α *rises from zero until they reach zero at some* $\alpha \leqslant 1$, *and remain equal to zero for all higher* α.

As A's stake in B grows, it internalizes more of the decline in B's profit caused by entry. More specifically, an increase in α causes A's no-entry payoff from Market 2 to grow by the change in α times monopoly profits. Its entry payoff also changes: its own facilities' profits are lower, but the profits of B's facilities, of which it receives the proportion α, increase as a result of the softening of competition.[9] The result implies that the increase in A's no-entry payoff is always larger than any increase in its entry payoff. As a result, entry is less attractive, and the threshold declines.

Proposition 1 also implies that B can always deter entry by selling a large enough equity stake. To understand this, consider the case in which $\alpha = 1$. In this situation, A is effectively a monopolist in Market 2 with the option of introducing a second facility at cost F to compete with itself. Because this is never optimal, there must exist a stake in [0, 1] that deters entry.

Corollary 1 is a direct implication of Proposition 1.

Corollary 1. *If there is an alliance, there exists some critical level,* $\alpha^*_{\mathrm{SA}} \in [0,1]$, *such that for all stake sizes* $\alpha \in [0, \alpha^*_{\mathrm{SA}})$, *A chooses to enter Market* 2, *so Case* 4 *is applicable. For all* $\alpha \in [\alpha^*_{\mathrm{SA}}, 1]$, *A chooses not to enter, so Case* 3 *is applicable. An analogous critical level,* $\alpha^* \in [0,1]$, *exists if there is no alliance.*

These critical levels are defined implicitly by A's no-entry constraint. Specifically, α^*_{SA} is defined by

$$\pi_A^{\mathrm{SA}}(\alpha^*_{\mathrm{SA}}) + \alpha^*_{\mathrm{SA}}\pi_B^{\mathrm{SA}}(\alpha^*_{\mathrm{SA}}) - \alpha^*_{\mathrm{SA}}\pi_m^{\mathrm{SA}} - F = 0, \tag{3}$$

and α^* is defined analogously. Using these definitions, Proposition 2 is derived.

Proposition 2. *An increase in F decreases* α^*_{SA} *and* α^*, *an increase in* γ *reduces* α^* *but does not affect* α^*_{SA}, *and an increase in* δ *raises* α^*_{SA} *but does not affect* α^*.

It is always more tempting for A to enter, implying a larger stake for deterrence, the smaller is the cost of entry, F. An increase in γ, the initial cost disadvantage, implies a higher marginal cost for A when no alliance is formed, making entry less profitable. It has no effect when there is an alliance because of the assumption that A becomes an equal competitor with B in that case. When there is an alliance, an increase in the efficiency enhancement, δ, increases duopoly profits for both firms, which makes entry more attractive.

3.1.2. Markets 1 and 3

The only relevant state variables for the Market 3 subgame are whether there is an alliance and the size of the equity stake, α. Let 1_{SA} be an indicator variable equaling one if an alliance is formed in Stage 1 and zero otherwise. Then the entrepreneur chooses his

[9] Farrell and Shapiro (1990) prove that the equity holder's profits fall in a more general setting.

effort level e in Stage 2 to maximize

$$(1-\alpha)e(V_3 + 1_{\text{SA}}\Delta V_3) - \frac{e^2}{2\beta}. \tag{4}$$

He thus chooses an effort level

$$e^* = \beta(1-\alpha)(V_3 + 1_{\text{SA}}\Delta V_3), \tag{5}$$

assuming an interior solution. For the remainder of the paper, I assume $\beta \leqslant \frac{1}{(V_3+\Delta V_3)}$, which ensures that there is always an interior solution for $e \in [0, 1]$. The total expected surplus generated in Market 3 can be written as

$$\frac{\beta}{2}(1-\alpha^2)(V_3 + 1_{\text{SA}}\Delta V_3)^2, \tag{6}$$

which is decreasing and concave in α. Equivalently, the cost of transferring equity is monotonically increasing and convex in α, which greatly simplifies the analysis.

If innovation is successful, the payoff $V_3 + 1_{\text{SA}}\Delta V_3$ is split between the firms in Stage 3 according to ownership of B's equity. The profit produced in Market 1, $V_1 + 1_{\text{SA}}\Delta V_1$, is fully retained by A.

3.2. *Analysis of Stage 1*

In Stage 1, A and B bargain over whether to form an alliance and the size of the equity stake (α) and monetary transfer (T). Denote the two firms' original shareholders' payoffs in the event of disagreement, or their threat points, as D_A and D_B. Denote their continuation payoffs conditional on the success of the negotiations, excluding the monetary transfer, as S_A and S_B. Because the bargaining game is modeled as generalized Nash bargaining, the bargainers maximize

$$[S_A - T - D_A]^{1-\theta}[S_B + T - D_B]^{\theta}. \tag{7}$$

This can be conceptualized as a two-step problem. In the first step the bargainers decide whether to form an alliance and how much equity to transfer to maximize their combined surplus, $S \equiv S_A + S_B$. In the second step they set the monetary transfer to divide the surplus according to their bargaining powers. Because the outcome of the negotiation is final, the disagreement payoffs equal the payoffs in the status quo.

The partners never have an incentive to transfer equity unless A would enter in Stage 2 with $\alpha = 0$. Thus, an equity transfer can occur only if the relevant fixed fee threshold exceeds F when $\alpha = 0$, i.e., if $f^{\text{SA}}(0) = \pi_A^{\text{SA}}(0) > F$ if there is an alliance and $f(0) = \pi_A(0) > F$ if there is not an alliance. The effects of the parameters on whether this holds thus follow directly from Proposition 2. Changes in parameters that make entry more attractive and thus increase the fixed fee thresholds also increase the probability that there is an incentive to transfer equity.

3.2.1. *Alliances with No Entry (SA/N)*

Depending on the parameter values, the equilibrium of the model can involve any one of the four classes of Market 2 subgame equilibria discussed in Section 3.1.1. This section focuses on cases in which it is optimal to form an alliance and deter entry (SA/N).

Proposition 3. *If $F \geqslant f^{SA}(0)$, then an alliance is formed and no equity is transferred. If $F < f^{SA}(0)$ and $V_3 + \Delta V_3$ is sufficiently small, then an alliance is formed and a deterrent stake of α^*_{SA} is transferred.*

Given that $F \geqslant f^{SA}(0)$ implies that no equity is required to deter entry, an alliance only has benefits and is always formed in this case. When there is a threat of entry [$F < f^{SA}(0)$], the only cost of transferring equity is the reduction of the entrepreneur's effort in Market 3. Thus, if this cost is small the firms always find it optimal to form an alliance and transfer enough equity to ensure that B remains a monopolist in Market 2. Also, there is a discrete upward jump in the bargainers' joint payoff when the critical no-entry stake size, α^*_{SA}, is reached, and there are no benefits to additional equity. Thus, for any specification of the remaining parameters, the no-entry equilibrium with a stake size of α^*_{SA} is the unique equilibrium for some range of the success payoff in Market 3, $V_3 + \Delta V_3$, as it increases from zero. This implies that the appropriate comparative statics for the optimal stake size are given by Proposition 2 in this case.

As the success payoff in Market 3 goes to zero, the equilibrium characterized in Proposition 3 approaches the firms' first-best contractual solution, in which A contractually agrees to stay out of Market 2, the firms form an alliance, and a monetary transfer is used to split the surplus. While such contracts could be legally unenforceable or impossible to write, the equity deal enables the firms to achieve a similar result because it makes the no-entry commitment strategically credible.

An important implication of this equilibrium is that an equity transfer paired with an alliance can be motivated by entry deterrence even if there would be no entry in the absence of a deal. To see this, note that the alliance fixed fee threshold, $f^{SA}(\alpha)$, always exceeds the no-alliance threshold, $f(\alpha)$, at $\alpha = 0$. The former is A's payoff as a symmetric Cournot duopolist with marginal cost $(1-\delta)c$, while the latter is A's payoff as a Cournot duopolist in a situation in which its marginal cost, $(1+\gamma)c$, is higher than its competitor's, c. It is well known that symmetric Cournot profits decrease as constant marginal costs increase, and that profits are lower for a Cournot competitor with a cost disadvantage. This directly implies Proposition 4.

Proposition 4. *Whenever $F \in [f(0), f^{SA}(0))$ and an alliance with a deterrent stake of α^*_{SA} is optimal, the deal deters entry that would not occur without an alliance.*

3.2.2. Mapping equilibria

In this Section I analyze more generally when the various alliance and entry configurations described in Section 3.1.1 occur in equilibrium. To make the analysis tractable, I assume that demand in Market 2 is linear. Also, for intuitive clarity I initially assume the alliance has no effect on Market 3, i.e., $\Delta V_3 = 0$, and provides no efficiency benefit to B in Market 2, i.e., $\delta = 0$. I discuss the effects of changes in ΔV_3 and δ following the formal analysis.

Choosing whether to have an alliance and how much equity to sell amounts to choosing among the four possible subgame equilibria, SA/N, SA/E, N/N, and N/E. As such, I analyze the problem as if the players find an optimal stake size for each possible subgame type, then compare the resulting payoffs to find the optimal choice.[10]

[10]When I analyze the scenarios involving accommodation, SA/E and N/E, for this purpose I assume that entry occurs for any stake size. If the optimal stake size assuming entry exceeds the relevant deterrent stake size, α^*_{SA} or α^*, then entry does not occur in the actual subgame. However, my assumption is innocuous because the corresponding deterrence equilibrium is always more attractive than any accommodation equilibrium in such a case.

Given the simplifying assumptions $\delta = 0$ and $\Delta V_3 = 0$, the three key drivers of the analysis are the success payoff in Market 3, V_3, the alliance benefit in Market 1, ΔV_1, and A's initial cost disadvantage, γ. The success payoff V_3 defines the cost of issuing equity to A. If this cost is high enough, the firms could decide to limit the stake size and accommodate entry to preserve the entrepreneur's innovative incentives. The benefit to Market 1, ΔV_1, is a fixed benefit and simply makes the alliance more attractive.

To understand the role of the cost disadvantage, γ, note that with $\delta = 0$ the only effect of the alliance in Market 2 is to reduce A's marginal cost from $(1+\gamma)c$ to c. Thus, if $\gamma = 0$ the alliance has no effect on Market 2. In this case an alliance is always optimal because it increases A's payoff in Market 1 by ΔV_1 with no countervailing cost. However, an increase in γ has two important effects that can cause the firms to avoid the alliance. First, entry becomes less attractive for A if there is no alliance because its marginal cost rises with γ. This implies that the amount of equity required to deter entry in the no-alliance, no-entry equilibrium, N/N, falls as γ rises, making that equilibrium relatively more attractive (see Proposition 2). In fact, there is always some value of γ, say γ^*, beyond which entry can never be profitable without an alliance. More formally, $\alpha^* = 0$ for all $\gamma > \gamma^*$. This ability to deter with a smaller stake is more valuable the greater is the cost of equity, indexed by V_3. Second, the duopoly profits enjoyed in the no alliance with entry equilibrium, N/E, change as A's cost disadvantage, γ, changes. They initially decrease as γ rises from zero because A becomes less efficient, which would imply that N/E becomes less attractive. However, duopoly profits can become increasing in γ as it continues to rise because A becomes such a weak (high cost) competitor that B is able to take advantage of near-monopoly status. Thus, the N/E equilibrium can be attractive at higher levels of γ.

Next I analyze the effect of V_3, ΔV_1, and γ on the alliance and accommodation decisions more formally. Then I bring the analysis together using a set of equilibrium maps that show when each equilibrium arises depending on the values of these parameters.

Consider the effect of the Market 3 success payoff, V_3. Proposition 3 implies that it is always optimal to form an alliance and sell a deterrent stake of α^*_{SA} for some range of V_3 as it rises from zero. Given this, I can define two threshold levels for V_3, one applicable to the alliance decision and the other to the deterrence or accommodation decision. Let V_3^{SA} be the maximum value of V_3 such that for all $\gamma \geqslant 0$, an alliance is optimal for all $V_3 < V_3^{SA}$. In other words, for some γ it becomes optimal to forgo the alliance as V_3 rises above V_3^{SA}. Similarly, let V_3^{E} be the maximum value of V_3 such that for all $\gamma \geqslant 0$, entry is optimally deterred when $V_3 < V_3^{E}$. So in this case, it becomes optimal to accommodate entry as V_3 rises above V_3^{E} for some γ.

The relationship between the two thresholds is key, so I specify two cases for the proceeding analysis. Case 1 is defined by $V_3^{E} > V_3^{SA}$, and Case 2 is defined by $V_3^{SA} > V_3^{E}$. The following result allows me to determine when these cases occur and, together with the definitions of the thresholds, narrow the range of possible equilibria for some values of V_3.

Lemma 1. *Assume demand in Market 2 is linear, $\delta = 0$, and $\Delta V_3 = 0$. Then $V_3^{SA} = 0$ when $\Delta V_1 = 0$, and an increase in ΔV_1 increases V_3^{SA} but does not affect V_3^{E}. Furthermore, for all $V_3 > V_3^{E}$, SA/E is preferred to SA/N for all γ.*

To understand this result, note that entry is always deterred if V_3 is small enough because there is minimal cost of issuing equity. However, if $\Delta V_1 = 0$ an alliance can only hurt because it increases the required stake size with no offsetting benefits. Together these imply $V_3^{SA} = 0$. Next, an increase in ΔV_1 simply makes the alliance more profitable with no effect on Markets 2 or 3. This makes alliances more likely at higher costs of equity, increasing V_3^{SA}. However, it does not affect the accommodation and stake size decision, which is driven by the trade-off between competitive and incentive effects in Markets 2 and 3. Thus, V_3^{E} stays the same. For the last sentence of the result, note that a change in the cost disadvantage, γ, does not affect the alliance equilibria, SA/N and SA/E, because A and B become equal competitors following the alliance. Thus, the accommodation decision is not affected by γ if there is an alliance.

Fig. 1 summarizes which equilibrium types are possible within different ranges of V_3 given the V_3 thresholds and Lemma 1. The first sentence of Lemma 1 implies that Case 1, $V_3^{E} > V_3^{SA}$, holds for low ΔV_1 and Case 2, $V_3^{SA} > V_3^{E}$, holds for high ΔV_1. Now consider the equilibria that are possible within the different ranges of V_3. The definitions of the thresholds directly imply that an alliance with deterrence, SA/N, is optimal for all $V_3 < \min[V_3^{SA}, V_3^{E}]$, that SA/E and N/E can be ignored for all $V_3 < V_3^{E}$, and that N/N and N/E can be ignored for all $V_3 < V_3^{SA}$. Furthermore, Lemma 1 implies that SA/N can be ignored for all $V_3 > V_3^{E}$. Thus, for Case 2 with $V_3^{SA} > V_3^{E}$, SA/E must be optimal between V_3^{E} and V_3^{SA}. This analysis significantly narrows the range of possible equilibria, as shown in Fig. 1.

I now turn to comparing the remaining equilibria within the ranges identified in the figure. These comparisons are driven mainly by A's cost disadvantage, γ. First consider no alliance with no entry (N/N) versus the other equilibria. N/N becomes relatively more attractive as γ rises because entry can be deterred using a smaller stake. Proposition 5 states this formally and defines cutoff levels of γ to compare N/N with the other equilibria wherever necessary.

Proposition 5. *Assume demand in Market 2 is linear,* $\delta = 0$, *and* $\Delta V_3 = 0$. *Then*

(a) *for all* $V_3 > V_3^{SA}$ *there exists a unique level of* γ, $\gamma^{SA/N}$, *such that SA/N is preferred to N/N whenever* $\gamma < \gamma^{SA/N}$ *and N/N is preferred otherwise*;
(b) *for all* $V_3 > \max[V_3^{E}, V_3^{SA}]$ *there exists a unique level of* γ, $\gamma^{SA/E}$, *such that SA/E is preferred to N/N whenever* $\gamma < \gamma^{SA/N}$ *and N/N is preferred otherwise*; *and*

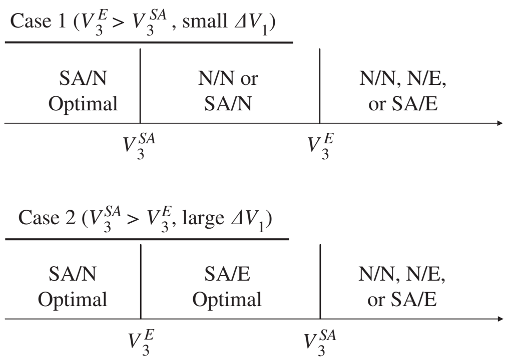

Fig. 1. Possible equilibrium types for various ranges of the success payoff in Market 3, V_3. N/N = No Alliance with No Entry; N/E = No Alliance with Entry; SA/N = Alliance with No Entry; SA/E = Alliance with Entry. Two cases are possible, differentiated by the relationship between two cutoff levels of V_3. V_3^{SA} is defined as the maximum value of V_3 such that for all $\gamma > 0$, an alliance is optimal when $V_3 < V_3^{SA}$, where γ is Firm A's initial cost disadvantage in Market 2. V_3^{E} is defined as the maximum value of V_3 such that for all $\gamma > 0$, entry is optimally deterred when $V_3 < {}_3^{E}$. Their relationship is determined by the magnitude of the alliance payoff in Market 1, ΔV_1.

(c) *for all* $V_3 > \max[V_3^{\mathrm{E}}, V_3^{\mathrm{SA}}]$ *there exists a unique level of* γ, $\gamma^{\mathrm{N/E}}$, *such that N/E is preferred to* N/N *whenever* $\gamma < \gamma^{\mathrm{N/E}}$ *and N/N is preferred otherwise.*

Part a reveals how the optimal equilibrium is determined in the middle range for V_3 in Case 1. Parts b and c describe how SA/E and N/E are compared with N/N in the high range for V_3 in both Cases 1 and 2.

It remains to determine how SA/E and N/E are compared. Both of these equilibria involve the operation of a duopoly, so this comparison depends on the overall profit potential of the duopoly. When there is no cost disadvantage, or $\gamma = 0$, SA/E and N/E are equivalent. However, duopoly profits in the N/E equilibrium initially decrease in γ as it rises from zero because A becomes less efficient. They can become increasing in γ only at higher γ, when A becomes such a weak competitor that B is able to take advantage of near-monopoly status. This trade-off leads to Proposition 6.

Proposition 6. *Assume demand in Market* 2 *is linear*, $\delta = 0$, *and* $\Delta V_3 = 0$. *Then*

(a) *for all* $V_3 > \max[V_3^{\mathrm{E}}, V_3^{\mathrm{SA}}]$ *such that* $\gamma^{\mathrm{SA/E}} > \gamma^{\mathrm{N/E}}$, *SA/E is preferred to N/E for all* $\gamma < \gamma^{\mathrm{SA/E}}$; *and*

(b) *for all* $V_3 > \max[V_3^{\mathrm{E}}, V_3^{\mathrm{SA}}]$ *such that* $\gamma^{\mathrm{SA/E}} < \gamma^{\mathrm{N/E}}$, *there exists a unique level of* γ, γ^{E}, *such that SA/E is preferred to N/E whenever* $\gamma < \gamma^{\mathrm{E}}$ *and N/E is preferred otherwise.*

To understand these results and the intuition behind them, see Fig. 2, which maps the optimal equilibrium type as γ and V_3 vary given different levels of ΔV_1. The results are translated into the figure by deriving a number of technical conditions on the various critical levels of γ and V_3. See Appendix B for a list of these conditions and their derivation.

Consider the upper map, in which $\Delta V_1 \approx 0$ and thus Case 1 is applicable. With this assumption, the only significant effect of the alliance is to make A more competitive in Market 2 by lowering its marginal cost from $(1+\gamma)c$ to c. In this case SA/N is generally inferior to N/N because a larger stake is required to deter entry with the alliance. In other words, with no fixed benefit to the alliance, SA/N can be optimal only when equity is nearly costless ($V_3 \approx 0$) or there is not much of a cost disadvantage without the alliance ($\gamma \approx 0$), i.e., the stake size difference is small. In terms of the thresholds, this means $V_3^{\mathrm{SA}} \approx 0$ and $\gamma^{\mathrm{SA/N}} \approx 0$. Therefore, N/N is generally optimal below V_3^{E}, which is defined as the point at which equity becomes so expensive that accommodation is sometimes optimal.

As the cost of equity, V_3, continues to rise, accommodation becomes more attractive because of the ever higher cost of a deterrent stake ($\gamma^{\mathrm{SA/E}}$ and $\gamma^{\mathrm{N/E}}$ rise from zero). However, because the alliance has no fixed benefits ($\Delta V_1 \approx 0$), N/N remains optimal if γ is sufficiently high. To understand this, consider a case in which A is so far behind in efficiency without an alliance that no equity is required to deter entry, or $\gamma \geqslant \gamma^*$. In this situation, surplus in Market 2 is maximized by N/N because entry is deterred, and surplus in Market 3 is maximized by N/N because there is no equity and thus no effort dilution.

The upper map also shows that SA/E is always preferred to N/E at low levels of γ when $V_3 > V_3^{\mathrm{E}}$. To understand this, note that these equilibria are equivalent when $\Delta V_1 \approx 0$ and $\gamma = 0$, i.e., when the alliance has no effect. Now note that N/E becomes less attractive as γ rises above zero because A becomes less efficient and duopoly profits fall, causing SA/E to

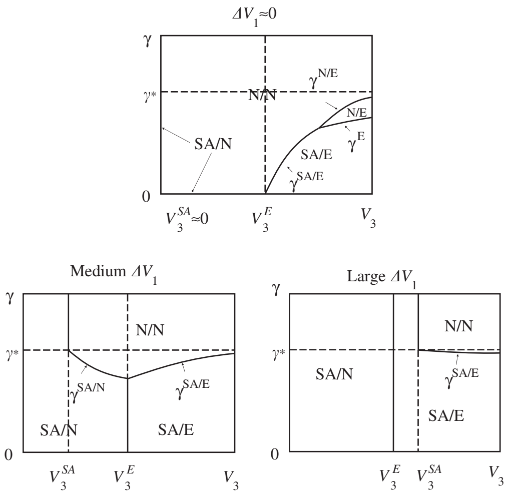

Fig. 2. Plots the equilibrium type for varying values of the success payoff in Market 3, V_3, Firm A's initial cost disadvantage, γ, and the alliance payoff in Market 1, ΔV_1. N/N = No Alliance with No Entry; N/E = No Alliance with Entry; SA/N = Alliance with No Entry; SA/E = Alliance with Entry. The solid lines delineate different equilibrium regions. $\gamma^{SA/N}$, $\gamma^{SA/E}$, and $\gamma^{N/E}$ are critical levels of γ defined such that the corresponding equilibrium type is superior to N/N for all γ below the critical level. γ^{E} is a critical level of γ defined such that SA/E is preferred to N/E whenever γ is below the critical level. The dashed lines represent other critical levels of V_3 and γ . V_3^{SA} is defined as the maximum value of V_3 such that for all $\gamma > 0$, an alliance is optimal when $V_3 < V_3^{SA}$. V_3^{E} is defined as the maximum value of V_3 such that for all $\gamma > 0$, entry is optimally deterred when $V_3 < V_3^{E}$. γ^* is the level of γ beyond which entry is always unprofitable without an alliance. The figure assumes no alliance efficiency enhancement in Market 2, or $\delta = 0$, and no alliance benefit in Market 3, or $\Delta V_3 = 0$.

dominate N/E. However, as γ continues to rise and A falls farther behind in efficiency, N/E can become more attractive as a result of B's ability to act as a near-monopolist.

The lower left map illustrates how these results change with an increase in the alliance benefit for Market 1, ΔV_1. First and foremost, this makes alliances more attractive and thus increases V_3^{SA} above zero, allowing a significant role for the SA/N equilibrium. In the example drawn here, ΔV_1 has increased only modestly, so this is still Case 1 with $V_3^{SA} < V_3^{E}$, implying, as shown in Fig. 1, that SA/N is always optimal below V_3^{SA}. As V_3 rises, but not enough to make accommodation optimal, N/N becomes increasingly attractive relative to SA/N ($\gamma^{SA/N}$ decreases from γ^*) as a result of the smaller stake size needed to deter entry with no alliance. When equity becomes costly enough (V_3 becomes high enough) that SA/E dominates SA/N, the partners compare SA/E with N/N. In the case drawn here, SA/E becomes relatively more attractive as the cost of equity grows because of its small stake size. Also note that the N/E equilibria do not appear in this figure. This is to reflect the fact that a significant alliance benefit for Market 1, ΔV_1, makes SA/E much more attractive relative to N/E ($\gamma^{SA/E}$ rises while $\gamma^{N/E}$ stays the same).

Finally, in the lower right map, the further increase in the fixed benefit ΔV_1 has made alliances very desirable and pushed V_3^{SA} above V_3^{E}. Thus, Case 2 is applicable and SA/N is always optimal until the cost of equity, V_3, rises so high that deterrence is too expensive. At this point, SA/E becomes optimal because the large ΔV_1 means it is still optimal to have an alliance. Finally, N/N plays a role as the extreme cost of equity makes the possibility of deterring entry with a smaller stake size attractive.

If the base assumptions that $\delta = 0$ and $\Delta V_3 = 0$ are relaxed, the analysis changes somewhat. An increase in the alliance efficiency enhancement, δ, does not affect the no alliance equilibria, N/E and N/N, and always makes SA/E relatively more attractive because duopoly profits increase as equal competitors' marginal costs decrease. However, its effect on the attractiveness of SA/N is more complicated, given that it both increases monopoly profits and increases the stake size required for deterrence. For small V_3 the former effect dominates, while for larger costs of equity the latter dominates. In contrast, an increase in ΔV_3 makes the alliance equilibria more attractive relative to the nonalliance equilibria and thus expands the area of the graph in which an alliance is formed.

Finally, comparative statics can be derived for the optimal stake size assuming that entry is accommodated.

Proposition 7. *Assume accommodation is optimal and demand in Market* 2 *is linear. Then the optimal stake size decreases in* V_3 *and* β *and is unaffected by* F. *The optimal stake size also increases in* δ *and decreases in* ΔV_3 *in the alliance case and decreases in* γ *in the no-alliance case.*

The results with respect to V_3, ΔV_3, and the sensitivity of the entrepreneur's effort, β, arise directly from the fact that the reduction of effort in Market 3 is the only cost of selling equity. The fixed fee, F, does not affect the marginal value of the stake once accommodation is assumed. An increase in the efficiency enhancement, δ, makes the market more profitable overall following an alliance, which increases the value of softening. Finally, an increase in γ reduces A's competitiveness if there is no alliance and reduces the value of softening.

3.3. Predictions and related empirical work

The empirical implications of the model are consistent with much of the extant evidence. Many papers have found that an equity stake is more likely to be transferred when an alliance involves R&D or technology sharing (see, e.g., Gulati and Singh, 1998; Oxley, 1997; Pisano, 1989; Robinson and Stuart, 2003). In a related vein, Oxley (1997) finds that more hierarchical alliances, including those with equity transfers, are associated with product or process design, multiple activities, and multiple technologies.[11] Oxley argues that product or process design activities are likely to facilitate transfers of both technology and know-how and that monitoring is more problematic with multiple activities or technologies. Finally, Gulati and Singh (1998) find that firms in industries with weak intellectual property rights are more likely to form equity alliances or joint ventures rather than simple contractual agreements. All of these findings confirm that equity stakes are more common in situations that involve knowledge transfer and in which appropriation is relatively easy.

While these results are consistent with my model, they are also consistent with explanations based on hold-up problems. It would be interesting for future empirical work to distinguish between contracting problems in trading relationships and problems related to potential competition as modeled here. Because most existing work considers only the equity versus no-equity decision and does not analyze the factors affecting stake sizes, the remaining comparative statics in Propositions 2 and 7 should be particularly useful for

[11] Pisano (1989) also finds that equity is more likely when an alliance involves multiple projects.

future empirical work. It is also important to note that equity could be used in alliances for other reasons, which makes tests of these and other predictions of the model more noisy. For example, since equity investments can be capitalized, the large firm could prefer equity to a simple cash transfer for accounting reasons.

An interesting implication of Proposition 2 is that a stake transfer is more likely (α^*_{SA} is more likely to be greater than zero) the greater is the efficiency enhancement, δ. This implies that equity alliances should be more profitable than nonequity alliances. This is consistent with the finding of Pablo and Subramaniam (2002) that issuers in equity alliances experience an average abnormal return of approximately 15% versus less than 2% for firms in nonequity alliances.

The results on when entry is accommodated and when alliances are formed (Lemma 1 and Propositions 5 and 6) also have several intuitive empirical implications. First, the size of the stake is smaller and entry more likely to be accommodated the more profitable or numerous are the entrepreneurial firm's future growth options. In other words, the value of the entrepreneurial firm's growth options is preserved by limiting the established firm's stake and sacrificing some profits in other markets. Second, an entrepreneurial firm is likely to forgo a profitable alliance opportunity to protect the value of its growth options only if its partner is far behind in competitiveness without the alliance (large γ).

4. Stake pricing

This section characterizes the model's predictions for the equilibrium price paid for the equity. Because the model is focused on explaining equity alliances, I restrict my attention to cases in which it is optimal to form an alliance.

It is a stylized fact that private placements of equity are sold for a discount to the pre-trade stock price of the issuing firm on average.[12] However, Barclay and Holderness (1989) and Barclay et al. (2001) find that sales of *existing* blocks generally occur at a premium to the issuing firm's post-trade stock price. Barclay et al. (2001) find that these premia reflect the intention of the block purchaser to be active in a relationship with the issuer, such as an alliance relationship or a control relationship. Similarly, Allen and Phillips (2000) find that block purchases by corporations, be they private placements or third-party trades, generally occur at a premium to the pre-trade price, although they do not have data for the equity alliances in their sample.

To derive results for the equilibrium price, I refer to the equilibrium monetary transfer, T^*, as the price paid for the equity stake. I compare the price paid per share with the post-trade fair value, or "price," of the entrepreneur's equity position in B. This is a common measure used in the literature on block trades.[13] In the present context, I define the post-trade price per share of B's stock as the expected continuation payoff per share for the entrepreneur, measured after the stake transfer and distribution of the monetary transfer. The results are the same if this is measured excluding the entrepreneur's effort cost.

Proposition 8. *If A will not enter in the status quo $[F \geqslant f(0)]$, the stake is always sold for a premium if θ is high enough and could sell for a premium or discount if θ is low. If A will enter*

[12] See, e.g., Hertzel et al. (2002).

[13] See, e.g., Barclay and Holderness (1989), Barclay et al. (2001), and Burkart et al. (2000).

in the status quo [$F < f(0)$], *a discount is possible for any* θ. *A premium is always more likely the larger is* θ *and the larger is* ΔV_1 *when* $\theta > 0$.

When the entrepreneur's bargaining power is high, he is always able to extract a premium from A if A's outside option in Market 2 is zero. The premium compensates the entrepreneur for his private effort cost and provides his share of A's benefit in Market 1. However, if A can profitably enter Market 2 in the status quo, this increase in its outside option can erode or eliminate the premium, as can a decrease in the entrepreneur's bargaining power. A large alliance benefit for A in Market 1, ΔV_1, always increases the chance of a premium because this benefit is available only if the entrepreneur agrees to the alliance.

5. Welfare analysis

The idea that equity transfers in cooperative relationships are motivated by entry deterrence or competitive softening brings their social efficiency into question. Antitrust officials are increasingly concerned about collaboration among competitors. In a joint report in 1999 by the Federal Trade Commission and the Department of Justice entitled *Antitrust Guidelines for Collaborations among Competitors*, US regulators state that such arrangements could be procompetitive to the extent that they improve economic efficiency but also could have anticompetitive effects when the partners are either current or potential competitors. They also acknowledge that financial interests in an alliance partner can affect the parties' competitive incentives. Here I address the model's implications for these questions. I focus on cases in which an alliance is formed.

The analysis relies on the implicit assumption that the entrepreneur's original decisions to invest in research and production capabilities are fixed. Also, because Markets 1 and 3 are modeled in reduced form, I focus on a narrow anti-trust perspective in the formal analysis and thus consider only welfare in Market 2. I then informally discuss the implications of considering welfare in Markets 1 and 3 in Section 5.1. Finally, I initially consider the equity transfer, monetary transfer, and alliance as a single deal. I discuss the social desirability of a rule disallowing equity transfers between alliance partners in Section 5.2.

The impact of the deal on social welfare depends on several well-known effects. First, there is the direct effect of increased production efficiency for both the incumbent small firm and the potential entrant, which is generally positive. Second, there is the anticompetitive effect of equity, which either softens competition (Reynolds and Snapp, 1986) or deters entry. Finally, there is the "business-stealing" effect (Mankiw and Whinston, 1986), whereby firm A fails to take into account the negative effect of its entry on firm B's profits, which can counterbalance the negative anticompetitive effect.

To begin the analysis, assume that SA/N is the equilibrium of the model. If A will not enter in the absence of a deal [$F \geqslant f(0)$], the deal must improve welfare in Market 2. Firm B is a monopolist with or without the deal, but it is a more efficient monopolist if there is an alliance. It is well known that total welfare increases when a monopolist's marginal cost decreases. There is no anticompetitive effect of equity or business-stealing effect in this case.

If A will enter in the absence of a deal [$F < f(0)$], selling a stake of α^*_{SA} has the direct anticompetitive effect of entry deterrence. However, because the alliance also increases efficiency and eliminates business-stealing, the effect on welfare is ambiguous, as reflected in Proposition 9.

Proposition 9. *Assume entry is optimally deterred. Then there exists a threshold fixed fee, F_W^N, such that the equity alliance is welfare-improving in Market* 2 *if* $F > \min[f(0), F_W^N]$ *and welfare-decreasing if* $F < \min[f(0), F_W^N]$. *As δ increases, F_W^N decreases. For sufficiently small values of γ, F_W^N decreases as γ increases.*

An equity alliance can improve welfare both by mitigating the business-stealing effect, which is more important the higher the fixed fee, and by increasing production efficiency, which explains the comparative static for the efficiency benefit δ. The proposition also implies that an equity alliance is more likely to improve welfare the larger is A's initial cost disadvantage. A larger cost disadvantage (higher γ) reduces total duopoly output and thus decreases consumers' surplus. However, it also increases B's market power and could raise its profits by more than the decrease in A's profits. When γ is small, the net result is unambiguously negative for status quo welfare and thus makes it more likely that the equity alliance is welfare-improving. However, this result does not always hold for larger γ.

For further intuition, consider the case of linear demand with $\delta = 0$ and $\gamma = 0$. In this case, it is easy to show that $F_W^N \in (0, f(0))$. Thus, there is a relevant range of fixed costs, $F \in (F_W^N, f(0))$, in which the equity alliance prevents entry that would have otherwise occurred but still increases social welfare even if there is no efficiency improvement. This is due to the mitigation of the business-stealing effect. As δ increases, implying efficiency enhancement, or γ increases, this interval expands, increasing the probability that the deal is welfare-improving.

If entry is optimally accommodated in equilibrium (SA/E), we have Proposition 10.

Proposition 10. *Assume entry is optimally accommodated. Then if A will enter in the status quo* $[F < f(0)]$, *the equity alliance is welfare-decreasing in Market* 2 *unless it sufficiently improves production efficiency for A or B. If A will not enter in the status quo* $[F \geqslant f(0)]$, *then there exists a threshold fixed fee, F_W^E, such that the equity alliance is welfare-improving if* $F_W^E > f(0)$ *and* $F < F_W^E$, *and welfare-decreasing otherwise.*

When entry is accommodated and there would be entry in the status quo, the softening effect is always bad for welfare, so increased production efficiency must be high enough to offset it. If there would not be entry in the status quo, then the business-stealing effect causes a reduction in welfare when the fixed fee is too high.

5.1. Welfare in markets 1 and 3

Markets 1 and 3 are modeled in reduced form throughout the paper, so it is impossible to perform a true welfare analysis. However, it is possible to informally analyze how considering welfare in those markets would affect the analysis. Welfare in Market 1 is affected by the deal only inasmuch as A becomes more efficient. Assuming this is positive for overall welfare in that market, then the deal is more likely to be welfare-improving when Market 1 is taken into account.

The effect of including Market 3 is more complicated. An alliance increases the overall payoff, but equity makes the effort choice less efficient. Again, a full analysis is impossible without an ad hoc structural specification for the market, but considering Market 3 should make the deal more likely to improve welfare if the effect on the success payoff in Market 3 is larger than the dilutive effect of the equilibrium stake size.

5.2. Disallowing equity transfers

The welfare consequences of a law disallowing equity transfers depend critically on the equilibrium of the base model. If entry is deterred in the base model (SA/N), a law disallowing equity transfers can be positive for welfare in Market 2 only if it results in entry and the fixed fee is not too high. Otherwise, the business-stealing effect dominates and causes a reduction in welfare. If entry is not deterred in the base model (SA/E), the law is more likely to increase welfare in Market 2 because the fixed fee is paid even in the base model, and the equity stake is used solely to soften competition. Thus, a law disallowing equity stakes improves welfare if an alliance is still formed under the law. If an alliance is no longer formed, the law improves welfare if the deal considered as a whole decreases welfare, which was analyzed above. A full analysis of this issue is available upon request.

6. Extensions

This section explores several extensions of the model.

6.1. Property rights and degree of knowledge transfer

The analysis thus far has assumed that the entrepreneurial firm's technology becomes fully available to the established firm in the alliance. In reality, one might expect some degree of property rights protection and some ability to minimize unnecessary transfers of sensitive knowledge. In this section I separately analyze the effects of these two factors.

6.1.1. Degree of property rights protection

Because A's use of acquired knowledge in Market 1 does not impact B, there is a natural way to model a change in the strength of intellectual property rights. Let $I \in [0, 1]$ index the strength of property rights protection, and let A's marginal cost in Market 2 following an alliance be $c_A^{SA} = I(1+\gamma)c + (1-I)(1-\delta)c$. In other words, under full protection ($I = 1$) A is unable to use its new knowledge against B in Market 2, and under no protection ($I = 0$) it can use the knowledge to be an equal competitor. The rest of the model remains the same, because B is happy to let A use its technology in markets where the firms do not compete.

Without an alliance, I has no effect. However, it has both direct and indirect effects on the optimal stake size.

Proposition 11. *The critical stake size for entry deterrence, α^*_{SA}, is*

(a) *decreasing in I; and*
(b) *decreasing in δ if I is sufficiently high and demand is linear.*

As property rights are strengthened, A's post-alliance marginal cost increases, which makes entry less attractive. This decreases the stake size required to deter entry. The strengthening of property rights can also change how the critical stake size varies with the efficiency enhancement, δ. In the base model, which has $I = 0$, an increase in δ makes entry more attractive. However, when property rights are strongly protected a larger δ implies that B becomes relatively more competitive, making entry less attractive for A.

The strength of property rights also affects the optimal stake size when entry is accommodated.

Proposition 12. *With linear demand, the optimal stake size conditional on accommodation is increasing in I.*

An increase in the strength of property rights increases the efficiency gap between the competing firms when entry is accommodated. This increases the stake size because the softening effect of equity is worth more on the margin when the competitors are more uneven.

Variations in I also affect the initial alliance decision. For the clearest intuition, consider the extreme protection case, with $I = 1$. Now consider the effect of an alliance in Market 2 for a *given* stake size, which is limited to a decrease in A's marginal cost. This implies $\alpha^*_{SA} < \alpha^*$. It is easier to deter entry with an alliance than without because A cannot use its new knowledge in the contested market, but B becomes more efficient. Thus, for a given stake size the alliance either maintains entry deterrence in the contested market (for $\alpha > \alpha^*$), deters entry that would occur without the alliance (for $\alpha \in [\alpha^*_{SA}, \alpha^*)$), or leads to a duopoly with a more efficient leading firm (for $\alpha < \alpha^*_{SA}$). Lemma 2 implies that an alliance results in higher profits in the latter case.

Lemma 2. *The sum of duopoly profits is decreasing in B's marginal cost for any stake size* $\alpha \in [0, \alpha^*]$.

The alliance also results in (weakly) greater profits in Market 2 when it either maintains entry deterrence or deters entry that would otherwise occur. Altogether this implies that the payoff in Market 2 is always at least as great with an alliance as without, so, given the other benefits of an alliance, an alliance is always optimal, as stated in Proposition 13.

Proposition 13. *An alliance is always formed when intellectual property is fully protected* ($I = 1$).

6.1.2. Degree of knowledge transfer

If the firms have control over the amount of knowledge transferred, it is natural to assume that both the value of cooperation, as indexed by δ, ΔV_3, and ΔV_1, and A's efficiency as an entrant in Market 2 are increasing in the amount transferred. I briefly explore such a model by assuming that the firms can choose a degree of transfer, K, from the set $\{0, 1\}$. To reflect increased value from additional knowledge, I let the efficiency enhancement parameters equal $\delta + K\widehat{\Delta\delta}$, $\Delta V_3 + K\widehat{\Delta V_3}$, and $\Delta V_1 + K\widehat{\Delta V_1}$. I model A's increasing ability to appropriate the technology by letting its marginal cost in Market 2 equal $c_A^{SA} = (1 - K)(1 + \gamma)c + K(1 - \delta - K\widehat{\Delta\delta})c$.

With this extension, the firms' Stage 1 task is more complicated. They must decide whether to have an alliance, how to structure the alliance (K), whether and how much equity to transfer (α), and the size of the monetary transfer (T). Given an alliance and a level of K, Proposition 1 implies that a unique stake size exists, say $\alpha^*_{SA}(K)$, such that an equity transfer of size $\alpha^*_{SA}(K)$ just suffices to deter entry. Lemma 2 and Proposition 13 imply that choosing a $K = 0$ alliance always dominates choosing not to have an alliance, leading to Proposition 14.

Proposition 14. *When the degree of knowledge transfer is endogenous, an alliance is always formed.*

For the rest of this section I assume that joint duopoly profits in Market 2 are larger for all $\alpha < \alpha^*_{SA}(0)$ when the firms are equal competitors with marginal cost $(1-\delta)c$ than when A's marginal cost is $(1+\gamma)c$ and B's is $(1-\delta)c$. It is easy to show that this is true whenever γ and δ are not too large and demand is linear. Now consider the case in which the incremental benefit of knowledge transfer is negligible, or, for intuitive clarity, $\widehat{\Delta\delta} = \widehat{\Delta V_3} = \widehat{\Delta V_1} = 0$. In this case, Proposition 15 holds.

Proposition 15. *Assume* $\widehat{\Delta\delta} = \widehat{\Delta V_3} = \widehat{\Delta V_1} = 0$. *Then*

(a) *if* $V_3 + \Delta V_3$ *is sufficiently small, the partners choose* $K = 0$ *and a stake of* $\alpha^*_{SA}(0)$; *and*
(b) *if* $V_3 + \Delta V_3$ *is sufficiently large, the partners choose* $K = 1$ *and a stake smaller than* $\alpha^*_{SA}(1)$.

If the firms' joint duopoly profits are higher when the firms are equal competitors, and additional knowledge transfer has no incremental benefits, the only effects of additional knowledge transfer are to make deterrence harder $[\alpha^*_{SA}(0) < \alpha^*_{SA}(1)]$ and increase duopoly profits. So if $V_3 + \Delta V_3$ is small and thus it is inexpensive to deter entry, the partners choose to deter, and choose $K = 0$ so they can do it at a smaller stake size. If $V_3 + \Delta V_3$ is too large, though, they do not want to deter entry and prefer to choose $K = 1$ and increase their duopoly profits.

Adding benefits of additional knowledge, i.e., $\widehat{\Delta\delta}, \widehat{\Delta V_3}, \widehat{\Delta V_1} > 0$, leads to Proposition 16.

Proposition 16. *If* $V_3 + \Delta V_3$, $\widehat{\Delta\delta}$, $\widehat{\Delta V_3}$, *and* $\widehat{\Delta V_1}$ *are all sufficiently small, the partners choose* $K = 0$ *and a deterrent stake of* $\alpha^*_{SA}(0)$. *Otherwise the partners choose* $K = 1$ *and the stake size and accommodation or deterrence decisions conform to the analysis in Section* 3.2.

The decision here is treated as a two-stage process. The partners determine the degree of knowledge transfer and then the optimal stake size. The choice of the degree of knowledge transfer boils down to comparing $K = 0$ with no entry versus the best $K = 1$ equilibrium, which I analyzed in Section 3.2. Full transfer is more attractive the greater are the benefits of additional knowledge. Relative to Proposition 15, the introduction of significant benefits to knowledge transfer means that it can be optimal to choose $K = 1$ and deter entry despite the larger required stake size.

6.2. A two market model

For tractability reasons, the base model assumes that the entrepreneur's effort applies to a new market in which the alliance partners are not potential competitors. This allows for a clean separation between the costs and benefits of equity. In some cases it is more realistic to assume that the entrepreneur's effort affects efficiency in the contested market. Here I briefly explore such a model.

Take the base model and assume that Market 3 does not exist. Furthermore, let the entrepreneur's effort choice e determine the alliance's degree of efficiency enhancement. Specifically, let $\delta = e$. In this model the exact timing of the effort and entry decisions is important because the value of the entrepreneur's effort depends on whether entry occurs. To be as realistic as possible, I assume that the effort and entry decisions are taken simultaneously in Stage 2. For tractability I also assume that inverse demand is linear and that β is such that there is always an interior solution for $e \in (0,1)$. This is henceforth called the two-market model.

An alliance is always optimal in this model because no efficiency enhancement is possible without the alliance. Given this, the entrepreneur's effort choice depends on whether he believes A will enter Market 2. Letting Π_B denote the expected profit of B conditional on the effort and entry decisions, the entrepreneur chooses his effort to maximize

$$(1-\alpha)\Pi_B - \frac{e^2}{2\beta}, \tag{8}$$

where $\Pi_B = r\pi_B(\alpha) + (1-r)\pi_m^{SA}$, r is the entrepreneur's perceived probability of entry by A, and $\pi_B(\alpha)$ and π_m^{SA} are as defined previously but now also depend on e via δ. The optimal effort choice solves $(1-\alpha)\frac{d\Pi_B}{de} - \frac{e}{\beta} = 0$. Let $e(r)$ denote the optimal effort choice conditional on r. Lemma 3 arises from an analysis of this solution.

Lemma 3. *In the two-market model, the entrepreneur's optimal effort choice is decreasing in α and r.*

For a given α and level of effort e, A's entry decision is just as in the base model; A enters if $F < f^{SA}(\alpha) = \alpha\pi_B^{SA}(\alpha) + \pi_A^{SA}(\alpha) - \alpha\pi_m^{SA}$ and otherwise does not enter. Holding e constant, Proposition 1 continues to hold in this setting, i.e., $f^{SA}(\alpha)$ is still decreasing in α. Lemma 4 also holds.

Lemma 4. *In the two-market model, $f^{SA}(\alpha)$ is increasing in e.*

This result is analogous to the result in the base model that α_{SA}^* is increasing in δ (Proposition 2) and implies that entry becomes less attractive as the entrepreneur's effort level falls. Putting this together with the entrepreneur's effort choice, the subgame equilibrium for each level of α can be determined. To start, assume for simplicity that $F < f^{SA}(0)$ for any level of e. In other words, A enters if $\alpha = 0$ regardless of the entrepreneur's effort choice.

As α increases there are two effects: Competition is softened and the entrepreneur's effort level for a given r falls. Because entry becomes less attractive as α rises and e falls, there are two relevant threshold stake sizes, labeled $\underline{\alpha}$ and $\overline{\alpha}$, where $\underline{\alpha} < \overline{\alpha}$. The lower threshold, $\underline{\alpha}$, is defined as the minimum stake size at which it is no longer profitable to enter if the entrepreneur chooses his effort level based on certain entry, i.e., chooses $e(1)$. The upper threshold, $\overline{\alpha}$, is defined as the minimum stake size at which it is no longer profitable to enter if the entrepreneur chooses his effort level based on no entry, i.e., chooses $e(0)$. The inequality $\underline{\alpha} < \overline{\alpha}$ follows from Lemmas 3 and 4.

This implies that there are three regions of interest. For all $\alpha < \underline{\alpha}$, entry occurs and the entrepreneur's equilibrium effort choice is accordingly $e(1)$. For all $\alpha \geqslant \overline{\alpha}$, entry does not occur and the equilibrium effort choice is $e(0)$. Finally, for all $\alpha \in [\underline{\alpha}, \overline{\alpha})$ there is no pure strategy equilibrium in the simultaneous effort and entry game. The entrepreneur can never be indifferent between different levels of effort, so this implies that the unique equilibrium has to involve a mixed entry strategy by A, in which the probability of entry declines from one to zero as α rises from $\underline{\alpha}$ to $\overline{\alpha}$.

The final task is to calculate the firms' joint expected surplus, S, as a function of α given the subgame equilibria derived above. This analysis yields Proposition 17.

Proposition 17. *In the two-market model, the firms' joint surplus S is* (a) *concave in* α *for all* $\alpha < \underline{\alpha}$ *and* (b) *increasing in* α *for all* $\alpha \in (\underline{\alpha}, \overline{\alpha})$.

This result implies that the model's solution is either a stake size of $\alpha = 0$ with entry, a stake size of $\alpha \in (0, \underline{\alpha})$ where the benefits of softening are weighed against the reduced efficiency enhancement, or a stake size of $\alpha = \overline{\alpha}$ with no entry. Proposition 18 gives some guidance on when each equilibrium prevails.

Proposition 18. *In the two-market model, the optimal stake size is weakly decreasing in* β.

As β rises, the entrepreneur has greater incentives to provide effort, which makes the dilutive effect of equity relatively more costly.

6.3. Capital constraints for B

The base model effectively assumes that B has deep pockets and thus no current financing needs. However, Lerner et al. (2003) show that alliances are often an important funding source for financially constrained firms. Here I explore the implications of financial constraints with a simple extension as follows. Assume that B must invest an amount $\kappa \leqslant F$ to remain in business. Also assume that the entrepreneur is penniless and that, other than A, the only source of capital for B is an outside equity investor. Finally, assume that forming an alliance is always optimal (ΔV_1 is large).

The firms' disagreement payoffs change with this adjustment to the model. Because B must take on an outside investor in disagreement, its bargaining position is worse and A's is unchanged relative to the case in which $\kappa = 0$ (assuming, as I do, that κ is small enough that B can raise it in the disagreement game). These changes arise both because of the payment κ and because of the reduced incentives in Market 3 as a result of the equity that must be issued in disagreement. This tends to reduce the monetary transfer, T^*. If no outside investor existed, B's disagreement payoff would be zero and A would be a monopolist in Market 2 in disagreement, which would further reduce T^*.

If the base model is solved ignoring κ and it turns out that $T^* \geqslant \kappa$, then the entrepreneur simply uses the proceeds from the monetary transfer to fund κ. If, however, $T^* < \kappa$, then B needs to secure additional cash to remain in business. It is optimal for B to remain in business as long as $\kappa \leqslant F$. To see this, note that transferring α^*_{SA} to A and deterring its entry, which also preserves some value in Market 3, dominates having a less efficient monopolist in Market 2 and completely destroying Market 3.

Consider possible contracts between A and B to provide the additional funding. To be made whole, A must hold a claim on B's Stage 3 cash flows over and above the equity stake that would be transferred in the model if $\kappa = 0$. Transferring additional equity is a possibility but is costly in terms of the entrepreneur's effort choice. Furthermore, Lerner et al. (2003) find that the portion of funding provided in biotechnology alliances via equity is small relative to the entire funding amount. Also, Robinson and Stuart (2002) find that debt contracts are not uncommon in alliances involving an equity stake. Therefore, I think of the equity stake as a strategic variable and focus here on the possibility of a nonequity claim that is more incentive-neutral. In particular, if a safe debt claim can be issued, this does not distort effort or entry incentives. In the context of the model I define "safe" by the notion that B can always guarantee a Stage 3 payout equal to the face value of the debt.

The use of such a contract is superior to taking on an outside equity investor because the latter would further dilute incentives in Market 3.

For example, assume A holds an equity stake of size α and $T^* \in [0, \kappa)$. For B to stay in business A needs to give B additional cash of $\kappa - T^*$. To keep A's payoff constant with this additional cash outflow, it can be given debt with face value $\frac{\kappa - T^*}{1-\alpha}$. The face value of the debt must be grossed up in this way because the payment of the debt instrument reduces the Stage 3 equity payout to A. Specifically, say the total Stage 3 cash flow of B is ρ. Then giving debt with face value $\kappa - T^*$ changes A's Stage 3 payout from $\alpha\rho$ to $(\kappa - T^*) + \alpha(\rho - (\kappa - T^*))$, for a total increase of $(\kappa - T^*)(1-\alpha)$. But to keep A whole it must be increased by $\kappa - T^*$, which implies a face value of $\frac{\kappa - T^*}{1-\alpha}$. It is easy to show that the face value is the same for the case $T^* < 0$.

Finally, if the funding need can be provided using safe debt, the Stage 1 equity and alliance decisions are unchanged. Thus, the only changes induced by the financing constraint are in the pricing of the stake.

The debt claim modeled here could be implemented in some other way, such as R&D funding provided in return for licensing rights on a particular product or in a particular market, or in combination with the equity stake via a convertible preferred equity security. If a larger debt issue is required that is not safe, the solution is affected somewhat because the dynamics of the debt contract affect the strategic impact of the equity stake. A full analysis of this case is beyond the scope of the paper, but the intuition that debt is preferred to additional equity is not lost.

6.4. Competition is other than Cournot

In this section I consider how the basic implications of the model change if the mode of competition in Market 2 is other than homogeneous-product quantity competition. It is not possible to solve the model using general reaction functions because the question of entry deterrence requires a structural model of underlying demand. However, it is possible to derive some results with a simple example of differentiated Bertrand competition.

Consider an economy with a representative agent whose utility for goods A and B (corresponding to those produced by firms A and B) is given by

$$U = N(q_A + q_B) - \frac{1}{2}(q_A + q_B)^2 - \frac{2((q_A - q_B)/2)^2}{1 + 2\lambda} - P_A q_A - P_B q_B.\text{[14]} \tag{9}$$

Here the parameter λ indexes the degree of substitutability between the products. If $\lambda = 0$ the products have independent demands, and they become perfect substitutes as $\lambda \to \infty$. When both products are being produced, demand for product $i \in \{A, B\}$ is given by

$$q_i = \tfrac{1}{2}[N - (1+\lambda)P_i + \lambda P_j]. \tag{10}$$

If only product B is produced (i.e., product A is not introduced), demand is given by

$$q_B = \frac{(1 + 2\lambda)(N - P_B)}{2(1+\lambda)}. \tag{11}$$

[14] This utility function is adapted from one introduced by Shubik (1980, p. 69).

The demand for a single-product industry cannot be derived from Eq. (10). Instead, one must start from the representative consumer's utility function and assume that $q_A = 0$. This illustrates the difficulty with solving the model using reduced-form profit or demand functions. Similar results could be obtained with a model based on spatial differentiation on a line or circle, but such an analysis would be more technically complex and would not add any important insights.

I assume that these are the appropriate demand functions for Market 2, that A and B compete in simultaneous price competition in Stage 3 if A enters, and that the rest of the model is as before. Also, for simplicity I assume that an alliance is always optimal.

A central difference in this case versus homogeneous Cournot is that deterring A's entry is not always optimal for joint surplus. With differentiated products, it can be jointly optimal to have both firms operating in the market. Also, a single firm owning and controlling the production of both products makes different production decisions than two separate firms even as A's ownership stake in B approaches $\alpha = 1$ (under the maintained assumption that A gains no control of B's decision process). These facts are clarified in the following preliminary results that focus exclusively on Market 2.

Lemma 5. *In the differentiated Bertrand case with joint ownership and control of both products, it is optimal to introduce product A at cost F if and only if* $\frac{(N-(1-\delta)c)^2}{8(1+\lambda)} > F$.

This implies that under joint ownership and control, it is optimal to introduce the second product only if the two products are not too substitutable (λ is not too high). Lemma 6 highlights the difference between joint ownership and control and passive equity ownership.

Lemma 6. *In the differentiated Bertrand case, if A holds a passive equity stake in B equal to* $\alpha \in [0, 1]$, *the firms' duopoly prices are always strictly lower than those that would be chosen under joint ownership and control as long as* $\lambda > 0$.

This implies that as long as the two products are substitutes, the two firms cannot approach the joint duopoly optimum using passive equity ownership. This differs markedly from the homogeneous Cournot case, in which a large enough passive stake can always achieve the joint optimum. It is a result of the dynamics of price competition and the fact that B always seeks to maximize its own profits. The implication is that entry deterrence could be more attractive under passive ownership than under joint control.

Lemmas 7 and 8 show that a passive stake can still be useful both for softening competition and, in some cases, entry deterrence.

Lemma 7. *In the differentiated Bertrand case, the sum of duopoly profits is strictly increasing in the stake size,* α.

This implies that when entry is accommodated, the firms could still transfer equity to soften competition. Now note that there is an alliance entry threshold in this case just as in the Cournot case. For consistency, I use the same notation: $f^{\mathrm{SA}}(\alpha) = \pi_A^{\mathrm{SA}}(\alpha) + \alpha\pi_B^{\mathrm{SA}}(\alpha) - \alpha\pi_m^{\mathrm{SA}}$, where it is understood that the duopoly profits are calculated under the Bertrand assumption and π_m^{SA} is B's monopoly profit if A stays out of the market.

Lemma 8. *In the differentiated Bertrand case, the alliance entry threshold,* $f^{\mathrm{SA}}(\alpha)$, *is strictly decreasing in the stake size,* α.

While entry is always less attractive as α increases, it is not always possible to deter entry at some $\alpha \leqslant 1$. To see this, note that as λ approaches zero and the products become independent, *B*'s duopoly profit becomes indistinguishable from its monopoly profit, and the equity stake can never cause *A* to choose not to enter if it otherwise will. However, Lemma 9 holds when entry deterrence is possible with a passive stake.

Lemma 9. *In the differentiated Bertrand case, if entry deterrence is possible at some* $\alpha \leqslant 1$, *then entry deterrence is optimal for Market 2 relative to any softening equilibrium with a lower* α. *Furthermore, if it is not optimal to introduce the second product under joint ownership and control, it is always possible to deter entry with a passive stake* $\alpha < 1$.

These results taken together imply that if mergers are ruled out, the optimal outcome for Market 2 in isolation is either entry with full softening at $\alpha = 1$ or entry deterrence with a sufficient passive stake. Also, if entry deterrence (or no introduction of product *A*) is globally optimal, even when mergers are allowed, then entry deterrence is always possible.

It is now possible to synthesize these results with the implications for Markets 1 and 3 and consider the equilibrium of the differentiated Bertrand case. For these purposes I assume that a merger causes Market 3 to become worthless.

Proposition 19. *In the differentiated Bertrand case,*

(a) *if mergers are possible, a merger is optimal whenever the potential payoff in Market* 3, $V_3 + \Delta V_3$, *is sufficiently small;*
(b) *if mergers are impossible and entry deterrence with a passive stake is impossible* (λ *is low*), *an optimal stake size is determined by trading off softening in Market* 2 *versus the cost to Market* 3; *and*
(c) *if mergers are impossible but entry deterrence with a passive stake is possible* (λ *is high*), *it is optimal either to transfer the minimum stake required to deter entry if* $V_3 + \Delta V_3$ *is sufficiently small, or accommodate entry and trade off market softening versus the cost to Market* 3 *if* $V_3 + \Delta V_3$ *is larger.*

Part a of the proposition reflects the fact that full joint ownership and control is more important in the Bertrand case than in the Cournot case. It is always optimal if the costs to the growth option market are small. If mergers are disallowed, then Part b shows that if the products are not substitutable enough for entry deterrence to be possible with a passive stake, then some equity could still be transferred to soften competition. Part c indicates that if the products are substitutable enough for entry deterrence to be possible, then the equilibrium of the Bertrand model is determined just as in the Cournot case. Entry is deterred if the costs of equity are not too great, and it is accommodated with a smaller softening stake otherwise.

7. Robustness

This section explores the importance of some of the model's key assumptions.

7.1. Security design

The analysis does not lend itself easily to a full security design exercise. As noted in Section 2.1, the model is based on the assumption that government regulations make certain types of contracting impossible, and as a result the firms resort to second-best contracting that obscures their true intentions. As such, it is impossible to say how far the firms could depart from standard securities without inviting regulatory attention. It is on this basis that I have restricted the parties to a standard equity contract. However, it is worthwhile to explore the subject more deeply.

It is important to realize first that giving A a risk-free debt claim on B does not affect their quantity-setting decisions or A's entry decision because none of A's actions affect the payoff of this security. However, given the lack of general uncertainty in the model, it is possible to design simple nonstandard securities that dominate straight equity in some cases. For example, assume that an alliance with no entry (SA/N) is optimal and Market 3 is small, such that $V_3 + \Delta V_3 < \pi_m^{SA} - [\pi_A^{SA}(0) + \pi_B^{SA}(0)]$. In this case a security that pays A a lump sum equal to $\pi_A^{SA}(0) - F$ if B's total profit exceeds π_m^{SA} and nothing otherwise deters entry. It also preserves the entrepreneur's incentive to efficiently exert effort in Market 3 because the probability of success in that market does not affect the probability of the payout to A. However, this is a highly nonstandard security.

Another set of securities that has the same effect in this simple case, but is closer to a standard set of securities, is an equity stake in B held by A together with a call option on B held by the entrepreneur that gives him all of the profits above π_m^{SA}. To be more specific, assume that A passively holds a proportion α of B's equity and the entrepreneur is given the option to buy back all of these shares at the price $\alpha\pi_m^{SA}$. If the equity stake is sufficient to deter entry but $V_3 + \Delta V_3$ is still small enough that B's profit exceeds π_m^{SA} only if A does not enter, then this set of securities also deters entry while fully preserving the entrepreneur's effort incentives. It is easy to show that when $V_3 + \Delta V_3 = 0$, the equity stake required to deter entry is the same with the option given to the entrepreneur as without it (the option is never exercised in this case). As $V_3 + \Delta V_3$ rises, so does the stake required to deter entry.

Neither of these sets of securities works as well when the potential payoff in Market 3 is larger or when additional uncertainty is added to the model. However, the point is not lost that it could be possible to improve on straight equity if permitted by the specific regulatory and contracting environment.

7.2. Renegotiation

Renegotiation has been ruled out thus far by assuming that it is prohibitively costly. If this assumption is relaxed in the static setting of the base model, several issues arise. In particular, the exact timing of the entry and effort decisions becomes important. If, as is currently assumed, the entry and effort decisions are simultaneous and production decisions come later, then costless renegotiation leads to an additional equity transfer before the production stage but after the entry and effort choices whenever there is entry. To see this, note that after the effort decision is made there is no longer any cost of transferring additional equity, so if A has entered the partners transfer additional equity up to the point where monopoly profits are achieved. Similarly, if the effort decision comes before the entry decision, the firms always transfer enough equity to deter entry following the effort choice. Finally, if the entry decision comes before the effort decision, there is always an incentive for A to sell any equity it holds back to B after the entry decision but before the effort decision to restore the entrepreneur's innovative incentives.

This analysis shows that no timing assumption can resolve the issue of renegotiation in a static model if renegotiation is inexpensive relative to profits. However, it does not imply that the model is irrelevant if renegotiation costs are small. To see this, note that it is the static nature of the model that makes it vulnerable to these concerns. But the static model is meant to serve as a reduced form of a more dynamic model, in which the firms are competing over time and the entrepreneur is innovating over time. The construction of a complete dynamic model along these lines is beyond the scope of this paper, but renegotiation is generally less of an issue in such a setting. As long as renegotiation costs exceed the small, short-term gains that could be created by opportunistically trading equity just before or after a particular effort or entry decision, renegotiation is not generally optimal once an initial stake size has been set. A more complete analysis of this issue in the context of an infinitely repeated game is available upon request.

7.3. A's incentive to sell

The model assumes that any equity given to A cannot be sold prior to A's entry and production decisions. Empirically, many equity alliances involve private issuers, and equity alliances with publicly traded issuers often include specific provisions limiting the partner's ability to dispose of the stake. However, if the commitment to hold the stake is not perfectly credible, this could have implications for the model's equilibrium. These implications differ depending on the exact timing of events in the model. Here I investigate some of these implications under the specific assumption that A has the opportunity to sell its stake to a third party after the alliance and the entrepreneur's effort choice but before any entry or production decision relevant to Market 2. I assume that the stake must be sold for a fair ex post price and there is no possibility of retrade between A and B following the sale. For intuitive clarity, and without loss of generality, I ignore the potential payoff in Market 3.

If A will not enter without a stake then no stake is transferred in the first place, so the only interesting case involves A selling its stake and then entering Market 2. The two partners thus compete in Stage 3 with no stake, yielding profits in Market 2 of $\pi_B^{\mathrm{SA}}(0)$ and $\pi_A^{\mathrm{SA}}(0)$. The stake can thus be sold to a third party for a price of $\alpha\pi_B^{\mathrm{SA}}(0)$. This implies that A's total continuation payoff if it sells the stake is $\pi_A^{\mathrm{SA}}(0)+\alpha\pi_B^{\mathrm{SA}}(0)-F$. If it holds the stake, A's continuation payoff (excluding Market 1) is either $\pi_A^{\mathrm{SA}}(\alpha)+\alpha\pi_B^{\mathrm{SA}}(\alpha)-F$ if it chooses to enter (the Alliance with Entry equilibrium of the standard model) or $\alpha\pi_m^{\mathrm{SA}}$ if it does not (the Alliance with No Entry equilibrium). For the former case, A wants to sell the stake if $\pi_A^{\mathrm{SA}}(0)+\alpha\pi_B^{\mathrm{SA}}(0)>\pi_A^{\mathrm{SA}}(\alpha)+\alpha\pi_B^{\mathrm{SA}}(\alpha)$. For the latter case, A wants to sell the stake if $\pi_A^{\mathrm{SA}}(0)+\alpha\pi_B^{\mathrm{SA}}(0)-\alpha\pi_m^{\mathrm{SA}}>F$, which effectively defines an alternative entry threshold that is higher than that of the original model if $\pi_A^{\mathrm{SA}}(0)+\alpha\pi_B^{\mathrm{SA}}(0)>\pi_A^{\mathrm{SA}}(\alpha)+\alpha\pi_B^{\mathrm{SA}}(\alpha)$. So whether the ability to sell the stake affects the equilibrium depends on this inequality for both cases.

Using a linear demand specification, it is easy to find examples in which $\pi_A^{\mathrm{SA}}(0)+\alpha\pi_B^{\mathrm{SA}}(0)>\pi_A^{\mathrm{SA}}(\alpha)+\alpha\pi_B^{\mathrm{SA}}(\alpha)$ holds for some α. However, an $\alpha<1$ still always exists such that entry can be deterred. To see this, note that the entry decision at $\alpha=1$ if $\pi_A^{\mathrm{SA}}(0)+\alpha\pi_B^{\mathrm{SA}}(0)>\pi_A^{\mathrm{SA}}(\alpha)+\alpha\pi_B^{\mathrm{SA}}(\alpha)$ is to enter if $\pi_A^{\mathrm{SA}}(0)+\pi_B^{\mathrm{SA}}(0)-\pi_m^{\mathrm{SA}}>F$, which can never hold. This implies that when entry would be deterred in the original model, it can still be deterred but could require a larger stake when resale is allowed. It is also possible to show that the same comparative statics from Proposition 2 apply to the new optimal stake size. When entry would be accommodated in the original model, the ability to resell the equity can eliminate some softening equilibria where $\pi_A^{\mathrm{SA}}(0)+\alpha\pi_B^{\mathrm{SA}}(0)>\pi_A^{\mathrm{SA}}(\alpha)+\alpha\pi_B^{\mathrm{SA}}(\alpha)$. However, because softening can increase joint profits, one might expect, instead, a renegotiation in which the stake is retained if the commitment to hold the equity can be made credible at the time of renegotiation.

7.4. Reverse equity sales

The analysis thus far has assumed that reverse equity sales, in which B buys a stake in A, are not allowed. To further support this assumption, note that a reverse sale cannot accomplish entry deterrence when A's manager maximizes profits to all stockholders. In this case, if B owns a stake in A, A's entry decision is to enter if $\pi_A^R(\alpha^R) - F > 0$, where α^R is the size of the reverse stake and $\pi_A^R(\alpha^R)$ is A's profit in the Cournot game with that stake size. The proof of Proposition 1 implies that $\frac{d\pi_A^R(\alpha^R)}{d\alpha^R} \geqslant 0$ always holds, which means that a reverse stake cannot accomplish entry deterrence.

Now consider whether a reverse sale would ever be used to soften competition in lieu of a regular sale when entry deterrence is too costly to Market 3. This could be optimal if B has deep pockets and a reverse sale has a significant softening effect, as is sometimes true. However, consider a case in which B does not have deep pockets and must issue equity to new investors to fund any purchase of A's equity. Assume B purchases a stake of size α^R in A, or, assuming A has one share of equity outstanding, $\frac{\alpha^R}{1-\alpha^R}$ new shares. Let V_A represent the total expected profit of A and define V_B likewise. If the proceeds of the sale are retained by A, then the price for the shares is $\frac{\alpha^R}{1-\alpha^R} V_A$, which means B must sell a stake in itself equal to $\frac{(\alpha^R/(1-\alpha^R))V_A}{V_B + (\alpha^R/(1-\alpha^R))V_A}$ to raise the required cash. When $V_B = V_A$ this reduces to α^R, and the required stake size rises as V_B falls. Thus, whenever $V_B < V_A$, B has to sell a stake in itself that is larger than the stake it purchases in A. As noted in Section 2.1, the established firm in an equity alliance is generally much larger than its entrepreneurial partner, so $V_B < V_A$ is a natural assumption.

Finally, if B has to issue a larger equity stake than it purchases in A, a regular sale is superior unless the reverse stake has a stronger softening effect. In the case of linear demand, it is easy to show that $\frac{d(\pi_B(\alpha)+\pi_A(\alpha))}{d\alpha} \geqslant \frac{d(\pi_B^R(\alpha^R)+\pi_A^R(\alpha^R))}{d\alpha^R}$ when A's marginal cost is weakly greater than B's, which implies that the opposite is true in all cases of interest. Thus, a reverse sale is never preferred to a regular sale.

The possibility of reverse equity sales would become much more important in a model in which both firms were potential entrants in each others' markets. Such a model would require a careful interpretation of the economics underlying mutual stakes and is beyond the scope of this paper.

8. Conclusion

The transfer of equity between partners in a strategic alliance is a common occurrence that can be motivated by several strategic and financial factors. The existing theoretical literature has focused mainly on the potential contracting benefits of equity linkages in the presence of specific investment. However, practitioners worry about the possibility of enabling a potential competitor if they share proprietary knowledge with a partner. Furthermore, regulators have voiced concern about the potential anticompetitive effects of interfirm cooperation and equity linkages among competitors or potential competitors.

Using a simple model based on Cournot competition, this paper shows that a stake transfer can be used to deter entry by a potential competitor when cooperation with that firm can improve an entrepreneurial firm's efficiency but also allows the partner to become a more effective competitor. When such deterrence is too costly for the entrepreneur's effort incentives, it could be optimal to sell no stake at all or sell a smaller stake to soften the inevitable future competition. The paper also analyzes how these factors are reflected in observed stake sizes and prices. The analysis has important implications for equity sales in strategic alliances, corporate venture capital investments, and the organization and financing of research activities.

This paper is intended to serve as a guide for further empirical work and to suggest that a more comprehensive theory of these transactions that incorporates multiple possible motivations is needed. The model's predictions for the equity versus no-equity and entry versus accommodation decisions, stake sizes, and market premia could be particularly useful for future empirical work.

Appendix A

Throughout Appendix A, total derivative notation is used to indicate that the effects of parameter changes on the equilibrium production quantities in Stage 3 are taken into account. Also, most algebraic calculations are excluded for brevity in cases with linear demand. Full details are available upon request.

Proof of Proposition 1. The proof of this and subsequent propositions requires additional notation for the equilibrium quantities produced in each subgame case. For Case 1, let q_m be the quantity that solves the standard first-order condition $P'(q_m)q_m + P(q_m) = c$. For the duopoly cases, let q_i denote the quantity choice for Firm i. For Case 2, B maximizes

$$\max_{q_B}(1-\alpha)[P(q_B+q_A)q_B - cq_B], \tag{12}$$

while A maximizes

$$\max_{q_A} P(q_B+q_A)(q_A+\alpha q_B) - \alpha c q_B - (1+\gamma)cq_A. \tag{13}$$

Let $q_B(\alpha)$ and $q_A(\alpha)$ be the quantities that solve the associated first-order conditions, $P'(q_B+q_A)q_B + P(q_B+q_A) = c$ and $P'(q_B+q_A)(q_A+\alpha q_B) + P(q_B+q_A) = (1+\gamma)c$. For Case 3, let q_m^{SA} be the quantity that solves B's first-order condition (derived as in Case 1). Finally, for Case 4, let $q_A^{\mathrm{SA}}(\alpha)$ and $q_B^{\mathrm{SA}}(\alpha)$ be the quantities that jointly solve the appropriate first-order conditions (derived as in Case 2).

The alliance case and the no-alliance case can be considered special cases of a general framework with two competitors having constant marginal costs of c_A and c_B with $c_A \geqslant c_B$. I prove the result for the no-alliance case replacing the two firms' specific marginal costs with the generic c_A and c_B, which suffices to prove both cases.

Consider how the optimal quantities vary with α. The two first-order conditions that must be satisfied in Case 2 whenever both firms' quantities are positive can be written as $q_B(\alpha)P' + P - c_B = 0$ for B and $(q_A(\alpha)+\alpha q_B(\alpha))P' + P - c_A = 0$ for A, where $P' = P'(q_A(\alpha)+q_B(\alpha))$ and $P = P(q_A(\alpha)+q_B(\alpha))$. Define $a_B = q_B(\alpha)P'' + 2P'$, $b_B = q_B(\alpha)P'' + P'$, $a_A = (q_A(\alpha)+\alpha q_B(\alpha))P'' + 2P'$, and $b_A = (q_A(\alpha)+\alpha q_B(\alpha))P'' + (1+\alpha)P'$, where $P'' = P''(q_A(\alpha)+q_B(\alpha))$. a_A, b_A, a_B, $b_B < 0$ by the assumption of stable equilibria, and

$|a_A|>|b_A|$ and $|a_B|>|b_B|$, implying that $a_Aa_B - b_Ab_B>0$. Using these definitions and totally differentiating the two first-order conditions with respect to $q_A(\alpha)$, $q_B(\alpha)$, and α yields, in matrix form,

$$\begin{bmatrix} a_B & b_B \\ b_A & a_A \end{bmatrix} \begin{bmatrix} \mathrm{d}q_B(\alpha) \\ \mathrm{d}q_A(\alpha) \end{bmatrix} = \begin{bmatrix} 0 \\ -q_B(\alpha)P'd\alpha \end{bmatrix}. \tag{14}$$

Applying Cramer's rule, I calculate

$$\frac{\mathrm{d}q_B(\alpha)}{\mathrm{d}\alpha} = \frac{b_Bq_B(\alpha)P'}{a_Aa_B - b_Ab_B} > 0 \quad \text{and} \quad \frac{\mathrm{d}q_A(\alpha)}{\mathrm{d}\alpha} = \frac{-a_Bq_B(\alpha)P'}{a_Aa_B - b_Ab_B} < 0, \tag{15}$$

where the inequalities follow from the assumptions that demand is downward sloping (i.e., $P'<0$), and the equilibrium is stable (i.e., b_B, $a_B<0$). Also,

$$\frac{\mathrm{d}q_B(\alpha)}{\mathrm{d}\alpha} + \frac{\mathrm{d}q_A(\alpha)}{\mathrm{d}\alpha} = \frac{q_B(\alpha)(b_B - a_B)P'}{a_Aa_B - b_Ab_B} < 0. \tag{16}$$

The fact that $\frac{\mathrm{d}q_A(\alpha)}{\mathrm{d}\alpha}<0$ when $q_B(\alpha), q_A(\alpha)>0$ together with the non-negative quantity assumption and an inspection of the first-order condition in Case 4 makes it clear that there is some critical level of $\alpha \leqslant 1$, say $\widetilde{\alpha}$, such that A's optimal production would be zero for all α at or above the critical level and positive otherwise. The critical level is less than one whenever A's marginal cost is greater than B's. When A's optimal quantity would be zero, the entry threshold clearly equals zero, so the relevant range of α over which the entry threshold can vary is $[0, \widetilde{\alpha})$, depending on the case. Thus it suffices to show that the entry threshold declines with α over its positive range.

Turning to the analysis of $f(\alpha)$ assuming $q_A(\alpha)>0$, directly calculating the derivative of interest yields

$$\frac{\mathrm{d}f(\alpha)}{\mathrm{d}\alpha} = [\pi_B(\alpha) - \pi_m] + \left[\frac{\mathrm{d}\pi_A(\alpha)}{\mathrm{d}\alpha} + \alpha\frac{\mathrm{d}\pi_B(\alpha)}{\mathrm{d}\alpha}\right]. \tag{17}$$

The first term in brackets in Eq. (17) is negative given the analysis above. Consider the second term in brackets. Calculating $\frac{\mathrm{d}\pi_A(\alpha)}{\mathrm{d}\alpha}$ directly yields

$$\frac{\mathrm{d}\pi_A(\alpha)}{\mathrm{d}\alpha} = (P - c_A)\frac{\mathrm{d}q_A(\alpha)}{\mathrm{d}\alpha} + q_A(\alpha)\left(\frac{\mathrm{d}q_A(\alpha)}{\mathrm{d}\alpha} + \frac{\mathrm{d}q_B(\alpha)}{\mathrm{d}\alpha}\right)P', \tag{18}$$

where $P = P(q_A(\alpha) + q_B(\alpha))$ and $P' = P'(q_A(\alpha) + q_B(\alpha))$. From A's first-order condition in Case 4, $q_A(\alpha)P' + (P - c_A) = -\alpha q_B(\alpha)P'$. Substitution yields

$$\frac{\mathrm{d}\pi_A(\alpha)}{\mathrm{d}\alpha} = \frac{\mathrm{d}q_A(\alpha)}{\mathrm{d}\alpha}(-\alpha q_B(\alpha)P') + q_A(\alpha)\frac{\mathrm{d}q_B(\alpha)}{\mathrm{d}\alpha}P'. \tag{19}$$

Direct calculation of $\frac{\mathrm{d}\pi_B(\alpha)}{\mathrm{d}\alpha}$ with an application of the envelope theorem yields

$$\frac{\mathrm{d}\pi_B(\alpha)}{\mathrm{d}\alpha} = \frac{\mathrm{d}q_A(\alpha)}{\mathrm{d}\alpha}q_B(\alpha)P'. \tag{20}$$

Using these expressions to calculate the second bracketed term in Eq. (17) yields

$$\frac{\mathrm{d}\pi_A(\alpha)}{\mathrm{d}\alpha} + \alpha\frac{\mathrm{d}\pi_B(\alpha)}{\mathrm{d}\alpha} = q_A(\alpha)\frac{\mathrm{d}q_B(\alpha)}{\mathrm{d}\alpha}P' < 0, \tag{21}$$

where the inequality follows from $P'<0$ and $\frac{dq_B(\alpha)}{d\alpha}>0$ when $q_A(\alpha)>0$. Thus, the term is negative over the relevant range. As the fixed fee threshold is continuous in α, and clearly converges to zero as α approaches $\widetilde{\alpha}$, the result follows.

Proof of Proposition 2. From the text, $\pi_A^{SA}(\alpha_{SA}^*)+\alpha_{SA}^*\pi_B^{SA}(\alpha_{SA}^*)-\alpha_{SA}^*\pi_m^{SA}-F=0$ implicitly defines α_{SA}^*, and α^* is defined analogously. Let $\Phi^{SA}\equiv f^{SA}(\alpha)-F$. Using implicit differentiation, $\frac{d\alpha_{SA}^*}{dx}=-\frac{d\Phi^{SA}/dx|_{\alpha=\alpha_{SA}^*}}{d\Phi^{SA}/d\alpha|_{\alpha=\alpha_{SA}^*}}$ where x is any parameter of interest. The denominator, $\frac{d\Phi^{SA}}{d\alpha}|_{\alpha=\alpha_{SA}^*}$, is negative (as proven in Proposition 1), thus $\frac{d\alpha_{SA}^*}{dx}$ has the same sign as $\frac{d\Phi^{SA}}{dx}|_{\alpha=\alpha_{SA}^*}$. Using similar logic, and letting $\Phi\equiv f(\alpha)-F$, $\frac{d\alpha^*}{dx}$ has the same sign as $\frac{d\Phi}{dx}|_{\alpha=\alpha^*}$. The statement that α_{SA}^* and α^* are decreasing in F follows because $\frac{d\Phi^{SA}}{dF}|_{\alpha=\alpha_{SA}^*}<0$ and $\frac{d\Phi}{dF}|_{\alpha=\alpha^*}<0$ must hold.

For the results with respect to γ, note that it plays no role following an alliance because the firms have equal marginal costs. Because γ does not affect π_m or F, $\frac{d\Phi}{d\gamma}|_{\alpha=\alpha^*}=\frac{d\pi_A(\alpha)}{d\gamma}|_{\alpha=\alpha^*}+\alpha^*\frac{d\pi_B(\alpha)}{d\gamma}|_{\alpha=\alpha^*}$. Directly calculating $\frac{d\pi_A(\alpha)}{d\gamma}|_{\alpha=\alpha^*}$ and substituting from A's first-order condition in Case 2 yields

$$\left.\frac{d\pi_A(\alpha)}{d\gamma}\right|_{\alpha=\alpha^*}=q_A(\alpha^*)\left.\frac{dq_B(\alpha)}{d\gamma}\right|_{\alpha=\alpha^*}P'-\alpha^*\left.\frac{dq_A(\alpha)}{d\gamma}\right|_{\alpha=\alpha^*}q_B(\alpha^*)P'-cq_A(\alpha^*), \tag{22}$$

where $P'=P'(q_B(\alpha^*)+q_A(\alpha^*))$ and $P=P(q_B(\alpha^*)+q_A(\alpha^*))$. Directly calculating $\frac{d\pi_B(\alpha)}{d\gamma}|_{\alpha=\alpha^*}$ and applying the envelope theorem yields

$$\left.\frac{d\pi_B(\alpha)}{d\gamma}\right|_{\alpha=\alpha^*}=\left.\frac{dq_A(\alpha)}{d\gamma}\right|_{\alpha=\alpha^*}q_B(\alpha^*)P'. \tag{23}$$

Together these imply

$$\left.\frac{d\Phi}{d\gamma}\right|_{\alpha=\alpha^*}=q_A(\alpha^*)\left.\frac{dq_B(\alpha)}{d\gamma}\right|_{\alpha=\alpha^*}P'-cq_A(\alpha^*)<0, \tag{24}$$

where the inequality follows from $P'<0$ and $\frac{dq_B(\alpha)}{d\gamma}|_{\alpha=\alpha^*}>0$. The latter follows from an analysis of the two first-order conditions similar to that employed in the proof of Proposition 1 for $\frac{dq_B(\alpha)}{d\alpha}$ and $\frac{dq_A(\alpha)}{d\alpha}$.

For the result with respect to δ, note that δ plays no role if there is no alliance. With an alliance, direct calculation and substitution from A's first-order condition in Case 4 yields

$$\begin{aligned}\left.\frac{d\pi_A^{SA}(\alpha)}{d\delta}\right|_{\alpha=\alpha_{SA}^*}&=q_A^{SA}(\alpha_{SA}^*)\left.\frac{dq_B^{SA}(\alpha)}{d\delta}\right|_{\alpha=\alpha_{SA}^*}P'\\&\quad-\alpha_{SA}^*\left.\frac{dq_A(\alpha)}{d\delta}\right|_{\alpha=\alpha^*}q_B^{SA}(\alpha_{SA}^*)P'+cq_A^{SA}(\alpha_{SA}^*).\end{aligned} \tag{25}$$

Directly calculating $\frac{d\pi_B^{SA}(\alpha)}{d\delta}|_{\alpha=\alpha_{SA}^*}$ and applying the envelope theorem yields

$$\left.\frac{d\pi_B^{SA}(\alpha)}{d\delta}\right|_{\alpha=\alpha_{SA}^*}=\left.\frac{dq_A^{SA}(\alpha)}{d\delta}\right|_{\alpha=\alpha_{SA}^*}q_B^{SA}(\alpha_{SA}^*)P'+cq_B^{SA}(\alpha_{SA}^*). \tag{26}$$

Putting these together yields

$$\left.\frac{\mathrm{d}\Phi^{\mathrm{SA}}}{\mathrm{d}\delta}\right|_{\alpha=\alpha^*_{\mathrm{SA}}} = q_A^{\mathrm{SA}}(\alpha^*_{\mathrm{SA}})\left.\frac{\mathrm{d}q_B^{\mathrm{SA}}(\alpha)}{\mathrm{d}\delta}\right|_{\alpha=\alpha^*_{\mathrm{SA}}} P' + c(q_A^{\mathrm{SA}}(\alpha^*_{\mathrm{SA}}) + \alpha^*_{\mathrm{SA}} q_B^{\mathrm{SA}}(\alpha^*_{\mathrm{SA}})). \tag{27}$$

Using Cramer's rule as above, it is easy to calculate

$$\frac{\mathrm{d}q_B^{\mathrm{SA}}(\alpha)}{\mathrm{d}\delta} = \frac{-c}{(q_A^{\mathrm{SA}}(\alpha) + q_B^{\mathrm{SA}}(\alpha))P'' + (3-\alpha)P'}. \tag{28}$$

Also, $q_A^{\mathrm{SA}}(\alpha^*_{\mathrm{SA}}) + \alpha^*_{\mathrm{SA}} q_B^{\mathrm{SA}}(\alpha^*_{\mathrm{SA}}) = q_B^{\mathrm{SA}}(\alpha^*_{\mathrm{SA}})$ must hold by inspection of the firms' first-order conditions in Case 4. Taking these into account, Eq. (27) can be rewritten as

$$\left.\frac{\mathrm{d}\Phi^{\mathrm{SA}}}{\mathrm{d}\delta}\right|_{\alpha=\alpha^*_{\mathrm{SA}}} = c q_B^{\mathrm{SA}}(\alpha^*_{\mathrm{SA}})\left[\frac{-(1-\alpha^*_{\mathrm{SA}})P'}{(q_A^{\mathrm{SA}}(\alpha^*_{\mathrm{SA}}) + q_B^{\mathrm{SA}}(\alpha^*_{\mathrm{SA}}))P'' + (3-\alpha^*_{\mathrm{SA}})P'} + 1\right] > 0, \tag{29}$$

where the inequality follows from the assumption that all equilibria are stable (i.e., $q_A^{\mathrm{SA}}(\alpha)$ $P'' + P' < 0$ and $q_B^{\mathrm{SA}}(\alpha)$ $P'' + P' < 0$).

Proof of Proposition 3. If $F \geqslant f^{\mathrm{SA}}(0)$, no stake is required to deter entry and optimize the Market 2 payoff, so no equity is sold. Also, the value of S can be written as

$$\Pi + \beta(1-\alpha^2)\frac{(V_3 + 1_{\mathrm{SA}}\Delta V_3)^2}{2} + V_1 + 1_{\mathrm{SA}}\Delta V_1, \tag{30}$$

where Π is the total surplus generated in Market 2. Applying Corollary 1, the surplus in Market 2 differs according to the Market 2 subgame equilibrium as follows: (1) $\Pi = \pi_m$ if $\alpha \geqslant \alpha^*$ and there is no alliance (N/N), (2) $\Pi = \pi_A(\alpha) + \pi_B(\alpha) - F$ if $\alpha < \alpha^*$ and there is no alliance (N/E), (3) $\Pi = \pi_m^{\mathrm{SA}}$ if $\alpha \geqslant \alpha^*_{\mathrm{SA}}$ and there is an alliance (SA/N), and (4) $\Pi = \pi_A^{\mathrm{SA}}(\alpha) + \pi_B^{\mathrm{SA}}(\alpha) - F$ if $\alpha < \alpha^*_{\mathrm{SA}}$ and there is an alliance (SA/E), corresponding to Cases 1 through 4 from the text, respectively. Because monopoly profits are always greater than the sum of duopoly profits and monopoly profits decrease in marginal cost, the largest achievable Market 2 surplus is π_m^{SA}, which requires no entry in Stage 2. Furthermore, Market 1 surplus is increased by $\Delta V_1 > 0$ if there is an alliance, so maximizing Market 1 and 2 joint surplus requires an alliance.

According to Proposition 1 and Corollary 1, if $F < f^{\mathrm{SA}}(0)$ the bargainers can ensure the maximum joint payoff in these two markets by forming an alliance and transferring an equity stake of α^*_{SA}. The only cost to joint surplus of transferring equity is the dilution of the entrepreneur's effort incentives in Market 3. Also, as long as $F > 0$ there is a discrete increase in joint payoff when the stake size reaches α^*_{SA}. Thus the optimal agreement involves entry deterrence with a stake size of α^*_{SA} when Market 3 is near worthless ($V_3 + \Delta V_3 \approx 0$). The discrete jump in payoff when α^*_{SA} is reached guarantees that this remains the equilibrium over some range as $V_3 + \Delta V_3$ rises above zero.

Proof of Lemma 1. With $\delta = 0$, $\Delta V_3 = 0$, and $\Delta V_1 = 0$, the only effect of the alliance is to reduce A's marginal cost to c if there is no alliance. Thus, $\alpha^*_{\mathrm{SA}} > \alpha^*$, implying that N/N is always weakly preferred to SA/N. Also, entry is always deterred at $V_3 = 0$ because there is no cost of equity. Together these imply $V_3^{\mathrm{SA}} = 0$. For the comparative statics, note that an increase in ΔV_1 raises the payoff of SA/N and SA/E for all γ, increasing V_3^{SA}. For V_3^{E}, ΔV_1 does not affect N/E or N/N but increases SA/N and SA/E by the same amount for all γ and thus does not affect the trade-off between them.

Consider the second part of the result. The optimal stake sizes for SA/N and N/N are α^*_{SA} and α^*, respectively. Let the optimal stake size for SA/E be $\alpha^{\mathrm{SA/E}}$ and define $\alpha^{\mathrm{N/E}}$ analogously. I first show that these stake sizes are well defined and unique. $\alpha^{\mathrm{N/E}}$ is determined by the maximization problem

$$\max_{\alpha}\left[\pi_A(\alpha)+\pi_B(\alpha)-F+\beta(1-\alpha^2)\frac{V_3^2}{2}\right]. \tag{31}$$

Using the properties of linear demand, it is easy to show that $\frac{\mathrm{d}(\pi_A(\alpha)+\pi_B(\alpha))}{\mathrm{d}\alpha}>0$, $\frac{\mathrm{d}^2(\pi_A(\alpha)+\pi_B(\alpha))}{\mathrm{d}\alpha^2}<0$ for all α if $\gamma=0$, $\frac{\mathrm{d}^2(\pi_A(\alpha)+\pi_B(\alpha))}{\mathrm{d}\alpha^2}>0$ for all $\gamma>0$ at $\alpha=0$, and $\frac{\mathrm{d}^3(\pi_A(\alpha)+\pi_B(\alpha))}{\mathrm{d}\alpha^3}<0$ whenever $q_A(\alpha)>0$. Taken as a whole, these properties imply that the maximand in Eq. (31) is initially increasing in α and is either globally concave if $\gamma=0$ or if $\gamma>0$, globally convex (and thus always increasing) or initially convex and then concave (and thus possibly decreasing at higher α) in α. Thus, when $V_3>0$, $\alpha^{\mathrm{N/E}}$ is unique and either equal to the maximum α such that $q_A(\alpha)\geqslant 0$ or is defined implicitly by

$$\frac{\mathrm{d}(\pi_A(\alpha)+\pi_B(\alpha))}{\mathrm{d}\alpha}-\alpha\beta V_3^2=0. \tag{32}$$

A similar analysis suffices to prove the same properties for $\alpha^{\mathrm{SA/E}}$ replacing $\pi_A(\alpha)+\pi_B(\alpha)$ with $\pi_A^{\mathrm{SA}}(\alpha)+\pi_B^{\mathrm{SA}}(\alpha)$ in Eq. (31).

This analysis implies that $\alpha^{\mathrm{SA/E}}\geqslant\alpha^*_{\mathrm{SA}}$ when $V_3=0$ and that $\alpha^{\mathrm{SA/E}}$ is decreasing in V_3. Thus there exists some cutoff level of V_3 such that for all V_3 below the cutoff $\alpha^{\mathrm{SA/E}}\geqslant\alpha^*_{\mathrm{SA}}$, and for all V_3 above the cutoff $\alpha^{\mathrm{SA/E}}<\alpha^*_{\mathrm{SA}}$. Clearly, SA/N is preferred to SA/E whenever $\alpha^{\mathrm{SA/E}}\geqslant\alpha^*_{\mathrm{SA}}$. Let $V^{\mathrm{SA/E}}\equiv\pi_A^{\mathrm{SA}}(\alpha^{\mathrm{SA/E}})+\pi_B^{\mathrm{SA}}(\alpha^{\mathrm{SA/E}})-F+V_1+\Delta V_1+\beta(1-(\alpha^{\mathrm{SA/E}})^2)\frac{V_3^2}{2}$ denote the maximized total surplus conditional on an alliance with entry and a stake size of $\alpha^{\mathrm{SA/E}}$, and define $V^{\mathrm{SA/N}}$, $V^{\mathrm{N/N}}$, and $V^{\mathrm{N/E}}$ analogously. By the envelope theorem, $\frac{\mathrm{d}V^{\mathrm{SA/E}}}{\mathrm{d}V_3}=(1-(\alpha^{\mathrm{SA/E}})^2)\beta V_3>\frac{\mathrm{d}V^{\mathrm{SA/N}}}{\mathrm{d}V_3}=(1-(\alpha^*_{\mathrm{SA}})^2)\beta V_3$whenever $\alpha^{\mathrm{SA/E}}<\alpha^*_{\mathrm{SA}}$. Thus, there is a cutoff level of V_3 such that $V^{\mathrm{SA/N}}>V^{\mathrm{SA/E}}$ below the cutoff and vice versa for all γ.

For all $\gamma\geqslant\gamma^*$, N/N is always preferred to N/E because it maximizes the Market 2 payoff with a stake of $\alpha^*=0$. An analysis similar to that above proves that for all $\gamma<\gamma^*$ there is a cutoff level of V_3 such that $V^{\mathrm{N/N}}>V^{\mathrm{N/E}}$ below the cutoff and vice versa for all γ. Furthermore, at $\gamma=0$, there is no difference between SA/N and N/N or SA/E and N/E other than ΔV_1, so the levels of the V_3 cutoffs at which $V^{\mathrm{SA/N}}=V^{\mathrm{SA/E}}$ and $V^{\mathrm{N/N}}=V^{\mathrm{N/E}}$ is the same. Given this, the second part of the result is proven by showing that the V_3 at which $V^{\mathrm{N/N}}=V^{\mathrm{N/E}}$, say V_3^{N}, is increasing in γ, which implies that $V^{\mathrm{SA/N}}=V^{\mathrm{SA/E}}$ defines V_3^{E}.

To see this, note that the critical V_3 at which $V^{\mathrm{N/N}}=V^{\mathrm{N/E}}$ is implicitly defined by $V^{\mathrm{N/E}}-V^{\mathrm{N/N}}=0$, which can be written as

$$X_1\equiv\beta((\alpha^*)^2-(\alpha^{\mathrm{N/E}})^2)\frac{V_3^2}{2}+\pi_A(\alpha^{\mathrm{N/E}})+\pi_B(\alpha^{\mathrm{N/E}})-\pi_m-F=0. \tag{33}$$

The result is proven by showing that $\frac{\mathrm{d}V_3^{\mathrm{N}}}{\mathrm{d}\gamma}>0$. Using implicit differentiation, $\frac{\mathrm{d}V_3^{\mathrm{N}}}{\mathrm{d}\gamma}=-\frac{\mathrm{d}X_1/\mathrm{d}\gamma}{\mathrm{d}X_1/\mathrm{d}V_3}|_{V_3=V_3^{\mathrm{N}}}$. By the envelope theorem, $\frac{\mathrm{d}X_1}{\mathrm{d}V_3}=\beta V_3((\alpha^*)^2-(\alpha^{\mathrm{N/E}})^2)>0$. Thus, it remains to show $\frac{\mathrm{d}X_1}{\mathrm{d}\gamma}<0$. $\frac{\mathrm{d}X_1}{\mathrm{d}\gamma}$ can be written as $\frac{\mathrm{d}V^{\mathrm{N/E}}}{\mathrm{d}\gamma}-\frac{\mathrm{d}V^{\mathrm{N/N}}}{\mathrm{d}\gamma}$. To see that this is negative when $V^{N/N} = V^{N/E}$, note that when $\gamma = 0$, $V^{N/E} > V^{N/N}$ holds for all $V_3 > V_3^N$. Using the properties of linear demand and the envelope theorem, it is easy to show that $\frac{d^2 V^{N/E}}{d\gamma^2} > 0$ and $\frac{dV^{N/E}}{d\gamma} < 0$ at $\gamma = 0$.Using the properties of linear demand and implicit differentiation, it is also easy to show that $\frac{dV^{N/N}}{d\gamma} > 0$ and $\frac{d^2 V^{N/N}}{d\gamma^2} < 0$. Finally, $V^{N/N} > V^{N/E}$ must hold at γ^*. Because $V^{N/E}$ starts above and finishes below $V^{N/N}$ as γ is raised from zero to γ^*, $V^{N/E}$ is convex in γ, and $V^{N/N}$ is concave and increasing in γ, they can cross only once, at which point $\frac{dV^{N/E}}{d\gamma} < \frac{dV^{N/N}}{d\gamma}$ must hold. This proves the second part of the result.

Proof of Proposition 5. For Part a, when $V_3 > V_3^{SA}$, $V^{N/N} > V^{SA/N}$ must hold for all $\gamma \geqslant \gamma^*$ (if no alliance is optimal anywhere, it must be optimal for all $\gamma > \gamma^*$ because this gives the highest payoff possible without an alliance), and $V^{N/N} < V^{SA/N}$ holds at $\gamma = 0$ (because the only difference between them is ΔV_1). Thus it suffices that $V^{N/N}$ is increasing in γ, as shown in the proof of Lemma 1, and $V^{SA/N}$ is unaffected by γ. For Part b it suffices that $V^{SA/E} > V^{N/N}$ holds at $\gamma = 0$ for all $V_3 > V_3^E$, as implied by Lemma 1, and $V^{N/N}$ is increasing in γ. Part c is implied by the analysis of V_3^N in the proof of Lemma 1.

Proof of Proposition 6. For Part a, $V^{SA/E} > V^{N/E}$ at $\gamma = 0$ because ΔV_1 is the only difference between them. From the proof of Lemma 1, $V^{N/E}$ is convex and initially decreasing in γ. Thus, for any V_3 such that $\gamma^{SA/E} > \gamma^{N/E}$, $V^{SA/E} > V^{N/E}$ at $\gamma^{SA/E}$, and therefore $V^{SA/E} > V^{N/E}$ for all $\gamma < \gamma^{SA/E}$. For Part b, $V^{SA/E} > V^{N/E}$ at $\gamma = 0$ always holds, and $V^{SA/E} < V^{N/E}$ at $\gamma = \gamma^{N/E}$ must hold if $\gamma^{SA/E} < \gamma^{N/E}$ because $V^{SA/E}$ is unaffected by γ, and $V^{N/E} = V^{N/N}$ holds at $\gamma = \gamma^{N/E}$. The result follows because γ^E is defined by $V^{SA/E} = V^{N/E}$, $V^{SA/E}$ is unaffected by γ, and $V^{N/E}$ is convex and initially decreasing in γ.

Proof of Proposition 7. With the assumption of accommodation and an alliance, total surplus S can be written as

$$\pi_A^{SA}(\alpha) + \pi_B^{SA}(\alpha) - F + \beta(1 - \alpha^2)\frac{(V_3 + \Delta V_3)^2}{2} + V_1 + \Delta V_1 \tag{34}$$

for the alliance case, and analogously for the no-alliance case. It follows from the proof of Lemma 1 that $\frac{dS}{d\alpha} = 0$ must hold at the optimal stake size when accommodation is optimal. The sign of $\frac{d\alpha}{dx}$ for any parameter x is then the same as the sign of $\frac{d^2 S}{d\alpha\, dx}$. The results for V_3, ΔV_3, F, and β follow directly from $\frac{dS}{d\alpha} = \frac{d(\pi_A^{SA}(\alpha) + \pi_B^{SA}(\alpha))}{d\alpha} - \alpha\beta(V_3 + \Delta V_3)^2$ in the alliance case and analogously for the no-alliance case. Because δ and γ affect only $\pi_A^{SA}(\alpha) + \pi_B^{SA}(\alpha)$ and $\pi_A(\alpha) + \pi_B(\alpha)$, respectively, δ has no effect in the no-alliance case and γ has no effect in the alliance case, the remaining results are easily verified by algebraically calculating and signing $\frac{d^2(\pi_A^{SA}(\alpha) + \pi_B^{SA}(\alpha))}{d\alpha\, d\delta}$and $\frac{d^2(\pi_A(\alpha) + \pi_B(\alpha))}{d\alpha\, d\gamma}$ under the assumption of linear demand.

Proof of Proposition 8. Let C denote the entrepreneur's continuation payoff as defined in the text. The equilibrium price paid per share, given than $\frac{\alpha}{1-\alpha}$ shares are transferred, is $\frac{(1-\alpha)T^*}{\alpha}$. With these definitions, the percentage premium (or discount if negative) paid can be expressed as

$$\frac{(1 - \alpha)T^*}{\alpha C} - 1. \tag{35}$$

The monetary transfer T^* is determined by maximizing Eq. (7) with respect to T given the alliance and stake size decisions. Thus, $T^* = \theta(S_A - D_A) - (1-\theta)(S_B - D_B)$, which is increasing in $\theta(\frac{\partial T^*}{\partial \theta} = S_B + S_A - D_B - D_A > 0)$. This implies that the largest possible premium is when $\theta = 1$. Let $\Upsilon \equiv \beta(1-\alpha^*_{\mathrm{SA}})(V_3 + \Delta V_3)^2$. Directly calculating the premium assuming $\theta = 1$ and entry is deterred yields

$$\frac{\alpha^*_{\mathrm{SA}}(\pi^{\mathrm{SA}}_m + \Upsilon) + \Delta V_1 - 1_{F<f(0)}(\pi_A(0) - F)}{\alpha^*_{\mathrm{SA}}(\pi^{\mathrm{SA}}_m + (1/2)\Upsilon)} - 1, \tag{36}$$

and assuming entry is accommodated yields

$$\frac{\alpha^*_{\mathrm{SA}}(\pi^{\mathrm{SA}}_B(\alpha) + \Upsilon) + \Delta V_1 + (\pi^{\mathrm{SA}}_A(\alpha) - F) - 1_{F<f(0)}(\pi_A(0) - F)}{\alpha^*_{\mathrm{SA}}(\pi^{\mathrm{SA}}_B(\alpha) + (1/2)\Upsilon)} - 1. \tag{37}$$

Both expressions are clearly positive when $F \geqslant f(0)$ and can be negative when $F < f(0)$ (consider cases in which $\Delta V_1 \approx 0$, $V_3 + \Delta V_3 \approx 0$). When $\theta = 0$, and letting $\Delta S \equiv S - D_A - D_B$, the expressions can be written as

$$\frac{\alpha^*_{\mathrm{SA}}(\pi^{\mathrm{SA}}_m + \Upsilon) + \Delta V_1 - \Delta S - 1_{F<f(0)}(\pi_A(0) - F)}{\alpha^*_{\mathrm{SA}}(\pi^{\mathrm{SA}}_m + (1/2)\Upsilon)} - 1 \tag{38}$$

and

$$\frac{\alpha^*_{\mathrm{SA}}(\pi^{\mathrm{SA}}_B(\alpha) + \Upsilon) + \Delta V_1 - \Delta S + (\pi^{\mathrm{SA}}_A(\alpha) - F) - 1_{F<f(0)}(\pi_A(0) - F)}{\alpha^*_{\mathrm{SA}}(\pi^{\mathrm{SA}}_B(\alpha) + (1/2)\Upsilon)} - 1, \tag{39}$$

respectively. The only difference is the subtraction of the total surplus created by the alliance, ΔS, in the numerator, which can clearly make the expressions negative even when $F \geqslant f(0)$ for some parameter constellations (consider cases in which $\Delta V_1 \approx 0$, $V_3 + \Delta V_3 \approx 0$ but δ is large). To see that a premium is still possible at $\theta = 0$, consider cases in which ΔV_1 is very large, δ is small, and γ is very large. The results with respect to θ and ΔV_1 follow from the fact that T^* is always increasing in θ and is increasing in ΔV_1 whenever the entrepreneur has some bargaining power, while C is not affected by either.

Proof of Proposition 9. Assuming A would enter in the absenzce of a deal [$F < f(0)$], the change in welfare induced by the equity alliance can be written as

$$\int_{q_A(0)+q_B(0)}^{q^{\mathrm{SA}}_m} P(y)\,\mathrm{d}y - c((1-\delta)q^{\mathrm{SA}}_m - q_B(0) - (1+\gamma)q_A(0)) + F. \tag{40}$$

The equity alliance deal is welfare-improving with respect to the status quo if this expression is positive. Using this result, I define

$$F^{\mathrm{N}}_{\mathrm{W}} = \int_{q^{\mathrm{SA}}_m}^{q_A(0)+q_B(0)} P(y)\,\mathrm{d}y + c((1-\delta)q^{\mathrm{SA}}_m - q_B(0) - (1+\gamma)q_A(0)), \tag{41}$$

so that the deal is welfare-improving with respect to the status quo if $F > F^{\mathrm{N}}_{\mathrm{W}}$. This plus the fact that welfare is always improved by the deal if $F \geqslant f(0)$ proves all but the comparative statics.

For the first comparative statics result, a change in δ affects welfare in the monopoly case but not in the status quo duopoly case. In terms of $F^{\mathrm{N}}_{\mathrm{W}}$, q^{SA}_m and δ are the only quantities that vary with δ. Totally differentiating B's first-order condition from Case 3 with respect to $(1-\delta)$ yields

$$\frac{\mathrm{d}q_m^{\mathrm{SA}}}{\mathrm{d}(1-\delta)} = \frac{c}{q_m^{\mathrm{SA}}P''(q_m^{\mathrm{SA}}) + 2P'} < 0, \tag{42}$$

where the inequality follows from $qP'' + P' < 0$. Calculating $\frac{\mathrm{d}F_{\mathrm{W}}^{\mathrm{N}}}{\mathrm{d}q_m^{\mathrm{SA}}} = -P(q_m^{\mathrm{SA}}) + (1-\delta)c$, $\frac{\mathrm{d}F_{\mathrm{W}}^{\mathrm{N}}}{\mathrm{d}q_m^{\mathrm{SA}}} < 0$ must hold. Now consider $\frac{\mathrm{d}F_{\mathrm{W}}^{\mathrm{N}}}{\mathrm{d}(1-\delta)} = \frac{\partial F_{\mathrm{W}}^{\mathrm{N}}}{\partial q_m^{\mathrm{SA}}}\frac{\mathrm{d}q_m^{\mathrm{SA}}}{\mathrm{d}(1-\delta)} + \frac{\partial F_{\mathrm{W}}^{\mathrm{N}}}{\partial(1-\delta)}$. The preceding proves $\frac{\partial F_{\mathrm{W}}^{\mathrm{N}}}{\partial q_m^{\mathrm{SA}}}\frac{\mathrm{d}q_m^{\mathrm{SA}}}{\mathrm{d}(1-\delta)} > 0$ and by direct calculation $\frac{\partial F_{\mathrm{W}}^{\mathrm{N}}}{\partial(1-\delta)} = cq_m^{\mathrm{SA}} > 0$, so the result follows.

The only objects in $F_{\mathrm{W}}^{\mathrm{N}}$ that vary with γ are $q_A(0)$, $q_B(0)$, and γ. Direct calculation yields $\frac{\mathrm{d}F_{\mathrm{W}}^{\mathrm{N}}}{\mathrm{d}\gamma} = P(q_A(0)+q_B(0))\frac{\mathrm{d}(q_A(0)+q_B(0))}{\mathrm{d}\gamma} - c\frac{\mathrm{d}(q_B(0)+(1+\gamma)q_A(0))}{\mathrm{d}\gamma}$. I calculate the two derivatives on the right hand side by totally differentiating the two first-order conditions with respect to $q_A(0)$, $q_B(0)$, and γ, applying Cramer's rule as in the proof of Proposition 1, and then using A's first-order condition to replace for $q_A(0)$ in the resulting expression for $\frac{\mathrm{d}(q_B(0)+(1+\gamma)q_A(0))}{\mathrm{d}\gamma}$. Inserting the results and simplifying yields

$$\begin{aligned}\frac{\mathrm{d}F_{\mathrm{W}}^{\mathrm{N}}}{\mathrm{d}\gamma} = {} & \frac{c}{(3P' + (q_A(0)+q_B(0))P'')P'}[4PP' + (q_A(0)+q_B(0))PP'' \\ & - c(2\gamma q_B(0) + q_A(0) + q_B(0) + \gamma q_A(0))P'' - c(4+5\gamma)P'].\end{aligned} \tag{43}$$

The sign of this expression is the same as the sign of the term in brackets because the leading fraction is positive (the denominator is positive because $P' + q_B(0)P'' < 0$, $P' + q_A(0)P'' < 0$, and $P' < 0$ by assumption). Evaluating the term in brackets at $\gamma = 0$ yields

$$(P-c)(4P' + (q_A(0)+q_B(0))P'') < 0, \tag{44}$$

where the inequality follows from $P' + q_B(0)P'' < 0$, $P' + q_A(0)P'' < 0$, and $P' < 0$. Because $\frac{\mathrm{d}F_{\mathrm{W}}^{\mathrm{N}}}{\mathrm{d}\gamma}$ is continuous in γ over the interval $[0,1]$, there must exist some $\gamma^* > 0$ such that $\frac{\mathrm{d}F_{\mathrm{W}}^{\mathrm{N}}}{\mathrm{d}\gamma} < 0$ for all $\gamma \in [0, \gamma^*)$. This proves the final comparative statics result.

Proof of Proposition 10. When entry is accommodated and A would enter in the status quo $[F < f(0)]$, the change in welfare as a result of the alliance can be written as

$$\int_{q_A(0)+q_B(0)}^{q_A^{\mathrm{SA}}(\alpha)+q_B^{\mathrm{SA}}(\alpha)} P(y)\,\mathrm{d}y - (1-\delta)c(q_B^{\mathrm{SA}}(\alpha) + q_A^{\mathrm{SA}}(\alpha)) + cq_B(0) + (1+\gamma)cq_A(0). \tag{45}$$

Assume $\delta = 0$ and $\gamma = 0$. This implies that the only effect of the alliance is the softening of competition, which implies that $q_B^{\mathrm{SA}}(\alpha) + q_A^{\mathrm{SA}}(\alpha) < q_B(0) + q_A(0)$. Then the integral is negative, and the remaining terms are reduced to $c(q_B(0) + q_A(0) - q_B^{\mathrm{SA}}(\alpha) - q_A^{\mathrm{SA}}(\alpha))$. Finally, because $P(y) > c$ over the relevant range, this term is smaller in absolute value than the integral, so welfare must be reduced by the alliance. This proves that sufficiently large δ or γ is required for the deal to increase welfare.

When entry is accommodated and A would not enter in the status quo $[F \geqslant f(0)]$, the change in welfare can be written as

$$\int_{q_m}^{q_A^{\mathrm{SA}}(\alpha)+q_B^{\mathrm{SA}}(\alpha)} P(y)\,\mathrm{d}y - F - (1-\delta)c(q_B^{\mathrm{SA}}(\alpha) + q_A^{\mathrm{SA}}(\alpha)) + cq_m. \tag{46}$$

Defining $F_{\mathrm{W}}^{\mathrm{E}} = \int_{q_m}^{q_A^{\mathrm{SA}}(\alpha)+q_B^{\mathrm{SA}}(\alpha)} P(y)\,\mathrm{d}y - (1-\delta)c(q_B^{\mathrm{SA}}(\alpha) + q_A^{\mathrm{SA}}(\alpha)) + cq_m$ proves the result.

Proof of Proposition 11. For Part a, because $\frac{\partial c_A^{SA}}{\partial I}>0$, $\frac{d\Phi^{SA}}{dI}|_{\alpha=\alpha^*_{SA}}$ (defined as in the proof of Proposition 2) has the same sign as $\frac{d\Phi^{SA}}{dc_A^{SA}}|_{\alpha=\alpha^*_{SA}}$. Because c_A^{SA} does not affect π_m^{SA} or F, $\frac{d\Phi^{SA}}{dc_A^{SA}}|_{\alpha=\alpha^*_{SA}} = \frac{d\pi_A^{SA}(\alpha)}{dc_A^{SA}}|_{\alpha=\alpha^*_{SA}} + \alpha^*_{SA}\frac{d\pi_B^{SA}(\alpha)}{dc_A^{SA}}|_{\alpha=\alpha^*_{SA}}$. Directly calculating this using the same techniques as in the proof of Proposition 2 yields

$$\left.\frac{d\Phi^{SA}}{dc_A^{SA}}\right|_{\alpha=\alpha^*_{SA}} = q_A^{SA}(\alpha^*_{SA})\left.\frac{dq_B^{SA}(\alpha)}{dc_A^{SA}}\right|_{\alpha=\alpha^*_{SA}} P' - q_A^{SA}(\alpha^*_{SA})<0, \tag{47}$$

where the inequality follows from $P'<0$ and $\frac{dq_B^{SA}(\alpha)}{dc_A^{SA}}|_{\alpha=\alpha^*_{SA}}>0$. The latter follows from an analysis of the two first-order conditions similar to that employed in the proof of Proposition 1 for $\frac{dq_A^{SA}(\alpha)}{d\alpha}$ and $\frac{dq_B^{SA}(\alpha)}{d\alpha}$. Thus, $\frac{d\alpha^*_{SA}}{dc_A^{SA}}<0$, which proves the result.

Part b is easily verified by calculating $\frac{d\Phi^{SA}}{d\delta}|_{\alpha=\alpha^*_{SA}}$ using the properties of linear demand.

Proof of Proposition 12. Because I affects only $\pi_A^{SA}(\alpha)+\pi_B^{SA}(\alpha)$ in the calculation of S, following the proof of Proposition 7, the result is easily verified by algebraically calculating $\frac{d^2(\pi_A^{SA}(\alpha)+\pi_B^{SA}(\alpha))}{d\alpha\, dI}$ under the assumption of linear demand.

Proof of Lemma 2. Assume that the two parties compete as in Case 2 from the text with marginal costs c_B for B and $c_A>c_B$ for A. I define $\Pi_d=\pi_A(\alpha)+\pi_B(\alpha)$, where $\pi_A(\alpha)+\pi_B(\alpha)$ are as in the text but using the generic constant marginal costs c_A and c_B. I assume that both $\pi_A(\alpha)$ and $\pi_B(\alpha)$ are positive. The result is proven by showing that $\frac{d\Pi_d}{dc_B}=\frac{d\pi_B(\alpha)}{dc_B}+\frac{d\pi_A(\alpha)}{dc_B}<0$. Calculating $\frac{d\pi_B(\alpha)}{dc_B}$ directly using the envelope theorem yields

$$\frac{d\pi_B(\alpha)}{dc_B}=q_B(\alpha)\frac{dq_A(\alpha)}{dc_B}P'-q_B(\alpha), \tag{48}$$

where $P'=P'(q_A(\alpha)+q_B(\alpha))$. Calculating $\frac{d\pi_A(\alpha)}{dc_B}$ yields

$$\frac{d\pi_A(\alpha)}{dc_B}=\frac{dq_A(\alpha)}{dc_B}(P-c_A+q_A(\alpha)P')+\frac{dq_B(\alpha)}{dc_B}q_A(\alpha)P', \tag{49}$$

where $P=P(q_A(\alpha)+q_B(\alpha))$. From A's first-order condition in Case 2, $P-c_A+q_A(\alpha)P'=-\alpha q_B(\alpha)P'$. Performing the replacement and adding the two derivatives together yields

$$\frac{d\Pi_d}{dc_B}=\left[(1-\alpha)q_B(\alpha)\frac{dq_A(\alpha)}{dc_B}+q_A(\alpha)\frac{dq_B(\alpha)}{dc_B}\right]P'-q_B(\alpha). \tag{50}$$

Totally differentiating the two first-order conditions with respect to $q_A(\alpha)$, $q_B(\alpha)$, and c_B and applying Cramer's Rule as in the proof of Proposition 1 yields

$$\frac{dq_A(\alpha)}{dc_B}=\frac{-((q_A(\alpha)+\alpha q_B(\alpha))P''+(1+\alpha)P')}{((q_B(\alpha)+q_A(\alpha))P''+(3-\alpha)P')P'} \tag{51}$$

and

$$\frac{dq_B(\alpha)}{dc_B}=\frac{(q_A(\alpha)+\alpha q_B(\alpha))P''+2P'}{((q_B(\alpha)+q_A(\alpha))P''+(3-\alpha)P')P'}. \tag{52}$$

Inserting these expressions into the expression for $\frac{d\Pi_d}{dc_B}$ and rearranging yields

$$\frac{d\Pi_d}{dc_B} = \frac{(q_A(\alpha) - \alpha(1-\alpha)q_B(\alpha))P'}{(q_B(\alpha) + q_A(\alpha))P'' + (3-\alpha)P'} + \frac{(q_A(\alpha) - (1-\alpha)q_B(\alpha))[(q_A(\alpha) + \alpha q_B(\alpha))P'' + P']}{(q_B(\alpha) + q_A(\alpha))P'' + (3-\alpha)P'} - q_B(\alpha) < 0. \quad (53)$$

To see the inequality, consider the second term. By inspection of the two first-order conditions, $q_A(\alpha) - (1-\alpha)q_B(\alpha) < 0$ and $(q_A(\alpha) + \alpha q_B(\alpha))P'' + P' < 0$ by assumption. Finally, the denominator is negative because $q_B(\alpha)P'' + P' < 0$ and $q_A(\alpha)P'' + P' < 0$ by assumption and $\alpha \leqslant 1$. Thus, the second term in Eq. (53) is negative. Now consider the first term. The denominator can be written as $[(q_A(\alpha) + \alpha q_B(\alpha))P'' + P'] + (1-\alpha)[q_B(\alpha)P'' + P'] + P' < P' < 0$, where the inequalities follow from the fact that the two bracketed terms are negative by assumption. Thus, the first term in Eq. (53) equals $q_A(\alpha) - \alpha(1-\alpha)q_B(\alpha)$ times a positive number less than one. Thus, if $q_A(\alpha) - \alpha(1-\alpha)q_B(\alpha)$ is negative the result is proven. If $q_A(\alpha) - \alpha(1-\alpha)q_B(\alpha)$ is positive, the first term is less than $q_A(\alpha) - \alpha(1-\alpha)q_B(\alpha)$, which is less than $q_A(\alpha)$. Because $q_B(\alpha) > q_A(\alpha)$ holds by inspection of the first-order conditions, the result follows.

Proof of Proposition 13. Follows from the preceding discussion in the text.

Proof of Proposition 14. Follows from the preceding discussion in the text.

Proof of Proposition 15. Follows from the preceding discussion in the text.

Proof of Proposition 16. The first statement follows from Proposition 15 and the continuity of the payoffs in the knowledge transfer benefit parameters. The second statement is trivial.

Proof of Lemma 3. Follows from $\frac{d(1-\alpha)(d\Pi_B/de)}{d\alpha} < 0$ and $\frac{d(1-\alpha)(d\Pi_B/de)}{dr} < 0$, which are easily verified using the properties of linear demand.

Proof of Lemma 4. This is easily verified by calculating $\frac{df^{SA}(\alpha)}{de}$ using the properties of linear demand.

Proof of Proposition 17. With linear demand parameterized as $P(Q) = N - bQ$, my assumption that there always exists an interior solution for e reduces to $\beta < \frac{2b}{Ac}$. To see this, simply compute the optimal e for each case using the properties of linear demand and set $\alpha = 0$. For Part a, the result follows from a direct calculation of $\frac{d^2S}{d\alpha^2}$ using the properties of linear demand and $S = \pi_A(\alpha) + \pi_B(\alpha) - F + V_1 + \Delta V_1$. For Part b, for a given $\alpha \in [\underline{\alpha}, \overline{\alpha})$, I use the properties of linear demand to calculate the e that would make A indifferent between entry and no entry as the e that sets $\alpha\pi_B(\alpha) + \pi_A(\alpha) - \alpha\pi_m^{SA} - F = 0$. I then use the formula for the entrepreneur's optimal effort to calculate r as a function of e and plug in the value of e derived above to get the equilibrium value of r. Finally, I calculate total expected surplus S as $r(\pi_A(\alpha) + \pi_B(\alpha) - F) + (1-r)\pi_m^{SA} + V_1 + \Delta V_1$ and take the derivative using the properties of linear demand to find $\frac{dS}{d\alpha} > 0$.

Proof of Proposition 18. Follows from the fact that $\frac{d^2S}{d\alpha\, d\beta} < 0$, which is easily verified using the properties of linear demand.

Proof of Lemma 5. When only product B is produced, B maximizes $q_B(P_B-(1-\delta)c)$ with respect to P_B using Eq. (11) for q_B. This results in a price of $\frac{N+(1-\delta)c}{2}$ and profit of $\frac{(1+2\lambda)(N-(1-\delta)c)^2}{8(1+\lambda)}$. Under joint ownership and control, the party in control maximizes $q_A(P_A-(1-\delta)c)+q_B(P_B-(1-\delta)c)$ with respect to P_B using Eq. (10) for q_A and q_B, with resulting prices of $P_i=\frac{N+(1-\delta)c}{2}$ for all i and total profit $\frac{(N-(1-\delta)c)^2}{4}$. It is thus optimal to introduce the second product if $\frac{(N-(1-\delta)c)^2}{4}-\frac{(1+2\lambda)(N-(1-\delta)c)^2}{8(1+\lambda)}>F$. Simplification yields the result.

Proof of Lemma 6. With entry and a passive equity stake, B maximizes $(1-\alpha)q_B(P_B-(1-\delta)c)$ with respect to P_B using Eq. (10), and A maximizes $q_A(P_A-(1-\delta)c)+\alpha q_B(P_B-(1-\delta)c)$ with respect to P_A using Eq. (10). This results in prices

$$P_B=(1-\delta)c-\frac{(N-(1-\delta)c)(2+3\lambda)}{-4+\lambda(-8+(\alpha-3)\lambda)} \tag{54}$$

and

$$P_A=\frac{c(1-\delta)(1+\lambda)(-2+(\alpha-3)\lambda)-N(2+(3+\alpha)\lambda)}{-4+\lambda(-8+(\alpha-3)\lambda)}. \tag{55}$$

It is easily verified that these are strictly lower than $\frac{N+(1-\delta)c}{2}$ for all $\lambda>0$.

Proof of Lemma 7. This is easily verified by calculating the derivative $\frac{\mathrm{d}(\pi_A^{SA}(\alpha)+\pi_B^{SA}(\alpha))}{\mathrm{d}\alpha}$ using the prices derived above.

Proof of Lemma 8. This is easily verified by calculating the derivative $\frac{\mathrm{d}(\alpha\pi_B^{SA}(\alpha)+\pi_A^{SA}(\alpha)-\alpha\pi_m)}{\mathrm{d}\alpha}$ using the prices derived above.

Proof of Lemma 9. If entry deterrence is possible at some $\alpha\leqslant 1$, then it is possible at $\alpha=1$. The optimal entry decision at this point is to not enter if $\pi_B^{SA}(\alpha)+\pi_A^{SA}(\alpha)-\pi_m<F$. This together with Lemma 7 implies that $\pi_m>\pi_A^{SA}(\alpha)+\pi_B^{SA}(\alpha)$ for all α, so deterrence must be optimal for Market 2. Finally, if it is not optimal to introduce the second product under joint ownership and control, then $\pi_B^{SA}(\alpha)+\pi_A^{SA}(\alpha)-\pi_m<F$ must hold because the sum of duopoly profits with a passive stake of $\alpha=1$ is lower than total profit under joint ownership and control according to Lemma 6. Thus, deterrence is possible for some range of $\alpha\leqslant 1$.

Proof of Proposition 19. For the first statement, the only cost of doing a merger is the loss of all surplus in Market 3, whereas a merger always implements the first best in Market 2. For the second statement, keeping product A out of the market is impossible, but equity softens the inevitable competition in Market 2, so some equity could be sold depending on the size of Market 3. The final statement follows from Lemma 9 and the fact that the only cost of transferring equity is the loss of surplus in Market 3.

Appendix B

The technical conditions used in the construction of Fig. 2 are provided and proven here. Throughout Appendix B, total derivative notation is used to indicate that the effects of parameter changes on the equilibrium production quantities in Stage 3 are taken into account.

- An increase in ΔV_1 increases $\gamma^{SA/N}$ and $\gamma^{SA/E}$ and does not affect $\gamma^{N/E}$. *Proof*: Follows from the fact that $V^{SA/E}$ and $V^{SA/N}$ increase in ΔV_1, while $V^{N/E}$ and $V^{N/N}$ are unaffected.
- An increase in V_3 decreases $\gamma^{SA/N}$, increases $\gamma^{N/E}$, and increases $\gamma^{SA/E}$ when ΔV_1 is small. *Proof*: For $\gamma^{SA/N}$, follows from $\frac{dV^{N/N}}{dV_3} = (1-(\alpha^*)^2)\beta V_3 > \frac{dV^{SA/N}}{dV_3} = (1-(\alpha^*_{SA})^2)\beta V_3$. For $\gamma^{N/E}$, it follows from the analysis of V_3^N in the proof of Lemma 1. For $\gamma^{SA/E}$, $\alpha^{SA/E} < \alpha^*$ when $\Delta V_1 = 0$ and $V^{SA/E} = V^{N/N}$, which follows from the fact that $\pi_m > \pi_A^{SA}(\alpha) + \pi_B^{SA}(\alpha)$. Thus, by the envelope theorem, $\frac{dV^{N/N}}{dV_3} < \frac{dV^{SA/E}}{dV_3}$ whenever $V^{SA/E} = V^{N/N}$.
- Neither $\gamma^{N/E}$ nor $\gamma^{SA/E}$ can ever exceed γ^*. *Proof*: For $\gamma^{N/E}$, follows from the fact that $V^{N/N} > V^{N/E}$ for all $\gamma \geqslant \gamma^*$. For $\gamma^{SA/E}$, $V^{N/N} > V^{SA/E}$ must hold for all $V_3 > V_3^{SA}$ (if no alliance is optimal anywhere, it must be optimal for all $\gamma > \gamma^*$ because this is the highest payoff possible with an alliance). Finally, $\frac{dV^{N/N}}{dV_3}|_{\gamma \geqslant \gamma^*} = \beta V_3 > \frac{dV^{SA/E}}{dV_3} = (1-(\alpha^{SA/E})^2)\beta V_3$.
- As $\Delta V_1 \to 0$, $V_3^{SA} \to 0$ and $\gamma^{SA/N} \to 0$ hold for all V_3, while $\gamma^{SA/E} \to 0$ and $\gamma^{N/E} \to 0$ hold when $V_3 = V_3^E$. *Proof*: For V_3^{SA} and $\gamma^{SA/N}$, $V^{N/N} \geqslant V^{SA/N}$ always holds when $\Delta V_1 = 0$. For $\gamma^{SA/E}$ and $\gamma^{N/E}$, $V^{SA/E} = V^{SA/N} = V^{N/E} = V^{N/N}$ holds if $\gamma = 0$ and $V_3 = V_3^E$ when $\Delta V_1 = 0$.
- As $\Delta V_1 \to 0$ and $\Delta V_3 \to \infty$, $\gamma^{SA/E} \to \gamma^*$ and $\gamma^{N/E} \to \gamma^*$. *Proof*: This follows from the fact that $V^{N/N} > V^{SA/E}$ and $V^{N/N} > V^{N/E}$ when $\gamma \geqslant \gamma^*$ and $\Delta V_1 = 0$, while $\frac{dV^{N/N}}{dV_3} < \frac{dV^{SA/E}}{dV_3}$ whenever $V^{SA/E} = V^{N/N}$ and $\Delta V_1 = 0$ (see above) and $\frac{dV^{N/N}}{dV_3} < \frac{dV^{N/E}}{dV_3}$ when $V^{N/E} = V^{N/N}$, which follows from $\alpha^{N/E} < \alpha^*$ at such points.
- $\gamma^{SA/E} > \gamma^{N/E}$ holds for some range of V_3 as it rises above $\max[V_3^E, V_3^{SA}]$. *Proof*: At $\gamma = 0$, $V^{SA/E} = V^{N/E} + \Delta V_1$ must hold. At V_3^E, $\gamma^{N/E} = 0$ must hold (see above). The result follows.
- Whenever $\gamma^{SA/E} = \gamma^{N/E}$, $\gamma^E = \gamma^{SA/E} = \gamma^{N/E}$ must hold. *Proof*: Follows from the fact that $V^{SA/E} = V^{N/E}$ holds at γ^E, while $V^{N/N} = V^{SA/E}$ at $\gamma^{SA/E}$ and $V^{N/N} = V^{N/E}$ at $\gamma^{N/E}$.

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