A GENERAL THEORY OF DEMAND IN A MULTI-PRODUCT MULTI-OUTLET MARKET

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Abstract

The overarching goal of this paper is to develop a robust methodology for deriving demand functions for multi-product, multi-outlet situations. The basic premise is that such functions are necessary if one wants to compare the implications of different channel systems across varying degrees of competition and/or varying number of available product offerings.

The paper starts by assuming a general utility function that captures four key factors felt to influence buyer behavior, i.e., the intrinsic value of the product, the price of the product, the lack of fit of the product's attributes compared to the buyer's ideal product attributes and travel costs. This leads to a simple spatial model that specifies the set of potential customers that consider a given product offering, i.e., a specific product sold at a specific retail outlet at a given price. Using this spatial model and assumptions about the distribution of potential customers with respect to tastes and physical location, we show how to derive demand functions for any given multi-product, multi-outlet situation.

We use this approach to derive demand functions for two, three and four product offering situations. In each case, we find that a reasonable set of assumptions concerning buyer behavior and market characteristics can lead to demand functions that are, for all intents and purposes, linear in price. The parameters of these demand functions are directly linked with the parameters of our underlying utility function and the degree of competition in the marketplace. Consequently, it is possible to compare different strategic alternatives and still be assured a consistent market environment.

We use our linear approximations to reanalyze two previously published studies that compare strategic alternatives that involve different numbers of product offerings. In contrast to our approach of starting with one underlying buyer behavior model and then deriving the demand functions, these two studies start by assuming demand functions for each product offering situation. We show that our results differ substantially from the existing published results. We attribute this difference to the fact that the prior analyses confound channel system differences with buyer behavior differences. In contrast our results are based upon one buyer behavior model for all the channel systems investigated.

Key Words: Demand Functions; Equilibrium Analyses; Channel System Analysis

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1. Introduction

Should competing manufacturers distribute their products through exclusive franchise systems or through a common channel of mass merchandisers? If a grocer carrying competing national brands adds a store brand to its product assortment in the same product category, how will it affect the prices and profits for the retailer and the national brand manufacturers? What would be the impact of a manufacturer starting a catalog (Internet) distribution channel in addition to using conventional bricks and mortar retailers?

These types of questions are representative of the serious strategic channel issues faced by today's marketing managers and the recent marketing literature has addressed such issues using analytic models (e.g., Choi 1993; Raju et al. 1995; Sayman et al. 2000; Trivedi 1998). The basic approach taken in many of these studies is to *begin by assuming a demand function* for each unique product sold by a specific retailer (hereafter referred to as a product offering) for each channel structure of interest. The researcher then specifies a set of rules that defines each player's strategic behavior and uses game theoretic analyses to derive equilibrium prices, profits, quantities, etc. for each strategic alternative being analyzed. These equilibrium values are then compared across alternatives to determine the impact of switching from one strategy to another.

It is crucial to note that the validity of such analyses is based upon the implicit assumption that the demand functions used to analyze each of the strategic alternatives map back to the same buyer behavior and market conditions. If such an assumption is violated, one cannot guarantee that any different effects observed across strategic alternatives are due to differences in channel members' strategic choices instead of being caused by differences in the underlying markets and/or buyer behavior. Consequently, without ensuring a consistent underlying market environment, one cannot rule out the possibility that such analyses might produce misleading strategic implications. Unfortunately, many of the analytic studies on channels do not directly addresses this issue. For instance, Trivedi's (1998) analysis is based on one set of four demand functions that represents a situation where two competing retailers carry both of two competing manufacturers' products and a second set of two demand functions that represent a situation where each manufacturer uses an exclusive retailer for distribution. She never shows that these two demand structures (e.g., the 4 demand equation structure and the 2 demand equation structure) arise from the same market conditions. Similarly, Raju, et al. (1995), in their store brand introduction analysis, never show that the buyer behavior, market size, and distribution of consumer tastes are held constant between the "before" store brand case (two demand functions) and the "after" store brand case (three demand functions).

We acknowledge that having a consistent set of customers with a consistent set of behaviors is not an issue when the same set of demand functions are used to compare the strategic alternatives. However, even here one needs to be careful if one is interested in analyzing the impact of competition within a channel system. Often this is done by assuming one or more of the demand parameters reflect competition between the product offerings, e.g., Choi (1993). Again, if these demand functions are not derived from an explicit model of underlying buyer behavior, there is an implicit assumption that varying one of these demand parameters while holding fixed the other parameters yields demand functions that reflect the same set of potential customers and the same underlying behavior. Otherwise, difference in prices and profits could be due to different market characteristics instead of different levels of competition. For example, as shown in McGuire and Staelin (1983), equilibrium prices and profits based on their rescaled demand functions increase with increases in product substitutability. However, as they point out, this increase is not due to changes in substitutability but instead to different underlying demand.

In this paper, we address these issues by developing a general theory of demand that enables the analyst to develop parsimonious demand models for different channel structures and market conditions based upon an explicit buyer behavior model for a specified set customers. We do this by developing an eight-step procedure as illustrated in Figure 1. In the first three steps we create a spatial model of the market by specifying 1) a buyer behavior model represented by a utility function for each potential customer, 2) the distribution of buyers in terms of tastes and physical location and 3) the positioning of each product offering as determined by product attributes and retail outlet locations. In the next three steps, we derive a demand function for each product offering by first 4) generating simulated demand data for each product offering for a large number of price levels for a specific setting, i.e., a specific degree of product and outlet competition. We then 5) use the simulated demand data to estimate a (linear) demand function for each product offering for this competitive situation. We repeat this procedure for a wide variety of market conditions, e.g., various degrees of product and outlet competition and various channel systems, thereby generating intercept and slope parameters for each of these market conditions. We next 6) determine how these linear demand parameters vary as a function of the market conditions. In the last two steps, we 7) use the linear functions as determined from steps 4 through 6 to investigate the effects of alternative channel strategies by deriving and analyzing equilibrium prices, profits, etc. Finally, in step 8), we verify that the obtained equilibria are within the range of prices applicable to our derived demand functions.

(Insert FIGURE 1 Here)

Note that the analyses by Choi (1993), Raju et al., (1995) and Trivedi (1998), like many other analytic studies on channel strategy, all start with step 6) and end at step 7) of our eight step procedure. In contrast, we maintain that it is crucial to begin from step 1) and to end with step 8) to ensure that the demand model is based on a consistent model of buyer behavior and market conditions. Later in the paper, we support this point by reanalyzing the problems investigated by Raju et al. and Trivedi and showing that our obtained results are substantially different from theirs.

We acknowledge that numerous demand formulations used in channel analyses are based upon explicit buyer behavior. One approach taken to derive demand functions and often referenced by channel analysts using linear demand functions, is found in the work of Dixit (1978) and Shubik and Levitan (1980). They do not explicitly model a situation with different retail outlets. Instead they model a two-product market by assuming a) all customers have identical tastes for a particular product and b) there are no travel costs. Dixit's formulation allows each customer to have two different utility functions, one for each product. Shubik and Levitan assume all customers have one utility function that holds for both products. From these assumptions they derive linear demand functions for both products sold in their respective duopoly models. While these analyses yield mathematically tractable linear demand functions, the formulations cannot be extended to situations where there is heterogeneity among buyers in terms of tastes and locations and, therefore, don't support the rich general setting normally associated with the analysis of complex strategic channel issues mentioned above.

A second general approach is exemplified by Purohit (1998) and is expanded on by Lal and his co-authors. The general idea is to place both buyers and the product offerings on a unit line. In the case of a duopoly, the two different store offerings are located at the two ends of the line. The distance between these offerings either represents physical distance of the stores, or attribute difference of the products. Buyers are then placed on the unit line in accordance to distance (either physical or taste) from the offerings. Linear demand functions are then derived by partitioning the unit line according to the location of buyers who are indifferent to the two offerings. Lal (1990) expands on this basic model by considering a situation where three different manufacturers sell three competing products to a market composed of three different types of customers. Based upon a set of assumptions concerning how these customers behave (e.g., loyal customers will always buy their preferred brand as long as the brand is at or below their reservation price), Lal derives a set of demand functions for each brand. Other examples include Lal and Rao (1997) who model a market composed of two types of buyers who consider both shopping costs and prices when buying from two different retailers and Lal, Little and Villas-Boas (1996) who model the behavior of three customers who potentially could buy one of two competing brands sold through a single retailer. While representing richer buyer behavior, models used in these latter studies yield demand functions that are not continuously differentiable and are sometimes non-linear. As a consequence, subsequent analyses for determining equilibrium conditions are "complex", requiring the analyst to check numerous boundary conditions. Perhaps more importantly, although the methodology used to derive the demand functions is "straightforward", their analysis approach provides few generalizable principles in terms of developing demand functions for other, somewhat different, and more complex marketing conditions.

Our approach expands the limits observed in these existing studies by blending the advantages of both types of approaches discussed above. Specifically, our model can accommodate the richness of buyer behavior beyond the small number of buyers (or segments) assumed by Purohit and Lal and his co-authors. At the same time, our model allows us to derive demand functions that are easy to apply in subsequent analyses. Therefore, this paper presents a general methodology for demand formulation that is applicable to a wide variety of multi-product, multi-outlet market situations.

In developing our model of buyer behavior, we posit that potential customers consider four factors when evaluating a product offering. These are the price of the product offering, the intrinsic value of the ideal product within the product class being considered, the degree to which the offering meets their ideal specifications and the shopping and travel costs associated with obtaining the product. Customer heterogeneity is assumed both in terms of tastes for the ideal product specification and buyers' travel costs. Assuming customers maximize their utility, we develop a spatial representation for the set of potential customers who are willing to consider a product offering at the given price. One of the outputs of this spatial representation is an ability to graphically specify the potential markets for any product offering at any given price. We find this spatial representation helpful in deriving the demand for each product offering over a number of different situations (e.g., monopoly, duopoly, etc.). It is also useful in providing new insights into the restrictions one needs to impose on any demand formulation used to represent a market composed of multiple products sold through multiple outlets.

One of the implications that flows from our procedure is that although it is possible to derive demand functions that are linear in prices for situations where one retailer sells one product within a category (i.e., a monopoly setting), the demand functions derived from this same underlying buyer behavior model are non-linear and at times not continuously differentiable when this product is sold in a multi-product and/or multi-outlet setting. However, as long as the prices seen by customers yield positive demand for all the available products and for each pair of competing offerings there are at least some customers willing to buy both of the competing offerings, we find the demand functions for all offerings to be extremely well approximated by linear demand functions. Moreover, we are able to develop relationships that define the intercepts and slopes of these linear demand functions as a function of the conditions within the market place, (e.g., number of competitors and the degree of competition between products and between outlets), and the parameters of our buyer behavior model, (e.g., the customer's underlying sensitivity to price and the intrinsic value the customer places on the ideal product offering). In this way we are able to specify general (linear) demand functions that are explicitly linked to an underlying buyer behavior model.

We show the usefulness and flexibility of our 8-step approach by applying it to two situations previously analyzed by other researchers who specified demand functions for different channel systems without explicitly linking these functions to an underlying buyer behavior model. We find our analyses yields significantly different results than those published.

In summary, we are able to accomplish the following:

1) Develop a methodology for deriving a general demand formulation based upon a rich buyer behavior model that captures key aspects of most purchase situations, allows for heterogeneity across customers and can be easily applied to numerous multi-product and/or multi-outlet situations. This methodology also yields a simple graphic representation of the market of interest that quickly conveys the competitive environment. Perhaps most importantly, since this demand formulation directly reflects our underlying buyer behavior model, any demand functions that are derived using this demand formulation, even if the buying environments or channel structure are different, are "logically consistent", in that they all map back to the same set of buyers. We believe this methodology should be of great interest to analytic channel modelers who are interested in analyzing and contrasting different marketing situations.

2) Show that these demand functions can be adequately estimated by linear demand functions where the parameters of these linear demand functions are a function of the underlying market conditions. This important result greatly enhances the value of our approach since such linear demands facilitates "easy" solution of equilibrium conditions.

3) Reanalyze the implications of two important channel management issues found in the current marketing literature using these "logically consistent" demand functions. Since we are assured that any differences in results across the different channel systems are due to channel structure and not buyer behavior differences, we believe our findings should be of great interest to channel scholars interested in these different policy choices (i.e., implications of introducing a store brand and the impact of manufacturers using intense (i.e., broad) channel coverage versus an exclusive channel system).

Basic Assumptions

While the proposed eight step process in Figure 1 can be applied to a much broader set of complex settings involving various channel strategy issues, we choose to limit our analysis to situations that involve no more than two manufacturers and two retailers. In this way, we are able to provide the intended insights with relative parsimony while maintaining a rich environment involving such factors as buyer heterogeneity, horizontal and vertical product differentiation, competition between spatially differentiated retailers, inter- and intra-brand competition, etc. Moreover, since almost all the models analyzed in the literature include no more than two manufacturers and two retailers (summarized as models 1, 2, 3, and 4 in Lee and Staelin 1997), this allows us to directly compare our results with those of existing studies. Below, we sketch out the main characteristics of the market/industry environment we analyze by listing our basic assumptions.

Assumption 1: There exist a large number of buyers in the market who are fully informed and rational.

We assume there exists a sufficiently large number of buyers to ensure that the resulting demand curve is reasonably continuous in price. By "fully informed" we assume every buyer's consideration set consists of all the product offerings available in the market and that there is no uncertainty involved in a buyer's evaluation of alternatives. Consequently, while buyers might differ in taste (i.e., ideal combination of product attributes), they are in complete agreement on product quality (thus the inherent value of the product). Buyers are also assumed to be "rational" in that they seek to maximize utility. This utility maximization behavior is specified by the next assumption.

Assumption 2: Each buyer purchases one unit of the product offering that maximizes his/her utility as long as it provides a utility level exceeding the individual's minimum required level. If none of the product offerings can provide greater utility than the buyer's minimum requirement, the potential buyer decides not to make a purchase in the product category.

The minimum required level of utility can be set at zero without loss of generality. Note also that, by assuming potential buyers have the option of not making a purchase in the product category, we allow the market demand to be expandable, departing from models assuming a fixed total demand (e.g., Hauser and Shugan 1983; Vandenbosch and Weinberg 1995; Lal and Rao 1997).

Assumption 3: A buyer's utility increases in product quality and decreases in price.

Assumption 4: A buyer has a set of ideal product attributes and the most preferred store location. Buyer utility decreases as the product attributes and the store locations move farther away from this "ideal point".

While Assumption 3 provides a basis to consider vertical differentiation, Assumption 4 captures the potential of horizontal differentiation between manufacturers and/or between retailers. Note that within the context of two competing manufacturers, horizontal differentiation based on any number of product attributes can be represented on a straight line called "product characteristic" without loss of generality. In the same way, two competing retailers might be differentiated in service, store atmosphere, location, etc., which can be captured in one summary dimension called "spatial location". This allows us to represent buyers' ideal points and perceptions of product offering positions in a two dimensional spatial model similar to an MDS perceptual map. For example, let the horizontal and vertical axes represent spatial location and product characteristic respectively in Figure 2. Also, let Z represent a particular product offering and ideal points *a* and *b* represent buyers *a* and *b* respectively. Then, buyer *a* is physically located next to the retail outlet but has a preference for a somewhat different set of attributes than provided by the product offering Z. On the other hand, buyer *b* finds the offering to provide his/her ideal attributes, but faces some physical distance to travel to reach the retail outlet. Therefore, a vertical distance between a product offering and a buyer's ideal point (between *a* and Z in the Figure) reflects the disutility caused by the product's inability to satisfy the buyer's taste, while a horizontal distance (from Z to *b*) reflects the disutility a buyer incurs traveling to an outlet.

(Insert FIGURE 2 Here)

Assumption 5: Buyers are uniformly distributed in terms of ideal product characteristic and spatial location of a retail outlet and these two distributions are uncorrelated.

This assumption is typical of spatial marketing models and can be relaxed without much difficulty. Coupled with Assumption 2, it enables us to represent demand graphically as an area in the two-dimensional space shown in Figure 2.

One important implication coming from the preceding five Assumptions is that the aggregate market demand is determined not only by price and product quality but also by product positions and store locations. Consider buyer *c* in Figure 2. If Z represents the only product offering available in the market, *c* might decide not to make a purchase due to the significant lack of fit in product characteristic and travel distance. What will happen if another manufacturer enters the market with the same price and quality level as the incumbent's? If the new product is also positioned at Z, it will not be able to attract any of non-buyers before its entry like *c*, leading to no increase in total market demand. On the other hand, if the new product is positioned close to *c* (represented by Z'), it will attract buyers such as *c*, leading to a market expansion. By explicitly modeling this type of phenomenon, we are able to capture the systematic impact of the positioning of existing or new products/stores on total market demand.

This is significant because many existing marketing models assume either a fixed total demand (e.g., Hauser and Shugan 1983; Vandenbosch and Weinberg 1995; Lal and Rao 1997) or total demand as a function of only product/service quality and price but not of product/store positions (e.g., Raju et al. 1995; Iyer 1999). Consequently, these models are unable to adequately capture strategic effects of positioning and differentiation that result in changes in total market size. There also exist demand models in which the aggregate demand does vary with product offering positions (e.g., McGuire and Staelin 1983; Choi 1991; Trivedi 1998). However, these models were constructed without any explicit consideration of buyers' reaction to changes in product offering positions and, consequently, the degree of competition between the offers. This lack of a link between buyer behavior and the demand function can lead to a counterintuitive relationship between the aggregate demand and product offering positioning (perhaps apart from the authors' intention), potentially leading to misleading results if one compares across different levels of competition. In contrast, Assumptions 1 through 5 provide us with a basis on which we build and analyze demand models without such problems.

Having laid out our basic assumptions on the demand side of the market (i.e., buyer behavior and heterogeneity), we now turn our attention to the supply side, stating basic assumptions concerning firms.

Assumption 6: There exist two partially differentiated products marketed by two competing national brand manufacturers through one or two retailers. The quality levels of the two national brands are identical and equal to or higher than that of a store brand, if it exists.

Assumption 7: Manufacturers and retailers are profit maximizers.

These assumptions create settings investigated by Raju et al. (1995) and Trivedi (1998), which we re-analyze using our demand model. As in their papers, we assume product positions and store locations are determined exogenously. We also assume that the horizontal competition is a Nash game at both the manufacturer and retailer levels when we solve for equilibrium prices, profits, etc. Using these assumptions, we build our demand model below.

Model Development

We start the modeling process by defining the key determinants of buyer utility based on Assumptions 3 and 4. First, define V to be the intrinsic value for each buyer's ideal product offering, measured by the maximum price each buyer is willing to pay (i.e., reservation price) if the product's characteristics and the location of the store are in perfect match with the buyer's ideal point. Second, define the "lack of fit" between buyer k's ideal point and manufacturer j's product to be $d_{\theta jk}$ where the subscript "θ" represents product characteristic dimension. Likewise, define the spatial distance between buyer k and outlet i to be $d_{\chi ik}$ where the "χ" subscript represents spatial distance. For example, in Figure 2 $d_{\chi ik} = 0$, $d_{\theta za} > 0$, $d_{\chi zb} > 0$ and $d_{\theta zb} = 0$. Finally, let the price of manufacturer j's product sold by retailer *i* be P_{ij}.

Next, we assemble these determinants of buyer utility to specify a utility function. We do this by finding a mathematical function that expresses the buyer's trade-offs of price, travel costs and lack of fit costs with the intrinsic product value consistent with Assumptions 3 and 4. Acknowledging there exists numerous utility functions that could capture these trade-offs, we choose to use the following functional form for its superior mathematical tractability in the subsequent analysis:

(1) $U_{ijk} = V(\frac{V-P_{ij}}{V})^{\gamma/2}$ - $\sqrt{d^2_{\chi ik} + d^2_{\theta jk}}$,

where U_{ijk} is buyer k's utility for the product offering i, j (i = outlet, j = manufacturer) and γ is a scaling parameter, $0 < \gamma < 2$.

Note that this function reflects all the characteristics required by Assumptions 3 and 4 describing utility as increasing in intrinsic product value and decreasing in price, lack of fit and travel cost. It also offers direct interpretation for each term of the equation. Thus, the latter term represents the Euclidean distance between buyer k's ideal point and the product offering's position in a two-dimensional space such as Figure 2.[1] The first term represents product offering ij's net utility (surplus) before subtracting the disutility of lack of fit and spatial distance. Thus, if the product offering is positioned ideally for buyer k, the utility is V at zero price and decreases as P_{ij} goes up. The term $(\frac{V-P_{ij}}{V})^{\gamma/2}$ represents the effect of price on the buyer's "surplus" utility. Since we assume $0 < \gamma < 2$, as P_{ij} decreases from its maximum feasible value of V, utility increases but at a decreasing rate, just as typical utility formulations assume initial increases in wealth provide the greatest increases in utility. If γ is close to 2, buyers value wealth almost linearly. When γ is close to 0, buyers value initial wealth increases much more than final increases in wealth.

Equation 1 and our assumptions of uniform distributions (Assumption 5) allow us to identify the set of buyers who will experience positive utility from purchasing a particular product offering at a particular price. Given Assumption 2 (maximum purchase quantity of one unit), the number of these buyers directly translates into the market potential for any given product offering at any given price. Consequently, the set of marginal buyers, who gain zero utility and are indifferent between purchasing the product offering and not buying, constitute the boundary condition between potential buyers and non-buyers at the given price.[2] Specifically, from equation 1 we see the following boundary condition (i.e., where $U_{ijk}=0$) when $P_{ij} = 0$:

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[1] A more general form than equation (1) can be obtained by multiplying weighting parameters, β_1 and β_2 to $d_{\chi ik}$ and $d_{\theta jk}$ respectively to acknowledge that utility, product characteristic and spatial distance are in different scales. However, it can be shown that one can normalize the scales of these factors to have "equal" weighting without loss of generality.

[2] In effect, this limits the market to this finite set of customers. Thus, we really don't assume an infinite market as implied by assumption 5.

(2) $V = \sqrt{d_{\chi ik}^2 + d_{\theta jk}^2}$, or

$$V^2 = d_{\chi ik}^2 + d_{\theta jk}^2 .$$

Since equation 2 defines a circle with radius V and center located at the product offering's position, any buyer located within this circle will consider buying the product offering assuming our underlying model of buyer behavior. (See Figure 3.)

[Insert FIGURE 3 Here]

Monopoly Demand

Deriving the demand function for a monopolist from our spatial model is straightforward. As Figure 3 shows, the number of buyers who are within the circle with radius V represents the market demand for the monopoly offering when the price is zero. Denote this number as $\overline{S}$. Since buyers are uniformly distributed over the two-dimensional space, $\overline{S}$ is proportional to πV^2, the area of the zero price circle. With no loss of generality, let $\overline{S} = \pi V^2$. Now let the price increase to $P_{ij} > 0$. From equation 1 we see that the radius of the circle now becomes $V(\frac{V - P_{ij}}{V})^{\gamma/2}$. Graphically, this has the effect of reducing the area of customers who will still have non-negative utility. (See Figure 3.) Specifically, the area of this (smaller) circle becomes $\pi V^2 (1 - \frac{1}{V} P_{ij})^{\gamma}$ or $\overline{S}\ (1 - \frac{P_{ij}}{V})^{\gamma}$.

Three observations can be drawn from the above development. First, if $\gamma = 1$ the change in the area when the price increases from zero to P_{ij} is proportional to the price change and equal to $\overline{S}\frac{P_{ij}}{V}$. More generally, the demand function for this monopoly product offering is linear in price with a negative slope of $\frac{\overline{S}}{V}$ and an intercept equal to $\overline{S}$. Mathematically, when $\gamma = 1$,

(3) $Q = \overline{S}(1 - \frac{1}{V} P_{ij})$,

where Q is the unit sales. Second, note that P_{ij}/V is less than, or equal to, 1 by definition, since any price greater than V would result in zero sales. Third, if $1 \leq \gamma < 2$ demand is concave in prices while if $0 < \gamma \leq 1$, demand is convex in prices. More specifically, if customers are almost linear in their evaluation of wealth increases, we note a concave demand function. As customers' value initial increases in wealth more than subsequent increases, we find the demand function becoming less concave and ultimately becoming convex. Given little empirical evidence to rely on in terms of knowing which behavior is more likely, we assume $\gamma = 1$ for the rest of the paper since it yields the mathematically tractable monopoly linear demand.

Duopoly Demand

We next expand our analysis to allow for any one of the following three different market situations where the market is composed of two products: a) two competing retailers selling competing products (e.g., McGuire and Staelin 1983), b) one retailer selling two competing products (Choi 1991), and c) an identical product being sold by two different outlets (Hotelling 1929). Within our spatial model, each of these channel systems can be represented with two overlapping circles, where each circle represents the market potential for the product offering assuming zero price and no competition as shown in Figure 4. For pedagogical convenience, we restrict our attention to the case of two competing retailers selling the same product (e.g., the Hotelling model). As will become immediately obvious, generalization of this case to the other two situations requires only a slight re-definition of terms. In addition, we designate the "western" most offering as product offering 1 and the "eastern" most offering as product offering 2.

[Insert FIGURE 4 Here]

In Figure 4, if prices are fixed at zero, any buyer located in the area where the circles overlap gets positive utility from both product offerings. As such, these two offerings constitute these customers' feasible choice set. Since the two retailers "share" these potential buyers, they find it in their best interest to "compete" in price to obtain a profitable share of them. This leads us to define competitive intensity in terms of the *degree of overlap* between the two zero-price circles. Note that the degree of overlap decreases with δ_χ, the distance between the two product offering positions and increases with the radius of the zero-price circle, V. When the two zero-price circles don't overlap (i.e., δ_χ exceed the sum of the two radii), there is no competition. Conversely, when there exists no spatial differentiation between the retailers (i.e., $\delta_\chi = 0$), the competitive intensity is at its maximum. We capture these end points by introducing the following measure of retail competitive intensity:

(4) $\chi = (1 - \frac{\delta_\chi}{2V})$ $\quad 0 \le \delta_\chi \le 2V$

$= 0$ $\quad$ otherwise,

where $\chi = 0$ indicates no competition and $\chi = 1$ implies perfect competition.

We next use this spatial representation and different combinations of P_1, P_2 and χ to produce six uniquely different competitive situations. We then analyze these six situations to determine the general shape of the conditional demand curve for product offering 1(i.e., the relationship between Q_1 and P_1) for given levels of P_2 and χ as shown in Figure 5. In the first case χ is low and P_2 is sufficiently high so that there is no overlap between the two circles even when P_1 is zero. (See the left-hand panel A in Figure 5.) In this situation, product-offering 1 is in a *localized monopoly* situation over the entire range of P_1 and consequently follows the monopoly demand function (equation 3). (See the right-hand panel of A in Figure 5.) In contrast, if both χ and P_2 are low, the two circles overlap when P_1 is low, leading to a duopoly in a strict sense (Figure 5-B). Note that *strict duopoly* demand is lower than monopoly demand because of the shared demand in the overlap area between the two product offerings. However, when P_1 is sufficiently high (represented by the dotted circle in Figure 5-B), the two circles do not overlap, leading to localized monopoly.

[Insert FIGURE 5 Here]

If χ is larger, we face more complicated situations. First, let P_2 be very high. Then the circle representing the product offering 2 is very small and is fully encircled by product offering 1's circle when $P_1 = 0$ (Figure 5, panel C). It can be shown that in such a case the utility from product offering 1 weakly dominates that from product offering 2 for *all* buyers, leading to a *dominant monopoly* case. When P_1 is sufficiently high (represented by the dotted circle in Figure 5, panel C), the two circles do not overlap, leading to localized monopoly. For medium levels for

P_1, the situation is strict duopoly. Combining these three ranges of P_1, we obtain a demand curve that first follows the monopoly demand, then change to a strict duopoly demand curve, and eventually change back to the monopoly demand as shown in the right-hand panel of C in Figure 5. Case D in Figure 5 is similar to Case C except that P_2 is lower and, therefore, product offering 2's circle includes customers located at the position of product offering 1, i.e., the center of circle 1 (see Figure 5-D). In this case, as P_1 increases from zero, product offering 1's demand equals the monopoly demand and then follows the strict duopoly demand until product offering 1 becomes *dominated* by offering 2 due to too high a P_1 (represented by the dotted circle in Figure 5-D). Cases E and F in Figure 5 can be interpreted in a similar fashion.

This discussion regarding Figure 5 reveals a number of characteristics of the duopoly demand function. First, it is evident that this demand function is seldom linear in own price (i.e., P_1) over the total range of feasible prices. Second, competitive price (P_2) affects both the intercept and the slope of the conditional demand function. Third, these effects of competitive price are moderated by χ. Specifically, note that the intercept increases with P_2 (for a fixed χ) and decreases with χ (for a fixed P_2). Based on these insights, we next explore how to specify a functional form for duopoly demand. We start with the knowledge that equation 3 correctly represents the demand for a duopolist in a localized or dominant monopoly situation. Consequently, we focus on obtaining proper mathematical forms to predict the demand parameters as functions of the competitive environment (i.e., χ) in the strict duopoly case, i.e., when both products have some demand and the two markets overlap.

Calculating Demand In a Duopoly

Our spatial model has an immediate implication for calculating demand for a product offering in the strict duopoly case. Specifically, it is defined as monopoly demand minus the number of buyers in the overlap area who purchase the competing product offering. The former comes directly from equation 3. The latter can be obtained by first solving for the indifference curve between the two product offerings, which, then, is used for calculating the area representing those who belong to the overlap region and purchase the competing product offering. Although this process is conceptually straightforward, it results in "messy" expressions that involve integrals and complex functions of P_1 and P_2 unless $P_1 = P_2$. Moreover, even though it is possible to solve these expressions for any specific set of parameters, the derived demand functions, since they involve integrals, are mathematically untractable for most subsequent game theoretic analyses that use standard optimization analyses. Not surprisingly, this problem worsens as the analysis expands to cases involving a larger number of product offerings. Consequently, we choose to take an alternative approach that relies on numerical demand evaluation.

Specifically, we wrote a (simple) computer program that searches over the potential market area (i.e., the two overlapping circles) in small steps and determines for each point (which represents one or more buyers), the product offering that would be selected based on maximizing this set of buyers' utility. By searching over the relevant market area, the program can count up the points (buyers) selecting each product offering, and in the process determine the demand for the product offering. In this way, we are able to easily determine the demand for any set of prices, underlying behavior, assumed distributions of buyer preferences, product and store positions, and channel structure.[3, 4]

We took this alternative approach and generated a large number of duopoly demand values over a broad range of parameters and price levels. We arbitrarily set $V = 100$, varied χ from .05 to .90 on 18 evenly spaced values and had γ take on the values of 0.5, 1 and 1.5. This 18x3 design captures a wide variety of different market conditions. For each of these 54 market conditions, we calculated the demand for the two products for a wide variety of prices. Specifically we use a 10x10 within market situation design where P_1 varies evenly from 0 to .9 V and P_2 varies evenly over the range where P_2 is small enough to insure that offering 1 does not dominate offering 2 but not so large that the two circles don't overlap. This approach produces strict duopoly demand quantities for up to 100 different pricing pairs for each of the 54 situations.[5] We then applied this simulated demand data set to linear regression to test if the

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[3] We acknowledge that such an approach is not elegant, nor does it provide closed form mathematical expressions. However, it is similar in spirit to solving complex functions using numerical integration (e.g., Gibbs sampling, etc.). It also allows us to easily relax our uniform distribution assumption and the assumption of one-dimensional product attribute space.

[4] We have subsequently written a more sophisticated program that greatly reduces the time necessary to generate the demand for any value of χ and values of P_1 and P_2. This was done by taking into consideration the geometry of the situation and thus limiting the search to the area where customers consider both offerings but purchase the other offering (_____________2000). However, both approaches yield identical results.

[5] Note that when χ is small, larger values of P_1 yield a monopoly setting regardless of the value of P_2. In these situations, we had less than 100 pricing points.

apparently nonlinear duopoly demand function can be reasonably approximated by a linear demand function (since it has much superior mathematical tractability).

Linear Approximation

Our linear approximation of duopoly demand uses a two-stage process. First we estimated the linear function using OLS for each of the 54 market conditions (as defined by χ and γ) with Q_1 as the dependent variable and P_1 and P_2 as independent variables. Next we analyzed the parameter estimates (i.e., the constant and the two price coefficients) from these 54 OLS results to identify the functional relationships between these slope and intercept demand parameters and the underlying market condition parameters, V, γ and χ.

The first stage of running 54 OLS regressions produced surprisingly high degrees of goodness of fit. For example, when χ=.6 and γ=1, the OLS results yielded an adjusted R^2 of .9961.[6] More generally, the average R^2s for the 18 different situations analyzed when γ= 1 was .9971. For γ=.5 and γ=1.5, the average R^2's were .9807 and .9755 respectively. The lowest R^2 value observed over our analyses was .9542 for γ=1.5 and χ=.90. These high levels of goodness of fit are particularly welcome results considering that we intentionally excluded from the data the price ranges leading to either dominant monopoly or localized monopoly, which is known to have a strictly linear demand function.

These regression results lead us to the following proposition:

PROPOSITION 1 – When potential customers have a utility function as described by equation 1, where $.5 \leq \gamma \leq 1.5$, and customers are uniformly distributed with respect to location and tastes, the duopoly demand function is, for all intents and purposes, linear.

Next, (i.e., step 6 of Figure 1) we analyzed the relationship between demand parameters and the underlying market condition parameters. We do this by first specifying the duopoly demand function in a form that is comparable to the monopoly demand in equation 3 as well as those used in existing studies (e.g., McGuire and Staelin 1983; Raju et al 1985; Trivedi 1998). Specifically we use the following form:

[6] We replicated these results by allowing the utility function to differentially weigh the lack of fit disutility (see footnote 1). Variations in this weighing did not affect the fit of the linear function but did affect the magnitude of the intercept term by rescaling V. Since this rescaling can be accounted for by redefining V, we do not incorporate β_1 and β_2 into our discussion.

(5) $Q_1 = S^* (1 - b^*_1 P_1 + b^*_2 P_2)$, and

$Q_2 = S^* (1 + b^*_2 P_1 - b^*_1 P_2)$,

where the * superscript denotes the duopoly case. Thus, S^* is the market potential for either product offering when $P_1 = P_2 = 0$ and $S^* b^*_1$ and $S^* b^*_2$ measures own- and cross-price sensitivities, respectively.

We obtained estimates of S^*, b_1^* and b_2^* directly from the OLS analyses, i.e., the intercept and the coefficients on P_1 and P_2 (after dividing these slope coefficients by the intercept). Then we looked for functional forms that predict S^*, b_1^* and b_2^* respectively in terms of V, χ and γ.[7] After several rounds of trials, tests and refinements with various functional forms, we selected those presented in Table 1. As shown in the table, each of the three demand parameters is a function of V and χ. (For editorial parsimony only, we present the cases where γ=1, although there is nothing unique about this choice.[8]) The predictive power of this linear approximation was very good, producing a correlation of .9976 between the true demand as determined from our spatial analysis and the predicted demand using the linear approximations given in Table 1. Moreover, an OLS regression of the true demand on the predictive demand resulted in a slope coefficient of 1.02, indicating these functional forms adequately capture how each parameter varies over the different market conditions analyzed. Therefore, we believe our general model is a good approximation to the true duopoly demand.

[Table 1 Here]

These analyses leads to our second proposition:

PROPOSITION 2 – There exists a general linear demand function for duopoly situations that adequately represents the demand of uniformly distributed buyers with a utility function given by equation 1. The parameters of this linear demand function assuming $\gamma = 1$ are given in

[7] It is not necessary to do this second stage analysis especially if one is primarily interested in using the derived demand functions as input to some game theoretic analysis. In such a case, one would just use the step 5 estimated demand functions and then look at the derived prices, profits, etc. as a function of the environmental factors. We discuss this more when we analyze Raju et al.

[8] We selected $\gamma = 1$ since it yields linear monopoly demand functions and also yielded the best fitting duopoly demand functions over the total range of overlap examined.

Table 1 and reflect not only the characteristics of the buyer's utility function, i.e., V and γ, but also the degree of overlap (competition) between the two product offerings. More specifically:

1) The market potential parameter, S* increases at an increasing rate with V and decreases at an increasing rate with the degree of overlap as measured by χ.
2) The cross price sensitivity parameter b_2^* increases with χ at an increasing rate.
3) Own price sensitivity parameter b_1^* is always larger than b_2^*, but the ratio b_2^* / b_1^* approaches 1 as χ approaches 1.
4) Own price sensitivity parameter b_1^* is an increasing function of χ, i.e., the market is more price sensitive when the two products are similar. However, this increased sensitivity does not necessarily come about because individual customers are attending more carefully to price. Instead the market becoming more sensitive because there are more marginalized customers, i.e., customers who are almost indifferent between the two offerings. Said differently, increases in own price sensitivity does not necessarily imply increased price sensitivity at the individual level. Instead, it can also imply that the competing product offerings are perceived to be more similar, i.e., δ_χ is smaller.
5) The intercept for industry demand decreases with χ. The industry price sensitivities, however, are only a function of V after adjusting for this changing intercept .

We find Proposition 2 and Table 1 to be very interesting in that it clearly specifies the conditions for the parameters of a linear demand function if one wants to hold fixed underlying buyer behavior while varying the marketplace (e.g., χ), For example, assume one wanted to investigate the effects of increased retail competition. This is often done via comparative statics, i.e., differentiating the solution as a function of χ holding fixed everything else. However, as pointed out above, everything else is not held fixed when χ changes. Thus, all three of the demand function parameters, S*, b_1^* and b_2^* will also change. One way of insuring that the assumed linear demand functions come from the same underlying buyer behavior model is to have the coefficients reflect the relationships as specified in Table 1 and outlined in Proposition 2. We point to numerous situations in the literature where these relationships were not maintained (e.g., Choi 1991; Lee & Staelin 1997). In such instances, it is not clear that the comparative static results are due to changing competition levels or changes in the underlying behavior of the buyers.

Although Propositions 1 and 2 imply that it is feasible to use our general linear demand function to capture duopoly demand, there is the still the issue of how much difference our approximate demand function is from the true demand function in terms of equilibrium prices, profits, etc. To provide some insights into this, we use both our Table 1 linear demand functions and the true demand function to solve the McGuire/Staelin model D1 (i.e., the manufacturer is Stackelberg leader). In the former case, we use closed form solutions based on a linear demand function with coefficients given by Table 1, while in the latter case we wrote computer code to conduct a numerical search where, using the true demand, we first determine the optimal retail prices conditional on a given set of wholesale prices, and then search for the optimal set of wholesale prices based on the previously determined retailer responses. We establish two general principles from this numerical search. First, although the retailer's reaction functions are not monotonically increasing over the total range of potential prices, there still exists only one equilibrium point for any given pair of wholesale prices. Second, as shown in Table 2, we found the equilibrium prices and quantities determined via the numerical search to be very similar to the equilibrium solutions derived using standard game theory analysis assuming the demand functions are linear and have coefficients given in Table 1.

This leads us to our third proposition.

PROPOSITION 3 – The equilibrium conditions obtained from our general linear demand equations are close approximations to those obtained from the true underlying demand function.

Finally, since we can compare solutions across different levels of competition (as measured by χ) it is possible to see how this factor influences prices and quantities. Four interesting observations can be made from Table 2. First, small values of χ (χ<.50) lead to prices that result in no customers considering both products, i.e., there is no overlap of the two circles. Second, moderate levels of competition (i.e., values of χ between .50 and .70) actually lead to retail prices above the monopolist price. Third, note that very high degrees of competition (e.g., $\chi \approx .95$ or higher) result in larger sales volume than the monopoly situation even though the market coverage is substantially less. This increased volume is directly associated with the almost 1/3 drop in retail prices. Fourth, these comparative static results are almost identical whether one uses the linear demand functions or the true demand.

Multiple Outlets and Multiple Product Offerings

We next extend our analysis to the situation where there are two competing retailers, each selling two substitutable products, i.e., there are four product offerings. We graphically denote this market with four overlapping circles as shown in Figure 6. Define the degree of overlap of any two product offering circles in a manner similar to our duopoly case. Let δ_θ be the distance between two competing offerings sold by the same retailer, and δ_χ be (as before) the distance between the two retailers. Then define the overlap between the competing offerings sold by the same retailer as follows:

(6) $\theta = (1 - \frac{\delta_\theta}{2V})$, $\quad 0 \leq \delta_\theta \leq 2V$

$= 0$ otherwise.

Define $\overline{\chi\theta}$, the degree of overlap between the competing products sold at competing retailer, in an analogous fashion. However, note that the distance between these two diagonal offerings, i.e., $\delta_{\overline{\chi\theta}}$, is determined once one specifies δ_χ and δ_θ. Thus, $\delta^2_{\overline{\chi\theta}} = \delta^2_\chi + \delta^2_\theta$. From equations 4 and 6, and the spatial relationship between $\delta^2_{\overline{\chi\theta}}$ and the other two distance constructs, i.e., δ^2_χ and δ^2_θ, it is easy to show that there is no overlap between the competing offerings sold by the competing retailers as long as $(1-\chi)^2 + (1-\theta)^2 \geq 1$. Moreover, the overlap between these two diagonal circles is uniquely determined once one knows χ and θ, and this overlap equals

(7) $\overline{\chi\theta} = 1 - [(1-\chi)^2 + (1-\theta)^2\ ]^{1/2}$ $\quad 0 \leq (1-\chi)^2 + (1-\theta)^2 < 1$

$= 0$ otherwise.

Consequently, $\overline{\chi\theta} < \chi$ and $\overline{\chi\theta} < \theta$ for all values of χ and θ except when $\chi = \theta = \overline{\chi\theta} = 1$.

[Insert FIGURE 6 Here]

These definitions and our previous development lead to the following observations:

1) When $\chi = 0$ and $\theta = 0$, all four offerings are completely separate. Consequently, we have the previously discussed monopoly situation.

2) When $\chi = 0$, and $\theta > 0$ or $\theta = 0$ and $\chi > 0$, we have the situation of two sets of overlapping circles, i.e., the duopoly case. From previous discussions we can immediately specify the two different sets of linear demand functions that capture this situation, where we measure

overlap in terms of the non-zero parameter, θ or χ.

3) When $\chi = 1$ and $\theta = 1$, the four circles completely overlap. Following our previous logic, demand for each offering at zero prices is $\overline{S}/4$ and decreases at the rate of $\overline{S}/4V$ assuming all prices move together.

4) Market demand for any product offering is zero when its price reaches V. This holds for all values of χ and θ.

5) When $(1-\chi)^2 + (1-\theta)^2 \leq 1$, all four zero price circles overlap. Now market demand will be influenced by χ, θ and $\chi\theta$, the three measures of overlap. Larger values imply larger degrees of overlap and, thus, lower market potential at zero prices and larger cross-price effects. However, the total number of customers who will switch from one product offering to another when one of the offerings changes price cannot be greater in the four product offering case than in a duopoly. This comes directly from our spatial model and the fact that now some customers in a given overlap area consider more than just the two brands. For example, some customers who would have switched from, say, offering 1 to 2 when 1 raises its price, in a two offering case, will stay with a third available offering because it still provides them with the highest utility, regardless of the change in price of offering 1.

Using these insights and the first six steps of our general methodology as outlined for the duopoly case, we again generated demand for each product for given sets of parameter values. To do this, we (arbitrarily) set $\gamma = 1$. We then systematically varied θ and χ over the range of .05 to .95 and prices P_1, P_2, P_3 and P_4 over the range that insured that each circle overlapped with the other circles but was not dominated. Our design used 7 equally spaced values for χ and θ and 5 equally spaced values for prices, resulting in $7^2 \cdot 5^4$ different demand calculations. These individual demands were then used as the dependent variable where the independent variables were a constant and the four prices. As before we ran separate regressions for each market condition. Since we had 7^2 different settings for χ and θ, this yielded 49 different regressions (market situations). This allows us to estimate the following linear demand function for each market situation:

(8) $Q_{ij} = S\,(1 - b_1P_{ij} + b_2P_{i,\,3-j} + b_3P_{i-3,\,j} + b_4P_{3-i\;3-j})$, i=1, 2; j=1, 2.

In this case, S is the size of the market for each offering when prices are zero and $b_1 - b_4$ are proportional to the four price sensitivities.

As with the duopoly case, we found the linear demand model provided a very good fit to the actual data for any given set of χ and θ values. For example, when χ=.65 and θ=.65, the adjusted R^2 was .989. The average over all analyzed pairs of χ and θ was .976.

Given the extremely good fit of the linear model and our desire to specify a general linear demand function for the four product offering case, we next searched for appropriate representations for each of the linear demand parameters as a function of the characteristics of the market. We again instituted curve-fitting procedures to obtain good approximations for each of the 5 parameter values of interest. These approximations are given in Table 3. As can be seen from this table, each parameter is a function of χ and θ (along with V). Finally, we used these approximations and the linear demand function specified in equation 8 to approximate the 25,000 plus actual demands that we used previously to estimate the original OLS regressions. We then regressed these estimated demands against the actual demands suppressing the constant. This time the slope parameter was .997 and the correlation coefficient was .986. These findings lead us to our fourth proposition:

PROPOSITION 4 – When potential customers have a utility function as defined by equation 1 where $\gamma = 1$ and these customers are distributed uniformly with respect to location and tastes, the demand functions for the two products sold through two common but competing retailers is for all intents and purposes linear. Moreover, the parameters of this linear demand function follow the relationships as specified in Table 3. More specifically:

1) The intercept is a function of the degree of competition within the market place. The greater the competition, the smaller the intercept.

2) After controlling for the intercept, own price sensitivity (as measured by b_1) increases with competition. However, the overall effect, i.e. Sb_1 decreases with competition. This own price sensitivity effect occurs even though the buyers' underlying behavior does not change.

3) The cross price effects increase with competition in very predictable ways. Interesting, the within store competition price sensitively parameter, b_2, is not only a function of the degree of competition between the two products sold in the store, i.e., θ, but also the degree of competition between outlets, i.e., χ.

4) The relationship between b_4 and the other two cross-price parameters is such that b_4 is less than or equal to b_2 and b_3. In general, this parameter is much smaller than either of the other two cross-price parameters and near zero when either χ or θ are near zero.

5) Total industry demand is most price sensitive (i.e., has the greatest slope) when there is no competition. However once one controls for the size of the total potential market, industry demand is not a function of the degree of competition within the market, i.e., χ

and θ.

As with Proposition 2, these results have strong implications in terms of the appropriate demand functions for a multi-product/multi-outlet marketplace, both in terms of specifying the appropriate 4 product offering demand functions and specifying how the individual coefficients vary with changes in χ and θ. We next use these propositions to reanalyze a set of channel structures first analyzed by Trivedi (1998). This allows us to detail the last two steps of our overall methodology, i.e., calculating equilibrium prices, profits, etc. and verifying that these prices are within the range of our derived demand functions. It also allows us to demonstrate that starting with a buyer behavior model and deriving demand functions instead of starting with demand functions that one intuits can lead to very different conclusions. Since we know all our demand functions are derived from one set of customers exhibiting one set of behaviors, we can attribute any differences in our results relative to those of Trivedi to the fact that her work confounds market differences with underlying buyer behavior differences.

Trivedi Analysis

Trivedi (1998) analyzes the benefits and costs of having an exclusive versus a broad distribution system. She does this by comparing the situation where two manufacturers distribute their competing products through a franchised system (she refers to this situation as a decentralized system) to a mass merchandising system where (two) competing retailers carry both products (she refers to this channel structure as a full system). Trivedi starts her analysis by assuming two different sets of linear demand functions, one for her decentralized situation and the other for her full channel system. Her specific forms are shown in Table 4.[9]

Three things should be noted about her four product offering demand equations shown in Table 4. First, the intercept term is a constant, i.e., is not a function of χ and θ. (This is also true for her duopoly functions.) Thus, she implicitly assumes that the market potential is not a function of the market coverage, i.e., the degree to which the outlets and/or products are differentiated. Second, b_2 (b_3) is less than b_4 when θ (χ) is greater than .5. This condition is not

[9] These forms are more general than those given in her published paper. However, she refers to the Table 4 formulation in her published paper saying that her assumed demand functions flow from the general demand functions given in Table 4. We use the general formulation since it better reflects how her assumed demand parameters vary by market conditions.

feasible under our assumption of buyer behavior. Third, her demand functions imply an industry demand function that has the own price coefficient, after controlling for the intercept, being a decreasing function of χ and θ (as opposed to our formulation where it is a constant). Consequently in Trivedi's analysis, a market environment representing a high degree of competition (i.e., high values of χ and θ) implicitly assumes an industry with less sensitivity to price than our analyses. We use these observed differences when we compare our results with her published results.

Results

We use our derived linear demand functions to calculate equilibrium profits and prices for each channel structure as a function of χ and θ. We do this as follows. First, using equation 5 and the Table 1 results, we solved for equilibrium conditions for the situation where each retailer only carries one product offering using the three different game rules analyzed by Trivedi; each manufacturer sets retail prices to maximize its total channel profits (denoted I, for the Integrative solution), the manufacturer acts as price leader (denoted DMS for Decentralized Manufacturer Stackelberg leadership), and the retailer acts as price leader (DRS). We then repeated this analysis for the Full channel system case using equation 8 and Table 3 results, allowing the manufacturer or the retailer to be price leader (FMS and FRS respectively). We then verify that the relevant circles overlap at these equilibrium prices, i.e., that we are using the applicable demand functions. If, for example, we find the duopoly prices are such that the two circles do not overlap for a given value of χ (i.e., there are no customers who are willing to consider both offerings), then we re-solve the channel system using the monopoly demand function. This time we check to see if the equilibrium retail prices are such that the relevant circles don't overlap.[10] In this way, we are assured that we are using the applicable demand function.

Following these procedures, i.e., steps 7 and 8 of our proposed methodology and comparing the derived solutions, we are able to provide a number of managerially relevant insights concerning:

1) the value to the manufacturer of being able to coordinate the total channel and capture all

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[10] If the circles overlap at the monopoly prices, then we know that the optimal solution is the corner solution, i.e., the intersection of the duopoly and monopoly demand functions (See Figure 5).

its profits (by comparing the solutions of I with D and F),

2) the value of being a price leader (MS versus RS),

3) the cost to the manufacturer of allowing the consumer to directly compare the two products within the same store (and the retailer to do product line pricing) versus the benefit of getting a wider distribution by letting all the retailers carry their product (F versus D), and

4) the value to the retailer of being able to carry both products versus the cost of losing market exclusivity (F versus D).

We present the ranking of these 5 profits for manufacturers and retailers as a function of χ and θ in Figures 7a and 7b respectively. A number of interesting conclusions can be drawn from these figures. The first is technical and relates to the fact that small values of χ and θ along with the (high) equilibrium prices result in some or all of the markets becoming separate. Consequently, the appropriate demand functions for the full system are no longer those given in Table 3, but instead Table 1 or the monopoly demand equation 3. This partitioning of the applicable space is shown via the dotted lines in both figures. Second, we find manufacturers are indifferent between the full system with price leadership (i.e., FMS) and an integrated system (i.e., a coordinated but exclusive distribution) when χ and θ are both small, i.e., region A. This occurs because the benefit of broader market coverage of the full system compensates for the loss of the integrated system's advantages in avoiding double marginalization and extracting the entire channel profits. In contrast, without acknowledging the market coverage expansion effect of the full channel system, Trivedi's Figure 2a suggests the integrated system should be most preferred when χ and θ are both small.

[Insert FIGURE 7 Here]

Third, we also note that when retail competition is very intense (a large χ), the manufacturer prefers a full system to the coordinated franchised system. (See region F.) We conjecture this is due to the fact that the intense retail competition restricts the retailer's ability to garner any monopolistic rents. Consequently, there is little opportunity for double marginalization to occur and thus, no need for the manufacturer to coordinate the channel. Since the lack of differentiation between the two retail outlets in this situation minimizes the market coverage difference between the exclusive and the full systems, it is not surprising that this result is consistent with Trivedi's finding.

Fourth, setting aside regions A and F, we see that in general, the manufacturer prefers an integrated solution (i.e., the coordinated channel system) to any other solution (see regions B, C, and D). However, as was first reported by McGuire & Staelin (1983), we still find the manufacturer would prefer to buffer itself from direct competition when retail and product competitions both intensify greatly (i.e., region E where DMS yields the largest manufacturer profits). This tendency to give up coverage to suppress direct comparison of products is also seen in region D where manufacturer profits under DMS are greater than under FMS. This result is counter to Trivedi's finding (shown in Figure 2: a and b in her paper) that a high degree of product competition and store competition generally cause manufacturers to favor the full structure (her Hypothesis 1). In contrast, our finding indicates that product competition (θ) and store competition (χ) affect a manufacturer's optimal choice of channel structure in very different ways.

As was first shown by Lee & Staelin (1997), we find that since demand is linear, it is always best for the manufacturer to be a price leader given a particular channel structure, i.e., Full or Decentralized. However, price leadership is not always as important as channel structure since in regions C and F (D) the manufacturer finds it best to be a price follower assuming the manufacturer can use a full (decentralized) instead of a decentralized (full) system. Trivedi's Hypothesis 2 is less specific, but states the manufacturer is willing to give up price leadership to get full distribution only at 'average' levels of competition.

The profit conclusions for the retailer also show a preference for a full system regardless of price leadership when retail competition is low (measured by χ). The applicable range over χ (as shown by regions A and B) increases as the products become less differentiated. For intermediate values of χ, we note a reversal of the order of FMS and DRS (region C). Thus, a retailer would prefer carrying only one product but having price leadership to carrying two products but being a price follower.[11] Still, under these conditions, the retailer prefers a full system assuming price leadership. Only when the retailer has very little spatial monopoly does it

[11] Trivedi, on the other hand, states in her Hypothesis 3 that retailers in a competitive market prefer FMS to DRS.

prefer foregoing being able to carry both products to reduce the head-to-head price competition that results when both outlets carry both products (regions D and E).[12]

We also overlay Figure 7a and Figure 7b to determine when both channel members prefer the same system. For parsimony, we only discuss situations where the manufacturer is the price leader. When we do this, we find that the two channel partners both want a full system for most of regions A, B, C and D of Figure 7b. However, for most of region D in Figure 7a the manufacturer prefers a decentralized system but the retailer prefers a full system, while the reverse is true for most of region E in Figure 7b. In such situations, one might anticipate more channel conflict.

For space reasons, we do not report equilibrium prices and quantities for all values of χ and θ. However, as might be expected, we find that prices and profits decrease as χ and θ increase (this is counter to Trivedi's results, see Hypothesis 7). In summary, our results differ from Trivedi's in a number of instances. This is due, conceptually, to three differences in the demand functions used:

1) Her industry demand function is less price sensitive than ours. Consequently her derived retail prices tend to be higher than ours when χ and θ are near 1. Her assumption of decreased sensitivity reduces competition and thus favors the full system.

2) Her demand functions do not allow for a decreased market potential when χ and θ increased, i.e., she does not model product coverage. Thus, she finds profits to rise when χ and θ approached 1. We find just the opposite, i.e., profits approach zero as χ and θ increases, partly because of decreased coverage, partly because of less differentiation between offerings.

3) Her demand functions allows $Sb_4 > Sb_3$ or Sb_2. This favors the full system over the decentralized for the manufacturer when χ and θ were close to one. Our demand function did not allow this to happen. Consequently, we find the decentralized system best for the manufacturer when θ approaches 1.

Multiple Product, Same Retailer

We also apply our 8-step approach to the two situations modeled by Raju, et al. (1995). The first represents one retailer selling three brands, two national brands and one store brand. The second has this retailer just selling two national brands, i.e., our duopoly case. They

[12] Trivedi's results indicate that the retailer always prefers a full system and this preference goes up with retail

postulate two different degrees of substitutability. The first, which they designate as θ, captures the degree of overlap between the two national brands. The second, which they designate as δ_i, is the degree to which the store brand competes with the i^{th} national brand. They fix total industry demand to be 1 at zero prices and treat the two national brands symmetrically. This leads them to assume the intercepts for the national brands are ½ when the retailer carries only the two national brands. They capture store brand quality (relative to the national brands) with the parameter, α. Thus, when the retailer also carries a store brand they assume the intercept is $1/(2+\alpha)$ for the national brands and $\alpha/(2+\alpha)$ for the store brand, $0 \leq \alpha \leq 1$, so that the store brand's market potential never can exceed that of either (symmetric) national brand. However, such an assumption also implies that the introduction of store brand does not increase market coverage.

We start our analysis by following Raju et al.'s lead and assume that the store brand competes equally with the two national brands. Consequently, under our framework, it must be located halfway between the two national brands. In order to capture quality differences between the national brands and store brand, we assume the store brand's zero price radius is a proportion of the national brands' zero price radius. We denote this proposition as ϕ, where $0 \leq \phi \leq 1$. In effect, we assume vertical (quality) differentiation as well as horizontal (θ) differentiation.

Using this spatial representation, we can make the following observations:

1) The degree of overlap between the national brands and the store brand is directly determined once θ, V and ϕ are specified. Since the degree of overlap determines cross-price sensitivities, it is not possible, assuming our buyer behavior model, to have two free price sensitivity parameters, one for the price sensitivity between the national brands (denoted θ by Raju, et al.) and the second for the price sensitivity between the national brands and the store brand (i.e., δ_i).

2) The market potential for the two (three) brand case depends on the breadth of market coverage (i.e., varies with θ). Thus, small values of θ imply more product differentiation and thus broader coverage. In contrast, breadth of coverage is assumed to be constant in Raju, et al. This implies industry demand does not decrease with increases in θ in their analysis and consequently their industry demand will be more price sensitive for high

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competition. (See her Figure 2.) This conclusion is not in accord with our results.

values of θ than would be expected under our assumed buyer behavior model.

Results

Using our general methodology we generated demand for the three brand situation using a wide variety of values for the two parameter values of interest, i.e., θ and ϕ. As before we used prices (in this case two prices for the national brand and one for the store brand) that varied over the complete range of feasible prices (i.e., prices that lead to overlapping circles) when we generated these demands. We then used the derived demand values as our dependent variable and the prices as the independent variables to estimate linear demands of the following form:

8) $q_{Ni} = S(1 - b_1 p_{Ni} + b_2 p_{N,3-i} + b_3 p_s)$ i = 1, 2

$q_s = \phi S(1 - b_4 p_s + b_5 p_{N1} + b_6 p_{N2})$,

where p_{Ni} is the price of the i^{th} national brand and p_s is the store brand price.

As in our previous result, we found the R^2 to be extremely high for all the estimated linear demand functions. (The average value for the national brand equations was over .99, the lowest value being .9649. For the store brand, the average exceeded .97 and the smallest R^2 was .9336.) We take these results to imply that specific (linear) approximations do an excellent job of capturing our assumed underlying buyer behavior for this store brand situation.

Next, we used the individual OLS estimates to solve for equilibrium prices, quantities and profits for each of the θ, ϕ situations previously analyzed using the same decision rules as outlined by Raju, et al., i.e., the manufacturer is Stackelberg leader and the retailer uses product line pricing. We then verified that at the equilibrium prices the two national brands competed. Next we compared our equilibrium results for the two different settings (i.e., 2 national brands versus 2 national brands and 1 store brand). Finally, we compared these results with those reported in Raju, et al. Below is a summary of some of the most interesting results.

1) In accordance with Raju, et al.'s Proposition 1, other things being equal, retailer profits are proportionally higher when they have a store brand versus not having a store brand when the two national brands are differentiated i.e., θ is low versus θ being high. This is due in part to greater coverage and in part to higher margins that result from carrying a store brand.

2) In accordance with Raju, et al.s' Proposition 3, other things being equal, retailer

profits increase when the quality level (as measured by ϕ) of its store brand is higher.

3) Contrary to Raju, et al's Proposition 5, which states that the store brand's market share is smaller for large θ, we find equilibrium store brand share is larger when the cross-price sensitivity among national brands (θ) is higher.

4) As would be expected, retail prices for the national brands decrease with θ. However, holding fixed θ there is almost no effect on these prices when the retailer introduces the store brand. Thus, consumers do not see the impact of the store brand in terms of lower retail prices for the national brands. That said, there is a major effect on the manufacturers' wholesale prices when the retailer introduces a store brand. Unless the store brand is of very poor quality, i.e., ϕ is near 0, the manufacturer lowers its wholesale price when the retailer introduces the store brand. This effect is most pronounced when θ is large. Thus, the manufacturer's profits are most hurt (proportionately) when the retailer introduces a store brand in an undifferentiated national brand market. (Also see point 1.)

5) Contrary to Raju, et al.'s Proposition 2, it is not possible to hold other things equal when one varies the cross-price sensitivity between the store brand and the national brands. Thus, either the quality level of the store brand has to increase or the differientation between the national brands has to decrease if the cross-price sensitivity is to increase. From point 1, 2 and 4, we see that these two underlying factors represent counterbalancing forces for the retailer. Thus, one needs to know if the cross-price sensitivity changes are due to changes in ϕ or changes in θ or both.

6) As would be expected, the store brand always sells for a discount even if its quality is equal to that of the national brands. This is due, in part, to the double marginalization associated with selling the national brands.

Discussion and Conclusion

The overarching result of this research is that it is possible to derive "linear" demand functions for multiple outlet, multiple product situations by starting with a comprehensive buyer behavior model. We use quotes around the word linear, since the obtained linear functions do not perfectly capture the actual demand functions. However, when we regress the actual demand against linear and additive prices, we normally observe R^2's in the range of .98 to .999. In addition when we calculated equilibrium prices using these linear functions we find that they are very similar to those obtained using the true demand function and they give the same comparative static results. We take this as evidence that these particular linear demand functions adequately capture our underlying buyer behavior.

A number of observations directly follow from our analyses. First, we are able to parsamoniously capture the demand implications of our buyer behavior model and the associated competitive environment with a simple spatial model of overlapping circles. Not only does this spatial model quickly convey the degree of competition for any particular market setting, it is also useful in determining the boundary conditions for prices and the size of the potential market. Second, we are able to specify the intercept and slope parameters of the linear demand function as a function of the competitive environment. For example, the intercept is a function of the buyer's reservation price V, and two characteristics of the market, i.e., the degree of overlap between products, θ, and retail outlets χ. More specifically, we find the intercept, which represents the zero-price market demand, increases in V and decreases in θ and χ. Similar relationships were found for each of the slope parameters. (See for example Tables 1 and 3.) Perhaps, more importantly these results point out that one is not free to choose any combination of intercept and slope parameters. Instead, there is a specific relationship between slope and intercept parameters that is directly related to the environment and channel structure. Thus, care must be taken when one does comparative static analyses, since varying one parameter, e.g., θ may require the variation of all of the parameters in the linear demand function. To date, most analytic analyses have not done this.

Third, we note that increased price sensitivity as measured by the demand function does not necessarily imply that consumers are paying more attention to price. Instead, it could just mean that that the products are less differentiated (i.e., there is more product overlap) and thus more consumers' choices are affected by small price changes. Therefore, one cannot infer individual behavior from longitudinal (or cross-sectional) analyses of aggregate demand level data without also knowing if the perceived distance between the product offerings has moved.

Fourth, changes in industry demand as a result of changes in price are only a function of V once one adjusts for market potential. Said differently, this intercept-adjusted industry price sensitivity does not increase (decrease) as a function of the competitive environment. This observation is often violated in channel analyses that do comparative statics on the degree of competition.

We also apply these buyer-behavior based demand functions to two previously analyzed problem setting, i.e., whether or not manufacturers (retailers) are better off when the retailers carry a complete product category (versus only the offering of one manufacturer), and whether or not manufacturers (retailer) are (is) better off when the mass merchandiser retailer also carries a store brand. In both instances, we found differences in our results compared to the previously published results. We believe these differences are due to the fact that prior analyses did not assure that their assumed demand functions are linked to an underlying buyer behavior model. These reversals in finding have a number of clear implications. For example, we show that for low competitive situations a manufacturer are indifferent to broad coverage (i.e., distributing through multiple mass merchandise retailers) and an integrated (coordinated) channel system. Also, we see that the effects of store competition (as measured by χ) are very different than the effects of product competition (as measured by θ) on prices and profits. In our analysis of the CPG industry, we show that the introduction of a store brand does not significantly lower retail prices of the national brand. Instead, it mainly affects the wholesale prices of these brands and, thus, transfers profits from the manufacturer to the retailer.

Finally, we point to the fact that our methodology opens up the possibility of exploring a number of channel structures and different buyer behavior segments here-to-fore not analyzed because of the difficulty of developing consistent demand systems and/or solving for equilibrium conditions for these demand systems. Specifically, our methodology allows the researcher to postulate rich buyer behavior and to capture heterogeneity and still specify linear demands. Thus, analytic modelers are no longer restricted to specifying only a few buyer segments (e.g., high and low) and/or non-representative markets. It is our hope that this paper will provide the impetus for analysis of such complex channel situations.

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Table 1

Estimation Equations For The Three

Parameters of our Generalized

Duopoly Linear Demand Functions

$$\hat{S}^{*} = \frac{\Pi V^{2}}{} \frac{1}{1+\chi^{2}}$$

$$\hat{b}_{1}^{*} = \frac{1}{V}(.90 + V\hat{b}_{2}^{*})$$

$$\hat{b}_{2}^{*} = \frac{1}{2.5V}\left(\frac{\chi}{1-\chi^{2}+\chi(1-\chi)}\right)$$

TABLE 2

Comparison of Equilibrium Wholesale Prices, Retail Prices and Quantities
Determined from True Demand and Linear Demand
For Different Values of χ

	True Demand			Linear Demand		
χ	W	P	Q	W	P	Q
0	50	75	7854	50	75	7854
.10	50	75	7854	50	75	7854
.20	50	75	7854	50	75	7854
.30	50	75	7854	50	75	7854
.40	50	75	7854	50	75	7854
.50	50	75	7854	50	75	7854
.55	51.9	76.5	7298	52.5	78.4	7089
.60	51.0	76.2	7143	52.0	77.7	6946
.65	50.5	75.6	6974	51.5	76.8	6825
.70	50.1	74.7	6802	50.8	75.6	6737
.75	48.2	73.1	6714	49.9	74.1	6702
.80	47.3	71.3	6600	48.6	71.9	6753
.85	43.8	67.4	6814	46.8	68.7	6957
.90	41.2	62.5	7109	43.5	63.2	7473
.95	36.6	52.6	8158	36.3	51.5	8850

TABLE 3

Estimating Equations for the

Demand Parameters For The

Two Outlets, Two Product Situation

$$\hat{S} = \frac{\pi V^2}{}\left(\frac{1}{1+\chi^2+\theta^2+(\chi\theta)^3}\right)$$

$$\hat{b}_1 = \frac{.8}{V} + \hat{b}_2 + \hat{b}_3 + \hat{b}_4$$

$$\hat{b}_2 = \frac{\chi}{(1-\chi^2)} \cdot \frac{(3-\theta^2)}{9V}$$

$$\hat{b}_3 = \frac{\theta}{(1-\theta^2)} \cdot \frac{3-\chi^2}{9V}$$

$$\hat{b}_4 = \frac{2\chi.\theta}{3(2-\chi^4)(2-\theta^4)V}$$

TABLE 4

Trivedi's Demand Functions

Duopoly Case (See Equation 5)

$S^* = 2S$

$$b_1^* = \frac{\beta}{(1-\theta)}$$

$$b_2^* = \frac{\beta\theta}{(1-\theta)}$$

4 Product Offering Case (See Equation 8)

$S = S$

$$b_1 = \frac{\beta}{(1-\theta)(1-\chi)}$$

$$b_2 = \frac{\beta\theta(1-\chi)}{(1-\theta)(1-\chi)}$$

$$b_3 = \frac{\beta\chi(1+\theta)}{(1-\chi)(1+\theta)}$$

$$b_4 = \frac{\beta\chi\theta}{(1-\chi)(1-\theta)}$$

Figure 1
Model Development and Analysis Process

Phase I: Development of a Spatial Model of Market

Step 1: Specify an individual buyer's utility function and decision rule.

Step 2: Specify the distribution of buyers in the product/location space.

Step 3: Specify the positions of product offerings in the spatial model.

Phase II: Modeling the Demand Structure

Step 4: Generate simulated demand data for each product offering at various price levels.

Step 5: Estimate demand functions using the simulated data.

Step 6: Analyze the relationships between market characteristics and demand parameters.

Phase III: Strategic Application and Validation

Step 7: Investigate channel strategy issues using the estimated demand functions.

Step 8: Verify the analytical results using the spatial model.

Figure 3
Monopoly Market

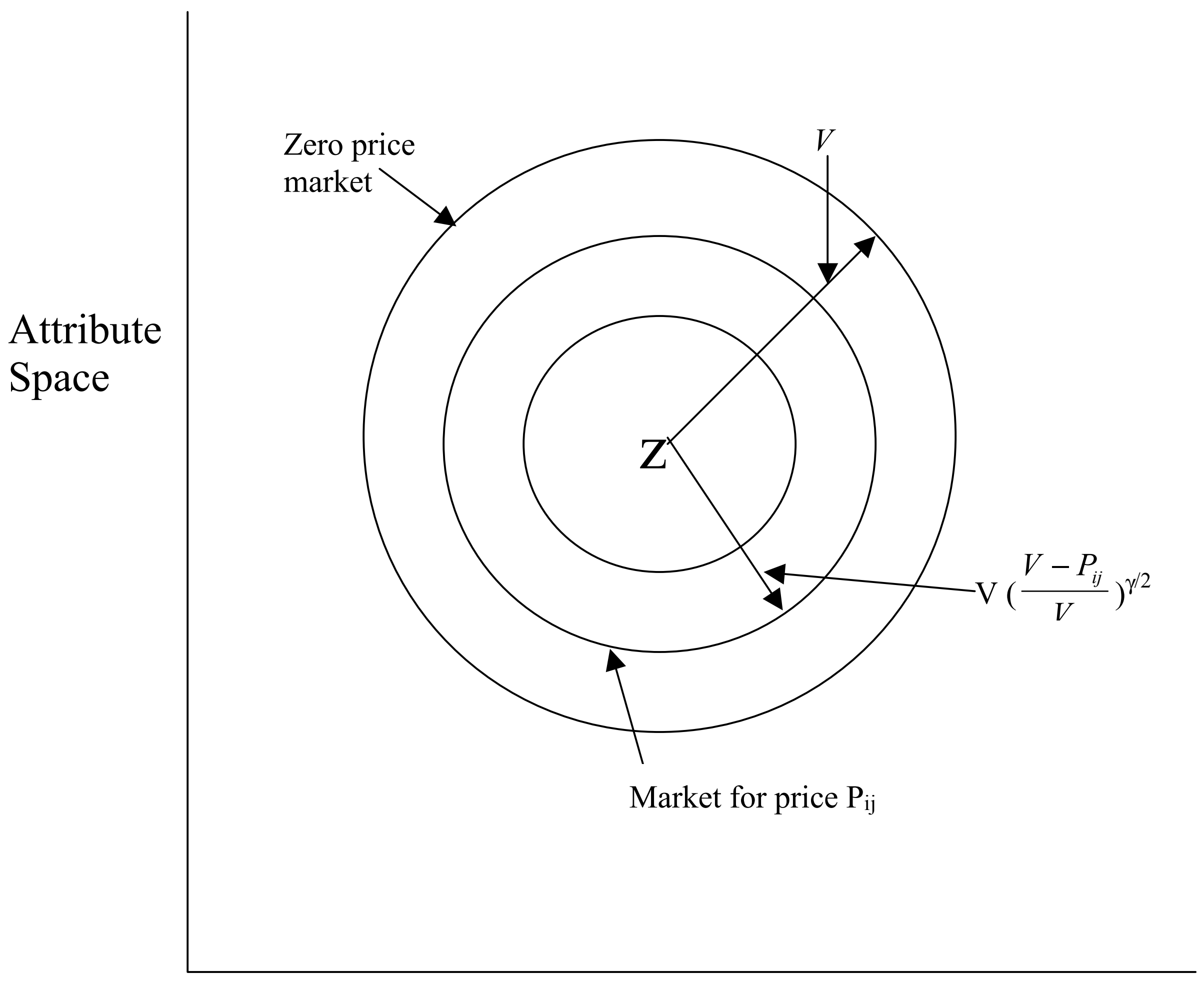


Figure 4

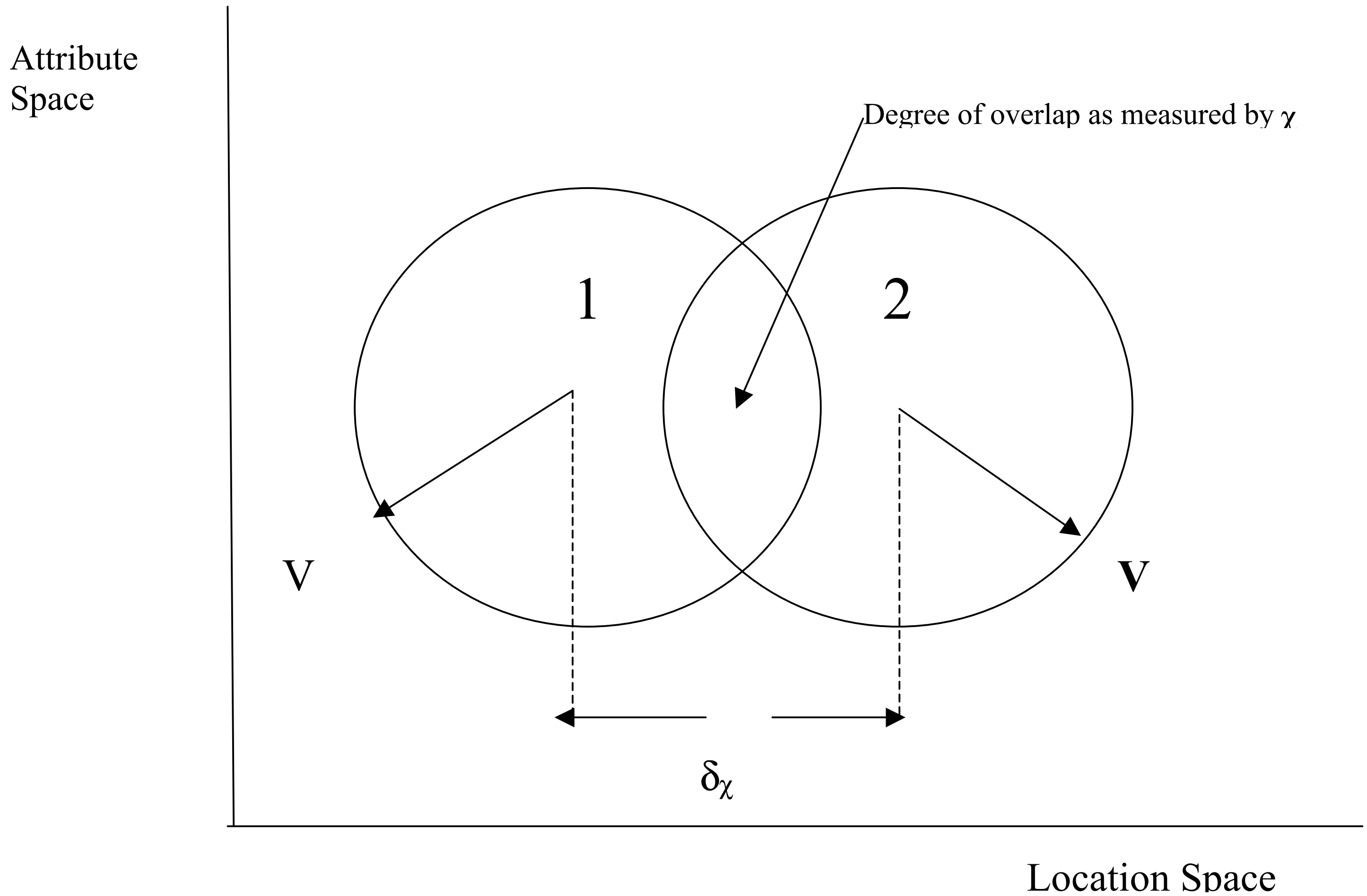
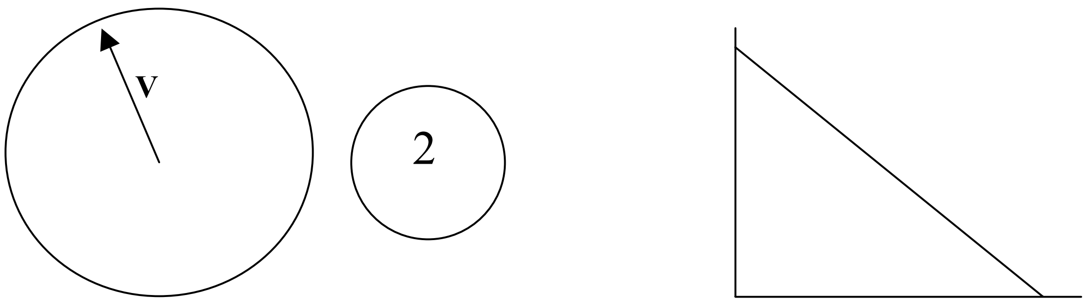
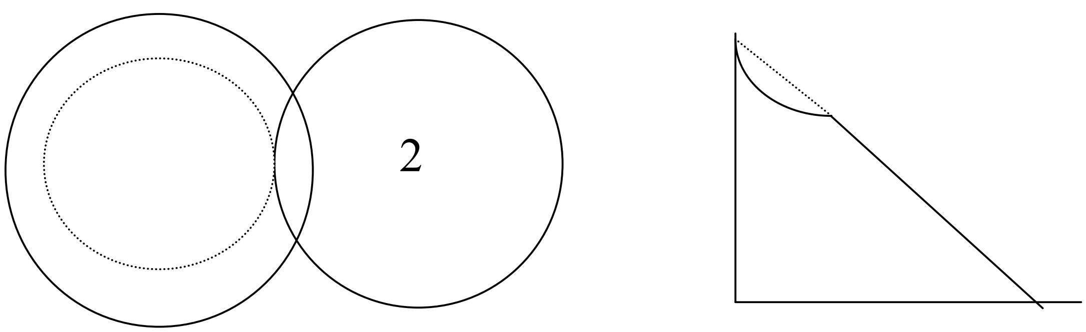

Attribute
Space
Degree of overlap as measured by γ
1
2
V
V
δχ
Location Space

FIGURE 5
Various Cases of Competition in a Duopoly

A. Low χ and High P_2: Localized Monopoly

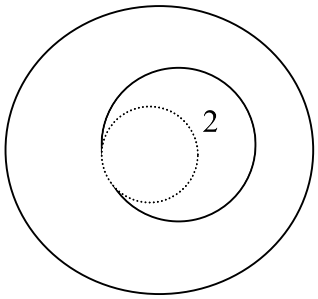


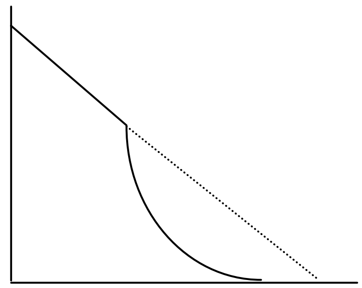
B. Low χ and Low P_2: Strict Duopoly and Localized Monopoly



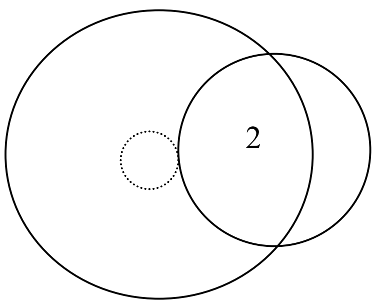
C. High χ and Very High P_2: Dominant Monopoly, Strict Duopoly and Localized Monopoly

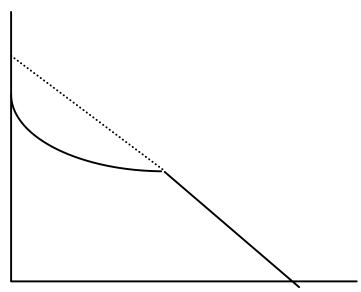
D. High χ and High P_2: Dominant Monopoly, Strict Duopoly and Dominated



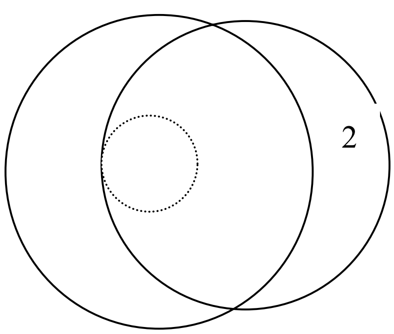


E. High χ and Medium P_2: Strict Duopoly and Localized Monopoly





F. High χ and Low P_2: Strict Duopoly and Dominated



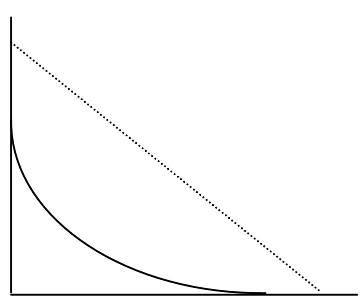


Figure 6
Multiple Outlet, Multiple Product Situation

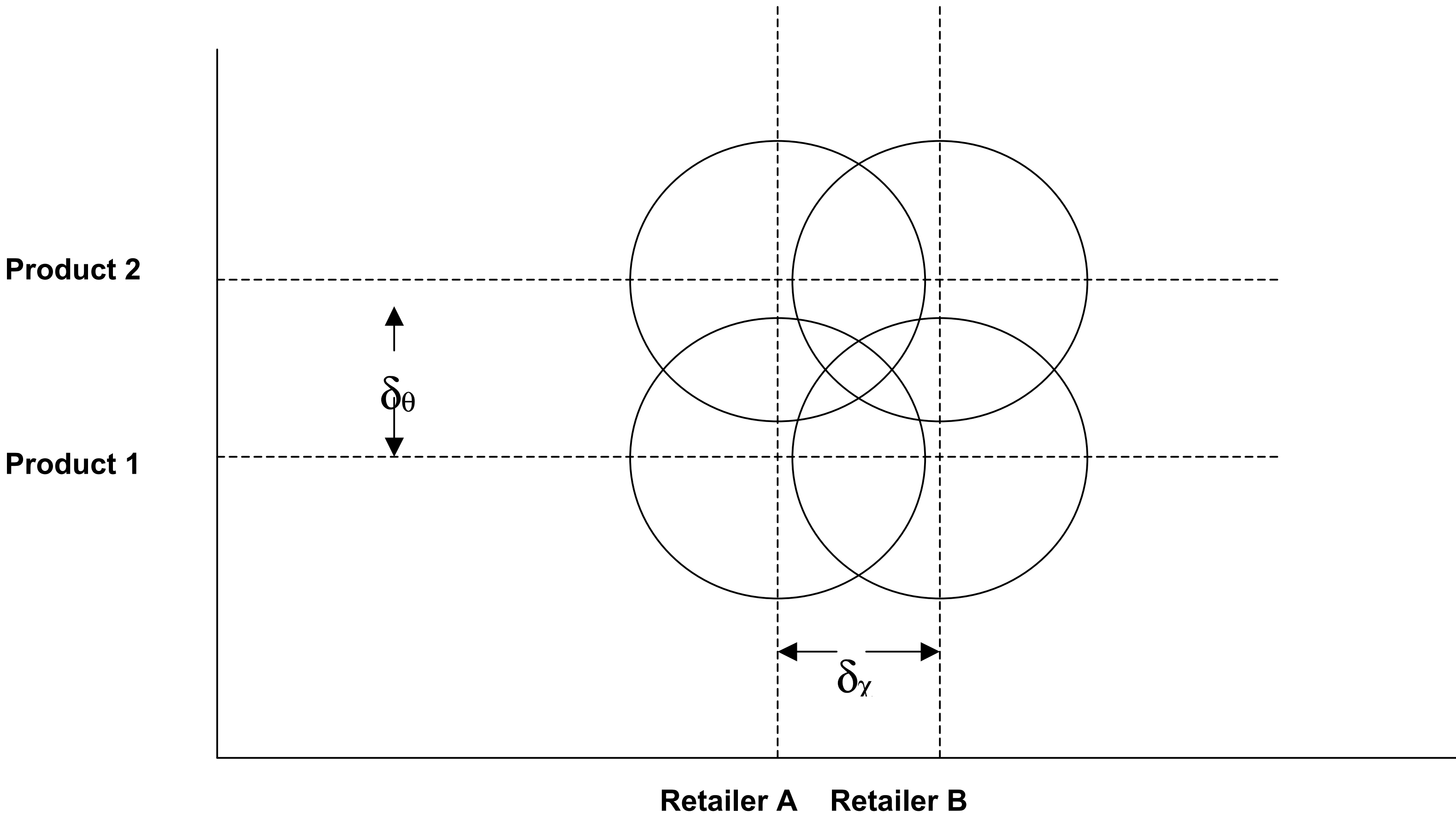
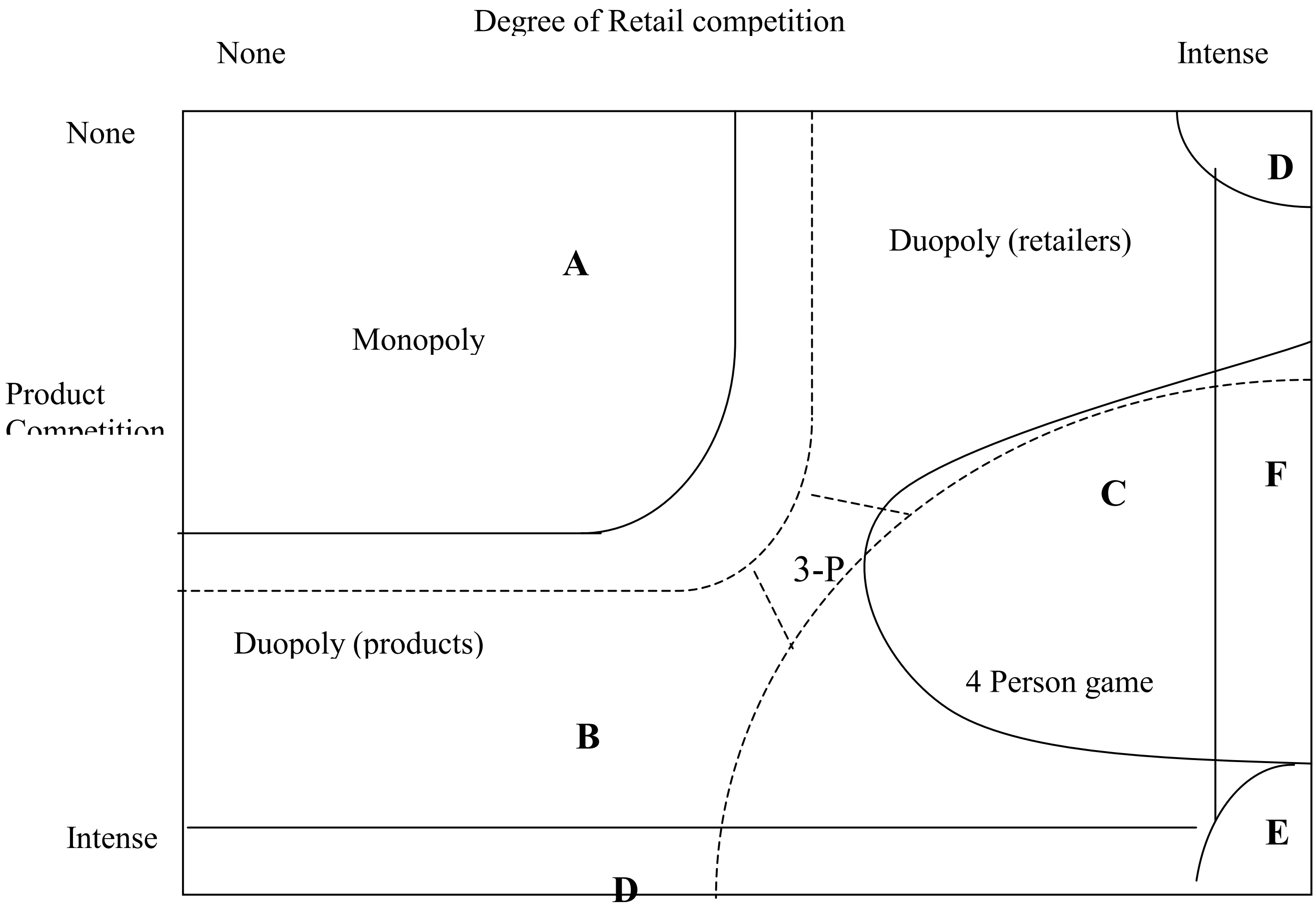


Figure 7a
Manufacturer's Profits as Function of Product (θ) and Retail (χ) Competition

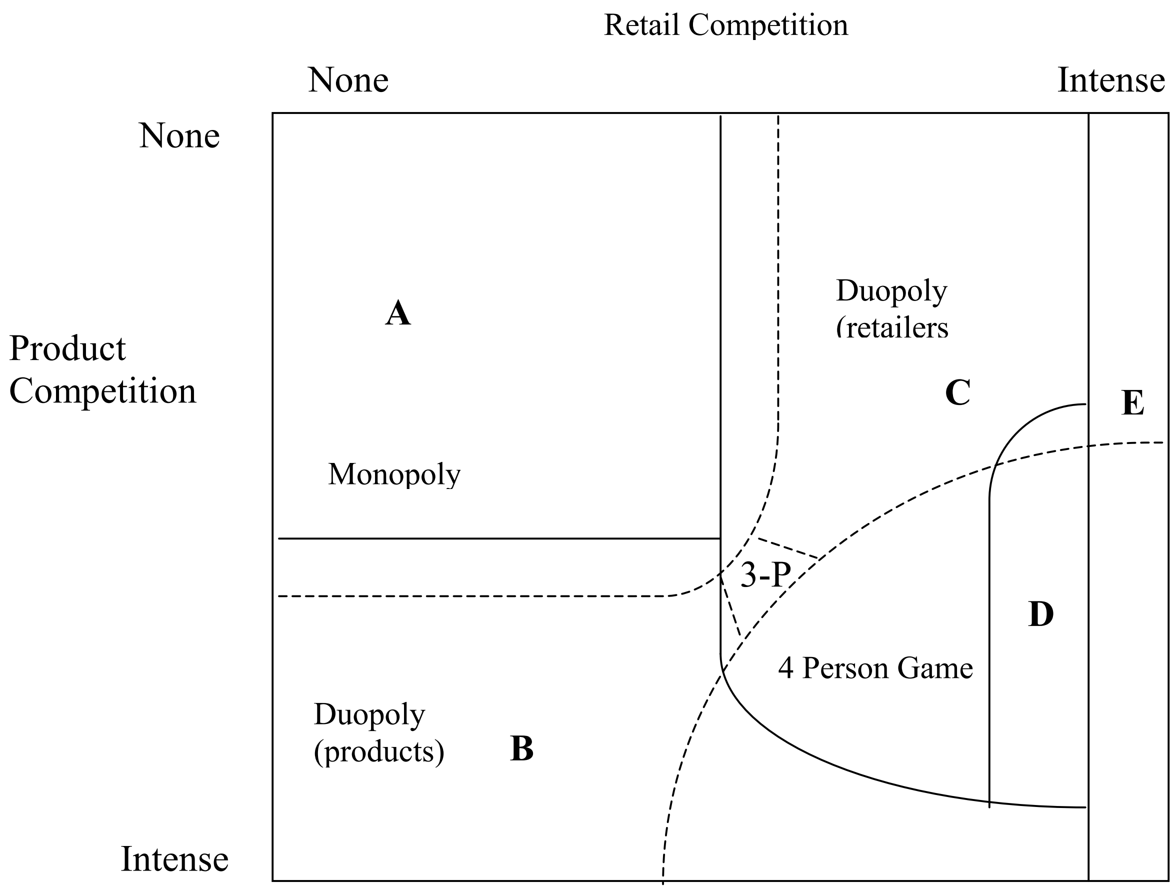


Legend:

A I=FMS>FRS=DMS>DRS
B I>FMS>DMS>FRS>DRS
C I>FMS>FRS>DMS>DRS
D I>DMS>DRS>FMS>FRS
E DMS>FMS>I>DRS>FRS
F FMS>FRS>I>DMS>DRS

3-P 3 Person game

Figure 7b
Retailer Profits as Function of Product (θ) and Retail (χ) Competition



Legend:

A	FRS>FMS=DRS>DMS
B	FRS>FMS>DRS>DMS
C	FRS>DRS>FMS>DMS
D	DRS>FRS>FMS>DMS
E	DRS>DMS>FRS>FMS
3-P	3 person game

Figure 2

Spatial Representation of an Offering
And Two Potential Customers