Appendix B.

Part 1. Relaxing the assumption $p_{i,j}(\theta_i, \theta_j) > 0$ for any $\theta_i \in \Theta_i, \theta_j \in \Theta_j$, i and j.

If this assumption does not hold, we proceed as follows. First, introduce the following definition.

Definition 6 *$T \equiv \{T^1, ..., T^q\}$ is the finest partition of the set $\{\theta \in \Theta | p(\theta) > 0\}$, i.e. the set of type profiles occurring with a positive probability, which satisfies the following property **F**: For all i and $\theta_i \in \Theta_i$, there exists $l \in \{1, ..., q\}$ s.t. $\{(\theta_i, \theta_{-i}) | \theta_{-i} \in \Theta_{-i}, p(\theta_i, \theta_{-i}) > 0\} \subset T^l$.*

T is well-defined because the trivial partition $\tilde{T}$ s.t. $q = 1$ and $\tilde{T}^1 = \{\theta \in \Theta | p(\theta) > 0\}$ satisfies property **F**, and a meet of two partitions T' and T'' possessing property **F** is also a partition satisfying this property. In fact, if for any pair of types θ_i and θ_j of any two agents i and j, there exists $\theta_{-i-j} \in \Theta_{-i-j}$ s.t. $p(\theta_{-i-j}, \theta_i, \theta_j) > 0$, then T is trivial, i.e. $q = 1$.

Also, note that since $p_i(\theta_i) > 0$ for all i and $\theta_i \in \Theta_i$, the projection of partition T on type space Θ_i is a partition of Θ_i which we denote by $\{T_i^1, ..., T_i^q\}$.

Definition 7 *The decision rule $x(\theta)$ is ex-ante socially rational for prior $p(.)$ ($EASR(p)$) if*

$$\sum_{\theta \in T_l} \sum_{i=1}^{n} u_i(x(\theta), \theta) p(\theta) \geq 0 \quad \textit{for all} \;\; l \in \{1, ..., q\} \tag{36}$$

Then it is easy to establish the following result:

Lemma 5 *If the allocation profile $(x(\theta), t(\theta))$ is incentive compatible and interim individually rational, then $x(\theta)$ is $EASR(p)$, i.e. ex ante socially rational for prior $p(.)$.*

Proof: Consider any agent i and θ_i s.t. $\theta_i \in T_i^l$ for some $l \in \{1, ..., q\}$, i.e. $(\theta_i, \theta_{-i}) \in T^l$ if $p(\theta_i, \theta_{-i}) > 0$. By Property **F**, $IIR_i(\theta_i)$ is equivalent to the following:

$$\sum_{\theta_{-i} : (\theta_i, \theta_{-i}) \in T^l} \left(u_i(x(\theta_{-i}, \theta_i), (\theta_{-i}, \theta_i)) + t_i(\theta_{-i}, \theta_i) \right) p(\theta_{-i}, \theta_i) \geq 0 \tag{37}$$

Summing (37) over all $\theta_i \in T_i^l$ and then adding the resulting inequalities together for all $i \in \{1, ..., n\}$, we obtain $\sum_{\theta \in T^l} \sum_{i \in \{1,...,n\}} (u_i(x(\theta), \theta) + t_i(\theta)) \, p(\theta) \geq 0$. Budget-balancing then implies (36). *Q.E.D.*

Careful reading of the proofs confirms that in this case Theorem 1 holds for any $EASR(p)$ decision rule. This property is required to establish modified Steps 4 and 5 in the proof. Furthermore, Theorem 2 holds with the following modification: the expected social surplus conditional on a particular element of the partition T can be allocated in an arbitrary way to the agent types within this element of the partition. All the results for the Informed Principal Problem continue to hold.

Part 2: Direct proof that $(x^*(\theta), t^*(\theta))$ is a neutral optimum.

By Theorem 7 of Myerson (1983), a mechanism $(x(\theta), t(\theta))$ is a neutral optimum if and only if for $\tau = 1, ..., \infty$ there exist collections of multipliers $\lambda^\tau(.) \in \mathbb{R}^{m_1}_{++}$, $\alpha_i^\tau(.|.) \in \mathbb{R}^{m_i^2}_{+}$, $\alpha_{i,0}^\tau(.) \in \mathbb{R}^{m_i}_{+}$ $i = 1, ..., n$, and 'warranted claims' $\omega^\tau(.) \in \mathbb{R}^{m_1}$ s.t. for all $\theta_1 \in \Theta_1$ and τ

$$\left(\lambda^\tau(\theta_1) + \sum_{\theta_1' \in \Theta_1} \alpha_1(\theta_1'|\theta_1)\right) \omega^\tau(\theta_1) - \sum_{\theta_1' \in \Theta_1} \alpha_1(\theta_1|\theta_1')\omega^\tau(\theta_1') =$$

$$\sum_{\theta_{-1}} \max_{\hat{x} \in X, \hat{t} \in \Delta^{n-1}} \left\{ \lambda^\tau(\theta_1) p(\theta_{-1}|\theta_1) \left\{ u_1(\hat{x}, (\theta_{-1}, \theta_1)) + \hat{t}_1 \right\} + \sum_{i=2,n} \alpha_{i,0}(\theta_i) \left(u_i(\hat{x}, (\theta_{-1-i}, \theta_1, \theta_i)) + \hat{t}_i\right) p(\theta_1, \theta_{-1-i}|\theta_i) \right.$$

$$\left. + \sum_{i=1,...,n; \theta_i' \in \Theta_i} \alpha_i(\theta_i'|\theta_i) \left(u_i(\hat{x}, (\theta_{-1}, \theta_1)) + \hat{t}_i\right) p(\theta_{-i}|\theta_i) - \alpha_i(\theta_i|\theta_i') \left(u_i(\hat{x}, (\theta_{-1-i}, \theta_1, \theta_i')) + \hat{t}_i\right) p(\theta_{-i}|\theta_i') \right\} \tag{38}$$

$$\lim_{\tau \longrightarrow \infty} \sup \omega^\tau(\theta_1) \leq \sum_{\theta_{-1} \in \Theta_1} (u_1(x(\theta_{-1}, \theta_1), (\theta_{-1}, \theta_1)) + t_1(\theta_{-1}, \theta_1)) \, p(\theta_{-1}|\theta_1) \tag{39}$$

To establish that $(x^*(\theta), t^*(\theta))$ is a neutral optimum, choose $\lambda^\tau(\theta_1) \equiv p_1(\theta_1)$, $\alpha_{i,0}^\tau(\theta_i) \equiv p_i(\theta_i)$, $\alpha_i^\tau(\theta_i'|\theta_i) \equiv 0$ for all $i = 2, ..., n$ and $\theta_i, \theta_i' \in \Theta_i$, and $\omega^\tau(\theta_1) = \sum_{\theta_{-1} \in \Theta_1} (u_1(x^*(\theta), \theta) + t_1^*(\theta)) \, p(\theta_{-1}|\theta_1)$ for all τ. Then (39) holds as an equality for all τ, while (38) becomes

$$p_1(\theta_1)\omega^\tau(\theta_1) = \sum_{\theta_{-1} \in \Theta_{-1}} \max_{\hat{x} \in X} \sum_{i=1,...,n} u_i(\hat{x}, (\theta_{-1}, \theta_1)) p(\theta_{-1}, \theta_1)$$

which holds because $x^*(\theta)$ is ex-post efficient and by definition of $\omega^\tau(.)$.