The Market Price of Capital Misallocation

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Abstract

We propose and estimate a new general equilibrium mechanism of how the distribution of idiosyncratic productivity shocks shape aggregate consumption and risk premia via the misallocation of capital. Our model features a representative household with Epstein-Zin preferences and a continuum of heterogenous firms, which face irreversible investment decisions. The key elements of our model are a productivity distribution obeying a power law and common idiosyncratic skewness. In the model, these features lead to large occasional inefficiencies in the allocation of capital across firms and it hinders the household's ability to smooth consumption. Despite the absence of firm-level granularity, the power law implies that large firms contributes disproportionally to aggregate consumption, so that negatively skewed shocks to their productivity are particularly painful.

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1 Introduction

A large body of research has shown that the cross-section of firms is characterized by a substantial degree of productivity and capital heterogeneity (e.g., Eisfeldt and Rampini (2006)). While the empirical facts about firm heterogeneity are well known, the aggregate consequences are not well understood. In this paper, we develop and estimate a simply general equilibrium model to illustrate how the dynamics of the cross-section of firms impact aggregate fluctuations and risk premia via the misallocation of capital resources. The key implication of our general equilibrium model is that idiosyncratic shocks do not integrate out at the aggregate level but instead generate cyclical movements in the higher moments of consumption growth and risk premia.

Our model is driven by a cross-section of heterogenous firms, which face irreversible investment decisions, exit, and permanent idiosyncratic and aggregate productivity shocks. The representative household has recursive preferences and consumes aggregate dividends. To generate aggregate consequences from a continuum of idiosyncratic shocks via capital misallocation, our model mechanism requires both a *power law distribution* as well as *common idiosyncratic skewness* in productivity.

While most of the literature assumes a log-normal idiosyncratic productivity distribution arising from mean-reverting Gaussian shocks, idiosyncratic shocks are permanent and follow random walks in our model. With firm exit, distributions of random walks generate power laws as emphasized by Gabaix (1999) and Luttmer (2007). Quantitatively, the endogenous power law for firm size is consistent with the data, as reported in Axtell (2001), such that the largest 5% of firms generate more than 30% of consumption and output in our model.

In addition to the random walk assumption, we model innovations to idiosyncratic productivity not only with Gaussian but also with negative Poisson shocks, which induce common idiosyncratic skewness. These negative Poisson shocks do not capture rare aggregate disaster, as in Gourio (2012), because they wash out at the aggregate level in a frictionless model.[1] Instead, time variation in the size of common idiosyncratic skewness allows us to capture the

[1]For disaster risk in consumption see Barro (2006), Gabaix (2012), and Wachter (2013).

cyclicality in the skewness of cross-sectional sales growth, consistent with the evidence in Salgado et al. (2015).

In the model, these features lead to large occasional inefficiencies in the allocation of capital across firms and it hinders the representative agent's ability to smooth consumption. Intuitively, in recessions aggregate productivity falls and the distribution of output growth becomes negatively skewed. Firms with negative idiosyncratic productivity draws are constrained because they cannot disinvest unproductive capital to raise dividends. At the same time, the representative household would like to reallocate capital to smooth consumption.

Because of the power law distribution in firm size, the share of output coming from large firms contributes disproportionally to aggregate consumption, so that negatively skewed shocks to their productivity are particularly painful. Consequently, the drop in dividends from the mass of constrained is large, given that they are large in size. While unconstrained firms increase dividends by reducing investment, they are smaller so that they are not able to offset the impact of large constrained firms on aggregate consumption. This effect implies that in recessions aggregate consumption falls by more than aggregate productivity, causing negative skewness and kurtosis, and it arises purely from the cross-sectional misallocation. In contrast, in models with log-normal productivity distributions the size difference between constrained and unconstrained firms is small so that the groups offset each other.

While the impact of capital misallocation on output and consumption are short lived under temporary mean-reverting shocks, permanent Poisson shocks render misallocation distortions long lasting. Quantitatively, output and consumption growth become more volatile and persistent, even though the model is only driven i.i.d. innovations. Importantly, consumption growth is left skewed and leptokurtic, as in the data. Because the household cares about long lasting consumption distortions due to Epstein-Zin preferences, the welfare costs of capital misallocation and aggregate risk premia are large.

Our mechanism to generate aggregate fluctuations from idiosyncratic shocks obeying a power law is distinct from the granular hypothesis of Gabaix (2011). While Gabaix also argues that the dynamics of large firms matters for business cycles, he relies on the fact that the number of firms is finite in an economy so that a few very large firms dominate aggregate output. The impact of these very large firms does not wash at the aggregate level when firm size follows a power law. In contrast, we model a continuum of firms so such each individual firm has zero mass. In our model, the power law in firm size generates aggregate fluctuations based on capital misallocation, arising from the investment friction, and not because the economy is populated by a finite number of firms. In reality, both effects are at work to shape business cycles.[2]

Methodologically, we build on Veracierto (2002) and Khan and Thomas (2008), who find that microeconomic investment frictions are inconsequential for aggregate fluctuations in models with mean-reverting idiosyncratic productivity.[3] We show that a model with permanent shocks and a more realistic firm size distribution not only breaks this irrelevance result, but also produces risk premia that are closer to the data. We are not the first to model permanent idiosyncratic shocks, e.g., Caballero and Engel (1999) and Bloom (2009) do so, but these paper study investment dynamics in partial equilibrium frameworks.

Starting with the influential paper by Bloom (2009), a growing literature emphasizes time varying uncertainty in productivity as driver of business cycles, e.g. Bloom et al. (2014), Bachmann and Bayer (2014), and Kehrig (2015). We differ from this literature by focusing on time varying skewness in the cross-section of sales growth. It is well-known that sales growth dispersion is strongly countercyclical. Less well-known is that this countercyclical dispersion is mainly driven by the left tail of the sales growth distribution. By just looking at the cyclicality of the IQR, one might conclude that in recessions, firms have more dispersed positive and negative productivity draws. But the cyclicality of Kelly skewness indicates that in recessions significantly more firms have extreme negative productivity draws. Our model is equipped to match this empirical fact because productivity is not only driven by Gaussian shocks but also by negative Poisson shocks.

This empirical fact is also reminiscent of Guvenen et al. (2014), who document that households' income shocks feature procyclical skewness. Constantinides and Ghosh (2015) and Schmidt

[2]Related to the granular notion, Kelly et al. (2013) derive firm volatility in sparse networks.
[3]Bachmann and Bayer (2014) show that the same irrelevance result holds with idiosyncratic volatility shocks.

(2015) show that procyclical skewness is quantitatively important for aggregate asset prices in incomplete market economies. Different from these papers, our paper focuses on heterogeneity on the productive side of the economy and analyzes the effect of skewed shocks on capital misallocation.

The first study to quantify capital misallocation is Olley and Pakes (1996). More recent contributions include Hsieh and Klenow (2009) and Bartelsman et al. (2013). We extend their measure of capital misallocation and derive a frictionless benchmark in a general equilibrium framework. The importance of capital misallocation for business cycles is illustrated by Eisfeldt and Rampini (2006).

Our study also relates to the literature on production-based asset pricing, including Jermann (1998), Boldrin et al. (2001), and Kaltenbrunner and Lochstoer (2010), which aims to make the real business cycle model consistent with properties of aggregate asset prices. While these models feature a representative firm, we incorporate a continuum of firms. This allows us to pay close attention to cross-sectional aspects of the data, thereby providing a more realistic micro foundation for the sources of aggregate risk premia. While Kogan (2001) and Gomes et al. (2003) also model firm heterogeneity, our model provides a tighter link to firm fundamentals such that we estimate model parameters.

Our model mechanism is also related to the works of Gabaix (1999) and Luttmer (2007). Gabaix (1999) explains the power law of city sizes with random walks reflected at a lower bound. Using a similar mechanism, Luttmer (2007) generates a power law in firm size in a steady-state model. We extend this literature by studying the impact of a power law in firm size in a business cycle model with common idiosyncratic skewness shocks.

Starting with the influential paper by Berk et al. (1999), there exist a large literature, which studies the cross-section of returns in the neoclassical investment framework, e.g., Carlson et al. (2004), Zhang (2005), Cooper (2006), and Gomes and Schmid (2010). For tractability, these papers assume an exogenous pricing kernel and link firm cash flows and the pricing kernel directly via aggregate shocks. In contrast, we provide a micro foundation for the link between investment frictions and aggregate consumption.

2 Model

Time is discrete and infinite. The economy is populated by a unit mass of firms. Firms own capital, produce output with a neoclassical technology subject to an investment irreversibility constraint, and face idiosyncratic as well as aggregate shocks. The representative household has recursive preferences and consumes aggregate dividends. This section elaborates on these model elements and defines the recursive competitive equilibrium of the economy.

2.1 Productivity

Aggregate productivity X follows a geometric random walk with standard normal innovations

$$X' = \exp\left\{g_x - \sigma_x^2/2 + \sigma_x \eta_x'\right\} X, \tag{1}$$

where g_x denotes the average growth rate of the economy, σ_x the volatility of log aggregate productivity growth, and η_x an i.i.d. standard normal innovation. Modeling aggregate productivity as a random walk as opposed to a mean-reverting process allows us to solve the model in normalized units, which has the advantage that it reduces the dimensionality of the state space. Additionally, it implies that the representative household's marginal utility features a permanent component, as documented empirically by Alvarez and Jermann (2005).

We assume that idiosyncratic productivity growth is a mixture of a normal and a Poisson distribution, allowing for rare but large negative productivity draws. These negative jumps capture, for instance, sudden drops in demand, increases in competition, the exit of key human capital, or changes in regulation. As we illustrate below, they are also essential for allowing the model to replicate the cross-sectional distribution of firms' sales growth.

Specifically, idiosyncratic productivity $\mathcal{E}$ follows a geometric random walk modulated with idiosyncratic jumps J and exit ξ:

$$\mathcal{E}' = (1-\xi')\exp\left\{g_\varepsilon - \sigma_\varepsilon^2/2 + \sigma_\varepsilon \eta' + \chi' J' - \lambda\left(e^{\chi'} - 1\right)\right\}\mathcal{E} + \xi' \exp\left\{g_0 - \sigma_0^2/2 + \sigma_0 \eta_0'\right\} \tag{2}$$

where g_ε denotes the average firm-specific growth rate of surviving firms, σ_ε the volatility of the normal innovations in firm-specific productivity, η an i.i.d. idiosyncratic standard normal shock, and J an i.i.d. idiosyncratic Poisson shock with intensity parameter λ.

The jump size χ varies with aggregate conditions η_x, which we capture with the exponential function

$$\chi(\eta_x) = -\rho_0 e^{-\rho_1 \eta_x} \tag{3}$$

with strictly positive coefficients ρ_0 and ρ_1. This specification implies that jumps are nonpositive and larger in worse aggregate times, i.e. for low values of η_x. It is important to note that the idiosyncratic jump χJ is compensated by its mean $\lambda(e^{\chi} - 1)$, so that the cross-sectional mean of idiosyncratic productivity is constant. This normalization implies that idiosyncratic jumps induce time-varying *common idiosyncratic skewness* in productivity, but they do not generate aggregate jumps in productivity as emphasized by, e.g. Gourio (2012).

Given the geometric growth in idiosyncratic productivity, the cross-sectional mean of idiosyncratic productivity is unbounded unless firms exit. To prevent this, we assume that at the beginning of a period each firm exits the economy with probability $\pi \in (0,1)$. This is captured in Equation (2) with the i.i.d. Bernoulli random variable ξ. New entrants draw their initial productivity level from a log-normal distribution with location parameter $g_0 - \sigma_0^2/2$ and scale parameter σ_0.

Due to the Poisson jumps and exit the distribution of $\mathcal{E}$ is unknown, but it is feasible to compute its moments. Let $\mathbb{M}_n$ denote the n-th cross-sectional raw moment of the idiosyncratic productivity distribution $\mathcal{E}$. It has the following recursive structure

$$\mathbb{M}_n' = (1-\pi)\exp\{ng_\varepsilon - n\sigma_\varepsilon^2/2 + n^2\sigma_\varepsilon^2/2 + \lambda(e^{n\chi'} - 1) - n\lambda(e^{\chi'} - 1)\}\mathbb{M}_n \tag{4}$$

$$+\pi\exp\{ng_0 - n\sigma_0^2/2 + n^2\sigma_0^2/2\}. \tag{5}$$

To make the model solution feasible, we ensure that the first cross-sectional moment, $\mathbb{M}_1$, is finite by imposing that

$$g_\varepsilon < -\ln(1-\pi) \approx \pi.$$

In words, the firm-specific productivity growth rate has to be smaller than the exit rate. In this case, the first raw moment is constant[4] and, for convenience, we normalize it to one by setting

$$g_0 = \ln(1 - e^{g_\varepsilon}(1-\pi)) - \ln(\pi). \tag{6}$$

[4]Note that the term involving χ' in Equation 4 cancels for the first central moment, i.e. for $n = 1$.

Our benchmark calibration further imposes that the second moment, $\mathbb{M}_2$, is finite by requiring that

$$\exp\{2g_\varepsilon + \sigma_\varepsilon^2 + \lambda(e^{2\chi} - 1) - 2\lambda(e^{\chi} - 1)\}(1 - \pi) < 1.$$

Importantly, the second and higher moments are time-varying, and the third and higher moments are potentially infinite.

2.2 Firms

Firms produce output Y with the neoclassical technology

$$Y = (X\mathcal{E})^{1-\alpha} K^{\alpha}, \tag{7}$$

where K denotes the firm's capital stock and α is the curvature in the production function. We summarize the distribution of firms over $(K, \mathcal{E})$ using the probability measure μ defined on the Borel algebra $\mathcal{B}(\mathbb{R}_+^2)$. The distribution of firms evolves over time according to a mapping, Γ, such that $\mu' = \Gamma(\mu, X)$.

To take advantage of higher productivity, firms make optimal investment decisions. Capital evolves according to

$$K' = (1 - \delta)K + I. \tag{8}$$

Without investment frictions, the detrended firm-level distribution collapses to a point mass and the model does not feature firm heterogeneity, as in a standard real business cycle model. To study the impact of firm heterogeneity on business cycles, we follow Veracierto (2002) and Khan and Thomas (2008) by modeling non-convex adjustment costs. Specifically, investment decisions are irreversible, such that $I \geq 0$. Implicitly, we assume that it is infinitely costly for firms to sell their capital. In the neoclassical investment model the benefits of capital sales and reallocation are strongly countercyclical, while in the data capital reallocation is strongly procyclical (see Eisfeldt and Rampini (2006)). By not allowing capital sales, the model better fits capital reallocation facts.

Because investment is irreversible, whenever firms exit, their capital stock is scrapped and entrants start with capital equal to initial productivity, $K_0 = X \exp\{g_0 - \sigma_0^2/2 + \sigma_0\eta_0\}$.

Firms maximize the present value of their dividend payments to shareholders by solving

$$V(K, \mathcal{E}, X, \mu) = \max_{I \geq 0} \left\{ Y - I + (1 - \pi)\mathbb{E}\big[M'V(K', \mathcal{E}', X', \mu')\big] \right\}, \tag{9}$$

where M' is the equilibrium pricing kernel based on the household's preferences defined in the next section. Given cum-dividend firm value V, returns are given by

$$R' = \frac{V'}{V - D}. \tag{10}$$

2.3 Household

The representative household of the economy maximizes recursive utility U over consumption C as in Epstein and Zin (1989):

$$U(X, \mu) = \max_{C} \left\{ (1 - \beta)C^{1-\frac{1}{\psi}} + \beta \left(\mathbb{E}\left[U(X', \mu')^{1-\gamma} \right] \right)^{(1-\frac{1}{\psi})/(1-\gamma)} \right\}^{1/(1-\frac{1}{\psi})} \tag{11}$$

where $\psi > 0$ denotes the elasticity of intertemporal substitution (EIS), $\beta \in (0, 1)$ the subjective discount factor, and $\gamma > 0$ the coefficient of relative risk aversion. In the special case when risk aversion equals the inverse of EIS, the preferences reduce to the common power utility specification.

The household's budget constraint is

$$\int sV \,\mathrm{d}\mu(K \times \mathcal{E}) \geq C + \int s'(V - D) \,\mathrm{d}\mu(K \times \mathcal{E}), \tag{12}$$

where s denotes the number of shares held of a specific firm.

The household's first order condition with respect to the optimal asset allocation implies the usual Euler equation

$$\mathbb{E}\big[M'R'\big] = 1$$

where M' is the pricing kernel and R' is the return on equity of a firm defined by (10). The pricing kernel is given by

$$M' = \beta^{\theta} \left(\frac{W'}{W - C} \right)^{\theta - 1} \left(\frac{C'}{C} \right)^{-\theta/\psi}, \tag{13}$$

where W denotes aggregate wealth and $\theta = \frac{1-\gamma}{1-1/\psi}$. Consistent with the Euler equation, wealth is defined recursively as the present value of future aggregate consumption:

$$W = C + \mathbb{E}\big[M'W'\big]. \tag{14}$$

Note that firm exit introduces a wedge between wealth and aggregate firm value. This stems from the fact that wealth captures the present value of both incumbents and entrants, whereas aggregate firm value relates to the present value of dividends of incumbent firms only.

2.4 Equilibrium

The model is closed by imposing the aggregate resource constraint. At the aggregate level, the production of firms can be either consumed by the representative household or invested in next-period capital. Thus, clearing the goods market requires

$$C = \int (X\mathcal{E})^{1-\alpha} K^{\alpha} + (1-\delta)K - K' \,\mathrm{d}\mu(K \times \mathcal{E}). \tag{15}$$

A recursive competitive equilibrium for this economy is a set of functions

$$(C, W, V, K, \Gamma)$$

such that

(*i*) Firm optimality: Taking M and Γ as given, firms solve (9) with policy function K.

(*ii*) Household optimality: Taking V as given, household maximize (11) subject to (12) with policy function C.

(*iii*) The good market clears according to (19).

(*iv*) Model consistency: The evolution of the cross-section Γ is induced by K and the exogenous processes (1) and (2).

3 Solution Method

Solving the model is challenging because it features (1) unit roots in both aggregate and idiosyncratic productivity and (2) an infinite-dimensional state space that results from μ. Our solution method extends the approximate aggregation approach of Krusell and Smith (1998) to handle nonstationarity environments, and it yields accurate results based on consumption as the single aggregate state variable summarizing μ, despite the fact that our model features substantial degrees of capital misallocation across firms. In the remainder of this section, we explain how the model is detrended and we discuss the numerical solution approach.

3.1 Firm Optimization

The homogeneity of the value function and the linearity of the constraints imply that we can detrend the firm problem by the permanent shocks $X\mathcal{E}$, as in Bloom (2009). Define $\kappa = K/(X\mathcal{E})$, $\tau = K'/(X\mathcal{E})$, and $v = V/(X\mathcal{E})$. The detrended value function becomes

$$v(\kappa, \mu) = \max_{\tau \geq (1-\delta)\kappa} \left\{ \kappa^\alpha - \tau + (1-\delta)\kappa + (1-\pi)\mathbb{E}\big[M'x'\varepsilon' v(\kappa', \mu')\big] \right\}, \tag{16}$$

where $\varepsilon' = \mathcal{E}'/\mathcal{E}$ and $x' = X'/X$.

Because the innovations ε and x are i.i.d., it follows that firms share a common time-varying optimal investment target $\mathcal{T}$, which is independent of their own characteristics, and given by

$$\mathcal{T}(\mu) = \arg\max_{\tau \geq (1-\delta)\kappa} \left\{ -\tau + (1-\pi)\mathbb{E}\big[M'x'\varepsilon' v(\kappa', \mu')\big] \right\}. \tag{17}$$

Consequently, firms' next period capital stock equals $K' = X\mathcal{E}\mathcal{T}$ when they are unconstrained. If the optimal target is below current depreciated capital, firms wait and undertake no investment, so that $K' = (1-\delta)K$. Written in deterended units, the law of motion for capital resulting from firms' optimal policies is given by

$$\kappa' = \max\{\mathcal{T}, (1-\delta)\kappa\}(x'\varepsilon')^{-1} \tag{18}$$

This formulation forms the basis of our numerical solution of the firm problem.

3.2 Aggregation

In contrast to the firm-level problem (9), the aggregate resource constraint (19) cannot be detrended with the idiosyncratic shock $\mathcal{E}$ because the optimal capital policy under irreversible investment implies that capital and idiosyncratic productivity are correlated. To induce stationarity, we therefore detrend it with aggregate productivity X, and define $c = C/X$ and $k = K/X \neq \kappa$. The detrended aggregate resource constraint reads

$$\begin{aligned} c &= \int \mathcal{E}^{1-\alpha} k^{\alpha} \, \mathrm{d}\mu(k \times \mathcal{E}) + (1-\delta) \int k \, \mathrm{d}\mu(k \times \mathcal{E}) \\ &\quad -(1-\pi) \int \max\{\mathcal{E}\mathcal{T}, (1-\delta)k\} \, \mathrm{d}\mu(k \times \mathcal{E}) - \pi e^{g_0}, \end{aligned} \tag{19}$$

where μ is the measure over the firm-level state space $(k, \mathcal{E}) \in \mathbb{R}_+^2$. The transition for μ depends on both the exogenous dynamics of $\mathcal{E}$ and the dynamics of k, which are in turn determined by firms' optimal investment policy and the initial capital stock of new entrants:

$$k' = (x')^{-1} \times \begin{cases} \max\{\mathcal{E}\mathcal{T}, (1-\delta)k\} & \text{incumbent,} \\ \exp\{g_0 - \sigma_0^2/2 + \sigma_0 \eta_0'\} & \text{entrant with probability } \pi. \end{cases} \tag{20}$$

3.3 Computation

As in Krusell and Smith (1998), we approximate the firm-level distribution μ with an aggregate state variable to make the model solution computable. Krusell and Smith and most of the subsequent literature used aggregate capital, $\bar{k} = \int k \, \mathrm{d}\mu(k \times \mathcal{E})$, and higher cross-sectional moments of capital to summarize the policy-relevant information in μ. Instead, we use (normalized) aggregate consumption c, and we argue that this approach is better suited for models with quantitatively important degrees of capital misallocation.

While aggregate capital (and higher moments of capital) depends only on the marginal distribution of capital, consumption depends on the joint distribution of capital and productivity. To illustrate the importance of this feature, consider the stylized example in Table 1, where both idiosyncratic productivity and capital can only take on two values. The table entries are the probability mass for each point in the support of μ. In Case I, productive firms hold a high capital stock, while unproductive firms hold a low capital stock. In Case II, the scenario is reversed and capital misallocated. In both cases, the aggregate capital stock equals $(k_{low} + k_{high})/2$. Consumption, however, will be much higher in Case I, where capital is not misallocated and productive firms hold a high capital stock. In this case, aggregate output is higher and likely to remain so for the foreseeable future due to the permanent nature of shocks and the investment friction. Therefore, consumption is better suited than aggregate

Case I

k_{low}	0.5	0
k_{high}	0	0.5
	ε_{low}	ε_{high}

Case II

k_{low}	0	0.5
k_{high}	0.5	0
	ε_{low}	ε_{high}

Table 1: Two stylized firm-level distributions

capital for summarizing the economically relevant aspects of μ in our model. We suspect that this advantage carries over to other cross-sectional models that feature substantial amounts of capital misallocation.

Methodologically, the main difference between aggregate capital compared to consumption as state variable arises when specifying their law of motions. Note that tomorrow's capital stock for each firm is contained in the current information set, which implies that tomorrow's *aggregate* capital stock is contained in the current information set as well. Consequently, it is possible to approximate the law of motion for aggregate capital with a deterministic function. On the contrary, tomorrow's consumption is not known today but depends on tomorrow's realization of the aggregate shock η_x'. We approximate the law of motion for consumption with an affine function in log consumption

$$\ln c' = \zeta_0(\eta_x') + \zeta_1(\eta_x') \ln c. \tag{21}$$

Note that these forecasting functions imply intercepts and slope coefficients that depend on the future shock to aggregate productivity, i.e., they yield forecasts conditional on η_x'. As we illustrate quantitatively in Section 5.1, this functional form for aggregate consumption is not very restrictive as it allows for time variation in conditional moments of log consumption growth

$$gC' = \log\left(\frac{C'}{C}\right) = \zeta_0(\eta_x') + (\zeta_1(\eta_x') - 1) \ln c + \ln x'.$$

In a model based on a representative household with power utility, the consumption rule (21) is sufficient to close the model. Because we model a representative household with recursive utility, we also have to solve for the wealth dynamics to be able to compute the pricing kernel (13). Khan and Thomas (2008) assume that marginal utility of consumption is a log linear function in aggregate capital and estimate the coefficients based on simulated data of the model. Instead, given consumption dynamics (21), we use the Euler equation for the return on wealth (14) to determine log wealth as a function log consumption, i.e., $w(c)$. To this end, we minimize the Euler equation error by iterating on the Euler equation. We use a fine grid for c and approximate $w(c)$ with cubic splines for off-grid values. As a result, wealth dynamics and optimal consumption satisfy the equilibrium Euler equation and the model does not allow for arbitrage opportunities.

To summarize, our algorithm works as follows. Starting with a guess for the coefficients of the equilibrium consumption rule (21), we first solve for the wealth rule and then the firm's problem (16) by value function iteration. To update the coefficients in the equilibrium rule (21), we simulate a continuum of firms. Following Khan and Thomas (2008), we impose market clearing in the simulation, meaning that firm policies have to satisfy the aggregate resource constraint (19). The simulation allows us to update the consumption dynamics and we iterate on the procedure until the consumption dynamics have converged.

4 Frictionless Case

4.1 Firms

To understand the impact of irreversible investment on aggregate consumption, we also solve the frictionless economy as a benchmark. Without investment and dividend frictions, the optimal firms' investment target can be solved for in closed-form

$$\mathcal{T}(\mu) = \left(\frac{(1-\pi)\alpha \mathbb{E}[M'(x'\varepsilon')^{1-\alpha}]}{1-(1-\pi)(1-\delta)\mathbb{E}[M']} \right)^{1/(1-\alpha)}$$

and optimal capital capital policy (18) simplifies to

$$K' = \begin{cases} X\mathcal{E}\mathcal{T} & \text{incumbent} \\ X\exp\{g_0 - \sigma_0^2/2 + \sigma_0\eta_0'\} & \text{entrant with probability } \pi \end{cases} \tag{22}$$

Intuitively, without investment and dividend constraints, firms are at their optimal capital target in every period.

4.2 Misallocation

In the full model, capital is misallocated across firms because unproductive firms cannot disinvest and productive firms are equity issuance constrained. To quantify the amount of capital misallocation, we propose the correlation between capital and productivity

$$\mathcal{M} = \text{Corr}(\ln K', \ln \mathcal{E}). \tag{23}$$

In the frictionless case, capital is never misallocated and $\mathcal{M} = 1$ in each period. In the full model, the more capital is misallocated across firms the lower is $\mathcal{M}$. In the extreme case, $\mathcal{M}$ could even turn negative. In Table 1, case I corresponds to no misallocation, $\mathcal{M} = 1$, and case II to full misallocation, $\mathcal{M} = -1$. Our misallocation measure is similar to Olley and Pakes (1996), which has been used more recently by Hsieh and Klenow (2009) and Bartelsman et al. (2013), who suggest the covariance between size and productivity.

4.3 Equilibrium

In the frictionless economy, it is feasible to derive a closed-form expression for the law of motion of aggregate capital. Specifically, by aggregating the optimal capital policy (22) it follows that

$$\bar{k}'x' = (1-\pi)\mathcal{T} + \pi e^{g_0} \tag{24}$$

Intuitively, aggregate capital is a weighted average of the investment target of incumbents and average capital of entrants. This aggregation result fails in the full model because the optimal capital policy under irreversible investment (8) implies that future capital is a function of past shocks, rendering capital and idiosyncratic shocks correlated.

Similarly, the detrended aggregate resource constraint (19) in the frictionless economy simplifies to

$$c = y + (1-\delta)\bar{k} - (1-\pi)\mathcal{T} - \pi e^{g_0}, \tag{25}$$

where output is given by

$$y' = A' \frac{(1-\pi)\mathcal{T}^\alpha}{(x')^\alpha} + \frac{\pi}{(x')^\alpha} e^{g_0}. \tag{26}$$

5 Quantitative Results

5.1 Empirical Facts

In this section, we estimate moments of the cross-sectional sales growth distribution. These estimates are then used as target moments for the model estimation. We use quarterly CRSP-Compustat data covering 1977 to 2014. We use 1977 as starting date because after this date the merged CRSP-Compustat sample contains at least 2,000 firms. We restrict the sample to firms with at least 5 years of data.

To measure the time-variation in cross-sectional heterogeneity, we use 4-quarter log sales growth rates, denoted by s, using Computstat item SALEQ. Following Salgado et al. (2015), we compute for each quarter the cross-sectional interquartile range (IQR) and Kelly skewness (KSK):

$$\text{IQR} = p_{75}(s) - p_{25}(s) \qquad \text{KSK} = \frac{p_{90}(s) - p_{50}(s)}{p_{90}(s) - p_{10}(s)} - \frac{p_{50}(s) - p_{10}(s)}{p_{90}(s) - p_{10}(s)}$$

where p_x denotes the x-th percentile of the sales growth rate distribution. Figure 1 depicts the time-series for IQR and KSK. It is well-known that sales growth dispersion is strongly countercyclical. Less well-known is that this countercyclical dispersion is mainly driven by the left tail of the sales growth distribution. By just looking at the cyclicality of the IQR, one might conclude that in recessions, firms have more dispersed positive and negative productivity draws. But the cyclicality of Kelly skewness indicates that in recessions significantly more firms have extreme negative productivity draws. Our model is equipped to match this empirical fact because productivity is driven by negative Poisson shocks.

Table 3 summarizes moments of IQR and KSK. In the data, IQR has a time-series average of 0.239 and standard deviation of 0.041. KSK is positive on average, 0.065, and fairly volatile, 0.101. Importantly, KSK is strongly negatively skewed, -0.894, and procyclical with respect to aggregate output growth, 0.607. As Figure 2 shows, the negative skewness of KSK and its strong positive correlation with aggregate output growth result largely from large negative spikes during recessions.

5.2 Calibration and SMM Estimation

We solve the model at a quarterly frequency. Our parameter choices are summarized in Table 2. The parameter values in Panel A are calibrated, while the ones in Panel B are estimated.

Following Bansal and Yaron (2004), we assume that the representative agent is fairly risk averse, $\gamma = 10$, and has a large EIS, $\psi = 2$. The time discount rate is set to achieve a low average risk-free rate, $\beta = 0.995$. Capital depreciates at a rate of 2.5% and the curvature of the production function equals 0.65, similar to Cooper and Haltiwanger (2006). Firms exit the economy with a rate of 2%, similar to the value reported in Dunne et al. (1988). The productivity draws of entrants has a mean pinned down by condition (6) and a volatility of 10%.

Given the calibrated parameters described in the previous section, we structurally estimate the remaining seven parameters, $(\lambda, \rho_0, \rho_1, g_E, \sigma_E, g_X, \sigma_X)$, with the simulated method of moments (SMM). The estimation attempts to fit the cross-sectional distribution of sales growth, while ensuring that aggregate quantity moments remain in line with the data. Specifically, we target three aggregate moments

1. average 4-quarter log aggregate output growth,
2. standard deviation of 4-quarter log aggregate output growth,
3. standard deviation of 4-quarter log aggregate consumption growth,

as well as seven moments of the cross-sectional distribution of 4-quarter log sales growth

4. average median,
5. average IQR,
6. standard deviation of the IQR,
7. average KSK,
8. standard deviation of KSK,
9. skewness of KSK,
10. correlation of KSK with 4-quarter log aggregate output growth.

The objective function consists of an equally-weighted average of the absolute percentage deviation between data and model moments. We employ a genetic algorithm to find the function's global minimum.

The benchmark parameter values are summarized in Table 2. At these parameter values, the approximate law of motion for detrended consumption in Equation (21) provides a good fit in the model, as the regression R^2s are at least 99.9% for all values of η'_x. The resulting capital targets $\mathcal{T}$ for the frictionless benchmark and the full model are illustrated in Figure 4, along with histograms for detrended consumption to indicate the relevant region of the target functions. In the full model, the optimal investment target is lower and less sensitive to fluctuations in (detrended) consumption, because the household takes into account the impact of the investment friction on consumption.

The parameters governing the drifts of idiosyncratic and aggregate productivity (g_ε and g_x) are identified by the median of sales growth and the mean of aggregate output growth. While the median of sales growth varies positively with both parameters, mean output growth only varies with g_X because we impose that the cross-sectional mean of idiosyncratic productivity is constant (see Equation (6)). Lastly, the parameters governing the volatilities of idiosyncratic and aggregate productivity (σ_ε and σ_x) are identified by the interquartile range of sales growth and the volatility or aggregate output and consumption.

5.3 Cross-sectional Distribution of Sales Growth

Figure 1 illustrated the defining features of the cross-sectional sales growth distribution, most notably the large negative spikes of its skewness during recessions. Panel A of Table 3 shows that the model does a good job in replicating most aspects of the cross-sectional distribution. In particular, Kelly skewness has a positive mean (0.07 in the model vs. 0.07 in the data), is fairly volatile (0.10 in the model vs. 0.11 in the data), strongly negatively skewed (-0.89 in the model vs. -0.89 in the data) and positively correlated with aggregate output growth (0.61 in the model vs. 0.59 in the data), while the IQR is large on average (0.16 in the model vs. 0.24 in the data).

Only the standard deviation of the IQR falls short of its data counterpart, because jumps are too rare to have a significant effect on firms in the center of the productivity distribution (the intensity parameter of the Poisson jumps is estimated as $\lambda = 0.06$), which is instead driven by the (constant) volatility of Gaussian productivity shocks. While the variability of the IQR could be matched by introducing time-varying volatility as in Bloom (2009), we decided against this to keep the model parsimonious.

To understand how the model replicates the features of Kelly skewness, note that a firm's annual log sales growth rate is a weighted average of the shocks to its productivity and its investment rates over the year. Using time subscript notation, annual sales growth is given by

$$gY_{t:t+4} = (1-\alpha)\sum_{h=1}^{4}(x_{t+h} + \varepsilon_{t+h}) + \alpha \sum_{h=1}^{4} \log\left(1 - \delta + \frac{I_{t+h}}{K_{t+h}}\right). \tag{27}$$

The (Kelly) skewness of the cross-sectional sales growth distribution thus depends on the marginal distributions of idiosyncratic productivity shocks and firms' investment rates. In particular, the shape of the sales growth distribution depends to a large extend on the relative magnitude of the two variables. For most aggregate shock realizations (medium to large values of η_x'), productivity shocks are small relative to investment rates, and sales growth is mostly determined by the latter. Because investment irreversibility induces right-skewness in investment rates – many firms are constrained and have zero investment rates – the sales growth distribution is right-skewed. For bad aggregate shock realizations, some firms receive large negative productivity draws, while a large fraction of firms is constrained. Productivity thus becomes the main determinant of the skewness in sales growth, making it strongly negative.

The parameters governing the Poisson jumps in idiosyncratic productivity $(\lambda, \rho_0, \rho_1)$ are identified by the four moments associated with Kelly skewness. Making jumps less frequent (by reducing λ) or smaller on average (by reducing ρ_0) increases the mean of Kelly skewness and reduces its volatility. The reason is that the left tail of the cross-sectional productivity distribution becomes thinner and less time-varying. On the other hand, reducing either the frequency of jumps or their cyclically (by reducing ρ_1) reduces the correlation between Kelly skewness and aggregate output growth.

Importantly, there exists a trade-off in the model between matching the cross-sectional dispersion in productivity as reflected in the IQR of sales growth on the one hand, and aggregate consumption volatility on the other hand. While larger, more frequent, and more cyclical jumps increase the mean and volatility of the IQR, they also increase consumption volatility relative to output volatility. The later effect arises because jumps increase misallocation and thereby hinder the household's ability to smooth consumption.

5.4 Power Law and Consumption Dynamics

The combination of a unit root in idiosyncratic productivity and random exit in our model results in a firm size distribution whose right tail exhibits a power law. Figure 5 shows that the log right tail probabilities (above the 50est percentile), with firm size measured by log capital, lie on a straight line with a slope of -1.73.[5] This means that the firm size distribution is well approximated in its right tail by a Pareto distribution with right tail probabilities of the form $1/S^{\xi}$, with a tail index ξ of around 1.7. To illustrate the economic effect of the power law, Figure 6 shows the degree of output concentration implied by our benchmark calibration. Specifically, it shows the fraction of aggregate output produced by firms in various percentiles of the capital distribution.[6] On average, the largest 5% of firms (in terms of capital) produce about 30% of aggregate output.

Due to the importance of large firms for output and consumption, permanent negative shocks to their productivity are particularly painful. Consequently, the drop in dividends from the mass of constrained firms is large, given that they are large in size. While unconstrained firms increase dividends by reducing investment, they are smaller so that they are not able to offset the impact of large constrained firms on aggregate consumption. This effect implies that in recessions aggregate consumption falls by more than aggregate productivity, causing negative skewness and kurtosis, and it arises purely from the cross-sectional misallocation. In contrast, in models with log-normal productivity distributions the size difference between constrained and unconstrained firms is small so that the groups offset each other.

Figure 7 illustrates this channel quantitatively via a comparative statics exercise that varies the exit probability π. A larger exit probability implies that firms survive shorter on average, which reduces the mass of large firms. The top panel shows that the power law coefficient ($-\xi$) decreases as π increases, meaning that the right tail of the firm size distribution becomes thinner. This implies that it becomes easier to smooth consumption by offseting the losses in consumption from large firms with the dividend payments of constrained (and relatively smaller) firms. As a consequence, the left skewness and kurtosis of consumption growth are

[5] The figure is generated based on a distrubution μ that occurred at an arbitrary point in time during the model simulation.

[6] To produce the figure, we simulate a continuum of firms for 2500 quarters and record the fraction of output produced by the 5% smallest firms (in terms of capital), firms between the 5th and 10th size percentiles, etc. in each period. We then average these fractions across all periods in the simulation.

reduced, as illustrated in the bottom two panels of Figure 7.

5.5 Procyclicality in the Dispersion of Investment Rates

Bachmann and Bayer (2014) documented that the cross-sectional dispersion in investment rates is strongly procyclical. The authors show that a heterogeneous firm model with countercyclical dispersion in idiosyncratic productivity shocks can replicate this feature, but they argue that "the procyclicality of investment dispersion places a robust and tight upper bound on the aggregate importance of firm-level risk shocks". In other words, models that produce aggregate fluctuations that originate in time-varying firm-level volatility (e.g. Bloom et al. (2014)) can only do so at the expense of having counterfactual implications for firms' investment behavior.

Our model provides a counterexample to this claim. In particular, the model produces a strongly procyclical investment rate dispersion despite the fact that firm-level risk shocks in the form of time-varying skewness are very pronounced. We now illustrate that this result is a consequence of the high value we assume for the representative households' EIS. This assumption stands in contrast to the previous literature, which relies on time-separable utility with log or power utility for the period utility function, and therefore implies low values for the EIS.

The left panel of Figure 8 shows the correlation between quarterly output growth and the investment rate dispersion as a function of the EIS. To produce the figure, we alter the EIS, but hold all other model parameters fixed. For the low values of the EIS assumed in the previous literature, the investment rate dispersion in our model is close to acyclical, which confirms the result of Bachmann and Bayer. As the EIS is increases, however, the correlation increases monotonically, reaching a value of about 0.35 at our benchmark EIS value of 2. The left panel of Figure 8 depicts the correlation between quarterly output growth and the fraction of constrained firms, i.e. the extensive margin of investment. The correlation is low for low values of the EIS, but reaches a value of about -0.4 at an EIS of 2. Bad aggregate times (low output growth) are therefore characterized by a low fraction of investing firms and therefore little dispersion in investment rates.

6 Conclusion

We study the impact of capital misallocation on business cycle dynamics and risk premia in a dynamic, stochastic general equilibrium model with firm heterogeneity. In our model economy, firms face irreversible investment decisions, exit, and persistent idiosyncratic and aggregate productivity shocks. The representative household has Epstein-Zin preferences. We solve for the equilibrium dynamics of the Epstein-Zin pricing kernel by aggregating dividends and firm-values across heterogenous firms to obtain consumption and wealth.

We differ from the existing literature by focusing on time varying skewness in the cross-section of sales growth. It is well-known that sales growth dispersion is strongly countercyclical. Less well-known is that this countercyclical dispersion is mainly driven by the left tail of the sales growth distribution. By just looking at the cyclicality of the IQR, one might conclude that in recessions, firms have more dispersed positive and negative productivity draws. But the cyclicality of Kelly skewness indicates that in recessions significantly more firms have extreme negative productivity draws. Our model is equipped to match this empirical fact because productivity is not only driven by Gaussian shocks but also by negative Poisson shocks.

Even though the model is only driven i.i.d. innovations, it replicates well the level, volatility, and persistence of capital misallocation in the US economy, which we define as correlation between size and productivity. In the frictionless benchmark model, capital and productivity is perfectly aligned across firms, implying a correlation of one, and output growth is not persistent. In the full model, investment is irreversible, implying that unproductive firms have excess capital. While the impact of capital misallocation on output and consumption are in the typical neoclassical model short lived, permanent Poisson shocks render misallocation distortions long lasting. Quantitatively, output and consumption growth become more volatile and output growth is more persistent because capital is sticky. Importantly, consumption growth is left skewed and leptokurtic, as in the data.

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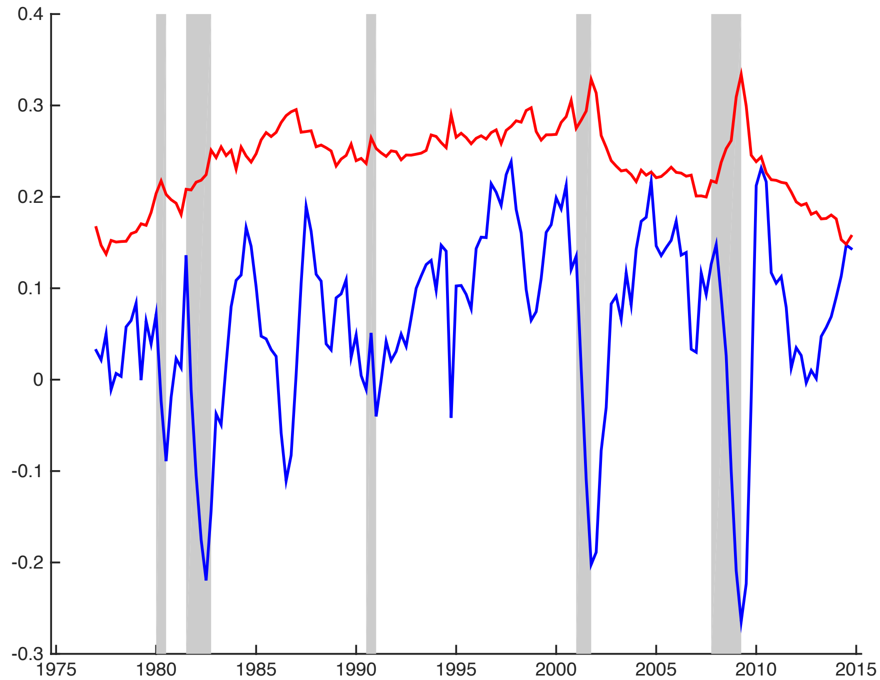

Figure 1: Interquartile Range (red) and Kelly Skewness (blue) of Annual Sales Growth

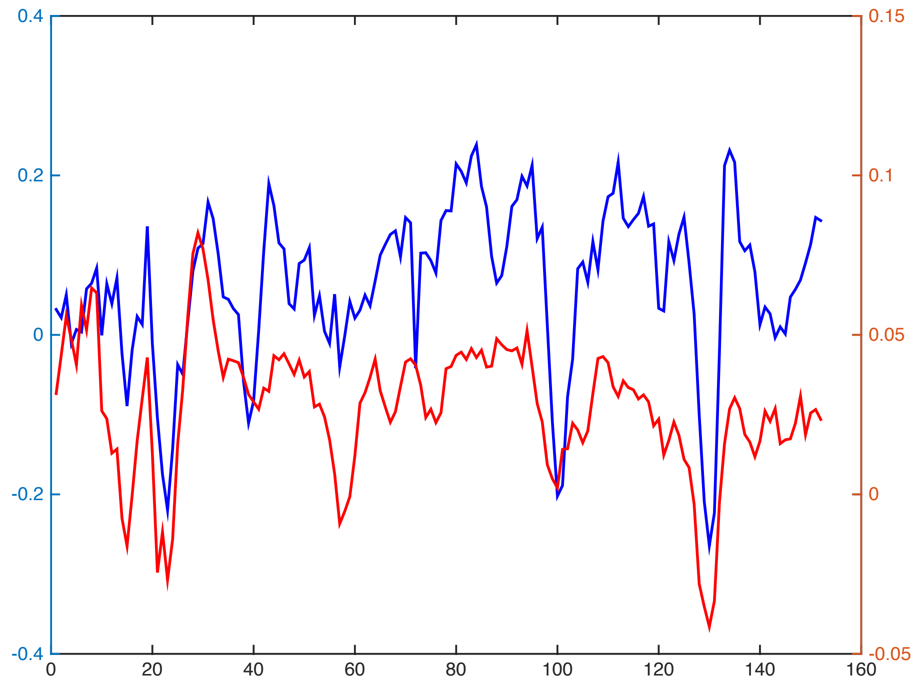

Figure 2: GDP Growth (red) and Kelly Skewness (blue) of Annual Sales Growth

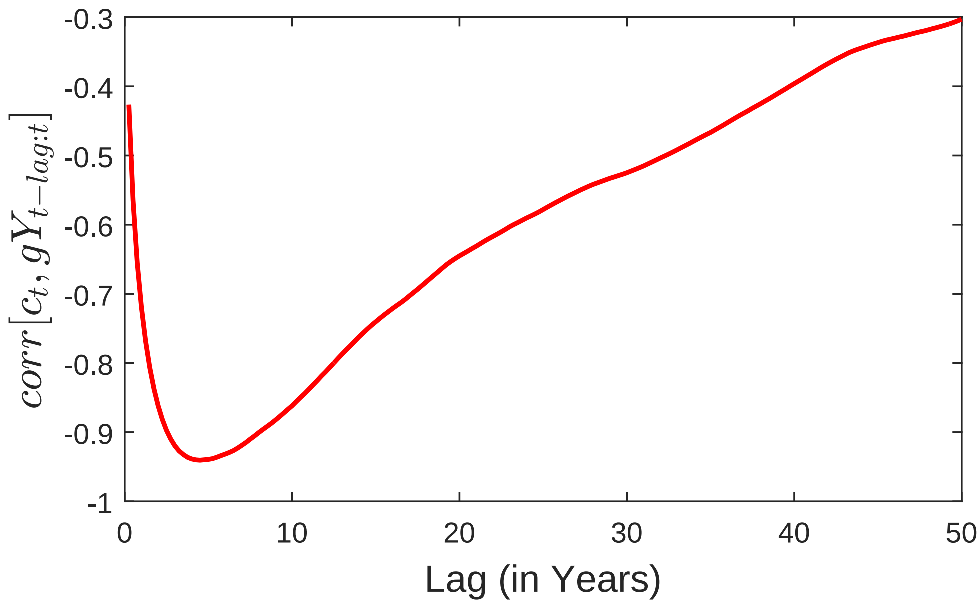

Figure 3: Normalized Consumption and History Dependence

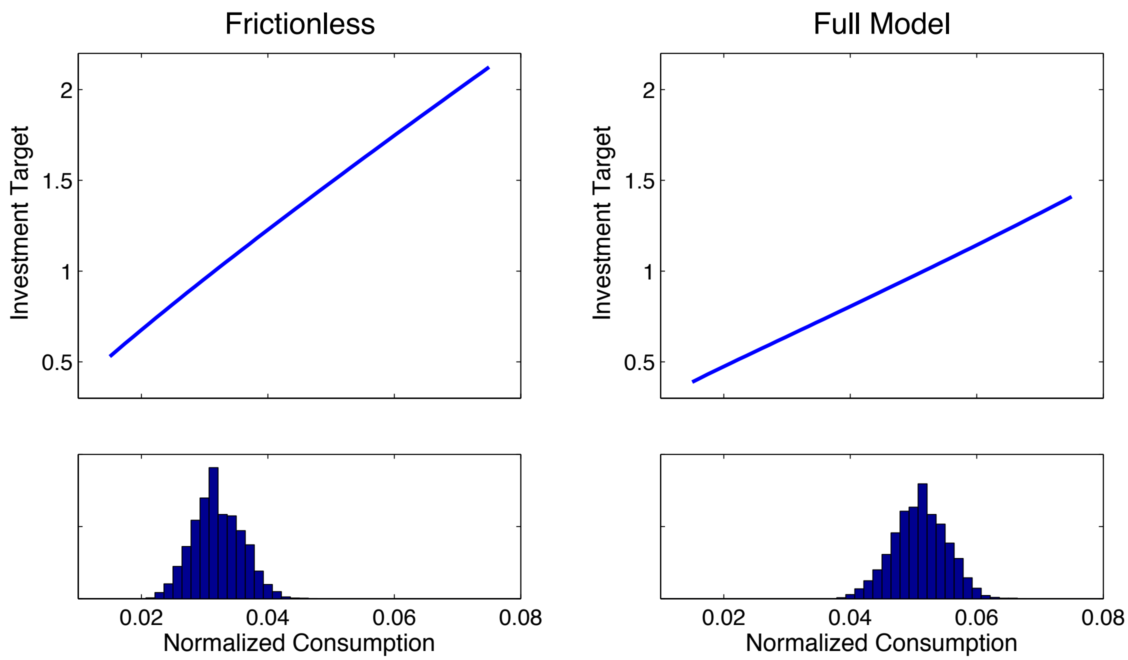

Figure 4: Optimal Investment Targets

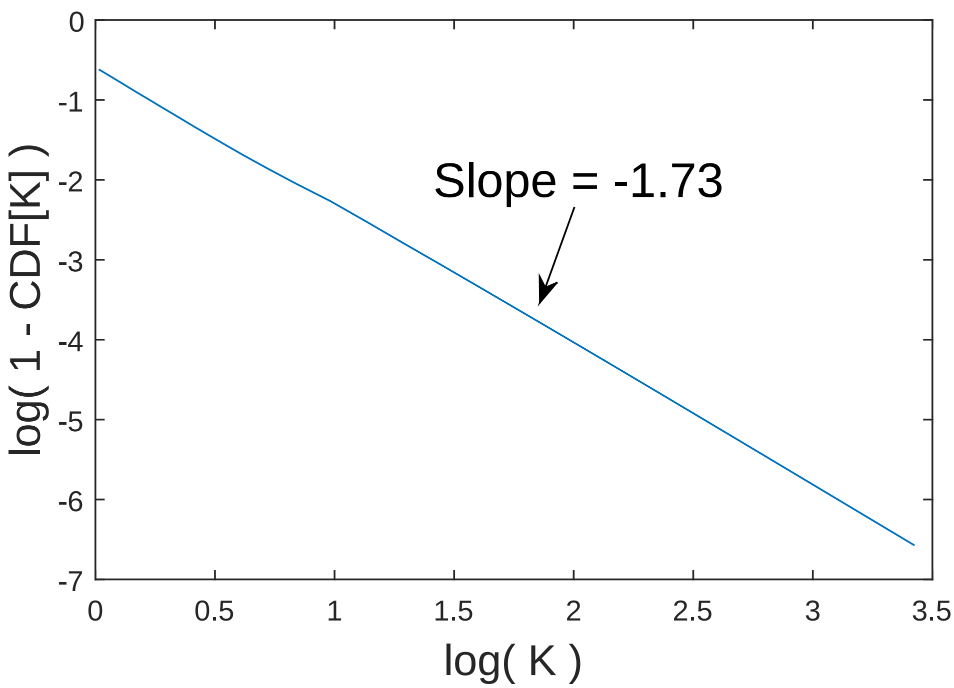

Figure 5: Power Law in Firm Size

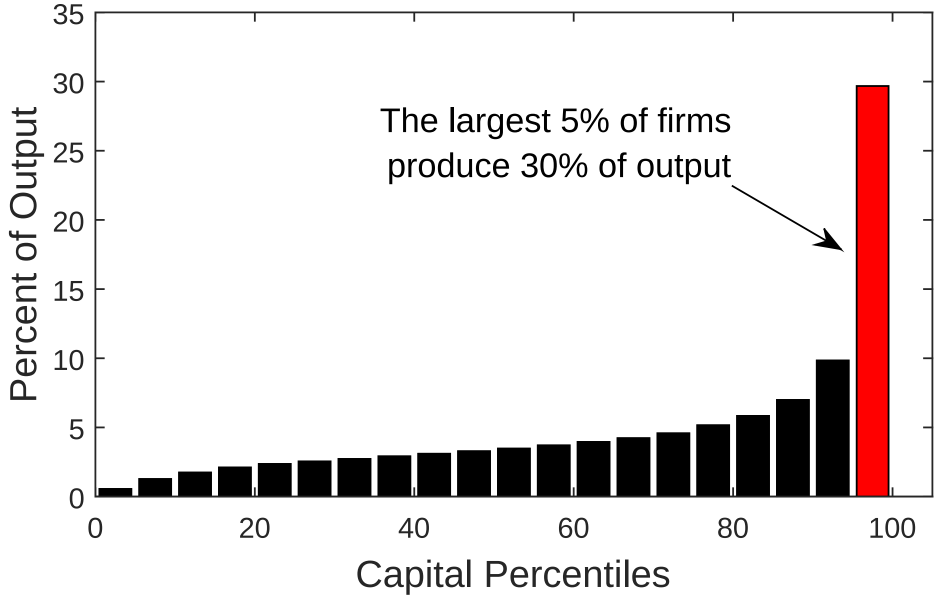

Figure 6: Output Concentration

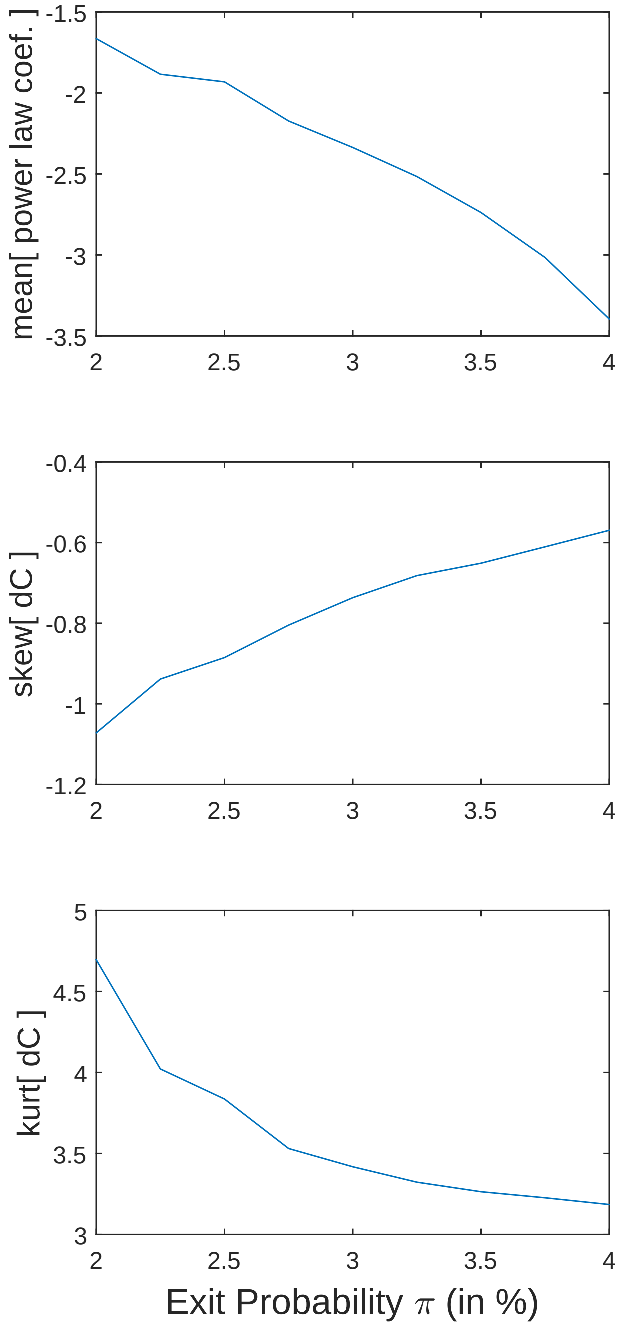

Figure 7: Comparative Statics: Power Law and Consumption Moments

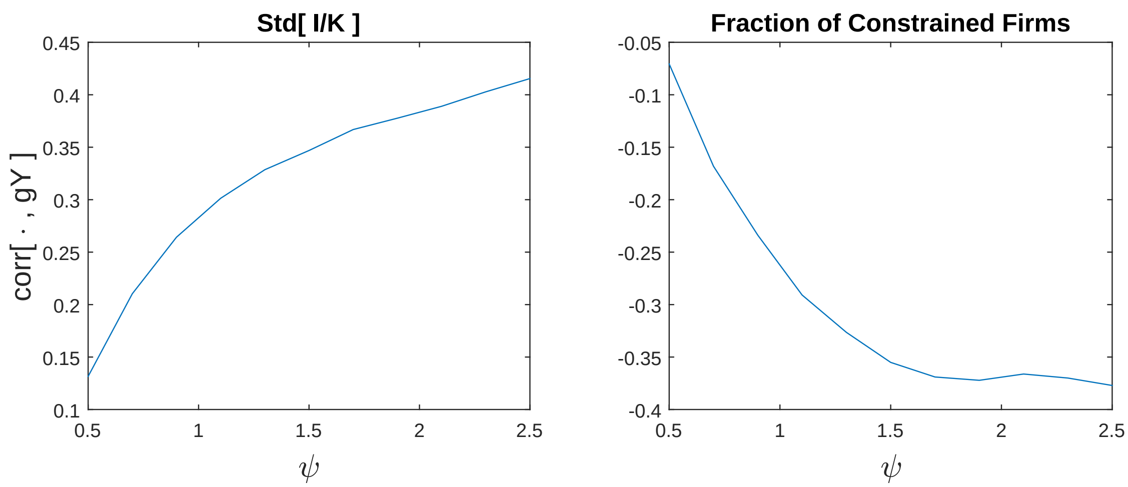

Figure 8: Comparative Statics: Cyclicality of the Investment Dispersion and the EIS

Table 2: Calibration and SMM Parameter Estimates

	Parameter	Value	Description
Panel A	δ	0.025	Depreciation rate
	α	0.65	Curvature in production function
	g_0	-0.3035	Mean productivity of new entrants
	σ_0	0.1	Volatility of productivity of new entrants
	π	0.02	Exit probability
	β	0.995	Time discount factor
	γ	10	Risk aversion
	ψ	2	Elasticity of intertemporal substitution
Panel B	ρ_0	0.3908	Parameter of jump size function
	ρ_1	0.6430	Parameter of jump size function
	λ	0.0412	Jump intensity
	g_E	0.0067	Mean idiosyncratic productivity growth rate
	σ_E	0.0481	Volatility of normal part of idiosyncratic productivity growth rate
	g_X	0.0073	Mean aggregate productivity growth rate
	σ_X	0.0302	Volatility of aggregate productivity growth rate

Panel A shows calibrated parameters and Panel B shows parameters estimated via SMM. The model is solved at a quarterly frequency.

Table 3: Moments Targeted in SMM Estimation

Moment	Data	Model
Panel A: Annual Sales Growth		
Mean(Median)	0.046	0.030
Mean(IQR)	0.239	0.158
SD(IQR)	0.041	0.008
Mean(Kelly)	0.065	0.065
SD(Kelly)	0.101	0.105
Skew(Kelly)	-0.894	-0.885
Corr(Kelly,gY)	0.607	0.594
Panel B: Aggregate Quantities		
Mean(gY)	0.027	0.027
SD(gY)	0.027	0.027
SD(gC)	0.018	0.018

The table summarizes the moments used in the SMM estimation. Panel A contains time series moments of moments that summarize the cross-sectional distribution of annual log sales growth. For example, $SD(Kelly)$ denotes the (time series) standard deviation of the (cross-sectional) Kelly skewness. Panel B reports time series moments of annual output growth (gY) and consumption growth (gC).

Table 4: Consmption Growth and Asset Prices

Moment	Data	Model
Panel A: Consumption Growth		
AC(gC)	0.304	0.509
skew(gC)	-1.414	-1.069
kurt(gC)	6.523	4.682
Panel B: Returns		
Excess return on wealth		0.017
Risk-free Rate		0.024
Sharpe Ratio		0.538

Panel A reports quarterly log consumption growth rate moments. Panel B shows annualized return moments.