Market Liquidity and Asset Prices under Costly Participation

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Abstract

In this paper, we develop an equilibrium model for market liquidity and its impact on asset prices when constant participation in the market is costly. We show that, even when agents’ trading needs are perfectly matched, costly participation prevents them from synchronizing their trades, which gives rise to the need for liquidity. Moreover, the endogenous liquidity need, when it occurs, can lead to market crashes in absence of any aggregate shock. We also show that the lack of coordination among agents in the demand and the supply of liquidity generates negative externalities, and the loss in social welfare can out-weigh the savings on participation costs.

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1. Introduction

It is well recognized that liquidity is of critical importance to the stability and the efficiency of the financial market.\(^1\) Yet, there is little consensus about exactly what liquidity is, what determines it, and how it affects asset prices.\(^2\) Market frictions have been considered as important determinants of liquidity and asset prices. But the precise nature of this link is not well understood due to the complexity in analyzing the interactions among diverse market participants under market frictions.\(^3\)

In this paper, we study how market frictions lead to market participants’ need for liquidity and how such a need can destabilize asset prices. We focus our attention on a specific form of market frictions, the cost to participate in the market.\(^4\) Participation costs prevent all agents from being in the market at all times. Their infrequent presence in the market causes non-synchronization in their trades even when their underlying trading needs are perfectly matched. Non-synchronized trades give rise to order imbalances and the need for liquidity. We show that this endogenous liquidity need, when it arises, generates excess sell orders of significant sizes. Consequently, it leads to large price drops in absence of any aggregate shock to the fundamentals. This result suggests that, in the presence of market frictions, endogenous surges in the need for liquidity can cause market crashes.

Two elements are essential for liquidity to be economically important: the need to trade and the cost to trade. In absence of any trading needs, there is no need for liquidity. In absence of any cost to trade, agents will trade in the market at all times in response to exogenous shocks. Although the equilibrium price does adjust to equate demand and supply, the market is perfectly liquid in the sense that the price reflects the “fair value” of the asset. Actual markets do not function in this “gigantic town meeting” style, as Grossman and Miller (1988) called it, where all potential buyers and sellers are present at all times and trades are conducted to balance the full demand and supply. Costs prevent potential participants from

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\(^1\)See, for example, CGFS (1999).


\(^4\)See, for example, Brennan (1975), Chatterjee and Corbae (1992), Hirshleifer (1988), and Merton (1987) for discussions of the importance of participation costs.
constantly being in the market. At any given instant, only a subset of traders are present in the market. When a trader arrives at the market, he only faces a “partial” demand/supply. Adjustments in prices fail to attract all potential buyers and sellers or to synchronize their trades. It is this non-synchronization in trading that gives rise to the need for liquidity, which in turn affects asset prices.

To formalize this intuition, we start with an economy in which potential traders receive idiosyncratic shocks to their total risk exposure. Those who receive positive shocks to their risk exposures are potential sellers while those receiving opposite shocks are potential buyers. They desire to trade in the market to offset their idiosyncratic shocks. Since idiosyncratic shocks always sum to zero across all potential traders, their trading needs are perfectly matched. In absence of participation costs, all traders are in the market at all times. Their trades, driven by off-setting shocks to their risk exposures, are fully synchronized. Sell orders are always accompanied by the same amount of buy orders. In this case, there is no need for liquidity and asset prices depend only on the “fundamentals”. The idiosyncratic trading needs of individual traders have no impact on prices.

In the presence of participation costs, however, potential traders will stay out of the market when expected gains from trading are small. They participate only when they are far away from their desired positions. The infrequent participation in the market has two consequences. First, potential traders, when hit by idiosyncratic shocks, cannot always offset them through trading. Having to bear some idiosyncratic risks, they become effectively more risk averse. As a result, their desired stock holdings decrease. This effect is independent of the realizations of idiosyncratic shocks. Second, given that their new desired holdings are lower than their initial holdings, potential traders who receive positive idiosyncratic shocks (i.e., potential sellers) becomes further away from their desired positions than those who receive negative shocks (i.e., potential buyers). The gains from trading are larger for potential sellers than for potential buyers. As a result, potential sellers are always more eager to enter the market than potential buyers, even though their idiosyncratic shocks perfectly match and they face the same participation costs. This asymmetry in participation between buyers and sellers implies that their trades are no longer synchronized, which gives rise to an order imbalance in the market and the need for liquidity. Moreover, the order imbalance is always skewed toward sell orders. Consequently, the stock price has to decrease in order to attract the market makers to absorb the excess sell orders.

Furthermore, we show that the order imbalance and the need for liquidity are highly nonlinear in the idiosyncratic shocks that drive agents’ trading needs. When the magnitude of idiosyncratic shocks is moderate, gains from trading are relatively small for all traders. As a result, they all stay out of the market and there is no need for liquidity. Only for large
enough idiosyncratic shocks, gains from trading exceed participation costs and potential traders may decide to enter the market. Thus, the order imbalance and the need for liquidity, when they occur, are often large in magnitude, causing the price to drop discretely in absence of any shocks to the fundamentals. As a result, “liquidity crashes,” which are market crashes driven purely by liquidity, can arise.

The inability of participants to coordinate their trades generates a negative externality. A trader who withdraws from the market to save costs not only gives up his own gains from trading but also reduces gains from trading for other traders. Thus, his withdrawal discourages other traders from participating in the market, which in turn further discourages the trader himself from participating. As a result of this negative externality, the overall welfare loss can be very high. In particular, it is possible that traders can be made better off if they are all forced to pay the cost to participate in the market at all times, rather than individually if they all have the option to participate only when the expected trading gain is larger than the participation cost.

Most of the existing literature on liquidity examines how market frictions affect the provision of liquidity and asset prices, given the need for liquidity.\(^5\) For example, Grossman and Miller (1988) consider the role of market makers in providing liquidity and reducing price volatility, taking as given the non-synchronization in trades. Such an approach ignores the fact that it is the same market frictions that give rise to the need for liquidity in the first place. Our analysis shows that endogenous liquidity need can lead to different predictions for its impact on prices and market efficiency.

In this regard, our paper shares the same spirit with Allen and Gale (1994), who consider the ex-ante participation decisions of agents with different future liquidity needs. They show that the ex-ante optimal level of participation can be inadequate ex post when the realized liquidity need is much larger than expected, causing additional volatility in prices. We focus more on the dynamic aspect of liquidity by allowing traders to make their participation decisions after observing new shocks to their trading needs over time. Thus, we are able to study how the need for liquidity rises endogenously in response to new idiosyncratic shocks to the traders. As we show, the properties of the endogenous liquidity need (e.g., one-sided and fat tailed) can be quite different from those assumed for exogenous liquidity shocks.\(^6\)

\(^5\)See, for example, Amihud and Mendelson (1980), Grossman and Miller (1988), Ho and Stoll (1981), Huang (2003), and Kyle and Xiong (2001). In most of the market micro-structure literature, which has liquidity as one of its central focus, the need for liquidity, as described by the order flow process, is often taken as given. See, for example, Glosten and Milgrom (1985), Kyle (1985), and Stoll (1985).

\(^6\)In addition, by making the trading needs perfectly matched between the traders, the liquidity we model purely comes from the non-synchronization in their trades. When the trading needs are not perfectly matched, there is also an aggregate shift in demand, which is the case in Allen and Gale (1988) as well as
Our model is closely related to the model of Lo, Mamaysky, and Wang (2004), who consider the impact of fixed transactions costs on trading volume and the level of asset prices. They show that, in a continuous-time stationary setting, gains from trading is in general asymmetric between traders with offsetting shocks when trading is infrequent. Such an asymmetry naturally leads to non-synchronization in trades, the need for liquidity, and price deviations. In order to focus on the impact of trading costs on price levels, they avoid order imbalances by allowing the participation cost to be allocated endogenously so that the trades of different market participants are always synchronized in equilibrium. As we show in this paper, it is the order imbalance that leads to changes in liquidity needs and the instability in asset prices.

By modelling costly participation, we capture an important aspect of the actual market. But the market structure we use, with the continuous presence of market makers, still takes the form of a centralized exchange. As Grossman and Miller (1988) point out, when the costs for such a market structure are significantly large, we may have to consider alternative market structures such as over-the-counter markets (see, e.g., Duffie, Gárleanu, and Pedersen (2004, 2005)).

The paper proceeds as follows. Section 2. describes the basic model. Section 3. solves for the intertemporal equilibrium of the economy. Section 5. analyzes how individual traders’ participation decisions give rise to the need for liquidity and the properties of equilibrium liquidity needs. In Section 5., we examine how the need for liquidity affects asset prices. In Section 6., we consider the welfare implications of liquidity. Section 7. considers the endogenous determination of liquidity provision in the market. Section 8. concludes. The appendix contains all the proofs.

2. The Model

We construct a model that captures several important factors in analyzing liquidity, especially, the need to trade and the cost to constantly participate in the market. For simplicity, we consider a finite-horizon setting, which can be embedded in an infinite-horizon framework. We maintain the parsimony of the model and return to extensions later in the paper.

in Campbell, Grossman, and Wang (1993), Campbell and Kyle (1993), and Kyle and Xiong (2001), among others, the distinction between shocks to liquidity and risk (and/or preference) becomes less clear.
2.1. Economy

A. Securities Market

There are four dates, \( t = 0, 1, 2, 3 \). Two securities traded in a competitive securities market, a bond and a stock. The bond gives a sure payoff of 1 at date \( t = 3 \) and the stock gives a risky terminal payoff \( V_3 \):

\[
V_3 = V + v_3
\]

where \( V \) is a positive constant and \( v_3 \) is a normal random variable with a mean of zero and a volatility of \( \sigma_v \). The bond will be used as the numeraire. Hence, its price is always one. The stock price at date \( t \) is denoted by \( P_t \). Absence of arbitrage insures that \( P_3 = V_3 \).

B. Agents

There is a continuum of agents in the economy. Each agent is initially endowed with zero units of the bond and \( \vartheta \) shares of the stock.

The agents consist of two types, who have different trading needs and face different trading costs. The first type of agents, denoted by \( m \), are “market makers”. They have no inherent trading needs, but are present in the securities market at all times, ready to trade with others. The second type of agents are “traders” who have inherent trading needs. There are two equal subgroups of traders, indexed by \( i = a, b \), respectively. They arrive at the market at date \( t = 1 \) and have to pay a cost to participate in the market. The population of the market makers and the traders are \( \mu \) and \( 2\nu \), respectively.

Agent \( i \) receives a non-traded income \( N^i_3 \) at date \( t = 3 \), where \( i = a, b, m \). The non-traded income is given by

\[
N^i_3 = X^i_2 n_3, \quad i = a, b, m
\]

\[
X^m_t = 0, \quad X^a_t = -X^b_t = X_t = X_{t-1} + x_t, \quad t = 1, 2
\]

where \( X_0 = 0 \), \( n_3 \) and \( x_t \) are mutually independent, normal random variables with mean of zero and volatility of \( \sigma_n \) and \( \sigma_x \), respectively. The non-traded income, \( n_3 \), is assumed to be correlated with the stock payoff \( v_3 \). For simplicity, we let \( n_3 = v_3 \).

The non-traded risk is purely idiosyncratic since \( \sum_{i=a,b,m} X^i_t = 0 \). Agent \( i \)'s idiosyncratic risk exposure at date \( t \), \( t = 1, 2 \), is given by \( X^i_t \). Specifically, the two groups of traders inherit offsetting idiosyncratic risks. In the absence of idiosyncratic risks, all agents are exposed to the same amount of aggregate risk (from their stock endowments) and there are no trading needs among them. In the presence of idiosyncratic risks, however, the two groups of traders...
will want to trade to share their risks. In particular, given the correlation between the non-traded risk and the stock payoff risk, they want to adjust their stock positions in order to hedge their non-traded risk. Thus, changes in the traders’ idiosyncratic risk exposures give rise to their inherent trading needs.\footnote{In our setting, heterogeneity in risk exposure is merely a device to introduce the need to trade for risk-sharing, as in Wang (1994), Huang and Wang (1997), Lo, Mamaysky, and Wang (2004). Other forms of heterogeneity among agents can also generate risk-sharing trading needs, such as difference in preferences (e.g., Dumas (1992) and Wang (1996)) or beliefs (e.g., Detemple and Murthy (1994)). Our modelling choice here is mainly motivated by tractability.}

By construction, the idiosyncratic risks sum to zero and $X_t^a = -X_t^b$. Thus, the traders’ underlying trading needs are perfectly matched. If all traders are present in the market at all times, a seller is always matched with a buyer and there is perfect synchronization in agents’ trades.

For tractability, we assume that all investors have the same constant absolute risk aversion (CARA) utility over their terminal wealth $W_t^i$ at date $t = 3$:

$$E \left[ -e^{-\alpha W_t^i} \right], \quad i = a, b, m. \quad (3)$$

Agent $i$’s terminal wealth consists of both his non-traded income and his financial wealth from his security holdings. For the model to be properly defined, we require $\alpha^2 \sigma_x^2 \sigma_v^2 < 1/2. \footnote{In the appendix, we show that if the constraint does not hold, the expected utility of traders becomes infinitely negative, which is unreasonable.}$

C. Trading Costs

The market makers can always trade in the market at no cost.\footnote{We later generalize the setting by introducing a setup cost for market makers and endogenously determine their participation decisions.} The traders, however, face a fixed cost $c \geq 0$ to participate in the market. There is a one-period lag between paying the cost and being able to trade in the market. This form of transactions costs is consistent with the costs of entering a market by setting up a trading operation, constantly monitoring the market, gathering and incorporating the information into their trading activities.\footnote{We introduce the lag mainly to emphasize that the participation cost we have in mind is more general than a simple fixed cost (such as commission fees). For the various costs mentioned above, it seems more sensible to allow a lag between participation decision and actual trading. Our results, however, do not depend on this lag. Also, we consider only the fixed setup costs since they are sufficient to generate non-synchronized trading and the need for liquidity.}

D. Time Line

At $t = 0$ and 1, only the market makers are in the market. At date $t = 1$, however, trader $i$, $i = a, b$, arrives and decides whether to pay a cost $c$ in order to trade at date $t = 2$. 

\[\]
Let $\eta_i^1$ be the discrete choice variable at $t = 1$ for trader $i$ of whether or not to participate, where $\eta_i^1 = 1$ denotes participation and $\eta_i^1 = 0$ denotes no participation. We use $\omega_i^1$ to denote the fraction of group $i$ traders who pay the participation cost (i.e., choose $\eta_i^1 = 1$) at date $t = 1$.

Let $\theta_i^t$ to denote the number of shares agent $i$ ($i = a, b, m$) holds after trading at date $t$. Before date 2, no trader is in the market and all agents simply hold their initial shares. Hence, $\theta_i^0 = \bar{\theta}$ for $i = a, b, m$ and $t = 0, 1$. At date $t = 2$, $\theta_i^t$ depends on $i$’s participation decision at date 1. In particular, $\theta_2^t(\eta_i^1 = 0) = \bar{\theta}$ for the nonparticipating traders, and $\theta_2^t(\eta_i^1 = 1)$ needs to be solved in equilibrium for the participating traders. The financial wealth $F_i^3$ at date 3 can be expressed as

$$F_i^3 = \bar{\theta}P_2 + \theta_2^t(\eta_i^1) (P_3 - P_2) - c^i \eta_i^1$$

where $c^i = c$ for $i = a, b$ and $c^i = 0$ for $i = m$. The total wealth at date 3 is then

$$W_i^3 = N_i^3 + F_i^3.$$  

Without confusion, we use $\theta_2^t$ to denote $\theta_2^t(\eta_i^1 = 1)$ in the future. The time-line of the economy is illustrated in Figure 1.

<table>
<thead>
<tr>
<th>Shocks</th>
<th>$X_1^i$</th>
<th>$X_2^i$</th>
<th>$V_3, N_3^i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choices</td>
<td>$\eta_1^i$</td>
<td>$\theta_2^i$</td>
<td></td>
</tr>
<tr>
<td>Equilibrium</td>
<td>$P_0$</td>
<td>$\omega_1^i, P_1$</td>
<td>$P_2$</td>
</tr>
</tbody>
</table>

Figure 1: The time line of the economy.

### 2.2. Definition of Equilibrium

The equilibrium of the economy requires three conditions. First, taking prices as given, all agents optimize with respect to their participation and trading decisions. Second, agents’ participation reaches an equilibrium. Third, the securities market clears.

At date 0 and 1, only the market makers are in the market. They determine the equilibrium price $P_0$ and $P_1$ through the market clearing condition $\theta_t^m = \bar{\theta}$.

At date 1, traders decide whether to pay a cost in order to trade next period. A participation equilibrium is reached if either all traders within the same group choose identical participation decisions (i.e., $\omega_i^1 = 0$ or 1), or they are indifferent between participating or not.

At date $t = 2$, both the market makers and the participating traders are in the mar-
ket. The total number of shares brought to the market by participating agents is \( \mu \bar{\theta} + \nu \sum_{i \in \{a,b\}} \omega_i \bar{\theta} \). Hence, the clearing of the securities market at date 2 requires that

\[
\mu \theta^m_2 + \nu \sum_{i \in \{a,b\}} \omega_i^2 \theta^i_2 = \left[\mu + \nu (\omega_a^i + \omega_b^i)\right] \bar{\theta}
\]  

(6)

which determines the securities market equilibrium at 2.

\subsection*{2.3. Equilibrium with Costless Participation}

Before solving for the equilibrium, we describe the case when participation costs are zero for all agents. This case serves as a benchmark when we examine the impact of participation costs on liquidity and stock prices. If \( c^i = 0 \forall i \), all traders and market makers always participate in the market, \( \omega_i^1 = 1 \forall i \). The equilibrium price and agents’ optimal trading policies are:

\[
P_3 = V_3, \\
P_2 = V - \alpha \sigma_v^2 \bar{\theta}, \quad \theta^i_2 = \bar{\theta} - X^i_2 \\
P_1 = V - \alpha \sigma_v^2 \bar{\theta}, \quad \theta^i_1 = \bar{\theta} \\
P_0 = V - \alpha \sigma_v^2 \bar{\theta}, \quad \theta^i_0 = \bar{\theta}
\]

(7)

where \( i = a, b, m \).

In this case, the price of the stock, \( P_t \), is determined by its expected payoff \( V \), the payoff risk \( \sigma_v \), and the aggregate (per capita) risk exposure \( \bar{\theta} \). We call these the “fundamentals” of the stock. \( P_t \) does not depend on the idiosyncratic risk exposure \( X^i_t \). Also, all traders choose to participate, and their order flows, as given by changes in \( \theta^i_t \) \((t = 1, 2)\), depend on changes in \( X^i_t \). Since their underlying trading needs are perfectly matched \((X^a_t = -X^b_t)\), so are their trades. The market is perfectly liquid in the sense that order flows have no price impact. The market makers perform no role in providing liquidity since there is no need for liquidity; and their holdings are always \( \theta^m_t = \bar{\theta} \).

\section*{3. Equilibrium}

We now solve for the equilibrium. First, taking the traders’ participation decisions at \( t = 1 \) as given, we solve the market equilibrium at \( t = 2 \). Next, we solve for individual traders’ participation policies \((\eta^i_t, i = a, b)\) and the participation equilibrium at \( t = 1 \) \((\omega^i_t, i = a, b)\), given the market equilibrium at \( t = 2 \). Finally, we solve the market equilibrium at \( t = 1 \) and \( t = 0 \).
3.1. Market Equilibrium at \( t = 2 \)

At \( t = 2 \), the state variables are \( \{\omega_i^t, X_i^t; i = a, b\} \), where \( \omega_i^t \) is the fraction of group \( i \) traders participating in the market at date 2 and \( X_i^t \) is the realization of his idiosyncratic risk exposure. For convenience, we define

\[
\delta_1 \equiv \frac{\nu(\omega_a^t - \omega_b^t)}{\mu + \nu(\omega_a^t + \omega_b^t)} \quad \text{and} \quad \delta_1^a = -\delta_1^b = \delta_1. \tag{8}
\]

The quantity \( \delta_1 \) measures the asymmetry in participation between group-\( i \) traders and their counterparts. For example, \( \delta_1^a > 0 \) means that more group-\( a \) traders are participating in the market than group-\( b \) traders. In fact, \( \delta_1 \) defines the market condition.

**Proposition 1.** The equilibrium price at date \( t = 2 \) is

\[
P_2 = V - \alpha \sigma_v^2 \bar{\theta} - \lambda_2 X_2 \tag{9}
\]

where \( \lambda_2 \equiv \alpha \sigma_v^2 \delta_1 \). The optimal stock holding for participating agent \( i \) is

\[
\theta_i^2 = \bar{\theta} - (1 - \delta_1)X_i^2, \quad i = a, b, m. \tag{10}
\]

When \( \delta_1 = 0 \), the participation of the two groups of traders is symmetric. Since their trades are driven only by the idiosyncratic shocks \( X_i^2 \), \( i = a, b \), and \( X_a^2 = -X_b^2 \), there is a perfect match in the buy and sell orders. As a result, the equilibrium price is not affected by the idiosyncratic shocks. Moreover, market makers hold their initial endowment \( \bar{\theta} \) and perform no role in providing liquidity. When \( \delta_1 \neq 0 \), the participation of the two trader groups becomes asymmetric. The idiosyncratic shock \( X_2 \) can affect the equilibrium price. Suppose, for example, \( \delta_1 > 0 \), i.e., more group-\( a \) traders participate than group-\( b \) traders. When \( X_2 > 0 \), \( X_a^2 = X_2 > 0 \) and \( X_b^2 = -X_2 < 0 \); trader \( a \) wants to sell while trader \( b \) wants to buy. Given that there are more group-\( a \) traders in the market, there will be more sellers than buyers. Consequently, the stock price has to decrease in order to attract the market makers to absorb the excess selling orders. Thus, even though traders face offsetting shocks, their asymmetric participation can give rise to a mismatch in their trades and cause the price to change in response to these shocks.

3.2. Optimal Individual Participation Decision at \( t = 1 \)

Given the market equilibrium at \( t = 2 \), we now consider the participation equilibrium at \( t = 1 \) in two steps. First, taking as a given the participation decision of other traders, we derive the optimal participation policy of an individual trader. Next, we find the competitive equilibrium for traders' participation decisions.
Trader $i$ makes his participation decision after observing his idiosyncratic shock $X_i^t$. He chooses to participate if and only if the expected gain from trading exceeds the cost of participation. Let $J^i_t(\eta^i_t = 1)$ and $J^i_t(\eta^i_t = 0)$ denote the value function of trader $i$ who chooses to participate or not to participate at date $t = 1$, respectively. Taking as given the participation decisions of other traders, we define the net gain from participation for trader $i$ by the corresponding certainty equivalence gain in wealth:

$$g(\theta^i_t, X^i_t, \delta^i_t) = -\frac{1}{\alpha} \ln \frac{J^i_t(\eta^i_t = 1)}{J^i_t(\eta^i_t = 0)}.$$  \tag{11}$$

The minus sign on the right-hand-side adjusts for the fact that $J^i_t < 0$. The optimal decision for trader $i$ is to participate if and only if $g(\cdot) \geq 0$. The following proposition describes the optimal participation policy for an individual trader.

**Proposition 2.** Given $X^i_t$, $\delta^i_t$, and $\theta^i_t$, trader $i$’s net gain from participation is given by

$$g(\theta^i_t, X^i_t, \delta^i_t) = g_1(\theta^i_t, X^i_t, \delta^i_t) + g_2(\delta^i_t) - c$$ \tag{12}$$

where

$$g_1(\theta^i_t, X^i_t, \delta^i_t) = h(\delta^i_t)(\theta^i_t - \hat{\theta}^i_t)^2$$ \tag{13a}$$

and

$$g_2(\delta^i_t) = \frac{1}{2\alpha} \ln \left[ 1 + (1-\delta^2_i)k/(1-k) \right]$$ \tag{13b}$$

and

$$k = \alpha^2 \sigma^2_x \sigma^2_v, \quad \hat{\theta}^i_t = \theta - \frac{1-\delta^i_t}{1-k \hat{\delta}^i_t}(k\hat{\theta} + X^i_t), \quad h(\delta^i_t) = \frac{\alpha \sigma^2_v (1-k \hat{\delta}^i_t)^2}{2(1-k)[1-k+k(1-\delta^i_t)^2]}.$$

Trader $i$ chooses to participate at date 1 iff

$$(\theta^i_t - \hat{\theta}^i_t)^2 \geq \left[ c - g_2(\delta^i_t) \right]/h(\delta^i_t).$$ \tag{14}$$

The net gain from participation consists of three terms. The first term, $g_1(\theta^i_t, X^i_t, \delta^i_t)$, represents expected gain from trading in response to the current shock $X^i_t$. This term is zero when current holding $\theta^i_t$ is equal to $\hat{\theta}^i_t$. Thus, we can interpret $\hat{\theta}^i_t$ as trader $i$’s desired stock holding at $t = 1$, given his current shock $X^i_t$ and the market condition $\delta^i_t$. The second term, $g_2(\delta^i_t)$, is the expected gain from offsetting future idiosyncratic shock, $x_2 = X_2 - X_1$. This term depends on the market condition $\delta^i_t$ (through its impact on $P_2$) and future trading needs, which is captured by $k$ and increases with the volatility of future shocks. The last term, $-c$, is simply the cost of participation.

Given the gain from participation, the optimal participation policy becomes very intuitive. When the participation cost is smaller than the expected gain from offsetting futures shocks, i.e., $c \leq g_2(\delta^i_t)$, (14) is always satisfied and trader $i$ always participates. The more
interesting case is when \( c > g_2(\delta_i^1) \) and trader \( i \) chooses to participate only if the gain from adjusting his current position is sufficiently large. This happens when his holding is sufficiently far away from the desired position, i.e., when \( |\theta_i^1 - \hat{\theta}_i^1| > \sqrt{[c - g_2(\delta_i^1)]/h(\delta_i^1)} \).

### 3.3. Participation Equilibrium at \( t = 1 \)

Given the individual participation policy, we now solve for the participation equilibrium. In the absence of participation costs, traders with positive idiosyncratic shocks want to sell the stock while traders with negative shocks want to buy. We thus refer to them as potential sellers and buyers, indexed by \( i \) and \(-i\), respectively. From Proposition 2, the gain from trading is always higher for sellers than for buyers. Because \( \theta_i^1 = \bar{\theta} \), we denote a trader’s net gain from participation by

\[
g_i^i(\delta_i^1) \equiv g(\bar{\theta}, X_i^i, \delta_i^1). \tag{15}
\]

Since traders’ trading needs are the same within the group and offsetting between groups, a trader’s gain from participation decreases as more traders from his own group participates but increases as more traders from the other group participates. In other words, \( g_i^i(\delta_i^1) \) decreases with \( \delta_i^1 \) and increases with \( \delta_{-i}^1 \) (since \( \delta_{-i}^1 = -\delta_i^1 \)). For convenience, we also define \( \bar{\delta} \equiv \frac{\nu}{\mu + \nu} \), which is the maximum asymmetry in participation between the two trader groups. Thus, \( g_i^i(-\bar{\delta}) \) gives maximum gain from participation for trader \( i \) and \( g_i^i(\bar{\delta}) \) gives the minimum.

**Proposition 3.** Given \( X_t \), there exists a unique participation equilibrium at date 1, which is fully specified as follows:

- **A.** If \( 0 \leq g_i^i(0) \leq g_i^i(\bar{\delta}) \), \( \omega_i^1 = \omega_{-i}^1 = 1 \) and all traders participate.
- **B.** If \( g_i^i(0) < 0 \) and \( g_i^i(0) \leq 0 \), \( \omega_i^1 = \omega_{-i}^1 = 0 \) and no trader participates.
- **C.** If \( g_i^i(-\bar{\delta}) < 0 < g_i^i(\bar{\delta}) \), \( \omega_i^1 = 1 \) and \( \omega_{-i}^1 = 0 \).

Otherwise, let \( \delta^* \) be the minimum \( \delta \in (0, \bar{\delta}] \) that violates \( g_i^i(-\delta) < 0 < g_i^i(\delta) \).

- **D.** If \( g_i^i(-\delta^*) = 0 \leq g_i^i(\delta^*) \), \( \omega_i^1 = 1 \) and \( \omega_{-i}^1 \in (0, 1) \).
- **E.** If \( g_i^i(-\delta^*) < 0 = g_i^i(\delta^*) \), \( \omega_i^1 \in (0, 1) \) and \( \omega_{-i}^1 = 0 \).

The partial participation levels \( \omega_{-i}^1 \) and \( \omega_i^1 \) in cases D and E are given in the appendix.\(^{11}\)

\(^{11}\)In Cases A and B, when the gain from trading is zero for a group of traders, their participation rate may not be unique, in which case we choose a particular value for convenience.
Case A and B describe two extreme situations. In Case A, all traders have (weakly) positive gains from trading when market participation is symmetric ($\delta_1 = 0$). Thus, they all choose to enter the market and the participation is indeed symmetric. In case B, the trading gain is negative for all traders when participation is symmetric. Hence, no participation is the only equilibrium outcome.

The other three cases fall between these two extremes, when the gain from participation is positive for potential sellers (group $i$) but negative for potential buyers (group $-i$) under symmetric participation (when $\delta_1 = 0$). In Case C, $g^i > 0$ and $g^{-i} < 0$ under all market conditions (i.e., for all $\delta_1$). As a result, all potential sellers participate but no buyers do. In Case D, however, more participation of potential sellers (i.e., an increase in $\delta^i_1$) can enhance the participation gain for potential buyers to eventually become positive. As a result, some buyers will be lured into the market, which in turn encourages more sellers to participate. An equilibrium is reached when all sellers and enough buyers are in the market so that the gain from further entry by buyers diminishes to zero. In Case E, participation of sellers is never sufficient to encourage any buyers to enter the market but enough to discourage other sellers from doing so at certain participation level. In equilibrium, only a fraction of potential sellers but no buyers do.

In all the cases other than A and B, more sellers, the group with higher trading gains, enter the market despite the fact that fewer buyers will do so. Their entry brings excess sell orders to the market and the need for liquidity, which is provided by the market makers.

3.4. Market Equilibrium at $t = 1$

At $t = 1$, only market makers are present in the market. Given the realizations of $X_1$, the participation equilibrium is fully determined in Proposition 3, which in turn determines the market equilibrium at $t = 2$. We can easily compute the market equilibrium at $t = 1$.

Proposition 4. Given $X_1$, the equilibrium price at $t = 1$ is

$$P_1 = V - \alpha \sigma_v^2 \bar{\theta} - \lambda_1 X_1 \quad \text{where} \quad \lambda_1 \equiv \alpha \sigma_v^2 \frac{\delta_1}{1+k \delta_1^2} \quad (16)$$

and $\delta_1$ is given by the participation equilibrium determined in Proposition 3.

When the participation is asymmetric between the two groups of traders, i.e., when $\delta_1 \neq 0$, $P_1$ depends not only on the fundamentals, i.e., $V$ and $\bar{\theta}$, but also on the idiosyncratic shock $X_1$. The quantity $\delta_1$ measures the degree of asymmetric participation between the two groups of traders and is fully determined by the realizations of $X_1$. We can interpret $\delta_1 X_1$ as the expected future order imbalances. Anticipating the order flow and its negative impact on
the price, the market makers demand an extra discount $\lambda_1 X_1$ on the stock for holding the stock at time 1.

3.5. Market Equilibrium at $t = 0$

At date 0, again only market makers are present in the market; and the equilibrium price can be calculated using their marginal valuation.

**Proposition 5.** The equilibrium price at date $t = 0$ is

$$P_0 = V - \alpha \sigma^2 \bar{\theta} - \lambda_0 \equiv \frac{E_0 \left[ \left( \sqrt{\frac{\lambda_1}{\delta_1}} e^{-\frac{1}{2} \alpha \lambda_1 \delta_1 X_1^2} \right) \lambda_1 X_1 \right]}{E_0 \left[ \sqrt{\frac{\lambda_1}{\delta_1}} e^{-\frac{1}{2} \alpha \lambda_1 \delta_1 X_1^2} \right]}.$$  

When participation is expected to be asymmetric, idiosyncratic shocks also affect the stock price at $t = 0$. As market makers expect to hold additional shares in the future when there is a mismatch in traders’ orders, they demand an extra discount $\lambda_0$.\(^{12}\)

4. Limited Participation and the Need for Liquidity

The equilibrium under costly participation shows several striking features. First, despite the fact that the two groups of traders have perfectly matching trading needs, their actual trades are not synchronized when participation in the market is costly. The non-synchronization in their trades gives rise to the need for liquidity in the market. A group of traders may bring their orders to the market while traders with off-setting trading needs are absent, creating an imbalance of orders. The stock price adjusts in response to the order imbalance in order to induce market makers to provide liquidity and to accommodate the orders. As a result, the price of the stock not only depends on the fundamentals (i.e., its expected future payoffs and total risk), but also depends on idiosyncratic shocks market participants face. In this section, we examine in more details these results and the economic intuition behind them.

4.1. Gains from Trading and Individual Participation Decisions

We start with the individual participation decision and the participation equilibrium. The traders’ participation decisions depend on the trade-off between the cost to be in the market and the gains from trading. The key result we have is that even though the two groups of

\(^{12}\)In a setting similar to ours, Lo, Mamaysky, and Wang (2004) provide a detailed analysis of this effect of liquidity on prices.
traders start with the same stock holdings and face off-setting idiosyncratic shocks, their gains from trading is different.

To understand the result, let us consider a simple situation when the participation is symmetric between the two groups, i.e., \( \delta_i^1 = 0 \). From Proposition 2, the gain from trading for trader \( i \) is \( g_1(\bar{\theta}, X_i^1, 0) + g_2(0) \). Since \( g_2(0) \) is the same for both traders, we only need to focus on \( g_1(\bar{\theta}, X_i^1, 0) \). The gain \( g_1(\bar{\theta}, X_i^1, 0) \) is fully determined by the distance between current stock holding \( \bar{\theta} \) and the desired holding \( \hat{\theta}_i^1 \), which reduces to \( (1-k)\bar{\theta} - X_i^1 \) when \( \delta_i^1 = 0 \).

It is important to note that, even when \( X_i^1 = 0 \), i.e., when the current idiosyncratic shock is zero, the gain from trading is not zero. The desired position, \( \hat{\theta}_i^1 = (1-k)\bar{\theta} \), is different from, in particular, lower than the initial position \( \bar{\theta} \). The reason that a trader now desires to hold less stock is as follows. Given the cost of participation, he only trades in the market at some times. As a result, he has to bear the idiosyncratic risk, at least sometimes. This extra risk he has to bear effectively reduces his risk tolerance and lowers his desired stock position. The percentage decrease in his desired position, which is \( k = \alpha^2\sigma_x^2\sigma_v^2 \), is proportional to the level of the idiosyncratic risk. Given that irrespective of a trader’s idiosyncratic shock, his desired stock position decreases with his effective risk tolerance, it becomes obvious why the gain from trading is asymmetric between the two groups of traders. A potential seller receives a positive shock to his risk exposure (\( X_i^1 > 0 \)). Thus, he becomes further away from his desired position than a potential seller who receives a negative shock. Consequently, the gain from trading is higher for the seller. Naturally, potential sellers are more likely to participate in the market than the buyers.

**Result 1.** The gain from trading is in general different between buyers and sellers even when their trading needs are perfectly matched.

It is important to recognize that Result 1 is a general phenomenon when trading is costly. When the traders can trade continuously, they will constantly maintain at the optimal position and the gains from trading is always symmetric for small deviations from the optimal position. Let \( u(\theta) \) denote the utility from holding \( \theta \), and let \( \theta^* \) be the optimal holding. Then, \( u'(\theta^*) = 0 \). For a small deviation \( x \) from the optimum, \( \theta = \theta^* + x \), the gain from trading is given by \( u(\theta^*) - u(\theta) \simeq -u''(\theta^*) x^2/2 \), which is symmetric for an opposite deviation \(-x\). Thus, at the margin, traders with offsetting shocks or trading needs always have the same gain from trading. This is no longer the case when trading is costly. Facing a cost to trade, traders no longer trade constantly. They only trade when the deviation from the optimal is sufficiently large. But for a finite deviation \( x = \theta - \theta^* \), \( u(\theta^*) - u(\theta^* + x) \neq u(\theta^*) - u(\theta^* - x) \) and the gains from trading are different between traders with perfectly matching trading
needs. Naturally, the asymmetry in gains from trading can in general lead to asymmetric participation between the traders. Moreover, in our setting sellers always have higher gains than buyers.

A trader’s gain from trading also depends on how many other traders are in the market. In particular, the gain from trading for an individual trader $i$ decreases with $\delta_i$. Since $\delta_i$ increases with $\omega_i^a$ and decreases with $\omega_i^{-a}$, we have the following result:

**Result 2.** The benefit from trading for a trader decreases with the population of participating traders from the same group and increases with the population of participating traders from the different group.

The fact that the gain from trading depends on the participation of other traders gives rise to the externality of a trader’s participation decision. As we will see later, this externality is an important driving force in the determination of participation equilibrium, liquidity and prices.

4.2. Non-Synchronized Trading and the Need for Liquidity

Given the individual trader’s entry policy, we now examine the participation equilibrium, which is stated in Proposition 3. Figures 2 shows the equilibrium participation decisions as a function of the idiosyncratic shock $X_1$. Panel (a) reports the fraction $\omega_i^a$ of traders within each group who choose to participate. The dotted line plots $\omega_i^a$ and the dashed line plots $\omega_i^{-a}$. Panel (b) reports the difference in participation ratio between the two groups of traders $\delta_1$, defined in equation (8), as a function of $X_1$. For a range of $X_1$ around 0, $\omega_i^a = 0$, that is, trader $a$ choose not to participate simply because the benefit from trading is too small. This is the no-participation region. It is worth pointing out that the no-participation region is not symmetric about 0, reflecting the fact that a trader’s gain from trading is asymmetric between positive and negative idiosyncratic shocks. As $X_1$ falls outside the no-participation region, $\omega_i^a$ starts to increase, indicating that more and more group $a$ traders choose to participate. Moreover, $\omega_i^a$ is larger when $X_1$ is positive. When $X_1$ exceeds certain thresholds, the gain from trading dominates the cost, all group $a$ traders choose to participate.

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13The gain from trading also depends on the initial position $\theta_i^1$. In our setting, $\theta_i^1 = \bar{\theta}$. In a stationary intertemporal setting, $\theta$ should be the optimal holding determined by the trader’s dynamic optimization problem. In a discrete setting like the one in this paper, $\theta_i^1$ is always different from $\bar{\theta}$ since the latter depends on the current shocks while the former does not. In setting similar to ours, Lo, Mamaysky, and Wang (2004) show that even in continuous-time the gain from trading is asymmetric around the optimal holding due to the fact that traders only trade infrequently.

14It is easy to show that $g_i'(\delta_i^1) \leq 0$. Since traders start with $\theta_i^1 = \bar{\theta}$ in our setting, $g_i(\bar{\theta}, X_i^1, \delta_i^1) = h(\delta_i^1)(\bar{\theta} - \bar{\theta})^2$. We can show that $h'(\delta_i^1) < 0$, and $(\bar{\theta} - \bar{\theta})^2$ decreases in $\delta_i^1$. Hence, $g_i$ decreases in $\delta_i^1$ as well.
and $\omega_a^1$ reaches 1. The participation rate of group $b$ traders, $\omega_b^1$, behaves similarly. In fact, $\omega_b^1$ is simply the mirror image of $\omega_a^1$ around the vertical axis because trader $a$ and $b$ face opposite idiosyncratic shocks. When $X_1 < 0$, the risk exposure of group-$a$ traders is below the average (which equals to the average endowment $\bar{\theta}$), their gains from trading is lower than that of group-$b$ traders. Thus, $\omega_a^1 \leq \omega_b^1$ and $\delta_1 < 0$, indicating that group $a$ traders are less likely to participate. When $X_1 > 0$, their total risk exposure is above average, and group $a$ traders are more likely to participate, or $\delta_1 > 0$.

(a) Participation Rate $\omega_a^1$ and $\omega_b^1$  
(b) Difference in Participation Rate $\delta_1$

Figure 2: Equilibrium Participation. The figure plots the equilibrium participation rate for the two trader groups for different values of idiosyncratic shock $X_1$. Panel A reports the equilibrium fraction of group $i$ traders who choose to participate, where the dotted line refers to trader $a$ ($\omega_a^1$) and the dashed line refers to trader $b$ ($\omega_b^1$). Panel B reports the difference in participation decisions, $\delta_1 = \nu(\omega_a^1 - \omega_b^1)/[\mu + \nu(\omega_a^1 + \omega_b^1)]$. Other Parameters are set at the following values: $\bar{\theta} = 1$, $\alpha = 4$, $\nu = \mu = \frac{1}{3}$, $c = 0.1$, $\sigma_v = 0.3$, and $\sigma_x = 0.5$.

To understand the nature of the participation equilibrium, we plot in Figure 3 the “phase diagram” for different realizations of the idiosyncratic shock $X_1$. Although the level of initial holdings $\bar{\theta}$ is not a state variable, it is an important parameter that impacts the equilibrium outcome. As indicated in equation (12), the quantity $k\bar{\theta} + X_1^1$, which measures the distance between the current holding $\bar{\theta}$ and the desired holding $\hat{\theta}_i^1$, is the main determinant of the gain from participation and hence the equilibrium decision. We only plot for the positive realizations of $X_1$ since the results for negative $X_1$ are symmetric.

There are five regions in the state and parameter space, corresponding to the five cases, A, B, C, D, and E in Proposition 3, respectively. When both $\bar{\theta}$ and $X_1^1$ are small, i.e., in the area close to the origin and below the downward sloping solid-line, the distance between current holding and the desired position $k\bar{\theta} + X_1^1$ is small and so is the gain from trading. As a result, no trader chooses to enter the market. This corresponds to case B in the proposition.

When either $\bar{\theta}$ or $X_1$ is large, and that $X_1$ is different enough from $k\bar{\theta}$, i.e., for the areas above the top upward sloping solid-line and below the bottom upward sloping solid-line, the distance $k\bar{\theta} + X_1^1$ is large for both traders, and so are the gains from trading. In equilibrium, all traders enter the market, i.e., $\omega_a^1 = \omega_b^1 = 1$ and we have Case A.
Whenever the value of $X_1$ is relatively close to $k\bar{\theta}$, i.e., in the area between the three solid lines, we have asymmetric participation between the two groups of traders. When $X_1$ is close to $k\bar{\theta}$, the distance $k\bar{\theta}+X_1^i$ is small for group-$b$ traders with $X_1^i=-X_1$, although it is large for group-$a$ traders with $X_1^i=X_1$. Thus, we have the situation where group $a$ traders are willing to enter the market while group-$b$ traders are less eager, leading to asymmetric participation between the two groups.

There are two possibilities under this situation. When $k\bar{\theta}+X_1$ is sufficiently large, in regions C and D, group-$a$ traders see large gains from trading, even if only with the market makers, and they will choose to enter, i.e., $\omega_a^1 = 1$. The high participation rate of group-$a$ traders increases the gains from trading for group-$b$ traders, given their offsetting trading needs. In region C, $k\bar{\theta}-X_1$ is too small and such an inducement does not increase the gain from trading for group-$b$ traders sufficiently. As a result, they remain out of the market. However, in region D where $k\bar{\theta}-X_1$ is not too small, the participation of group-$a$ traders increases the gains from trading sufficiently for group-$b$ traders so that at least some of them choose to participate as well. When $k\bar{\theta}+X_1$ is modest as in region E, the gain from trading is only large enough to induce a fraction of group-$a$ traders to participate. It is not enough, even with the participation of some group-$a$ traders, to induce any group-$b$ traders to enter the market. In this case, we have $0 < \omega_a^1 < 1$ and $\omega_b^1 = 0$.

Clearly, even with perfectly matching trading needs, the traders fail to synchronize their trades under costly participation. In particular, for certain realizations of the shock $X_1$, the participation is asymmetric, causing a mismatch in the buy and sell orders in the market. This order imbalance then creates the need for liquidity. Thus, we have the following result.

**Result 3.** *In equilibrium, participation can be asymmetric among traders even when their*
trading needs are perfectly matched, giving rise to non-synchronization in their trades and the endogenous need for liquidity.

Proposition 3 indicates that $\delta_i^i$ is always positive for trader $i$ with positive idiosyncratic shock $X_i^i$ (i.e., potential sellers) and negative for negative $X_i^i$ (i.e., potential buyers). We confirm the result in Figure 3 where $\omega_i^a \geq \omega_i^b$ for $X_i^a = -X_i^b = X_1 > 0$. Hence, $\delta_i^i X_1 = \delta_i^i X_i^i$ is always positive, and the aggregate order imbalance is always skewed towards sell orders. The result is summarized as follows:

**Result 4.** In absence of any aggregate shocks, potential sellers always participate more in the market than potential buyers in equilibrium. The aggregate order imbalance always takes the form of an excess supply.

5. **Liquidity and Equilibrium Stock Price**

Our analysis above suggests that participation costs prevent traders from always being in the market, and the benefit from trading is different for different traders. As a result, self interest fails to coordinate their participating decisions and synchronize their trades even when their trading needs perfectly match. This non-synchronization in trades gives rise to imbalances in asset demand and the need for liquidity. Such exogenous order imbalances are the starting point of Grossman and Miller (1988) and market microstructure models like Ho and Stoll (1981) and Glosten and Milgrom (1985). In our model, by explicitly modelling the motives and the costs to be in the market, we endogenously derive the order imbalance. In particular, we show that, in absence of any aggregate shocks, the order imbalance always takes the form of an excess supply rather than an excess demand. This directional prediction leads to interesting implications on equilibrium prices, which we now turn our attention to.

From Equations (9) and (16), the equilibrium stock price consists of two components: the risk-adjusted “fundamental value,” $V - \alpha\sigma^2 v \theta$, and the liquidity effect. Naturally, we focus on the second component,

$$\tilde{p}_t \equiv P_t - (V - \alpha\sigma^2 v \theta) = -\lambda_t X_t.$$  (18)

When traders face no participation costs, they all participate in the market and there is no need for liquidity. The stock price equals the fundamental value and does not depend on the idiosyncratic shocks individual traders face. Hence, the liquidity component $\tilde{p}_t = 0$, $t = 1, 2$. In the presence of participation costs, partial participation leads to non-synchronized trades among traders and the need for liquidity. The stock price has to adjust to attract the market
makers to provide the liquidity and to accommodate the trades. In general, $\tilde{p}_t \neq 0$ and the stock price becomes dependent on the idiosyncratic shocks of individual traders.

As Result 4 states, potential sellers are more willing to enter the market to sell the stock. Thus, the order imbalance, as captured by $-\delta_1 X_1$, is negative, which leads to a negative $\tilde{p}_t$, $t = 1, 2$. It is important to note that the sign of $\tilde{p}_t$ is independent of the sign of $X_1$. In other words, it does not depend on the distribution of idiosyncratic shocks among the traders. Therefore, we have the following result:

**Result 5.** The impact of liquidity needs always decreases asset prices.

The magnitude of the liquidity effect on price depends on $X_1$. Figure 4(a) plots $\tilde{p}_1$ against $X_1$. First, we note that at $t = 1$, both groups of traders receive their idiosyncratic shocks but are not trading in the market. However, in anticipation of their trades at date 2 and the need for liquidity due to the order imbalances, the current price already adjusts. Second, the liquidity effect on the price is always negative, as mentioned before. Third, the impact of liquidity on the stock price is highly non-linear in $X_1$, the idiosyncratic shocks to the traders. In particular, for small values of $X_1$, gains from trading are small for all traders and they do not enter the market. As a result, there is no need for liquidity and price equals the fundamental. For large values of $X_1$, gains from trading are sufficiently large for all traders and they all enter the market. As a result, their traders are synchronized and there is no need for liquidity. The stock price also equals the fundamental. For intermediate ranges of $X_1$, the gains from trading are large enough for some traders to enter the market, but not for all traders. It is in this case when trades are non-synchronized and liquidity is needed in the market, which will in turn affect the stock price. As Figure 4 shows, the price impact of liquidity reaches the maximum for a certain magnitude of the idiosyncratic shock.

The result that the price impact of liquidity is one-sided and highly non-linear arise from the fact that liquidity needs are endogenous in our model. In most of the existing models of liquidity, such as Grossman and Miller (1988), liquidity needs are exogenously specified. Consequently, its price impact is linear in the exogenous liquidity needs and symmetrically distributed. Our analysis shows that modelling the liquidity needs endogenously is important to understand the impact of liquidity on prices. After all, it is the same economic factor, namely, the cost to participating in the market, that drives both the liquidity needs of the traders and the liquidity provision of market makers.

The non-linearity in the price impact of liquidity leads to another interesting result: large and frequent price movements in absence of any aggregate shocks. Figure 4(b) plots the probability distribution of $\tilde{p}_1$. As a benchmark, when participation is costless, the idiosyncratic shock $X_1$ does not affect stock prices and there is no liquidity effect. The
(a) Liquidity Factor in Price ($\tilde{p}_1$) (b) Probability Distribution of $\tilde{p}_1$

Figure 4: The liquidity factor in price. The figure reports the date $t=1$ liquidity factor, $\tilde{p}_1 \equiv P_1 - (V - \pi \bar{\theta})$, which captures the price movement in excess of the “fundamentals”. Panel (a) plots the conditional liquidity factor as a function of the idiosyncratic liquidity shocks ($X_1$). Panel (b) reports the probability density function of $\tilde{p}_1$, except at the point $\tilde{p}_1 = 0$ where the value corresponds to the total probability mass at the point (since the density function should be infinity at the point). For ease of exposition, except at the point of $\tilde{p}_1 = 0$, we scale down the probability density function by a factor of 200 in the plot. Other parameters are set at the following values: $\bar{\theta} = 1$, $\alpha = 4$, $\nu = \mu = \frac{1}{3}$, $c = 0$, $\sigma_v = 0.3$, and $\sigma_x = 0.5$.

distribution is simply a delta-function at zero. When traders face costs to participate in the market, however, the stock price also depends on the idiosyncratic shock $X_1$. Moreover, even though the underlying idiosyncratic shocks that drive the individual traders’ trading needs are normally distributed, their price impact as measured by $\tilde{p}_1$ is always negative and has a fat-tailed distribution. Aside from a non-zero probability mass at the origin, the distribution peaks at a finite and negative value, reflecting the fact that liquidity becomes important and affects the price for a range of finite shocks. Moreover, the impact of liquidity gives rise to the possibility of a large price movement in absence of any shocks to the fundamentals of the stock. Since such a price movement is associated with a large imbalance in trades and a surge of liquidity needs, it is a market crash driven purely by liquidity needs. We call it “liquidity crashes.” Summarizing the results above, we have the following:

**Result 6.** The impact of liquidity can lead to “liquidity crashes” in which large price drops occur in absence of any shocks to the fundamentals.

### 6. Externality in Trading and Welfare

The participation equilibrium as described in Proposition 3 has the interesting feature that it is always a corner equilibrium. That is, the participation rate is either zero or one for at least one trader group. This result is driven by the externality in traders’ participation decisions. In particular, by trading in the market, a trader generates a positive externality since he provides liquidity to other traders, especially those with off-setting trading needs. On the
contrary, withdrawing from the market when faced with a cost brings negative externality as it reduces the market liquidity and the incentive for other traders to participate in the market. In this section, we examine how the externality in traders’ participation decisions gives rise to the corner nature of the participation equilibrium and its welfare implications.

6.1. Externality of Individual Participation

To analyze the externality of individual participation, we plot in Figure 5 the gains from trading for the two groups of traders as a function of $\delta_1$, the relative excess participation of group-$a$ over group-$b$ traders, where the solid line is for group $a$ and the dashed line is for group $b$. Consistent with Result 2, the gain from trading, say, for group-$a$ traders, decreases as more of them participate (i.e., when $\delta_1$ becomes more positive), but increases as more group-$b$ traders participates (i.e., when $\delta_1$ becomes more negative). The fact that the participation of one group of traders can increase the participation of the other group of traders reflects the externality of their participation decisions and determines the nature of the equilibrium.

In Figure 5(a), at $\delta_1 = 0$ the gain from trading is negative for both groups of traders. Consequently, no traders choose to enter the market (Case B in Proposition 3). In Figure
5(b), at $\delta_1 = 0$ the gain from trading is positive for group-\textit{a} traders but negative for group-\textit{b} traders. Thus, a subset of group-\textit{a} traders will choose to enter the market. However, for $\delta_1$ sufficiently large, the gain from trading remains negative for group-\textit{b} traders and they remain out of the market. Given that no group-\textit{b} traders enters the market to provide additional liquidity, the gain from trading quickly diminishes as more group-\textit{a} traders enter the market to consume the limited amount of liquidity the market makers provide. As a result, we have the situation in which only a fraction of group-\textit{a} traders enter to trade with the market makers while no group-\textit{b} traders enter. This corresponds to Case E in Proposition 3.

Figure 5(c) best illustrates the positive participation externality between the two groups. At $\delta_1 = 0$, the gain from trading is positive for group-\textit{a} traders and negative for group-\textit{b} traders. However, as group-\textit{a} traders enter the market, the gain from trading starts to increase for group-\textit{b} traders and quickly becomes positive. This induces group-\textit{b} traders to enter the market. Moreover, their participation in the market keeps the gain from trading positive for group-\textit{a} traders, which lures more group-\textit{a} traders into the market. The positive feedback stops only when all group-\textit{a} are in the market. The equilibrium is reached when enough group-\textit{b} traders enters the market such that the gain from trading becomes zero. This corresponds to Case D in which $\omega_1^a = 1$ and $0 < \omega_1^b \leq 1$. The positive externality is reflected by the fact that the equilibrium is reached at a level of participation higher than the level when either group is alone in the market. Obviously, without the participation of group-\textit{a} traders, it is not feasible for any group-\textit{b} traders to participate since their gain only becomes positive when there are more group-\textit{a} traders in the market (when $\delta_1^a > 0$). Interestingly, without the participation of group-\textit{b} traders, it is not feasible for all group-\textit{a} traders to participate either, since the trading gains for group-\textit{a} trader is negative at the point $\delta_1^a = \bar{\delta} = \frac{1}{2}$, which corresponds to the case when $\omega_1^a = 1$ and $\omega_1^b = 0$. Therefore, some group-\textit{a} traders would have optimally stayed out of the market had all group-\textit{b} traders chosen not to participate. Figure 5(d) shows the simple situation when gain from trading is positive for both groups at $\delta_1 = 0$, in which case they all participate in the market. The result corresponds to Case A in Proposition 3 where $\omega_1^a = \omega_1^b = 1$.

The discussion above illustrates how the externality of traders’ participation decisions gives rise to the corner nature of the equilibrium.

6.2. Welfare Implications

In the market equilibrium we consider, individual traders’ participation decisions are based on their own trading gains, which do not take into account their impact on social gains. Thus, the resulting outcome may not be socially optimal. To understand the welfare implication of
the individual participation decisions, we compare the welfare achieved by different agents in the market equilibrium, in which traders can choose to participate, with that in the equilibrium in which all traders are forced to participate, i.e., to all pay the cost $c$ at date 1 so they can all trade at date 2. Here, the welfare of an agent is measured by the certainty equivalence of his ex-ante value function.\footnote{Let $J^i_0$ and $J^i_{0,FP}$ denote the ex ante value function agent $i$ achieves in the market equilibrium and the forced-participation equilibrium, respectively, where $i = a, b, m$. Given the form of the value function in both cases, which is negative exponential in financial wealth, the certainty equivalence of agent $i$’s value function is defined as follows: $CE^i = -\ln (-J^i_0) / \alpha$ and $CE^i_{FP} = -\ln (-J^{i}_{0,FP}) / \alpha$, $i = a, b, m$.} In particular, we examine the “welfare gain under forced participation”, defined by

$$\Delta^i \equiv CE^i_{FP} - CE^i_i, \quad i = a, b, m$$

where $CE^i_i$ and $CE^i_{FP}$ denote agent $i$’s welfare in the market equilibrium and in the forced participation equilibrium, respectively. Since the certainty equivalence measures the ex-ante expected utility, it is the same for both groups of traders, but different between traders and market makers.

Market makers do not pay participation costs but are always in the market. When all traders are forced to participate, their trades are perfectly matched and market makers have no role to play. When, instead, traders optimally choose when to participate, non-synchronous participation arises. Market makers then provide liquidity service and are compensated for it. Apparently, they are worse off when traders are forced to participate, i.e., $\Delta^m \leq 0$.

Since $CE^i_0$ measures the welfare achieved when traders choose their participation optimally, one might expect that they will be worse off when forced to participate unconditionally, i.e., $\Delta^i$ should always be negative for $i = a, b$. This is certainly true if we ignore the externality generated by traders’ participation decisions. After observing their own shocks, traders can avoid the participation cost if the gain from trading is small. However, the withdrawal from the market by some traders will have a negative impact on market liquidity and the gains from trading for other traders. Despite the price adjustment, the potential gain from trading cannot always be fully internalized and appropriately allocated among traders in a competitive market. As a result, the market equilibrium can fail to achieve what is socially optimal. In fact, for certain parameter values, we find that $\Delta^a$ and $\Delta^b$ become positive, indicating that traders are ex-ante better off when forced to always participate rather than individually making the choice by themselves. Moreover, the welfare gains for the traders can significantly out weight the losses for market makers.

Figure 6 plots the welfare gains for individual agents under forced participation relative
to that under optimal participation. Panel (a) reports the welfare gains for a trader ($\Delta^a = \Delta^b$), the solid line, and a market maker ($\Delta^m$), the dotted line, for different values of the participation cost $c$. Panel (b) reports the social welfare improvement, $2\nu\Delta^a + \mu\Delta^m$, which is the weighted average of welfare gains for all agents.

The difference between the forced and the optimal participation equilibrium comes from the balance between paying the participation cost when ex-post trading needs are small and the additional risk sharing benefit with more counter-parties when ex-post trading needs are large. As shown in Proposition 3 and discussed in Section 4., when the cost is small, all traders will choose to participate unconditionally. The equilibrium under optimal participation is identical to that under forced participation. Thus, $\Delta^m = \Delta^a = 0$, as Figure 6(a) shows. When the cost is prohibitively high, traders are clearly worse off if forced to participate, and $\Delta^a < 0$. The interesting case is for intermediate levels of the participation cost. Some traders optimally choose not to participate when the expected gain from trading is small. Their withdrawal from the market reduces the risk sharing capacity in the market. As Figure 6 shows, there exists a range of participation costs when the welfare gain of forced participation is positive for the traders, indicating that the benefit of improved risk sharing dominates the cost of participation. Although, as previously discussed, the market makers are always worse off, socially it is still beneficial to force participation when the gains to the traders are sufficiently large. This is shown Figure 6(b). The positive social welfare gain of forced participation illustrates a situation of market failure. Optimal behavior at the individual level within a conventional market setting does not necessarily lead to optimal risk-sharing. In summary, we have

**Result 7.** In a competitive equilibrium, the inability of individual traders to coordinate their participation choices can generate negative externalities. The social welfare loss can outweigh the total participation costs.
Despite the simplicity of our setting, the conclusion that a competitive market fails to coordinate potential traders under costly participation is rather general: Each market participant not only benefits from his own trades but also brings liquidity to the market. Bearing the full cost alone, each trader may not be able to effectively internalize the benefit he creates for the market. As a result, the traders’ participation decisions, while optimal at the individual level, may well be socially sub-optimal.\textsuperscript{16}

7. Endogenous Participation of Market Makers

So far, we have treated the population of market makers in the market as given. But as shown in Grossman and Miller (1988), their presence is also endogenous, depending on the cost of participating in the market. We now extend our model to incorporate the market makers’ participation decisions in the overall equilibrium. In general, market makers face both an initial cost of setting up the operation and an on-going cost for running the business. For simplicity, we consider only on the up-front cost. We assume all potential market makers (with total population weight $\bar{\mu}$) can pay a cost $c^m$ at time 0 to be in the market at all times ($t = 0, 1, 2$) or simply stay out.\textsuperscript{17}

The solution of the equilibrium with endogenous participation of market makers is a straightforward extension of our previous solution. Let $\omega^m$ denote the fraction of potential market makers participating in the market. Figure 7 reports the equilibrium $\omega^m$ for different values of their participation cost $c^m$, given traders’ participation cost $c$. The solid line is for $c = 0.15$, the dashed line is for $c = 0.09$ and the dotted line is for $c = 0.25$.

To be specific, let us consider the case when $c = 0.15$, i.e., the solid line in Figure 7. When $c^m$ is small, all potential market makers will be in the market (i.e., the participation rate is 1). When $c^m$ becomes large enough, the participation rate falls below 1. It continues to decrease with $c^m$ until a threshold. At a threshold value of $c^m = 0.0044$, the participation

\textsuperscript{16}Allen and Gale (1988) considered situations when agents need to pay a fixed cost to create market for new securities. Duffie and Jackson (1989) considered the introduction of new futures contracts. In these cases, they also show that the free-rider problem can lead to sub-optimal outcomes of the economy. However, the situation considered in these papers are different from the situation here. There the decisions made by the agents, what securities to introduce into the market, are macroscopic by nature. The externality involved is more explicit as changes in market structure can drastically change the equilibrium and the allocation. In our case, the participation decision of individual traders is microscopic by nature. The externality is more implicit and endogenous. It arises from the interaction among the agents through the feedback of their individual actions on each other.

\textsuperscript{17}The assumption that market makers decide to participate at time 0 instead of time 1 is intended to capture the characteristic of a market maker as someone who maintains a presence in the market at all times.
rate reaches 0.4 and then drops drastically to zero as \( c_m \) goes beyond 0.0044.\(^{18}\) This suggests that the market making capacity in the market can be very sensitive to small changes in the participation cost of market makers. This result is in contrast to that in Grossman and Miller (1988), where the benefit for market makers decrease smoothly with their total population, and the number of market makers decreases gradually as the cost increases. The difference comes from how liquidity needs are modelled. Grossman and Miller (1988) take the liquidity need as exogenously given. We, however, model the liquidity need endogenously, together with the endogenous liquidity provision by the market makers. When there are few market makers, traders can only expect to trade with each other. As a result, the participation decisions of the two groups of traders become highly correlated and their trades become more synchronized. Their synchronized trading reduces potential order imbalances and further diminishes the need for market makers. Such an interaction between endogenous liquidity needs and market makers’ participation decisions puts a lower bound on the viable market making capacity as Figure 7 illustrates.

Another interesting feature of the equilibrium market making capacity, as measured by the fraction of participating market makers, is that it is not monotonic in the participation cost for traders, \( c \). Figure 7 shows that market makers participate more when \( c = 0.15 \), the solid line, than when either \( c = 0.09 \), the dashed line, or 0.25, the dotted line. This result is intuitive. When traders face small cost \( c = 0.09 \), they participate most of the time, and hence, the need for liquidity is small. Similarly, when cost is large \( (c = 0.25) \), all traders stay out of the market most of the time and the need for liquidity is again small. In both

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\(^{18}\)Strictly speaking, when the participation rate of potential market makers reaches zero, the nature of the equilibrium, if it exists, becomes different from the type of equilibrium we have been analyzing, which assumes the presence of some market makers. We can avoid this change of equilibrium by assuming that there is always market makers present, with arbitrarily small population.
cases, the need for market makers is smaller and so is their participation rate.

It is worth pointing out that the threshold participation cost for the market makers is much lower than the participation cost for the traders. For example, at a cost of $c = 0.15$ for traders, the market makers stops participating when their participation cost is as low as $c^m = 0.0044$. The difference in magnitude of the two participation costs comes from two reasons. First, traders have inherent trading needs (due to idiosyncratic shocks) while market makers do not. Second, traders make their decision after observing their trading needs, while market makers make their decisions before observing the liquidity need, which arises only in certain states. For these two reasons, the ex ante expected gain from being in the market becomes very low for potential market makers and so is the threshold cost for participating.\(^{19}\)

8. Conclusion

In this paper, we show that frictions such as participation costs can induce non-synchronization in agents’ trades even when their trading needs are perfectly matched. Each trader, when arriving at the market, faces only a partial demand/supply of the asset. The mismatch in the timing and the size of trades creates temporary order imbalances and the need for liquidity, causing asset prices to deviate from the fundamentals. Purely idiosyncratic liquidity shocks can affect prices, introducing additional price volatility. Moreover, the price deviations tend to be highly skewed and of large sizes. In particular, the shortage of liquidity always causes the price to decrease and when it happens, the price tends to drop significantly, resembling a crash due to a sudden surge in liquidity needs. We further show that partial participation in the market by a subset of traders can have important welfare implications. The withdrawal of some traders from the market reduces market liquidity which further reduces the incentives for others to participate in the market. The fact that agents cannot fully internalize the benefit from their participation and liquidity provision leads to sub-optimal allocations in the economy despite the optimizing behavior at the individual level.

\(^{19}\)The discussion above merely provides the intuition for the low threshold cost for market makers within the context of our model. It remains an empirical question whether the large difference in the costs between traders and market makers, if interpreted literally, is reasonable empirically. An alternative way to justify the cost difference is to allow difference in risk tolerance. Several authors (see, for example, Kyle and Xiong (2001) and Vayanos (2004)) have argued that traders are effectively much more risk averse than the market makers during liquidity events, due either to their wealth constraints or to additional risk exposures from the rest of their portfolios. If this is true, we can derive comparable threshold participation costs for traders and potential market makers, and our qualitative results regarding the equilibrium market-making capacity will remain the same.
Appendix

Proof of Proposition 1.

Given any stock price $P_2$, the participating agent $i$’s expected utility is given by

$$J^i_2(\eta^i_1 = 1) = \max_{\theta^i_2} \mathbb{E}_2 \left[ -e^{-\alpha W^i} \right] = \max_{\theta^i_2} -e^{-\alpha \left[ F^i_2-c^i+N^i_2(V-P_2)-\frac{1}{2}\sigma^2_0(\theta^i_2+X^i_2)^2 \right]}.$$

His optimal stock holding is easily calculated by solving the first order condition with respect to $\theta^i_2$ for all agents,

$$\theta^i_2 = \frac{1}{\alpha\sigma^2_0} (V - P_2) - \bar{\theta} - X^i_2, \quad i = a, b, m.$$

The market clearing condition (6) implies an equilibrium price at date 2 of

$$P_2 = V - \alpha\sigma^2_0 \bar{\theta} - \frac{\alpha\sigma^2_0}{\mu + \nu(\omega^a_1 + \omega^b_1)} \left( \nu \sum_{i \in \{a,b\}} \omega^i_1 X^i_2 \right).$$

Further plugging in the definition of liquidity shock $X^a_2 = -X^b_2 = X_2$ and $X^m_2 = 0$ yields the result.

Proof of Proposition 2.

Calculating the expected value function for the participating and non-participating traders, $J^i_2(\eta^i_1 = 1)$ and $J^i_2(\eta^i_1 = 0)$, and plugging into equation (11) yield the result. Trader $i$ chooses to participate in the market if and only if gains from trading is positive, or $g(\theta^i_1, X^i_1, \delta^i_1) > 0$.

Proof of Proposition 3.

Case A describes a situation when all traders have (weakly) positive gains from trading if market participation rate is symmetric ($g^i(0) \geq 0, \forall i$). Since both $g_1$ and $g_2$ in equation (13) are decreasing functions of $\delta^i_1$, $g^i(\delta^i_1)$ is a decreasing function of $\delta^i_1$. If in equilibrium the participation is not symmetric, in particular, there are more group $j$ (for any $j = a$ or $b$) traders participating. Then $\delta^{-j}_1 < 0$, and

$$g^{-j}(\delta^{-j}_1) > g^{-j}(0) \geq 0.$$

Hence, more group $-j$ traders would like to participate in equilibrium. The only equilibrium is reached when $\delta_1 = 0$ and all traders participate.

Similarly in case B, the trading gain is negative for all traders when participation is symmetric. Hence, no participation is the only equilibrium outcome.
If \( g^{-i}(0) < 0 < g^i(0) \), let \( \delta = \frac{\nu}{\mu + \nu} \). Then it corresponds to the value of \( \delta^i \) when \( \omega^i = 1 \) and \( \omega^{-i} = 0 \), the case \( \delta^i = \bar{\delta} \) reflects a market situation when only traders from group \( i \) participate, hence is the least favorable market condition for trader \( i \). In contrast, it also is the most favorable market condition for an individual trader \( -i \) since he can unload all his idiosyncratic risks to group \( i \) traders in the market. If \( g^{-i}(-\bar{\delta}) < 0 < g^i(\bar{\delta}) \), then traders of group \( i \) have positive gains under the least favorable market condition while traders of group \( -i \) have negative gains even under the most favorable market condition. Naturally, the only equilibrium outcome is that all group \( i \) traders participate and no one from group \( -i \) participates. This is case C.

If on the other hand, the condition \( g^{-i}(\bar{\delta}) < 0 < g^i(\bar{\delta}) \) is violated. Since this condition is satisfied at \( \delta = 0 \) and violated at \( \delta = \bar{\delta} \), and that \( g^i(\delta^i) \) (and \( g^{-i}(-\delta^i) \)) are monotonically decreasing (and increasing) continuous functions in \( \delta^i \), there always exists a \( \delta^* \in (0, \delta] \) that violates the condition. As we gradually increase \( \delta \) from 0 until the condition is violated, either case D or E has to be true.

We first solve the following equation to derive the optimal \( \delta^* \),

\[
g^{-i}(-\delta^*) = 0.
\]

If \( g^i(\delta^*) > 0 \), then we have case D and \( \omega^i = 1 \) in equilibrium. The equilibrium \( \omega^{-i} \) can be derived by solving \( \delta^* = \frac{\nu(1-\omega^{-i})}{\mu + \nu(1+\omega^{-i})} \). Hence, the equilibrium participation is

\[
\omega^i = 1, \quad \omega^{-i} = \max \left[ 0, \min \left[ 1, \frac{\nu + (\mu + \nu)\delta^*}{\nu(1-\delta^*)} \right] \right].
\]

If instead \( g^i(\delta^*) < 0 \), then we have case E. We need to solve

\[
g^i(\delta^{**}) = 0.
\]

for the optimal \( \delta^{**} \). Clearly, \( g^{-i}(-\delta^{**}) < 0 \) in this case and the optimal \( \omega^{-i} = 0 \) in equilibrium. The equilibrium \( \omega^i \) can be derived by solving \( \delta^{**} = \frac{\nu(\omega^i - 0)}{\mu + \nu(\omega^i + 0)} \). Hence, the equilibrium participation is

\[
\omega^i = \max \left[ 0, \min \left[ 1, \frac{\mu\delta^{**}}{\nu(1-\delta^{**})} \right] \right], \quad \omega^{-i} = 0.
\]

**Proof of Proposition 4.**

Given the realization of shock \( X_1 \), individual participation decisions are fully determined in Proposition 3, and so is the equilibrium price \( P_2 \) in equation (9). We can calculate the
expected value function of the market maker at date \( t = 1 \) if they choose to hold \( \theta_1^m \) stocks:

\[
J_1^m = E[J_2^m \mid \theta_1^m, P_1, X_1] = -\frac{1}{\sqrt{1 + k\delta_1^2}} e^{-\alpha F_1^m + \theta_1^m (V - P_1) - \frac{1}{2} \alpha \sigma_\theta^2 (\theta_1^m)^2 + \frac{1}{2} \alpha \sigma_\theta^2 (\theta_1^m - \delta_1 X_1)^2}
\]

where \( F_1^m \) is total financial wealth at beginning of period. Taking first order condition with respect to \( \theta_1^m \) and applying market clearing condition \( \theta_1^m = \bar{\theta} \) yields the equilibrium price \( P_1 \).

**Proof of Proposition 5.**

The expected value function of the market maker at date \( t = 0 \) if they choose to hold \( \theta_0^m \) stocks:

\[
J_0^m = E[J_1^m \mid \theta_0^m, P_0] = E \left[ -\sqrt{\lambda_1/(\alpha \sigma_\theta^2 \delta_1)} e^{-\alpha [\theta_0^m (V - P_0 - \alpha \sigma_\theta^2 \bar{\theta} - \lambda_1 X_1) - \frac{1}{2} \alpha \sigma_\theta^2 \bar{\theta}^2 + \frac{1}{2} \delta_1 \lambda_1 X_1^2]} \right].
\]

The first order condition with respect to \( \theta_0^m \) is

\[
\frac{\partial J_0^m}{\partial \theta_0^m} = E \left[ -\alpha \sqrt{\lambda_1/(\alpha \sigma_\theta^2 \delta_1)} (V - P_0 - \alpha \sigma_\theta^2 \bar{\theta} - \lambda_1 X_1) e^{-\alpha [\theta_0^m (V - P_0 - \alpha \sigma_\theta^2 \bar{\theta} - \lambda_1 X_1) - \frac{1}{2} \alpha \sigma_\theta^2 \bar{\theta}^2 + \frac{1}{2} \delta_1 \lambda_1 X_1^2]} \right].
\]

Solving the first order condition and plugging in \( \theta_0^m = 0 \) in equilibrium, we have

\[
P_0 E \left[ -\sqrt{\lambda_1/(\alpha \sigma_\theta^2 \delta_1)} \left( V - \alpha \sigma_\theta^2 \bar{\theta} - \lambda_1 X_1 \right) e^{\frac{1}{2} \alpha \sigma_\theta^2 \bar{\theta}^2 - \frac{1}{2} \alpha \delta_1 \lambda_1 X_1^2} \right] = E \left[ -\sqrt{\lambda_1/(\alpha \sigma_\theta^2 \delta_1)} \left( V - \alpha \sigma_\theta^2 \bar{\theta} - \lambda_1 X_1 \right) e^{\frac{1}{2} \alpha \sigma_\theta^2 \bar{\theta}^2 - \frac{1}{2} \alpha \delta_1 \lambda_1 X_1^2} \right].
\]

Note that only \( X_1 \) is random variable at time 1. Solving for \( P_0 \) yields the result.
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