Overinvestment and Corporate Fraud in Efficient Capital Markets

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Abstract

Overinvestment in certain firms or sectors induced by corporate fraud, where informed insiders strategically manipulate outside investors’ beliefs by exaggerating financial performance and economic prospects, has been endemic historically, and has recently attracted much attention. Building on the coordination problem faced by dispersed and changing owners of widely-held firms, we present a theoretical framework that reconciles corporate fraud and overinvestment with efficient capital markets. Shareholders attempt to design managerial compensation contracts to both elicit valuable information from insiders and to ameliorate their own investment coordination problem. However, for a wide range of conditions, the optimal contract induces overstatements by managers, i.e., exaggerated disclosures regarding future investment opportunities, and subsequent overinvestment by rational investors. Our framework helps explain why instances of corporate fraud and overinvestment tend to follow the introduction of new technologies, concentrate in industries with valuable growth options, and intensify when firms have better access to external capital markets. We also link managerial career concerns to the likelihood of overinvestment, compare information precision in internal versus external capital markets, and provide a new perspective on the design of corporate charters.

Keywords: Overinvestment; Investor Coordination; Fraud; Managerial Control

JEL classification codes: G31, D23, D82
1 Introduction

Corporate fraud has attracted much attention recently because of prominent cases of corporate malfeasance where insiders were able to attract investment through overly-optimistic representations of financial performance and economic prospects (e.g., Worldcom and Enron). An important consequence of this type of fraud is overinvestment in certain industries or sectors when uninformed investors direct capital flows to not only the firms manipulating investor beliefs but the entire industry. For example, Worldcom’s claim in 1996—since proved fraudulent—that Internet traffic was doubling every 100 days not only bolstered its own stock price but also appears to have led to a glut in fiber-optic capacity (see, e.g., Dreazen (2002) and Sidak (2003)). But while Worldcom and Enron are recent examples, manipulation of outside investors by strategic insiders through excessively optimistic portrayals of investment prospects has a long history, and appears to have existed from the onset of organized trading and investment.\(^1\)

There is empirical evidence in the literature suggesting that manipulations of performance measures by insiders lead to overinvestment. For example, Teoh, Welch and Wong (1998a,b) find that earnings management prior to IPO’s and SEO’s can explain long-term underperformance. Dechow et al. (1996) find that firms that commit fraud tend to have higher ex ante needs for additional funds. Similarly, Wang (2004) finds that fast growing firms with large external financing needs are likely to commit fraud.

The Worldcom and Enron cases point to the role of asymmetric information in inducing overinvestment. For example, Worldcom’s misrepresentations on the rate of growth of Internet traffic had credibility because such information is highly proprietary to carriers. In fact, Sidak (2003, page 230) notes that “...Worldcom used [the] asymmetry of information to exaggerate the value of its stock by overstating the growth of Internet volumes.” But this assertion, along with empirical findings emphasizing overinvestment as an attendant cost of corporate fraud, appear puzzling because in efficient capital markets strategic managerial disclosures aimed to inflate firm prospects should be discounted by rational market participants (e.g., Stein (1989) and Narayanan (1985)).

Indeed, corporate finance models with adverse selection often predict underinvestment rather than overinvestment, because of the adverse selection problem in equity financing (e.g., Myers and Majluf (1984) and Greenwald, Stiglitz and Weiss (1984)) or capital rationing by debt markets (e.g.,

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\(^{1}\)Notable early examples include the South Sea Trading Company in the seventeenth century and Utilities in mid-western states in the 1920s. Skeel (2005) provides and interesting discussion and numerous other examples of this phenomenon.
Stiglitz and Weiss (1983)).

Our contribution is to develop a theory where fraud and overinvestment jointly occur in a rational expectations equilibrium, i.e., firms with low expected returns receive more than their efficient levels of investment capital even when investors follow their Bayes-rational or individually efficient investment policies. Our framework is built on a fundamental property of modern corporations: the separation of ownership and control between dispersed owners and self-interested managers (Berle and Means (1933) and onwards). Because owners of widely-held firms are dispersed and even change over time, it is difficult for them to credibly precommit to an aggregate investment response to information provided by insiders in the future. That is, shareholders of public corporations as a group cannot commit to investment policies that are ex post inefficient for individual shareholders and lead to a transparent undervaluation of the firm for a sustained period of time, even if such policies may be incentive-efficient ex ante.

Our main insight is that the investor coordination constraint (so to speak) substantially affects the allocation of capital when managers have asymmetric information. This is because shareholders cannot provide incentives for truth-telling to informed managers by committing to investment distortions that are incentive-efficient ex ante, but inefficient (from a productive or portfolio perspective) ex post. Faced with their coordination constraints, owners of widely held firms attempt to design managerial compensation contracts to both elicit valuable information from insiders and to ameliorate their investment coordination problem. However, inducing truth-telling is not always incentive-efficient when shareholders cannot coordinate the optimal design of investment and managerial incentive contracts. Intuitively, the incentive cost of inducing truth-telling now falls entirely on the managerial compensation contracts rather than on both investment and wage contracts. Our analysis demonstrates that for a wide-range of situations, the optimal compensation contract induces misreporting by insiders with a positive probability, and this results in overinvestment in some states by rational investors. This misreporting is how we define fraud in our model.

To explicate, suppose that there is a preference conflict between outsiders and insiders on investment policy. Such a conflict can arise for a variety of reasons; in particular, we suppose that insiders receive private benefit from higher investment, i.e., value “empire building” (e.g., Stulz (1990) and Hart (1995)). When management is privately informed of the quality of the firm’s investment opportunities, they have an incentive to mis-report or strategically communicate their

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2Other agency models that emphasize the asset substitution problem (Jensen and Meckling (1976)) and the debt overhang problem (Myers (1977)) also lead to underinvestment. See Stein (2001) for a very useful survey.
private information in order to manipulate (outside) investors toward their (i.e., the management’s) desired investment level. When investors can collectively coordinate their investment response to managerial disclosures and courts can enforce managerial compensation contracts, the optimal contract induces truth telling by tying investment and compensation, to managerial disclosures and firm performance. But, as is well known, the optimal mechanism will typically impose (ex-post) inefficient investment whenever the truth-telling constraints are binding (see, e.g., Baron and Myerson (1982) and Holmstrom and Weiss (1985)).

But ex-post inefficient investment levels imply a common knowledge undervaluation of the firm’s assets; enforcing such an investment policy, therefore, requires a substantial degree of commitment by not just future owners but all capital market participants. This is because transparently inefficient investment policies—either by the firm or by the shareholders—are likely to be either removed by boards of directors that represent and serve current shareholders’ interests or by the market for corporate control because a portfolio strategy that purchases the firm to remove the inefficiency has a positive NPV.

However, coordination failure per se does not rule out truthful reporting from strategic and informed insiders. After all, investors have other instruments to provide incentives for truth-telling, for example, tying managerial compensation to previous reports and observed outcomes (e.g., Dybvig and Zender (1991)). In the presence of coordination failure, however, it is more costly to implement truth telling. Consequently, it might be optimal for owners to elicit less precise information from insiders in order to reduce expected compensation. For example, the optimal contract might include overstatement of firm prospects with some probability. Then, owners, while unaware of the true prospects of the firm, invest based on their equilibrium beliefs. The cost of such strategy is, of course, embedded in the overinvestment that follows overstatements.³

Intuitively, there are three types of equilibrium: one in which disclosures are truthful, one in which disclosures reveal no information, and finally an equilibrium in which disclosures are not truthful but still have valuable content. In the latter case, managers of firms with inferior prospects (with some probability) make false and excessively optimistic projections of their firm’s economic prospects. Which equilibrium prevails will depend on the cost of implementing truth telling and the benefits from efficient investment, and in our setting this trade-off depends on the opportunity cost.

³The general point is that in with imperfect investor coordination the revelation principle (e.g., Myerson (1982)) need not apply (see, Mookherjee (2003)) and, loosely speaking, “all bets are off” regarding the equilibrium information content of disclosures by informed insiders.
of capital, properties of the investment technology, and managers’ private benefits from control.

One important empirical implication of our analysis is that, corporate fraud and overinvestment are positively related to firms’ growth options. This implication is consistent with the observation that incidences of corporate fraud are common following the introduction of new technologies (such as global trade in the late eighteenth century, railroads during the nineteenth century, and the Internet more recently), the exploration of new business strategies (such as mass production in the 1920s), and among fast growing firms with large external capital needs (Dechow et al. (1996), Wang (2004)). Another important implication is that corporate fraud and overinvestment are negatively related to firms’ cost of raising capital (or investors’ risk premia), which may help explain why instances of corporate fraud have followed financial innovations that have increased liquidity in financial markets and presumably reduced firm’s cost of capital; for e.g., the introduction of publicly held corporate debt in the 1860’s (Skeel (2005)) and the introduction of closed-end funds in the 1920’s (Shleifer (2000)). Our framework also suggests an association between business cycles and the likelihood of corporate fraud through investors’ risk premia.5

In light of the formal analysis, we discuss how managerial career concerns in competitive managerial labor markets can lead to noisy revelation and overinvestment, and slow the pace of learning about managerial talent. A somewhat surprising implication of our model is that the corporate charter (as a coordination mechanism) can sometimes increase firm value ex ante by making it more difficult for the firm to minimize its cost of external financing ex post. Finally, we compare information precision and investment allocation efficiency between internal and external capital markets. Our analysis provides a framework to potentially reconcile the argument in the literature that internal capital markets generate higher information quality or precision (Alchian (1969) and Williamson (1975)) with evidence that indicates that multi-division corporations tend to overinvest in bad divisions and underinvest in good ones (e.g., Scharfstein (1998)).

The manager’s optimal compensation contract also has interesting characteristics. The manager is rewarded following disclosures of poor firms prospects. Such down-side-risk protection can be interpreted as the provision of severance packages, e.g., golden parachutes. The model predicts that managers of firms with valuable growth options enjoy more substantial down side risk protection. Somewhat counter intuitively, such down-side risk protection is associated with less accurate

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4We discuss these implications of the model and the empirical evidence in more detail in the section 7.
5Anecdotal evidence suggestive of this link include the 1920’s and the 1990’s - see discussion in Povel, Singh and Winton (2005).
disclosures in equilibrium.

In addition, our analysis points to an interesting role for performance-contingent compensation contracts for managers of widely held firms, one that has not been emphasized in the literature. Our model only has adverse selection (i.e., there is no hidden action), the agent is risk-averse, and the shareholders are risk-neutral. In the standard principal-agent framework, performance-contingent contracts will not be incentive-efficient because random compensation worsens risk-sharing without endowing any incentive benefits. However, such contracts may benefit the principals (or shareholders in our model) by relaxing the coordination constraints by influencing their objective function ex post, thereby making their Bayesian-rational investment strategy closer to the ex ante incentive-efficient investment policy.

More generally, our framework suggests that fraud aimed to manipulate future investors is more likely to be observed as the control of a corporation becomes less concentrated: a refutable prediction that would be interesting to test in future research. For example, Coffee (2005) attributes differences in firms’ propensity to manage earnings, while comparing the U.S. and Europe, to differences in firms’ concentration of ownership.

We organize the remainder of the paper as follows. We discuss the related literature in Section 2. Section 3 sets out the basic model and Section 4 presents some benchmark outcomes, with alternative informational and contractual assumptions. Section 5 considers the general relationship between investor coordination and equilibrium fraud, and Section 6 provides a detailed analysis of an equilibrium with fraud for a parameterized version of the model. In section 7 we discuss the empirical implications of our results. Section 8 concludes and discusses avenues for future research.

2 Related Literature

The desire to understand the causes and consequences of (corporate) fraud as an equilibrium phenomenon at least dates back to Becker (1968). Stein (1989) argues that myopic managers emphasize short-term financial performance (e.g. stock price and earnings) at the expense of long-term economic value, and therefore (inefficiently) transfer cash flows from future periods to boost current earnings. In equilibrium, however, the market rationally reacts to such behavior by insiders and discounts the current earnings while evaluating the firm. Others who emphasize the role of managerial myopia are Narayanan (1986), Von Thadden (1995), Fischer and Verrecchia (2000), and Kedia and Philippon (2005). Such myopia is sometimes motivated through imperfections in
long-term incentive contracting. In our analysis, however, we do not rely on managerial myopia, nor do we impose restrictions on the ability to design long-term incentive contracts.

Our analysis is, thus, more closely related to papers that study optimal compensation contracts when managers can distort information. Goldman and Slezak (2003) present a model where optimal managerial compensation contracts balance the provision of incentives to exert effort against the agent’s desire to commit fraud. In their setting, there is no residual uncertainty regarding firm productivity. In our setting, however, the level of residual uncertainty following managerial disclosures and the subsequent overinvestment play a key role. Dye (1988) and Demski (1998) present models in which the optimal contract is designed to overcome problems of limited communication. In Dye (1998) there is asymmetric information of two dimensions while the informed agent can only provide a one-dimensional report; there is thus non-truthful reporting (with some probability) in equilibrium. In Demski (1998), earnings management is actually beneficial since that is the only way the manager can communicate the level of future earnings. Our result do not rely on an assumption that limits the ability of insiders to efficiently communicate their information. In particular, under certain parameter regions insiders choose not to truthfully communicate their information even though they can. More importantly, all three analyses mentioned above de-emphasize the role of capital markets through which managers have damaged their companies in several corporate scandals - an element we pay more attention to.\textsuperscript{6}

Instead of relying on managerial myopia, limitations on communication, multi-tasking, or investor irrationality, the primary friction we focus on is imperfect investment coordination amongst dispersed investors. In our analysis we allow the design of optimal long-term managerial incentive or compensation contracts. Perfect coordination among a large number of owners, dispersed in space and time, is quite implausible, and therefore our perspective appears to rest on rather unexceptionable foundations.

Our analysis is not the first to provide a rationale for the clustering of corporate fraud overtime. However, as we will discuss below, we add a new perspective on the issue. In Povel et al. (2005) and Hertzberg (2003), beliefs regarding firms’ productivity play a key role in determining the likelihood of fraud. When investors are optimistic about firm prospects, Povel et al. (2005), show that auditors have weaker incentives to monitor and Hertzberg (2003) shows that owners are more likely

\textsuperscript{6}There is also a more recent literature that assumes some market irrationality in reconciling fraud with market equilibrium. Notable examples include Jensen (2004) and Bolton et al. (2004). As we do not rely on investor irrationality, our analysis is quite different.
use pay-for-performance contracts. Both analyses predict more frequent incidents of fraud when investors are (rationally) optimistic. Thus, it is the cyclicality in productivity of firms that results in the cyclicality of corporate fraud. We, however, link investors’ risk premia to the likelihood of fraud. In our framework, the incidence of fraud would be time-varying, following the pattern of time-variation in risk premia (e.g., Lettau and Ludvigson (2001)).

Our paper is also linked to a broader agency literature where constraints, on the principal’s ability to credibly precommit ex-ante, limit the amount of information he possesses ex-post (e.g., Crawford and Sobel (1982) and Laffont and Tirole (1988)). Two related applications of this idea are Arya et al. (1998) and Krasa and Villamil (2000). In Arya et al. (1998), the owner cannot commit to a long term employment contract with the manager; earnings manipulation by the manager is then sometimes optimal because it reduces the owner’s propensity to replace the manager following low reported earnings. Krasa and Villamil (2000) consider a costly state verification model when the lender cannot commit to an auditing schedule. The optimal contract resembles a simple debt contract, where auditing occurs in the case of a default, and in equilibrium the entrepreneur falsely reports cash flows with some probability. These analysis, however, do not address the link between corporate fraud and investment. This allows us to address the cyclicality of corporate fraud incidences and its tendency to concentrate in industries with valuable growth options.

3 The Model

3.1 Technology

There are three time periods in the model, \( t = 0, 1, 2 \). The firm has a point input-point output technology that stochastically converts investment at time \( t = 1 \), \( k \), to (operational) earnings at time \( t = 2 \), \( y \). For given investment, the expected earnings depend on the capital productivity of the firm, which is uncertain ex-ante. To make our point in the simplest possible way, we suppose that earnings take only two possible values: a high value, \( y = 1 \), and a low value \( y = 0 \). However, the probability distribution of earnings is influenced by the firm’s economic prospects (or productivity), \( s \in \{ s_h, s_\ell \} \), \( 1 > s_h > s_\ell \), and investment. Specifically, the probability of high earnings is given by \( sf(k) \), where \( f \) is an increasing, concave, and non-negative valued function defined on the feasible investment set \( [0, \bar{k}] \), such that \( f(0) = 0 \) and \( f(\bar{k}) \leq 1 \). The firm is liquidated upon the realization of \( y \).
3.2 Ownership and Control

Ownership of the firm is dispersed (i.e., the firm is widely held), but all owners have a common opportunity cost of investment which is the gross return $R > 1$. There is a separation of ownership and control because the firm is controlled by a manager. The manager receives two types of utility from controlling the firm. He receives utility from consuming wages, $w$, that are paid at the time of firm’s liquidation. The manager also receives benefits from controlling the firm which are generally a mixture of subjective utility and non-contractible pecuniary benefits; these benefits increase with the size of the capital assets. The manager’s attitudes toward risk are represented by an increasing and concave von-Neumann-Morgenstern expected utility function $u(w)$ (with $u(0) = 0$), and his private benefits from control are represented by a strictly increasing and concave function $b(k)$ (with $b(0) = 0$).

3.3 Information and Contracting

The manager privately observes the productivity ($s$) before investment takes place, i.e. before time $t = 1$. The ex-ante probability of observing $s_h$ ($s_{\ell}$) is $\mu (1 - \mu)$. It will sometimes be convenient to put, $\mu_h = \mu$, and $\mu_{\ell} = (1 - \mu)$. We will allow the manager to communicate with the owners regarding his private information on the firm’s productivity.

Owners, or their representatives, can write a wage contract with the manager, at time $t = 0$, that is contingent on the manager’s communication of the productivity, at time $t = 1$, and the observed earnings, at time $t = 2$. From an institutional perspective, shareholders delegate the responsibility of wage contracting to the board of directors. These contracts are enforceable in the sense that managers can move the courts to enforce prior wage contracts even though the share ownership of the firm changes on a daily basis.

Specifically, at date $t = 0$, and prior to the realization of the manager’s private information, owners (or their representatives) can present a managerial incentive or wage menu, $w = \{w^+_r, w^0_r\}_{r=\ell}$, that determines the manager’s output-contingent compensation as a function of his communication; here, $w^+_r$ ($w^0_r$) denotes the compensation when earnings are positive (zero) and the productivity $r$ was communicated. Realized compensation must be non-negative, because we assume that managers have limited liability.\(^7\)

\(^7\)We could also consider possible non-pecuniary penalties (e.g., jail); however, this will not alter our main results so long as the expected utility costs from such penalties are not so substantial to essentially wipe out the agency problem.
But while owners of widely held firms typically write enforceable performance-contingent wage contracts with the firm’s employees (including the top-management), they generally can not so easily write enforceable contracts with respect to the specific projects the firm will invest in and the dollar amount allocated to each project. More precisely, owners of a widely-held firm can not credibly pre-commit to an arbitrary investment menu, say $k_r, r \in \{\ell, h\}$, that binds future financial market participants to a personal investment or portfolio strategy that is not time-consistent, or is \textit{ex post} inefficient. Therefore, we require that investment be Bayes-rational or ex-post efficient for investors in equilibrium.\footnote{Since outsiders are risk-neutral and share the same information, their Bayes-rational consistent investment policy is identical in our model. Therefore, ex post efficient investment is the level of investment that maximizes the value of the firm—given investors’ (current) posterior beliefs on the firm’s productivity.}

To see this point, consider a candidate long-term investment contract for a corporation that specifies an investment over some pre-specified duration in the future. If this investment is to be financed externally through seasoned equity offerings, then the contract must credibly enforce future equity offerings by the firm that may be transparently inefficient. Such a policy may not only be unacceptable to new shareholders, but more importantly the inefficient financial policy will lead to an undervaluation of the firm’s assets because of an inefficient physical investment and a value-destroying equity dilution. However, a transparent under-valuation is not a viable situation since there is a clear positive NPV opportunity: purchasing the firm at the under-valued price, re-setting the investment to the ex post efficient level, and selling (or holding) the firm. In sum, an ex post inefficient investment policy can only be implemented by coordinating the behavior of all financial market participants or by legally enforcing contracts that restrict trading in transparently mis-valued firms. These are patently unrealistic restrictions.

Similarly, there are problems in writing wage contracts that depend arbitrarily on future investment by shareholders. In practice, such investment is difficult to specify ex ante and costly to verify ex post. For example, does the investment refer to external financing by equity holders or investment outlays in projects by the firm based on a combination of retained earnings and external financing? Since external financing and outlays only get consummated over time, the time-frame for the investment also needs to be specified ex ante. Ex post, there may be verification issues if the measure of investment involves internal financing. In our analysis, we therefore assume that investment contingencies are not verifiable by courts and the level of investment is not contractible.
3.4 Timing Conventions

The timing conventions of the model are described in Figure 1 below.

<table>
<thead>
<tr>
<th>Compensation contract determined</th>
<th>Manager learns private signal</th>
<th>Signal communicated to investor</th>
<th>Investment occurs</th>
<th>Output realized</th>
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<tr>
<td>$t = 0$</td>
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<td>$t = 1$</td>
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Figure 1

4 Benchmark Outcomes

Prior to the main analysis, we discuss two benchmarks that will facilitate comparison with, and intuition for, our main results below. We consider the cases of complete information, and asymmetric information with full commitment from the side of owners. We will say that investment is contractible if it is costlessly verifiable and there is perfect coordination among investors so that they can credibly pre-commit to investment levels.

4.1 Complete Information and Contractible Investment

Suppose there is complete information on the productivity and investment is contractible. In this case, the efficient investment policy is denoted by $k_j^*, j = h, \ell$, such that,

$$s_j f'(k_j^*) = R. \quad (1)$$

From concavity and the fact that $s_h > s_\ell$, it follows that $k_h^* > k_\ell^* > 0$. Clearly, the manager is not given any wage payment with complete information since he enjoys benefits of $b(k_j^*) > 0$ in each productivity state.\(^9\)

4.2 Asymmetric Information and Contractible Investment

As another benchmark, we examine the situation where managers have private information but investment is contractible in the sense described above. The complete information allocation is

\(^9\)Throughout the analysis we assume that $k_h^* < \bar{k} - $ that is to insure that the first best level of investment defined above is feasible. Since the equilibrium levels of investment we establish subsequently will fall short of the first best level of investment $k_h^*$ we disregard this constraint.
transparently not feasible when the manager has private information on capital productivity, since
the low productivity manager can do strictly better by pretending to be the high productivity
manager. We therefore characterize the information-constrained efficient outcome.

Since owners can credibly pre-commit to both wage and investment policies (by assumption),
the revelation principle applies. Thus, without loss of generality we can restrict attention to direct
mechanisms, where the agent’s message space is \( \{ \ell, h \} \), and truth-telling is an optimal strategy.
A typical direct mechanism is specified as a wage rule that is contingent on the manager’s report
and the revenue outcome, \( w \), and an investment rule that is contingent on the manager’s report,
\( k \equiv \{ k_r \}_{r=\ell}^h \). Given this mechanism, the agent’s payoffs when the true productivity is \( j \), but he
reports \( r \), for \( j, r \in \{ \ell, h \} \) is,

\[
U(j, r \mid w, k) = s_j \mu_j \left[ u(w_r^+) - u(w_r^0) \right] + u(w_r^0) + b(k_r).
\]

(2)

The optimal (information-constrained) mechanism is then the solution to the following program
(P1):

\[
\max_{(w, k) \in R_+^6} \sum_{j=\ell}^h \mu_j \left[ s_j f(k_j)(1 - w_j^+ + w_j^0) - w_j^0 - Rk_j \right], \quad s.t.
\]

\[
U(j, j \mid w, k) \geq U(j, r \mid w, k), \quad r \neq j, \quad r, j \in \{ \ell, h \}, \quad (4)
\]

\[
w_r^+ \geq 0, \quad w_r^0 \geq 0, \quad r \in \{ \ell, h \}. \quad (5)
\]

Here, (4) are the truth-telling (incentive compatibility) constraints and (5) are the non-negativity
constraints on managerial wages. We denote the solution to (P1) as \( \hat{\delta} \equiv \{ \hat{w}_r^+, \hat{w}_r^0, \hat{k}_r \}_{r=\ell}^h \).

It is clear that the manager’s participation constraint will not bind in either productivity state.
Since the marginal productivity of capital becomes unbounded for infinitesimal investment levels,
the information-constrained efficient investment levels will be positive in both productivity states.
Hence, the manager will obtain positive benefits which can not be taxed away by the owners due
to the non-negativity restriction on wages.

Turning to incentive issues, a pooling investment policy where the investment level is the same
for each (reported) productivity would trivially be incentive compatible. However, such a policy
imposes substantial investment inefficiency costs. Therefore, there will be dispersion in the invest-
ment levels across productivity states; i.e., \( k_h > k_\ell \). But intuition suggests that this dispersion
will be lower with asymmetric information relative to the complete information dispersion in order to
relax the truth-telling constraints; i.e., \( (k_h - k_\ell) < (k_\ell^* - k_h^*) \).
Of course, a mechanism with \( k_h > k_\ell \) is not incentive compatible with zero wages. Some introspection suggests that it would be inefficient to offer a positive wage if the manager reports high productivity. The reason is that such a mechanism will only tighten the truth telling incentive constraints for the low productivity managers. Hence, the optimal mechanism will give a positive wage to managers reporting low productivity that makes them just indifferent between truthfully revealing their true state and pretending to be the high-productivity manager.

**Theorem 1** In the optimal mechanism \( \hat{\delta} \equiv \{\hat{w}_r, \hat{w}_r^0, \hat{k}_r \}_{r=\ell}^h \),

1. \( \hat{w}_r^+ = \hat{w}_r^0 = 0; \)
2. \( \hat{w}_\ell^+ = \hat{w}_\ell^0 = w > 0; \)
3. \( u(w) + b(\hat{k}_\ell) = b(\hat{k}_h) > 0; \)
4. \( k^*_\ell < \hat{k}_\ell < \hat{k}_h < k^*_h. \)

It is useful to characterize \( k_h \) and \( k_\ell \) more explicitly to serve as a benchmark for equilibrium investment when investors can not perfectly coordinate. Put \( v(y) \equiv u'(u^{-1}(y)). \) Then,

**Corollary 1** In the optimal mechanism \( \hat{\delta}, \hat{k}_h \) and \( \hat{k}_\ell \) are implicitly determined by,

\[
s_h f'(\hat{k}_h) - R - \left( \frac{1 - \mu}{\mu} \right) \frac{b'(\hat{k}_h)}{v(b(\hat{k}_h) - b(\hat{k}_\ell))} = 0,
\]
\[
s_\ell f'(\hat{k}_\ell) - R + \frac{b'(\hat{k}_\ell)}{v(b(\hat{k}_h) - b(\hat{k}_\ell))} = 0
\]

It is easy to see that the second-best or the optimal mechanism with perfect investor coordination distorts investment in the high-productivity (low-productivity) state downward (upward) by a factor that depends on the manager’s marginal private benefit from controlling higher investment. This verifies the intuition that the extent of investment distortion in the second best will be positively associated with the extent of managerial moral hazard for over-investment.

## 5 Investment Coordination and Equilibrium Fraud

We now examine the equilibrium outcomes when investors can not coordinate investment allocations. In this case, investors follow their optimal Bayesian investment strategies, based on their
beliefs on the manager’s true capital productivity and his (possibly randomized) reporting strategy. Recall, however, that the owners can still precommit to an output-contingent wage policy. Therefore, we first define the hierarchical equilibrium concept that appears best suited to the assumptions at hand (see, e.g., Krasa and Villamil (2000)). We then show that implementing truth telling is feasible, even though, the revelation principle does not apply since the construction of equivalent truthful equilibria (see, e.g., Myerson (1979)) is not generic. Afterwards, in section 6, we demonstrate the existence of a hierarchical equilibrium that involves, with positive probability, false reporting by the manager of the firm’s productivity.

5.1 Equilibrium Definition

An equilibrium consists of a compensation contract, a reporting strategy by the manager, and owner’s investment response to managerial reports. This triple is a hierarchical Bayesian equilibrium if the compensation contract maximizes the (ex-ante) value of the firm to the owners under the constraints that the reporting strategy is optimal for the manager (given owner’s investment response to managerial reports), and the owner’s investment response to managerial reports is optimal given manager’s reporting strategy.

Let investors’ beliefs on the manager’s reporting strategy be given by the probabilities $0 \leq \pi_{jr} \leq 1$, $j, r \in \{\ell, h\}$, where $\pi_{jr}$ is the probability that a manager with an actual productivity $j$ reports the productivity $r$. It is notationally convenient to put $\pi_{jj} \equiv \pi_j$ and $\pi = (\pi_\ell, \pi_h)$. Then, the ex-ante probability of receiving the report $r \in \{\ell, h\}$ is:

$$q_r(\pi) = \mu \pi_{hr} + (1 - \mu) \pi_{\ell r}, \quad r \in \{\ell, h\}, \quad r \neq j.$$  

Hence, by Bayes rule, the conditional expectation of $s$ given report $r$ and strategy $\pi$ is,

$$E(s|r, \pi) \equiv \frac{\mu \pi_{hr} s_h + (1 - \mu) \pi_{\ell r} s_\ell}{q_r(\pi)}, \quad r \in \{\ell, h\}.$$  

Given a wage compensation contract $w$, reporting strategy $\pi$, and investment response strategy $k \equiv (k_h, k_\ell)$, the expected utility of the manager when the productivity is $j \in \{\ell, h\}$, is given by (8), the value of the firm to the owners following a report $r$ by the manager is given by (9), and the (ex-ante) value of the firm to the owners is given by (10):

$$U_j(\pi_j, w, k) = \pi_j U(j, j \mid w, k) + (1 - \pi_j) U(j, r \mid w, k), \quad r, j \in \{\ell, h\}, r \neq j.$$  

\footnote{While investors may generally have heterogeneous beliefs on reporting strategies, for notational ease we suppress this type of investor-dependence.}
\[ V_r(\pi, w, k_r) = E(s|r, \pi) f(k_r)[1 - w_r^+ + w_r^0] - w_r^0 - Rk_r \quad r \in \{\ell, h\}. \]  

\[ V(\pi, w, k) = q_h V_h(\pi, w, k) + q_\ell V_\ell(\pi, w, k) \]  

The equilibrium is defined by the solution to program (P2),

\[
\max_{\pi, w, k} V(\pi, w, k), \quad s.t.
\]

\[ \pi_j \in \arg \max_{\tilde{\pi} \in [0,1]} U_j(\tilde{\pi}, w, k), \quad j \in \{\ell, h\}, \]  

\[ k_r \in \arg \max_{k \geq 0} V_r(\pi, w, \tilde{k}). \]  

\[ w_r^+ \geq 0, \quad w_r^0 \geq 0, \quad r \in \{\ell, h\}. \]  

Here, (12) insures that the manager’s reporting strategy is optimal given owner’s investment response to managerial reports, \(k\), and wages, \(w\). Constraint (13) insures that investment is ex-post optimal given manager’s reporting strategy \(\pi\), and wages, \(w\). Finally, (14) represents manager’s limited liability.

We denote the solution to (P2) (i.e., the hierarchical Bayesian equilibrium) by \(\sigma \equiv (w, \pi, k)\), and denote the expected utility of the manager along the equilibrium path by \(U(\sigma) = \mu U_h(\pi, w, k) + (1 - \mu) U_\ell(\pi, w, k)\); similarly, the expected value of the firm along the equilibrium path is denoted by \(V(\sigma)\).

### 5.2 The Feasibility of a Truth-Telling Mechanism

The revelation principle applies when, for any given communication equilibrium between the principal and the agent, we can construct another equilibrium where truth-telling is a best-response strategy for the agent and both the principal and the agent receive expected payoffs equivalent to the original equilibrium. However, because of the lack of coordination amongst owners, i.e., constraint (13) added to the mechanism design program, there is effectively no precommitment on the investment response, and the revelation principle does not apply in our setting (e.g., Poitevin (2000)). This, however, does not mean that a truth telling mechanism does not exist in our setting. Rather, it implies that it is no longer guaranteed that the solution to program (P2) will satisfy truth telling.
To gain further intuition on why the revelation principle does not hold in our setting, consider an equilibrium that induces misreporting with some probability. And, suppose that a low type manager reports that she is of high type with the probability $0 < \pi_{th} < 1$. Further, suppose that the manager’s compensation following a high report is zero, i.e., $w_h^+ = w_h^0 = 0$ (this will actually turn out to be the optimal policy). This implies that investment following the high report is strictly lower than the first best investment level $k_h^*$ because in equilibrium owners rationally anticipate the manager’s reporting strategy. Moreover, it implies that in the low-productivity state the owners (effectively) randomize between a high level of investment, following a high report, and a lower level of investment, following a low report. Under a truth telling equilibrium, however, such randomization would never occur, as the owners would be investing in an ex-post efficient manner.

Before solving for the optimal contract, we would like to demonstrate that a truth telling mechanism does exist in our setting. In particular, suppose that $w_{\ell}^+ = w_{\ell}^0 = u - 1(b(k_{\ell}^*) - b(k_{\ell}^*))$, and $w_{h}^+ = w_h^0 = 0$. This implies that the manager is willing to report truthfully and that the owner invests the first best levels of investment $k_j^*$ in state $j \in \{\ell, h\}$.

**Theorem 2** If investors cannot coordinate investment strategies, i.e., (13) applies, then the Revelation Principle does not apply to program (P2). However, truth telling is still a feasible solution candidate for program (P2).

Theorem 2 implies that, in the absence of investment coordination, we have to examine the existence of non-truthful equilibria. As we will show in the following section, it might be beneficial for the owners to reduce the informativeness of managerial disclosures and suffer overinvestment in the low-productivity state with some probability in order to reduce the expected wage costs.

### 5.3 Noisy Revelation Equilibrium

In the hierarchical (Bayesian) equilibria, owners have significant influence on the information content or accuracy of the manager’s reports through the initial design of the wage policy. A main result of this paper is that, for an open set of parameters, there is a unique hierarchical equilibrium that has the manager mis-reporting the firm’s true economic state with positive probability. That is, there is noisy revelation of the firm’s economic state in equilibrium. Before deriving this result formally we would like to provide further intuition.

As specified in Theorem 1, when investors can commit to an investment response to managerial disclosures, the investment policy satisfies $k_h^* > \hat{k}_h$ and $k_{\ell}^* < \hat{k}_l$, where $\hat{k}_j$ is the efficient (or
information-constrained) investment in a firm with productivity \(j\). This point facilitates intuition for the noisy revelation equilibria. In such an equilibrium, owners’ best response investment strategy is lower (higher) than \(k^*_h (k^*_l)\) when the manager reports a high (low) state, because with positive probability the actual state is low (high).

Then consider a candidate equilibrium in which the manager truthfully reports the firm’s capital productivity and investment is at the first best level, \(k^*_l, k^*_h\). Clearly, truth-telling is an equilibrium strategy for the low-productivity manager if and only if,

\[
s_{\ell}f(k^*_\ell)(u(w^*_\ell) - u(w^0_\ell)) + u(w^0_\ell) + b(k^*_\ell) \geq s_{\ell}f(k^*_h)(u(w^*_h) - u(w^0_h)) + u(w^0_h) + b(k^*_h). \tag{15}
\]

Due to the non-negativity constraint on managerial wages and managerial risk aversion, (15) is satisfied only if,

\[
s_{\ell}f(k^*_\ell)(w^*_\ell - w^0_\ell) + w^0_\ell \geq b(k^*_h) - b(k^*_\ell). \tag{16}
\]

Thus, the managerial wage cost in a truth telling equilibrium can be very high if the first-best or optimal investment gap between high and low productivity states is high. Extending this argument further, if there is some way to make it individually rational (or incentive compatible) for investors to reduce (inflated) by a small amount their investment relative to \(k^*_h (k^*_l)\) in response to good (bad) productivity reports, then investors’ welfare will be improved. We know this because, by definition, there exists no investment (and managerial compensation) policy that is more incentive-efficient than the second best arrangement (\(\hat{\delta}\)).

A natural possibility for making investors’ individually rational response to managerial reports closer to the second-best investment policy is an equilibrium where the manager strategically mis-reports productivity with a positive probability in the high state. In equilibrium, this fraud strategy is common knowledge. This dampens investors’ individually rational response to a high-productivity report, and reduces the investment gap between the high- and low-productivity states. However, this “solution” is costly because it leads to over investment in the bad state whenever mis-reporting occurs. We now analyze this tradeoff and derive conditions under which investors optimally choose the wage-incentive mechanism to support noisy revelation or fraud in equilibrium.\(^{11}\)

\(^{11}\)Alternative mechanisms for dampening investment in the high state are also analyzed.
6  Equilibrium Fraud and Investment Distortion

In this section, we demonstrate the existence of, and characterize, a noisy revelation equilibrium where investors optimally design an optimal wage contract that induces fraudulent reporting with positive probability by the low-productivity manager, i.e., \( 0 < \pi_{lh} < 1 \), but truthful reporting by the high-productivity managers, i.e., \( \pi_{hh} = 1 \).

Focusing attention to overstatements of economic performance is of particular interest. For example, while exploring data of accounting restatements over the period 1995-2001, Burns and Kedia (2004) report that 93% of restatements involved overstating net income in the year of misreporting. Such overstatement of the firm’s economic prospects, by the manager, in the low productivity state, leads to overinvestment relative to the first best.

To facilitate intuition, we work with a parameterization of technology and preferences that allows us to constructively characterize the noisy revelation equilibrium. Let the production technology be given by, \( f(k) = 2\sqrt{k} \), and the private benefit be, \( b(k) = \psi\sqrt{k} \). Furthermore, we set \( u(w) = w^{1/\gamma}, \gamma > 1 \), so that \( u(\cdot) \) belongs to the CRRA family.\(^{12}\)

As demonstrated earlier in Theorem 1, output contingent payoffs play no role in reducing expected agency costs with coordination. In particular, under the second best (optimal) contract \( \hat{\delta} \), payoffs are contingent on reports only. However, without coordination, contingent wages not only affect managerial reports but alters the share of profits allocated to owners, which in turn affects owners’ best response investment to managerial reports. We therefore conduct the analysis in two stages, first considering only short term performance contracts that do not specify wages contingent on realized output but only on managerial reports. Second, while considering long-term performance contracts we complete the analysis by providing conditions under which the former approach is without loss of generality.\(^{13}\)

6.1  Short-term Performance Contracts

We first analyze the optimal compensation contract while excluding the possibility of output contingent wages, i.e.,

\[
  w_j^+ = w_j^0 = w_j, \quad j \in \{\ell, h\}.
\]

\(^{12}\)In order to make sure that \( s_2\sqrt{k} < 1 \) in equilibrium we suppose that \( s_h^2 < R/2 \).

\(^{13}\)Thus, we do not restrict attention to short-term contracts.
As we mentioned above, we will construct an equilibrium where the high-productivity manager reveals his state truthfully, but the low-productivity manager over-states his productivity with positive probability. Therefore, in equilibrium, the low-productivity manager must be indifferent between truthful and untruthful reports, i.e.,

\[ u(w_h) + \psi \sqrt{k_h} = u(w_\ell) + \psi \sqrt{k_\ell} \tag{18} \]

Hence, both manager-types are indifferent between reporting truthfully and untruthfully. There is an intuition that the optimal wage-policy will set zero wages when the manager reports the high-productivity state. To see this, suppose to the contrary that, \( w_h > 0 \). Then, by reducing \( w_h \) and \( w_\ell \) concurrently, the outsiders can still maintain the incentive compatibility condition (18), and increase expected profits. Thus, at the optimum we have,

\[ u(w_\ell) = \psi (\sqrt{k_h} - \sqrt{k_\ell}). \tag{19} \]

With the annunciated reporting strategies, investors’ sequentially rational investment response is,

\[ k_\ell = \left( \frac{s_\ell}{R} \right)^2, \]
\[ k_h = \left( \frac{\mu s_h + (1 - \mu)\pi_{th}s_\ell}{R [\mu + (1 - \mu)\pi_{th}]} \right)^2. \tag{20} \]

Then substituting (20) into (19) and re-arranging terms, yields,

\[ \pi_{th} = \frac{\mu (\psi (s_h - s_\ell) - u(w_\ell)R)}{(1 - \mu)u(w_\ell)R}. \tag{21} \]

It is now transparent how the wage-policy \((w_\ell)\) influences the low-productivity manager’s (randomized) reporting strategy (in equilibrium), because \( \pi_{th} \) depends on \( w_\ell \) through the \( u(w_\ell) \) terms in the right hand side of (21). An interesting aspect of this randomized reporting strategy is that, in equilibrium, \( \pi_{th} \) and \( w_\ell \) are negatively related; i.e., a higher wage in response to the low-productivity report is associated with a lower probability of over-reporting by the low-productivity manager. Intuitively, shareholders can purchase more truthfulness from the manager in the bad productivity state by increasing his compensation. As discussed later, however, this partial equilibrium intuition, will not hold at the optimum when variables are endogenous. In other words, higher compensation following low managerial reports is positively associated with higher levels of fraud, in equilibrium.

In sum, the optimal contract to implement over-reporting by the low type manager solves,

\[ \max_{\pi, w, k} V(\pi, w, k) \tag{22} \]
s.t. \( \pi_{hh} = 1, (17), (19) \) and (20)

For computational ease, it is useful to view \( u \equiv u(w_\ell) \) as the control variable, and express \( k_h \) and \( \pi_{lh} \) as functions of \( u \), using (20) and (19). The optimization problem (22) can then be summarized as,

\[
\max_{u \geq 0} \{ \mu (s_h f(k_h) - Rk_h) + (1 - \mu)(\pi_{lh}(s_\ell f(k_\ell) - Rk_\ell) + (1 - \pi_{lh})(s_\ell f(k_\ell) - Rk_\ell - u^\gamma)) \}. \tag{23}
\]

And the optimal \( u \) is implicitly defined by the first order condition,

\[
F \equiv \gamma Ru^{\gamma-1} - [\mu \psi(s_h - s_\ell)] \left( u^{\gamma-2}[\gamma - 1] + \frac{R}{\psi^2} \right) = 0. \tag{24}
\]

In equilibrium, the optimal \( u \) determines the low-productivity manager’s reporting strategy \( \pi_{th} \) through (21). Comparative statics on (22) then clarify the determinants of \( w_\ell \).

**Proposition 1** If, in equilibrium, \( 0 < \pi_{th} < 1 \), then the certain wage for the low-productivity manager, \( w_\ell \), is increasing in \( s_h \), but is decreasing in \( R \).

In general, higher is \( w_\ell \), the less the incentive for the manager to over-state the true productivity of the firm; in effect, \( w_\ell \), is the manager’s compensation for truthfully reporting the low-productivity state and accepting a lower investment level. Therefore, \( w_\ell \) will be higher, other things held fixed, if the costs of false reporting of capital productivity are severe for the shareholders. Proposition 1 is therefore consistent with this intuition. Both an increase in value of the growth option, \( s_h \), and a decrease investors’ opportunity cost of capital \( R \) increase firm productivity and lead to higher levels of investment (as formally shown in proposition 3 subsequently). Thus, intuitively, the model predicts that \( w_\ell \) will increase as a result (see (19)). An increase in managements’ private benefit from control, however, is counter productive, as it leads (only) to stronger incentives for manipulation (all else equal). This, in turn, reduces investment (see proposition 3) and the overall effect on the certain wage for the low-productivity manager is, thus, ambiguous.

The comparative statics on \( u \) (or \( w_\ell \)) also clarify the determinants of the equilibrium noisy revelation strategy, \( \pi_{th} \) (cf. 21). This is because \( \pi_{th} \) is influenced by the choice of \( w_\ell \). As standard, for any given parameter \( x \),

\[
\frac{d\pi_{th}}{dx} = \frac{\partial\pi_{th}}{\partial x} + \frac{\partial\pi_{th}}{\partial u} \frac{\partial u}{dx}
\]

(25)

In general, \( \pi_{th} \) is determined through the incentive-compatibility condition (for noisy revelation) given in (19). Specifically, if a change in a model parameter increases the manager’s propensity to
over-state the firm’s productivity, then $\pi_{th}$ will rise because inducing truthfulness becomes more costly. Thus, $\pi_{th}$ will be increasing in $\psi$, because the manager’s incentives for attracting higher investment through mis-reporting are greater if his private control benefits are higher. Meanwhile, $s_h$ increases or $R$ falls, the manager receives higher investment from mis-reporting, and hence we expect that $\pi_{th}$ will be increasing in $s_h$ and decreasing in $R$.

**Proposition 2** *If, in equilibrium, $0 < \pi_{th} < 1$, then it is increasing in $s_h$ and $\psi$, and decreasing in $R$."

We turn now to an analysis of equilibrium investment in the noisy revelation equilibrium. We see from (20) that there is no investment distortion when a low-productivity state is announced under the noisy revelation equilibrium that we consider\(^{14}\); i.e., $k_\ell = k^*_\ell$. However, there is clearly overinvestment in the low-productivity state with probability $\pi_{th}$. To be clear, we view overinvestment as the extent of capital mis-allocation when disclosures are false, i.e., the difference between the equilibrium level of investment, $k_h$, and the first best level of investment, $k^*_\ell$. The extent of overinvestment, thus, depends on investors’ beliefs regarding managements’ reporting strategy (which are consistent in equilibrium). The effects of changes in the cost of capital ($R$), the value from investment in the good state ($s_h$), and managements’ private benefits from control ($b(k)$) on the optimal level of overinvestment are deduced from (19).

**Proposition 3** *If, in equilibrium, $0 < \pi_{th} < 1$, then the extent of overinvestment $k_h - k^*_\ell$ is increasing in $s_h$, and decreasing in $\psi$ and $R$."

Thus, even in the noisy revelation equilibrium, the investment response to a high-productivity report maintains certain efficiency features: it is negatively related to the opportunity cost ($R$), positively related to the expected capital productivity in that state, but negatively associated with the manager’s private benefit from investment ($\psi$). This implies that when growth options are high or access to capital is at lower cost, then both the probability of overstatements and the extent of overinvestment that follows are higher.

We now turn to examine the feasibility of a noisy revelation equilibrium where $0 < \pi_{th} < 1$. Intuitively, the equilibrium $\pi_{th}$ will lie strictly between 0 and 1 if the costs of inducing information from the low productivity manager are neither too low nor too high. For, if these costs are too low, then it would be optimal to induce the correct information, i.e., $\pi_{th} = 0$. On the other hand, if

\(^{14}\)We consider a noisy revelation equilibrium with underreporting in the extensions section.
these costs are very high, then it may be optimal not to induce any information at all, i.e., $\pi_{th} = 1$. In our analysis above, we have related the cost of inducing information to the manager’s private benefit of control ($\psi$) and the cost of capital ($R$). Indeed, we find that,

**Theorem 3** There exist $(\underline{R}, \bar{R})$ and $(\underline{\psi}, \bar{\psi})$ such that,

(i) If $R > \bar{R}$ (or $\psi < \underline{\psi}$), then managerial reports are truthful.

(ii) If $R < \underline{R}$ (or $\psi > \bar{\psi}$), then managerial reports are not informative.

(iii) Otherwise, there exists a noisy revelation equilibrium where $u$ and $\pi_{th}$ are given by (24) and (21), respectively, and $0 < \pi_{th} < 1$.

In the absence of investor coordination, owners have the choice *ex ante* to set a wage-policy that induces truth-telling from the high-productivity manager, regardless of the opportunity cost of capital, $R$, and managements’ private benefit from control, $\psi$. An important implication of Theorem 3 is, therefore, that investors’ welfare *ex ante* is not necessary maximized under a truth telling mechanism. In particular, truth telling is no longer optimal when investors’ required rate of return is low or managements’ private benefit from control is high. In such cases, the credible or consistent way for investors to restrict their response to managerial disclosures is to suffer the likelihood that the low-productivity manager will fraudulently over-state the investment prospects of the firm. This in turn reduces managements’ expected compensation. Theorem 3 explicates the conditions under which savings from lower managerial compensation costs under noisy revelation offset the investment efficiency loss from the manager’s fraudulent reporting.

**Corollary 2** In the noisy revelation equilibrium, a truth telling mechanism results in lower (and thus suboptimal) firm value, $V$.

### 6.2 Example with an Analytic Solution

As an example of Theorem 3, consider the case where $\gamma = 2$ so that $u(w) = \sqrt{w}$. In this case, we can obtain an analytic solution for the optimal wage as,

$$w = \left[ \frac{\mu(s_h - s_l)(R + \psi^2)}{2\psi R} \right]^2. \quad (26)$$

Substituting (26) into (21) yields,

$$\pi_{th} = \frac{\psi^2(2 - \mu) - \mu R}{(1 - \mu)(R + \psi^2)}. \quad (27)$$
For this special value of $\gamma$, the equilibrium $\pi_{th}$ is independent of $(s_h - s_\ell)$, while the equilibrium $w$ does depend on these expected productivity parameters.

Returning to Theorem 3, it is easily seen from (27) that,

$$\psi = \sqrt{\frac{\mu R}{2\mu - \mu}} \quad \bar{\psi} = \sqrt{R}$$

$$\bar{R} = \psi^2 \quad \bar{\bar{R}} = \psi^2 (2 - \mu)/\mu$$

Figure 2 below plots the equilibrium $\pi_{th}$ against $\mu$ and $R$ when $\psi = 0.75$. Apparently, the likelihood of fraud in equilibrium increases as $\mu$ falls, ceteris paribus. That is, fraud is more likely if investors prior expectations of high productivity is low. Meanwhile, $\pi_{th}$ also falls as the cost of capital rises, which is consistent with Proposition 3. Next, Figure 3 plots the equilibrium $\pi_{th}$ against $R$ and $\psi$. We see that the equilibrium likelihood of fraud rises as the managerial benefit of control increases. Interestingly, variations in $\psi$ appear to have a greater effect on the equilibrium $\pi_{th}$, at the margin, compared to variations in with respect to $R$. Next, Figure 4 plots $\pi_{th}$ against $R$ and $D \equiv (s_h - s_\ell)$, the difference in the capital productiveness across the two states. The equilibrium likelihood of fraud rises as $D$ increases, and again the impact of changes in $D$ on $\pi_{th}$, at the margin, exceed those of changes in $R$.

Finally, Figure 5 graphs the disclosure information regions, in terms of $(R, \psi)$, delineated in Theorem 3, for a parameterization that takes $\mu = 0.05$ and sets $s_h/D = 1.2$. That is, for each value of $(R, \psi)$ in a suitable interval, we compute the corresponding $(\bar{\psi}, \bar{\psi})$ and $(\bar{R}, \bar{\bar{R}})$ values (cf. Theorem 3) to identify the pooling, noisy revelation, and separating (or truth telling) regions.\textsuperscript{15}

### 6.3 Long-Term Performance Contracts

In our earlier analysis we have assumed away any possibility of output contingent wages. This was done in order to illustrate the basic point of our paper, in the most intuitive manner. In this section, we complete the analysis to include general compensation contracts. It is clear from Theorem 1 that output contingent wages do not mitigate the manager’s incentive problem and therefore are not optimal. Long-term performance contracts are, however, at least a potential avenue to mitigate investors’ coordination problem. In particular, any level of investment implemented in the noisy revelation equilibrium (e.g., Theorem 3) can be implemented under a truth telling mechanism with

\textsuperscript{15}The return $R$ in the graph represents the annual return on a project spanning over the duration of five years, i.e., equals the fifth root of the return over the life of the project (which represents one period in our model).
output contingent wages. We show, however, that ruling out such contracts does not limit the implications of our analysis.

To illustrate, consider first a truth telling mechanism and recall that $1 - w_h^+$ is the net output received by owners. Ex-post investment in the high state, then, satisfies $s_h f'(k_h)(1 - w_h^+) = R$. Thus, by altering the manager’s output contingent compensation, any level of (under) investment can be implemented via this mechanism. More generally, in a noisy revelation mechanism the perceived Bayesian updated probability of the high state following a high report is $\frac{\mu s_h + (1 - \mu) \pi_{\text{th,sl}}}{\mu + (1 - \mu) \pi_{\text{th}}}$ and the optimal level of investment solves,

$$\max_k \left( \frac{\mu s_h + (1 - \mu) \pi_{\text{th,sl}}}{\mu + (1 - \mu) \pi_{\text{th}}} \right) f(k)[1 - w_h^+] - Rk$$

While the potential advantage of providing output contingent wages lies in the reduction in investment ex-post which in turn leads to lower rents to the manager, it is not without cost. Providing compensation in the good state reduces the owners’ claim on realized output. Both mechanisms, non-truthful reporting (or noisy revelation) and, output contingent wages, can mitigate investors’ coordination problem at a cost. One would therefore expect the optimal mechanism to include both.

Our goal in this section is to provide conditions, e.g., regarding the private benefit from control, under which the use of output contingent payoffs is not optimal. This would put in perspective our foregoing results. We start by providing a necessary condition for the non-optimality of long-term performance contracts.

**Proposition 4** There exists $\tilde{\psi}$ such that for $\psi < \tilde{\psi}$, $w_h^+ = w_h^0 = 0$ and $w_k^+ = w_k^0$.

Intuitively, the rational to use output contingent wages is to decrease investment following a good report and reduce expected wages paid to the manager through the reduction in the investment gap $k_h, k_t$. The larger are the private benefits from control, the more beneficial it is to reduce the investment gap. Thus, it would be beneficial when private benefits from control exceed the threshold $\tilde{\psi}$. It is worth noting that the condition threshold provided above is a necessary condition and the actual private benefit threshold above which output contingent wage contracts become optimal might be higher.

Recall that noisy revelation by the manager is optimal when private benefits from control are
neither too high nor too low, i.e., the interval \((\psi, \bar{\psi})\) as defined in Theorem 3. In order to better understand the optimality of output contingent wages and noisy revelation by the manager, we next explore the relation between the threshold \(\tilde{\psi}\) and the aforementioned interval.

**Proposition 5** The interval \((\psi, \tilde{\psi})\) is non-empty, and \(\bar{\psi} \leq \tilde{\psi}\) provided that \(\frac{s_h}{s_h - s_l} \geq \frac{\gamma(1 - \mu)}{2\mu^2 - 1}\); where \(\tilde{\psi}\) is as defined in Proposition 4, and \(\bar{\psi}\) and \(\bar{\psi}\) are as defined in Theorem 3.

The above proposition confirms that the noisy revelation equilibrium does not rely on any limitation to short-term performance contracts. In particular, there will always exist an interval (in the spirit of Theorem 3) for which the optimal contract will involve noisy revelation by the manager. Moreover, under the aforementioned condition, disregarding long-term contracts for characterizing the properties of the noisy revelation equilibrium, is done without loss of generality.\(^{16}\) This is summarized as follows.

**Theorem 4** In the general case where long-term performance contracts are considered Theorem 3 holds. In particular, \(\pi_{th} \in (0, 1)\) for \(\psi \in (\bar{\psi}, \min(\bar{\psi}, \tilde{\psi}))\).

We conclude that fraud—where insiders over-state the economic prospects of the firm with positive probability—can occur in equilibrium, irrespective of whether performance-contingent contracts can be written or not. Interestingly, performance contracts that induce compensation uncertainty for a risk-averse agent may be optimal even when such contracts worsen risk-sharing without relaxing the agent’s incentive constraints. This is because such contracts may mitigate the investors’ ex post coordination constraints by influencing their objective function ex post, thereby making their Bayesian-rational investment strategy closer to the ex ante incentive-efficient investment policy.

### 7 Discussion and Extensions

#### 7.1 Corporate Fraud, Overinvestment, and Firm Growth

In the past, certain industries (and individual firms) suffered more than others during episodes of corporate fraud and overinvestment. Such overinvestment took place, for example, during the South Sea Bubble among firms specializing in trade; during the 1920’s among firms exploiting mass

\(^{16}\)To illustrate the above condition, suppose that \(\mu = 1/2\). The condition, then implies that \(\frac{s_h}{s_h - s_l} \geq \frac{\gamma(1 - \mu)}{2\mu^2 - 1}\). For \(\gamma = 2\) this is always the case, while for \(\gamma = 3\) this implies \(\frac{s_h}{s_h - s_l} \geq 3\).
production and utilities companies; and, more recently, among Internet related firms in the 1990s. In each case, overinvestment and fraud was triggered by investors’ expectations for high growth either because of the introduction of new technologies or because of exploration of new business opportunities. Such high expectations were often sustained by information provided by insiders (Shleifer (2000) and Sidak (2003)). In addition, there is systematic evidence for the relation between corporate fraud, overinvestment, and the value of growth options. (see, e.g., Wang (2004))

In our model, the probability of success in the model can be viewed as a measure of the value of the firm’s growth option: the larger is $s_h$, the more valuable is the option to invest with accurate information. Thus, our model predicts a positive relation between the value of growth options and the likelihood of fraud (Proposition 2), which is consistent with the announced empirical observations.

### 7.2 Corporate Fraud, Overinvestment, and Access to Capital

The likelihood of misreporting in our model is negatively related to the investors’ required rate of return, i.e., our model predicts that fraud and overinvestment are more (less) likely to occur in periods where firms have easy (costly) access to capital. Indeed, easy access to external funds has been an integral part of several corporate finance scandals throughout history. For example, the South Sea Company obtained the right from the government to convert a large portion of the government debt (held by the public) into company shares, which not only provided substantial access to capital but also motivated the company to mislead investors (Feber (2002)). Similarly, in the 1860’s, railroad entrepreneurs copied a major financial innovation of the time—namely, the selling of government bonds to the public rather than only to financial institutions—to finance their rapidly growing industry. As in the case of the South Sea Company, this access to capital was followed by fraudulent disclosures to investors, e.g., the Union Pacific Railroad company (Skeel (2005)). Similarly, during 1920’s, a period in which it was "...easier than ever before to raise huge amounts of capital..." (Skeel (2005) page 79), Wall Street played an increasing role in satisfying corporations financing needs.

Thus, our model may help explain why instances of corporate fraud and overinvestment have

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17Wang (2004) uses Securities Class Action Clearinghouse database to explore a sample of 656 U.S. public companies during the years 1996 to 2003 for which lawsuits involving allegations of accounting irregularities where filed, and finds that instances of corporate fraud are more likely in certain industries, such as Telecommunications and Services and among firms with valuable growth options.
seemed to be clustered throughout history - often referred to as waves. As capital markets play a central role in satisfying firms' financing needs during such waves, some researchers have attributed waves of corporate fraud to periods of overly optimistic investors, or investor mania, in which managers take advantage of naive investors by overstating the prospects of their firms (e.g., Jensen (2004)). While our theory does not have much to say about such irrationalities and their link to corporate fraud, it suggests that during periods of low required rates of return by investors, disclosures will be less informative. Low costs of raising capital can arise for a number of reasons that include, more liquid financial markets, financial innovations, or low risk premia.

If there is systematic time-variation in risk-premia, then our model predicts that the incidence of fraud and overinvestment will also have a predictive component. In fact, a growing literature emphasizes systematic time-variation in risk-premia (see, e.g., Fama and French (1988)). In particular, there is evidence of cyclicality in risk-premia (Lettau and Ludvigson (2001)). With cyclical risk-premia, our model also delivers the prediction that the likelihood of fraud will be cyclical, complementing similar predictions in Hertzberg (2003) and Povel et al. (2005).

However, the asymmetric information foundations of our framework yield a richer set of dynamic predictions on the likelihood of fraud and overinvestment than cyclicity. Arguably, asymmetric information regarding investment opportunities is the greatest when there are new type of investment opportunities; for example, through technological innovations or trading in new markets. Because the likelihood of fraud is positively related to the extent of asymmetric information in equilibrium, our framework suggests that the incidence of fraud and overinvestment would be higher at the onset of new technologies or new trading opportunities. The effect of permanent shocks like technological innovations on fraud distinguishes our analysis from models that view fraud as a cyclical phenomena.

7.3 Communication in Internal and External Capital Markets

While our model has been couched in terms of allocation of capital by external capital markets, it has implications for within-firm capital allocation by internal capital markets, as well. A fundamental distinction between internal and external capital markets is that total and unconditional control rights are allocated to the capital provider (i.e., the CEO) in internal capital markets, while the capital providers in external markets can not exercise control (in corporations), as emphasized by Berle and Means (1933).
An old line of argument, due to Alchian (1969) and Williamson (1975), is that an internal capital market results in higher quality of information than an external capital market. The literature emphasizes the incentives to produce information as an extension of this argument. In particular, by learning about their organizations, CEO’s can implement appropriate value-enhancing actions (unlike other financiers that lack control rights), which in turn improves the decision-making process in internal capital markets (Gertner, Scharfstein and Stein (1994) and Stein (2001)).

But there is also considerable evidence that conglomerates tend to overinvest in bad divisions and underinvest in good ones. For example, Scharfstein (1998) finds that division in high-q industries tend to invest less than their stand-alone industry peers, while Rajan Servaes and Zingales (2000) show that diversified firms tend to allocate more capital, on average, toward low-q segments compared to high-q segments. In a related vein, Gertner, Powers and Scharfstein (2002) find that spin-offs tend to cut investment in low-q industries and increase investment in high-q industries.

These findings raise the natural question: if internal capital markets result in a higher quality of information (cf. Alchian (1969) and Williamson (1975)), why then do they overinvest? Interestingly, our analysis is consistent with both superior information generation and overinvestment by internal capital markets in low productivity units. However, the reasons for the informational advantages in our model are different from those advanced in the literature: in our framework, internal capital markets are more informative because of the ability of CEOs to use their control rights and credibly commit to an investment response to division managers’ strategic disclosures.

To fix ideas, suppose that a CEO of a multi division firm is less informed than a division manager regarding a certain project. While the division manager might have preferences to increase the level of investment in her own division the CEO must elicit this information in order to efficiently invest. As we have discussed above, the optimal mechanism may involve (ex-post) inefficient levels of investment in order to reduce the rents received by the (division) manager. Unlike dispersed owners (over space and time), the CEO faces a repeated relationship with the division managers and has an incentive to build a reputation. Such a repeated game may support ex-post inefficient levels of investment executed by the CEO to increase the ex-ante value of the firm. For example, the CEO may build a reputation to underinvest in good projects—as suggested by the second-best contract described in Theorem 2. To the extent that such commitment can be supported in internal capital markets by reputation, the noisy revelation equilibrium is less beneficial and internal capital markets become more informative.
### 7.4 Communication in Competitive Managerial Labor Markets

It turns out that if we interpret “private benefits” of control in terms of career concerns of self-interested managers, then our framework suggests a higher likelihood of fraud—including earnings manipulations by CEOs and window-dressing by money managers—because of the inability of labor market participants and capital providers to coordinate.\(^\text{18}\) Empirically, career concerns seem important in the markets for CEO’s and money managers. For example, CEO turnover tends to follow poor stock market performance (Coughlan and Schmidt (1985) and Weisbach (1988)) and those that perform poorly are less likely to become outside members on the Boards of Directors of other firms (Gibbons and Murphy (1992)). Similarly, past performance of various types of institutional fund managers influences their fund-raising ability (Lakonishok, Shleifer, Thaler, and Vishny (1991), Sirri and Tufano (1998)). Meanwhile, Holmstrom (1982) and Holmstrom and Ricart I Costa (1986) and theoretically examine the implications for capital budgeting and managerial actions when managers are concerned about the effects of firm performance on the labor market’s perceptions of their managerial skills.

To incorporate career concerns, we need not significantly augment our model, but rather only slightly reinterpret the parameters. When managers and markets learn about managers’ type and talented managers are scarce, one would expect talented managers to earn a premium over nontalented managers in equilibrium. Suppose that talent is correlated across periods; and, in the context of our model, more talented managers have a higher probability of the good state occurring and are therefore more likely to be productive. In such an environment, reporting a high signal is beneficial to managers as it alters investor’s updated beliefs about the manager’s talent. The extent to which a high report will lead to higher future wages for the manager, however, depends on the informativeness of the signal.

In an equilibrium with reporting strategy \(\pi_{lh}\), let \(\mu_h\) be the probability that the state is high, given a high signal reported by the manager, i.e., \(\mu_h = \frac{\mu}{\mu + (1-\mu)\pi_{lh}} \geq \mu\). We also recall that the probability that the state is high given a low report is zero. Let the manager’s future compensation as a function of investors’ beliefs \(z\) be \(\hat{b}(z)\). The manager will prefer to report a high signal over a low signal, as \(\hat{b}(\mu_h) - \hat{b}(0) \geq 0\). This leads to a problem that is mathematically equivalent to the one we have analyzed above, as long as \(\hat{b}(z)\) is an increasing function - in particular when

\(^{18}\text{Lakonishok et al. (1991) show that funds (in particular small) sell poorly performing stocks in a manner that is consistent with a window-dressing strategy, and Gompers and Lerner (1999) provide evidence of grandstanding by Venture Capitalists.}\)
Under this interpretation, the parameter $\psi$ represents the extent to which learning about managerial type is productive in future periods and the results we have discussed follow directly from Theorem 3 and the subsequent analysis.

Interestingly, this reinterpretation of the theory also suggests that career concerns may lead to slower learning about managerial ability, in particular, in times when this information is most valuable. Thus, changes in the market for CEO’s during the last two decades (such as high turnover) may have contributed to the increase in fraudulent behavior.

### 7.5 Coordination via the Corporate Charter

Lack of coordination among dispersed owners is the central friction we have explored in this paper, which results in implementation of non-truthful reporting by insiders and subsequent overinvestment in equilibrium. While such a remedy involves inefficiencies and is costly to owners, one would expect that mechanisms that facilitate such coordination should have emerged over time. Indeed, one such example is the Corporate Charter. The corporate charter specifies the list of conditions that provides the corporation with the right to exist and operate. Owners in control of the corporation in future subsequent periods then must follow the guidelines put forward in the charter.

Corporate charters exhibit substantial cross sectional differences, especially with respect to the strength of corporate governance (Gompers, Ishii, and Metrick (2003)). For example, while some firms provide guidelines that secure the interests of shareholders (*democratic*), other firms provide more freedom to management (*dictatorship*). *Prima facie*, this type of heterogeneity is puzzling because a common objective of corporate charters is to provide a set of guidelines to secure subsequent efficient operation of the corporation; one would therefore expect corporate charters to uniformly aim towards minimizing firms’ cost of capital.

Our model highlights a somewhat surprising advantage for firms to restrict themselves ex-ante to higher costs of capital in the future. That is, firms optimally set guidelines in the corporate charter that *impede* the minimization of the cost of raising capital. To see the main intuition, consider a marginal increase in a firm’s cost of capital. The clear disadvantage to owners is that they now need to allocate higher fractions of firm output to their financiers (all else equal). The more subtle advantage, however, is that this assists in mitigating the coordination problem. In particular, when owners are faced with higher costs of capital then insiders are less inclined to overstate firm
performance as the implications for investment are dampened. One can make this intuition more precise and show that when the private benefits from control exceed a certain threshold, it will be optimal to voluntarily increase the firms cost of capital. That is, for high enough private benefits of control, a possible substitute for implementing misreporting by insiders is a somewhat "inefficient" corporate charter.

7.6 Robustness: Equilibrium Under-Reporting

Up to this point, we have focused our attention to the possibility of over-reporting by management. As we mentioned earlier, our approach is primarily motivated by the empirical evidence regarding the misreporting practices of corporations (see, e.g., Burns and Kedia (2004) and Bogle (2005)). However, there is also evidence suggesting that firms sometimes understate earnings (e.g., Burns and Kedia (2004)). The theory presented above indicates that understating performance may increase investment efficiency when investors interpret earnings as signals for future performance. In the following proposition, we provide conditions under which an equilibrium with under-reporting dominates a truth-telling equilibrium and a pooling equilibrium.

Proposition 6 There exists $\psi_1, \psi_2$ such that an equilibrium with under-reporting dominates a truth-telling equilibrium and a pooling equilibrium. Moreover, when $\mu < \frac{1}{2}$, $(\psi_1, \psi_2) \subset (\psi, \psi)$, that is, under a wider set of parameters overreporting is optimal. Finally, as $\mu \to 0$, $(\psi_1, \psi_2) \to \emptyset$ and $(\psi, \psi) \to \mathbb{R}_+^+$. 

8 Summary and Conclusions

Overinvestment in certain firms or sectors, induced by corporate fraud where informed insiders manipulate the beliefs of uninformed investors through exaggerations of economic prospects, has
been historically prevalent, and has recently attracted much attention because of some prominent cases (e.g., Enron and World Com). However, reconciling overinvestment and corporate fraud with rational capital markets poses obvious challenges. We provide a new theory of overinvestment, based on the dominant characteristics of modern corporations: a dispersed and changing ownership. Because of the dispersed and changing ownership, corporate owners cannot credibly precommit to (portfolio) investment policies that are ex post inefficient.

We develop and analyze an agency model, with insiders who are privately informed of the firm’s expected returns on investment, in a plausible institutional setting: dispersed outsiders follow individually rational portfolio investment strategies, but the firm can write long-term incentive contracts with managers. For a wide range of conditions, outside shareholders optimally suffer a positive probability of fraud in equilibrium, because the lack of investor coordination increases the incentive compensation costs of inducing truthfulness from informed insiders. In equilibrium, shareholders optimally determine the probability of fraud and the extent of overinvestment.

Our analysis indicates that the likelihood of fraud is higher when the cost of capital or equity risk-premium is low; it is higher when growth-options become more valuable (as when significantly new technologies are introduced); and, it is higher when management values control more. Moreover, protecting management compensation from downside risk of investment performance increases the likelihood of fraud in equilibrium. These refutable implications of our model appear consistent with historical evidence, and our framework also yields other refutable implications: comparing the likelihood of fraud in internal versus external capital markets, linking the likelihood of fraud to managers’ career concerns, and providing a new perspective on the role of corporate charter.
Appendix

Proof of Theorem 1: We solve program (P1) by first neglecting the incentive compatibility constraint for the high type and verifying that the solution to the more relaxed program, we refer to as (P1’), satisfies this constraint:

\[
(P1') \max_{\{w,k\} \in \mathcal{W}_+} \sum_{j=\ell}^{h} \mu_j \left[ s_j f(k_j)(1 - w_j^+ + w_j^0) - w_j^0 - Rk_j \right] \\
\text{s.t.} \\
U(\ell, \ell | w, k) \geq U(\ell, h | w, k) \\
w_j^+ \geq 0, \ w_j^0 \geq 0, \ r \in \{\ell, h\}
\]

First note that at the optimum \(k_h \geq k_\ell\). To see this, suppose by contradiction that \(k_h < k_\ell\). Then, the incentive compatibility constraint is satisfied for a compensation schedule that pays the manager zero in every state. Thus, under the optimal contract it must be that the manager receives zero compensation. It must also be that the proposed optimal \(k_\ell^*\) and \(k_h^*\) belong to the interval \([k_\ell^*, k_h^*]\). For example, if \(k_\ell^* > k_\ell > k_h^*\), then increasing investment in both states to the level \(k_\ell\) would increase the objective function and still satisfy all constraints in (P1’). When \(k_\ell^* \leq k_h < k_\ell \leq k_h^*\), however, an increase in the objective function is achieved under the alternative \(k_h' = k_\ell' = k_\ell\) (while wages are still zero). Thus, we reach a contradiction and it, therefore, must be that \(k_h \geq k_\ell\) at the optimum.

Next note that the solution to program (P1’) satisfies \(w_h^+ = w_h^0 = 0\). To see this, suppose by contradiction that either \(w_h^+\) or \(w_h^0\) are strictly positive. Then an examination of the incentive compatibility constraint for the low type reveals that it is not violated following a reduction in this wage, say to zero. Thus, the objective function can be increased and we reach a contradiction.

Moreover, note that a flat compensation of \(w'\) following a low report, where \(u(w') = s_\ell f(k_\ell)(u(w_\ell^+) - u(w_\ell^0)) + u(w_\ell^0)\) is optimal. This result directly follows from managerial risk aversion. Thus, \(w_\ell^+ = w_\ell^0 = w\) for some \(w \geq 0\) at the optimum.

Finally, note that the incentive compatibility constraint (i.e., of the low type) binds at the optimum. To see this, recall that \(k_h \geq k_\ell\) at the optimum and suppose by contradiction that there is slack in the incentive compatibility constraint. This amounts to supposing that \(u(w) + b(k_h) > b(k_h)\). It is clear, then, that a reduction in \(w\) would be feasible (i.e., \(w > 0\)) and that such a reduction would increase the objective function. Thus, we reach a contradiction.

It remains only to verify that the incentive compatibility constraint for the high type (which appears in program (P1) but has been neglected in program (P1’)) is satisfied under the solution to program (P1’). This is trivial, however, as \(w_h^+ = w_h^0 = 0, w_\ell^+ = w_\ell^0 = w, \) and \(u(w) = b(k_h) - b(k_\ell)\). Therefor, the solution to program (P1’) coincides with the solution to program (P1).
Proof of Corollary 1: It follows from Theorem 1 that program (P1) can be rewritten as below and the Corollary follows from deriving the first order conditions.

\[
\max_{(k_h, k_{\ell}, w) \in \mathbb{R}^3_+} \mu(s_h f(k_h) - R k_h) + (1 - \mu)(s_{\ell} f(k_{\ell}) - R k_{\ell} - w)
\]

\[
s.t. \quad u(w) = b(k_h) - b(k_{\ell})
\]

Proof of Proposition 1: Differentiating the objective function in (23) (labeled ‘OBJ’ for convenience) yields,

\[
\frac{dOBJ}{du} = (1 - \mu) \left[ \left( s_{\ell} - \frac{R}{4} (f(k_h) + f(k_{\ell})) \right) (f(k_h) - f(k_{\ell})) + u^\gamma \right] \frac{\partial \pi}{\partial u} - \gamma u^{\gamma - 1} \frac{Ru - \mu \psi(s_h - s_{\ell})}{Ru}
\]

\[
= (1 - \mu) \left[ \left( -\frac{uR}{2\psi} \frac{2u}{\psi} + u^\gamma \right) \left[ -\frac{\mu \psi (s_h - s_{\ell})}{(1 - \mu)u^2R} \right] - \gamma u^{\gamma - 1} \frac{Ru - \mu \psi(s_h - s_{\ell})}{Ru} \right]
\]

(28)

Setting (28) equal to zero implies,

\[
\left[ u^{\gamma - 2} - \frac{R}{\psi^2} \right] [\mu \psi(s_h - s_{\ell})] = \gamma u^{\gamma - 2} [\mu \psi(s_h - s_{\ell}) - Ru]
\]

\[
\Rightarrow F = \gamma Ru^{\gamma - 1} - [\mu \psi(s_h - s_{\ell})] \left( u^{\gamma - 2} [\gamma - 1] + \frac{R}{\psi^2} \right) = 0.
\]

Now, using the implicit function theorem, from any parameter \(x\),

\[
\frac{\partial u}{\partial x} = -\frac{\partial F}{\partial x} \cdot \frac{\partial F}{\partial u} \quad (29)
\]

But, at the optimum,

\[
\frac{\partial F}{\partial u} = \gamma (\gamma - 1)Ru^{\gamma - 2} - (\gamma - 1)(\gamma - 2)u^{\gamma - 3} [\mu \psi(s_h - s_{\ell})]
\]

\[
= \frac{\gamma - 1}{u} (\gamma Ru^{\gamma - 1} - (\gamma - 2)u^{\gamma - 2} [\mu \psi(s_h - s_{\ell})])
\]

\[
\geq \frac{\gamma - 1}{u} (\gamma Ru^{\gamma - 1} - (\gamma - 1)u^{\gamma - 2} [\mu \psi(s_h - s_{\ell})]) > 0.
\]

Furthermore, and again at the optimum,

\[
\frac{\partial F}{\partial R} = \gamma u^{\gamma - 1} - \frac{1}{\psi^2} [\mu \psi(s_h - s_{\ell})] \propto \gamma Ru^{\gamma - 1} - \frac{R}{\psi^2} [\mu \psi(s_h - s_{\ell})] > 0
\]

\[
\frac{\partial F}{\partial \psi} = [\mu(s_h - s_{\ell})] \left( \frac{R}{\psi^2} - u^{\gamma - 2} [\gamma - 1] \right)
\]

\[
\frac{\partial F}{\partial s_h} = -u^{\gamma - 2} [\gamma - 1] \mu \psi - \frac{R}{\psi^2} \mu \psi < 0.
\]

The results then follow from (29).
Proof of Proposition 2: We can compute,
\[
\frac{\partial \pi}{\partial \psi} = \frac{\partial \psi}{u \frac{\mu (s_h - s_l)}{(1 - \mu)R}} > 0
\]
\[
\frac{\partial \pi}{\partial R} = \frac{\partial u R}{\partial R} \left( \frac{\mu \psi (s_h - s_l)}{(1 - \mu) [u R]^2} \right) < 0
\]
\[
\frac{\partial \pi}{\partial s_h} = \frac{\partial (s_h - s_l) / u}{(1 - \mu) R} > 0.
\]

Proof of Proposition 3: From \( \frac{\partial u}{\partial R} < 0 \), \( k_h - k_l = (\sqrt{k_h} - \sqrt{k_l}) (\sqrt{k_h} + \sqrt{k_l}) = \frac{u}{\psi} (\sqrt{k_h} + \sqrt{k_l}) \) and the derivatives below the results follow.
\[
\frac{\partial k_h}{\partial \psi} = \frac{\partial k_h}{\partial \pi} \frac{\partial \pi}{\partial \psi} < 0
\]
\[
\frac{\partial k_h}{\partial R} = 2 \sqrt{k_h} \left( -\frac{s_l}{R^2} + \frac{1}{\psi} \frac{\partial u}{\partial R} \right) < 0
\]
\[
\frac{\partial k_h}{\partial s_h} = 2 \sqrt{k_h} \left( \frac{1}{\psi} \frac{\partial u}{\partial s_h} \right) > 0.
\]

Proof of Theorem 3: A feasible probability of fraud implied by the first order conditions implies that \( u R \in (\pi, u) \) where \( \pi = \frac{\psi (s_h - s_l)}{R} \) and \( u = \mu \pi \). Next, notice that the (strictly positive) solution to the first order conditions is unique as the second order conditions satisfy,
\[
\frac{d^2 OBJ}{du^2} \bigg|_{u=\pi} = -\gamma (\gamma - 1) R u^{\gamma - 2} + u^{\gamma - 3} (\gamma - 2) [\mu \psi (s_h - s_l)]
\]
\[
\propto -\gamma R u + (\gamma - 2) [\mu \psi (s_h - s_l)]
\]
\[
\Rightarrow \begin{cases} 
> 0, & u < u^0 = \frac{(\gamma - 2) [\mu \psi (s_h - s_l)]}{\gamma R} \\
< 0, & u > u^0
\end{cases}
\]
Note that \( u^0 < u < \pi \) as \( \frac{\gamma - 2}{\gamma} < 1 \). This implies that the solution to the first order conditions, \( u^* \), is feasible if and only if \( \frac{d OBJ}{du} \bigg|_{u=\pi} > 0 > \frac{d OBJ}{du} \bigg|_{u=u^0} \).
\[
\frac{d OBJ}{du} \bigg|_{u=\pi} = -\left( \frac{\psi (s_h - s_l)}{R} \right)^{\gamma - 1} R (\gamma - (\gamma - 1) \mu) + \frac{R}{\psi^2} [\mu \psi (s_h - s_l)]
\]
Thus,
\[
\frac{d OBJ}{du} \bigg|_{u=\pi} < 0 \iff \frac{(\gamma - 1) \mu}{\psi^2 (s_h - s_l)^{\gamma - 2}} < \psi^\gamma
\]
Also,
\[
\frac{d OBJ}{du} \bigg|_{u=u} = -\left( \frac{\mu \psi (s_h - s_l)}{R} \right)^{\gamma - 1} R (\gamma - (\gamma - 1)) + \frac{R}{\psi^2} [\mu \psi (s_h - s_l)]
\]
Thus,
\[
\left. \frac{dOBJ}{du} \right|_{u=u^*} > 0 \iff \left( \frac{R^{\gamma-1}}{(s_h-s_l)^{\gamma-2}} \right) \frac{1}{\mu^{\gamma-2}} > \psi^\gamma
\]

Thus, the solution is feasible whenever,
\[
\psi \in (\psi, \bar{\psi}) \equiv \left( \left( \frac{R^{\gamma-1}}{(s_h-s_l)^{\gamma-2}} \right) \frac{1}{\mu^{\gamma-2}} \right)^{1/\gamma} \left( \mu^{s_h} \gamma (\gamma-1) \right) \left( \mu^{s_l} R \right) \left. \mu^{s_l} \gamma \frac{1}{\gamma} \right)
\]

This set is non-empty for all \( \gamma \geq 2 \). Similarly, the interval \((\underline{R}, \overline{R})\) can be defined.

**Proof of Corollary 2:** We note that there exists a \( w_\ell > 0 \) that sets \( \pi_{\ell h} \) as described in (21) to 0. Hence, under the condition annunciated in Theorem 1, that choice of \( w_\ell \) is dominated by another wage, given which, the manager’s best response reporting strategy sets \( \pi_{\ell h} < 1 \).

**Proof of Proposition 4:** It follows from optimal ex-post investment that the only wage contingent plan to induce under investment following a high report satisfies \( w_h^+ > w_h^0 \). Further, it is not optimal for the owners to set \( w_h^0 > 0 \). In the following we derive the sufficient conditions for the optimal contract not to include output contingent wages. As we are interested in deriving a sufficient condition we disregard the utility of the manager from compensation following a high report. This relaxes the incentive compatibility constraint and increases the gains from using output contingent wages as a disciplinary device. Thus, if we find that the use of output contingent wages is not optimal in the problem to follow, it is definitely not optimal in the original problem. Finally, notice that we can disregard the incentive compatibility constraint of the high type manager which will strictly prefer to report truthfully as long as the low type manager is indifferent.

\[
\max_{w_h^+} \mu((1-w_h^+)^2s_h\sqrt{k_h}-Rk_h) + (1-\mu)[\pi((1-w_h^+)^2s_h\sqrt{k_h}-Rk_h) + (1-\pi)(2s_\ell\sqrt{k_\ell}-Rk_\ell-u^\gamma)]
\]

**s.t.**
\[
u + \psi\sqrt{k_\ell} = \psi\sqrt{k_h}
\]
\[
\sqrt{k_h} = \frac{(1-w_h^+)(\mu s_h + (1-\mu)s_l)}{\mu + (1-\mu)\pi R}
\]
\[
\sqrt{k_\ell} = \frac{s_\ell}{R}
\]
\[
s_h = \frac{\mu s_h + (1-\mu)s_l}{\mu + (1-\mu)\pi}
\]

We can substitute and get,
\[
u = \frac{\psi}{R} \left( \frac{(1-w_h^+)(\mu s_h + (1-\mu)s_l)}{\mu + (1-\mu)\pi} - s_\ell \right)
\]

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Thus we can write the objective function,

$$
\max_{w^+_h} \quad \mu \left[ (1 - w^+_h)^2 s_h \left( \frac{1 - w^+_h}{R} \right)^2 \right] - R \left[ \left( \frac{1 - w^+_h}{R} \right)^2 \right] + (1 - \mu)(1 - \pi) \left( 2 s_i \frac{s_i}{R} - \left( \frac{\psi}{R} \right) \left( (1 - w^+_h) s_h - s_i \right) \gamma \right) + (1 - \mu) \pi \left( (1 - w^+_h)^2 s_h - R \left[ \left( \frac{1 - w^+_h}{R} \right)^2 \right] \right) = \\
(\mu + (1 - \mu)\pi) \left( \frac{(1 - w^+_h)^2 s_h^2}{R} \right) + (1 - \mu)(1 - \pi) \gamma \left( \frac{\psi}{R} \left( (1 - w^+_h) s_h - s_i \right) \right) \gamma^{-1} \psi s_h \frac{R}{R}$$

We now derive first order conditions for optimality with respect to $w^+_h$,

$$FOC_{w^+_h} : \quad -2(\mu + (1 - \mu)\pi) \left( \frac{(1 - w^+_h)^2 s_h^2}{R} \right) + (1 - \mu)(1 - \pi) \gamma \left( \frac{\psi}{R} \left( (1 - w^+_h) s_h - s_i \right) \right) \gamma^{-1} \psi s_h \frac{R}{R}$$

We will now analyze the above first order condition in order to obtain a condition under which neglecting output contingent wages is without loss of generality. We seek to find conditions under which the above derivative is negative. It is worth noting at this point that we should restrict attention to $w^+_h \leq 1 - \frac{s_i}{s_h}$ as no rents are being paid out to the manager at this point and the incentive compatibility constraint applies when investment in the high state is larger than investment in the low state. Thus, it suffices to explore the sign of the above derivative for $w^+_h \in [0, 1 - \frac{s_i}{s_h}]$. For a given $\pi$ and $\gamma > 2$, the FOC is a convex function of $w^+_h$. Furthermore, the FOC are negative for $w^+_h = 1 - \frac{s_i}{s_h}$. Thus, it suffices to explore the derivative at the extreme value $w^+_h = 0$. For this case the FOC can be rewritten as,

$$FOC_{w^+_h=0} : \quad -2(\mu + (1 - \mu)\pi) \left( \frac{s_h^2}{R} \right) + (1 - \mu)(1 - \pi) \gamma \left( \frac{\psi}{R} (s_h - s_i) \right) \gamma^{-1} \psi s_h \frac{R}{R}$$

Notice that $FOC_{w^+_h=0}$ is decreasing in $\pi$ (this is because $s_h$ is decreasing in $\pi$). Thus, it suffices to verify that the FOC is negative $\pi = 0$.

$$FOC_{w^+_h=0, \pi=0} : \quad -2\mu \left( \frac{s_h^2}{R} \right) + (1 - \mu) \gamma \left( \frac{\psi}{R} (s_h - s_i) \right) \gamma^{-1} \psi s_h \frac{R}{R} \\
\propto -2\mu \left( \frac{s_h}{R} \right) + (1 - \mu) \gamma \left( \frac{\psi}{R} \right) \gamma (s_h - s_i) \gamma^{-1} \psi s_h \frac{R}{R} \\
\propto -\frac{2}{\gamma (1 - \mu)} \left[ s_h - s_i \right] \gamma^{-1} + \psi$$

Thus, the use of output contingent wages is not optimal when $\psi < \psi$ where, $\psi \equiv \frac{2}{\gamma (1 - \mu) \left[ s_h - s_i \right] \gamma^{-1} + \psi}$.}

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Proof of Proposition 5: We first provide conditions under which \( \tilde{\psi} \geq \bar{\psi} \),

\[
\tilde{\psi} \equiv \frac{2}{\gamma (1 - \mu)} \frac{\mu s_h}{|s_h - s_l|} R^{\gamma - 1} \quad \geq \quad \left( \frac{R^{\gamma - 1}}{(s_h - s_l)^{\gamma - 2}} \right) \frac{1}{\mu^{\gamma - 2}} \equiv \bar{\psi} \\
\Rightarrow \quad \frac{2}{\gamma (1 - \mu)} \frac{\mu^{\gamma - 1} s_h}{|s_h - s_l|} \quad \geq \quad 1
\]

It remains to show that there always exists an interval for which a noisy revelation equilibrium without output contingent wages is optimal. Thus, by contradiction, suppose that \( \tilde{\psi} \leq \bar{\psi} \),

\[
\tilde{\psi} \equiv \frac{2}{\gamma (1 - \mu)} \frac{\mu s_h}{|s_h - s_l|} R^{\gamma - 1} \quad \leq \quad \left( \frac{R^{\gamma - 1}}{(s_h - s_l)^{\gamma - 2}} \right) \frac{\mu}{\gamma - (\gamma - 1)\mu} \equiv \bar{\psi} \\
\Rightarrow \quad \frac{2}{\gamma (1 - \mu)} \frac{s_h}{|s_h - s_l|} \quad \leq \quad \frac{1}{\gamma - (\gamma - 1)\mu} \\
\Rightarrow \quad \frac{s_h}{|s_h - s_l|} \quad \leq \quad \frac{\gamma (1 - \mu)}{\gamma (1 - \mu) + \mu} < 1
\]

We can conclude that there always exists an interval in which the equilibrium in characterized by noisy revelation equilibrium without output contingent wages.

Proof of Proposition 6: The problem can be rewritten as follows where \( p \equiv \pi_{hl} \) (while \( \pi_{lh} = 0 \)).

Here we also focus on the case without output contingent wages. As before, one could extend the analysis further in this case also.

\[
\max_u \mu((1 - p)(s_h f(k_h) - Rk_h) + p(s_h f(k_l) - Rk_l - u^\gamma)) + (1 - \mu)(s_l f(k_l) - Rk_l - u^\gamma)
\]

\[
s.t. \quad u \equiv u(w) = w^{\frac{1}{\gamma}} \quad \quad \\
\quad \quad u = \psi(\sqrt{k_h} - \sqrt{k_l}) \quad \\
\quad \quad k_h = \left( \frac{s_h}{R} \right)^2 \Rightarrow f(k_h) = 2 \left( \frac{s_h}{R} \right) \quad \\
\quad \quad k_l = \left( \frac{s_l(1 - \mu) + s_h \mu p}{R((1 - \mu) + \mu p)} \right)^2 \Rightarrow f(k_l) = 2 \left( \frac{s_l(1 - \mu) + s_h \mu p}{R((1 - \mu) + \mu p)} \right) \quad \\
\quad \quad p = \frac{(1 - \mu)(\psi(s_h - s_l) - uR)}{\mu \nu R}
\]

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\[
\frac{dOBJ}{du} = \sum_{i \in \{h, l\}} \frac{\partial OBJ}{\partial k_i} \frac{\partial k_i}{\partial u} + \frac{\partial OBJ}{\partial p} \frac{\partial p}{\partial u} + \frac{\partial OBJ}{\partial u}
\]

37
Thus, this solution implies that, thus we have,

\[ \psi = 1 \]

\[ dOBJ \quad \frac{du}{\partial u} = \mu \left( R \frac{1}{4} \right) (f(k) + f(k)) - s_h(f(k) - f(k)) - u^\gamma \frac{\partial p}{\partial u} - \gamma u^{\gamma - 1} \left( \frac{1 - \mu}{\psi} \right) \]

Note that,

\[ \frac{R}{4} (f(k) + f(k)) = \frac{R}{2} (\sqrt{k} + k) = R \left( \frac{2}{2} \right) \left( \sqrt{k} - u \right) = R \left( \frac{2}{2} \right) \left( \frac{s_h - u}{\psi} \right) = s_h - \frac{uR}{2} \]

\[ f(k) - f(k) = 2(\sqrt{k} - k) = 2u \frac{u}{\psi} \]

\[ \frac{\partial p}{\partial u} = -\frac{(1 - \mu)\psi(s_h - s_l)}{\mu u^2 R} \]

Thus, we have,

\[ dOBJ \quad \frac{du}{\partial u} = \mu \left( R \frac{1}{4} \right) (f(k) + f(k)) - s_h(f(k) - f(k)) - u^\gamma \frac{\partial p}{\partial u} - \gamma u^{\gamma - 1} \left( \frac{1 - \mu}{\psi} \right) \frac{\psi(s_h - s_l)}{R} u \]

Thus,

\[ \frac{dOBJ}{du} = 0 \]

\[ \Rightarrow (1 - \mu)\psi(s_h - s_l) \left( u^{\gamma - 2} - \frac{R}{\psi^2} \right) = 0 \]

\[ \Rightarrow u = \left( \frac{R}{\psi^2} (\gamma - 1) \right) \]

This solution implies that,

\[ p = \frac{1 - \mu}{\mu} \left[ \frac{\psi(s_h - s_l)}{R \left( \frac{R}{\psi^2} (\gamma - 1) \right) \gamma - 2} - 1 \right] \]

This solution satisfies \( p \in (0, 1) \) when,

\[ \psi^\gamma \in \left( \frac{R^{\gamma - 1}}{(s_h - s_l)^{\gamma - 2} \gamma - 2 (\gamma - 1)} \right) \left( \frac{R^{\gamma - 1}}{(s_h - s_l)(1 - \mu)^{\gamma - 2} (\gamma - 1)} \right) \]

\[ \text{\blacksquare} \]
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Figure 4
Figure 5
Equilibrium Regions

ψ

Pool
Noisy Revelation
Separating

R

1
1.05
1.1
1.15
1.2
1.25