Levered Returns

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Abstract

In this paper we investigate the theoretical relation between financial leverage and stock returns in a dynamic world where both the corporate investment and finance decisions are endogenous. We find that the link between leverage and stock returns is more complex than the static textbook examples suggest and will usually depend on the investment opportunities available to the firm. In the presence of financial market imperfections leverage and investment are generally correlated so that highly levered firms are also mature firms with relatively more (safe) book assets and fewer (risky) growth opportunities. We show that a quantitative version of our model can replicate both the evidence in Bhandari (1988) and Fama and French (1992) about the effects of leverage on returns and the results in Welch (2004) about the impact of returns on leverage ratios.

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1 Introduction

Standard finance textbooks propose a relatively straightforward link between capital structure and the expected returns on equity: increases in financial leverage directly increase the risk of the cash flows to equity holders and thus raise the required rate of return on equity. This remarkably simple idea has proved extremely powerful and has been used by countless researchers and practitioners to examine returns across and within firms with varying capital structures.

Unfortunately, despite, or perhaps because of, its extreme clarity, this relation between leverage and returns has met with only mixed success empirically. Notable early papers (Bhandari (1988) and Fama and French (1992)) were somewhat inconclusive and a negative relation between stock returns and various measures of financial leverage is documented in several more recent studies (George and Hwang (2006), Penman, Richardson, Tuna (2005), Korteweg (2004)).

This paper suggests that the link between financial leverage and stock returns is generally more complex and depends crucially on how debt is used and on its impact on the firm’s investment opportunities. Extant literature generally assumes that debt will be used to fund changes in equity, a tradition that is rooted both in the static trade off view of optimal leverage (Miller (1977)) and the Modigliani-Miller theorem decoupling the firm’s investment and financing strategies.

Our analysis focuses instead on the effects of debt on the left side of the balance sheet as firms use debt to finance capital spending. Since this expansion naturally increases the value of assets in place to growth options it may reduce the underlying (total) risk of the firm and thus its equity risk as well. While these effects can be dismissed in the benchmark Modigliani-Miller
setting, they become of paramount importance in the presence of financial frictions, when investment and financing strategies must be examined jointly.

Our theoretical results can be used to interpret the contradictory empirical evidence about the role of leverage in determining expected returns. In a world of financial market imperfections leverage and investment are often strongly correlated. This, in turn, implies that highly levered firms are also more mature firms with (relatively safe) book assets and fewer (risky) growth opportunities. As a result, cross-sectional studies that fail to control for the interdependence of leverage and investment decisions are unlikely to be very informative.

Clearly real life decisions by corporations will reflect both the existing textbook analysis and our new view. Nevertheless this subtle new link between leverage and expected equity return raises some doubts about the usefulness of the standard textbook formulas in real world applications. This is particularly true when changes in the asset side of the balance sheet are important such as when making cross-sectional comparisons across firms, or when constructing the cost of capital for new projects within a firm.

We begin by constructing a very simple continuous time real options model that formalizes our basic intuition and delivers closed form expressions linking expected returns and corporate decisions on investment and financing. Although stylized the only key assumptions in this example are that debt and investment decisions are linked and, that growth options are relatively less important for large mature firms. If both assumptions are satisfied then highly levered firms will face less underlying (asset) risk and, possibly, also less equity risk as well.

This simple example is very useful to develop intuition for our key insights but it is necessarily far too stylized to be taken directly to the data. To
do this we then proceed to construct a more detailed quantitative model that inherits the key properties of our simple example. Thus, this model introduces features such as endogenous borrowing constraints, investment costs, and equity issues. We use this model to create an artificial panel of firms and use it to replicate some of the key empirical studies about leverage and returns. Specifically we show that simulated data from the model can successfully replicate both the evidence in Bhandari (1988) and Fama and French (1992) about the effects of leverage on returns and the results in Welch (2004) about the impact of returns on leverage ratios.

Our work is at the center of several converging lines of research. First, it builds on the growing theoretical literature that attempts to link corporate decisions to the behavior of asset returns (a partial list includes Berk, Green and Naik (1999), Gomes, Kogan and Zhang (2003), Carlson, Fisher and Gianmarino (2004), Cooper (2005) and Zhang (2005)). From this point of view the novelty in our work is the fact that we explicitly allow for deviations of the Modigliani-Miller theorem so that corporate financing decisions will affect investment and thus asset prices.

Our paper also adds to the recent literature on dynamic models of the capital structure that attempt to link the corporate investment and leverage policies of firms (a partial list includes Hennessy and Whited (2004, 2006), Miao (2005) and Sundaresan and Wang (2006)). Here the key novelty of our work is allowing for exposure to systematic risk and our specific focus on the asset pricing implications of these models.

Finally, this work is also related to recent attempts to examine the link between financing constraints and stock returns (work along these lines includes Gomes, Yaron, and Zhang (2004, 2007), Whited and Wu (2005) and Livdan, Sapraiza, and Zhang (2006)).
This paper is organized as follows. Section 2 provides a simple example where we can derive in closed form the effects of endogenous leverage on expected returns. Section 3 builds on this intuition to construct a general model where we can develop quantitative implications in a more realistic setting. Section ?? explores the optimal investment and leverage decisions in this general setting while section 4 examines the model’s implications for the cross-section return and leverage regressions. Finally section 5 offers a few concluding remarks.

2 Leverage, Investment, and Returns: A Simple Example

In this section we construct a simple continuous time real options model that formalizes our basic insights and delivers closed form expressions linking expected returns and corporate decisions on investment and financing. These ideas are then integrated in the more general model developed in the next section.

2.1 Profits and Dividends

We consider the problem of value maximizing firms, indexed by the subscript $i$, that operate in a perfectly competitive environment. The instantaneous flow of (after tax) operating profits, $\Pi_i$, for each firm $i$ is completely described by the expression

$$\Pi_i = (1 - \tau)X_t K_i^\alpha, \quad 0 < \alpha < 1$$

where $K_i$ is the productive capacity of the firm, $\tau$ is the corporate tax rate, and the variable $X$ is an exogenous state variable that captures the state of aggregate demand (or productivity).
As usual we think of this profit function as that resulting from the determination of the optimal choices for all other (static) inputs such as labor and raw materials for example. This combination of perfect competition with decreasing returns to scale can be shown to be equivalent to that of a monopolist facing a downward slopping demand curve for its output so that our assumptions are not too restrictive.

The state variable, $X_t$, is assumed to follow the stochastic process

$$dX_t/X_t = \mu dt + \sigma d\varepsilon_t$$

where we assume for simplicity that $\varepsilon_t$ is a standard Brownian motion under a risk-neutral measure.\(^1\)

2.2 Investment and Financing

A typical firm is endowed with an initial capacity $K_0$ and one option expand this capacity to $K_1$ by purchasing additional capital in the amount $I = K_1 - K_0 > 0$. We assume that the relative price of capital goods is one and that there are no adjustment costs to this investment. In what follows we will use say that the firm is “young” if it has not yet exercised this growth option and “mature” if this option has already been exercised.

For this example we assume that to finance this investment opportunity a firm needs to raise debt in the amount of $I$. Formally this requires us to make two simplifying assumptions. First, we need to assume that a young firm will distribute its entire earnings in every period. Second we also rule out new equity issues at the time of investment by assuming that the costs of doing so for a young firm are prohibitive.

\(^1\)As is well known this measure may or may not be unique depending on whether financial markets are assumed to be complete or not. At this stage however we only require the existence of one such measure.
While these are convenient assumptions for the purposes of our illustration neither of them is really essential and they will both be relaxed in the more general model below. Our basic insights will survive as long as at least some of the investment is financed with debt. Given the tax benefits of debt this will always be the case.

Given our simplifying assumptions debt then will have a face value of $I$. We assume also that this debt takes the form of a consol bond that pays a fixed coupon $c$. Young firms have no debt outstanding so that $c$ represents the total flow of interest commitments per period for a mature firm.

2.2.1 The Problem for Mature Firms

Given our assumptions it follows that the instantaneous dividends for the equity holders of a mature firm are equal to

$$(1 - \tau) \left( X_t K_t^a - c \right).$$

Given debt, $I$, and its associated coupon payment, $c$, the value of a mature firm, $V_1(X; c)$, satisfies the following Bellman equation

$$V_1(X; c) = (1 - \tau) \left( X_t K_t^a - c \right) dt + (1 + r dt)^{-1} E[V_1(X + dX; c)] \quad (1)$$

Here our choice of notation, $V_1(X; c)$, emphasizes the dependence of equity value on the firm’s leverage.

Equation (1) holds only as long as the firm meets its obligations to the debt holders. However it is reasonable to assume that equity-holders will choose to close the firm and default on their debt repayments if the prospects for the firm are sufficiently bad. If equity holders have no outside options this (optimal) default occurs whenever $V_1(X; c)$ reaches zero. Alternatively, default occurs as soon as the value of $X$ reaches some (endogenous) default threshold $X_D$. 

2.2.2 The Problem for Young Firms

Young firms have no leverage, but they have an option to expand their productive capacity and become mature firms. For young firms the flow of operating profits (and dividends) per unit of time is then given by the expression

$$(1 - \tau) X K_0^\alpha.$$  

This yields the following Bellman equation for equity value, $V_0(X)$:

$$V_0(X) = \max \left\{ V_1(X; c), (1 - \tau) X K_0^\alpha dt + (1 + rd)^{-1} E[V_0(X + dX)] \right\}$$  \hspace{1cm} (2)

The maximum in equation (2) now reflects the existence of an investment opportunity for the young firm. If demand grows sufficiently, so that $X$ is above an investment threshold $X_I$, the firm will choose to expand its productive capacity to $K_1$. At this investment threshold firm value must obey the usual boundary conditions:

$$V_1(X_I; c) + B(X_I; c) - I = V_0(X_I)$$  \hspace{1cm} (3)

$$V'_1(X_I; c) + B'(X_I; c) = V'_0(X_I)$$  \hspace{1cm} (4)

where $B(X_I; c)$ denotes the value of debt issues at the time of investment. Given our assumption that all investment is financed through debt issuance $B(X_I; c) = I$ and the value matching condition (3) collapses to

$$V_1(X_I; c) = V_0(X_I).$$

2.2.3 Debt Value and Coupon Payments

Before computing the value of each firm explicitly it is helpful to construct the market value of the debt outstanding and the instantaneous coupon payments, since both of these values are linked to the firm’s decision to investment.
The possibility of default will naturally induce a deviation between the market, \( B \), and the book value of debt, \( I \), at any point in time. As in Leland (1994), as long as the firm does not default this market value satisfies the Bellman equation

\[
B(X; c) = cdt + (1 + rdt)^{-1} E[B(X + dX; c)]
\]

Bankruptcy costs are assumed large enough so that the firm is liquidated and no value is left for bondholders. Again this is extreme but not really important. Formally this implies that, at default, \( B(X_D; c) = 0 \). Given this boundary condition at default we can easily construct the expression for the market value of debt, \( B(X; c) \). This is given by

\[
B(X; c) = \frac{c}{r} \left( 1 - \left( \frac{X}{X_D} \right)^{v_1} \right)
\]

where \( v_1 < 0 \) so that the market value converges to \( c/r \) as \( X \) approaches infinity.

To determine the value of the periodic coupon payment, \( c \), we use the fact that the initial debt issue must be enough to finance investment, so that \( B(X_I; c) = I \). Replacing in the expression for the market value of debt, (5), we obtain

\[
c = \frac{r}{1 - \left( \frac{X_I}{X_D} \right)^{v_1} I}
\]

Hence the value of the coupon payment depends both on the face value of debt as well as default and investment thresholds \( X_D \) and \( X_I \), respectively. The impact of the former is fairly standard and is due to the fact that the possibility of future default raises the required coupon payments. Holding the face value of debt, \( I \), fixed, the effect of the investment threshold is related to its impact of the probability of future default. The larger the threshold the less likely the firm is to default. As we will see below this is something that a young firm will take into account when making investment decisions.
2.3 Valuation

We are now ready to compute the value of equity for both young and mature firms. To compute the value of a mature firm, given a pre-determined coupon payment, $c$, we use Ito’s Lemma in equation (1) and impose default when $X = X_D$ to solve the associated second order differential equation.

This procedure implies that the value of a mature firm satisfies the expression

$$V_1(X; c) = \frac{(1 - \tau)XK_1^\alpha}{r - \mu} - \frac{(1 - \tau)c}{r} + A_1X^{v_1} \tag{7}$$

where $v_1 < 0$, and the value for the constant $A_1 > 0$ can be obtained using the relevant boundary conditions at the default threshold.$^2$

The first term in equation (7) is the present value of the future cash flows generated by existing assets, $K_1$. To this value we must then deduct the present value of all future debt obligations, which is captured by the term $\frac{(1 - \tau)c}{r}$. Finally, the last term shows the impact of default on the value of the firm to its shareholders.

In the case of a young firm we apply Ito’s Lemma to the Bellman equation (2) and solve the associated differential equation to obtain the expression

$$V_0(X) = \frac{(1 - \tau)XK_0^\alpha}{r - \mu} + A_0X^{v_0} \tag{8}$$

where $v_0 > 1$, and $A_0 > 0$ is determined by imposing the boundary conditions at $X_I$.

The first term in equation (8) for the equity value of young firms, is the present value of the future cash flows generated by existing assets and is

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$^2$In this case we obtain that

$$A_1 = -\left(\frac{(1 - \tau)X_DK_1^\alpha}{r - \mu} - \frac{(1 - \tau)c}{r}\right) \left(\frac{1}{X_D^{v_1}}\right).$$
essentially the same as that in the equation for the value of mature firms in (7).

More importantly equation (8) shows that the value of young firms, $V_0$, differs from that of mature firms, $V_1$, in a number of ways. First, the equity value of young firms will depend on the (positive) value of future growth options, here captured by the term $A_0X^{v_0}$. In this simple example this piece is entirely missing from the expression for the value of mature firms. While this is clearly too extreme, it is nevertheless plausible to expect that the value of growth options to be relatively more important for young firms.

Second, mature firms are larger ($K_1 > K_0$) and precisely for that reason they are also more levered. These additional effects of debt are captured in the last two terms in equation (7).

2.4 Leverage and Risk

Expected returns can be recovered by looking directly at the equity betas implied by the valuation expressions (7) and (8). In our simple example these conditional betas can be computed in closed form by examining the elasticities of the value functions with respect to $X_t$.

We will express conditional equity betas $\beta_{it}$, for any firm, young ($i = 0$) or old ($i = 1$), in a general form as

$$\beta_{it} = 1 + \frac{(1 - \tau)c}{rV_{it}} + \frac{V_{it}^D}{V_{it}}(v_1 - 1) + \frac{V_{it}^G}{V_{it}}(v_0 - 1), \quad i = 0, 1$$

Here we use $V_{it}^G = A_0X^{v_0}$ to denote the value of the young firm’s growth options and $V_{it}^D = A_1X^{v_1}$ is the value of the default option for the mature firm.

The first term in this expression is common to both young and old firms and is simply the firm’s revenue beta, which captures the (unlevered) riskiness
of assets in place. Since operating profits are linear in the aggregate state of demand, this term is here effectively normalized to 1.

The next two components of equity risk are directly tied to leverage and, in our simple example are only relevant for mature firms. Together they capture the traditional effects of leverage on returns so often emphasized in the static literature. The second term, \( \frac{(1-\tau)c}{rV_0} \), shows the effects of levering up equity cash flows on expected returns, even in the absence of any default risk.\(^3\) The third term on the other hand reflects the impact of default on equity risk. Together these two terms imply the usual positive relation between leverage and expected equity returns that is described in most finance textbooks.\(^4\)

The novelty however is the last term in equation (9). This term reflects the effect of growth options and depends on the relative importance of these options to the equity value of the firm. In our simple example this term will add to the underlying risk of the young firm, since mature firms no longer have any growth options, since \( v_0 > 1 \).

Thus, our expression for equity betas, (9), illustrates the potential pitfalls of searching for simple mappings between leverage and equity risk. This equation implies that, all else constant, financial leverage clearly increases equity risk. However this simple rule only holds in a static world when leverage is already pre-determined.

In a richer dynamic setting leverage is itself endogenous and generally related to investment decisions of varying degrees of risk. And because leverage tends to be generally higher for mature, low growth, firms which are otherwise less risky, simple correlations between discount rates and leverage

\(^3\)Note that \( \frac{(1-\tau)c}{rV_0} \) is simply the value of a riskless perpetuity.

\(^4\)Here the endogenous nature of default limits the firm's downside risk \( (A_1 > 0) \). This may change however if we allow for more sophisticated default mechanisms in which the firm may be liquidated sub-optimally due to covenant violations.
are unlikely to produce meaningful results (see Barclay, Morellec and Smith (2006) for example). More precisely, equation (9) suggests that accounting for the importance of growth (and default) options is crucial when examining this relation between leverage and returns.

2.5 Numerical Illustration

Our key insights can now be developed with a numerical example. Since our focus is no longer on obtaining closed form solutions we can also begin to relax some of our more restrictive assumptions about the environment. The most significant change is that we now allow the firm to finance investment with both debt and newly issued equity. Hence a firm is now simultaneously choosing optimal investment and financing policies at the investment threshold $X_I$.

Mathematically this implies that the boundary condition $B(X_I; c) = I$ is no longer required.

A less important but nevertheless realistic change concerns the assumed recovery rate on assets. We now assume that debt holders will be able to recover a fraction, $\phi > 0$, of the asset value of the firm upon default. Formally we now impose the boundary condition on debt

$$B(X_D; c) = \phi \frac{(1 - \tau)XK^\alpha}{r - \mu}.$$  

Effectively this assumes that, after accounting for some transaction costs, debt holders will take over the firm and will be entitled to the entirety of its future cash flows.

2.5.1 Leverage

Figure 1 shows the betas for several hypothetical mature firms as a function of alternative levels of (book) leverage as measured by their periodic coupon
payment – the dashed line. Because these firms differ only in their leverage the curve is upward sloping, conforming with the static view that, if all else is constant, higher leverage will raise expected equity returns.

The figure also shows the betas of unlevered young firms – the solid line. However since young firms are not levered the beta is just a constant here. Because of the role of growth options however this beta will be relatively high particularly when compared with moderately levered firms. In this case it is quite possible that unlevered young firms will have higher expected returns than levered mature firms. These two lines then provide an effective graphical illustration of the basic intuition from equation (9) and the limitations of the usual textbook intuition.

2.5.2 Business Cycle Effects

Figure 2 provides additional insights into the role of leverage in determining equity risk. This figure plots equity betas for both (optimally) levered and unlevered firms as a function of the state of demand, $X$.

As before the dashed line shows the beta for mature firms, while the solid line shows the beta for the young firms. Not surprisingly we see that expected returns rise with $X$ for the young firms because this increases the relative importance of their growth options in total firm value. Also intuitive is the pattern for mature firms. Here risk increases as demand conditions, $X$, worsen since this makes it more likely that the firms will find itself in default.

Figure 2 also confirms our findings that expected returns will not in general be monotonic in leverage. Depending on demand conditions it is possible for unlevered firms to be either more or less risky (as measured by expected

\footnote{Here we hold the value of the state variable $X$ fixed.}
Another implication of this result is that it suggests that the relationship between leverage and returns is conditional in nature: In bad times the contribution of default and cash flow risk is greater, while in good times the investment channel dominates. Thus when default risk is rather small, the figures suggest that expected returns are decreasing at least in book leverage, a finding that seems consistent with the recent empirical literature.

Finally this cyclical pattern of equity risk across firms is also interesting because it shows how financial leverage can generate endogenously the kind of variation in equity returns that is often required to replicate the value premium (See for example Carlson, Fisher and Gianmarino (2004), Cooper (2005), or Zhang (2005)). Unlike the existing literature however, our mechanism does not rely on exogenous technological assumptions but is instead linked to the capital structure of the firm.

3 The General Model

The simple example in the section 2 provides much of the intuition for our findings although at the cost of some loss of generality. The model is also too stylized to allow for a more serious quantitative investigation of its key predictions.

In this section we embed the key ideas from our example in a more general environment that allows for more complex investment and financing strategies. Specifically, we now let firms have access to multiple investment options, while also relaxing the assumption that investment and financing must be perfectly coordinated. Firms can now issue debt (and equity) at any point in time and in any amount, subject to the natural financing constraints.

In addition we now allow for additional cross sectional firm heterogeneity
in the form of firm specific shocks to both current profitability and the value of growth options. Moreover, aggregate shocks to the state of demand now impact both firm profitability and the discount rates as we no longer conduct our analysis under risk-neutral valuation.

Although this more general environment contains several additional ingredients its basic features are very similar, and our notation is, when possible, identical to that in the section 2.

**3.1 Firm Problem**

**3.1.1 Profits and Investment**

As before we begin by considering the problem of a typical value maximizing firm in a perfectly competitive environment. Time is now discrete. The flow of after tax operating profits per unit of time for each firm $i$ is described by the expression

$$\Pi_{it} = (1 - \tau)(Z_{it}X_tK_{it}^\alpha - f), \quad 0 < \alpha < 1$$  \hspace{1cm} (10)

where $Z_i$ captures a firm specific component of profits and the variables $X_t$ and $K_{it}$ denote, as before, the aggregate state of productivity and the book value of the firm’s asset. We use $f \geq 0$ to denote a (per-period) fixed cost of production.

Both $X$ and $Z$ are assumed to be lognormal and obey the following laws of motion

$$\log(X_t) = \rho_x \log(X_{t-1}) + \sigma_x \varepsilon_t$$

$$\log(Z_{it}) = \rho_z \log(Z_{it-1}) + \sigma_z \eta_{it}$$

and both $\eta_i$ and $\varepsilon$ are (standard) normal variables.

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6This is slightly incorrect. To ensure the existence of a solution to the firm’s problem the shocks must be finite. We accomplish this by imposing a (very large) upper bound on the $\varepsilon$ and $\eta$. 

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$Z_{it}$ is firm specific requires that

$$E\varepsilon_t \eta_{it} = 0$$
$$E\eta_{jt} \eta_{it} = 0, \text{ for } i \neq j$$

The firm is now allowed to scale operations by picking between any level of productive capacity in the set $[K_0, K_N]$. This can be accomplished through (irreversible) investment, $I_{it}$, which is linked to capacity by the standard capital accumulation equation

$$I_{it} = K_{it+1} - (1 - \delta)K_{it} \geq 0$$ (11)

where $\delta > 0$ denotes the depreciation rate of capital per unit of time.

### 3.1.2 Financing

Corporate investment as well as any distributions, can be financed with either the internal funds generated by operating profits or net new issues which can take the form of new debt (net of repayments) or new equity.

As before we assume that debt $B$ can take the form of a consol bond that pays a periodic coupon $c$ per unit of time. However each firm is now allowed to renegotiate the terms of any outstanding issue at any point in time. This is then equivalent to letting the firm refinance the entire value of outstanding liabilities in every period. Formally, letting $B_{it}$ denote the book value of outstanding liabilities for firm $i$ at the beginning of period $t$ we define the value of net new issues as

$$B_{it+1} - (1 - c_{it})B_{it}.$$

where $c_{it}$ is again the coupon payment on $B_{it}$ which will in general depend on a number of firm and aggregate variables. Note that now both debt and coupon payments will exhibit potentially significant time variation.
The firm can also raise external finance by means of seasoned equity offerings. For added realism however we now assume that these issues entail additional costs so that firms will never find it optimal to simultaneously pay dividends and issue equity. Following the existing literature we assume that these costs include both fixed and variable components, which we denote by $\lambda_0$ and $\lambda_1$, respectively.\(^7\) Thus, letting $D_{it}$ denote the net payout to equity holders, issuance costs, per unit of equity raised, are given by the function:

$$\Lambda(D_{it}) = \left(\frac{\lambda_0}{D_{it}} + \lambda_1\right) I\{D_{it} < 0\}$$

where the indicator function implies that these costs apply only in the region where the firm is raising new equity finance so that net payout, $D_{it}$, is negative.

Investment, distribution, and financing decisions must meet the following identity between uses and sources of funds

$$D_{it} + I_{it} = \Pi_{it} + \tau \delta K_{it} + B_{it+1} - (1 + (1 - \tau)c_{it})B_{it} \quad (12)$$

where again $D_{it}$ denotes the equity payout net of new issues and flotation costs. The resource constraint (12) recognizes the tax shielding effects of both depreciated capital and interest expenditures.

3.1.3 Valuation

The equity value of the firm, $V$, is defined as the discounted sum of all future equity distributions. Here again we assume that equity-holders will choose to close the firm and default on their debt repayments if the prospects for the firm are sufficiently bad, i.e., whenever $V$ reaches zero.

To discount future cash flows we directly parameterize the discount factor

\(^7\)See Gomes (2001) and Hennessy and Whited (2006).
applied to future cash flows as a stochastic process given by the expression

$$\log M_{t+1} = \log \beta - \gamma \log (X_{t+1}/X_t)$$

with $\gamma > 0$. Although this pricing kernel is exogenous its basic properties seem plausible, most notably, the idea that the risk premium is directly related to aggregate growth in cash flows.\(^8\)

The complexity of the problem is reflected in the dimensionality of the state space necessary to construct the equity value of the firm. This includes both aggregate and idiosyncratic components of demand, productive capacity, and total debt commitments, defined as

$$\hat{B}_{it} \equiv (1 + (1 - t)c_{it-1})B_{it}$$

To save on notation we henceforth use the $S_{it} = \{K_{it}, \hat{B}_{it}, X_t, Z_{it}\}$ to summarize our state space.

We can now characterize the problem facing equity holders, taking coupon payments as given. These payments will be determined endogenously below. The equity value of each firm can then be computed as the solution to the following dynamic program

$$V(S_{it}) = \max\{0, \max_{K_{it+1}, \hat{B}_{it+1}} \{D(S_{it}) + E[M_{t,t+1}V(S_{it+1})]\}\} (13)$$

s.t. $K_{it+1} \geq (1 - \delta)K_{it}$

where the expectation in the left hand side is taken by integrating over the conditional distributions of $X$ and $Z$. Note that the first maximum captures the possibility of default at the beginning of the current period, in which case the shareholders will get nothing. Finally, aside from the budget constraint, the only significant constraint on this problem is the requirement that investment is irreversible.

\(^8\)See Berk et all (1999) and Zhang (2005) for similar applications and in-depth explorations of this assumption.
3.1.4 Default and Bond Pricing

We now turn to the determination of the required coupon payments, taking into account the possibility of default by equity holders. Assuming debt is issued at par, the market value of new issues must satisfy the following condition

\[ B_{it+1} = E \left[ M_{t,t+1}((1 + c_{it+1})B_{it+1}I_{\{V_{it+1} \geq 0\}} + R_{it+1}(1 - I_{\{V_{it+1} \geq 0\}})) \right] \]

where \( R_{it+1} \) denotes the recovery on a bond in default and \( I_{\{V_{it+1} \geq 0\}} \) is an indicator function that takes the value of 1 if the firm remains active and 0 when equity chooses to default.

Finally, we follow Hennessy and Whited (2006) and specify the dead-weight losses at default to consist of a fixed and a proportional component.

Upon default creditors can recover a fraction of the firm’s current assets and profits net of fixed cost. Formally we assume that

\[ R_{it} = \Pi_{it} + \tau \delta K_{it} + \xi_1(1 - \delta)K_{it} - \xi_0 \]

Since the equity value \( V_{it+1} \) is endogenous and itself a function of the firms’ debt commitments this equation cannot be solved explicitly to determine the value of the coupon payments, \( c_{it} \). However, using the definition of \( \hat{B} \) we can rewrite the bond pricing equation as

\[ B_{it+1} = E \left[ M_{t,t+1}((1 + \frac{1}{1 - \tau} \hat{B}_{it+1}I_{\{V_{it+1} \geq 0\}} + R_{it+1}(1 - I_{\{V_{it+1} \geq 0\}})) \right] \]

\[ = B(K_{it+1}, \hat{B}_{it+1}, X_t, Z_{it}) \]

Given this expression and the definition of \( \hat{B} \) we can easily deduce the implied coupon payment as

\[ c_{it+1} = \frac{1}{1 - \tau} \left( \frac{\hat{B}_{it+1}}{B_{it+1}} - 1 \right) \]
Using $\hat{B}$ as a state variable and constructing the bond pricing schedule $B(\cdot)$ offers several important computational advantages. Because equity and debt values are mutually dependent (since the default condition affects the bond pricing equation) we would normally need to resort to a two-step procedure that iterates on both the interest rate schedule (or bond prices) and value function. Our present specification eliminates this problem and reduces it to one simple function evaluation during the value function iteration. Using this formulation automatically nests the debt market equilibrium in the calculation of equity values and greatly reduces computational complexity.

### 3.2 Optimal Firm Behavior

Given our assumptions, the dynamic programming problem (13) has a unique solution, that can be characterized efficiently by the optimal distribution, financing, and investment, policies. However since this problem cannot be solved in closed form we must resort to numerical methods which are detailed in Appendix ??.

We now investigate some of the properties of optimal strategies implied by the solution to our more general problem.

Figures 3 and 4 illustrate the optimal financing and investment policies of the firm and their implications for equity values, default probabilities, credit spreads and betas in the model. In both pictures the dashed line corresponds to a high realization for the aggregate state of demand (an economic boom) while the solid line shows the results when demand is relatively weak (a recession). The idiosyncratic profitability shock, $Z_{it}$, is set to its mean. Figure 3 looks at the effects of asset size, as measured by productive capacity, $K_{it}$, in determining firm decisions and asset values. Figure 4 isolates the role of leverage. It shows the same functions but now plotted against the value of

---

9It is straightforward but cumbersome to show this formally. The interested reader is referred to Gomes (2001) and Hennessy and Whited (2006) for similar proofs.
current liabilities, $\hat{B}_{it}$.

3.2.1 Investment Policies

The plot labelled “investment policy” depicts the optimal choice of next period capital, $K_{it+1}$, as a function of the underlying variables. Naturally this value is considerably higher in good times due to the persistence of the aggregate shock.

More importantly, this picture neatly illustrate the interaction of financing and investment decisions, particularly for small firms. With unlimited access to external funds, the optimal choice of capacity would be independent of this period capital stock, at least for low values of $K$ as the irreversibility constraint only binds on disinvestment. Here however small firms’ capacity choice is clearly dependent on the value of their current assets. This is because an increase in $K_{it}$ generates higher internal cash flows and more collateral, thus alleviating the financing constraint for these firms.

Equally interesting is the fact that the optimal capacity choice is declining in $\hat{B}_{it}$, as in seen in the first panel of figure 4. Although reminiscent of the popular debt overhang effect our result is worthy of note since our model allows firms to renegotiate the terms of its debt in every period.

3.2.2 Debt Policies

The 'debt policy' panels show the optimal choice for new debt issues, $B_{it+1}$. Two features are noteworthy here. First, optimal issuance is, at least over a large range, decreasing in firm size a fact that seems consistent with empirical evidence. Whereas this goes against the usual static trade-off theory, it is a natural consequence of our dynamic setting where growth options are declining in size and is also one the key findings of Hennessy and Whited (2005).
The second notable feature is the strong positive relation between current and lagged leverage, a phenomenon sometimes dubbed as 'hysteresis', and that suggests that our model is also consistent with the well documented finding that financial leverage is extremely persistent. In the context of our model this is again a noteworthy since we have abstracted from all the usual arguments about market timing or non-convex costs in debt issues. Here, persistence in leverage is due almost exclusively to the nature of investment decisions of the firm. The intuition for our result however is fairly simple. As we have seen from the investment policies discussed above the firm’s choice of capacity will be closely related to its recent choices. With financial frictions, debt issues are also closely tied to these optimal investment decisions, so that optimal leverage choices will be quite persistent as well.

3.2.3 Betas

Next we consider betas. As figure 4 shows we find that, controlling for size and profitability, leverage increases the systematic risk to equity holders. Once again this is precisely the result identified in traditional static models and discussed in section 2.

Once again however we also find that equity risk declines fairly quickly in firm size. Figure 3 shows that equity risk is significant smaller for larger firms. Again the intuition is precisely the same that we identified in section 2: with decreasing returns to scale, large firms also have fewer growth options which reduces their risk.\textsuperscript{10}

As before this finding implies that, to the extent that leverage and investment policies are jointly determined, the link between expected returns

\textsuperscript{10}In the general model the value of growth options is tied to firm size due to the presence of decreasing returns to scale. These ensure that the marginal value of new additions to productive capacity is significantly lower for large firms.
and leverage is likely to be more subtle than what is traditionally suggested in the literature. In fact if decreasing returns are sufficiently strong it is actually possible that the relation between returns and debt will be downward slopping.

3.2.4 Other Implications

The remaining panels show additional implications of these policies. Not surprisingly equity values are increasing in both $X$ and $K$ while decreasing in $\hat{B}$. As for the credit spreads we note that the model gives rise to a sizable spread. The intuition here is straightforward: What matters for credit spreads are not so much the actual default probabilities depicted but much more the risk-adjusted pricing kernel weighted default probabilities. In recessions risk adjusted default probabilities are much higher than the historical probabilities delivering a high credit spread while simulations are consistent with historical default probabilities. Finally, notice that both credit spreads and default probabilities are countercyclical.

Moreover, risk rises substantially when the firm is very small and leverage is high, since this scenario leads to a dramatic increase in the probability of default.

4 Cross-Sectional Regressions

This section compares the quantitative implications of our model with some well documented findings in the empirical literature on leverage and asset returns. To accomplish this we construct an artificial cross-section of firms by simulating the investment and leverage rules implied by our model. This procedure is described in detail in Appendix A

To discipline our analysis we restrict our parameters values so that our
model matches the key unconditional moments of investment, returns, and cash flows both in the cross-section and at the aggregate level. These parameter choices, summarized in Table 1 are discussed in more detail in Appendix ??.

4.1 Cross-Section of Expected Returns

Bhandari (1988) and Fama and French (1992) provide detailed empirical examinations of the links between equity returns and indicators of financial leverage. In this section we compare the findings of our model with their evidence by applying their econometric procedures to our artificial data.

We begin by constructing theoretical counterparts to the empirical measures of returns, beta, book-to-market, and leverage. Equity returns between \( t \) and \( t + k \) are defined in a straightforward fashion by the identity

\[
 r_{t,t+k} = \frac{V_{t+k} + D_{t+k}}{V_t}
\]

To construct a measure of beta that is comparable to empirical estimates (as opposed to the model’s theoretical beta which is unobservable) we follow the procedure detailed in Fama and French (1992). In our model the book value of assets is simply given \( K \), while the book value of equity is \( BE = K - B \). To facilitate comparisons with these studies we will henceforth use \( ME = V \) to denote the market value of equity. Thus book leverage is measured by the ratio \( B/K \), while book-to-market equity is defined as \( BE/ME \).

Table 3 compares our model’s implications with the findings in Bhandari (1988) empirical study about leverage and returns. Specifically this table reports the results of estimating the following equation

\[
 r_{t+1} = \alpha_0 + \alpha_1 \beta_t + \alpha_2 \log(ME_t) + \alpha_3 \log(B_t/ME_t)
\]
Each column reports a different variation of (14) that is obtained by imposing alternate parameter restrictions.

Table 3 shows that the model is quite successful at replicating both the unconditional correlations between each variable and equity returns, and the conditional correlations implied by multiple regression coefficients. Moreover this match is also quantitatively significant as almost all coefficients are broadly in line with the empirical estimates in Bhandari (1988) and only the coefficients on beta are somewhat smaller in the model than in the data.

The intuition for these results is fairly simple and not really novel to this paper. The one factor structure of our model, combined with the assumptions of decreasing returns to scale, will always induce a negative relation between size, as measured by $ME$, and risk. Thus we expect any measure of size (including market leverage) to exhibit a strong correlation with returns.

This correlation survives the inclusion of beta in the regression equations because empirical estimates of beta are only noisy proxies for the true theoretical beta. Thus, although less significant, the coefficients on size and leverage remain important in these regressions.

Table 4 complements these findings by comparing our model’s results with the evidence in Fama and French (1992). These are obtained from running regressions of equity returns on various combinations of size, book to market, and leverage.

As before we find that our model matches the observed correlation between firm size and equity returns. This correlation is also the key to understanding the positive relation between returns and book-to-market since this variable is mostly driven by variations in the market value of equity. Collinearity between these two variables explains why regression coefficients and statistical significance drop somewhat when both variables are included.
in cross-sectional regressions, although this drop is more pronounced in the model than in the data.

The final column examines the effect of book and market leverage on equity returns. As in Fama and French (1992) these measures are defined by the (log) ratios $K/BE$, for book leverage, and $K/ME$, for market leverage. Although our coefficients are a little lower than those in the data, the model is able to produce fairly realistic estimates. As in the data, we find that the coefficients on book and market leverage have opposite signs. Moreover, as suggested by Fama and French (1992), the coefficients on book and market leverage are also of similar magnitudes suggesting that the effects of leverage on returns can be effectively summarized by the book to market ratio.

Collectively the findings in Tables 3 and 4 confirm that our model produces plausible implications for the relation between equity returns and leverage, as well as the standard size and book to market effects studied by previous authors.

### 4.2 Leverage Regressions

Given that the model is so successful in replicating the cross-section of returns it is important to ensure that its predictions for the key investment and corporate finance decisions are not unrealistic. In this section we compare our model’s implications for the leverage findings in Welch (2004).

Specifically, we construct the Welch measures of actual (ADR) and implied leverage (IDR) as follows:

$$ ADR_t = \frac{B_t}{V_t + B_t} $$

$$ IDR_{t,t-k} = \frac{B_{t-k}}{V_{t-k}(1 + r_{t-k,t}^e) + B_{t-k}} $$

where $r_{t-k,t}^e$ denotes the stock return between $t - k$ and $t$, net of dividends.
We then construct the following regression of actual leverage on lagged actual leverage and implied leverage:

\[ ADR_t = \alpha_0 + \alpha_1 ADR_{t-k} + \alpha_2 IDR_{t,t-k} \] (15)

Table 5 reports the results of estimating the equation (15) in our artificial dataset and compares them with those in Welch (2004). The results are quite striking. Both at the 1, 3, and 5 year horizons our coefficient estimates are broadly in line with those in the data. In particular we find that, as in Welch (2004), most of the variation in actual leverage is induced by movements in equity returns, suggesting a relatively small role for net changes in issues and/or retirements of existing debt.

Empirically, this finding is often interpreted as evidence of significant persistence in leverage ratios and is usually attributed to either market timing or to the presence of large adjustment costs to adjustments in leverage.

The fact that our model can capture this persistence is the more striking since we have none of these features. But the intuition is nevertheless simple: leverage is persistent because capital is persistent. Since discrete adjustment in leverage are associated with changes in investment policy, capital structure choices will be also affected by the relative persistence in capital.

5 Conclusion

In this paper we investigate the theoretical relation between financial leverage and stock returns in a dynamic world where both the corporate investment and finance decisions are endogenous. We find that in general the link between leverage and stock returns is more complex than the static textbook examples suggest and will generally depend on the investment opportunities available to the firm. In the presence of financial market imperfections lever-
age and investment are generally correlated so that highly levered firms are also mature firms with relatively more (safe) book assets and fewer (risky) growth opportunities. A quantitative version of our model can replicate both the evidence in Bhandari (1988) and Fama and French (1992) about the effects of leverage on returns and the results in Welch (2004) about the impact of returns on leverage ratios.
References


[20] Livdan, Dmitry, Horacio Sapriza and Lu Zhang, Financially Constrained Stock Returns, Mimeo, University of Michigan


[23] Sundaresan, Suresh, and Neng Wang, 2006, Dynamic Investment, Capital Structure, and Debt Overhang, Mimeo, Graduate School of Business, Columbia University


A Appendix: Computation Details

We use a standard value function iteration algorithm on a discretized state space to solve the model. The major advantage of this approach, in spite of being relatively time-consuming, is that it makes the inequality constraints in the model (investment irreversibility and dividend non-negativity) straightforward to handle. Additionally, the method is very robust and precise.

To that end we discretize all variables in the model to lie on finite grids. The capital stock $K$ is constrained to lie on a equally-spaced grid with $n_k = 50$ elements. Similarly, the face value of debt $B$ lies on a grid with $n_b = 50$ elements. The lower and upper bounds of the grids are chosen to ensure that they never bind.

The state variables $X$ and $Z$ are defined on continuous state spaces and need to be transformed into discrete state spaces as well. We use the well-known Tauchen-Hussey procedure to do that. This transforms the autoregressive processes into finite Markov Chains. Specifically, we use $n_x = 3$ points for the aggregate shock $x$, $n_z = 5$ points for the idiosyncratic shocks $Z$ and $n_\theta = 3$ for $\theta$.

On this state space with $n_k \times n_b \times n_\theta \times n_x \times n_z$ elements we guess an initial value function at every point. We then iterate until convergence on the Bellman equation to find the value function and the optimal investment and financing policies. To do so, we restrict the control variables $K'$ and $B'$ to lie on equally-spaced grids with 250 elements each. Since the value function is defined on a smaller grid, we use linear interpolation extensively to find values on non-grid points. We implement the inequality constraints by setting the corresponding values to a very large negative number whenever the constraints are violated, making sure that those values of the controls are never chosen.
B Appendix: Parameter Choices

To illustrate some of the properties of the more general model we must first choose parameter values. Our choices follow closely the existing literature (e.g. Gomes (2001), Cooley and Quadrini (2001), Hennessy and Whited (2005), Zhang (2005)). Parameter values are picked so that the model matches key unconditional moments of investment, returns, and cash flows both in the cross-section and at the aggregate level.

The persistence, $\rho_x$, and conditional volatility, $\sigma_x$, of aggregate productivity, are set to 0.95 and 0.025 values close to the (annualized) values report in Cooley and Prescott (1995). For the persistence, $\rho_z$, and conditional volatility, $\sigma_z$, of firm-specific productivity, we choose the values constructed by Gomes (2001) to match the cross-sectional properties of firm investment and valuation ratios.

The depreciation rate of capital, $\delta$, is set equal to 0.1 which provides a good approximation to the average annual rate of investment found in both macro and firm level studies. For the degree of decreasing returns to scale we use 0.65, which is the number used in Cooper and Ejarque (2003) as well as documented in several recent micro studies.

We set $\xi_1$ which equal one minus the proportional cost of bankruptcy are set to 0.9, in line with recent empirical estimates in Hennessy and Whited (2006) as well as typical estimates of the direct costs of bankruptcy in the literature. We then choose $\xi_0$, the fixed cost of bankruptcy, such that we match average market-to-book values in the economy. The costs of equity issuance $\lambda_0$ and $\lambda_1$ are chosen as in Gomes (2001).

Following Zhang (2005) we choose the pure time discount factor $\beta$ and the pricing kernel parameter $\gamma$ such that the model approximately matches two key moments of asset markets, namely the mean risk free rate and the
Sharpe ratio. This pins down $\beta$ at 0.995 and we set $\gamma$ equal to 20. We note that this parameterization pins down *aggregate* risk characteristics, whereas our emphasis is on cross-sectional risk characteristics.
This table reports parameter choices for our general model. The model is calibrated to match annual data both at the macro level and in the cross-section. The persistence, $\rho_x$, and conditional volatility, $\sigma_x$, of aggregate productivity, are set close to the corresponding values reported in Cooley and Prescott (1995). The persistence, $\rho_z$, and conditional volatility, $\sigma_z$, of firm-specific productivity, are close to the corresponding ones constructed by Gomes (2001) to match the cross-sectional properties of firm investment and valuation ratios. The parameter $\delta$ is equal to the depreciation rate of capital and is set to approximate the average monthly investment rate. For the degree of decreasing returns to scale we use 0.65 which is the value in Cooper and Ejarque (2003). Finally the pricing kernel parameter $\gamma$ is chosen as in Zhang (2005) to match average asset market data.
This figure presents betas for young and mature firms as a function of an exogenously chosen coupon $c$. The beta for young firms is represented by the solid line, while the beta for mature firms is represented by the dashed line. Parameter values in the example are $r = 0.05, \mu = 0.03, \tau = 0.35, \sigma = 0.2$, recovery rate on debt $\phi = 0.9, I = 10, \alpha = 0.3, K_0 = 1, K_1 = 11$. The value of the shock $X$ is chosen such that it is below the investment trigger for the young firm for every choice of the coupon.
Figure 2: Beta and Business Cycles

This figure presents betas for young and mature firms as a function of the shock $X$ for an optimally chosen coupon. The beta for young firms is represented by the solid line, while the beta for mature firms is represented by the dashed line. Parameter values in the example are $r = 0.05, \mu = 0.03, \tau = 0.35, \sigma = 0.2$, recovery rate on debt $\phi = 0.9, I = 10, \alpha = 0.3, K_0 = 1, K_1 = 11$. This gives an investment trigger $x_I = 1.55$ for the young firm. The dashed line represents the mature firm.
This figure summarizes optimal firm policies and its consequences as a function of the capital stock $K$ for two values of the aggregate shock $X$. The dashed line refers to a realization of the shock one standard deviation above its mean, the other to one below the mean. The value of $\hat{B}$ is chosen such firms approach the default boundary but do not cross it in the figure.
This figure summarizes optimal firm policies and its consequences as a function of the capital stock $\hat{B}$ for two values of the aggregate shock $X$. The dashed line refers to a realization of the shock one standard deviation above its mean, the other to one below the mean. The value of $K$ is chosen such firms approach the default boundary but do not cross it in the figure.
Table 2: Sample Moments

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual risk-free rate</td>
<td>0.018</td>
<td>0.023</td>
</tr>
<tr>
<td>Annual volatility of risk-free rate</td>
<td>0.03</td>
<td>0.015</td>
</tr>
<tr>
<td>Annual Sharpe ratio</td>
<td>0.43</td>
<td>0.41</td>
</tr>
<tr>
<td>Investment-to-asset ratio</td>
<td>0.14</td>
<td>0.15</td>
</tr>
<tr>
<td>Market-to-book ratio</td>
<td>1.49</td>
<td>1.62</td>
</tr>
<tr>
<td>Book leverage</td>
<td>0.59</td>
<td>0.65</td>
</tr>
<tr>
<td>Market leverage</td>
<td>0.230</td>
<td>0.27</td>
</tr>
<tr>
<td>Frequency of Equity Issuance</td>
<td>0.09</td>
<td>0.15</td>
</tr>
<tr>
<td>Default rate</td>
<td>0.02</td>
<td>0.03</td>
</tr>
</tbody>
</table>

This table reports unconditional sample moments generated from the simulated data of some key variables of the model. The first two entries are related to the specification of the pricing kernel and make sure that the model is consistent with aggregate asset market data. The corresponding data are from Campbell, Lo, and McKinlay (1997). The data moments on the investment-to-asset ratio and the market-to-book ratio are taken from Gomes (2001) and Zhang (2005) respectively. Leverage and aggregate default rate are taken from Covas and Den Haan (2006). All data are annualized.

Table 3: Bhandari Regressions

<table>
<thead>
<tr>
<th>Returns</th>
<th>Actual Regressions</th>
<th>Simulated Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.63 (2.15)</td>
<td>0.14 (3.72)</td>
</tr>
<tr>
<td>log(ME)</td>
<td>-0.15 (-3.29)</td>
<td>-0.09 (-7.13)</td>
</tr>
<tr>
<td>log(B/ME)</td>
<td>0.16 (3.97)</td>
<td>0.12 (5.66)</td>
</tr>
<tr>
<td>log(ME)</td>
<td>0.25 (0.92)</td>
<td>0.11 (4.82)</td>
</tr>
<tr>
<td>log(B/ME)</td>
<td>-0.13 (-3.33)</td>
<td>-0.10 (-9.24)</td>
</tr>
<tr>
<td>log(ME)</td>
<td>0.49 (1.68)</td>
<td>0.19 (6.35)</td>
</tr>
<tr>
<td>log(B/ME)</td>
<td>-0.13 (-2.93)</td>
<td>-0.12 (-5.15)</td>
</tr>
<tr>
<td>log(ME)</td>
<td>0.17 (0.60)</td>
<td>0.17 (0.60)</td>
</tr>
<tr>
<td>log(B/ME)</td>
<td>-0.11 (-3.00)</td>
<td>-0.11 (-3.00)</td>
</tr>
</tbody>
</table>

This table corresponds to table II in Bhandari (1988). It reports Fama-Macbeth regressions on actual data and on simulated data of realized returns $R_{t+1}$ on $\beta$, size as measured by the logarithm of market equity, log ME, and the book debt-to-market equity ratio log(B/ME). The numbers are obtained by 100 simulations of 1000 firms over 100 years and then averaged across simulations.
Table 4: Fama-French Regressions

<table>
<thead>
<tr>
<th>Returns</th>
<th>Actual Data</th>
<th>Simulated Data</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log(ME\textsubscript{t})</td>
<td>-0.15 (-2.58)</td>
<td>-0.11 (-1.99)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>log(BE\textsubscript{t}/ME\textsubscript{t})</td>
<td>0.50 (5.71)</td>
<td>0.35 (4.44)</td>
<td>0.50 (5.69)</td>
<td>-0.57 (5.34)</td>
</tr>
<tr>
<td>log(K\textsubscript{t}/ME\textsubscript{t})</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log(K\textsubscript{t}/BE\textsubscript{t})</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This table corresponds to table III in Fama and French (1992). It reports Fama-Macbeth regressions on actual data and on simulated data of realized returns $R_{t+1}$ on firm size as measured by the logarithm of market equity, log ME, the ratio of book equity-to-market equity ratio log(BE/ME), the ratio of book assets-to-market equity log(K/ME) and the ratio of book assets-to-book equity log(K/BE). The numbers are obtained by 100 simulations of 1000 firms over 100 years and then averaged across simulations.
Table 5: Welch Regressions

<table>
<thead>
<tr>
<th>Market leverage $c_t$</th>
<th>Actual Data</th>
<th>Simulated Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Horizon $k$</strong></td>
<td>1 year</td>
<td>3 years</td>
</tr>
<tr>
<td>$IDR_t$</td>
<td>1.014</td>
<td>0.944</td>
</tr>
<tr>
<td></td>
<td>(1.3)</td>
<td>(1.5)</td>
</tr>
<tr>
<td>Market leverage $c_{t-k}$</td>
<td>-0.053</td>
<td>-0.042</td>
</tr>
<tr>
<td></td>
<td>(1.2)</td>
<td>(1.4)</td>
</tr>
<tr>
<td>$IDR_t$</td>
<td>1.103</td>
<td>1.013</td>
</tr>
<tr>
<td></td>
<td>(9.4)</td>
<td>(11.8)</td>
</tr>
<tr>
<td>Market leverage $c_{t-k}$</td>
<td>-0.364</td>
<td>-0.107</td>
</tr>
<tr>
<td></td>
<td>(-5.6)</td>
<td>(-3.4)</td>
</tr>
</tbody>
</table>

This table reports regressions corresponding to Welch (2004), table III. It regresses market leverage in period $t$ on market leverage in period $t - k$ and the implied debt ratio $IDR_{t,t-k}$ over the periods $t - k$ to $t$, where $IDR_{t,t-k} = \frac{B_{t-k}}{ME_{t-k}(1+x_{t-k,t}) + B_{t-k}}$. $x_{t-k,t}$ is the stock return between period $t - k$ and $t$, net of dividends. The numbers are obtained by 100 simulations of 1000 firms over 100 years and then averaged across simulations. The data are annualized.