Time Varying Corporate Capital Stocks and the Cross Section and Intertemporal Variation in Stock Returns

Jacob Sagi†, Matthew Spiegel‡, and Masahiro Watanabe*

October 10, 2008

We thank Sugato Bhattacharyya, Ines Chaieb, Nobuya Takezawa, Akiko Watanabe, seminar participants at Columbia University and University of South Carolina, session participants at the 2008 European Finance Association Meetings, the 2008 Asian/Nippon Finance Association Meetings, and the 2008 Singapore International Conference on Finance for helpful comments. This is an extensively revised version of the previous paper circulated under the title "Equity Issuance and Expected Returns: Theory and New Evidence."

†Owen Graduate School of Management, Vanderbilt University, 401 21st Ave S., Nashville, TN 37203-2422. Phone: 615.343.9387, fax: 615.343.7177, email: Jacob.Sagi@owen.vanderbilt.edu.
‡Yale School of Management, 135 Prospect Street, New Haven, CT 06511. Phone: (203) 432-6017, fax: (203) 432-8931, matthew.spiegel@yale.edu.
*Jones Graduate School of Management, Rice University - MS531, 6100 Main St., Houston, TX 77252-2932. Phone: (713) 348-4168, fax: (713) 348-6331, watanabe@rice.edu.

JEL Classification: G12

Keywords: overlapping generations model, capital investment, equity issuance, return predictability, supply shocks, excessive volatility.

Time Varying Corporate Capital Stocks and the Cross Section and Intertemporal Variation in Stock Returns

Abstract

This paper uses a general equilibrium model to examine an economy in which firm managers seek to maximize their individual firm's value through the costly adjustment of their capital stock in response to economic shocks. These economic shocks impact both the number of capital units each firm has and how productive each unit is. The ultimate value of these corporate assets is determined by risk averse investors that trade in a competitive multiple security market. Because capital stocks change slowly over time, the relative return to owning them does as well. This generates both cross sectional and intertemporal return patterns in which economic shocks lead to large returns, followed by what appear to be long term abnormal returns in the other direction.

Stock returns appear to display a number of long run cross sectional and intertemporal patterns. One of the most studied is probably the tendency for returns to increase in a firm's book-to-market ratio and decrease in its size (Fama and French (1992)). But there are others. Marsh (1982), Asquith and Mullins (1986), Mikkelson and Partch (1986), Jung, Kim and Stulz (1996), and Baker and Wurgler (2002) all find that following a new equity issue a stock's return is lower than one might otherwise forecast. At the opposite end Ikenberry, Lakonishok, and Vermaelen (1995) find that after a firm engages in share repurchases it tends to have above normal returns. The negative relationship between net equity issuance and subsequent returns is further confirmed in both the U.S. (Fama and French (2007) and Pontiff and Woodgate (2008)) and international (McLean, Pontiff, and Watanabe (2008)) markets. Most importantly for this paper, Titman, Wei and Xie (TWX, 2004) show that the equity issue and repurchase findings are in fact tied to a firm's investments. Firms that invest today tend to have lower returns going forward and visa versa (also see Lyandres, Sun, and Zhang (2007) and Xing (2007)). The goal of this paper is to provide an explanation for this phenomenon in a tractable general equilibrium framework.

In the model both firms and investors play an active role in the determination of equilibrium prices and thus expected returns. Firms create goods and services across a number of industries by employing industry specific capital that varies over time. One source of this variation comes from employing individuals. These employees sell their human capital to the firm which then converts it to corporate capital. Another source of variation comes through the direct purchase and sale of capital in the financial markets. It is assumed that if a firm adds or subtracts from its capital stock in this manner it is relatively less expensive to do so slowly. The demand side of the model comes from risk adverse investors that own shares in the industries and trade them in a competitive market.

By using an overlapping generations framework based on Spiegel (1998) and related to those in Watanabe (2008), and Biais, Bossaerts, and Spatt (2008) the model is not only tractable but can easily be validated against readily available data sources. Another nice feature of the model is that the CAPM holds. However, while it holds period-by-period the model is not static. The return an investor can expect to earn by investing in an industry varies over time as the firms vary their capital levels.

One can think of capital units in this setting as being represented by a set of ATMs. Firms find that the number of ATMs they have varies over time as their employees are able to build more or less of them each period. At the same time the amount of money produced by each machine also varies. In reaction to these events the firm then creates (or sells) additional ATMs by employing financial capital. But, shocks to their capital stock are undone slowly as that is the most economical way to do so. With regard to stock returns this results in large returns in one direction, due to a shock, leading to lower expected returns in the other direction as the firms seek to undo the shock by adjusting their capital levels. This then leads to patterns similar to those in TWX as well as the prior literature on stock sales and repurchases. But, as in TWX the return phenomena are tied to changes in corporate capital levels and not stock sales and repurchases *per se*.

This paper is not the first to theoretically examine the relationship between stock returns and both real and financial corporate capital adjustments. In response to the findings in the empirical literature on new share issues a number of authors have proposed behavioral explanations in which managers take advantage of overvalued shares to raise capital (Loughran, Ritter, and Rydqvist (1984), Ritter (1991), Laughran and Ritter (1995), Rajan and Servaes (1997), Pagano, Panetta, and Zingales (1998), Baker and Wurgler (2000) and Lowry (2003)). In contrast, recent models by Pastor and Veronesi (2005) and Dittmar and Thakor (2007) both offer rational explanations.

Like Pastor and Veronesi (2005) and Dittmar and Thakor (2007) this paper also proposes a rational model that generates return series similar to what is seen in the data. In Pastor and Veronesi market conditions change exogenously over time along three dimensions: expected returns, aggregate profitability, uncertainty regarding future profitability. This leads to a number of phenomena including IPO waves and post-IPO returns that are lower than one might expect in a static model. In Dittmar and Thakor a firm's managers and the investing public may not agree on the value associated with a new investment. When the divergence is large firms finance with debt, and when it is small with equity. What drives their result is that a firm's stock value is likely to be higher when investors and managers share the same beliefs. That occurs because when the beliefs are similar the investors think it is less likely that management will engage in wasteful investment. This in turn increases the appeal of equity financing as well but it also means that going forward shareholder returns are likely to be lower as the level of agreement between them and management has nowhere to go but down.

This paper contributes to the above articles by also seeking to explain the phenomena between investment and returns documented in TWX. Another contribution is to do so within a general equilibrium framework. That allows the model to examine not only time variation in returns, but betas, cross sectional patterns, and the relationship these all bear to variables like industry productivity. In the Pastor and Veronesi (2005) paper market conditions are exogenous and firms react to them, here they are endogenous and influenced by the firms. This interplay allows the model to also make some predictions regarding how overall capital investment impacts the future trajectory of the economy. Also, where Dittmar and Thakor (2007) look at how heterogeneous beliefs influence returns in this article everyone has identical beliefs.

Other related models are those by Berk, Green and Naik (1999) and Carlson, Fisher, and Giammarino (2004, 2006). These authors use real options models to examine how a firm's expected return will vary over time and focus on the relationship between a firm's book-to-market and size that they generate. As they show, it tends to induce patterns that look like those found in Fama and French (1992). The firms in this paper's model have a much simpler investment problem. Another difference is in the data needed to corroborate each model's predictions. Using commonly available data sources it is often difficult to know where and to what degree real option values are influencing a firm's current stock price. In the model developed here one only needs information like the firm's current capital and investment levels. While that does not make the model any more or less likely to be "right" it does make it easier to test and potentially refute.

The paper is structured as follows. Section 1 presents the model. Section 2 contains the analysis. Section 3 the conclusion.

1. A Competitive Model with Capital Adjustments

1.1 Setting

There are K production factors which the paper will also refer to as industry sectors. Each production factor is used by a continuum of competitive equity value maximizing price taking firms with mass of unity. There is a single risk free bond that pays r per period and serves as the numeraire with a constant value of 1. The production factors evolve over time via:

$$N_t = N_{t-1} + \eta_t + Y_t \tag{1}$$

where N_t equals the $K\times1$ vector of production factors, t the time period. The η_t represents the influence of human capital on the total supply of corporate capital. In the model people are born with a human capital endowment which in aggregate equals η_t. Through their employment this human capital is then converted into corporate capital and has the impact shown in (1). From the perspective of investors η_t is a normally distributed random vector with mean zero and variance-covariance matrix Σ_η. The Y_t term is a $K\times1$ vector of capital created by firms in addition to what they get from the amount generated by their employees in the normal course of their business.

In each period the production factors pay a $K\times1$ dividend vector D_t that evolves via:

$$D_t = D_{t-1} + G\left(\bar{D} - D_{t-1}\right) + \delta_t. \tag{2}$$

Here G is a $K\times K$ matrix of constants representing the speed at which asset payouts mean revert, $\bar{D}$ a $K\times1$ vector of constants representing the long run payout per asset class, and the term δ_t is a $K\times 1$ normally distributed random vector with zero mean and variance-covariance matrix Σ_δ.

1.2 Firms

Each firm's output comes from a single production factor.[1] Firm *fk*, (i.e., firm *f* using factor *k*) seeks to maximize its current equity value as follows:

$$\max_{y_{fk,t}} \left(n_{fk,t-1} + \eta_{fk,t} + y_{fk,t}\right) p_{k,t} - c_{k1} y_{fk,t} - \eta_{fk,t} p_{k,t} - \frac{1}{2} c_{k2} y_{fk,t}^2, \tag{3}$$

where $p_{k,t}$ is the period t market price of a unit of capital associated with the k^{th} production factor. The expression $n_{fk,t} = n_{fk,t-1} + \eta_{fk,t} + y_{fk,t}$ is the date-*t* capital employed by firm *fk*. The human capital it employs to create additional corporate capital is represented by the $\eta_{fk,t}$ term and $y_{fk,t}$ is the new capital deployed beyond what is created by the employee base in the normal course of business.[2] Thus, the term $\eta_{fk,t} p_{k,t}$ in (3) implies that firms have to pay their employees the full market value of the capital they create. Implicitly, this means there is a competitive labor market. The constants c_{k1} and c_{k2} represent capital adjustment costs for the k^{th} production factor. All firms in an industry are assumed to face the same costs c_{k1} and c_{k2}.

Each of the c_k terms represent a different aspect of the costs associated with creating productive capital. The c_{k1} parameter captures the base line cost of constructing a unit of production. For example, consider a poultry producer like Tyson. For it c_{k1}

[1] In principle, firms could produce more than a single type of capital output. Assuming that the cost of building or liquidating capital is assessed at the firm level for each production factor separately, there is no loss of generality in considering firms that specialize only in a single type of output.

[2] There is no physical limit to the amount of new capital that can be deployed. Also, to maintain tractability new capital is financed only through the issuing (repurchase in the case of negative deployment) of equity. One could also allow for the use of riskless debt without any fundamental change to the model's results.

equals the cost of building a new chicken farm. This ultimately depends on the price of raw materials like wood, wire, trucks, and the like and not the market value of Tyson's own assets. Thus, the firm can potentially profit by building new farms when their market value exceeds their construction value and by selling them off when the reverse is true. The c_{k2} parameter captures the cost of increasing the speed with which assets are created or sold. Presumably, rushing the construction of a new chicken farm increases its ultimate cost but does allow the firm to generate cash flows from it earlier on. Naturally, whether a firm wishes to rush production of a new facility depends upon how much it expects to earn on it.

Differentiating (3) with respect to $y_{fk,t}$, recalling that the firms take the price vector as given, and then solving for $y_{fk,t}$ yields for each production factor a total capital issuance of

$$y_{fk,t} = \frac{p_{k,t} - c_{k1}}{c_{k2}}. \tag{4}$$

Or, integrating both sides over f and recalling that the total mass is unity,

$$y_{k,t} = \frac{p_{k,t} - c_{k1}}{c_{k2}}, \tag{5}$$

where $y_{k,t} = \int y_{fk,t}\, df$ is the total amount of new capital deployed in factor k. For reference, let $N_{k,t} = \int n_{fk,t}\, df$ and $\eta_{k,t} = \int \eta_{fk,t}\, df$. Writing equation (5) in vector form:

$$y_t = C_{2D}^{-1}(P_t - C_1), \tag{6}$$

where C_1 is the vector of linear costs with the k^{th} element c_{k1}, and C_{2D} is a $K \times K$ matrix with the k^{th} diagonal element equal to c_{k2} and zeros elsewhere thus:

$$C_{2D} = \begin{bmatrix} c_{12} & & & 0 \\ & c_{22} & & \\ & & \ddots & \\ 0 & & & c_{K2} \end{bmatrix}. \tag{7}$$

1.3 Population

Investors, like firms, are assumed to take prices as given. A continuum of investors with unit mass is born in period t, consume and then die in period t+1. Each investor has a negative exponential utility function with risk aversion parameter θ. The only endowment an investor begins life with is his or her human capital. In their first period of life they sell their human capital to firms (that convert it to corporate capital) and buy and sell securities to fund their retirement.

Let $X_{i,t}$ represent the K×1 portfolio of share holdings of investor i in period t. Each share is assumed to represent one unit of a production factor. Let $w_{i,t}$ be the wealth with which investor i is born at date t. The assumption that people are born only with human capital implies $w_{i,t}$ equals the market value of that capital. Furthermore, because investors have negative exponential utility functions and all of the random variables are normally distributed the initial allocation of human capital does not impact the model's equilibrium results. Thus, all that is needed to proceed is knowledge that in the aggregate the incoming human capital equals η_t and that those with skills associated with industry k will earn $\eta_{k,t} p_{k,t}$.

Based on the above discussion and letting R=1+r an investor's period t+1 consumption equals:

$$X'_{i,t}\left(P_{t+1} + D_{t+1} - RP_t\right) + Rw_{i,t} \tag{8}$$

because it is assumed that he or she sells the portfolio prior to death. Again using the assumption that all of the random vectors are normally distributed, and that the investors have negative exponential utilities investors maximize their expected utility by solving the following mean-variance problem:

$$\max_{X_{i,t}} E_t\left[X'_{i,t}\left(P_{t+1}+D_{t+1}-RP_t\right)+Rw_{i,t}\right]-\frac{\theta}{2}\operatorname{var}_t\left[X'_{i,t}\left(P_{t+1}+D_{t+1}-RP_t\right)+Rw_{i,t}\right]. \tag{9}$$

This reduces to,

$$\theta \operatorname{var}_t\left[Q_{t+1}\right]X_{i,t}=E_t\left[Q_{t+1}\right], \tag{10}$$

where

$$Q_{t+1}=P_{t+1}+D_{t+1}-RP_t \tag{11}$$

is the excess payoff vector from a unit position in each type of capital, and $\operatorname{var}_t\left[Q_{t+1}\right]$ is its variance-covariance matrix. Integrating over the continuum of investors and setting the market clearing condition $N_t=\int X_{i,t}\,di$ yields,

$$\theta \operatorname{var}_t\left[Q_{t+1}\right]N_t=E_t\left[Q_{t+1}\right]. \tag{12}$$

1.4 Equilibrium

Investors conjecture that prices are determined via the following formula:

$$P_t=A_0+A_1N_t+A_2D_t \tag{13}$$

where A_0 is a $K\times 1$ vector, while A_1 and A_2 are $K\times K$ matrices. Next, update the time subscripts in (13) to $t+1$ and then plug equations (1), (2) and (6) into equation (13) in order to solve for P_{t+1} in terms of the parameter values known at time t and the unknown $t+1$ shocks:

$$P_{t+1} = \left(I - A_1 C_{2D}^{-1}\right)^{-1} \left\{A_0 + A_1\left(N_t + \eta_{t+1} - C_{2D}^{-1} C_1\right) + A_2\left[D_t + G\left(\bar{D} - D_t\right) + \delta_{t+1}\right]\right\}. \quad (14)$$

Using (13), equation (14) can be rewritten as

$$P_{t+1} = (I - A_1 C_{2D}^{-1})^{-1} \left\{P_t + A_1\left(\eta_{t+1} - C_{2D}^{-1} C_1\right) + A_2\left[G\left(\bar{D} - D_t\right) + \delta_{t+1}\right]\right\}, \quad (15)$$

implying that the price vector follows a VAR(1) process. With some algebra, use (11) and (15) to write

$$\begin{aligned} E_t\left[Q_{t+1}\right] = &\left[\left(I - A_1 C_{2D}^{-1}\right)^{-1} - RI\right] P_t \\ &+ \left(I - A_1 C_{2D}^{-1}\right)^{-1}\left[A_2 G\left(\bar{D} - D_t\right) - A_1 C_{2D}^{-1} C_1\right] + D_t + G\left(\bar{D} - D_t\right). \end{aligned} \quad (16)$$

Similarly,

$$\begin{aligned} &\operatorname{var}_t\left[Q_{t+1}\right] = \operatorname{var}_t\left[\left(I - A_1 C_{2D}^{-1}\right)^{-1}\left(A_1 \eta_{t+1} + A_2 \delta_{t+1}\right) + \delta_{t+1}\right] \\ &= \left[\left(I - A_1 C_{2D}^{-1}\right)^{-1} A_1 \Sigma_\eta A_1' \left(I - A_1 C_{2D}^{-1}\right)^{-1\prime} + \left(\left(I - A_1 C_{2D}^{-1}\right)^{-1} A_2 + I\right) \Sigma_\delta \left(\left(I - A_1 C_{2D}^{-1}\right)^{-1} A_2 + I\right)'\right] \\ &\equiv V. \end{aligned} \quad (17)$$

To solve for the equilibrium values of the A's, replace $E_t\left[Q_{t+1}\right]$ and $\operatorname{var}_t\left[Q_{t+1}\right]$ in equation (12) with the corresponding terms in equations (16) and (17). The coefficients of N_t and D_t must vanish separately as well as those that do not multiply any time varying parameters. This yields for the terms that do not multiply either N_t or D_t,

$$\left[\left(I - A_1 C_{2D}^{-1}\right)^{-1} - RI\right] A_0 + \left(I - A_1 C_{2D}^{-1}\right)^{-1}\left[A_2 G\bar{D} - A_1 C_{2D}^{-1} C_1\right] + G\bar{D} = 0, \quad (18)$$

while for the terms multiplying N_t,

$$\begin{aligned} &\left[\left(I - A_1 C_{2D}^{-1}\right)^{-1} - RI\right] A_1 - \\ &\theta\left[\left(I - A_1 C_{2D}^{-1}\right)^{-1} A_1 \Sigma_\eta A_1' \left(I - A_1 C_{2D}^{-1}\right)^{-1\prime} + \left(\left(I - A_1 C_{2D}^{-1}\right)^{-1} A_2 + I\right) \Sigma_\delta \left(\left(I - A_1 C_{2D}^{-1}\right)^{-1} A_2 + I\right)'\right] = 0, \end{aligned} \quad (19)$$

and finally for the terms multiplying D_t,

$$\left[\left(I - A_1 C_{2D}^{-1}\right)^{-1} A_2 + I\right]\left(I - G\right) - RA_2 = 0. \tag{20}$$

2. Analysis

2.1 Steady state

The economy is defined to be in a steady state in period t if firms do not actively seek to change their capital stock and if the expected change in the payout per unit of capital is expected to remain unchanged. This is a useful base case as it yields the model's predictions regarding unconditional moments in the data. From there it is then possible to see how various shocks to the system will impact estimated returns, risk factors and other financial and economic variables of interest.

Firm's do not actively change their capital stock in period t if Y_t equals a $K\times 1$ vector of zeros and if dividends are also expected to remain unchanged implying $E[D_{t+1}]=D_t$. From equation (6) the vector Y_t will equal zero if and only if $P_t = C_1$. Similarly, asset payouts are expected to remain unchanged if and only if $D_t = \bar{D}$. The unconditional expected return to an investor from holding a claim in one unit of corporate asset k equals

$$E\left[r_{k,t+1}\right] = \frac{E\left[p_{k,t+1}\right] - p_{k,t} + d_{k,t}}{p_{k,t}}, \tag{21}$$

where $d_{k,t}$ represents the k'th element of the vector D_t. Employing the condition that $p_{k,t} = c_{k,1}$ and $d_{k,t} = \bar{d}_k$ in (15) and using the result in (21) shows that if the economy is in steady state then

$$E\left[r_{k,t+1}\right] = \frac{\bar{d}_k}{c_{k1}}. \tag{22}$$

Equation (22) implies that the unconditional mean return to an investor holding a share of stock if a firm that uses asset class *k* equals the long run ratio of that asset's ability to generate cash flows per unit cost of creating that asset. Empirically, one should thus find in the cross section that long run equity returns are linearly related to this ratio. Further note, the right hand side of (22) can (at least in principle) be calculated with data commonly available. For a firm it should equal the long run average change in per period earnings divided by the change in per period book value or similar measures of a firm's cash flow and productive assets.

2.2 Steady State Disrupted by a One Time Shock to Capital

Imagine the economy is in its long run steady state as of period $t-1$ and there is a one time shock to capital (η) or cash flows (D) in period t. To simplify the notation needed to conduct the analyses define the following variables:

$$\begin{aligned} &\hat{P}_t \equiv P_t - C_{1,} \\ &\Delta D_t \equiv D_t - D_{t-1} = G\left(\bar{D} - D_{t-1}\right) + \delta_t, \text{ and} \\ &F \equiv I - A_1 C_{2D}^{-1}. \end{aligned} \tag{23}$$

Subtracting C_1 from both sides of (15) and making the above substitutions yields:

$$\hat{P}_t = F^{-1}\left(\hat{P}_{t-1} + A_1\eta_t + A_2\Delta D_t\right). \tag{24}$$

Rolling (24) back and then substituting out $\hat{P}_t$ for P_t produces the equilibrium price vector that investors expect to occur going forward:

$$P_t = C_1 + \sum_{s=0}^{\infty} F^{-s-1}(A_1\eta_{t-s} + A_2\Delta D_{t-s}) \tag{25}$$

implying the impulse response τ periods after a time t supply shock is given by $F^{-\tau-1}A_1\eta_t$. Similarly, the impulse response τ periods after a time t dividend change is given by $F^{-\tau-1}A_2\Delta D_t$. Since $F \equiv I - A_1 C_{2D}^{-1}$ as long as A_1 is negative definite equation (25) implies that a capital or cash flow shock decays roughly at the rate of $0 < F^{-1} < 1$ (in some matrix norm) per period. The next proposition says that this will always occur in an economy with a large quadratic adjustment cost (C_{2D}).

Proposition 1: As C_{2D}^{-1} approaches zero, A_1 tends to a negative definite matrix in an equilibrium in which it is finite.

Proof. See the Appendix for the proof of this and all other propositions.

In fact, it is straightforward to confirm that the equilibria with finite A_1 converge to those of Spiegel (1998) as $C_{2D}^{-1} \to 0$. Under this assumption C_{2D}^{-1} equals the zero matrix and (20) simplifies to,

$$-A_2(rI+G)+I-G=0 \tag{26}$$

and thus A_2 equals $(I-G)(rI+G)^{-1}$. Next (19) reduces to,

$$rA_1+\theta\left[A_1\Sigma_\eta A_1'+R^2(rI+G)^{-1}\Sigma_\delta(rI+G)^{-1\prime}\right]=0, \tag{27}$$

which can now be solved for A_1, while using (18), $A_0=\dfrac{R}{r}(rI+G)^{-1}G\bar{D}$.

Assuming A_1 is negative definite, equation (25) provides a number of empirical predictions. At time 0 suppose a shock creates a large positive price move across stocks. Equation (25) shows that this will then be followed by a declining price series. Note, this does not mean returns are negative as investors continue to receive a cash flow stream from the assets. But it does mean returns are lower than they are on average. Looking at returns, the implication is that a large return in one direction will lead to lower future returns in the other. Also, note what this implies about the relationship between capital expenditures and future returns. When an industry capital unit fetches a value above its long run equilibrium value firms in that industry increase their holdings of it (see equation (6)). Thus, if a shock generates a large price increase that will in turn generate new investment by firms in the industry. This will be followed by lower equilibrium returns for investors, lower capital prices for the industry, and reduced investment. The process continues on like this until the steady state equilibrium is restored.

2.3 Other Limits of Interest

Two other limits also yield simplified equilibrium expressions and will prove useful for developing the model's implications. The first occurs as investors become more and more risk neutral: $\theta \to 0$. From equation (19), there are two possibilities for A_1. Either it also tends to zero or, alternatively, $\left[\left(I - A_1 C_{2D}^{-1}\right)^{-1} - RI\right] \to 0$ meaning that $A_1 \to \frac{r}{1+r} C_{2D} > 0$. The latter has the undesirable equilibrium implication of upward sloping demand curves for employed capital. Thus, the only economically sensible equilibrium is one in which $\lim_{\theta \to 0} A_1 \to 0$. In turn, this implies that $\lim_{\theta \to 0} A_2 \to (I - G)(rI + G)^{-1}$. Notice that near this limit A_1 is negative definite as $-rA_1 \approx \theta V$ from equation (19). Thus, one has yet another set of sufficient conditions for Proposition 1 to hold with regard to cash flow shocks.

A second limit examined in this section that will prove useful occurs as the variance of the cash flow shocks tends to zero: $\Sigma_\delta \to 0$. In this case, equation (19) becomes

$$\left[\left(I - A_1 C_{2D}^{-1}\right)^{-1} - RI\right] A_1 = \theta\left(I - A_1 C_{2D}^{-1}\right)^{-1} A_1 \Sigma_\eta A_1' \left(I - A_1 C_{2D}^{-1}\right)^{-1\prime}. \tag{28}$$

One obvious solution has A_1 approach 0 from below, which is desirable and in turn implies $\lim_{\Sigma_\delta \to 0} A_2 \to (I - G)(rI + G)^{-1}$. Thus, one has the result that Proposition 1 holds in this limit as well. One way to justify concentrating on this equilibrium is that it holds if A_1 has a power series solution in θ. See the Appendix for a proof.

2.4 Capital Investment and Expected Return

Generally in a model with a downward sloping demand curve, we expect a negative supply shock to increase the current price and decrease expected return, and vice versa. In our model, this is additionally associated with an observable change in capital investment in the same direction as the price change. Therefore, we expect a negative relation between expected return and capital investment. To analyze this formally, define firm k's excess return as

$$r_{k,t+1}^e = \frac{q_{k,t+1}}{p_{k,t}}, \tag{29}$$

where $q_{k,t+1}$ is the k^{th} element of Q_{t+1}. Throughout the rest of the paper, we assume that the price and supply of capital are positive.[3] The next proposition asserts that the

[3] While both the price and supply are normally distributed in our model, one can arbitrarily reduce the probability of their assuming negative values. The distribution of the ratio of normals is called the Fieller distribution and its application is abundantly found in the statistics literature.

expected excess return decreases with capital investment as long as the quadratic adjustment cost is sufficiently large, as assumed in Proposition 1.

Proposition 2: In the limit of Proposition 1, a firm's expected excess return decreases with capital investment caused by supply shocks:

$$\lim_{C_{2D}^{-1}\to 0} \frac{\partial E\,[r_{k,t+1}^{e}]}{\partial y_{k,t}} < 0. \tag{30}$$

2.5 Capital Investment and CAPM Beta

Since the model's random variables are normally distributed and since the stock market is assumed to be competitive and frictionless the CAPM must hold. To verify this rewrite the equilibrium condition in equation (12) as

$$E_t[Q_{t+1}] = \theta\,\mathrm{cov}_t(Q_{t+1}, Q_{M,t+1}), \tag{31}$$

where $Q_{M,t+1} \equiv Q'_{t+1}N_{t+1}$ is the excess payoff on the market portfolio. Pre-multiply the N_t' to obtain

$$E_t[Q_{M,t+1}] = \theta\,\mathrm{var}_t(Q_{M,t+1}). \tag{32}$$

Dividing these two expressions side by side and rearranging, we have

$$E_t[Q_{t+1}] = \frac{\mathrm{cov}_t(Q_{t+1}, Q_{M,t+1})}{\mathrm{var}_t(Q_{M,t+1})} E_t[Q_{M,t+1}]. \tag{33}$$

Define the vector of excess returns and the excess market return as

$$\begin{aligned} r_{t+1}^{e} &= Q_{t+1} \div P_t, \\ r_{M,t+1}^{e} &\equiv \frac{Q_{M,t+1}}{P_{M,t+1}} = \frac{Q_{M,t+1}}{P'_{t+1}N_{t+1}}, \end{aligned} \tag{34}$$

where ÷ denotes the elementwise division operator, and rewrite equation (33) in terms of excess returns:

$$E_t[r_{t+1}^e] = \frac{\mathrm{cov}_t(r_{t+1}^e, r_{M,t+1}^e)}{\mathrm{var}_t(r_{M,t+1}^e)} E_t[r_{M,t+1}^e] \equiv \beta_t E_t[r_{M,t+1}^e]. \tag{35}$$

Here, the vector of betas can be written as

$$\beta_t = \frac{\mathrm{cov}_t(r_{t+1}^e, r_{M,t+1}^e)}{\mathrm{var}_t(r_{M,t+1}^e)} = (VN_t) \div P_t \bullet \frac{N_t' P_t}{N_t V N_t}. \tag{36}$$

Its k'th element is

$$\beta_{k,t} = \frac{e_k' V N_t}{p_{k,t}} \bullet \frac{N_t' P_t}{N_t V N_t} = \frac{E_t[r_{k,t+1}^e]}{E_t[r_{M,t+1}^e]}, \tag{37}$$

where e_k is the choice vector with 1 in its k'th element and 0 elsewhere. Since a firm's expected return decreases with supply-induced capital investment as long as the quadratic adjustment cost is sufficiently large (see Proposition 2), we expect that the CAPM beta will also decrease. The next proposition shows that this is true in a large economy with independent industries:

Proposition 3: Consider a large economy with independent industries (V and A_1 are diagonal). In the limit of Proposition 1, a firm's CAPM beta decreases with capital investment caused by a supply shock:

$$\lim_{\substack{C_{2D}^{-1} \to 0, \\ K \to \infty}} \frac{\partial \beta_{k,t}}{\partial y_{k,t}} < 0. \tag{38}$$

Intuitively, the assumption of cross-sectional independence ensures that the supply shock does not cause market wide price movement. Therefore, in a large economy the firm k

shock only affects its own price and has a negligible effect on the expected market return. Thus, the result on the expected return in Proposition 2 translates into the beta.

3. Conclusion

Traditionally the asset pricing literature has taken the set of corporate assets as given and then asked what the equilibrium returns should be to those that hold them. Recently a number of papers have begun to look at the problem when corporate assets change over time. Articles by Spiegel (1998), Watanabe (2008), Biais, Bossaerts, and Spatt (2008), Pastor and Veronesi (2005), Dittmar and Thakor (2007), Berk, Green and Naik (1999), and Carlson, Fisher, and Giammarino (2004, 2006) all fall into this category. This paper seeks to add to this literature a general equilibrium view of the problem. Rather than take the pricing kernel as given or the movement in asset supplies both are under the population's control to at least some degree.

In this paper asset prices are endogenously determined period by period via market clearing conditions. At the same time corporate capital stocks are impacted by both random fluctuations and firms as they seek to add and subtract from their capital base in response to market conditions. The end result is a tractable model that yields a number of empirical predictions many of which are consistent with the data. Among these are the following:

- Stock returns should be positively correlated with the earnings yield on a firm's capital stock.
- Large returns (price moves) in one direction will be followed by a decaying series in the opposite direction.

- Capital expenditures will be negatively correlated with future returns.
- Small company stocks will generate higher average returns than those of larger companies.
- Since the CAPM holds, period-by-period in the model, the above relationships regarding returns also hold for period-by-period betas. This, however, also implies that empirical models that do not allow betas with time trends will be incorrectly specified.

We plan to calibrate our model and empirically examine these predictions in future work.

4. Bibliography

Asquity, Paul, and David Mullins, 1986, "Equity Issues and Offering Dilution," *Journal of Financial Economics*, 15, 61-89.

Baker, Malcolm and Jeffrey Wurgler, 2000, "The Equity Share in New Issues and Aggregate Stock Returns," *Journal of Finance*, 55, 2219-2257.

Baker, Malcolm and Jeffrey Wurgler, 2002, "Market Timing and Capital Structure," *Journal of Finance*, 57, 1-32.

Berk, Jonathan, 1995, "A Critique of Size-Related Anomalies," *Review of Financial Studies*, 8, 275-286.

Berk, Jonathan, Richard Green, and Vasant Naik, 1999, "Optimal Investment, Growth Options, and Security Returns," *Journal of Finance*, 54, 1553-1607.

Biais, Bruno, Peter Bossaerts, and Chester Spatt, 2008, "Equilibrium Asset Pricing and Portfolio Choice Under Asymmetric Information," *Review of Financial Studies*, forthcoming.

Carlson, Murray, Adlai Fisher, and Ron Giammarino, 2004, "Corporate Investment and Asset Price Dynamics: Implications for the Cross-section of Returns," *Journal of Finance*, 59, 2577-2603.

Carlson, Murray, Adlai Fisher, and Ron Giammarino, 2006, "Corporate Investment and Asset Price Dynamics: Implications for SEO Event Studies and Long-Run Performance," *Journal of Finance*, 61, 1009-1034.

Dittmar, Amy and Anjan Thakor, 2007, "Why Do Firms Issue Equity?," *Journal of Finance*, 62, 1-54.

Fama, Eugene F., and Kenneth R. French, 1992, "The Cross Section of Expected Stock Returns," *Journal of Finance*, 47, 427-466.

Fama, Eugene F., and Kenneth R. French, 2007, "Average Returns, B/M, and Share Issues," *Journal of Finance*, forthcoming.

Jung, Kooyul, Yong Cheol Kim, and Rene M. Stulz, 1986, "Timing, Investment Opportunities, Managerial Discretion, and the Security Issue Decision," *Journal of Financial Economics*, 42, 159-185.

Ikenberry, David, Josef Lakonishok, and Theo Vermaelen, 1995, "Market Underreaction to Open Market Share Repurchases," *Journal of Financial Economics*, 39, 181-208.

Lyandres, Evgeny, Le Sun, and Lu Zhang, 2007, "The New Issues Puzzle: Testing the Investment-Based Explanation," *Review of Financial Studies*, forthcoming.

Loughran, Tim, Jay Ritter, and Kristian Rydqvist, 1984, "Initial Public Offerings: International Insights," *Pacific-Basin Finance Journal*, 2, 165-199.

Loughran, Tim, and Jay Ritter, 1995, "The New Issues Puzzle," *Journal of Finance*, 50, 23-51.

Lowry, Michael, 2003, "Why Does IPO Volume Fluctuate So Much?," *Journal of Financial Economics*, 67, 3-40.

McLean, David R., Jeffrey Pontiff, and Akiko Watanabe, 2008, "Share Issuance and Cross-sectional Returns: International Evidence," *Journal of Financial Economics*, forthcoming.

Mikkelson, Wayne H. and Megan Partch, 1986, "Valuation Effects of Security Offerings and Issuance Process," *Journal of Financial Economics*, 15, 31-60.

Pagano, Marco, Fabio Panetta, and Luigi Zingales, 1998, "Why Do Companies Go Public? An Empirical Analysis," *Journal of Finance*, 53, 27-64.

Pastor, Lubos and Pietro Veronesi, 2005, "Rational IPO Waves," *Journal of Finance*, 60, 1713-1757.

Pontiff, Jeffrey, and Artemiza Woodgate, 2008, "Share Issuance and Cross-sectional Returns, *Journal of Finance*, 63 (2), 921-945.

Rajan, Raghuram, and Henri Servaes, 1997, "Analyst Following of Initial Public Offerings," *Journal of Finance*, 52, 507-529.

Ritter, Jay, 1991, "The Long Run Performance of Initial Public Offerings," *Journal of Finance*, 46, 3-27.

Spiegel, Matthew, 1998, "Stock Price Volatility in a Multiple Security Overlapping Generations Model," *Review of Financial Studies*, 11, 419-447.

Titman, Sheridan, K. C. John Wei, and Feixue Xie, 2004, "Capital Investments and Stock Returns," *Journal of Financial and Quantitative Analysis*, 39, 677-700.

Watanabe, Masahiro, 2008, "Price Volatility and Investor Behavior in an Overlapping Generations Model with Information Asymmetry," *Journal of Finance*, 63, 229-272.

Xing, Yuhang, 2007, "Interpreting the Value Effect Through the Q-Theory: An Empirical Investigation," *Review of Financial Studies*, forthcoming.

5. Appendix

5.1 Proofs

Proposition 1: As C_{2D}^{-1} approaches zero, A_1 tends to a negative definite matrix in an equilibrium in which it is finite.

Proof. Rewrite equation (19) as

$$\left[\left(I - A_1 C_{2D}^{-1}\right)^{-1} - RI\right] A_1 = \theta V \, , \tag{39}$$

where V is the covariance matrix of excess payoffs defined in equation (17). If A_1 is finite, $A_1 C_{2D}^{-1}$ in the left hand side approaches zero as $C_{2D}^{-1} \to 0$. In the limit, we have

$$-r \lim_{C_{2D}^{-1} \to 0} A_1 = \theta V \, . \tag{40}$$

Since the right hand side of this equation is positive definite by construction, A_1 must converge to a negative definite matrix. ■

Proposition 2: In the limit of Proposition 1, a firm's expected excess return decreases with capital investment caused by supply shocks:

$$\lim_{C_{2D}^{-1} \to 0} \frac{\partial E\,[r_{k,t+1}^e]}{\partial y_{k,t}} < 0.$$

Proof. From equation (5), there is a positive relation between the price and capital investment of each firm. That is, the denominator of equation (29) increases with capital investment. Thus, it suffices to show that its numerator decreases with capital investment and equivalently with the price. Invert the price conjecture in (13) for N_t,

$$N_t = A_1^{-1}(P_t - A_0 - A_2 D_t) \tag{41}$$

and rewrite the market-clearing condition in (12) as

$$\begin{aligned} E_t\left[Q_{t+1}\right] &= \theta V N_t \\ &= \theta V A_1^{-1}(P_t - A_0 - A_2 D_t) \\ &= [(I - A_1 C_{2D}^{-1})^{-1} - RI](P_t - A_0 - A_2 D_t), \end{aligned} \tag{42}$$

where we have used equation (39) in the last line. For a change in P_t caused by supply shocks (and not by dividend shocks), equation (42) implies that

$$\frac{\partial E_t\left[Q_{t+1}\right]}{\partial P_t'} = (I - A_1 C_{2D}^{-1})^{-1} - RI \xrightarrow{C_{2D}^{-1} \to 0} -rI \ . \tag{43}$$

The k'th diagonal element of this derivative shows that $\lim_{C_{2D}^{-1} \to 0} \dfrac{\partial E\,[Q_{k,t+1}]}{\partial p_{k,t}} = -r < 0.$ This completes the proof. ■

Proposition 3: Consider a large economy with independent industries (V and A_1 are diagonal). In the limit of Proposition 1, a firm's CAPM beta decreases with capital investment caused by a supply shock:

$$\lim_{\substack{C_{2D}^{-1} \to 0, \\ K \to \infty}} \frac{\partial \beta_{k,t}}{\partial y_{k,t}} < 0.$$

Proof. Since all elements of V are nonnegative, each term in equation (37) is strictly positive as long as prices and supply are, and one can take its logarithm:

$$\log \beta_{k,t} = \log e_k' V N_t - \log P_{k,t} + \log N_t' P_t - \log N_t V N_t . \tag{44}$$

Again, due to the positive relationship between the price and capital investment of each firm (see equation (5)), it suffices to show that this quantity decreases with an increase in firm k'th price caused by supply shocks. Noting that N_t is a fuction of P_t (see equation (41)), differentiate the above expression with respect to P_t:

$$\frac{\partial \log \beta_{k,t}}{\partial P_t} = \frac{A_1'^{-1} V e_k}{e_k' V N_t} - \frac{e_k}{p_{k,t}} + \frac{A_1'^{-1} P_t + N_t}{N_t' P_t} - \frac{2 A_1'^{-1} V N_t}{N_t V N_t}. \tag{45}$$

The k'th element is

$$\frac{\partial \log \beta_{k,t}}{\partial p_{k,t}} = \frac{e_k' A_1'^{-1} V e_k}{e_k' V N_t} - \frac{1}{p_{k,t}} + \frac{e_k' A_1'^{-1} P_t + n_{k,t}}{N_t' P_t} - \frac{2 e_k' A_1'^{-1} V N_t}{N_t V N_t}. \tag{46}$$

The first two terms are the derivative of the log expected firm return and the last two terms the derivative of the log expected market return. If the last two terms vanish in a large economy, we are left with the first two terms, which we expect to be negative given the result in Proposition 2. This indeed happens when both V and A_1 are diagonal, which allows us to write:

$$\frac{\partial \log \beta_{k,t}}{\partial p_{k,t}} = \frac{a_{1kk}^{-1}}{n_{k,t}} - \frac{1}{p_{k,t}} + \frac{a_{1kk}^{-1} p_{k,t} + n_{k,t}}{\sum_{j=1}^{K} n_{j,t} p_{j,t}} - \frac{2 a_{1kk}^{-1} v_{kk} n_{k,t}}{\sum_{j=1}^{K} v_{jj} n_{j,t}^2} \xrightarrow[C_{2D}^{-1} \to 0]{K \to \infty} \frac{a_{1kk}^{-1}}{n_{k,t}} - \frac{1}{p_{k,t}} < 0. \tag{47}$$

where a_{1kk} and v_{kk} are the k'th diagonal element of A_1 and V, respectively, and we have used the fact that the two summations in the denominator are sums of positive terms and therefore diverge to infinity in the limit. The condition that $C_{2D}^{-1} \to 0$ ensures the negative definiteness of A_1 and hence $a_{1k,k} < 0$ as presented in Proposition 1. ■

5.2 Series Solution to the Equilibrium Matrices

Conjecture that

$$Z(\theta) \equiv A_1 C_{2D}^{-1} = \sum_{n=1}^{\infty} z_n \frac{\theta^n}{n!}. \tag{48}$$

Henceforth, suppress the θ-dependence of *Z*. Let $F \equiv (I-Z)$, use equation (20) to write $F^{-1}A_2 + I = R\left(F^{-1}-RI\right)^{-1}$, and plug this into (19) to get

$$Z = \theta\left[(F^{-1}-RI)^{-1}F^{-1}ZC_{2D}\Sigma_\eta C_{2D}Z'F^{-1\prime} + R^2(F^{-1}-RI)^{-2}\Sigma_\delta(F^{-1}-RI)^{-1\prime}\right]C_{2D}^{-1}. \qquad (49)$$

The idea is now to apply the differential operator, $\frac{\partial^n}{\partial\theta^n}$ to each side of equation (49), set θ to zero, and solve for z_n. This yields a ***unique*** solution for *Z* because the right side of equation (49) is multiplied by θ and thus the application of this solution procedure yields an equation of the form,

$$z_n = f(R, \Sigma_\eta, \Sigma_\delta, C_{2D}, \{z_i\}_{i=1}^{n-1}). \qquad (50)$$

It is straight forward to solve for the first few coefficients. For the fourth order coefficients or higher, the procedure becomes tedious. It should be immediately obvious that the first and second order coefficients of *Z* do not depend on Σ_η.

Solving for z_1: Writing,

$$z_1 = \left[R^2(I-RI)^{-2}\Sigma_\delta(I-RI)^{-1\prime}\right]C_{2D}^{-1}, \qquad (51)$$

thus

$$z_1 = -\frac{(1+r)^2}{r^3}\Sigma_\delta C_{2D}^{-1}. \qquad (52)$$

In particular, the coefficient is negative, implying downward sloping demand curves, as desired.

Solving for z_2: Write,

$$z_2 = \frac{\partial}{\partial\theta}\left[R^2(F^{-1} - RI)^{-2}\Sigma_\delta(F^{-1} - RI)^{-1\prime} \right]_{\theta=0} C_{2D}^{-1}, \tag{53}$$

and employ the identity $\frac{\partial}{\partial\theta}G^{-1}(\theta) = -G^{-1}(\theta)\left(\frac{\partial}{\partial\theta}G^{-1}(\theta)\right)G^{-1}(\theta)$ to eventually yield

$$z_2 = 2\frac{(1+r)^4}{r^7}(\Sigma_\delta C_{2D}^{-1})^2. \tag{54}$$

It would probably be worthwhile to work out the next term (later). What should be clear, is that z_3 and, in fact, all higher order coefficients vanish if z_1 and z_2 vanish. In other words, as conjectured earlier, this solution has the property that $\lim_{\Sigma_\delta \to \infty} Z \to 0$, and thus $\lim_{\Sigma_\delta \to \infty} A_2 \to \frac{1}{r}$.