Smart Buyers*

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Abstract

In many bilateral transactions, the seller fears to be underpaid because its outside option is better known to the buyer. We rationalize a variety of observed contracts as solutions to such smart buyer problems. The key to these solutions is to grant the seller upside participation. In contrast, the lemons problem calls for offering the buyer downside protection. Yet in either case, the seller (buyer) receives a convex (concave) claim. Thus, contracts commonly associated with the lemons problem can equally well be manifestations of the smart buyer problem. Nevertheless, the information asymmetries have opposite cross-sectional implications. To avoid underestimating the empirical relevance of adverse selection problems, it is therefore critical to properly identify the underlying information asymmetries in the data.

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1 Introduction

The highly successful 2010 US reality documentary *American Pickers* follows antique hunters on expeditions through people's homes, barns, and sheds to purchase collectibles to supply antique dealers. The story line portrays the thrill of discovering items of value unbeknown to their current owners. In a similar spirit, the British television show *Bargain Hunt* challenges contestants to generate as much profit as possible by buying antiques at a fair and reselling them in an auction.

Buyers outsmart less informed sellers not only on television. In 1981, Seattle Computer sold its rights to QDOS, a microcomputer operating system, to Microsoft for a flat price of about \$50,000. After some modifications, Microsoft licensed its own version, called MS-DOS, to IBM for a per-copy royalty fee. MS-DOS became the standard for IBM's hugely successful personal computer (PC), turning Microsoft's founder, Bill Gates, into a billionaire by 1986. Presumably, if Seattle Computer had grasped the potential of QDOS, or Microsoft's intentions, it would have asked for more than \$50,000. Microsoft got a bargain precisely because it had a better idea of the value locked in QDOS. This is a dramatic but by no means isolated incident. Better informed buyers are present in many markets: a real estate developer buying land, a pharmaceutical company buying patents, a private equity fund buying out widely held firms, a venture capital firm buying into start-ups, a producer buying the movie rights to a novel, and collectors of all sort, to name a few.

When facing a better informed buyer, sellers may suspect that the terms of trade are unfavorable. Such fear of being shortchanged makes them reluctant to sell, and therefore engenders trade frictions that are the inverse of Akerlof (1970)'s famous lemons problem. In the lemons problem, the seller is privately informed about the quality of the good, which affects the buyer's payoff from *entering* the trade (inside option). The buyer hesitates because the seller is likely to *over*state the inside option in order to raise the price. In the reverse constellation, which we refer to as the "smart buyer problem," the buyer is privately informed about the value of the good, which affects the seller's payoff from *rejecting* the trade (outside option). Here, the seller hesitates because the buyer is likely to *under*state the outside option in order to lower the price.

These differences can also be cast in terms of signaling incentives. Consider

Spence (1973)'s classic labor market example, in which workers sell human capital to firms, but are privately informed about the quality of that capital. Confronted with a lemons problem, the firm hesitates to commit to a high "price." Conversely, to avoid selling their services below par, workers must credibly reveal high skill levels. In some labor market situations, the information asymmetry is arguably the reverse. A record producer signing up a new garage band, a movie company signing a promising actress, tenured faculty recruiting PhDs fresh out of graduate school, employers making exploding offers to rookie candidates, and other adept buyers of human capital are often in a better position to evaluate a candidate's potential (during the hiring process) than the candidate itself. In such situations, the challenge for the buyer is not how to match a candidate's outside option but rather how to signal that the latter is indeed matched.

The present paper analyzes contractual solutions to the smart buyer problem with three aims in mind: First, we seek to draw attention to the prevalence of this problem. To this end, we consider various bilateral trade examples, all plagued by the smart buyer problem. We demonstrate that the information asymmetry gets resolved by observed contractual arrangements, such as royalties, cash-equity bids, earn-out clauses, debt-equity swaps, "gross points" (for actors), and concessions. Second, we systematically explore how a smart buyer can signal its information to the seller, and compare these solutions to those of the lemons problem. The latter leads to downward-sloping demand and necessitates downside protection for the uninformed party. By contrast, the smart buyer problem creates upward-sloping supply and calls for granting the uninformed party upside participation. Third, and perhaps most importantly, we show that – despite these differences – contractual solutions to these two information asymmetries look intriguingly similar. Indeed, this deceptive similarity can lead to erroneous conclusions in empirical contract studies.

These points are best illustrated by an example. Consider a company that wants to purchase a patent from a scientist to develop a new product. Gallini and Wright (1990) show that, if the company is concerned about the quality of the patent, the scientist can signal high quality by accepting royalties. This signal is credible because a scientist with less confidence in the patent is less willing to participate in revenues in case the product is a failure. By taking on upside exposure, the scientist effectively provides the company with downside protection. It is equally plausible, however, that the company, rather than the scientist, can better assess the latent value of (products based on) the patent. The scientist is then reluctant to enter into an outright sale for fear of giving up a gem. We show that the company can alleviate such concerns by conceding royalties. This signal is credible because a more optimistic company is not as willing to share revenues in case the product is a success. By surrendering such gains, the company grants the scientist upside participation. So royalties can be a manifestation of (either) the smart buyer problem or the lemons problem.

Despite implying identical contract forms, the two information asymmetries yield opposite predictions in the cross-section. When the scientist uses royalties as a signal, a greater reliance on royalties implies a higher patent value. Conversely, when it is the company that uses royalties as a signal, a greater reliance on royalties implies a lower patent value. Neglecting this subtlety – identical contract form but contrary cross-sectional predictions – can lead to false conclusions. For instance, imagine an empirical study that relates patent valuations or revenues to royalties and does not find a positive relation between patent value and royalties. While this would indeed contradict the existence of a lemons problem, such evidence need not rule out that information problems shape the contract choice (royalties). It may be that the patent sales under consideration are plagued by the smart buyer problem in which case lower patent values go together with more royalties. Alternatively, some transactions may be subject to the lemons problems but others to the smart buyer problem, and their countervailing effects attenuate the average effect in the data. The general point is that confounding the two problems can make asymmetric information appear less relevant for contract design than it is.

We derive these insights in an informed principal model (Maskin and Tirole, 1992). Our key assumption is that the buyer has superior information not only about its own valuation of the good, but also about the common value component. To trade, the buyer must convince the seller that the latter's participation constraint is met. Our model captures the seller's fear of being short-changed as well as the notion that certain sophisticated buyers are "cherry pickers" and have the upper hand in bargaining. While these features make the informed principal framework appealing, our main insights could also be established in a screening model.[1]

[1]Shifting bargaining power changes the allocation of rents between seller and buyer, rather than the shape of the optimal contracts or the distribution of rents among the different types of the informed party. Thus, the results that obtain from a comparison between the smart

Our simple bilateral trade model allows us to cover a wide variety of transactions involving indivisible and divisible goods, as well as assets that generate verifiable or non-verifiable returns. For each type of goods transaction, we explore whether the smart buyer can signal the common value of the good. In general, a fully revealing equilibrium exists, provided that the buyer's information advantage is one-dimensional, or equivalently, provided that the common value of the good is a sufficient statistic for the buyer's valuation of the good. In case of a two-dimensional type space, the information advantage reaches an extent that severely undermines the buyer's ability to signal. Finally, we also study equilibrium selection, which allows us to address the choice between "standardized" (pooling) and "customized" (separating) purchase offers, and to identify additional differences to the lemons problem.

There exist, of course, numerous papers that examine bilateral trade under one-sided asymmetric information in a variety of economic contexts. In many of the cases, it is predominantly – if not exclusively – the lemons problem that is given consideration.[2] Our paper connects to this body of research in several ways. First, it argues that the smart buyer problem is prevalent in practice, in some markets even more salient than the lemons problem. Second, it proposes contract solutions that previous studies associate (solely) with the lemons problem. Prominent examples of such studies include Leland and Pyle (1977), Myers and Majluf (1985), Gallini and Wright (1990), Eckbo et al. (1991), and Duffie and DeMarzo (1999). Third, our analyses synthesizes and generalizes the results of existing, but isolated, applications of the smart buyer problem, such as Shleifer and Vishny (1986), Hirshleifer and Titman, (1990) or Daughety and Reinganum (2010). Most closely related is the contemporaneous paper by Dari-Mattiacci et al. (2010) who also consider a bilateral trade setting with an informed buyer. Their analysis concentrates on the pooling equilibrium and shows that buyer

buyer problem and the lemons problem are, for all intents and purposes, orthogonal to the allocation of bargaining power. This is not true when private information only pertains to private value components, in which case giving the informed party all the bargaining power eliminates all trade frictions (see Bolton and Dewatripont, 2005).

[2] Riley (2001) and Horner (forthcoming) provide recent reviews of the signaling literature. Most applications cited in these surveys – in marketing, industrial organization, finance, labor markets, politics, and biology – involve lemons problems; firms want to signal high quality to customers, competitors, or investors, workers want to signal high skill levels, politicians want to signal that they are highly attractive to voters, and animals want to signal high levels of fitness. A notable exception is Banks (1990)'s (non-trade) model of a political agenda setter who wants to signal that rejecting a proposal, that is, reversion to the status quo is undesirable. Note that the agenda setter wants to signal a low outside option.

information advantage leaves only high quality goods on the market and pushes up the equilibrium price. They informally discuss contractual and legal remedies to the smart buyer problem such as buy-back options or duties to disclose. By contrast, we study both pooling and separating equilibria and examine in detail how (and when) the buyer can structure the purchase offer to credibly reveal its private information.

Another related strand of the literature is auction theory where buyers with private information are the norm. Since multiple bidders compete for the good, there are, from the seller's perspective, no latent opportunities; all "options" are present in the bidding contest. Consequently, the seller's concern is how to maximize the expected auction revenues rather than how to avoid the risk of being short-changed. In contrast, the smart buyer problem presupposes that seeking alternative buyers or organizing an auction is costly, and that the informed buyer knows more about the costs and benefits of pursuing these outside "options." Put differently, a buyer faces actual competition in auctions which it must defeat, whereas in the smart buyer problem a buyer must convince the seller that its offer surpasses any latent alternative.

DeMarzo et al. (2005) study auctions in which buyers compete for a good by bidding with securities. In their model, the seller prefers to receive bids in "steep" claims; it prefers, for example, equity over cash. Steeper claims intensify bidding competition, thereby enabling the seller to extract more revenues.[3] By the same token, the buyers prefer to submit bids in "flat" claims lest they give up rents to the seller. Indeed, if the buyers design their offers, they choose to bid in cash. (This contrasts with the smart buyer problem where the buyer voluntarily offers steep claims to appease the seller with upside participation.) In a similar spirit, Axelson (2005) and Garmaise (2007) examine how a firm designs securities that it wants to sell in an auction to privately informed investors.

Like these papers, we explore optimal contracts in the presence of informed buyers but focus on the buyer's design problem within bilateral trade settings. Inderst and Mueller (2006) take a similar perspective. In their model, a better-informed lender commits to a security design before screening loan applicants;

[3] This insight builds on Hansen (1985) and Rhodes-Kropf and Viswanathan (2000). To the contrary, Che and Kim (2010) show that sellers may prefer flatter securities when bidders have private information about the investment cost. Similarly, Gorbenko and Malenko (forthcoming) show that competition among sellers may lead to auctions in flatter securities, as sellers must relinquish rents to attract bidders.

the optimal design is a commitment to minimize inefficient lending decisions. In contrast, our model applies to situations in which the buyer is already informed and not yet committed to any particular contract.[4] In this sense, our paper is more closely related to the literature on the (choice of) means of payment in takeovers, which we refer to later in the paper.

The paper proceeds as follows. Section 2 outlines the model and gives examples of smart buyer situations. Section 3 examines separating equilibria. Section 4 compares the smart buyer problem to the lemons problem. Section 5 introduces pooling offers and addresses the issue of equilibrium selection. Concluding remarks are set forth in Section 6, and the mathematical proofs are presented in the Appendix.

2 Framework

2.1 Model

A buyer approaches a seller, who possesses one unit of a tradable good. A transaction is characterized by a pair (x, t) where $x \in \mathfrak{X} \subset [0, 1]$ is the traded quantity and t is the total (net) cash transfer from the buyer to the seller. The buyer's and the seller's payoffs are $V(x; \cdot) = v(x; \cdot) - t$ and $U(x; \cdot) = u(1 - x; \cdot) + t$, respectively, where u and v are differentiable functions with $u(0; \cdot) = v(0; \cdot) = 0$, $u_x > 0$, and $v_x > 0$. To focus on the adverse effects of asymmetric information, we assume that the buyer has no wealth constraints, whereas the seller is penniless.

The buyer's valuation of the good can be written as the sum of two components, $v(x; \cdot) = [u(1; \cdot) - u(1 - x; \cdot)] + z(x; \cdot)$. The first term $u(1; \cdot) - u(1 - x; \cdot)$ represents the seller's loss from giving up x of the good, and the second term $z(x; \cdot)$ are the gains from trade. In other words, the function $v(x; \cdot)$ can be decomposed into a common value component $u(1; \cdot) - u(1 - x; \cdot)$ and a private benefit component $z(x; \cdot)$.

For tractability, we let the seller's valuation be constant per unit of the good,

[4] In fact, the setting in Inderst and Mueller (2006) is more akin to the lemons problem, since the lender privately learns the firm's inside option (payoff from borrowing), while all outside options are commonly known. The lender de facto sells a loan product that may or may not be good for the borrower. Viewed in this light, we are the first to study security design by a smart buyer.

$u(x) = \theta_x x$ with $\theta_x \geq 0$. The coefficient θ_x reflects the seller's outside option. For example, θ_x could be the (expected) price a latent alternative buyer would pay, in which case $z(x;\cdot)$ would be the value-added by the present buyer relative to the latent alternative buyer. The linearity simplifies the analysis but is not crucial for the qualitative insights.

Given the decomposition and the linearity, the preferences can be written as

$$U(x;\cdot) = \theta_x (1-x) + t \qquad \text{(Seller)}$$
$$V(x;\cdot) = \theta_x x + z(x;\cdot) - t. \qquad \text{(Buyer)}$$

We assume that $z = z(x;\theta_x,\theta_z)$. That is, the buyer's private benefits depend on the traded quantity x, factors determining the common value (captured by θ_x), and other factors (captured by θ_z). For example, θ_x is the objective quality of a car as the determinant of its market price, and θ_z is the buyer's idiosyncratic pleasure from driving that car.

Assumption 1 $z \geq 0$, $z_x \geq 0$, $z_{\theta_x} \geq 0$, $z_{\theta_z} \geq 0$, $z_{x\theta_x} \geq 0$, *and* $z_{x\theta_z} \geq 0$.

The buyer's private benefits are non-negative and increase in x, θ_x, and θ_z. Further, they increase marginally more in x when θ_x or θ_z are higher. Assumption 1 implies that the efficient outcome is full trade, $x = 1$.[5]

The parameters are continuously distributed on $\Theta_x \times \Theta_z = [\underline{\theta}_x, \overline{\theta}_x] \times [\underline{\theta}_z, \overline{\theta}_z]$ according to a commonly known distribution. The true parameters (θ_x, θ_z) are realized prior to the offer. We now introduce our central assumption.

Assumption 2 *Only the buyer observes* θ_x.

Put differently, the seller knows less about her outside option than the buyer. As a result, the seller is concerned that the buyer's offer might be too low. This in turn puts the burden on the buyer to convince the seller that the offer indeed matches, or exceeds, the latter's true outside option.[6]

[5] This assumption is the opposite of, but also akin to, Assumption 5 in Riley (1979) in the following sense: It is the opposite in that, for the equilibrium analysis, it will imply that increasing the level of the "signal" is cheaper for *lower* common value types; however, it fulfils the same function in that it will allow us to achieve *separation*.

[6] Unless the buyer's private information is about θ_x, there are no frictions in this model; the buyer would offer $t = E[\theta_x]$ for $x = 1$, which the seller would always accept. That is, being better informed does not matter to the buyer as long as she can guarantee to pay the seller's outside option. Note that, if the seller, rather than the buyer, had private information about θ_x, the buyer would face the lemons problem.

With respect to θ_z, the bulk of our analysis presumes symmetric information, so that the buyer's type (information advantage) is one-dimensional and given by $\theta_x \in \Theta_x$. For simplicity, we assume that θ_z is common knowledge. (Assuming instead that neither party observes θ_z does not change the qualitative results.) Only in Section 5.3, we assume that θ_z is known exclusively by the buyer, in which case the type (information advantage) is two-dimensional and given by $(\theta_x, \theta_z) \in \Theta_x \times \Theta_z$. In either case, the buyer has superior information both about its private benefits and about the seller's outside option. The difference is that, in the one-dimensional case, θ_x is a sufficient statistic for both.

Contracting follows the informed principal model of Maskin and Tirole (1992). The informed buyer moves first and makes a take-it-or-leave-it offer to the seller. The offer consists of a menu of contracts. The seller decides whether to accept the menu or not. If the seller accepts the menu, the buyer chooses one contract from the menu, which is then implemented.

We introduce two conditions that render our model applicable to a wide range of bilateral trade situations, as the examples in Section 2.2 illustrate.

Condition V $v(x)$ is verifiable.

Condition V (for "verifiability") is satisfied when a third party, such as a court, can verify the buyer's valuation (only) after trade has taken place. If Condition V holds, the buyer can commit to monetary transfers that are contingent on the buyer's valuation; otherwise, only fixed monetary transfers are feasible. Note that, under Condition V, the buyer's valuation is verifiable as a whole, but not the individual components of the buyer's valuation. Otherwise, payments could be made contingent on the common value θ_x, which would trivially resolve the smart buyer problem.

Condition D $\mathfrak{X} = [0, 1]$.

Condition D (for "divisibility") states that the traded good is (perfectly) divisible. When Condition D is not satisfied, the parties can either trade the entire good or not at all. Unless otherwise stated, Conditions V and D are not satisfied.

2.2 Applications

We now present several examples of smart buyer constellations, including some of those mentioned in the introduction.

A0: Art collector. A famous art collector wants to buy a work from a novice artist. The collector derives non-verifiable (hedonic) utility from complementing her own existing collection with the work, but also has more experience to assess the potential (future) value of the artist's work. Here, θ_z captures the collector's idiosyncratic hedonic utility, while θ_x reflects the latent market value of the work. Neither Condition V nor Condition D is satisfied.

A1: Securities trading. A sophisticated investor wants to buy securities from a market maker. The investor gains from the trade partly because it hedges risk exposures that are specific to her current portfolio. These hedging gains are non-verifiable and cannot be transferred to the market maker. The investor also has private information about the fundamental value of the securities. Here, θ_z captures the investor's idiosyncratic hedging demand, while θ_x reflects the securities' fundamentals. Condition D is satisfied.

A2: Real estate. A well-known real estate developer wants to buy some property to build a hotel in lieu of the existing buildings. The future cash flow from operating the hotel can be shared. The current owner is unable to develop the land in the same way. She is also less informed about (valuations on) the real estate market. Here, θ_z captures the developer's capabilities, while θ_x reflects the value of the "location." Condition V is satisfied.

A3: Patent. A company wants to buy a patent from a scientist to improve its products. The scientist knows less about how valuable the patent is for improving such products. Here, θ_z captures the company's product market share, while θ_x reflects the patent's latent market value. Condition V is satisfied.

A4: Restructuring. The controlling shareholder of a firm under bankruptcy protection offers to inject new capital in exchange for partial debt forgiveness. While there is consensus that continuation is better, the controlling shareholder has superior information about the going concern value and the liquidation value. The creditors question the proposed terms. Here, θ_z captures the shareholder's managerial ability, while θ_x reflects the firm's liquidation value. Conditions V and D are satisfied.

A5: Takeover. A small firm with promising ideas is approached by a large industry peer. The target management deems the buy-side valuations of their firm suspiciously low. Here, θ_z captures acquirer characteristics, while θ_x reflects target characteristics. Conditions V and D are satisfied.

A6: Venture capital. A seasoned venture capitalist wants to invest in a start-up firm, help develop its business, and finally take it public. The venture capitalist contributes useful experience to the start-up but also knows more about its potential market value. The firm founders fear conceding too large a stake. Here, θ_z captures the venture capitalist's experience, while θ_x reflects the potential of the firm. Conditions V and D are satisfied.

A7: Movie rights. A Hollywood studio wants to buy the movie rights to a series of novels. Compared to the seller (writer and/or publishing company), the studio can better assess the box office potential of the novels and whether another studio may be interested. The seller is concerned about giving up a "hidden gem." Here, θ_z captures the studio's movie-making capacities, while θ_x reflects the novels' box office potential. Condition V is satisfied.

A8: Hiring talent. A music producer wants to sign up a new band. The contract would confer exclusive rights to produce the band for several years. While the band is inexperienced, the producer has a track record of developing new talent. The band members have reservations about some of the contract terms and wonder whether they could get a better deal elsewhere or later. Here, θ_z captures the producer's capability, while θ_x reflects the potential of the band. Condition V is satisfied.

A9: Legal counsel. A defendant (or plaintiff) considers hiring a specialist attorney, as opposed to seeking standard counsel. The specialist attorney claims that, without its help, the chances of winning the case are slim. The defendant is unsure about the validity of this claim, particularly given the high(er) legal fees. Here, θ_z reflects the quality of the specialist lawyer, while θ_x reflects the chances of winning the case with a run-of-the-mill attorney. Condition V is satisfied.

3 Persuasive purchase offers

An actual trade contract takes the form

$$\mathcal{C} = [x, t_0, \tau(v)]$$

where x is the quantity traded, t_0 is a fixed monetary transfer, and $\tau(v)$ is a monetary transfer contingent on the ex post realization of v. Thus, the total payment is $t = t_0 + \tau(v)$. When Condition V is violated, $\tau(v) = 0$, whereas $x = 1$ for any non-trivial buy offer when Condition D is violated. Let $\mathcal{C}_\emptyset \equiv [0, 0, 0]$ denote the null contract. A buy offer consists of a set of contracts $\mathfrak{C}$, henceforth referred to as a contract *menu*.

In the absence of private benefits ($\overline{z} = 0$), the unique equilibrium outcome is the absence of trade. This result follows directly from the no-trade theorem (e.g., Milgrom and Stokey, 1982). Without gains from trade, any buy offer reveals that the buyer deems the common value of the asset (weakly) higher than the offered price. Hence, rejecting the buyer's offer is the dominant strategy for any offered contract (menu). In the presence of private benefits, trade is feasible in equilibrium.

In this section, we focus on Perfect Bayesian equilibria and restrict the buyer to make separating offers. A separating offer is a contract menu $\mathfrak{C}$ such that, if the menu is accepted, the buyer's selection from the menu fully reveals her type unless the selected contract is $\mathcal{C}_\emptyset$. That is, a separating offer is a function $\mathcal{C} = \mathcal{C}(\theta_x)$ that attributes a particular contract to each buyer type. We focus on differentiable $\mathcal{C}(\theta_x)$ without loss of generality (Mailath and von Thadden, 2010). The restriction to separating offers is made for expositional convenience; most equilibria under this restriction remain Perfect Bayesian equilibria even when pooling offers are allowed (see Section 5).

3.1 Upward-sloping supply

Suppose Conditions V and D are violated; the good is indivisible, and transfers cannot be contingent. Thus, the buyer can only acquire the good through a contract of the form $\mathcal{C} = [1, t, 0]$. It is straightforward to see that such contracts rule out any separating offer, since all bidder types would select the contract with the lowest t from any given contract menu.

To construct separating equilibria in this case, we need to allow for *stochastic* contracts. Under a stochastic contract $\tilde{\mathcal{C}}_g$, a deterministic contract $\mathcal{C}$ is randomly implemented according to a probability distribution $g(\mathcal{C})$.

For a buyer of type θ_x, the expected payoff from a stochastic contract $\tilde{\mathcal{C}}_g$ is

$$\begin{aligned}\Pi(\tilde{\mathcal{C}}_g;\theta_x) &= p_g[\theta_x + z\,(1;\theta_x,\theta_z) - \bar{t}_g^1] - (1-p_g)\bar{t}_g^0 \\ &= p_g[\theta_x + z\,(1;\theta_x,\theta_z)] - \bar{t}_g \end{aligned} \tag{1}$$

where $p_g \equiv \Pr_g\,(x=1)$, $\bar{t}_g^1 \equiv E_g(t\,|x=1)$, $\bar{t}_g^0 \equiv E_g(t\,|x=0)$, and $\bar{t}_g \equiv E_g(t)$ under the probability distribution g. The payoff-relevant characteristics of $\tilde{\mathcal{C}}_g$ are thus summarized by p_g and $\bar{t}_g$, which allows us to express a stochastic contract in reduced form as $\tilde{\mathcal{C}}_g = [p_g, \bar{t}_g]$.

Proposition 1 (Trade failures) *Suppose Conditions V and D are violated. No deterministic fully revealing equilibrium exists. There exist stochastic fully revealing equilibria, in all of which buyer type $\theta_x \in \Theta_x$ trades with probability*

$$p_g(\theta_x,\theta_z) = \exp\left[-\int_{\theta_x}^{\overline{\theta}_x}[z(1;s,\theta_z)]^{-1}ds\right] \tag{2}$$

and the expected transfer to the seller is $\bar{t}_g(\theta_x,\theta_z) = p_g(\theta_x,\theta_z)\theta_x$.

Both success probability $p_g(\theta_x,\theta_z)$ and expected transfer $\bar{t}_g(\theta_x,\theta_z)$ are increasing in buyer type θ_x. A lower-valued buyer can credibly reveal its type by accepting a higher risk of trade failure. With less to gain, lower-valued types are less keen on trading and so bid less aggressively. Effectively, this implies a (stochastic) upward-sloping supply curve; the seller is more willing to supply the good, the higher the price.

The expected transfer equals the seller's true reservation price. So, in expectation, the buyer appropriates no part of the common value, which implies that the buyer does not signal its type by forgoing common value through its contract choice. Rather, it is through relinquishing expected *private benefits* that the buyer reveals its type when accepting a higher risk of trade failure.

The following corollary highlights the importance of private benefits for mitigating the smart buyer problem.

Corollary 1 *A reduction in θ_z decreases the trade probability of all buyer types except type $\overline{\theta}_x$.*

A marginal reduction in the private benefits of all buyer types increases the risk of trade failure for all but the highest type. Intuitively, a lower θ_z means that the buyer — or, more precisely, its capacity to derive utility from the good — is less "special," which increases the seller's suspicion that the buyer is merely after a good bargain.

Example 1 (Art). Consider the art collector setting (A0). A famous art collector approaches a young artist to buy a painting. The painting is virtually indivisible (since its artistic value would be destroyed), and the collector's hedonic pleasure from owning the painting is non-verifiable.

Under the stochastic contract

$$\tilde{\mathcal{C}}_g = \begin{cases} [1, \theta_x, 0] & \text{with probability } p_g(\theta_x, \theta_z) \\ \mathcal{C}_0 & \text{with probability } 1 - p_g(\theta_x, \theta_z) \end{cases}, \tag{3}$$

the collector commits to a mechanism that results in trade under the deterministic contract $[1, \theta_x, 0]$ with probability $p_g(\theta_x, \theta_z) \in [0, 1]$ and in no trade with probability $1 - p_g(\theta_x, \theta_z)$. By construction, $\bar{t}_g(\theta_x, \theta_z) = p_g(\theta_x, \theta_z)\theta_x$, and provided $p_g(\theta_x, \theta_z)$ satisfies (2), the stochastic contract $\tilde{\mathcal{C}}_g$ implements a stochastic fully revealing equilibrium. The collector's personal interest in the work ($z \geq 0$) facilitates trade. If its intentions were predominantly commercial ($\theta_z \to \underline{\theta}_z$), the artist would be more prone to reject low bids.

Key to signaling is that private benefits can be forgone in a way that reveals information about the common value. Stochastic contracts allow the buyer to forgo private benefits by accepting failure with a self-selected probability. As we shall see, such randomization is no longer necessary for signaling when Condition V or Condition D holds. Save for a few remarks, we henceforth abstract from stochastic contracts.

Now suppose only Condition D holds; while the good can be divided, transfers still cannot be contingent. For a buyer of type θ_x, the payoff from a deterministic contract $\mathcal{C} = [x, t, 0]$ is

$$\Pi(\mathcal{C}; \theta_x) = x\theta_x + z\,(x; \theta_x, \theta_z) - t. \tag{4}$$

Proposition 2 (Trade rationing) *Suppose only Condition D is satisfied. There exists a unique deterministic fully revealing equilibrium in which buyer type $\theta_x \in \Theta_x$ acquires quantity $x(\theta_x, \theta_z)$ at price $t_0(\theta_x, \theta_z) = x(\theta_x, \theta_z)\theta_x$, where*

$x(\theta_x, \theta_z)$ satisfies the differential equation

$$\frac{x'(\theta_x, \theta_z)}{x(\theta_x, \theta_z)} = [z_x(x(\theta_x, \theta_z); \theta_x, \theta_z)]^{-1} \tag{5}$$

and the boundary condition $x(\overline{\theta}_x, \theta_z) = 1$.

The trade quantity $x(\theta_x, \theta_z)$ and the unit price $t(\theta_x, \theta_z)/x(\theta_x, \theta_z)$ are increasing in the buyer type θ_x. The buyer signals a lower valuation by trading a smaller quantity. Quantity rationing is a means to relinquish private benefits, analogous to lowering trade probability in the stochastic separating equilibrium. The exact quantity schedule $x(\theta_x, \theta_z)$ depends on the private benefit function $z(x; \theta_x, \theta_z)$, which we illustrate using a linear example. (A logarithmic example is provided in the proof of Proposition 2.)

Again, trade is de facto characterized by an upward-sloping supply curve; the seller is willing to supply more of the good when the price is higher. In contrast, the lemons problem leads to a downward-sloping demand curve. In Duffie and DeMarzo (1999), for example, the securities market suffers illiquidity in the form of downward-sloping demand. This provides the backdrop for our next example.

Example 2 (Liquidity). Consider a simple two-period model of financial trade (A1). There are a buyer and a seller, both of whom are endowed with (zero-interest) cash to support trade. In addition, the seller is endowed with one unit of a long-term security that yields an uncertain payoff $\tilde{\theta}_x \in \Theta_x$ later at date 1, where $\Theta_x = (1, \overline{\theta}_x]$.

The seller's and the buyer's consumption utilities are

$$u(c) = c_0 + c_1 \quad \text{and} \quad v(c) = c_0 + (1 + \theta_z)c_1,$$

respectively; c_t denotes date-t consumption, and $\tilde{\theta}_z \in \{-\theta_z, \theta_z\}$ is a consumption preference shock. If $\tilde{\theta}_z = -\theta_z$, the buyer is impatient and prefers consumption at date 0. If $\tilde{\theta}_z = \theta_z$, the buyer is patient and prefers consumption at date 1. By contrast, the seller is indifferent with respect to the timing of consumption.

When $\tilde{\theta}_z = -\theta_z$, the buyer uses her wealth to consume at date 0, and there is no demand for trading the security. However, when $\tilde{\theta}_z = \theta_z$, the buyer would like to invest some of its wealth in the security to increase date 1 consumption. If both knew the realization of θ_x at date 0, the buyer would simply offer $t = \theta_x$

and would enjoy additional benefits of $z(1;\theta_x,\theta_z)=\theta_z\theta_x$. When only the buyer learns the true return θ_x, there exists a unique deterministic fully revealing equilibrium characterized by Proposition 2.

To determine the equilibrium quantity schedule, note that (5) becomes

$$\frac{x'(\theta_x,\theta_z)}{x(\theta_x,\theta_z)}=[\overline{\theta}_z\theta_x]^{-1}, \tag{6}$$

since $z(1;\theta_x,\theta_z)=\theta_z\theta_x x$ in this example. Integrating on both sides, and using $x(\overline{\theta}_x,\theta_z)=1$ to determine the integration constant, yields

$$x(\theta_x,\theta_z)=\left(\theta_x/\overline{\theta}_x\right)^{1/\overline{\theta}_z}. \tag{7}$$

Since the equilibrium per-unit price is θ_x, (7) also describes an upward-sloping supply curve. One can invert (7) to derive an equilibrium price function

$$P=\overline{\theta}_x x^{\theta_z}.$$

The slope of this function $\partial P/\partial x=\theta_z\overline{\theta}_x x^{\theta_z-1}$ reflects the price impact of a given quantity order, similar to Kyle's (1985) λ, though not a constant.

As in Corollary 1, the traded quantity $x(\theta_x,\theta_z)$ is strictly increasing in θ_z for all $\theta_x\in\Theta_x\backslash\{\overline{\theta}_x\}$. (Recall that $x(\overline{\theta}_x,\theta_z)=1$ irrespective of θ_z.) When non-informational trade motives become less important ($\theta_z\to 0$), the seller becomes more reluctant to trade which translates here into less liquidity: trade quantity decreases, and price impact increases.

3.2 Upside participation

Now suppose only Condition V holds; while the good cannot be divided, transfers can be made contingent on $v(\cdot)$. Feasible deterministic contracts now take the form $\mathcal{C}=[1,t_0,\tau(v)]$. We first consider a simple category of contingent transfers: linear sharing rules where $\tau(v)=(1-\alpha)v$. Further below, we allow for more general contracts.

For a buyer of type θ_x, the payoff from a revenue sharing contract is

$$\Pi(\mathcal{C};\theta_x)=\alpha[\theta_x+z\left(1;\theta_x,\theta_z\right)]-t_0. \tag{8}$$

It is straightforward to see that (8) is isomorphic to (1), and Proposition 1 therefore applies.

Proposition 3 (Revenue sharing) *Suppose only Condition V is satisfied and contingent transfers are restricted to linear sharing rules. There exists a unique deterministic fully revealing equilibrium in which buyer type* $\theta_x \in \Theta_x$ *acquires the good in exchange for a fixed transfer* $t_0(\theta_x, \theta_z) = \alpha(\theta_x, \theta_z)\theta_x$ *and a fraction* $1 - \alpha(\theta_x, \theta_z)$ *of the buyer's total revenues, where*

$$\alpha(\theta_x, \theta_z) = \exp\left[-\int_{\theta_x}^{\bar{\theta}_x} [z(1; s, \theta_z)]^{-1} ds\right]. \tag{9}$$

When the good has a low(er) common value, the buyer offers a large(r) fraction of revenues. The intuition behind the inverse relationship is that the buyer signals a low common value with granting the seller upside participation. Buyers do not want to mimic lower-valued types because the gains from paying a lower price are (more than) offset by the cost of conceding more revenues. Conversely, overstating the value of the good is not profitable since the gains from a larger share of revenues do not compensate for the higher cash price.

Note that the buyer relinquishes exactly the same amount of (expected) private benefits as in the stochastic fully revealing equilibrium (Proposition 1), yet through revenue sharing rather than through trade failure. The above equilibrium is efficient (since trade always occurs), and Pareto-dominates the stochastic fully revealing equilibrium (since the seller is strictly better off).

Example 3 (Royalties). Consider the setting of a patent sale (A3). A company needs a scientist's invention to develop a new product cycle. Let θ_z reflect the size of the company's product market, whereas θ_x reflects the market valuation of (products based on) the patent. Concerned that the company is better informed about market valuations, the scientist is reluctant to enter into an outright sale. Aware of these concerns, the company offers a contract that combines a fixed payment of $t_0(\theta_x, \theta_z) = \alpha(\theta_x, \theta_z)\theta_x$ with royalties that give the scientist a share $1 - \alpha(\theta_x, \theta_z)$ of the revenues from the final product.

Intuitively, if the company were to offer only a low fixed price, the trade might fail (as Example 1). Implicitly, the scientist's reluctance to sell is a request for a higher price. If the patent value — or, more precisely, θ_x — is, however, indeed low, the company is unwilling to increase the fixed price. Instead, it can concede a share of the revenues to placate the scientist's fear of being short-changed. Like the lemons problem (cf. Gallini and Wright, 1990), the smart buyer problem thus provides a rationale for royalties; the difference being that, in the smart buyer problem, they are a means to reassure the seller, as opposed to the buyer.

Cash-equity offers in corporate control transfers are another example. Again, our explanation based on the seller's fear of underpricing stands out against the extant literature (Hansen, 1987; Eckbo et al., 1990).[7] An exception is Berkovitch and Narayanan (1990), where the seller's outside option is to wait for a competing bid, the value of which depends on the initial buyer's privately observed quality relative to potential competitors. Our smart buyer framework parsimoniously subsumes their setting (in reduced form). Further corporate finance examples include venture capital investments (cf. Garmaise, 2007), equity issues (discussed in Section 4), and our next example of bankruptcy restructuring.

Example 4 (Debt-equity swap). Consider an owner-managed firm in financial distress (A4). Everyone agrees that continuation is efficient, and the owner-manager is willing to inject fresh capital in exchange for partial debt forgiveness. Such a transaction amounts to "buying" back control from the creditors.

The problem is that the owner-manager is in a better position to assess both the firm's liquidation value θ_x and its continuation value $v(1;\theta_x,\theta_z)$. This creates disagreement: On one hand, the creditors question the low liquidation value estimates. On the other hand, the owner-manager deems creditor demands too high. One solution is to "settle" the debt not only in cash but also in equity, whereby creditors benefit from a cash infusion $t_0(\theta_x,\theta_z)$ *and* receive a $1-\alpha(\theta_x,\theta_z)$ equity stake in the restructured firm.

A standard explanation for the use of debt-equity-swaps in financial distress is debt overhang. The current example shows that smart buyer problems provide an alternative explanation for debt-equity-swaps. In fact, while debt overhang problems can be resolved by means of debt forgiveness for *cash only*, this is not true for smart buyer problems.[8]

[7] Burkart and Lee (2010) study tender offers, in which the bidder is better informed about the post-takeover share value improvement which is a private value component. However, dispersed target shareholders free-ride and therefore behave as if they are entitled to this private value. That is, they treat the share value improvement de facto as a common value, thereby inducing a smart buyer problem.

[8] The restructuring of Marvel in the mid-1990s provides anecdotal evidence of smart buyer problems in bankruptcy (Harvard Business School Case No: N9-298-059). Ron Perelman,

We now relax the restrictions of linear sharing rules and let the buyer choose the *form* of contingent transfers $\tau(\cdot)$ that maximizes its payoff. More precisely, a buyer of type θ_x chooses $\mathcal{C} = [1, t_0, \tau(v)]$ to maximize

$$\Pi(\mathcal{C}; \theta_x) = \theta_x + z\,(x; \theta_x, \theta_z) - t_0 - E[\tau(v)\,|\theta_x]. \tag{10}$$

Given that θ_z is commonly known, the buyer knows exactly how large the contingent payment will be. In fact, $E[\tau(v)\,|\theta_x] = \tau[v(1; \theta_x, \theta_z)]$. This simplifies the optimal contracting problem greatly.

Proposition 4 (Security design) *Suppose Condition V is satisfied, and contingent transfers are unrestricted. There exists a deterministic fully revealing equilibrium in which buyer type θ_x acquires the good in exchange for a fixed transfer $t_0(\theta_x, \theta_z) = \theta_x$ and a contingent transfer*

$$\tau(v) = \begin{cases} 0 & \text{if } v \leq v(1; \theta_x, \theta_z) \\ v - v(1; \theta_x, \theta_z) & \text{if } v > v(1; \theta_x, \theta_z) \end{cases}, \tag{11}$$

thereby retaining the entire trade surplus, as under symmetric information.

For given θ_z and $x = 1$, the buyer's total valuation v is a one-to-one mapping from Θ_x to $[v(1; \underline{\theta}_x, \theta_z), v(1; \overline{\theta}_x, \theta_z)]$. Moreover, v is strictly increasing in θ_x. Consequently, one can use a simple scheme to punish the buyer for understating θ_x: Let the buyer's reported type be $\hat{\theta}_x$. Then, the buyer incurs a penalty if and only if verification reveals that the true v is larger than $v(1; \hat{\theta}_x, \theta_z)$. The penalty specified in Proposition 4 satisfies limited liability ($\tau \leq v - t_0$); it requires the buyer to pay the seller the difference between actual total valuation and valuation implied by $\hat{\theta}_x$. This amounts to a call option that allows the seller to extract more from the buyer when v is high. Thus, the penalty scheme grants the seller upside participation.[9]

as debtor-in-possession, plays the role of a smart buyer. His attempt to regain control of the financially distressed Marvel company at too low a price is eventually thwarted by the appearance of another smart player, Carl Icahn.

[9] The penalty need neither have this specific form nor be payable to the buyer to ensure incentive compatibility. Also, a similar penalty scheme is effective even when θ_z is unobserved by either party. Under Assumption 1, the support of v would differ across buyer types. In fact, the maximum of the support $\overline{v}(\theta_x, \theta_z)$ would be strictly increasing in θ_x. But even if the support of v were identical for all types, an incentive-compatible penalty scheme exists if the distributions $\{f_{\theta_x}(v)\}_{\theta_x \in \Theta_x}$ satisfy the monotone likelihood property (Burkart and Lee,

Example 5 (Earnout). Consider a private equity buyout (A5). A well-known buyout firm wants to acquire a small private firm. The buyout managers present low current valuations for the target (low estimates of θ_x) and argue that post-takeover improvements will primarily result from their managerial skill (θ_z). The current owners take a hard bargaining stance, arguing that the buyout firm purposely undervalues the target. Yet, the buyout firm refuses to increase its cash bid $t = \theta_x$.

To persuade the current owners, the buyout firm includes a so-called "earnout" clause, whereby it must pay $v - v(1; \theta_x, \theta_z)$ (only) if the target's post-takeover market value v exceeds $v(1; \theta_x, \theta_z)$. In practice, earnout clauses specify supplementary payments when the target's operational or financial performance (for example, revenue, earnings, or stock price) exceeds pre-determined threshold levels within a given time period after the takeover. According to practitioners, earnout clauses allow the buyer "to pay a lesser guaranteed amount to the sellers of the target business on the closing date" (Gallant and Ross, 2009).

Interestingly, earnouts can also be motivated on the grounds that the target owners have private information (Datar et al., 2001). Put differently, as in the case of royalties, it is a priori unclear whether a given earnout clause is a manifestation of the lemons problem or the smart buyer problem.

The next example applies the model to the "purchase" of human capital.

Example 6 ("20-against-20"). Consider an example of "hiring talent" (in the spirit of A8). A film studio wants an actress for a lead role in a new movie. The studio is better informed about industry factors that determine the actress' latent outside options (θ_x), and it can better estimate the movie's box office potential ($v(1; \theta_x, \theta_z)$). A producer and a director are already signed up, both well-known and experienced (high θ_z).

The actress bargains for a high salary (t'_0), otherwise reluctant to commit to the project in hopes of better options. Finding her demands too high ($t'_0 > \theta_x$), the studio pays the larger of a cash salary t_0 and a fraction of the revenues αv; $\max\{t_0, \alpha v\}$. In essence, this compensation package amounts to a fixed salary supplemented by a fraction of revenues, *provided that the revenues exceed a certain threshold.*

2010). The basic principle remains punishing the buyer for understating θ_x by granting the seller upside participation in v.

Such convex salaries exist in the film industry. One better-known example is the so-called "20-against-20" contract, whereby a movie star effectively gets the larger of \$20 million and 20 percent of the movie's gross revenues.[10] This creates upside participation; the payoff is flat until the revenues reach \$100 million but thereafter increases linearly with further revenues. Weinstein (1998) argues that one explanation for such contracts is that the studio is better informed than the star, but acknowledges that such contracts are also consistent with the information asymmetry being the opposite.[11]

Propositions 3 and 4 illustrate the two effects that the verifiability of v (Condition V) and the use of contingent transfers have on the solution to the smart buyer problem. First, trade becomes efficient, since revenue sharing replaces more wasteful means of relinquishing private benefits, such as rationing. Second, the buyer appropriates more of the trade surplus in the absence of restrictions on the contract form, since security design enhances the buyer's ability to commit to truthful behavior.

A noteworthy proviso is that both results rely on the (implicit) assumption that contingent transfers do not affect the trade surplus. This may be debatable in some applications.[12] Earnout clauses, for example, can dampen the acquirer's incentives to increase the target's post-takeover value, creating tension between signaling and incentive provision.

4 Fooled buyers or short-changed sellers?

4.1 Observationally equivalent contracts

The smart buyer problem presumes that the buyer knows more than the seller, and that this knowledge pertains to the seller's outside option. By contrast, the lemons problem presumes that the seller has superior knowledge of the buyer's

[10] For example, Tom Cruise signed a 20-against-20 contract for *Valkyrie*.

[11] Goetzmann et al. (2007) study pricing and contracts in sales of screenplays. In our view, their evidence is consistent with the possibility that film studios know more than inexperienced screenwriters. In particular, they show that experienced screenwriters more often receive fixed payments, and that studios forecast box office success well. Further, the motivating example on page 8 – studios offer contingent contracts when less optimistic – and the cross-sectional evidence – better scripts coincide with higher prices and less contingent payments – are more consistent with the smart buyer problem than with the lemons problem (cf. Section 4.2 below).

[12] Also, revenue-sharing may not implement efficient trade in the presence of exogenous transaction costs, such as administrative expenses.

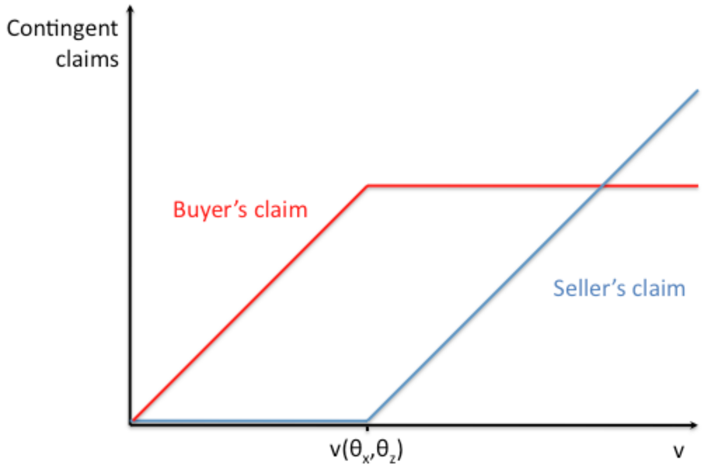

Figure 1: The graph illustrates the security design solution (Proposition 4) under limited liability. It plots the value of contingent claims granted to the buyer and the seller (vertical axis) as functions of total realized value v (horizontal axis). The seller receives equity (blue line), and the buyer receives debt (red line).

inside option. These opposite points of departure become manifest in the signaling incentives. In the lemons problem, the informed party wants to convey a *high* value, whereas it wants to convey a *low* value in the smart buyer problem.

This difference also distinguishes the rationales behind the solutions. Signaling a high value calls for *downside protection*, whereby the uninformed party is recompensed if expectations are ex post not met. Signaling a low value calls for *upside participation*, whereby the uninformed party is recompensed if expectations are ex post surpassed. This is most evident in the security design solution with $\tau(v) = \max\{0, v - v(1; \theta_x, \theta_z)\}$ (Proposition 4). As v increases from zero, the payoff from this claim is flat until $v = v(1; \theta_x, \theta_z)$ and then increases linearly. The convexity is designed to give the uninformed seller upside participation. The buyer's payoff is accordingly concave, increasing linearly until $v = v(1; \theta_x, \theta_z)$ and flat thereafter.

As Figure 1 illustrates, the above claims represent standard securities, straight debt for the buyer and equity for the seller. This is compelling in that the same claim structure is optimal in security design models with better-informed sellers (Duffie and DeMarzo, 1999). That is, optimal contracts in lemons problems and optimal contracts in smart buyer problems can be *observationally equivalent*. The conventional view, starting with Myers and Majluf (1984) links debt issuance to private information of the issuer: In the typical pecking order model, debt best protects less uninformed investors from buying overvalued securities. As our analysis shows, debt is also optimal when the issuer faces better-informed investors. It best protects the issuer from selling undervalued securities. Thus, empirical tests built on the presumption that debt issuance is more likely (only) when the issuer has better information than the market may capture only half the picture.[13]

The above also explains why other contractual provisions, such as royalties, earnouts, and "20-against-20," can be ascribed to either of the two opposite information asymmetries (see Examples 3, 5, and 6). The patent example provides a lucid illustration. If the company is concerned about the quality of the patent, the scientist can signal high quality by *accepting* royalties, that is, through its willingness to share "losses" in case the product is a flop (downside protection). Conversely, if the scientist is wary of the terms of trade, the company can alleviate such concerns by *conceding* royalties, that is, through its willingness to share "profits" in case of success (upside participation). In either case, the scientist receives royalties. Analogously, cash-equity bids in takeovers can reflect the seller's willingness to accept equity as a signal that the target is not a lemon or the buyer's willingness to concede equity as a signal that the offer is adequate (we revisit this example further below).

This coincidence can be attributed to the fact that, as the mode of signaling switches between downside protection and upside participation, so does the identity of the informed principal. The eventual claim structure is the same regardless of whether the seller grants the buyer the downside protection or the buyer grants the seller the upside participation. The more fundamental reason is that both the lemons problem and the smart buyer problem can be reduced to asymmetric information about a common value component. Crucially, conditions for incentive compatibility and separation depend on neither the identity of the informed party nor whether it wants to overstate or understate the value. Conditional on separation, the informed principal's identity affects only the surplus division (between buyer and seller and across common value types).[14]

[13] Chen et al. (2007) provide empirical evidence that investors may sometimes know more about a firm's fundamentals than its managers. See fn. 16 for papers that formalize this idea in the context of primary capital markets.

[14] Similarly, neither do the conditions for uninformative equilibria impose contractual differences between the two settings. Uninformative equilibria in the lemons problem and the

4.2 Opposite cross-sectional predictions

Because of identical contractual solutions, real-world contracts are unlikely to be sufficient to identify empirically the underlying information problem. One may have to look beyond the contract shape and take into account the division of surplus. For example, signaling costs are borne by the seller in the lemons problem, whereas they are borne by the buyer in the smart buyer problem. Hence, the identity of the party who is willing to pay for third-party verification, such as due diligence or fairness opinions, differs. However, in practice, it is difficult to attribute such expenses to one or the other party, because they may be laid out by one party but accounted for in the transaction price.

Alternatively, one can study how contracts relate to (revealed) common value, which reflects the distribution of rents across common value types. This relation changes with the identity of the informed party. Let us extend the financial trade application (Examples 2 and 5) by giving the informed party an endowment that it wants to *sell* in case of impatience, thereby introducing a lemons problem. Figure 2a depicts the relation between trade quantity x, which is the signaling instrument in this setting, and common value θ_x. It is positive when the informed party wants to buy (green line) but negative when it wants to sell (grey line).[15] Identifying trade "direction" is therefore important, as commonly done in market microstructure research (e.g., Lee and Ready, 1991).

Accounting for trade direction also matters in other settings. Figure 2b illustrates this point for linear sharing rules, where the seller's equity stake α is the signaling instrument. In smart buyer problems, α and θ_x are inversely related (green line); sellers *receive* more equity when common values are lower. In lemons problems, they are positively related (grey line); sellers *retain* more equity when common values are higher. For instance, consider a firm that wants to issue equity. Leland and Pyle (1977) presume that the issuer is better informed than the investors, who are therefore unwilling to pay a high(er) price unless the issuer retains a large(r) stake. However, it can also be that, for example, institutional investors know more than the issuer,[16] who therefore wants to retain

smart buyer problem differ only in whether high-valued types or low-valued types are shut out of the market. In a sense, buying something "bad" suffers from the same difficulties as selling something "good."

[15] The derivation of the lines in Figure 2a is available from the authors upon request.

[16] Indeed, Benveniste and Spindt (1989) argue that IPO bookbuilding processes are a way for underwriters to elicit information from sophisticated investors. Subrahmanyam and Titman

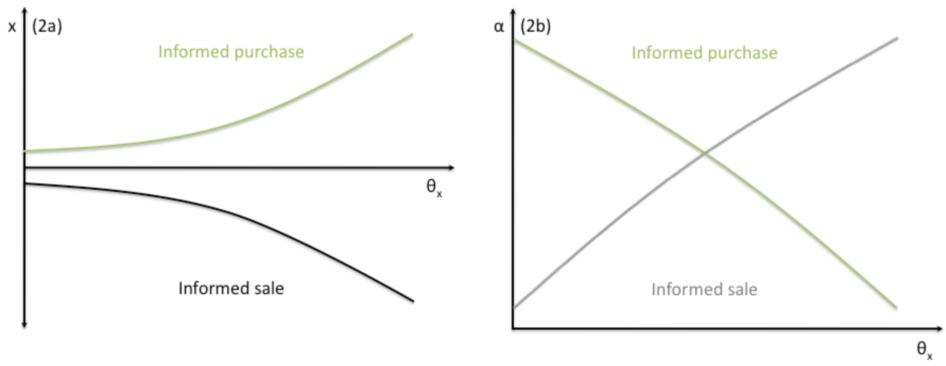

Figure 2: The two graphs illustrate that the smart buyer problem (informed purchase) and the lemon's problem (informed sale) can make opposite predictions about the relationship between the underlying common value and the signaling instrument. The left graph shows this for trade rationing, while the right graph shows this for linear sharing rules.

a large(r) stake if investors insist on a low(er) price. In the former case, the uninformed investors adapt price to quantity, paying less for more; in the latter case, the uninformed issuer adapts quantity to price, selling less for less. Thus, the correlations between quantity and price are the opposite.

Analogously, consider a bidder that wants to acquire a target (firm). If current owners) are better informed than the bidder, the latter is unwilling to pay a high(er) price unless the former are willing to accept a large(r) part of the consideration in post-takeover equity. Conversely, if current (dispersed) shareholders are less informed than the bidder, the latter must pay either a high(er) price or offer a large(r) part of the consideration in post-takeover equity. Similar to above, the correlations between equity consideration and (total) price are the opposite. Confounding, or incorrectly classifying, the lemons problem and the smart buyer problem can hence lead to erroneous conclusions. Suppose a takeover study searches for evidence of the lemons problem, but finds takeover premia (or post-takeover performance) not to be decreasing in the share of equity consideration. Yet, this does not warrant the conclusion that asymmetric information is negligible for the choice of consideration. In fact, the evidence could be consistent with the smart buyer problem; or the average effect could be weak because both information asymmetries, with their countervailing effects,

(1999), Axelson (2007), Lyandres et al. (forthcoming) also presume primary equity markets in which managers want to extract information from the investors.

are present in the data. Incidentally, the existing evidence on contractual signaling in external financing, or in mergers and acquisitions, is rather mixed. The above suggests one possible reason why past empirical studies may have been inconclusive.

4.3 Classification by way of intermediary contracts

In some markets, none of the aforementioned methods may help discriminate lemons problems from smart buyer problems. In case of the art collector (Example 1), one would have to observe, possibly counterfactual, unsuccessful offers to determine the underlying problem from data. Fortunately, in many such markets, information frictions are mitigated by expert intermediaries, whose role is to merely buy and resell goods. The role of such intermediaries is not entirely obvious. How can the presence of another, say, smart buyer resolve an uninformed seller's fear of being short-changed?

We argue that this is indeed possible, and that intermediary contracts can help infer the underlying information problem. In fact, the solution is rather simple. Suppose a person wants to sell an inherited antique but lacks expertise to assess its value (θ_x). To evade bargain hunters, the seller enters into a contract with a specialized antiques dealer: The dealer buys the antique for θ_x and makes supplementary payments if the antique is resold for more than $\theta_x + \varepsilon$. Since the dealer is smart, the ultimate buyer will not be able to buy the antique for less than θ_x. The increment ε can be interpreted as a dealer commission. In fact, a similar deal can be implemented through a percentage commission.

This example illustrates that informed intermediation helps for two reasons. First, the intermediary may not buy the good for own consumption. Otherwise, the seller is back to the original problem. Second, the resale generates *verifiable* information about θ_x. This allows for contracts that resemble the security design solution or revenue sharing, thereby obviating inefficient signaling through trade failures. In practice, such *informed* intermediation is provided by agents (for performing artists), galleries (for visual artists), market-makers (in stock exchanges), or underwriters (in capital markets). It would be interesting to explore these ideas on intermediation more thoroughly, but this is beyond the scope of this paper.

Relevant for the present discussion is merely the fact that the intermediary may enter into different contracts with the informed and the uninformed party. In a smart buyer problem, the intermediary would enter into a contingent contract with the seller, such as a commission, while engaging in a simple cash transaction with the buyer. By contrast, in a lemons problem, the intermediary would enter into a contingent contract with the buyer, such as a warranty, while engaging in a simple cash transaction with the seller. This asymmetry in the way the intermediary interacts with both sides of the market may shed light on the underlying information problem.

5 Standardized vs. customized offers

In this section, we introduce pooling offers. We first establish equilibrium existence under the restriction that the buyer can make only pooling offers. Next, we derive Perfect Bayesian equilibria in the absence of restrictions on the offer form, and turn to the issue of equilibrium selection. Finally, we study how the equilibrium outcome changes when the buyer has additional private information about the private benefits.

The results shed light on the prevalence of standardized contracts (uninformative offers) and customized contracts (partially or fully revealing offers). On one hand, the buyer prefers standardized contracts when the seller's outside option is *on average* low but customized contracts otherwise. On the other hand, additional private information about private benefits diminishes the buyer's signaling ability, and hence forces standardization. In fact, some signaling devices, such as revenue-sharing and security design, become completely ineffective. We develop these insights in settings where they are most clear-cut, and comment on how they extend to other settings.

5.1 Uninformative offers under trade rationing

A pooling offer is a contract menu $\mathfrak{C}$ containing *inter alia* $\mathcal{C}_\emptyset$ and some contract $\mathcal{C}_P \neq \mathcal{C}_\emptyset$ such that, if the menu is accepted, the buyer selects either $\mathcal{C}_\emptyset$ or $\mathcal{C}_P$ for all $\theta \in \Theta$. It is therefore without loss of generality to focus on contract menus with only two elements. In an uninformative equilibrium, every type $\theta \in \Theta$ submits the same pooling offer $\mathfrak{C}_P = \{\mathcal{C}_P, \mathcal{C}_\emptyset\}$ and selects the same contract from this menu. In a partially revealing equilibrium, offers may but need not contain more than two elements. The defining feature is that not all buyer types choose the same contract although some contracts are chosen by more than one type. For example, an equilibrium in which all types submit the same offer $\mathfrak{C}_P = \{\mathcal{C}_P, \mathcal{C}_\emptyset\}$ and both contracts are sometimes chosen is partially revealing.

Since the security design solution in Proposition 4 implements the first-best outcome, we must exclude it to create a role for pooling offers. In this subsection, we focus on trade rationing contracts; only Condition D is satisfied, and contracts take the form $\mathcal{C} = [x, t, 0]$. In this setting, an equilibrium is *efficient* if all buyer types trade the full quantity $x = 1$. Note that efficient equilibria in this setting must be uninformative.

A pooling equilibrium offer must meet both parties' participation constraints. As regards individual contracts in the menu, $\mathcal{C}_\emptyset$ trivially satisfies this condition. By contrast, $\mathcal{C}_P$ meets the buyer's participation constraint only when the latter's type $\theta \in \Theta$ satisfies

$$x_P\theta_x + z(x_P; \theta_x, \theta_z) \geq t_P. \tag{12}$$

For a given $\mathcal{C}_P$, let Θ^P denote the subset of buyer types for whom (12) holds.

Similarly, $\mathcal{C}_P$ satisfies the seller's participation constraint if and only if

$$t_P \geq x_P E\left[\theta_x \middle| \mathfrak{C}\right]. \tag{13}$$

In words, the seller must deem the fixed transfer larger than the forgone common value, given beliefs that are conditional on the observed pooling offer.

Two considerations determine how the expectations in (13) are formed. First, the seller must conjecture what subset of Θ would make such an offer $\mathfrak{C}_P = \{\mathcal{C}_P, \mathcal{C}_\emptyset\}$. Let Θ_1^P denote this subset. Second, the seller must infer what subset of Θ_1^P prefers $\mathcal{C}_P$ over $\mathcal{C}_\emptyset$. Let Θ_2^P denote this subset. Suppose the seller conjectures $\Theta_1^P = \Theta$, that is, it believes that all types $\theta \in \Theta$ make the observed offer (as must be true in an uninformative equilibrium). Then, $\Theta_2^P = \Theta^P$ and (13) can be written as

$$t_P \geq x_P E\left[\theta_x \middle| \theta \in \Theta^P\right]. \tag{14}$$

An uninformative equilibrium offer $\mathfrak{C}_P$ must satisfy (14) and further requires out-of-equilibrium beliefs that prevent deviations from $\mathfrak{C}_P$.

Lemma 1 *Suppose only Condition D is satisfied and the buyer is restricted to pooling offers. There exist multiple equilibria, among which there is an efficient* *equilibrium (only) when* $\underline{\theta}_x + z(1; \underline{\theta}_x, \theta_z) \geq E(\theta_x)$.

These equilibria have the same structure as in Shleifer and Vishny (1986). There is a unique cut-off type $\theta_x^c \in \Theta_x$ such that $\Theta^P = \left[\theta_x^c, \overline{\theta}_x\right]$; only buyer types $\theta_x \geq \theta_x^c$ prefer $\mathcal{C}_P$ over $\mathcal{C}_\emptyset$, that is, engage in actual trade. The pooling price at least matches the *average* common value that the seller forgoes, $P_P \equiv t_P/x_P \geq E(\theta_x | \theta_x \geq \theta_x^c)$. By the same token, the price typically does not equal the buyer's *true* common value. Indeed, all types $\theta_x \in [\theta_x^c, E(\theta_x | \theta_x \geq \theta_x^c))$ overpay, while all types $\theta_x \in (E(\theta_x | \theta_x \geq \theta_x^c), \overline{\theta}_x]$ underpay in equilibrium. All types $\theta_x \in [\underline{\theta}_x, \theta_x^c)$ choose $\mathcal{C}_\emptyset$, that is, refrain from trade because the overpricing is so severe that their participation constraint is violated.

5.2 Equilibrium selection

We now allow the buyer to freely choose the offer form. First, note that the separating outcome of Proposition 2 remains an equilibrium outcome even when the buyer is no longer restricted to separating offers. Since the seller's participation constraint is *binding* for every type under the separating offer, any alternative offer that some types find more attractive, if attributed to the highest of those types, is unacceptable to the seller. Thus, there always exists a fully revealing equilibrium, in which all types submit the separating offer from Proposition 2.

By contrast, uninformative equilibria do not always exist. Since (a deviation to) the above separating offer is always acceptable to the seller, uninformative equilibria cannot exist unless *every* buyer type weakly prefers some uninformative offer over the separating offer. This is only the case when the seller's average outside option, $E(\theta_x)$, is so low as to warrant a sufficiently low pooling price. Indeed, skewing the probability distribution toward type $\underline{\theta}_x$ raises every type's pooling payoff but leaves their separating payoff unchanged.

Finally, there can be partially revealing equilibria. For example, given some pooling offer $\mathcal{C}_P$, let $\Theta_x^+(\mathcal{C}_P) \subset \Theta_x^+$ denote the subset of buyer types that prefers $\mathcal{C}_P$ over the separating offer. If $\mathcal{C}_P$ is acceptable to the seller under the premise that it is made by all types in $\Theta_x^+(\mathcal{C}_P)$, there is an equilibrium in which all types in $\Theta_x^+(\mathcal{C}_P)$ submit $\mathcal{C}_P$ and all other types submit the separating offer.

Lemma 2 *Suppose only Condition D is satisfied. In the absence of restrictions on the offer form,*

- *There always exists one fully revealing equilibrium.*
- *There is a unique threshold value $\underline{\mu}_x \in (\underline{\theta}_x, \overline{\theta}_x)$ such that an uninformative equilibrium exists if and only if $E(\theta_x) \leq \underline{\mu}_x$.*
- *A partially revealing equilibrium exists if and only if there exists a pooling offer $\mathcal{C}_P = [x_P, t_P, 0]$ such that $t_P \geq x_P E\left[\theta_x \middle| \theta_x \in \Theta_x^+(\mathcal{C}_P)\right]$.*

As is common in signaling games, the equilibrium can but need not be unique. The fully revealing equilibrium always exists. Hence, any unique equilibrium is fully revealing. In contrast, partially revealing or uninformative equilibria exist only under certain conditions. The proclivity for pooling increases as the seller's average outside option decreases, that is, as pooling becomes more lucrative (less expensive) for high (low) types. Indeed, all buyer types are weakly better off in partially revealing or uninformative equilibria than in the separating equilibrium; else, some type would deviate to the separating offer, which invariably succeeds. In the limit, as $E(\theta_x) \to \underline{\theta}_x$, every buyer type prefers the efficient (uninformative) equilibrium over any other equilibrium.

An appealing conjecture is that, under such conditions, standard refinement criteria select the efficient uninformative equilibrium. Yet, this is not the case. On the contrary, Cho and Kreps (1987)'s intuitive criterion uniquely selects the fully revealing equilibrium, even though it is every type's least preferred.

Lemma 3 *Only the fully revealing equilibrium survives the intuitive criterion.*

Under the intuitive criterion, there is always some, however small, deviation that causes pooling offers to collapse — unless deviations are somehow bounded. For example, suppose the buyer must purchase a minimum quantity. In this case, uninformative equilibria, in which all types buy the minimum quantity, survive the intuitive criterion. Moreover, the cheapest among these is the only equilibrium that survives Grossman and Perry's (1986) credible beliefs criterion.[17]

Proposition 5 *Suppose the buyer must purchase some minimum quantity $\underline{x}$. There is always some $\underline{x} \in (x(\underline{\theta}_x, \theta_z), 1)$ such that, when $E(\theta_x)$ is sufficiently low, there exists an uninformative equilibrium with $\mathcal{C}_P = [\underline{x}, \underline{x}E(\theta_x), 0]$. If existent, it is also the unique Perfect Sequential Equilibrium.*

[17] Riley (2001) shares our affinity for the credible beliefs criterion: To him, "the simplest way out of the maze [of refinement criteria] is to employ the stronger Grossman-Perry approach and then seek conditions that are sufficient to ensure existence" (p. 451).

Unlike the intuitive criterion, the credible beliefs criterion requires the seller to attribute a deviation to *all* types that would like the deviation to succeed. In particular, it forces the seller to acknowledge that all buyer types prefer an uninformative equilibrium, if this is the case.

Proposition 5 speaks to the relative merits of the contrasting analyses of takeovers under asymmetric information by Shleifer and Vishny (1986) and Hirshleifer and Titman (1990); the former's (latter's) focus on pooling (separating) offers seems appropriate when the expected post-takeover value improvement is low (high). An analogous argument can be used to explain different forms of illiquidity.

Example 7 (Impact vs. spread). Let us introduce a minimum quantity $\underline{x}$ into the financial trade example (Example 2). In the fully revealing equilibrium, type θ_x acquires quantity $x(\theta_x, \theta_z) = \left(\theta_x/\overline{\theta}_x\right)^{1/\overline{\theta}_z}$ of the good in exchange for a fixed payment of $t(\theta_x, \theta_z) = x(\theta_x, \theta_z)\theta_x$, *provided that* $x(\theta_x, \theta_z) \geq \underline{x}$. All types for which $x(\theta_x, \theta_z) < \underline{x}$ are shut out of the market. Conditional on $x(\theta_x, \theta_z) \geq \underline{x}$, the buyer's payoff is

$$z\left(x(\theta_x, \theta_z); \theta_x, \theta_z\right) = \left(\theta_x/\overline{\theta}_x\right)^{1/\overline{\theta}_z} \theta_x \theta_z.$$

There exists an uninformative equilibrium with $\mathcal{C}_P = [\underline{x}, \underline{x}E(\theta_x), 0]$ if the offer satisfies every type's participation constraint and, furthermore,

$$\underline{x}\left[\theta_x + \underline{x}\theta_x\theta_z - E(\theta_x)\right] \geq \left(\theta_x/\overline{\theta}_x\right)^{1/\overline{\theta}_z} \theta_x \theta_z$$

for all types that can resort to a profitable separating offer. For $\underline{x} \to 1$ and $E(\theta_x) \to \underline{\theta}_x$, the inequality converges to $\theta_x - \underline{\theta}_x + \theta_x\theta_z \geq \left(\theta_x/\overline{\theta}_x\right)^{1/\overline{\theta}_z} \theta_x\theta_z$, which is satisfied for any $\theta_x \in \Theta_x$. Thus, the uninformative equilibrium exists if $\underline{x}$ is sufficiently high *and* $E(\theta_x)$ is sufficiently low. If so, it is the only equilibrium that survives the credible beliefs criterion.

In the separating equilibrium, there is price impact; as shown previously, the price function, $P = \overline{\theta}_x x^{\theta_z}$, is increasing in the order size x. By contrast, in the uninformative equilibrium, both price and order size are "standardized." Proposition 5 suggests that (a) security trades where the seller's fear of being short-changed is large exhibit significant variance in order size and price, and illiquidity primarily takes the form of price impact; whereas (b) security trades where this fear is small exhibit less variance in order size and price, and illiquidity is manifest mostly in spreads. As regards spreads, it is simple to extend the model such that a buyer's *arrival* raises seller expectations about θ_x. For example, when $\Theta_x = [-\overline{\theta}_x, \overline{\theta}_x]$, mere buy interest causes seller expectations to jump from $E(\theta_x)$ to $E(\theta_x \,|\theta_x \geq 0)$, creating an "ask spread."[18]

The formal results above apply to trade rationing. Similar results hold in the case of stochastic offers or linear sharing rules. While there can be uninformative equilibria in either case, they do not survive the intuitive criterion unless there is some friction (such as the minimum trade quantity in the case of trade rationing). For example, the efficient uninformative equilibrium, which exists for sufficiently low $E(\theta_x)$, uniquely survives the credible beliefs criterion when there is, however small, a fixed cost of writing financial contracts or enforcing contingent claims. This has a simple interpretation in the context of "hiring talent." Consider linear sharing rules in the film studio example (Example 6). An analogue of Proposition 5 would suggest that actors with high average outside options ("stars") receive customized contracts, combining cash and revenue shares, whereas actors with low average outside options ("extras") are paid a standardized cash salary.

Proposition 5 points to another difference between the smart buyer problem and the lemons problem. Applied to the latter problem, similar arguments would imply that the uninformative equilibrium or, put differently, standardized offers prevails when the expected common value is sufficiently high rather than low. This mirrors the cross-sectional differences discussed already in Section 4.

5.3 Inscrutable private information

We now turn to the case of two-dimensional private information, in which the buyer observes both θ_x and θ_z but the seller observes neither. First, we note that the proof of Lemma 1 is still valid, except that the condition for the existence of efficient uninformative equilibria becomes $\underline{\theta}_x + z(1; \underline{\theta}_x, \underline{\theta}_z) \geq E(\theta_x)$. At the same time, one expects the additional private information about θ_z to make separating offers more difficult. The reasons is that all aforementioned signaling devices — stochastic offers, trade rationing, revenue sharing, and security design — exploit the relation between θ_x and z. Being less informed about θ_z, and therefore

[18] One can integrate the lemon's problem into this simple setup, in which case one obtains a bid-ask-spread as in Glosten and Milgrom (1985). See Figure 2a in Section 4.

about this relation, impairs the seller's ability to infer the common value from the private benefits.

To illustrate this point in a stark way, we demonstrate how two-dimensional private information affects the security design solution with linear sharing rules.

Proposition 6 *Suppose only Condition V is satisfied and contingent transfers are unrestricted. If the buyer has private information about both θ_x and θ_z, there exists no equilibrium in which θ_x is fully revealed to the seller.*

It is instructive to consider why the specific signaling mechanisms in Propositions 3 and 4 collapse. Linear sharing rules enable the buyer to signal its type by relinquishing a particular fraction of $v = \theta_x + z$. The willingness to do so is informative because there is symmetric information about $\partial z/\partial\theta_z$. Private information about $\partial z/\partial\theta_z$ on part of the buyer obstructs the seller's inference: It is unclear to what extent the relinquished part of v is private benefits or common value.[19]

The collapse of the security design solution illustrates the problem even more starkly. With θ_z known, there is a one-to-one mapping from common value θ_x to total value $v(1;\theta_x,\theta_z)$. This allows the buyer to signal θ_x, and to offer $t = \theta_x$, by accepting a commensurate penalty in the event of $v > v(1;\theta_x,\theta_z)$. When less informed about θ_z, the seller must be wary of another type (θ'_x,θ'_z) with larger common value $\theta'_x > \theta_x$ but identical total value $v\left(1;\theta'_x,\theta'_z\right) = v\left(1;\theta_x,\theta_z\right)$. Under the above contract, type (θ'_x,θ'_z) would earn (precisely $\theta'_x - \theta_x$) more than under full information, and hence more than under a fully revealing contract. In fact, since these types are ex post indistinguishable, there is no penalty scheme that can discriminate between them based on ex post information.[20]

[19]While the same problem undermines signaling via stochastic offers, signaling via trade rationing can remain feasible. Unlike the other signaling devices, rationing quantity need not reduce common value and private benefits in the same proportions for all types since $\partial z/\partial x$ can vary across types. Due to this variation, trade rationing retains discriminatory power under two-dimensional private information. Still, full revelation can break down as for instance when $\partial v(x;\theta_x,\theta_z)/\partial x > \partial v(x;\theta'_x,\theta'_z)/\partial x$ for some $\theta_x < \theta'_x$ and $\theta_z > \theta'_z$.

[20]When v is a random variable drawn from a conditional distribution $h(v\,|\theta_x,\theta_v\,)$, differences in $h(v\,|\theta_x,\theta_v\,)$ across types with identical $E(v\,|\theta_x,\theta_z\,)$ facilitate separation. Still, if (a subset of) different types have identical $h(v\,|\theta_x,\theta_v\,)$, the above result holds. For example, this is the case when v is normal, and θ_x and θ_z only affect the conditional mean. If $h(v\,|\theta_x,\theta_v\,)$ is different for each type, types can be described by a single parameter; though, this need not recover full revelation. Within the "redefined" type space, separation may be infeasible, even though it is feasible when private information pertains only to θ_x.

Intuitively, Proposition 6 can be explained as follows. The smart buyer problem arises because the seller is less informed about the common value. The buyer can overcome this problem contractually by sharing trade surplus, provided that the seller knows how trade surplus relates to common value. Yet, this signaling mechanism breaks down when the informational disadvantage is so severe that the seller, for any given common value, cannot fathom the buyer's ability to add value; that is, communicating private information becomes more difficult for the buyer when the seller does not "understand the business."

Example 8 (Merger). Consider a takeover of a small firm by a larger industry peer (A5). The acquirer paints a bleak picture of the target's standalone future (low estimates of θ_x) but a rosy one of the potential merger synergies (high estimates of z). With the target being wary of low cash offers, the acquirer considers an offer that includes an equity stake in the merged firm.

The problem is that the post-merger value (v) also depends on the quality of the acquirer's assets (θ_z), about which the target is not well-informed. Suspicious again, the target demands a large stake, to be on the safe side. This demand in turn makes equity payments less attractive to the acquirer. The acquirer is caught in a dilemma: It needs to concede equity to overcome the smart buyer problem (private information about θ_x), but issuing equity suffers from the lemons problem (private information about θ_z).

6 Concluding remarks

Our analysis of bilateral trade frictions, and their contractual resolution, premises that the buyer is better informed about the seller's outside option. This outside option, which we posit in *reduced form*, could be the seller's (counterfactual) payoff either when retaining the good indefinitely or when seeking out alternative buyers to eventually sell the good. In the latter case, our implicit assumption is that searching for alternative buyers is costly, and that the initial buyer has private information about the costs and benefits of doing so. Clearly, a natural extension is to embed the current model into a search market, in which participants on one side of the market are informed about each other's valuations, whereas participants on the other side of the market only know their individual valuations. In such a setting, every meeting between potential trading partners results in a smart buyer problem, since one has private information about the other's *outside* option. What contracts would arise in equilibrium, and how would they depend on the (severity of the) search frictions?

Another promising avenue, touched upon in Section 4, is to explore the role of intermediaries in brokering trade. In practice, laypeople frequently employ experts as agents to negotiate trades with the other (better informed) side of the market, often motivated by the fear of otherwise being shortchanged. Conversely, better informed parties sometimes use "front men" to trade on their behalf in order to avoid suspicion. This use of third parties by both buyers and sellers has possibly interesting implications for market structure, intermediary contracts, and firm boundaries. These issues as well as more specific applications of the smart buyer framework are left for future research.

Appendix

Proof of Proposition 1

The inexistence of a deterministic separating equilibrium is explained in the text. It remains to be shown that a stochastic separating equilibrium must satisfy the properties stated in the proposition. We can represent a stochastic contract in reduced form as

$$\tilde{\mathcal{C}}_g = \begin{cases} [1, t_g^1, 0] & \text{with probability } p_g \\ [0, t_g^0, 0] & \text{with probability } 1 - p_g \end{cases} \tag{15}$$

where $t_g^1 \equiv E_g(t\,|x = 1)$, $t_g^0 \equiv E_g(t\,|x = 0)$, and $p_g \equiv \Pr_g(x = 1)$ under g. For a buyer of type θ_x, the expected payoff from $\tilde{\mathcal{C}}_g$ is

$$p_g[\theta_x + z(1; \theta_x, \theta_z) - t_g^1] - (1 - p_g)t_g^0 = p_g[\theta_x + z(1; \theta_x, \theta_z)] - E_g(t). \tag{16}$$

Consider two arbitrary buyer types, θ_x^h and $\theta_x^l < \theta_x^h$. Conjecture a separating equilibrium and, with slight abuse of notation, let $\tilde{\mathcal{C}}_i$ denote type θ_x^i's stochastic contract. The implied downstream incentive compatibility constraint is

$$\begin{aligned} p_h[\theta_x^h + z\left(1; \theta_x^h, \theta_z\right)] - E_h(t) &\geq p_l[\theta_x^h + z\left(1; \theta_x^h, \theta_z\right)] - E_l(t) \\ \Delta_p[\theta_x^h + z\left(1; \theta_x^h, \theta_z\right)] &\geq E_h(t) - E_l(t) \end{aligned} \tag{17}$$

where $\Delta_p \equiv p_h - p_l$. The constraint implies $\Delta_p > 0$ if $E_h(t) > E_l(t)$ and $E_h(t) < E_l(t)$ if $\Delta_p < 0$. Similarly, the implied upstream incentive compatibility constraint is

$$\begin{aligned} p_l[\theta_x^l + z\left(1; \theta_x^l, \theta_z\right)] - E_l(t) &\geq p_h[\theta_x^l + z\left(1; \theta_x^l, \theta_z\right)] - E_h(t) \\ \Delta_p[\theta_x^l + z\left(1; \theta_x^l, \theta_z\right)] &\leq E_h(t) - E_l(t). \end{aligned} \tag{18}$$

This constraint, in turn, implies $\Delta_p < 0$ if $E_h(t) < E_l(t)$ and $E_h(t) > E_l(t)$ if $\Delta_p > 0$. Taken together, (17) and (18) allow for two possibilities: either $E_l(t) < E_l(t)$ and $\Delta_p < 0$ or $E_h(t) > E_l(t)$ and $\Delta_p > 0$. In the former case, (17) and (18) can be combined into

$$\theta_x^h + z\left(1; \theta_x^h, \theta_z\right) \leq \frac{E_h(t) - E_l(t)}{\Delta_p} \leq \theta_x^l + z\left(1; \theta_x^l, \theta_z\right), \tag{19}$$

which is impossible given that $\theta_x^h + z\left(1; \theta_x^h, \theta_z\right) > \theta_x^l + z\left(1; \theta_x^l, \theta_z\right)$ by Assumption 1. In the latter case, (17) and (18) can be combined into

$$\theta_x^h + z\left(1; \theta_x^h, \theta_z\right) \geq \frac{E_h(t) - E_l(t)}{\Delta_p} \geq \theta_x^l + z\left(1; \theta_x^l, \theta_z\right), \tag{20}$$

which is possible. Thus, a stochastic separating equilibrium — if it exists — implies $E_h(t) > E_l(t)$ and $\Delta_p > 0$.

The buyer's problem can be represented as a maximization problem under a direct mechanism, $\max_{\hat{\theta}_x} p(\hat{\theta}_x)[\theta_x + z\,(1; \theta_x, \theta_z)] - \bar{t}(\hat{\theta}_x)$, where $\hat{\theta}_x$ is the buyer's self-reported type and $\tilde{\mathcal{C}}_g(\hat{\theta}_x) = [p(\hat{\theta}_x), \bar{t}(\hat{\theta}_x)]$ is the stochastic contract associated with type $\hat{\theta}_x$.

A contract menu $\mathfrak{C} = \{\mathcal{C}_0, \tilde{\mathcal{C}}_g(\theta_x)\}_{\theta_x \in \Theta_x}$ is a separating offer if $\tilde{\mathcal{C}}_g(\theta_x)$ is a bijection (so that each contract uniquely identifies a type) and the solution to the buyer's maximization problem under $\mathfrak{C}$ is $\hat{\theta}_x = \theta_x$ for all $\theta_x \in \Theta_x$. Note that, for any given $\mathfrak{C}$, if the objective function is continuous in Θ_x, which holds if $p(\hat{\theta}_x)$ and $\bar{t}(\hat{\theta}_x)$ are continuous in Θ_x, a solution to the maximization problem exists by the Weierstrass Theorem.

First-order condition: Truthful revelation implies that $\arg\max_{\hat{\theta}_x} p(\hat{\theta}_x)[\theta_x + z\,(1; \theta_x, \theta_z)] - \bar{t}(\hat{\theta}_x) = \theta_x$, which in turn implies the first-order condition

$$p'(\theta_x)[\theta_x + z\,(1; \theta_x, \theta_z)] = \bar{t}'(\theta_x) \quad \text{for all } \theta_x \in \Theta_x. \tag{21}$$

Given that $p(\hat{\theta}_x)$ and $\bar{t}(\hat{\theta}_x)$ are continuously differentiable, (21) is a *necessary* condition for truthful revelation.

Quasi-concavity: Using the first-order condition to substitute for $\bar{t}'(\hat{\theta}_x)$ in the first derivative of the objective function and rearranging yields

$$p'(\hat{\theta}_x)[\theta_x + z\,(1; \theta_x, \theta_z)] - \bar{t}'(\hat{\theta}_x)\Big|_{\bar{t}'(\hat{\theta}_x) = p'(\hat{\theta}_x)[\hat{\theta}_x + z\left(1; \hat{\theta}_x, \theta_z\right)]} = \\ p'(\hat{\theta}_x)[\theta_x - \hat{\theta}_x + z\,(1; \theta_x, \theta_z) - z(1; \hat{\theta}_x, \theta_z)] \tag{22}$$

Since $p'(\theta_x) > 0$ (Claim I) and $z_{\theta_x}(x; \theta_x, \theta_z) \geq 0$ (Assumption 1), (22) is positive for all $\hat{\theta}_x \leq \theta_x$ but negative for all $\hat{\theta}_x \geq \theta_x$. Thus, (21) implies quasi-concavity of the buyer's objective function and is therefore also a *sufficient* condition for truthful revelation.

Equilibrium rents: Consider the buyer's equilibrium payoff $\Pi^*(\theta_x) = p(\theta_x)[\theta_x +$

$z(1; \theta_x, \theta_z)] - \bar{t}(\theta_x)$. By the envelope theorem,

$$\frac{\partial \Pi^*}{\partial \theta_x} = p(\theta_x)[1 + z_{\theta_x}(1; \theta_x, \theta_z)]. \tag{23}$$

In equilibrium, type $\overline{\theta}_x$ must trade with certainty. Otherwise, it prefers (offering) a simple contract $[1, \overline{\theta}_x + \varepsilon, 0]$, which is certainly accepted by the seller and, for some $\varepsilon \to 0$, is at least as lucrative for type $\overline{\theta}_x$ as any contract $\tilde{\mathcal{C}}_g$ that the seller would accept. Thus, type $\overline{\theta}_x$'s payoff is the same as under symmetric information: $\Pi^*(\overline{\theta}_x) = z\left(1; \overline{\theta}_x, \theta_z\right)$. Using (23), type θ_x's payoff can therefore be expressed as

$$\Pi^*(\theta_x) = z\left(1; \overline{\theta}_x, \theta_z\right) - \int_{\theta_x}^{\overline{\theta}_x} p(s)[1 + z_{\theta_x}(1; s, \theta_z)]ds. \tag{24}$$

Importantly, note that the buyer — irrespective of her type — prefers separating offers that involve lower levels of $p(\cdot)$.

Equilibrium offer: While the buyer prefers separating offers with lower levels of $p(\cdot)$, the separating offer she chooses must be consistent with the participation constraints.

We begin by examining what low levels of $p(\cdot)$ imply for the expected transfers $\bar{t}(\cdot)$. Since $p(\overline{\theta}_x) = 1$, type θ_x's probability of trading can be expressed as

$$\begin{aligned} p(\theta_x) &= 1 - \int_{\theta_x}^{\overline{\theta}_x} p'(s)ds \\ &= 1 - \int_{\theta_x}^{\overline{\theta}_x} \frac{\bar{t}'(s)}{s + z(1; s, \theta_z)} ds \end{aligned} \tag{25}$$

where the last equality follows from (21). Evidently, lower levels of $p(\cdot)$ can be achieved by increasing $\bar{t}'(\cdot)$. Since $\bar{t}(\overline{\theta}_x) = \overline{\theta}_x$, type θ_x's expected transfer can be expressed as

$$\bar{t}(\theta_x) = \overline{\theta}_x - \int_{\theta_x}^{\overline{\theta}_x} \bar{t}'(s)ds,$$

which shows that increasing $\bar{t}'(\cdot)$ amounts to decreasing $\bar{t}(\cdot)$. Thus, the buyer's most preferred separating offer is the one that minimizes the expected transfers $\bar{t}(\cdot)$, subject to participation constraints.

In other words, the buyer wants to reduce $\bar{t}(\cdot)$ until the seller's participation constraint,

$$\bar{t}(\theta_x) \geq p(\theta_x)\theta_x, \tag{26}$$

binds. If this constraint is slack for some $\theta_x \in \Theta_x$, the buyer can marginally lower $\bar{t}(\cdot)$ (for some types) to reduce $p(\cdot)$ without affecting the seller's participation, thereby (weakly) increasing $\Pi^*(\theta_x)$ for all $\theta_x \in \Theta_x$.

Choosing $\bar{t}(\cdot)$ such that the seller's participation constraint is binding imposes another joint restriction on $\bar{t}(\theta_x)$ and $p(\theta_x)$:

$$\bar{t}(\theta_x) = p(\theta_x)\theta_x \tag{27}$$

Differentiating on both sides with respect to θ_x and substituting in (21) yields the first-order differential equation

$$\frac{p'(\theta_x)}{p(\theta_x)} = [z\,(1; \theta_x, \theta_z)]^{-1}. \tag{28}$$

Integrating on both sides yields

$$p(\theta_x) = K \exp\left[\int_{\underline{\theta}_x}^{\theta_x} [z(x; s, \theta_z)]^{-1} ds\right] \tag{29}$$

where K is an integration constant. Using $p(\overline{\theta}_x) = 1$ in (29) yields the boundary condition

$$K = \exp\left[-\int_{\underline{\theta}_x}^{\overline{\theta}_x} [z(x; s, \theta_z)]^{-1} ds\right]. \tag{30}$$

so that (29) becomes

$$p(\theta_x) = \exp\left[-\int_{\theta_x}^{\overline{\theta}_x} [z(x; s, \theta_z)]^{-1} ds\right]. \tag{31}$$

Note that $p(\theta_x)$ is differentiable. It also satisfies the properties of a probability, that is, $p(\theta_x) \in [0, 1]$. Finally, in accordance with Claim I, $p'(\theta_x) > 0$. ■

Proof of Proposition 2

To characterize a contract it is convenient to use the per-unit price $P \equiv t/x$. For a buyer of type θ_x, the expected payoff from $\mathcal{C} = [x, P, 0]$ is

$$x\theta_x + z(x; \theta_x, \theta_z) - xP. \tag{32}$$

Consider two arbitrary buyer types, θ_x^h and $\theta_x^l < \theta_x^h$. Conjecture a separating equilibrium and, with slight abuse of notation, let $\mathcal{C}_i = [x_h, P_h, 0]$ denote type θ_x^i's contract. The implied downstream incentive compatibility constraint is

$$\begin{aligned} x_h\theta_x^h + z\left(x_h; \theta_x^h, \theta_z\right) - x_hP_h &\geq x_l\theta_x^h + z\left(x_l; \theta_x^h, \theta_z\right) - x_lP_l \\ z\left(x_h; \theta_x^h, \theta_z\right) - z\left(x_l; \theta_x^h, \theta_z\right) + \Delta_x\theta_x^h &\geq x_hP_h - x_lP_l \end{aligned} \tag{33}$$

where $\Delta_x \equiv x_h - x_l$. Since $z_x(x; \theta_x, \theta_z) \geq 0$ (Assumption 1), the constraint implies $x_h > x_l$ if $x_hP_h > x_lP_l$ and $x_hP_h < x_lP_l$ if $x_h < x_l$. Similarly, the implied upstream incentive compatibility constraint is

$$\begin{aligned} x_l\theta_x^l + z\left(x_l; \theta_x^l, \theta_z\right) - x_lP_l &\geq x_h\theta_x^l + z\left(x_h; \theta_x^l, \theta_z\right) - x_hP_h \\ z\left(x_h; \theta_x^l, \theta_z\right) - z\left(x_l; \theta_x^l, \theta_z\right) + \Delta_x\theta_x^l &\leq x_hP_h - x_lP_l \end{aligned} \tag{34}$$

This constraint, in turn, implies $x_h < x_l$ if $x_hP_h < x_lP_l$ and $x_hP_h > x_lP_l$ if $x_h > x_l$. Taken together, (33) and (34) allow for two possibilities: either $x_hP_h < x_lP_l$ and $x_h < x_l$ or $x_hP_h > x_lP_l$ and $x_h > x_l$. In the former case, (33) and (34) in combination imply

$$\frac{z\left(x_l; \theta_x^h, \theta_z\right) - z\left(x_h; \theta_x^h, \theta_z\right) - \Delta_x\theta_x^h}{x_lP_l - x_hP_h} \leq \frac{z\left(x_l; \theta_x^l, \theta_z\right) - z\left(x_h; \theta_x^l, \theta_z\right) - \Delta_x\theta_x^l}{x_lP_l - x_hP_h}. \tag{35}$$

However, this is impossible. If $x_l > x_h$, then $z\left(x_l; \theta_x^h, \theta_z\right) - z\left(x_h; \theta_x^h, \theta_z\right) \geq z\left(x_l; \theta_x^l, \theta_z\right) - z\left(x_h; \theta_x^l, \theta_z\right)$ because $z_{x\theta_x} \geq 0$ (Assumption 1). This together with $-\Delta_x\theta_x^h > -\Delta_x\theta_x^l$ (for $x_l > x_h$) implies that the left-hand side is strictly larger than the right-hand side. For $x_hP_h > x_lP_l$ and $x_h > x_l$, (33) and (34) in combination imply

$$\frac{z\left(x_h;\theta_x^h,\theta_z\right)-z\left(x_l;\theta_x^h,\theta_z\right)+\Delta_x\theta_x^h}{x_hP_h-x_lP_l}\geq \frac{z\left(x_h;\theta_x^l,\theta_z\right)-z\left(x_l;\theta_x^l,\theta_z\right)+\Delta_x\theta_x^l}{x_hP_h-x_lP_l}, \quad (36)$$

which is possible. Thus, a separating equilibrium with deterministic trade rationing — if it exists — implies $x_h > x_l$ and $x_hP_h > x_lP_l$ or, put differently, that lower buyer types are less likely to acquire the good and pay less in total.

The buyer's problem can be represented as a maximization problem under a direct mechanism, $\max_{\hat{\theta}_x} x(\hat{\theta}_x)\theta_x + z(x(\hat{\theta}_x);\theta_x,\theta_z) - t(\hat{\theta}_x)$, where $\hat{\theta}_x$ is the buyer's self-reported type and $\mathcal{C}(\hat{\theta}_x) = [x(\hat{\theta}_x), t(\hat{\theta}_x), 0]$ is the contract associated with type $\hat{\theta}_x$. A contract menu $\mathfrak{C} = \{\mathcal{C}_0, \mathcal{C}(\theta_x)\}_{\theta_x\in\Theta_x}$ is a separating offer if $\mathcal{C}(\theta_x)$ is a bijection (so that each contract uniquely identifies a type) and the solution to the buyer's maximization problem under $\mathfrak{C}$ is $\hat{\theta}_x = \theta_x$ for all $\mathcal{C}(\theta_x)$. Note that, for any given $\mathfrak{C}$, if the objective function is continuous in Θ_x, which holds if $x(\hat{\theta}_x)$ and $t(\hat{\theta}_x)$ are continuous in Θ_x, a solution to the maximization problem exists by the Weierstrass Theorem.

First-order condition: Truthful revelation implies that $\arg\max_{\hat{\theta}_x} x(\hat{\theta}_x)\theta_x + z(x(\hat{\theta}_x);\theta_x,\theta_z) - t(\hat{\theta}_x) = \theta_x$, which in turn implies the first-order condition

$$x'(\theta_x)\theta_x + z_x(x(\theta_x);\theta_x,\theta_z)x'(\theta_x) = t'(\theta_x) \quad \text{for all } \theta_x \in \Theta_x. \quad (37)$$

Given that $x(\hat{\theta}_x)$ and $t(\hat{\theta}_x)$ are continuously differentiable, (37) is a *necessary* condition for truthful revelation.

Quasi-concavity: Substituting the first-order condition into the first derivative of the objective function and rearranging yields

$$x'(\hat{\theta}_x)\theta_x + z_x(x(\hat{\theta}_x);\theta_x,\theta_z)x'(\hat{\theta}_x) - t'(\hat{\theta}_x)\Big|_{t'(\hat{\theta}_x)=x'(\hat{\theta}_x)\hat{\theta}_x+z_x(x(\hat{\theta}_x);\hat{\theta}_x,\theta_z)x'(\hat{\theta}_x)} = x'(\hat{\theta}_x)[\theta_x - \hat{\theta}_x + z_x(x(\hat{\theta}_x);\theta_x,\theta_z) - z_x(x(\hat{\theta}_x);\hat{\theta}_x,\theta_z)] \quad (38)$$

Since $z_{\theta_x}(x;\theta_x,\theta_z) \geq 0$, $z_x(x;\theta_x,\theta_z) \geq 0$ (Assumption 1) and $x'(\theta_x) > 0$ (Claim I), (38) is positive for all $\hat{\theta}_x \leq \theta_x$ but negative for all $\hat{\theta}_x \geq \theta_x$. Thus, (37) implies quasi-concavity of the buyer's objective function and is therefore also a *sufficient* condition for truthful revelation.

Equilibrium rents: Consider the buyer's equilibrium payoff $\Pi^*(\theta_x) = x(\theta_x)\theta_x + z(x(\theta_x);\theta_x,\theta_z) - t(\theta_x)$. By the envelope theorem,

$$\frac{\partial \Pi^*}{\partial \theta_x} = x(\theta_x) + z_{\theta_x}(x(\theta_x);\theta_x,\theta_z). \tag{39}$$

In equilibrium, a buyer of type $\overline{\theta}_x$ must trade $x = 1$. Otherwise, it prefers (offering) a simple contract $[1, \overline{\theta}_x + \varepsilon, 0]$, which is certainly accepted by the seller and, for some $\varepsilon \to 0$, is at least as lucrative for type $\overline{\theta}_x$ as any fully revealing contract with $x < 1$ that the seller would accept. That is, type $\overline{\theta}_x$'s payoff must be the same as under symmetric information: $\Pi^*(\overline{\theta}_x) = z\left(1;\overline{\theta}_x,\theta_z\right)$. Using (39), type θ_x's payoff can therefore be expressed as

$$\Pi^*(\theta_x) = z\left(1;\overline{\theta}_x,\theta_z\right) - \int_{\theta_x}^{\overline{\theta}_x} \left[x(s) + z_s(x(s);s,\theta_z)\right] ds. \tag{40}$$

Importantly, note that the buyer — irrespective of her type — prefers separating offers that involve lower levels of x, that is, lower quantities of trade.

Equilibrium offer: While the buyer prefers separating offers with lower levels of $x(\cdot)$, the separating offer it chooses must be consistent with the participation constraints.

We begin by examining what low levels of $x(\cdot)$ imply for the transfers $t(\cdot)$. Since $x(\overline{\theta}_x) = 1$, type θ_x's trading quantity can be expressed as

$$\begin{aligned} x(\theta_x) &= 1 - \int_{\theta_x}^{\overline{\theta}_x} x'(s)ds \\ &= 1 - \int_{\theta_x}^{\overline{\theta}_x} \frac{t'(s)}{s + z_x(x(s);s,\theta_z)} ds \end{aligned} \tag{41}$$

where the last equality follows from (37). Evidently, lower levels of $x(\cdot)$ can be achieved by increasing $t'(\cdot)$. Since $t(\overline{\theta}_x) = \overline{\theta}_x$, type θ_x's transfer can be expressed as

$$t(\theta_x) = 1 - \int_{\theta_x}^{\overline{\theta}_x} t'(s)ds,$$

which shows that increasing $t'(\cdot)$ amounts to decreasing $t(\cdot)$. Thus, the buyer's most preferred separating offer is the one that minimizes the transfers $t(\cdot)$, subject to participation constraints.

In other words, the buyer wants to reduce $t(\cdot)$ until the seller's participation constraint,

$$t(\theta_x) \geq x(\theta_x)\theta_x, \tag{42}$$

binds. If this constraint is slack for some $\theta_x \in \Theta_x$, the buyer can marginally lower $t(\cdot)$ (for some types) to reduce $x(\cdot)$ without affecting the seller's participation, thereby (weakly) increasing $\Pi^*(\theta_x)$ for all $\theta_x \in \Theta_x$.

Choosing $t(\cdot)$ such that the seller's participation constraint is binding imposes another joint restriction on $t(\theta_x)$ and $x(\theta_x)$:

$$t(\theta_x) = x(\theta_x)\theta_x. \tag{43}$$

Differentiating on both sides with respect to θ_x and substituting in (37) yields the first-order differential equation

$$\frac{x'(\theta_x)}{x(\theta_x)} = [z_x(x(\theta_x); \theta_x, \theta_z)]^{-1} \tag{44}$$

Example: Let $z(x(\theta_x); \theta_x, \theta_z) = \ln \alpha x(\theta_x)\theta_x$. So, $z_x(x(\theta_x); \theta_x, \theta_z) = [x(\theta_x)]^{-1}$, and equation (44) yields

$$\begin{aligned} \frac{x'(\theta_x)}{x(\theta_x)} &= x(\theta_x) \\ x'(\theta_x) &= [x(\theta_x)]^2 \\ x(\theta_x) &= -\frac{1}{\theta_x} + K \end{aligned}$$

The boundary condition, $x(\overline{\theta}_x) = 1$, yields

$$K = 1 + \frac{1}{\overline{\theta}_x}.$$

Substituting for K, we finally get

$$x(\theta_x) = 1 + \frac{1}{\overline{\theta}_x} - \frac{1}{\theta_x}.$$

Note that, for very low θ_x, incentive-compatibility requires negative trade, which means that very low types are shut out of the market. ■

Proof of Proposition 4

A buyer of type θ_x receives the payoff $z(1;\theta_x,\theta_z)$ when making a truthful offer with the fixed transfer θ_x. Now consider its payoff when mimicking a lower valued type $\theta_x' < \theta_x$. By Assumption 1, $v(1;\theta_x',\theta_z) < v(1;\theta_x,\theta_z)$. Hence, when mimicking type θ_x', type θ_x would incur a penalty $\tau > \theta_x - \theta_x'$ and its payoff would be less than $z(1;\theta_x,\theta_z)$. Now consider the payoff from mimicking any type $\theta_x'' > \theta_x$. By Assumption 1, $v(1;\theta_x'',\theta_z) > v(1;\theta_x,\theta_z)$. Hence, mimicking would not trigger a penatly, but type θ_x would pay a fixed transfer of θ_x'', which is higher than the fixed transfer θ_x under its truthful offer. ■

Proof of Lemma 1

Rewriting the buyer's participation constraint (12) in terms of the per-unit pooling price $P_P = t_P/x_P$ yields

$$\theta_x + \frac{z(x_P;\theta_x,\theta_z)}{x_P} \geq P_P. \tag{45}$$

We make three observations: (i) for $P_P = \underline{\theta}_x$, (45) holds for all types $\theta \in \Theta$; (ii) for $P_P = \overline{\theta}_x$, (45) holds for at least some types $\theta_x \leq \overline{\theta}_x$ since $z(x_P;\theta_x,\theta_z) \geq 0$; (iii) as P_P increases, the buyer needs a higher v, and hence a higher θ_x, to satisfy (45).

Now, let us rewrite the seller's participation constraint (14) as

$$P_P \geq E\left[\theta_x | \, \theta \in \Theta^P\right]. \tag{46}$$

Observation (i) implies that (46) is violated for $P_P = \underline{\theta}_x$. Observation (ii) implies that (46) always holds for $P_P = \overline{\theta}_x$. Thus, observation (iii) implies that, for *any* given x_P, there is a unique per-unit price $P_P^c(x_P) \in [\underline{\theta}_x, \overline{\theta}_x)$ such that the seller's participation constraint holds for some $P_P \in [P_P^c(x_P), \overline{\theta}_x]$. In other words, for any x_P, there exists at least one $P_P < \overline{\theta}_x$ such that $[x_P, x_P P_P, 0]$ satisfies (14). At least some of these offers can be supported against deviations to other (more attractive) pooling offer. Consider, for example, offers with $x_P = 1$ and $P_P \in [P_P^c(x_P), \overline{\theta}_x]$. Any such offer can be supported against deviations to $(x_P^d, x_P^d P_P^d, 0)$ where $P_P^d < \overline{\theta}_x$ by off-equilibrium beliefs that associate such deviations with buyer type $\overline{\theta}_x$.

Finally, efficient pooling equilibria (in which all buyer types purchase the entire good) exist in the case of one-dimensional private information if (45) holds for $\theta_x = \underline{\theta}_x$, $P_P = E_{\theta_x}(\theta_x)$ and $x_P = 1$. The same is true in the case of two-dimensional private information if (45) holds for $(\theta_x, \theta_z) = (\underline{\theta}_x, \underline{\theta}_z)$, $P_P = E_{\theta_x,\theta_z}(\theta_x)$ and $x_P = 1$. ■

Proof of Lemma 2

Existence of single fully revealing equilibrium: Consider an arbitrary type θ'_x and denote its preferred contract under the separating offer by $\mathcal{C}' = [x', t', 0]$. We know from Proposition 2 that the seller breaks even under the separating offer, irrespective of buyer type. We show that type θ'_x trading less than x' or more than x' cannot be supported as an equilibrium. Consider any (off-equilibrium) contract $\mathcal{C}_1^d = \left[x_1^d, t_1^d, 0\right]$ that type θ'_x strictly prefers to $\mathcal{C}'$ where $x_1^d \leq x'$. The total surplus is weakly smaller under $\mathcal{C}_1^d$ than under $\mathcal{C}'$ because $z_x \geq 0$ for all types (see Assumption 1). Hence, the fact that type θ'_x prefers $\mathcal{C}_1^d$ over $\mathcal{C}'$ implies that the seller's payoff is smaller under $\mathcal{C}_1^d$ than under $\mathcal{C}'$ (for buyer type θ'_x). Given the latter payoff is zero, off-equilibrium beliefs that assign the contract $\mathcal{C}_1^d$ to type θ'_x cause the seller to reject $\mathcal{C}_1^d$, thereby deterring such a deviation.

Consider now any (off-equilibrium) contract $\mathcal{C}_2^d = \left[x_2^d, t_2^d, 0\right]$ that type θ'_x strictly prefers to $\mathcal{C}'$ where $x_2^d > x'$. In this case, x^d must be equal to the quantity that some higher type $\theta''_x \in (\theta'_x, \overline{\theta}_x]$ trades under the separating offer. Denote type θ''_x's preferred contract under the separating offer by $\mathcal{C}'' = [x'', t'', 0]$ where $x'' = x_2^d$. Since type θ'_x prefers $\mathcal{C}'$ over $\mathcal{C}''$ (by incentive-compatibility), it also prefers $\mathcal{C}_2^d$ over $\mathcal{C}''$ (by transitivity). For $x'' = x_2^d$, this implies that $t'' > t^d$. This in turn implies that type θ''_x must also prefer $\mathcal{C}_2^d$ over $\mathcal{C}''$. Furthermore, since $x'' = x^d$, the fact that type θ''_x prefers $\mathcal{C}_2^d$ over $\mathcal{C}''$ implies that the seller's payoff is smaller under $\mathcal{C}_2^d$ than under $\mathcal{C}'$ (for type θ''_x). Given the latter payoff is zero, off-equilibrium beliefs that assign the contract $\mathcal{C}_2^d$ to type θ''_x cause the seller to reject $\mathcal{C}_2^d$, thereby deterring such a deviation. Defining off-equilibrium beliefs in this manner, one can deter all potential deviations of every buyer type to support a fully revealing equilibrium, in which every buyer type makes the separating offer derived in Proposition 2. Finally, there can be no other fully revealing equilibrium because any other incentive-compatible schedule implies a (strictly) lower payoff for (at least some of) the buyer types.

Existence of an uninformative equilibrium: An uninformative equilibrium exists when a pooling offer is both (a) acceptable to the seller and (b) preferred by every buyer type over the separating offer in Proposition 2. Condition (a) implies that, if such an equilibrium exists, it is efficient in that all buyer types participate. Condition (b) accordingly implies that the lowest acceptable pooling offer price is $P_P = E(\theta_x) \equiv \mu_x$. For any given x_P, arbitrary type θ'_x's payoff under this pooling offer would then be

$$\Pi_P(\theta'_x) = x_P \theta'_x + z(x_P; \theta'_x, \theta_z) - \mu_x.$$

Crucially, unlike the buyer's payoff under the separating offer, the buyer's payoff under the pooling offer depends on the probability distribution of θ_x over Θ_x. Specifically, note that $\partial \Pi_P(\theta'_x)/\partial \mu_x < 0$ for all $\theta'_x \in \Theta_x$. This implies that one can always construct a pooling offer that satisfies both of the above conditions, (a) and (b), by letting $\mu_x \to \underline{\theta}_x$. To see this, consider the limit $\lim_{\mu_x \to \underline{\theta}_x} \Pi_P(\theta'_x)$ in the case of $x_P = 1$:

$$\lim_{\mu_x \to \underline{\theta}_x} \Pi_P(\theta'_x) = \theta'_x + z(1; \theta'_x, \theta_z) - \underline{\theta}_x.$$

For any given type, (the value of) this limit is weakly larger than the buyer's full information payoff, and a fortiori larger than the buyer's payoff under the separating offer. Thus, there must exist some threshold $\underline{\mu}_x \in (\underline{\theta}_x, \overline{\theta}_x]$ such that, when $\mu_x < \underline{\mu}_x$, every type's payoff under the pooling offer is higher than under the separating offer for some $x_P \in (0, 1]$. By construction, deviations to acceptable separating offers are unattractive. So are, by implication, deviations to offers that contain only contracts with $P \geq \overline{\theta}_x$ because they are less attractive than type $\overline{\theta}_x$'s preferred contract in any separating offer. Finally, deviations to any (non-separating) offer that contains some contract with $P < \overline{\theta}_x$ can be deterred by off-equilibrium beliefs that attribute that offer to type $\overline{\theta}_x$.

Existence of partially revealing equilibrium: A partially revealing equilibrium can only exist if there is a pooling offer that is both (a') acceptable to the seller and (b') preferred by some buyer types over the separating offer in 2. For any possible pooling offer $\mathcal{C}_P$, one can find a set $\Theta_x^+(\mathcal{C}_P) \subset \Theta_x$ that contains all buyer types who prefer $\mathcal{C}_P$ *over their respective contract under the separating offer*. There always exist some $\mathcal{C}_P$ such that $\Theta_x^+(\mathcal{C}_P)$ is non-empty, and hence satisfy condition (b'). Such a pooling offer $\mathcal{C}_P = [x_P, t_P, 0]$ also satisfies condition (a') if

$$t_P \geq x_P E\left[\theta_x | \, \theta_x \in \Theta_x^+(\mathcal{C}_P)\right]. \tag{47}$$

Whenever there exists a pooling offer $\mathcal{C}_P$ with non-empty $\Theta_x^+(\mathcal{C}_P)$ that satisfies (47), there exists a partially revealing offer of the following kind: All types $\theta_x \in \Theta_x^+(\mathcal{C}_P)$ make the pooling offer $\mathcal{C}_P$, whereas all other types make the separating offer. Off-equilibrium beliefs associated with any other (potential deviating) offer are chosen as in the proof of Claim I.

6.1 Proof of Lemma 3 and Proposition 5

Claim I: If $E(\theta_x)$ is sufficiently low, every buyer type receives a higher payoff in the efficient uninformative equilibrium than in any other equilibrium.

Proof of Claim I: In a fully revealing equilibrium, a buyer type's payoff does not depend on the probability distribution of buyer types. In contrast, in an efficient uninformative equilibrium, type θ_x's payoff is given by $\Pi_P(\theta_x) = \theta_x + z(1; \theta_x, \theta_z) - \mu_x$. As $\mu_x \to \underline{\theta}_x$, this payoff approaches to $\lim_{\mu_x \to \underline{\theta}_x} \Pi_P(\theta_x) = \theta_x - \underline{\theta}_x + z(1; \theta_x, \theta_z)$, which is clearly higher than the fully revealing payoff.

Claim II: Suppose the buyer must acquire some minimum quantity $\underline{x}$. There is always some $\underline{x} \in (x(\underline{\theta}_x, \theta_z), 1)$ such that, if $E(\theta_x)$ is sufficiently low, there exists an uninformative equilibrium with $\mathcal{C}_P = [\underline{x}, \underline{x}E(\theta_x), 0]$.

Proof of Claim II: Again, in a fully revealing equilibrium, the type distribution does not matter for payoffs. Type θ_x's payoff in an uninformative equilibrium with $x_P = \underline{x}$ is $\Pi_P(\theta_x) = \underline{x}\theta_x + z(\underline{x}; \theta_x, \theta_z) - \underline{x}\mu_x$. As $\mu_x \to \underline{\theta}_x$, this payoff approaches $\lim_{\mu_x \to \underline{\theta}_x} \Pi_P(\theta_x) = \underline{x}\theta_x - \underline{\theta}_x + z(\underline{x}; \theta_x, \theta_z)$, which for some $\underline{x} > x(\underline{\theta}_x, \theta_z)$ is higher than the fully revealing payoff.

Claim III: The equilibrium in Claim II survives the intuitive criterion, and uniquely survives the credible beliefs criterion.

Proof of Claim III: Consider the uninformative equilibrium in Claim II. By construction, a deviation to the separating offer is unattractive. Consider the potential deviation (x', t'). A deviation where $x' \geq \underline{x}$ and $t' \geq \underline{x}\mu_x$ can *only* be attractive to types that are undervalued in the uninformative equilibrium, that is, for $\theta_x > \mu_x$. These types also find a deviation $x' \geq \underline{x}$ and $t' < \underline{x}\mu_x$ attractive. That is, there is no attractive deviation that the undervalued types do not find attractive. Thus, off-equilibrium beliefs that attribute the deviation to all types that find the deviation attractive make the deviation unacceptable to the seller, and hence deter such deviations. Moreover, such beliefs are admissible under the intuitive criterion and the credible beliefs criterion. We now turn to fully and partially revealing equilibria, in which some types either make a separating offer or, at least, do not pool with the lowest types. Under the credible beliefs criterion, a deviation from the fully revealing equilibrium to $(\underline{x}, \underline{x}\mu_x)$ must be attributed to all types, and hence succeeds. In a partially revealing equilibrium, under the credible beliefs criterion, the lowest types must pool at some offer $(\underline{x}, t_P)$ with $t_P \leq \underline{x}\mu_x$; otherwise, at least type $\underline{\theta}_x$ deviate to quantity $\underline{x}$ or transfer $\underline{x}\mu_x$. But under conditions where the uninformative equilibrium with $x_P = \underline{x}$ exists, such an offer is at least as attractive as $(\underline{x}, \underline{x}\mu_x)$, and will therefore be mimicked. ■

6.2 Proof of Proposition 6

For the proof, it is convenient to define buyer types in the θ_x-z-space. We proceed as follows: (I) We characterize necessary conditions for separating types with different θ_x but the same total valuation v. (II) We then characterize necessary conditions for separating types with different v. (III) We demonstrate that the conditions in (I) and (II) cannot be reconciled with each other.

(I) Define $\Theta_v \equiv \{(\theta_x, z) : \theta_x + z = v_i\}$, which contains some type $\left(\overline{\theta}_x, \hat{z}\right)$. Using $z = v - \theta_x$, we can express any type in Θ_v in terms of θ_x only, namely $(\theta_x, v - \theta_x)$. Let $\mathfrak{C}_v(\theta_x)$= $\{[1, t_v(\theta_x), [1 - \alpha_v(v; \theta_x)]v]\}$ denote an offer that separates all types in Θ_v, where $\alpha_v(v; \theta_x)v$ is a claim contingent on the realized v. For a given contract in this offer, type θ_x's payoff is

$$\Pi(\theta_x) = E\left[\alpha_v\left(v; \theta_x\right) v \,|v\right] - t_v\left(\theta_x\right) = \alpha_v\left(v; \theta_x\right) v - t_v\left(\theta_x\right), \tag{48}$$

which is deterministic because the buyer knows its v. To achieve separation in Θ_v, the payoff must satisfy the invariance condition

$$\Pi(\theta_x) = \overline{\Pi} \quad \text{for all } \theta \in \Theta_v. \tag{49}$$

Otherwise, some types in Θ_v would be mimicked by other types in the set. Furthermore, note that $\overline{\Pi} = \hat{z}$. Otherwise, type $\left(\overline{\theta}_x, \hat{z}\right)$ would deviate to the contract $[1, \overline{\theta}_x, 0]$. Using this, merging (48) and (49), and simplifying yields

$$t_v(\theta_x) = \overline{\theta}_x - \beta_v(v; \theta_x)v \tag{50}$$

where $\beta_v(v; \theta_x)v \equiv [1 - \alpha_v(v; \theta_x)]\, v$ denotes the contingent claim paid to the seller. For $\partial \beta_v(v; \theta_x)/\partial \theta_x \neq 0$, (50) characterizes all offers $\mathfrak{C}_v(\theta_x)$ that achieve separation within Θ_v, that is, separation of all types that generate the same total value v.

The invariance condition pins down a buyer's payoff as a function of total valuation. This function is given by, with slight abuse of notation, $\Pi(v) = \hat{z} = v - \overline{\theta}_x$. That buyer profits follow this function *across* Θ_vs is a necessary condition for separation *within* Θ_vs. Importantly, note that the function is *linear* in v, that is,

$$\partial \Pi / \partial v = 1. \tag{51}$$

(II) We now consider separation across different v, which is a necessary condition for achieving separation across all $\theta_x \in \Theta_x$. (If marginally different v are not separated, some θ_x-types are pooled.) A direct mechanism that separates different v must yield $\arg\max_{\hat{v}} \{E[\alpha(v; \hat{v})\, v \,|v] - t(\hat{v})\} = v$ for all v. The corresponding first-order condition is

$$\frac{\partial \alpha(v; v)}{\partial v} v = \frac{\partial t(v)}{\partial v}. \tag{52}$$

By the envelope theorem, separation across v requires that equilibrium payoffs must vary across v according to

$$\partial \Pi / \partial v = \alpha(v; v). \tag{53}$$

(III) Conditions (51) and (53) can only hold simultaneously if $\alpha(v; v) = 1$. This already shows that separation cannot simultaneously hold within each Θ_v and across v. Indeed, substituting $\alpha(v; v) = 1$ — more precisely, $\partial \alpha(v; v)/\partial v = 0$ — into (52) yields $\partial t(v)/\partial v = 0$, which in turn implies that $t(v) = K$ where K is some constant. It is obvious that $\alpha(v; v) = 1$ and $t(v) = K$ cannot achieve separation across v.

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