Feedback Effects of Credit Ratings

Gustavo Manso*

February 18, 2011

Abstract

Rating agencies are often criticized for being biased in favor of borrowers, for being too slow to downgrade following credit quality deterioration, and for being oligopolists. Based on a model that takes into account the feedback effects of credit ratings, I show that: (i) a rating agency should focus not only on the accuracy of its ratings but also on the effects of its ratings on the probability of survival of the borrower; (ii) even when a rating agency pursues an accurate rating policy, multi-notch downgrades or immediate default may occur in response to small shocks to fundamentals; (iii) increased competition between rating agencies can lead to rating downgrades, increasing default frequency and reducing welfare.

JEL Classification: G24, G28, G32, G01, L43.

Keywords: Credit rating agencies; rating triggers; performance-sensitive debt; financial regulation; credit-cliff dynamic;

---

*MIT Sloan School of Management, 100 Main Street E62-635, Cambridge, MA 02142 (e-mail: manso@mit.edu). I thank Nittai Bergman, Hui Chen, Darrell Duffie, and Darren Kisgen for helpful discussions and comments and Yan Ji for outstanding research assistance.

1 Introduction

Rating agencies are supposed to provide an independent opinion on the credit quality of issuers. However, if market participants rely on credit ratings for investment decisions, then credit ratings themselves affect the credit quality of issuers. For example, a rating downgrade may lead to higher cost of capital for the borrowing firm because it induces a deterioration in investors' perceptions about the credit quality of the borrowing firm, because of regulations that restrict investors' holdings of lower rated bonds, or because of rating triggers in financial contracts.[1] Rating agencies face thus the problem of setting credit ratings that accurately represent the credit quality of a particular issuer taking into account the effect of these ratings on the credit quality of the issuer.

Based on a model that incorporates the feedback effects of credit ratings, I show that: (i) a rating agency should focus not only on the accuracy of its ratings but also on the effects of its ratings on the probability of survival of the borrower; (ii) even when a rating agency pursues an accurate rating policy, multi-notch downgrades or immediate default may occur in response to small shocks to fundamentals; (iii) increased competition between rating agencies can lead to rating downgrades, increasing default frequency and reducing welfare. These findings call into question the recent criticism directed at rating agencies for being biased in favor of borrowers, for being too slow to downgrade following credit quality deterioration, and for being oligopolists.

The model is based on the *performance-sensitive-debt* (PSD) model introduced by Manso, Strulovici, and Tchistyi (2010). Cash flows of the firm follow a general diffusion process. The firm has debt in place in the form of a ratings-based PSD obligation, which promises a non-negative interest pay-

[1] Kisgen (2007) describes in more detail the channels through which credit ratings affect the cost of capital for a borrower.

ment rate that decreases with the credit rating of the firm. Equityholders choose the default time that maximizes the equity value of the firm. The rating agency's objective is to set accurate ratings that inform investors about the probability of default over a given time horizon. In this setting, the interaction between the borrowing firm and the rating agency produces feedback effects. With a ratings-based PSD obligation, the rating determines the interest rate, which affects the optimal default decision of the issuer. This, in turn, influences the rating.

The interaction between the rating agency and the borrowing firm is a game of strategic complementarity (Topkis 1979, Vives 1990, Milgrom and Roberts 1990). Typically, games of strategic complementarity exhibit multiple equilibria. In the smallest equilibrium, which I call the *soft-rating-agency equilibrium*, the rating agency assigns high credit ratings, leading to lower interest rates for the borrowing firm, and consequently, a lower default probability. In the largest equilibrium, which I call the *tough-rating-agency equilibrium*, the rating agency assigns low credit ratings, leading to higher interest rates for the borrowing firm, and consequently, a higher default probability. The soft-rating-agency equilibrium is associated with the lowest bankruptcy costs and consequently the highest welfare among all equilibria.

Given the welfare implications of the different equilibria, it is important to understand how rating agencies set their rating policies in practice. To deal with the feedback effects introduced by rating triggers, rating agencies have proposed the use of stress tests.[2] In such tests, the company with exposure to rating triggers needs to be able to survive stress-case scenarios in which the triggers are set off. When the tough-rating-agency equilibrium involves immediate default, the borrowing firm will fail the stress test, potentially inducing rating agencies to select the tough-rating-agency equilibrium, the

[2] "Moody's Analysis of US Corporate Rating Triggers Heightens Need for Increased Disclosure," Moody's, July 2002.

worst equilibrium in terms of welfare.

The best equilibrium in terms of welfare is the soft-rating-agency equilibrium, since it is the equilibrium with the lowest probability of default over any given time horizon. To implement such equilibrium, a credit rating agency should be concerned not only with the accuracy of its ratings, but also with the survival of the borrowing firm. One way in which this can be achieved is by having rating agencies collect a small fee from the firms being rated. Under this scheme, rating agencies become interested in the survival of the borrowing firm, inducing them to select the soft-rating-agency equilibrium.

The fact that rating agencies are paid by issuers has received intense criticism. The concern is that this practice may induce bias in favor of issuers. While this is a valid concern, the results of this paper suggest that if the fee is small relative to the reputational concerns of rating agencies, it only introduces small distortions while inducing rating agencies to select the Pareto-preferred soft-rating-agency equilibrium.

Stability of an equilibrium may play an important role in equilibrium selection and in the dynamics of credit ratings. The paper shows that if equilibrium is unique, then it is globally stable, so that small shocks to fundamentals lead to gradual changes in credit ratings. If there are multiple equilibria, however, some of them may be unstable. Small shocks to fundamentals may thus lead to multi-notch downgrades or even immediate default, in what has been called a "credit-cliff dynamic."

The effect of competition between rating agencies on equilibrium outcomes depends crucially on how credit ratings from different agencies affect interest payments by the borrowing firm. If interest payments depend on the minimum (maximum) of the available ratings then only the equilibrium with the highest (lowest) probability of default survives. If interest payments depend on some average of the available ratings, I provide conditions under which the only equilibrium that survives is one with immediate default. Therefore, increased competition may lead to the selection of the tough-rating-agency equilibrium, reducing welfare.

The model specification is flexible to capture realistic cash-flow processes, potentially allowing rating agencies and other market participants to incorporate the effects of rating triggers into debt valuation and rating policies. Because we have a game of strategic complementarity, we can use iterated best-response to compute the soft-rating-agency equilibrium and the tough-rating-agency equilibrium. To calculate best-responses in the case of a general diffusion process, we need to solve an ordinary differential equation and compute the first-passage-time distributions of a diffusion process through constant threshold. I compute equilibria of the game for the case of mean-reverting cash flows. For the base-case example, the present value of losses due to bankruptcy costs is approximately 14% of asset value under the tough-rating-agency equilibrium and zero under the soft-rating-agency equilibrium.

The paper is organized as follows. Section 2 reviews the related literature. Section 3 introduces the model. Section 4 shows existence of equilibrium in Markov strategies. Section 5 discusses equilibrium selection and the role of stress tests and fee structures in the credit rating industry. Section 6 studies equilibrium stability and discusses the "credit-cliff dynamic." Section 7 studies competition between rating agencies. Section 8 provides some comparative statics results. Section 9 studies the numerical computation of equilibria. Section 10 concludes. All proofs are in the appendix.

2 Related literature

Previous theoretical literature on credit rating agencies has overlooked potential feedback effects of credit ratings, focusing instead on how incentive problems of information intermediaries may reduce the quality of the information disclosed to the market. Lizzeri (1999) considers the optimal disclosure policy of an information intermediary who can perfectly observe the type of the seller at zero cost, and finds that in equilibrium the information intermediary does not disclose any information. Doherty, Kartasheva, and Phillips (2009) and Camanho, Deb, and Liu (2010) study how competition between rating agencies affects information disclosed to investors. Bolton, Freixas, and Shapiro (2009) and Skreta and Veldkamp (2009) develop models in which rating inflation emerges due to investors behavioral biases. Opp, Opp, and Harris (2010) study rating inflation due to preferential-regulatory treatment of highly rated securities. Fulghieri, Strobl, and Xia (2010) study the welfare effects of unsolicited credit ratings.

An exception is Boot, Milbourn, and Schmeits (2006) who consider a model in which credit ratings have a real impact on the firm's choice between a risky and a safe project. In their model, if some investors base their decisions on the announcements of rating agencies, then rating agencies can discipline the firm, inducing first-best project choice.

Also related to the current paper is the literature on credit risk models, which can be divided into two classes. In some models, such as Black and Cox (1976), Fischer, Heinkel, and Zechner (1989), Leland (1994), and Manso, Strulovici, and Tchistyi (2010) default is an endogenous decision of the firm. In other models, default is exogenous. There is either an exogenously given default boundary for the firm's assets (Merton 1974, Longstaff and Schwartz 1995), or an exogenous process for the timing of bankruptcy, as described in Jarrow and Turnbull (1995), Jarrow, Lando, and Turnbull (1997) and Duffie and Singleton (1999). My paper belongs to the class of models with endogenous default, which is essential to capture the feedback effect of credit ratings.

The closest paper in the credit risk literature is Manso, Strulovici, and Tchistyi (2010), who study performance-sensitive debt (PSD) with general performance measures. In contrast to Manso, Strulovici, and Tchistyi (2010), I restrict attention to ratings-based PSD and focus on the strategic interaction between rating agencies and the borrowing firm. This allows me to study multiple equilibria and their implications for rating agencies policies and industry regulation.

At a broader level, the paper is also related to the literature linking financial markets to corporate finance and demonstrating the real effects of financial markets. Fishman and Hagerthy (1989), Leland (1992), Holmstrom and Tirole (1993), Dow and Gorton (1997), Subrahmanyam and Titman (1999), Fulghieri and Lukin (2001), and Goldstein and Guembel (2008) are examples of papers in this literature.

An important assumption in the model is that credit ratings affect the cost of capital for a borrower. Several studies provide empirical evidence on this link. West (1973) and Ederington, Yawitz, and Roberts (1987) find that credit ratings predict bond yields beyond the information contained in publicly available financial variables and other variables that predict spread. Hand, Holthausen, and Leftwich (1992) document negative average excess bond and stock returns upon the announcement of downgrades of straight debt. Kliger and Sarig (2000) study the impact of credit ratings on yields using a natural experiment. In April 1982, Moody's added modifiers to their ratings, increasing the precision of their rating classes (e.g. an A-rated firm then became an A1-, A2-, or A3-rated firm). This exogenous change in the information produced by Moody's ratings affected bond yields in the direction implied by the modification.

Focusing on the regulatory-based explanation for the impact of ratings on yields, Kisgen and Strahan (2010) study the recent certification of ratings from Dominion Bond Rating Service for regulatory purposes. They find that after certification bond yields fell for firms that had a higher rating from Dominion than from other certified rating agencies. Chen, Lookman, Schurhoff, and Seppi (2010) exploit a 2005 change in the eligibility of split-rated bonds for inclusion in the Lehman Brothers bond indices to study the impact of credit ratings on bond yields. Bonds that were mechanically upgraded from high yield to investment grade after the Lehman rule announcement experienced positive abnormal returns.

There is also indirect evidence that credit ratings affect cost of capital. Kisgen (2006, 2009) finds that credit ratings directly affect firms' capital structure decisions. Kraft (2010) finds that rating agencies are reluctant to downgrade borrowers whose debt contracts have rating triggers.

3 The Model

The model is based on the performance-sensitive debt model introduced by Manso, Strulovici, and Tchistyi (2010). A firm generates non-negative after-tax cash flows at the rate δ_t, at each time t. I assume that δ is a diffusion process governed by the equation

$$d\delta_t = \mu(\delta_t)dt + \sigma(\delta_t)dB_t, \tag{1}$$

where μ and σ satisfy the classic assumptions for the existence of a unique strong solution to (1) and B is the standard Brownian motion.

Agents are risk neutral and discount future cash flows at the risk-free interest rate r. The expected discounted value of the firm at time t is

$$A_t = E_t \left[\int_t^\infty e^{-r(s-t)}\, \delta_s \, ds \right]. \tag{2}$$

The firm has debt in place in the form of a ratings-based performance-sensitive debt (PSD) obligation, which promises a non-negative payment rate that may vary with the credit rating of the firm. Credit ratings are represented by a stochastic process R taking values in $\mathcal{I} = \{1, \dots, I\}$, with 1 the lowest ("C" in Moody's ranking) and I the highest ("Aaa" in Moody's ranking). Formally, a ratings-based PSD obligation $C(\,\cdot\,)$ is a function C :

$\mathcal{I} \to \mathbb{R}^{+}$, such that the firm pays $C(R_t)$ to its debtholders at time t with $C(i) \geq C(i+1)$.[3]

Given a rating process R, the firm's optimal liquidation problem is to choose a default time $\widehat{\tau}$ to maximize its initial equity value W_0^C, given the debt structure C. That is,

$$W_0 \equiv \sup_{\widehat{\tau} \in \mathcal{T}} E\left[\int_0^{\widehat{\tau}} e^{-rt}[\delta_t - (1-\theta)C(R_t)]\,dt\right], \tag{3}$$

where $\mathcal{T}$ is the set of $\mathcal{F}_t$ stopping times, θ is the corporate tax rate, and $(1-\theta)C(\pi_t)$ is the after-tax effective coupon rate. If τ^* is the optimal liquidation time, then the market value of the equity at time $t < \tau^*$ is

$$W_t = E_t\left[\int_t^{\tau^*} e^{-r(s-t)}[\delta_s - (1-\theta)C(R_s)]\,ds\right]. \tag{4}$$

Analogously, the market value U_t^C of the ratings-based PSD obligation C at time t is

$$U_t \equiv E_t\left[\int_t^{\tau^*} e^{-r(s-t)}C(R_s)\;ds\right] + E_t\left[e^{-r(\tau^*-t)}\left(A_{\tau^*} - \rho(A_{\tau^*})\right)\right], \tag{5}$$

where $\rho(A)$ is the bankruptcy cost. I assume that $\rho(A)$ is increasing in A and is less than the asset level at time of default.

The rating agency is concerned about its reputation, which depends on the accuracy of its ratings. An accurate rating informs investors about the probability of default over a given time horizon. Given a default policy $\widehat{\tau}$, a rating process R is *accurate* if

$$R_t = i \text{ whenever } P(\widehat{\tau} - t \leq T \mid \mathcal{F}_t) \in [G_i, G_{i-1}), \tag{6}$$

[3] The ratings-based PSD obligation C represents the total debt payment of the borrower. If the firm has a complex capital structure that includes various issues of ratings-based PSD obligations and also fixed-coupon debt, then $C(R_t)$ is the sum of the payments for each of the firm's obligations at time t given the rating R_t at time t. In other words, a complex capital structure consisting of a combination of ratings-based PSD obligations is a ratings-based PSD obligation.

where $\{G_i\}_{i=0}^I$ with $G_0 = 1$, $G_I = 0$, and $G_i \geq G_{i+1}$ are the target rating transition thresholds. Higher ratings correspond to lower default probabilities.

In this setting, the interaction between the borrowing firm and the rating agency produces important feedback effects. With a ratings-based PSD obligation, the rating determines the coupon rate, which affects the optimal default decision of the issuer. This, in turn, influences the rating.

Definition 1 *An equilibrium (τ^*, R^*) is characterized by the following:*

1. *Given the rating process R^*, the default policy τ^* solves (3).*

2. *Given the default policy τ^*, the rating process R^* satisfies (6).*

4 Equilibrium in Markov Strategies

The cash flow process δ is a time-homogeneous Markov process. Therefore, the current level δ_t of cash flows is the only state variable in the model. I will thus focus on equilibrium in Markov strategies that are a function of the current level δ_t of cash flows.

A Markov default policy takes the form $\tau(\delta_B) = \inf\{s : \delta_s \leq \delta_B\}$. Under such policy, default is triggered the first ("hitting") time that the cash flow level hits the threshold δ_B.

A Markov rating policy takes the form of rating transition thresholds $H = \{H_i\}_{i=0}^I$ such that $R_t = i$ if $\delta_t \in [H_i, H_{i-1})$ with $H_{i+1} \geq H_i$, $H_0 = 0$, and $H_I = \infty$. Under such policy, rating transitions happen when the cash flow process crosses specific cash-flow thresholds.

Given a Markov rating policy H, a best-response default policy for the firm is a Markov strategy. Under a Markov rating policy H, the ratings-based PSD obligation C is equivalent to a step-up PSD obligation C^H promising coupon payment $C^H(\delta_t) = C(i)$ if $\delta_t \in [H_i, H_{i-1})$. Manso, Strulovici, and Tchistyi (2010) show that, under a step-up PSD obligation C^H, the optimal default policy of the firm takes the form $\tau(\delta_B)$, and provides the following algorithm to compute the optimal default boundary δ_B:

Algorithm 1 *1. Determine the set of continuously differentiable functions that solve the following ODE*

$$\frac{1}{2}\sigma^2(x)W''(x) + \mu(x)W'(x) - rW(x) + x - (1-\theta)C^H(x) = 0. \quad (7)$$

at each of the intervals $[H_i, H_{i-1})$. It can be shown that any element of this set can be represented with two parameters, say L_1^i and L_2^i.

2. Determine δ_B, L_1^i, and L_2^i using the following conditions:

a. $W(\delta_B) = 0$ and $W'(\delta_B) = 0$.

b. $W(H_i-) = W(H_i+)$ and $W'(H_i-) = W'(H_i+)$ for $i = 1, \ldots, I$.

c. W' is bounded.

The above conditions give rise to a system of $2I+1$ equations with $2I+1$ unknowns ($L_j^i, j \in \{1,2\}, i \in \{1, \ldots I\}$ and δ_B).

On the other hand, for a fixed Markov default policy $\tau(\delta_B)$, an accurate ratings policy is also a Markov strategy. This is due to the fact that δ_t is a sufficient statistic for $P(\tau(\delta_B) - t \leq T \mid \mathcal{F}_t)$ for any $t \leq T$. Therefore, the best-response rating transition thresholds H are such that

$$P(\tau(\delta_B) - t \leq T \mid \delta_t = H_i) = G_i. \quad (8)$$

Because $P(\tau(\delta_B) - t \leq T \mid \delta_t)$ is strictly decreasing and continuous in δ_t, the thresholds H, as defined by (8), exist and are unique. Solving for rating transition thresholds H amounts to computing first-passage time $\tau(\delta_B)$ distributions, which is a classical problem in statistics.[4]

[4] See, for example, Ricciardi, Sacerdote, and Sato (1984) for a characterization of this distribution in terms of an integral equation, and Giraudo, Sacerdote, and Zucca (2001) for a method to compute the distribution using Monte Carlo simulation.

Since best responses to Markov strategies are also Markov strategies, when characterizing the Markov equilibria of the game, without loss of generality, I restrict attention to deviations that are Markov strategies. Therefore, from here on, I represent the default and ratings policies as Markov strategies. A default policy is thus given by some $\delta_B : \mathbb{R}^{I+1} \mapsto \mathbb{R}$ that maps rating transition thresholds into a default boundary $\delta_B(H)$. A rating policy is given by some $H : \mathbb{R} \mapsto \mathbb{R}^{I+1}$ that maps a default boundary into rating transition thresholds $H(\delta_B)$.

For given rating transition thresholds H, the equityholders' optimal problem is to choose the default threshold δ_B that maximizes:

$$\widetilde{W}(\delta_B, H) \equiv E\left[\int_0^{\tau(\delta_B)} e^{-rt}\left[\delta_t - (1-\theta)C^H(\delta_t)\right] dt\right],$$

The function $\widetilde{W}(\delta_B, H)$ represents the equity value if the rating agency chooses rating transition thresholds H and equityholders default at the threshold δ_B.

The set $\mathcal{E}$ of Markov equilibria of the game is given by:

$$\mathcal{E} = \{(x, y) \in \mathbb{R} \times \mathbb{R}^{I+1}; (x, y) = (\delta_B(y), H(x))\}. \tag{9}$$

I now prove existence of Markov equilibria in pure strategies. The key for existence is to establish that best-responses are increasing in the other player's strategy. The next two propositions establish these results.

Proposition 1 *The best-response default policy $\delta_B(H)$ is increasing in the rating transition thresholds H.*

Higher rating transition thresholds H imply lower credit ratings and consequently higher coupon payments. As a result, it is optimal for the firm to default earlier by setting a higher default threshold δ_B.

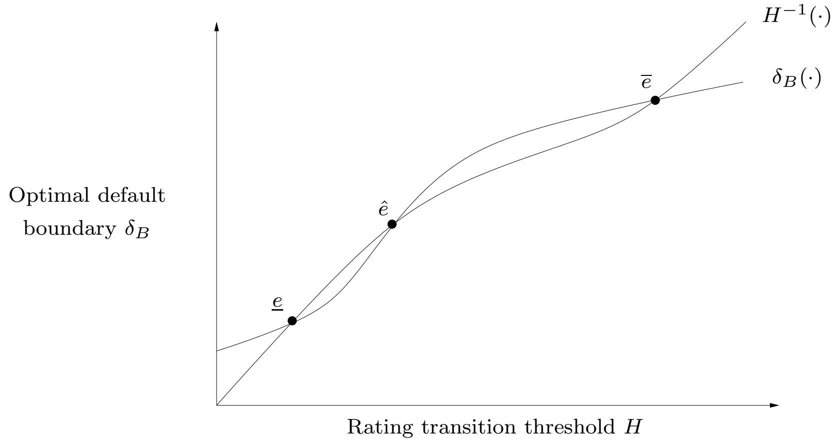

Figure 1: The figure plots best-response functions of the rating agency and the borrowing firm. Points $\underline{e}$, $\hat{e}$, and $\overline{e}$ are Markov equilibria of the game. The soft-rating-agency equilibrium is given by $\underline{e}$, while the tough-rating-agency equilibrium is given by $\overline{e}$. The point $\hat{e}$ corresponds to an intermediate equilibrium.

Proposition 2 *The best-response rating policy $H(\delta_B)$ is increasing in the default threshold δ_B.*

A higher default threshold δ_B translates into earlier default. To remain accurate, the rating agency needs to set higher rating transition thresholds H.

Propositions 1 and 2 show that the game between the rating agency and the borrowing firm is a game of strategic complementarity. The next theorem relies on the results of these two propositions to show existence of pure strategy equilibrium in Markov strategies.

Theorem 1 *The set $\mathcal{E}$ of Markov equilibria has a largest and a smallest equilibrium.*

Theorem 1 shows not only existence of equilibrium, but also that there exist a smallest and a largest equilibrium. Since the smallest equilibrium of the game has a low default boundary and low rating thresholds, I will call it the *soft-rating-agency equilibrium.* Since the largest equilibrium of the game has high rating thresholds and a high default boundary, I will call it the *tough-rating-agency equilibrium.* Figure 1 plots the best response functions of the rating agency and the borrowing firm as well as the corresponding equilibria of the game. The tough-rating-agency equilibrium $\overline{e}$ has higher default and rating transition thresholds than the soft-rating-agency equilibrium $\underline{e}$.

The following algorithm will be useful in computing equilibria of the game:

Algorithm 2 *Start from* x_0*.*

1. *calculate* $x_n = \delta_B(H(x_{n-1}))$*.*
2. *If convergence has been achieved (*$|x_n - x_{n-1}| \leq \epsilon$*), output* $(x_n, H(x_n))$*. Otherwise, return to step 1.*

Proposition 3 *Algorithm 2 always converges to an equilibrium of the game. It converges to the soft-rating-agency equilibrium, if started from* $x_0 = \delta_B(0, \ldots, 0)$*, and to the tough-rating-agency equilibrium, if started from* $x_0 = \delta_B(\infty, \ldots, \infty)$

Algorithm 2 can thus be used to find out whether the game has a unique equilibrium.

Corollary 1 *The game has a unique Markov equilibrium if and only if Algorithm 2 yields the same equilibrium if started from* $x_0 = \delta_B(0, \ldots, 0)$ *or* $x_0 = \delta_B(\infty, \ldots, \infty)$*.*

Convergence of the algorithm to the same equilibrium point when started from $x_0 = \delta_B(0, \ldots, 0)$ or $x_0 = \delta_B(\infty, \ldots, \infty)$ is a necessary and sufficient condition for uniqueness.

If the capital structure of the firm can be represented by a fixed-coupon consol bond, there is no feedback effect of credit ratings on the firm. The following proposition shows that in this case equilibrium is unique.

Proposition 4 *If C is a fixed-coupon consol bond (i.e. $C(i) = c$ for all i), then the equilibrium is unique.*

The case of a fixed-coupon consol bond is a benchmark model in the credit risk literature (Black and Cox 1976, Leland 1994). In this benchmark model, there are no feedback effects of credit ratings. Rating agencies are merely observers trying to estimate the first-passage-time distribution through a constant threshold. The main departure of the current paper from this benchmark model is that ratings affect credit quality, creating a circularity problem that makes the task of rating agencies more difficult. When credit ratings affect credit quality, multiple equilibria may exist, in which case there is more than one accurate rating policy that can be selected by the rating agency.

5 Social Welfare and Equilibrium Selection

The previous section shows that multiple equilibria may result from the interaction between the rating agency and the borrowing firm. An important question is which equilibrium is more likely to be selected in practice and what are the implications for social welfare.

Since in equilibrium ratings are always accurate, the only welfare losses arise from bankruptcy costs. A higher equilibrium default boundary is thus associated with lower welfare due to higher bankruptcy costs. The following proposition summarizes this result.

Proposition 5 *Equilibria of the game are Pareto-ranked. The tough-rating-*

agency equilibrium is the worst equilibrium, while the soft-rating-agency equilibrium is the best equilibrium.

In both the soft-rating-agency equilibrium and tough-rating-agency equilibrium, ratings are accurate, providing a correct estimate of the probability of default of the firm. Therefore, accuracy cannot be the only criterion guiding rating agencies in choosing their policies.

To maximize total welfare, a rating agency should always select the soft-rating-agency equilibrium. In practice, though, rating agencies may fail to select the soft-rating-agency equilibrium. One reason could be simply because correctly understanding and incorporating the feedback effects of credit ratings is difficult. For example, in December 2001, a few days after the collapse of Enron, which had exposure to several rating triggers, Standard and Poor's issued a report explaining its policy on rating triggers:[5]

> How is the vulnerability relating to rating triggers reflected all along in a company's ratings? Ironically, it typically is not a rating determinant, given the circularity issues that would be posed. To lower a rating because we might lower it makes little sense – especially if that action would trip the trigger!

Almost three years later, in October 2004, S&P republished the same report, with a correction to reflect its more recent view that vulnerability relating to rating triggers can be reflected all along in a company's ratings, but that there remains questions over circularity.

Moody's, on the other hand, has clearly indicated in the aftermath of Enron's collapse that it would take rating triggers into account when assigning credit ratings. In a July 2002 report,[6] Moody's explains that it will require

[5] "Playing Out the Credit-Cliff Dynamic," Standard and Poor's, December 2001.

[6] "Moody's Analysis of US Corporate Rating Triggers Heightens Need for Increased Disclosure," Moody's, July 2002.

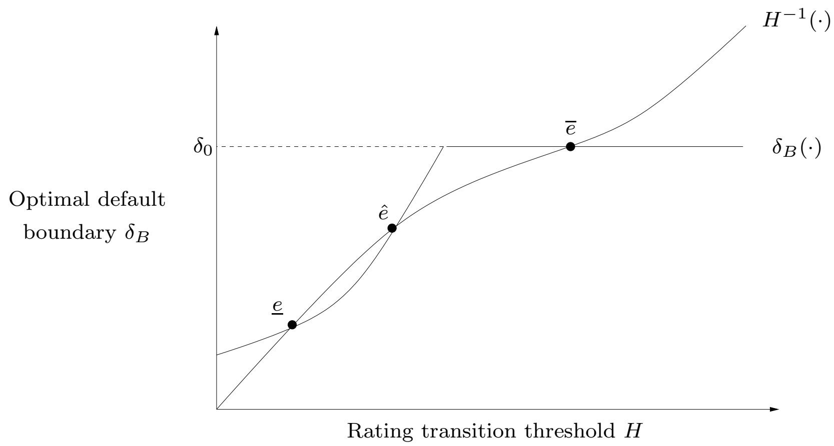

Figure 2: The figure plots best-response functions of the rating agency and the borrowing firm and the corresponding equilibria. In this case, the borrowing firm would fail a stress test, since the tough-rating-agency equilibrium $\overline{e}$ involves immediate default. The firm would survive if the rating agency selected the soft-rating-equilibrium.

issuers to disclose any rating triggers and will incorporate the serious negative consequences of rating triggers in its ratings by conducting stress tests with firms that have exposure to such triggers. In these stress tests, firms need to be able to survive stress-case scenarios in which rating triggers are set off.

According to the analysis in the current paper, however, failure in a stress test does not imply that the issuer should be downgraded. Figure 2 illustrates a situation in which downgrades can be avoided even though under a stress-case scenario the firm would immediately default. In the figure, the tough-rating-agency equilibrium $\overline{e}$ involves immediate default. When performing a stress test in this situation, the rating agency will find that under the rating thresholds associated with the tough-rating-agency equilibrium the borrowing firm would default immediately, failing thus the stress test. In this example, welfare would be higher and ratings would still be accurate under the soft-rating-agency equilibrium.

The above discussion makes it clear that, to obtain the Pareto-preferred soft-rating-agency equilibrium, the objective function of the rating agency should incorporate, in addition to accuracy, some other concern. Among all equilibria, the soft-rating-agency equilibrium has the lowest default threshold, and consequently the lowest probability of default over a given horizon. Therefore, a concern about the survival of the borrowing firm may lead the rating agency to select the soft-rating-agency equilibrium.

One way this can be implemented in practice is by having the borrowing firm pay a small fee to the rating agency in exchange for its services. The rating agency would receive this fee continuously until the borrowing firm defaults. In the limit, as this fee gets close to zero, the rating agency's preference becomes lexicographic, so that it is concerned about rating accuracy in the first place and minimizing the probability of default of the borrowing firm in the second place. Under this scheme, rating agencies would select the soft-rating equilibrium, since, among all accurate rating policies, it is the one that minimizes the probability of default, and thus maximizes the present value of fee payments.

The above scheme may in fact be close to how the credit ratings industry is currently organized. For a rating agency, potential reputational losses from setting inaccurate ratings are likely to be much more important than the fees they receive from any individual issuer.[7] As noted by Thomas McGuire, former VP of Moody's, "what's driving us is primarily the issue of preserving

[7] Using corporate bond prices and ratings, Covitz and Harrison (2003) find evidence supporting the view that rating agencies are motivated primarily by reputation-related incentives. In contrast, He, Qian, and Strahan (2010) find that rating agencies reward large issuers of mortgage-backed securities by granting them unduly favorable ratings. In mortgage-backed securities markets, there are a small number of large issuers, weakening the reputational incentives.

our track record. That's our bread and butter."[8]

The fact that rating agencies are paid by the firms they rate has received intense criticism. The concern is that this practice may induce bias in favor of issuers. While this is a valid concern, the results of this paper suggest that small fees paid by issuers to the rating agencies may induce rating agencies to select the Pareto-preferred soft-rating-agency equilibrium, without introducing significant biases.

6 Stability and the Credit-Cliff Dynamic

In this section, I study equilibrium stability and its implications for credit ratings. The following proposition analyzes the special case in which equilibrium is unique.

Proposition 6 *If the game has a unique Markov equilibrium, it is globally stable in terms of best-response dynamics.*

Proposition 6 asserts that if the equilibrium is unique then it is globally stable in terms of best-response dynamics. This means that if one starts from any Markov strategy, iterative best-response dynamics will lead to the unique equilibrium of the game. Milgrom and Roberts (1990) show that stability also holds with respect to several other types of learning dynamics. Therefore, when the equilibrium is unique, small perturbations to the parameters of the model or to the responses of players will only have a small impact on the equilibrium outcome, so that changes in credit ratings will be gradual.

As shown in the previous sections, however, the model does not always produce a unique equilibrium. Because this is a game of strategic complementarity there will typically exist multiple equilibria. When there are multiple equilibria, some of them may be unstable. As such, small perturbations to

[8] Institutional Investor, 10-1995, "Ratings Trouble."

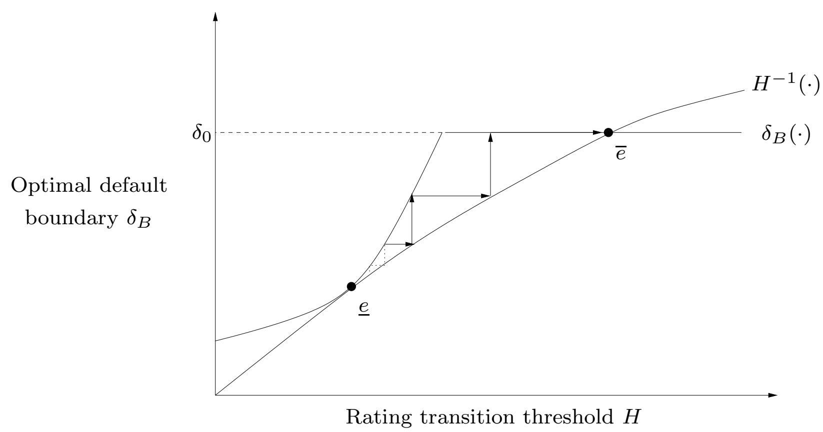

Figure 3: The figure plots best-response functions of the rating agency and the borrowing firm and the corresponding equilibria. The soft-rating-agency equilibrium $\underline{e}$ is unstable. Small shocks may produce a "credit-cliff dynamic" that leads to the tough-rating-agency equilibrium $\overline{e}$, which in this case involves immediate default.

the parameters of the model or to the responses of players may lead to large shifts in the equilibrium outcome. Multi-notch downgrades or even immediate default of highly rated corporations as response to small shocks are thus possible.

Figure 3 illustrates one situation in which this happens. In the figure, the soft-rating-agency equilibrium is locally unstable. Small perturbations to the best-response of either players may generate best-response dynamics that resemble what has been described as "credit-cliff dynamic." Starting from the soft-rating-agency equilibrium $\underline{e}$, if the rating agency becomes slightly tougher by increasing its ratings transition thresholds H, the firm's optimal response is to increase its default threshold δ_B. This in turn makes rating-agencies increase ratings thresholds even further. The credit-cliff dynamic only stops when the tough-rating-agency equilibrium is reached. In the sit-

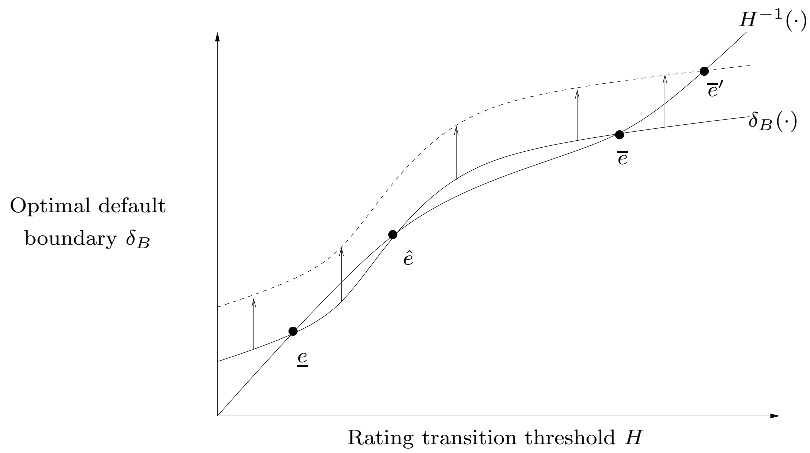

Figure 4: The figure plots best-response functions of the rating agency and the borrowing firm and the corresponding equilibria. A small shock to fundamentals may eliminate all equilibria except for the tough-rating-agency equilibrium $\overline{e}'$, leading to a multi-notch downgrade or even immediate default.

uation depicted in Figure 3, the tough-rating-agency equilibrium involves immediate default. Therefore, in this case, the credit-cliff dynamic produces a "death spiral."

One may argue that situations such as the one illustrated by Figure 3 are not generic because they require $H^{-1}(\cdot)$ to be exactly tangent to $\delta_B(\cdot)$ at the soft-rating-agency equilibrium point. Figure 4 depicts a situation in which both the soft-rating-agency and the tough-rating-agency equilibrium are locally stable, but a small unanticipated shock to some parameter of the model (such as an increase in the discount rate r) makes the soft-rating-agency equilibrium $\underline{e}$ and the intermediate equilibrium $\hat{e}$ disappear. The only remaining equilibrium is the tough-rating-agency equilibrium. Small shocks to fundamentals may thus lead to multi-notch downgrades or even immediate default of a highly rated firm.

7 Competition Between Rating Agencies

In this section, I consider competition between rating agencies. The model is similar to the model considered in previous sections except that there are now two rating agencies $k \in \{1, 2\}$, who compete for market share. The objective of each rating agency is to have more accurate ratings than the other rating agency.

Rating agency k assigns a rating R_t^k to the borrowing firm at each time t. The ratings-based PSD obligation C promises payments $C(R_t^1, R_t^2)$ from the borrowing firm to debtholders at each time t. The promised coupon payments are assumed to be decreasing in the credit ratings R_t^1 and R_t^2. Firms with higher ratings face lower coupon payments.

As in the previous sections, I focus on Markov equilibria of the game. The choice of rating transition thresholds $H = (H^1, H^2)$ by rating agencies 1 and 2 induces a step-up PSD obligation C^H promising payments $C^H(\delta_t) = C(i, j)$ whenever $\delta_t \in [H_i^1, H_{i-1}^1) \cap [H_j^2, H_{j-1}^2)$. The optimal default threshold is of the form $\tau(\delta_B)$ and depends on the rating transition thresholds $H = (H^1, H^2)$ of both rating agencies.

Lemma 1 *With a ratings-based PSD obligation C whose coupon depends on R_t^1 and R_t^2, any equilibrium involves rating agencies choosing symmetric rating transition thresholds ($H^1 = H^2$). The firm default boundary δ_B and the rating transition thresholds H^1 or H^2 are in the equilibrium set $\mathcal{E}$ of the game with a single rating agency.*

If the two agencies could perfectly coordinate on ratings, the analysis would be similar to the one in the previous section. Any equilibrium in which both rating agencies select the same rating transition thresholds H in $\mathcal{E}$ would be sustainable.

In practice, however, rating agencies are independent and have discretion to select ratings. Some equilibria in $\mathcal{E}$ may not survive deviations by a single rating agency. To study this issue it becomes important to understand how coupon payments are determined when ratings are split (i.e. $R_t^1 \neq R_t^2$).

If the ratings-based PSD obligation is induced by explicit contracts such as in the case of rating triggers, it is easy to find out the criterion to be applied when ratings are split. For a sample of bank loan contracts containing explicit rating triggers between 1993 and 2008, Wiemann (2010) manually checked 50 randomly selected contracts and found that 22 contracts used the highest rating, 20 contracts used the lowest rating, and the remaining 8 contracts used an average rating.[9]

Formally, the ratings-based PSD obligation C relies on the minimum rating if its promised payment depends only on $\min[R_t^1, R_t^2]$. It relies on the maximum rating if its promised payment depends only on $\max[R_t^1, R_t^2]$. The next proposition studies equilibria of the model with rating agency competition when the ratings-based PSD contract relies on the minimum or maximum of the two ratings.[10]

Proposition 7 *If the ratings-based PSD obligation C relies on the minimum (maximum) of the ratings, then the unique Markov equilibrium of the game is the tough-rating-agency (soft-rating-agency) equilibrium.*

Therefore, the effects of competition depend on how the rating triggers are specified in the contract. In particular, the way in which rating splits are resolved has an important impact on the equilibrium outcome. Under contracts that rely on the minimum of the ratings, rating agencies cannot coordinate on any equilibrium other than the tough-rating-agency equilibrium. If they try to coordinate on any other equilibrium, one rating agency would

[9] According to Wiemann (2010), the most common average is $(R_t^1 + R_t^2)/2$ rounded to the higher rating.

[10] For the above results, the restriction to Markov Perfect Equilibrium is important. If one considers strategies that depend on the whole history of the game, sufficiently patient rating agencies would be able to sustain coordination of any equilibrium in $\mathcal{E}$.

have an incentive to deviate to a rating policy associated with a tougher equilibrium, affecting the default threshold of the borrowing firm and making the rating policy of the other agency inaccurate. Therefore, only the tough-rating-agency equilibrium survives under contracts that rely on the minimum of the two credit ratings. By a similar argument, under contracts that rely on the maximum of the two ratings, only the soft-rating-agency equilibrium survives.

Even though, according to Wiemann (2010), the vast majority of the contracts rely on either the maximum or the minimum credit rating, there are reasons why one may want to understand the general case in which $C(i, j)$ depends on both ratings. As discussed previously, ratings-based PSD is not always explicitly given by a contract. It can, for example, be induced by the rollover of short-term debt. If the firm is performing well and has high credit ratings it will pay a lower interest rate when rolling over its maturing debt. If the firm is performing poorly and has low credit ratings it will pay a higher interest rate when rolling over its maturing debt. The interest that the firm pays on the new debt could depend in this case on both credit ratings assigned to the firm.

The following proposition partially characterizes equilibrium in this more general case.

Proposition 8 *Let $\overline{H}$ be the rating transition associated with the tough-rating-agency equilibrium and $\widehat{H} \equiv H(\delta_B(0, \dots, 0))$. If*

$$\delta_B(\overline{H}, \widehat{H}) > \delta_0 \quad \text{and} \quad \delta_B(\widehat{H}, \overline{H}) > \delta_0 \tag{10}$$

then the unique Markov equilibrium of the game is the tough-rating-agency equilibrium, which involves immediate default.

If a single rating agency can drive the firm to immediate default by adopting the rating transition thresholds associated with the tough-rating-agency equilibrium, then the only equilibrium that survives is the tough-rating-agency equilibrium. The intuition for this result is similar to the one in Proposition 7.

8 Comparative Statics

In this section, I study how the tough-rating-agency equilibrium and the soft-rating-agency equilibrium respond to changes in some of the parameters of the model.

Proposition 9 *The equilibrium default boundary δ_B and rating transition thresholds H associated with the tough-rating-agency equilibrium and the soft-rating-agency equilibrium are*

1. *increasing in the coupon payments C.*
2. *increasing in the interest rate r.*
3. *decreasing in the drift $\mu(\cdot)$ of the cash flow process.*
4. *decreasing in the target rating transition thresholds G.*

9 Equilibrium Computation

In this section I compute the best-response functions δ_B and H and equilibria when the cash flow process δ is a geometric Brownian motion or a mean-reverting process. The computation of the default threshold δ_B involves solving an ordinary differential equation, while the computation of the rating transition thresholds H involves computing the first-passage time distribution through a constant threshold. Equilibria of the game can then be computed by best-response iteration as explained in Algorithm 2.

Geometric Brownian Motion When the cash flow process δ of the firm follows a geometric Brownian motion,

$$d\delta_t = \mu\delta_t dt + \sigma\delta_t dB_t, \tag{11}$$

equilibrium of the game is unique and can be solved in closed-form. This example is discussed in Manso, Strulovici, and Tchistyi (2010).

To obtain the optimal default threshold δ_B, I apply Algorithm 1. As shown in Appendix B, the optimal default threshold δ_B solves

$$0 = -(\gamma_1 + 1)\frac{\delta_B}{r-\mu} + \frac{\gamma_1}{r}\left(c_1 - \sum_{i=1}^{I-1}(c_i - c_{i+1})\left(\frac{\delta_B}{H_{i+1}}\right)^{-\gamma_2}\right) \tag{12}$$

where $\gamma_1 = \dfrac{m + \sqrt{m^2 + 2r\sigma^2}}{\sigma^2}$, $\gamma_2 = \dfrac{m - \sqrt{m^2 + 2r\sigma^2}}{\sigma^2}$, $\quad m = \mu - \dfrac{\sigma^2}{2}$, and $c_i \equiv (1-\theta)C(i)$.

To derive the best-response $H(\delta_B)$ one needs to study the first-passage time distribution of the process δ. Since δ is a geometric Brownian motion, its first-passage time distribution is an inverse Gaussian:

$$P(\tau(\delta_B) - t \leq T \,|\, \mathcal{F}_t) = 1 - \Phi\left(\frac{m(T-t) - x}{\sigma\sqrt{T-t}}\right) + e^{\frac{2mx}{\sigma^2}}\Phi\left(\frac{x + m(T-t)}{\sigma\sqrt{T-t}}\right),$$

where, $x = \log\left(\frac{\delta_B}{\delta_t}\right)$, $m = \mu - \frac{1}{2}\sigma^2$, δ_t is the current level of assets, and Φ is the normal cumulative distribution function. Since $P(\tau(\delta_B) \leq T \,|\, \mathcal{F}_t)$ depends on δ_t only through $\frac{\delta_B}{\delta_t}$, we have the linearity of $H(\,\cdot\,)$.

$$H(\delta_B) = \delta_B\, h, \tag{13}$$

where $h \in \mathbb{R}^{I+1}$ is such that $h_0 = 0$, $h_I = \infty$, and $h_{i+1} \geq h_i$.

Equilibrium needs to satisfy $(x, y) = (\delta_B(y), H(x))$, or alternatively, $x = \delta_B(H(x))$. Plugging (13) into (12) and solving for δ_B one obtains the unique equilibrium default threshold δ_B^*, which is given by:

$$\delta_B^* = \frac{\gamma_1(r-\mu)}{(\gamma_1+1)r}\widehat{C}, \tag{14}$$

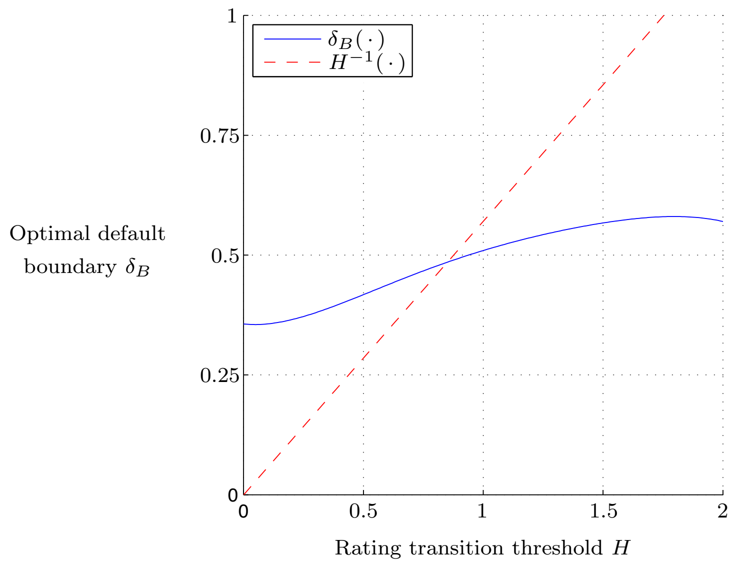

Figure 5: The figure plots best-response functions of the rating agency and the borrowing firm and the corresponding equilibrium when the cash flow process follows a geometric Brownian motion. The parameters used to plot the figure are $r = 0.06$, $\mu = 0.02$, $\sigma = 0.25$, $c_1 = 1$, $c_2 = 1.5$, and $G = 2\%$.

where

$$\widehat{C} = \sum_{i=1}^{I} \left[\left(\frac{1}{h_{i+1}} \right)^{-\gamma_2} - \left(\frac{1}{h_i} \right)^{-\gamma_2} \right] c_i.$$

The equilibrium rating transition thresholds H^* are thus given by:

$$H^* = \frac{\gamma_1 (r - \mu)}{(\gamma_1 + 1) r} \widehat{C} h$$

Figure 5 plots the best-response and the corresponding unique equilibrium of the game when the cash flow process is a geometric Brownian motion. As shown above, there is always a unique equilibrium in this case.

Mean-reverting process I now assume that the cash-flow process δ follows a mean-reverting process with proportional volatility:

$$d\delta_t = \lambda(\mu - \delta_t)dt + \sigma\delta_t dB_t \tag{15}$$

where λ is the speed of mean reversion, μ is the long-term mean earnings level to which δ reverts, and σ is the volatility. Sarkar and Zapatero (2003) study the optimal default decision of equityholders when cash flows follow a mean-reverting process and the firm issues a consol bond with fixed coupon payments c.

As Bhattacharya (1978) notes, "...mean-reverting cash flows are likely to be more relevant than the extrapolative random walk process in Myers and Turnbull (1977) and Treynor and Black (1976) for sound economic reasons. In a competitive economy, we should expect some long-run tendency for project cash flows to revert to levels that make firms indifferent about new investments in the particular type of investment opportunities that a given project represents, rather than 'wandering' forever." Several empirical studies indeed find that earnings are mean-reverting (Freeman, Ohlson, and Penman 1982, Kormendi and Lipe 1987, Easton and Zmijewski 1989, Fama and French 2000).

Here I consider the situation in which the firm issues a ratings-based PSD obligation C. Using the algorithms provided in this paper, I compute numerically the best response functions δ_B and H and then find the equilibria of the game.

For a given step-up PSD obligation C^H with transition thresholds H, I compute the best-response δ_B using Algorithm 1. As shown in Appendix B, the optimal default threshold δ_B solves

$$0 = \frac{\frac{1}{\lambda+r}g_1(\delta_B) - (\frac{1}{\lambda+r}\delta_B + \frac{\lambda\mu}{(\lambda+r)r} - \frac{c_1}{r})g_1'(\delta_B)}{g_2(\delta_B)g_1'(\delta_B) - g_2'(\delta_B)g_1(\delta_B)} + \frac{1}{r}\sum_{i=1}^{I-1}\frac{g_1'(H_{i+1})(c_{i+1} - c_i)}{g_2(H_{i+1})g_1'(H_{i+1}) - g_2'(H_{i+1})g_1(H_{i+1})}$$

where

$$g_i(x) = x^{\eta_i}M_i(x),$$

$$M_i(x) = M(-\eta_i, 2 - 2\eta_i + 2\lambda/\sigma^2; 2\lambda\mu/\sigma^2 x),$$

M is the confluent hypergeometric function given by the infinite series

$$M(a, b; z) = 1 + az/b + \{[a(a+1)]/[b(b+1)]\}(z^2/2!) + \{[a(a+1)(a+2)]/(b(b+1)(b+2)]\}(z^3/3!) + \dots,$$

η_1 and η_2 are roots of the quadratic equation

$$\frac{1}{2}\sigma^2\eta(\eta - 1) - \lambda\eta - r = 0,$$

and $c_i \equiv (1-\theta)C(i)$.

In the case of mean-reverting cash flows, there is no closed-form solution for the first-passage-time distribution. Therefore, I compute the best-response rating transition thresholds H using Monte Carlo simulation.

Figure 6 plots the best response functions in case the cash flows follow the mean-reverting process (15). For this particular example there are three possible equilibria. The soft-rating-agency equilibrium in this case involve zero default boundary, and consequently zero bankruptcy costs. In contrast, in the tough-rating-agency equilibrium, the present value of bankruptcy costs corresponds to 13.8% of the firm asset value when upon bankruptcy 20% of the firm asset value is lost ($\rho(x) = 0.2x$). This shows that the selection of equilibria by the rating agency can have a big impact on welfare.

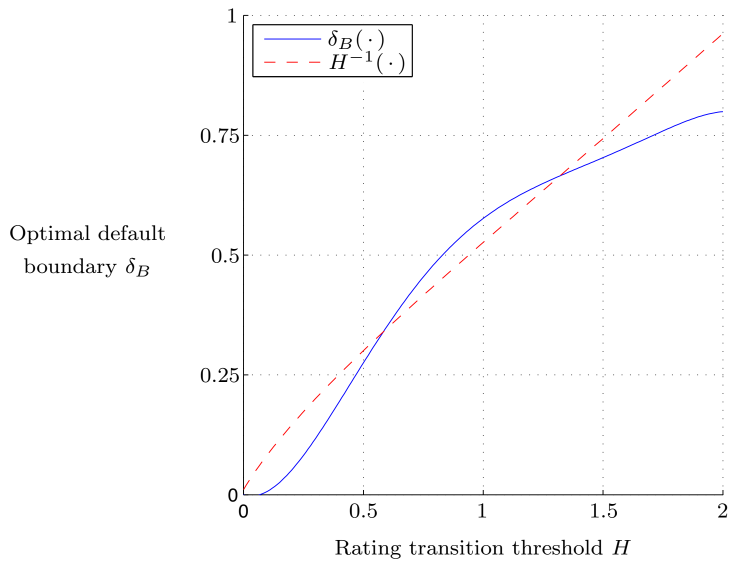

Figure 6: The figure plots best-response functions of the rating agency and the borrowing firm and the corresponding equilibrium when the cash flow process follows the mean-reverting process (15). The parameters used to plot the figure are $r = 0.06$, $\lambda = 0.15$, $\mu = 1$, $\sigma = 0.4$, $c_1 = 0.5$, $c_2 = 1.5$, and $G = 10\%$.

In figure 6, the soft-rating-agency equilibrium involves a zero probability of default. It is possible to construct examples under the mean-reverting cash-flow process (15) such that the soft-rating-agency equilibrium involves non-zero probability of default. Figure 7 provides one such example. The situation resembles the one analyzed in Figure 3.

10 Conclusion

This paper develops a dynamic credit risk model that incorporates feedback effects of credit ratings. It shows that feedback effects of credit ratings have

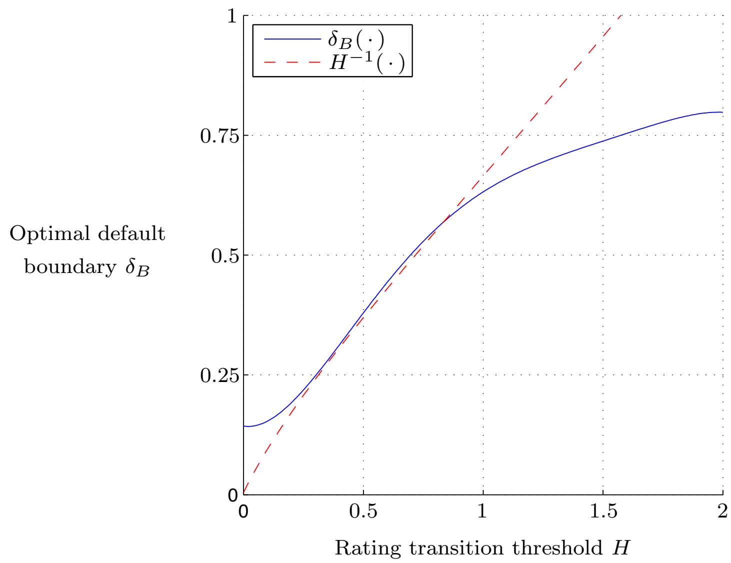

Figure 7: The figure plots best-response functions of the rating agency and the borrowing firm and the corresponding equilibrium when the cash flow process follows the mean-reverting process (15). The parameters used to plot the figure are $r = 0.06$, $\lambda = 0.15$, $\mu = 1$, $\sigma = 0.4$, $c_1 = 0.75$, $c_2 = 1.5$, and $G = 30\%$.

important implications for the regulation of the credit rating industry. Rating agencies that have a small bias towards the survival of the borrower, which can be achieved via the issuer-pay model, are likely to select the Pareto-preferred soft-rating-equilibrium. Stress tests, on the other hand, may lead to the selection of the Pareto-dominated tough-rating-agency equilibrium. Even if the rating agency pursues an accurate rating policy, multi-notch downgrades or immediate default may occur as responses to small shocks to fundamentals. Increased competition between rating agencies may lead to rating downgrades, increasing default frequency and reducing welfare.

The model specification is flexible to capture realistic cash-flow processes, and thus potentially allows rating agencies and other market participants to incorporate the feedback effects of credit ratings into debt valuation and rating policies. There may be important welfare implications. In numerical examples with mean-reverting cash flows, I find that the present value of bankruptcy losses in the tough-rating-agency equilibrium is substantially higher than in the soft-rating-agency equilibrium.

There are several unanswered questions. One question involves the effects of rating agencies on systemic risk. Rating downgrades of one firm could create pressure for the downgrades of other firms, in a form of feedback not studied in the current paper. It would also be interesting to study the capital structure decision of the firm, and the interaction of this decision with the rating policy of the credit rating agency. I leave these questions for future research.

Appendices

A Proofs

Proof of Proposition 1: It is enough to show that the firm's equity value $\tilde{W}(\delta_B, H)$ has increasing differences in δ_B and H. If $H' \geq H$,

$$\widetilde{W}(\delta, \delta_B, H') - \widetilde{W}(\delta, \delta_B, H) = E_x\left[\int_0^{\tau(\delta_B)} e^{-rt}\left[(1-\theta)C^H(\delta_t) - C^{H'}(\delta_t)\right] dt\right] \quad (16)$$

is increasing in δ_B, since $C^H(\delta_t) - C^{H'}(\delta_t) \leq 0$. ■

Proof of Proposition 2: It follows from the fact that $P(\tau(\delta_B) \leq T \,|\, \mathcal{F}_t)$ is increasing in δ_B. ■

Proof of Theorem 1: Let the function $F : \mathbb{R}^{I+1} \times \mathbb{R} \mapsto \mathbb{R} \times \mathbb{R}^{I+1}$ be such that $F(x, y) = (\delta_B(y), H(x))$. From Propositions 1 and 2, F is monotone. The set $\mathcal{E}$ correspond to fixed points $(x, y) = F(x, y)$. Let Y be such that

$$Y = \{(x, y) \in \mathbb{R} \times \mathbb{R}^{I+1}; 0 \leq x \leq \delta_B(\infty, \dots \infty) \text{ and } (0, \dots, 0) \leq y \leq H(\delta_B(\infty, \dots \infty))\}.$$

The set Y is a complete lattice with the usual partial order on Euclidean spaces. The function $G = F|_Y$ maps Y into Y and is monotone. By the Tarski fixed point theorem, the set $\mathcal{E}$ of Markov equilibria is a complete lattice. ■

Proof of Proposition 3: Because δ_B and H are increasing, the sequence $\{x_n\}$ produced by Algorithm 2 is either increasing or increasing. Since the sequence is bounded above by $\delta_B(\infty, \dots, \infty)$ and bounded below by 0, it must converge to some point e. The claim is that $(e, H(e))$ is an equilibrium of the game. Let $y \in \mathbb{R}$ be any other default strategy for the borrowing firm and take any sequence $\{y_n\}$ converging to y. By construction,

$$W(y, H(e)) = \lim_{n\to\infty} W(y_n, H(x_{n-1}) \leq \lim_{n\to\infty} W(x_n, H(x_{n-1})) = W(e, H(e))$$

where the first and last equality follow from the continuity of H and W. Therefore $(e, H(e))$ is an equilibrium of the game.

It remains to show that if $x_0 = \delta_B(0, \ldots, 0)$, then the algorithm converges to the lowest equilibrium $(\underline{e}, H(\underline{e}))$ of the game. If $(e, H(e))$ is any other element of $\mathcal{E}$, $x_0 \leq e$, and $x_n \leq e$ implies $x_{n+1} = \delta_B(H(x_n)) \leq \delta_B(H(e)) = e$. By induction, $(\underline{e}, H(\underline{e}))$ is the smallest element in $\mathcal{E}$.

The proof of convergence of the algorithm to the largest equilibrium when $x_0 = \delta_B(\infty, \ldots, \infty)$ is symmetric. ■

Proof of Proposition 4: If C is a fixed-coupon consol bond paying coupon c, then

$$\widetilde{W}(\delta, \delta_B, H) \equiv E_x \left[\int_0^{\tau(\delta_B)} e^{-rt} \left[\delta_t - (1-\theta)c\right] dt \right],$$

does not depend on H. Therefore, the default policy $\delta_B(H)$ that maximizes $\widetilde{W}(\delta, \delta_B, H)$ does not depend on H, and Algorithm 2 must converge to the same point in one iteration when started from either $x_0 = \delta_B(0, \ldots, 0)$ or $x_0 = \delta_B(\infty, \ldots, \infty)$. ■

Proof of Proposition 6: From Proposition 3, the sequence produced by an algorithm that iterates best-response functions converges to an equilibrium if started from any default threshold x_0. Therefore, if the equilibrium of the game is unique, it is globally stable. ■

Proof of Lemma 1: The proof is by contradiction. Suppose there was an equilibrium in which $H^1 \neq H^2$. Then it must be the case that $H^1 \neq$

$H(\delta_B(H^1, H^2))$ or $H^2 \neq H(\delta_B(H^1, H^2))$. Suppose, without loss of generality, that rating agency 1 is inaccurate (i.e. $H^1 \neq H(\delta_B(H^1, H^2)))$. One needs to show that it can improve its ratings.

For a fixed H^2, $\delta_B(H^1, H^2)$ is increasing in H^1 since $C(i, j)$ is decreasing in i, and the problem becomes similar to the one studied in Section 4. For a fixed H^2, let $\widehat{\mathcal{E}}$ be the set of equilibria δ_B and H^1. It follows from Theorem 1 that $\widehat{\mathcal{E}}$ is non-empty. Therefore, given H^2, there exists an accurate policy for rating agency 1, making this a profitable deviation. ■

Proof of Proposition 7: Suppose that ratings-based PSD obligation C relies on the minimum of the ratings. From Lemma 1, the only possible equlibria are in the set $\mathcal{E}$ and involve rating agencies playing symmetric strategies. Let $\overline{e} = (\overline{\delta_B}, \overline{H})$ correspond to the tough-rating-agency equilibrium. Suppose that there exists an equilibrium of the game with $(\widetilde{\delta_B}, \widetilde{H}) \neq (\overline{\delta_B}, \overline{H})$. Rating agency 1 could then deviate and choose $H^1 = \overline{H}$. Because C relies on the minimum of the ratings, and $\overline{H} \geq \widetilde{H}$, under this deviation, rating agency 1 would have accurate ratings while rating agency 2 would have inaccurate ratings.

It remains to show that the tough-rating-agency equilibrium is indeed an equilibrium. If agency 2 selects ratings thresholds $H^2 = \overline{H}$, then agency 1 cannot do better than selecting $H^1 = \overline{H}$. Any deviation $H^1 \leq \overline{H}$ would make its ratings inaccurate, since the default boundary would stay at $\overline{\delta_B}$. Any deviation $H^1 \geq \overline{H}$ would also make its ratings inaccurate, since even though it could move the default boundary to a level higher than $\overline{\delta_B}$, H^1 would not be accurate by the definition of the tough-rating-agency equilibrium. Finally, deviations in which $H^1_i < \overline{H_i}$ for some i and $H^1_i \geq \overline{H_i}$ for some i cannot lead to accurate ratings either since they would move the default boundary to a higher level than $\overline{\delta_B}$, but for some i the rating transition threshold H^1_i would be lower than $\overline{H_i}$, the accurate rating transition threshold under $\overline{\delta_B}$.

The proof for when the ratings-based PSD obligation C relies on the maximum of the ratings is similar. ■

Proof of Proposition 8: The proof is similar to the proof of Proposition 7. Condition (10) guarantees that if one agency deviates to the tough-rating-agency equilibrium policy the firm defaults immediately, destroying all equilibria but the tough-rating-agency equilibrium. Condition (10) also guarantees that under the tough-rating-agency equilibrium no rating agency wants to deviate to a softer policy since that will not be enough to save the firm from bankruptcy. ■

Proof of Proposition 9: It is enough to show that the best-response functions δ_B and H increase when there is an increase in the parameter of interest. If this is the case, the sequence produced by Algorithm 2 under the higher parameter will be greater than or equal to the sequence produced by Algorithm 2 under the lower parameter. Since the soft-rating-agency and the tough-rating-agency equilibrium are the limits of such sequences, they will also be higher under the higher parameter.

I first study comparative statics with respect to C. To show that the best response function δ_B is increasing in C it is enough to show that the firm's equity value $\tilde{W}(\delta_B, H; C)$ has increasing differences in δ_B and C. If $\hat{C} \geq C$,

$$\widetilde{W}(\delta_B, H; \hat{C}) - \widetilde{W}(\delta_B, H; C) = E\left[\int_0^{\tau(\delta_B)} e^{-rt}\left[(1-\theta)C^H(\delta_t) - \hat{C}^H(\delta_t)\right] dt\right] \quad (17)$$

is increasing in δ_B, since $C^H(\delta_t) - \hat{C}^H(\delta_t) \leq 0$. On the other hand, the best-response function H is unaffected by changes in C.

Next, I study comparative statics with respect to r. Theorem 2 of Quah and Strulovici (2010) guarantees that δ_B is increasing in r. On the other hand, the best-response function H is not affected by changes in r.

Next, I study comparative statics with respect to $\mu(\cdot)$. To show that δ_B is decreasing in $\mu(\cdot)$ it is enough to show that the firm's equity value $\tilde{W}(\delta_B, H; \mu)$ has increasing differences in δ_B and $-\mu$. Let $\hat{\mu} \geq \mu$ and δ_t $(\hat{\delta}_t)$ be the cash-flow process under μ $(\hat{\mu})$. We then have that

$$\widetilde{W}(\delta_B, H; \hat{\mu}) - \widetilde{W}(\delta_B, H; \mu) = \\ E\left[\int_0^{\tau(\delta_B)} e^{-rt} \left\{ \left[\hat{\delta}_t - \delta_t\right] + (1-\theta)\left[C^H(\delta_t) - C^H(\hat{\delta}_t)\right]\right\} dt\right],$$

is decreasing in δ_B, since C^H is decreasing and $\hat{\delta}_t \geq \delta_t$ in every path of B_t. The rating transition thresholds H are decreasing in $\mu(\cdot)$ since $\hat{\delta}_t \geq \delta_t$ for every path of B_t.

Finally, I study comparative statics with respect to G. The best-response function δ_B is unaffected by changes in G. The rating transition thresholds H are decreasing in G, since $P(\tau(\delta_B) \leq T \,|\, \mathcal{F}_t)$ is decreasing in δ_t. ■

B Particular Cash-Flow Processes

Geometric Brownian Motion Based on Algorithm 1, the equity value W and default threshold δ_B under a step-up PSD obligation C^H with transition thresholds H solve:

$$W(x) = \begin{cases} 0, & x \leq \delta_B\,, \\ L_1^i x^{-\gamma_1} + L_2^i x^{-\gamma_2} + \frac{x}{r-\mu} - \frac{(1-\theta)C(i)}{r}, & H_i \leq x \leq H_{i+1}\,, \end{cases} \tag{18}$$

for $i = 1, \ldots, I$, where $\gamma_1 = \dfrac{m + \sqrt{m^2 + 2r\sigma^2}}{\sigma^2}$, $\gamma_2 = \dfrac{m - \sqrt{m^2 + 2r\sigma^2}}{\sigma^2}$, $m = \mu - \dfrac{\sigma^2}{2}$, and where δ_b, L_1^i and L_2^i solve the following system of equations:

$$W(\delta_B) = 0, \quad W'(\delta_B) = 0\,, \tag{19}$$

and for $i = 1, \ldots, I-1$,

$$W(H_i-) = W(H_i+), \quad W'(H_i-) = W'(H_i+). \tag{20}$$

Because the market value of equity is non-negative and cannot exceed the asset value,[11]

$$L_2^I = 0. \tag{21}$$

The system (19)–(21) has $2I+1$ equations with $2I+1$ unknowns (L_j^i, $j \in \{1,2\}$, $i \in \{1,\ldots,I\}$, and δ_B). Substituting (18) into (19)–(21) and solving gives

$$\begin{aligned}
L_1^1 &= \frac{(\gamma_2+1)\frac{\delta_B}{r-\mu} - \gamma_2 \frac{c_1}{r}}{(\gamma_1-\gamma_2)\,\delta_B^{-\gamma_1}}, \\
L_2^1 &= \frac{-(\gamma_1+1)\frac{\delta_B}{r-\mu} + \gamma_1 \frac{c_1}{r}}{(\gamma_1-\gamma_2)\,\delta_B^{-\gamma_2}}, \\
L_1^j &= L_1^1 + \frac{\gamma_2}{(\gamma_1-\gamma_2)r} \sum_{i=1}^{j-1} \frac{c_i - c_{i+1}}{H_{i+1}^{-\gamma_1}}, \; j = 2, \ldots, I, \\
L_2^j &= L_2^1 - \frac{\gamma_1}{(\gamma_1-\gamma_2)r} \sum_{i=1}^{j-1} \frac{c_i - c_{i+1}}{H_{i+1}^{-\gamma_2}}, \; j = 2, \ldots, I,
\end{aligned}$$

$$0 = -(\gamma_1+1)\frac{\delta_B}{r-\mu} + \frac{\gamma_1}{r}\left(c_1 - \sum_{i=1}^{I-1}(c_i - c_{i+1})\left(\frac{\delta_B}{H_{i+1}}\right)^{-\gamma_2}\right), \tag{22}$$

where, for convenience, I let $c_i \equiv (1-\theta)C(i)$. Therefore, the best response $\delta_B(H)$ is given by the solution of (22).

[11] Since $\gamma_1 > 0$ and $\gamma_2 < 0$, the term $L_2^I x^{-\gamma_2}$ would necessarily dominate the other terms in the equation (18) violating the inequality $0 \le W(x) \le x/(r-\mu)$, unless $L_2^I = 0$.

Mean-Reverting Process The equity value W that solves (7) for the mean-reverting process (15) can be written as:

$$W(x) = \begin{cases} \hfill 0, \quad x \leq \delta_B \ , \\ L_1^i x^{-\eta_1} M_1(x) + L_2^i x^{-\eta_2} M_2(x) \\ \hfill + \frac{x}{\lambda+r} + \frac{\lambda\mu}{(\lambda+r)r} - \frac{(1-\theta)C(i)}{r}, \quad H_i \leq x \leq H_{i+1} \ , \end{cases} \tag{23}$$

for $i = 1, \ldots, I$, where η_1 and η_2 are roots of the quadratic equation

$$\frac{1}{2}\sigma^2\eta(\eta-1) - \lambda\eta - r = 0,$$

$M_1(x) = M(-\eta_1, 2-2\eta_1+2\lambda/\sigma^2; 2\lambda\mu/\sigma^2 x)$, $M_2(x) = M(-\eta_2, 2-2\eta_2+2\lambda/\sigma^2; 2\lambda\mu/\sigma^2 x)$, and where M is the confluent hypergeometric function given by the infinite series $M(a,b;z) = 1+az/b+\{[a(a+1)]/[b(b+1)]\}(z^2/2!)+\{[a(a+1)(a+2)]/(b(b+1)(b+2)]\}(z^3/3!)+\ldots$

The default threshold δ_b, and constants L_1^i and L_2^i thus solve the following system of equations:

$$W(\delta_B) = 0, \quad W'(\delta_B) = 0 \ , \tag{24}$$

and for $i = 1, \ldots, I-1$,

$$W(H_i-) = W(H_i+), \quad W'(H_i-) = W'(H_i+) \, . \tag{25}$$

Because the market value of equity is non-negative and cannot exceed the asset value,

$$L_2^I = 0. \tag{26}$$

The system (24)–(26) has $2I+1$ equations with $2I+1$ unknowns (L_j^i, $j \in \{1,2\}$, $i \in \{1,\ldots,I\}$, and δ_B). Substituting (23) into (24)–(26) and solving numerically gives the best-response δ_B to any rating transition thresholds H.

The solution to this system of equations is:

$$
\begin{aligned}
L_1^1 &= \frac{\frac{1}{\lambda+r}g_2(\delta_B) - (\frac{1}{\lambda+r}\delta_B + \frac{\lambda\mu}{(\lambda+r)r} - \frac{c_1}{r})g_2^{'}(\delta_B)}{g_1(\delta_B)g_2^{'}(\delta_B) - g_1^{'}(\delta_B)g_2(\delta_B)} \\
L_2^1 &= \frac{\frac{1}{\lambda+r}g_1(\delta_B) - (\frac{1}{\lambda+r}\delta_B + \frac{\lambda\mu}{(\lambda+r)r} - \frac{c_1}{r})g_1^{'}(\delta_B)}{g_2(\delta_B)g_1^{'}(\delta_B) - g_2^{'}(\delta_B)g_1(\delta_B)} \\
L_1^j &= L_1^1 + \frac{1}{r}\sum_{i=1}^{j-1}\frac{g_2^{'}(H_{i+1})(c_{i+1}-c_i)}{g_1(H_{i+1})g_2^{'}(H_{i+1}) - g_1^{'}(H_{i+1})g_2(H_{i+1})},\ j=2,...I \\
L_2^j &= L_2^1 + \frac{1}{r}\sum_{i=1}^{j-1}\frac{g_1^{'}(H_{i+1})(c_{i+1}-c_i)}{g_2(H_{i+1})g_1^{'}(H_{i+1}) - g_2^{'}(H_{i+1})g_1(H_{i+1})},\ j=2,...I
\end{aligned}
$$

$$
\begin{aligned}
0 = {} & \frac{\frac{1}{\lambda+r}g_1(\delta_B) - (\frac{1}{\lambda+r}\delta_B + \frac{\lambda\mu}{(\lambda+r)r} - \frac{c_1}{r})g_1^{'}(\delta_B)}{g_2(\delta_B)g_1^{'}(\delta_B) - g_2^{'}(\delta_B)g_1(\delta_B)} \\
& + \frac{1}{r}\sum_{i=1}^{I-1}\frac{g_1^{'}(H_{i+1})(c_{i+1}-c_i)}{g_2(H_{i+1})g_1^{'}(H_{i+1}) - g_2^{'}(H_{i+1})g_1(H_{i+1})} \qquad (27)
\end{aligned}
$$

where

$$g_i(x) = x^{\eta_i}M_i(x).$$

and $c_i \equiv (1-\theta)C(i)$. Therefore, the best response $\delta_B(H)$ is given by the solution of (27).

References

Acharya, V., S. Das, and R. Sundaram, 2002, A Discrete-Time Approach to No-arbitrage Pricing of Credit Derivatives with Rating Transitions, *Financial Analysts Journal* May-June, 28–44.

Bhattacharya, S., 1978, Project Valuation with Mean-Reverting Cash Flow Streams, *Journal of Finance* 33, 1317–1331.

Black, F., and J. Cox, 1976, Valuing Corporate Securities: Some Effects of Bond Identures Provisions, *Journal of Finance* 31, 351–367.

Bolton, P., X. Freixas, and J. Shapiro, 2009, The Credit Ratings Game, Working paper, Columbia Business School.

Boot, A., T. Milbourn, and A. Schmeits, 2006, Credit Ratings as Coordination Mechanisms, *Review of Financial Studies* 19, 81–118.

Camanho, N., P. Deb, and Z. Liu, 2010, Credit Rating and Competition, Working paper, London School of Economics.

Chen, Z., A. Lookman, N. Schurhoff, and D. Seppi, 2010, Why Ratings Matter: Evidence from the Lehman Brothers' Index Rating Redefinition, Working paper, Tepper School of Business at Carnegie Mellon.

Covitz, D., and P. Harrison, 2003, Testing Conflicts of Interest at Bond Ratings Agencies with Market Anticipation: Evidence that Reputation Incentives Dominate, Working paper, Federal Reseve Board.

Das, S., and P. Tufano, 1996, Pricing Credit Sensitive Debt When Interest Rates, Credit Rates and Credit Spreads Are Stochastic, *Journal of Financial Engineering* 5, 161–198.

Doherty, N., A. Kartasheva, and D. Phillips, 2009, Does Competition Improve Ratings?, Working paper, University of Pennsylvania Wharton School.

Dow, J., and G. Gorton, 1997, Stock Market Efficiency and Economic Efficiency: Is There a Connection?, *Journal of Finance* 52, 1087–1129.

Duffie, D., and K. Singleton, 1999, Modelling the Term Structure of Defaultable Bonds, *Review of Financial Studies* 12, 687–720.

Easton, P., and M Zmijewski, 1989, Cross-Sectional Variation in the Stock Market Response to Accounting Earnings Announcements, *Journal of Accounting and Economics* 11, 117–141.

Ederington, L., J. Yawitz, and B. Roberts, 1987, The Informational Content of Bond Ratings, *The Journal of Financial Research* 10, 211–226.

Fama, E., and K. French, 2000, Forecasting Profitability and Earnings, *Journal of Business* 73, 161–176.

Fischer, E., R. Heinkel, and J. Zechner, 1989, Dynamic Capital Structure Choice: Theory and Tests, *Journal of Finance* 44, 19–40.

Fishman, M., and K. Hagerthy, 1989, Disclosure Decisions by Firms and the Competition for Price Efficiency, *Journal of Finance* 44, 633–646.

Freeman, R., J. Ohlson, and S. Penman, 1982, Book Rate of Return and Prediction of Earnings Changes: an Empirical Investigation, *Journal of Accounting Research* 2, 639–653.

Fulghieri, P., and D. Lukin, 2001, Information Production, Dilution Costs, and Optimal Security Design, *Journal of Financial Economics* 61, 3–42.

Fulghieri, P., G. Strobl, and H. Xia, 2010, The Economics of Unsolicited Ratings, Working paper, University of North Carolina at Chapel Hill.

Giraudo, M., L. Sacerdote, and C. Zucca, 2001, A Monte-Carlo Method for the Simulation of First Passage Times of Diffusion Processes, *Methodology and Computing in Applied Probability* 3, 215–231.

Goldstein, I., and A. Guembel, 2008, Manipulation and the Allocational Role of Prices, *Review of Economic Studies* 75, 133–164.

Graham, J., and J. Harvey, 2001, The Theory and Practice of Corporate Finance: Evidence from the Field, *Journal of Financial Economics* 60, 187–243.

Hand, J., R. Holthausen, and R. Leftwich, 1992, The Effect of Bond Rating Agency Announcement on Bond and Stock Prices, *Journal of Finance* 47, 733–752.

He, J., J. Qian, and P. Strahan, 2010, Credit Ratings and the Evolution of the Mortgage-Backed Securities Market, Working paper, Boston College.

Holmstrom, B., and J. Tirole, 1993, Market Liquidity and Performance Monitoring, *Journal of Political Economy* 101, 678–709.

Houweling, P., A. Mentink, and T. Vorst, 2004, Valuing Euro Rating-Triggered Step-Up Telecom Bonds, *Journal of Derivatives* 11, 63–80.

Jarrow, R., D. Lando, and S. Turnbull, 1997, A Markov Model for the Term Structure of Credit Risk Spreads, *Review of Financial Studies* 10, 481–523.

Jarrow, R., and S. Turnbull, 1995, Pricing Derivatives on Financial Securities Subject to Credit Risk, *Journal of Finance* 50, 53–86.

Kisgen, D., 2006, Credit Ratings and Capital Structure, *Journal of Finance* 61, 1035–1072.

Kisgen, D., 2007, The Influence of Credit Ratings on Coporate Capital Structure Decisions, *Journal of Applied Corporate Finance* 19, 56–64.

Kisgen, D., 2009, Do Firms Target Credit Ratings or Leverage Levels?, *Journal of Financial and Quantitative Analysis* 44, 1323–1344.

Kisgen, D., and P. Strahan, 2010, Do Regulations Based on Credit Ratings Affect a Firm's Cost of Capital?, *Review of Financial Studies* 23, 4324–4347.

Kliger, D., and O. Sarig, 2000, The Information Value of Bond Ratings, *Journal of Finance* 55, 2879–2903.

Kormendi, R., and R. Lipe, 1987, Earnings Innovations, Earnings Persistence, and Stock Returns, *Journal of Business* 60, 323–346.

Kraft, P., 2010, Do Rating Agencies Cater? Evidence from Rating-Based Contracts, Working paper, NYU Stern.

Lando, D., and A. Mortensen, 2005, On the Pricing of Step-Up Bonds in the European Telecom Sector, *Journal of Credit Risk* 1, 71–111.

Leland, H., 1992, Insider Trading: Should it be Prohibited?, *Journal of Political Economy* 100, 859–887.

Leland, H., 1994, Corporate Debt Value, Bond Covenants, and Optimal Capital Structure, *Journal of Finance* 49, 1213–1252.

Lizzeri, A., 1999, Information Revelation and Certification Intermediaries, *Rand Journal of Economics* 30, 214–231.

Longstaff, F., and E. Schwartz, 1995, Valuing Risky Debt: A New Approach, *Journal of Finance* 50, 789–820.

Manso, G., B. Strulovici, and A. Tchistyi, 2010, Performance-Sensitive Debt, *Review of Financial Studies* 23, 1819–1854.

Merton, R., 1974, On the Pricing of Corporate Debt: The Risk Structure of Interest Rates, *Journal of Finance* 2, 449–470.

Milgrom, P., and J. Roberts, 1990, Rationalizability, Learning, and Equilibrium in Games with Strategic Complementarities, *Econometrica* 58, 1255–1277.

Myers, S., and S. Turnbull, 1977, Capital Budgeting and the Capital Asset Pricing Model: Good News and Bad News, *Journal of Finance* 32, 321–332.

Opp, C., M. Opp, and M. Harris, 2010, Rating Agencies in the Face of Regulation: Rating Inflation and Regulatorty Arbitrage, Working paper, University of Pennsylvania Wharton School.

Quah, J., and B. Strulovici, 2010, Discounting and Patience in Optimal Stopping and Control Problems, Working paper, Northwestern University.

Ricciardi, L, L. Sacerdote, and S. Sato, 1984, On an Integral Equation for First-Passage-Time Probability Densities, *Journal of Applied Probability* 21, 302–314.

Sarkar, S., and F. Zapatero, 2003, The Trade-off Model with Mean Reverting Earnings: Theory and Empirical Tests, *Economic Journal* 111, 834–860.

Skreta, V., and L. Veldkamp, 2009, Ratings Shopping nad Asset Complexity: A Theory of Ratings Inflation, *Journal of Monetary Economics* 56, 678–695.

Subrahmanyam, A., and S. Titman, 1999, The Going-Public Decision and the Development of Financial Markets, *Journal of Finance* 54, 1045–1082.

Topkis, D., 1979, Equilibrium Points in Nonzero-Sum n-Person Submodular Games, *Siam Journal of Control and Optimization* 17, 773–787.

Treynor, J., and F. Black, 1976, *Corporate Investment Policy.* (Praeger Publishing Westport, CT).

Vives, X., 1990, Nash Equilibrium with Strategic Complementarities, *Journal of Mathematical Economics* 19, 305–321.

West, R., 1973, Bond Ratings, Bond Yields and Financial Regulation: Some Findings, *Journal of Law and Economics* 16, 159–168.

Wiemann, M., 2010, Rating triggers in loan contracts – how much influence have credit rating agencies on borrowers – in monetary terms, Working paper, Frankfurt School of Finance & Management.