

Beyond Q: Investment without Asset Prices

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Abstract

Empirical corporate finance studies often rely on measures of Tobin's Q to control for "fundamental" determinants of investment. However, since Tobin's Q is a good summary of investment behavior only under very stringent conditions, it is far better to instead use the underlying state variables directly. In this paper we show that under very general assumptions about the nature of technology and markets, these state variables are easily measurable and greatly improve the empirical fit of investment models. Even a general first or second order polynomial that does not rely on additional details about the nature of the investment problem accounts for a substantially larger fraction of the total variation in corporate investment than standard Q measures.

Keywords: Investment, Firm Size, Tobin's Q .

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Hayashi's (1982) famous elaboration of Brainard and Tobin's Q-theory has influenced the theory and practice of corporate and aggregate investment for nearly three decades. The prediction that Tobin's Q is a sufficient statistic to describe investment behavior has proved immensely popular among researchers, and the simple investment regressions implied by the linear-quadratic versions of the model form the basis for a myriad of empirical studies in economics and finance. Despite a long-standing consensus that Q is poorly measured and that the homogenous linear-quadratic model which motivates its use is misspecified, linear Q-based investment regressions still form the basis for most of inferences about corporate behaviors.¹ This practice is even harder to justify on empirical grounds since Tobin's Q accounts for very little of the variation in firm level investment.

In this paper we propose an alternative procedure to estimate investment equations under very general assumptions about the nature of technology and markets. Our methodology is not only superior theoretically and empirically, but also easier to implement and even applicable to private firms. Like others, our starting point is also a structural model of corporate investment behavior, but without the usual often counterfactual assumptions about homogeneity and perfect competition.² We exploit the fact that the optimal investment policy is always function of key state variables of the firm and can be approximated by a low order polynomial. Unlike marginal q, many of the state variables are directly observable or can be readily constructed from observables, under fairly general conditions.

Empirically, the main novelty of our approach is to identify firm size and sales (or cash flows) as the key state variables for optimal investment. Surprisingly, given its popularity in other empirical applications, firm size is often ignored in the investment literature, and if used, it usually shows up only as sorting variable for identification of

¹Q-based investment regressions, often augmented by various ad-hoc measures of cash flows, have been used to, among other purposes, test the importance of financial constraints, the effects of corporate governance, the consequences of having bad CEOs and the efficiency of market of signals.

²Possible departures from homogeneity due to technological and/or financial frictions include market power or decreasing returns to scale in production (Gomes, 2001; Cooper and Ejarque, 2003; Abel and Eberly, 2010), inhomogeneous costs of investment (Abel and Eberly, 1994, 1997; Cooper and Haltiwanger, 2006), and inhomogeneous costs of external financing (Hennessy and Whited, 2007).

financially constrained firms.³ We show, instead, that firm size naturally becomes an important determinant of investment whenever Tobin's Q is not a sufficient statistic, even in the absence of financial market frictions.

Our approach also clarifies the role of sales or cash flow variables. Contrary to the once popular use of these variables in tests of financing constraints, we show that they matter because they capture shocks to productivity and demand as well as any variations in factor prices. This interpretation is also suggested in Gomes (2001), and Cooper and Ejarque (2003), while Abel and Eberly (2010) focus instead on differences between marginal and average Q. In those studies however, the role of cash flow is often limited to rationalize the evidence based on misspecified Q-type investment regressions, and often considered marginal relative to Tobin's Q. Instead, we argue that cash flow or sales should always be treated as a primary determinant of investment, regardless of Tobin's Q, and even in the absence of capital market imperfections.

An important corollary of our paper is that investment can be studied without using market values. As a result, our methodology can be readily applied to study the behavior of private as well as public firms. As data on private firms is becoming more widely available this provides a significant advantage over existing Q-theory.⁴ By avoiding market values we also minimize the serious measurement concerns induced by potential stock market misvaluations (Blanchard, Rhee and Summers, 1993; Erickson and Whited, 2000), and approximations of unavailable market values of debt securities and nonphysical assets such as human capital, intangibles, and goodwill (Erickson and Whited, 2006). No doubt many of our proposed variables are also subject to measurement error, but this is likely to be far smaller than in Tobin's Q.

Our approach is not the first to propose the study of investment models without the use of data on market values. For example several authors have focused on using the Euler equation for investment alone.⁵ Another set of studies uses VAR-based forecasts for

³A notable exception is Gala and Julio (2011).

⁴Asker, Farre-Mensa and Ljungqvist (2011) offer an example on the difficulties of using Q-theory with non-traded firms.

⁵E.g. Shapiro (1986), Whited (1992), and Bond and Meghir (1994) among others.

the profitability of capital to estimate marginal q , which is then included in investment regressions.⁶ The relative merit of our approach, which is based on direct approximations of the true investment policy function, is that it imposes almost no restrictions on the functional forms of the stochastic discount factor and firm technologies, and allows for simple estimation methods.

Empirically, our state variable representation of the optimal investment policy is noticeably superior to that of standard Q-type investment regressions. Both firm size and sales account for more than twice as much as Tobin's Q of the within-variation in investment, and more than four times as much as Tobin's Q of the total explained investment variation. The empirical performance is even stronger in first differences: our core state variables can explain as much as eight times more than Tobin's Q of the overall variation in investment changes, and account for about 96 percent of the total explained variation in investment changes.

Finally, although we focus mainly on frictionless investment models, our approach can easily accommodate financial frictions. Intuitively, most deviations from the Modigliani-Miller theorem imply that the optimal investment policy often depends on an augmented set of state variables, which also includes financial leverage. We show how to pursue this extension by including different measures of financial leverage as state variables, in addition to firm size and sales. Similarly, we show how our methodology can easily handle more complex adjustment cost specifications like in Eberly, Rebelo and Vincent (2011), by including lagged investment as additional state variable for the optimal investment policy. Our polynomial approximation approach can also account for the impact of aggregate variation on investment, above and beyond the variation already incorporated in the measured firm level state variables, by augmenting the polynomial state variables with a complete set of time dummies.

In many ways our paper follows logically from the work of Erickson and Whited (2000, 2006 and 2011) and their ultimate conclusion that "Tobin's Q contains a great deal of measurement error because of a conceptual gap between true investment opportunities

⁶E.g. Abel and Blanchard (1986) and Gilchrist and Himmelberg (1995, 1998).

and observable measures”. Our suggestion here is to simply avoid the notoriously difficult problem of treating measurement error in the market value of a firm’s assets and limit any potential errors to the measurement of the firm’s capital stock, which contains substantially less noise (Erickson and Whited, 2006, 2011).

We believe our paper contributes to the literature in three significant ways. First, and foremost, it provides a superior empirical methodology to characterize firm level investment behavior. With a better empirical description at hand, we are able to quantify through a statistical variance decomposition the importance of various state variables, and corresponding class of investment models, for the overall variation in investment. Second, we provide a clearly articulated justification for the use of firm size as well as alternative flow variables, such as sales and operating profits, as determinants of investment. Although the latter are often informally motivated, there are very few formal arguments in the context of a very general investment model. Third, unlike misspecified Q-type investment regressions, our direct approximation of investment policy functions provides naturally more informative empirical moments for the identification and inference of the underlying structural parameters of the model. This is particularly useful for estimation of structural models via indirect inference methods.

The rest of our paper is organized as follows. The next section describes our general model, the implied optimal investment policies, and how they can be approximated empirically as function of the key state variables. We describe the data in Section 3, and present the main findings in Section III.. We discuss how to generalize the basic approach to accommodate labor market shocks, capital markets imperfections and alternative adjustment cost specifications in Section IV.. We conclude in Section V. with a brief discussion of the role of asset prices in estimating investment.

I. Modeling Investment

This section describes a general structural model of corporate investment that is suitable for empirical work. It provides guidance and impose discipline on the identification of

relevant state variables. Moreover, by being explicit about our detailed assumptions this section also serves to clarify and emphasize the exact differences with respect to the more restrictive Q-theory environments. Our model can be seen as a generalized version of Abel and Eberly (1994, 1997) and Caballero and Engel (1999), which allows for asymmetric, non-convex and possibly discontinuous adjustment costs, together with a general weakly concave technology that allows for decreasing returns to scale. This environment is flexible enough to include the large majority of investment models in the literature as special cases. For exposition purposes, we delay the introduction of additional features such as financial market imperfections, which are instead discussed in Section IV..

A. The General Model

We start by examining the optimal investment decision of a firm that seeks to maximize current shareholder value in the absence of any financing frictions, V . For simplicity, we assume that the firm is financed entirely by equity and denote by D the value of periodic distributions net of any securities issuance.

The operating cash flows or profits of this (representative) firm are summarized by the function Π defined as sales revenues net of operating costs. We formalize this relation as follows:

$$\Pi(K_t, A_t, W_t) = \max_{N_t} \{A_t F(K_t, N_t) - W_t N_t\}. \quad (1)$$

The function $Y_t = A_t F(K_t, N_t)$ denotes the value of sales revenues in period t , net of the cost of any materials. These revenues depend on the firm's capital stock and labor input, denoted by K_t and N_t , respectively. The variable A_t captures the exogenous state of demand and/or productivity in which the firm operates. W_t denotes unit labor costs, including wages, taxes and other employee benefits. Both W_t and A_t are allowed to vary stochastically over time, thus accommodating any variations to the state of the economy, or industry in which our firm operates. We now summarize our main assumptions about revenues and profits.

Assumption 1. Sales. The function $F : K \times N \rightarrow R_+$, (i) is increasing and concave in both K and N ; (ii) is twice continuously differentiable; (iii) satisfies $F(hK, hN, A) \leq hF(k, N)$ for all (K, N) ; and (iv) obeys the standard Inada boundary conditions.

The key item is (iii) which is a departure from the standard linear homogeneous model and explicitly allows for the possibility of decreasing returns to scale. It is straightforward to show that the function $\Pi(K, A, W)$ is also increasing and weakly concave in K .

Accumulation of physical capital requires capital expenditures, or investment, which is denoted by I_t , and takes some time. We formalize this idea of time to build, by assuming that current investment spending does not affect the current level of installed capacity and becomes productive only at the beginning of the next period:

$$K_{t+1} = (1 - \delta) K_t + I_t. \quad (2)$$

Moreover, there may be costs to adjusting the stock of capital. These are assumed to reduce operating profits and are summarized by the function $\Phi(\cdot)$. They depend on the amount of investment and the current stock of capital. Our assumptions about the adjustment cost function are described below.

Assumption 2. Adjustment Cost. The adjustment cost function $\Phi(\cdot) : I \times K \rightarrow R_+$ obeys the following conditions: it is (i) twice continuously differentiable for $I \neq I^*(K)$; (ii) $\Phi(I^*(K), K) = 0$; (iii) $\Phi_I(\cdot) \times (I - I^*(K)) \geq 0$; (iv) $\Phi_K(\cdot) \leq 0$; and (v) $\Phi_{II}(\cdot) \geq 0$.

Items (ii) and (iii) together imply that adjustment costs are non negative and minimized at the natural rate of investment $I^*(K)$. In most cases this is assumed to be either 0 or δK depending on whether one intends adjustment costs to apply to gross or net capital formation. Part (i) of Assumption 2 allows for the possibility of very general non-convex and indeed discontinuous adjustment costs. A general function that satisfies

these assumptions is:

$$\Phi(I, K) = \begin{cases} a^+ + p^+I + \frac{b^+}{v} \left(\frac{I - I^*(K)}{K} \right)^v & \text{if } I > I^*(K) \\ 0 & \text{if } I = I^*(K) \\ a^- + p^-I + \frac{b^-}{v} \left(\frac{I - I^*(K)}{K} \right)^v & \text{if } I < I^*(K) \end{cases} \quad (3)$$

where a^+ , a^- , p^+ , p^- , b^+ and b^- are all non-negative, and $v \in \{2, 4, 6, \dots\}$. We have non-convex and discontinuous fixed cost of investment when a^+ and/or a^- are positive. The linear cost of investment are discontinuous when the purchase price of capital, p^+ , differs from the sale price p^- , with $p^+ > p^- \geq 0$ reflecting fire-sales. We have asymmetric and convex costs of investment when b^+ differs from b^- and $v > 1$, with $b^- > b^+ \geq 0$ reflecting costly reversibility. The standard smooth quadratic adjustment costs are obtained as special case of (3) with $v = 2$, $a^+ = a^- = 0$, $p^+ = p^- > 0$, and $b^+ = b^- > 0$.

B. The Investment Decision

We can now define the sequence of optimal investment decisions by the firm as the solution to the following problem:

$$V(K_t, A_t, W_t, \Omega_t) = \max_{\{I_{t+s}, K_{t+s+1}\}_{s=0}^{\infty}} \mathbb{E}_t \left[\sum_{s=0}^{\infty} M_{t,t+s} D_{t+s} \right] \quad (4)$$

$$\text{s.t.} \quad \Pi(K_{t+s}, A_{t+s}, W_{t+s}) = D_{t+s} + \Phi(I_{t+s}, K_{t+s}) \quad (5)$$

together with the capital accumulation equation (2). $M_{t,t+s}$ is the stochastic discount factor between periods t and $t+s$, and Ω_t denotes the set of *aggregate* state variables summarizing the state of the economy. Although not explicitly modeled, the set of *aggregate* state variables may include *aggregate* shocks to productivity, wages, capital adjustment costs, relative price of investment goods, and representative household preferences.

B.1 Smooth Policies

Under the general conditions above, the value function, V , is not generally differentiable everywhere unless $a^+ = a^- = 0$, which we now impose.

When first-order conditions can be used to characterize the solution to (4), the optimal investment policy obeys:

$$q_t = \Phi_I(I_t, K_t) \quad (6)$$

where q_t is the marginal value of installed capital, or *marginal* q , which follows the law of motion:

$$q_t = \mathbb{E}_t [M_{t,t+1} (\Pi_K(K_{t+1}, A_{t+1}, W_{t+1}) + (1 - \delta) q_{t+1} - \Phi_K(I_{t+1}, K_{t+1}))]. \quad (7)$$

B.2 Investment Equations

Computing the optimal investment policies requires combining the expressions in (6) and (7), which does not generally yield an explicit closed form solution under the general conditions assumed here. Nevertheless, these policies can be further characterized by rewriting (6) as:

$$I_t/K_t = \tilde{G}(K_t, q_t). \quad (8)$$

Most of the literature follows Hayashi (1982) and assumes linear homogeneity for the functions $\Pi(\cdot)$ and $\Phi(\cdot)$ to obtain a linear investment equation from (6) under quadratic adjustment costs:

$$I_t/K_t = \alpha_0 + \alpha_1 q_t. \quad (9)$$

Under these assumptions q_t equals the Q-ratio between the market value and replacement cost of capital, and this equation can be estimated directly from the data. Unfortunately, this simple linear equation offers a poor fit to firm level data.

Under less restrictive conditions, however, marginal q is not directly observable. Nevertheless, as long as the process for the stochastic variables is Markov, the law of motion

(7) implies that the marginal value of installed capital can be written as $q_t = q(K_t, Z_t)$, where the vector Z captures possible shocks to firm productivity, firm output demand, firm wages, and aggregate state variables, i.e. $Z_t = (A_t, W_t, \Omega_t)$.

In general, the optimal rate of investment can then still be described by the following parsimonious state variable representation:

$$I_t/K_t = G(K_t, Z_t) \tag{10}$$

where the explicit form for the function $G(\cdot)$ depends on the specific functional forms of $\Pi(\cdot)$ and $\Phi(\cdot)$, and may not be readily available in most circumstances. However, given the measurability of investment, the unknown function $G(\cdot)$ can be directly approximated by polynomials in K and Z as long as these underlying state variables are also measurable.

B.3 Non-Smooth Policies

When there are also non-convex adjustment costs, i.e. $a^+ \neq a^- \neq 0$, the optimal investment policy still has the state variable representation in (10), but it is now discontinuous. In this case, the optimal investment policy may either be estimated in two branches, together with the endogenous point of discontinuity, or we may require several higher order polynomial terms to better capture the nonlinearities. We choose the latter to preserve uniformity in the presentation.⁷

C. Our Estimation Approach

Given the widely acknowledged failure of simple linear models, some authors have proposed slightly modified versions of equation (10) by relaxing some assumptions about technology and costs. This approach has often yielded improved results, but has generally remained close to the basic linear model.

⁷More generally, the optimal investment policy can also be estimated using a full non-parametric approach.

Instead of imposing additional conditions beyond those in Assumptions 1 and 2, we choose instead to *approximate* globally the general investment equation by using a polynomial version of the general function $G(K, Z)$. Specifically, the approximate tensor product representation for the optimal investment policy is:

$$\frac{I}{K} \simeq \sum_{i_k=0}^{n_k} \sum_{i_z=0}^{n_z} c_{i_k, i_z} k^{i_k} z^{i_z} \quad (11)$$

where $z = \log(Z)$ and $k = \log(K)$. We estimate the coefficients c_{i_k, i_z} via ordinary least squares approximation. These coefficients can then be used to further infer the underlying structural parameters of the model, for instance using indirect inference methods, or at the very least, place restrictions on the nature of technology and adjustment costs.

Several practical questions arise when implementing these approximations in empirical work. The first issue concerns the order of the polynomial. As shown below, in most cases we find that a second order polynomial in k and z is often sufficient, and higher order terms are generally not necessary to improve the quality of the approximation. Practically, we focus on tensor product polynomials of second order in k and z .

A related issue is whether to use natural or orthogonal polynomial terms in the approximation. We find that empirical estimates generally work better when we use orthogonal polynomials. Indeed, when using higher order polynomials multicollinearity can become a problem when attempting to obtain precise estimates for the parameters c_{i_k, i_z} . However, the use of orthogonal polynomials makes it more difficult to interpret the estimated coefficients and also to establish a link to the underlying structural model. Since we prefer to emphasize the overall fit of our model and are less concerned about the significance of individual coefficients, we report only the results based on natural polynomials.

C.1 Measurement

The final and most important issue concerns measurement of the state variables, particularly of the exogenous state z . Although this is not directly observable, we can use the theoretical restrictions imposed by our model to estimate it.

We discuss how to account for variation in the aggregate state variables, Ω , both in the next subsection and with further details in the “extensions” section. Similarly, we address firm-specific wage shocks, W , in the “extensions” section. For now, let us suppose that the only sources of uncertainty are in firm technology and demand (i.e. $z = \ln A$). In this case, we can estimate these shocks directly from observed sales in a way that emulates the literature on the construction of Solow residuals as:

$$z = y - \ln F(K, N) \tag{12}$$

where $y = \ln Y$. Effectively, this approach implies a two-stage estimation for our investment equation. First, we obtain estimates of the sales shocks z and then use these in the investment equation (11). In practice, this may be problematic for a number of reasons. First, we would need to assume a particular functional form for $F(\cdot)$ or at a minimum estimate the labor and capital elasticities, α_N and α_K . Second, we would also need a correction for the endogeneity bias in estimating z (see Olley and Pakes, 1996). This is because, as long as z is persistent, there is an endogenous correlation between capital input (the accumulation of past investments optimally chosen in response to past z 's) and current productivity so that estimates of α_K and α_N are generally inconsistent.

Fortunately however, we are only interested in estimating investment equations and not in identifying the capital and labor elasticities. Hence, we can avoid both of these problems by using (12) to replace the unobservable variable z in the investment equation (11) directly and work instead with:

$$\frac{I}{K} \simeq \sum_{i_k=0}^{n_k} \sum_{i_y=0}^{n_y} \sum_{i_n=0}^{n_n} b_{i_k, i_y, i_n} k^{i_k} y^{i_y} n^{i_n}. \tag{13}$$

Now the empirical investment equation is just a direct function of three observable variables including capital, sales and labor, and can be readily estimated from the data.⁸

Moreover, since the right hand side variables are all in logs, we can scale employment and sales by the size of the capital stock and estimate a version of (13) using $\ln(Y/K)$ and $\ln(N/K)$. This transformation generally allows us to make our results more directly comparable with the existing literature and is without loss of generality.

C.2 Time and Firm Fixed Effects

As discussed above, the *aggregate* state variables, Ω , can also be part of the exogenous state Z . To the extent that variation in these variables affects all firms equally, it can be easily captured by allowing the constant term $b_{0,0,0}$ in (13) to be time-specific.⁹ For easy of exposition and comparison with the existing literature, we focus our analysis on unobserved aggregate variation that enters only linearly the investment equation. In the section “Extensions”, we also allow for unobserved aggregate variation to enter in a non-additive fashion the investment equation, which we account for by introducing not only time-fixed effects, but also time-specific slope coefficients.

It is natural to expect differences in firms’ natural rate of investment, $I^*(K)/K$, mainly due to variations in the depreciation rates on their assets. We can capture firm heterogeneity in depreciation rates, i.e. $\delta = \delta_j$ in the capital accumulation (2), by allowing the constant term $b_{0,0,0}$ in (13) to be also firm-specific.

D. Discussion

The general polynomial representation (13) expresses optimal investment rates as function of firm size, employment and sales. Undoubtedly these variables are all measured with some error, but they are readily available and, we think, they are more reliable than

⁸The coefficients b_{i_k, i_z, i_n} are now convolutions of the elasticities α 's and approximation coefficients c 's.

⁹Of course additional industry-level fixed effects can also be used to capture industry-level state variables.

any available estimate of marginal q . Even more importantly, this empirical specification is very general and does not rely on the knife-edge and counterfactual assumptions that firms exhibit constant returns to scale in production and adjustment costs, while operating in perfectly competitive markets.

An immediate benefit of (13) is that it clearly identifies firm size and sales (or cash flows in later sections) as the core determinants of optimal investment, regardless of financial market imperfections. Except in Gala and Julio (2011), firm size is often neglected in the investment literature, and if used, it usually stands for either a catch-all variable to mitigate omitted variable bias in traditional investment regressions, or as sorting variable for identification of financially constrained firms.^{10,11} Here we show how size arises naturally as a key determinant of investment when Tobin's Q is no longer a sufficient statistic. On the other hand, sales (cash flows) capture variations in firm productivity, input prices and output demand. With few exceptions in the literature (for instance, Abel and Eberly, 2010), cash flow variables are generally used either as ad-hoc proxy for a firm's financial status (Fazzari, Hubbard, and Petersen, 1988; Hubbard, 1998) or interpreted as the by-product of mismeasurement in marginal q (see Erickson and Whited, 2000; Gomes, 2001; and Cooper and Ejarque, 2003).

The intuitive appeal of Tobin's Q relies on the belief that it serves as a forward-looking measure of investment opportunities that reflects information about expected future profitability and discount rates. Not often emphasized is the fact that this information is also incorporated in the characterization of the optimal investment policy as function of the underlying state variables.¹² The key difference resides on the fact that the policy function characterization is always correct, while the use of average Q relies on a very stringent set of assumptions.

¹⁰Gala and Julio (2011) exploit the empirically important inverse relationship between firm size and investment ("convergence") to identify the impact of technological decreasing returns. Using simulated method of moments, they estimate a neoclassical model of investment to replicate quantitatively the observed relationship between convergence in investment and decreasing returns to scale across industries.

¹¹From a pure econometric point of view, we may be concerned about the fact that firm size may seem to exhibit a trend in small samples. Theoretically, however, firm size always converges to a stationary long-run level with decreasing returns.

¹²Mathematically, the policy function approximation effectively parameterizes expectations of future marginal return to capital and discount rates as functions of current state variables.

Methodologically, our approach can also be viewed as a more straightforward implementation of earlier attempts to approximate marginal q with other measurable characteristics.¹³ However, approximating the optimal investment policy directly recognizes the empirical irrelevance of obtaining a preliminary estimate of marginal q . This helps increase the efficiency of our estimators. Unlike structural estimation of investment models via the Euler equation and/or simulated methods of moments, which also do not rely on measures of Tobin's Q , our direct investment policy function approximation imposes very few restrictions on the functional forms of the stochastic discount factor, operating profit and adjustment costs, and can be readily estimated using standard OLS and IV regressions rather than resorting to nonlinear estimation techniques.

Our methodology builds on the consideration that any model is described not only by its restrictions on functional forms, but also, and most importantly, by its state variables. Different classes of investment models do often correspond to different sets of state variables. As such, our approach can help quantifying the importance of various classes of investment models through a statistical variance decomposition of their corresponding state variable representation of investment. Such variance decomposition also provides a useful empirical benchmark to compare with the model-implied variance decomposition and assess the validity of alternative investment models.

Moreover, even within classes of models sharing the same set of state variables, our representation offers a more informative set of moments for the estimation of structural models than misspecified Q -type investment regressions: the direct estimation of the model-implied investment policy function provides empirical estimates that are likely to be more informative to infer underlying structural parameters using indirect inference methods.

¹³Abel (1980) and Shapiro (1986) parameterize the marginal cost of investment to estimate marginal q using the Euler equation in (7). Abel and Blanchard (1986), and Gilchrist and Himmelberg (1995, 1999) propose solving (7) forward and estimate q using VAR-based forecasts of the expected future marginal revenue product of capital and marginal adjustment cost.

***E.* Econometric Issues: Simultaneity, Measurement Error, and Omitted Variables**

Direct estimation of the policy function implied by a structural model has another important benefit: it avoids many of the endogeneity problems that are endemic in reduced form methods (see Roberts and Whited, 2012).

In theory, the firm’s market value and its investment are determined simultaneously as managers respond optimally to the underlying economic fundamentals. Therefore, simultaneity concerns naturally arise in standard Q-type investment regressions. By construction however all state variables are *pre-determined* when current investment decisions are made. Therefore, direct estimates of the policy function do not suffer from any direct endogeneity problems. From this perspective, our approach is also more suitable than Q-type investment regressions as a baseline for establishing causal effects.¹⁴

In common with alternative methodologies for estimating investment models via the Euler equation and/or simulated methods of moments, our methodology also minimizes the measurement error concerns induced by potential stock market misvaluations (Blanchard, Rhee and Summers, 1993; Erickson and Whited, 2000), and approximations of unavailable market values of debt securities and nonphysical assets such as human capital, intangibles, and goodwill (Erickson and Whited, 2006).

As any structural model and/or econometric specification of investment suffers from omitted variables problems to one degree or another, there may also be important state variables omitted from our policy function approximation. Failing to include an important state variable however simply means that we need to enrich the baseline structural model in Section I.. Section IV. investigates the relative importance of additional state variables as implied by alternative class of investment models. For instance, if the true structural model was a version of Hennessy and Whited (2007), we would need to add financial leverage as a state variable.

Finally, is there a reason to still look at Tobin’s Q? Theoretically, if the structural

¹⁴More formally, state variables are sequentially exogenous (see Wooldridge, 2002, Chapter 11).

model is well specified the answer is clearly no. Empirically, however, we may want to consider that Tobin's Q remains an endogenous variable in any class of investment models, and as such it may contain valuable information about any omitted state variables.

Thus, whenever measurement error in market values does not represent a major concern, one could consider using Tobin's Q to complement the structural policy function approximation and diagnose errors in model specification or capture additional variation in unobserved firm-specific and time-varying state variables.

II. Data

Our data comes from the combined annual research, full coverage, and industrial COMPUSTAT files. To facilitate comparison with much of the literature our initial sample is made of an unbalanced panel of firms for the years 1972 to 2010, that includes only manufacturing firms (SIC 2000-3999) with at least five years of available accounting data. Later, we also perform the analysis on a broader cross-section firms and using different time periods.

We keep only firm-years that have non-missing information required to construct the primary variables of interest, namely: investment, I , firm size, K , employment, N , and sales revenues, Y . We also include standard measure of Tobin's Q and cash flow, CF , to report comparative results. These variables are constructed as follows. Firm size, or the capital stock, is defined as net property, plant and equipment. Investment is defined as capital expenditures in property, plant and equipment. Employment is the reported number of employees. Sales are measured by net sales revenues. Cash flow is defined as earnings before extraordinary items plus depreciation. In our implementation these last four variables are all scaled by the beginning-of-year capital stock.¹⁵ Finally, Tobin's Q is measured by the market value of assets (defined as the book value of assets plus the market value of common stock minus the book value of common stock) scaled by the

¹⁵Below we discuss extensions that include several measures of cash flow, CF/K , and leverage, B/K .

book value of assets.^{16,17}

Our sample is filtered to exclude observations where total capital, book value of assets, and sales are either zero or negative. To ensure that our measure of investment captures the purchase of property, plant and equipment, we eliminate any firm-year observation in which a firm made an acquisition. Finally, all variables are trimmed at the 1st and 99th percentiles of their distributions to reduce the influence of any outliers, which are common in accounting ratios. This procedure yields a base sample of 32,890 firm-years observations. Table I reports summary statistics including mean, standard deviation and main percentiles for the variables of interest.

III. Findings

We now describe our main findings. We first examine the variation of investment rates across portfolios sorted by firm size, sales-to-capital ratio and employment-to-capital ratio. We proceed performing formal tests of our state variable representation for investment.

A. Investment Rates by State-Variables Portfolios

To gain some insights about the role of size, sales and employment in determining investment rates, we first sort all firms into separate decile portfolios, which are rebalanced every year. Table II reports both equal-weighted and capital-weighted mean annual investment rates for each of these portfolios. Panel A shows that investment rates decline significantly with size, with smaller firms growing about 70 percent faster than larger ones. Panel B shows that investment rates increase monotonically with the sales-to-

¹⁶We obtain similar results when using alternative proxies of Tobin's Q including its definition as market value of equity plus debt minus inventories scaled by capital. We measure the capital stock using (i) its book value (i.e. property, plant and equipment), and (ii) its replacement value estimated with the perpetual inventory algorithm as in Whited (1992). Results are available upon request.

¹⁷Erickson and Whited (2006) show that using a perpetual inventory algorithm to estimate the replacement cost of capital and/or a recursive algorithm to estimate the market value of debt barely improves the measurement quality of the various proxies for Q, while depleting the number of usable observations.

capital ratio. The equal-weighted average investment rate for firms in the highest decile is nearly 3 times that of firms in the lowest decile. Panel C shows that investment rates also increase with the employment-to-capital ratio. The equal-weighted average investment rate ranges from 20 percent for firms in the lowest decile to about 37 percent for firms in the highest decile.

Table III reports the variation in the equal-weighted average investment rates across portfolios double-sorted on the empirical distribution of the variables of interest. Specifically, Panel A shows the average investment rates across 25 portfolios double-sorted on the empirical distribution of the sales-to-capital ratio conditional on firm size. For each firm size quintile, investment rates increase monotonically with the sales-to-capital ratio. Within the smallest firm size quintile, the average investment rate for firms in the highest sales-to-capital ratio quintile is nearly 3 times that of firms in the lowest quintile. Within the largest firm size quintile, firms with the highest sales-to-capital ratio grow about 50 percent faster than firms in the lowest quintile. Panel B shows the average investment rates across 25 portfolios double-sorted on the empirical distribution of the employment-to-capital ratio conditional on firm size. The pattern in average investment rates across portfolios is similar to that of portfolios sorted on sales-to-capital ratio and firm size. These relations are statistically and economically significant across portfolios.

Thus, both single-sort and double-sort portfolio analysis reinforce our belief that our underlying state-variable representation captures a substantial variation in investment rates. We now turn to formally testing the state-variable representation of investment in a regression framework.

B. Investment Equations

In this section we investigate the empirical relevance of the investment polynomial representation and compare its performance with conventional Q investment regressions.¹⁸

¹⁸Since we prefer to emphasize the overall fit of our model and its variance decomposition, and are less concerned about establishing causal effects, we report OLS rather than IV estimates of the polynomial approximations. Such a choice eliminates any degrees of freedom in affecting the measure of overall fit

In so doing, we can assess quantitatively the importance of deviations from the standard assumptions about homogeneity and perfect competition that motivate the use of Tobin’s Q as sufficient statistic for investment.

B.1 Choice of Investment Polynomial

We now formally estimate and test the state-variable representation of investment in (13). Several practical issues arise when implementing the investment approximation empirically. The first issue concerns the number of variables with “independent” information about the true state variables to include in the approximation. The second issue concerns the order of the polynomial. Our goal is to choose a parsimonious polynomial representation both in terms of variables and order of approximation that provides the best overall fit for investment empirically.

Table IV reports the estimates for various specifications of the investment polynomial regression always including firm and time fixed effects. The first three columns present results of the investment approximation using only firm size and sales-to-capital ratio for first order, second order (excluding interaction term) and second order complete polynomials, respectively. Including second order terms in firm size and sales-to-capital ratio, except for the interaction term that is not statistically significant, increases the overall fit for investment. The next three columns report instead the investment approximation using firm size, sales-to-capital ratio, and employment-to-capital ratio for first order, second order (excluding interaction term) and second order complete polynomials, respectively. First and second order terms in all variables are strongly statistically significant, except for the second order term in the employment-to-capital ratio, which is significant only at the 10 percent level. Interaction terms among the variables are generally not statistically significant and do not improve much the quality of the approximation as witnessed by the virtually unchanged adjusted R^2 . We omit higher order terms in the polynomial representation because they are not statistically significant and are generally not necessary to improve the quality of the approximation. Overall, while including sec-

by cherry-picking the set of available instruments.

ond order terms improves the approximation regardless of the variables selection, adding the employment-to-capital ratio in the polynomial leaves instead virtually unaffected the overall fit for investment. Hence, we focus on the second order polynomial approximation in firm size and sales-to-capital ratio (column 2) as the best parsimonious state-variable representation of investment empirically. We now provide comparisons with standard Q-type regressions estimated using both fixed effect and first difference estimators.

B.2 Fixed Effect Estimators

We compare the empirical performance of the investment polynomial and standard Q regressions. Table V reports the fixed effect (within-transformation) estimates for various specifications of the investment regression always including firm and time fixed effects. The first column presents results for the standard investment specification including only Tobin’s Q. According to the neoclassical theory of investment (Hayashi, 1982; Abel and Eberly, 1994), homogeneity of equal degree of a firm’s operating profit and investment cost functions makes Tobin’s Q proportional to marginal q , and hence a sufficient statistic for investment. The coefficient estimate on Tobin’s Q is positive and significant as predicted by the neoclassical investment theory, but variation in Q can only account for 12 percent of the within-variation in investment rates.

To account for potential nonlinearities in the relationship between investment and Tobin’s Q, we add higher order terms in Q. As shown in the second column, including a quadratic term in Q, while statistically significant, has only a negligible impact on explanatory power with an adjusted R^2 of about 13 percent. We omit for brevity statistically insignificant higher order terms.

In the third column we report for easy of comparison our parsimonious state-variable representation of investment including only linear and quadratic terms in firm size and sales-to-capital ratio. Consistently with the convergence findings in Gala and Julio (2011), firm size is negatively related to investment rates.¹⁹ The coefficient on the sales-

¹⁹In the baseline polynomial approximation we scale end-of-year investment by beginning-of-year capital on the left hand side, and include the log of beginning-of-year capital on the right hand side of the

to-capital ratio - our proxy for productivity shocks - is positively related to investment as predicted by the neoclassical theory of investment. The quadratic terms are also significant and positively related to investment. The polynomial representation of investment as function of the underlying state-variables spanned by firm size and sales-to-capital ratio can account alone for up to 24 percent of the within-variation in investment rates. Thus, our state-variable representation of investment outperforms by an order of magnitude the conventional Q-investment regression (about 100 percent increase in adjusted R^2) implied by the standard homogeneous model.²⁰

In the fourth and fifth columns, we add linear, and linear plus quadratic terms in Q to our state-variable representation of investment, respectively. The inclusion of these variables does not affect the significance of the polynomial representation of investment and only leads to an increase in the adjusted R^2 of about 15 percent starting from a value of 24 percent up to 28 percent. Hence, Tobin's Q, while not redundant, incorporates only little valuable information for investment beyond the information already captured by the measurable fundamental state variables. We further quantify the contribution of Tobin's Q to the overall firm level investment variation in the next subsection.

B.3 First Difference Estimators

In Table VI we report the first-difference estimates for the same investment regressions reported in Table V. The first difference specification mitigates concerns about the presence of serially correlated disturbance terms in empirical investment equations when run in levels. When estimated in first-differences the overall significance of the state-variable representation of investment is further reinforced. The polynomial representation of

regression. One potential concern is that the negative coefficient on log of firm size may be mechanically driven. However, we obtain similar results, which we omit for brevity, when using past lags of capital stock either in place of or as instrument for beginning-of-period capital stock. All results are available upon request.

²⁰Even when we follow the extant literature and arbitrarily add ad-hoc cash flow measures to the conventional Q-investment regression, the overall explanatory power raises up only to 19 percent. As such, misspecified cash-flow augmented Q-investment regressions also underestimate (relative to our state-variable approximation) the overall empirical relevance of deviations from conventional homogeneity assumptions. All results are available upon request.

investment as function of the underlying state-variables spanned by firm size and sales-to-capital ratio can account alone for up to 30 percent of the variation in investment rate changes. This is in sharp contrast with Q-investment regressions, which can only account for up to 4 percent of the variation in investment rate changes. Hence, the gain in adjusted R^2 when using the state-variable representation of investment is even larger in first-differences: about 7.8 times higher than the Q-investment specification. Furthermore, adding linear and/or quadratic terms in Q to our state-variable representation of investment (specification 4 and 5) leaves virtually unchanged the overall significance and explanatory power.

C. Variance Decomposition

To understand the relative importance of various variables in capturing investment variation we follow the analysis of covariance (ANCOVA) in Lemmon and Roberts (2008). To do so we estimate the empirical model of investment:

$$\frac{I_{jt+1}}{K_{jt}} = \beta X_{jt} + \delta_j + \eta_t + \varepsilon_{jt+1} \quad (14)$$

where δ_j is a firm fixed effect and η_t is a year fixed effect. X denotes a vector of explanatory variables that includes various combinations of Tobin's Q, cash flow, and the state-variable polynomial in firm size, $\ln K$, and sales-to-capital ratio, $\ln Y/K$.

Table VII reports the results of this covariance decomposition for several specifications. Each column in the table corresponds to a different specification for investment. The numbers reported in the table, excluding the adjusted R^2 reported in the last row, correspond to the fraction of the total Type III partial sum of squares for a particular model.²¹ That is, we normalize the partial sum of squares for each effect by the aggregate partial sum of squares across all effects in the model, so that each column sums to one. Intuitively, each value in the table corresponds to the fraction of the model sum of

²¹We use Type III sum of squares because (i) the sum of squares is not sensitive to the ordering of the covariates, and (ii) our data is unbalanced (some firms have more observations than others).

squares attributable to a particular effect (i.e. firm, year, Q, cash flow, etc.). Panel A of Table VII provides estimates based on fixed effect estimators, while Panel B reports estimates based on first difference estimators.

As shown in Panel A, column 1, firm and year fixed effects capture 28 percent of the variation in investment rates, of which 84 percent can be attributable to firm fixed effects alone. As discussed above, firm fixed effects capture cross sectional variation in the depreciation rate, δ_j , which is equal to the long run or steady-state investment rate. As such, we should expect firm fixed effects to account for a large variation in investment rates.²² Moreover, unlike studies on the empirical determinants of leverage, for example, these fixed effects have strong economic (as opposed to purely statistical) content supporting their empirical significance. Year fixed effects, which capture unobserved aggregate variation, account instead for, at most, only 16 percent of the total explained variation in investment.

Augmenting the firm and time fixed effects specification with Tobin's Q increases the total adjusted R^2 for investment from 28 percent to 33 percent. However, only 8 percent of the explained sum of squares captured by the included covariates can be attributed to Tobin's Q.

Column 3 shows the variance decomposition associated with the state-variable polynomial, including only fixed effects. To highlight the significant incremental contribution of the state-variable representation in accounting for variation in investment, we notice that the adjusted R^2 increases from 28 percent to 42 percent. About 1/4 of this overall variation can now be attributed to the state-variable polynomial. And in fact, adding Tobin's Q to our core state variables only increases the adjusted R^2 marginally from 42 percent to 45 percent. Most importantly, only 7 percent of the overall variation can now be attributed to Tobin's Q, while 32 percent is attributable to the state-variable polynomial. Overall, our parsimonious state-variable polynomial accounts for more than four times as much as Tobin's Q of the total explained variation in investment.

²²In fact, most investment models predict that in the long run *all* cross-sectional variation in investment is captured by δ_j alone.

The variance decomposition of investment rates changes in Panel B contributes to strengthen the previous results. Year fixed effects alone can only capture about 1 percent of the variation in investment rate changes. Augmenting the time fixed effects specification with Tobin’s Q increases the adjusted R^2 for investment changes only marginally from 1 to 4 percent.

Instead, our state-variable polynomial including only fixed effects accounts for up to 30 percent of the total variation in investment changes, of which 98 percent are attributable to the state-variable polynomial alone. Adding Tobin’s Q to our core state variables only marginally increases the adjusted R^2 to 31 percent. Most importantly, Tobin’s Q accounts only for 2 percent of the overall explained variation in investment changes, which is as much as year fixed effects, while 96 percent is attributable to our core state variables. Overall, Tobin’s Q incorporates only very little “independent” information for investment beyond the information already captured by our fundamental state variables.

D. Alternative Samples

Capital-intensive manufacturing firms form probably the most reliable panel for this study, but it is nevertheless interesting to examine how both approaches would perform on different samples. Table VIII reports the results for three alternative panels. The first column looks at a panel that now includes all firms except those in the financial sector, regulated utilities and public services. The second shows the results for a panel covering only the period between 1982-2010, which many authors focus on.²³ The third column reports the results for a balanced panel of manufacturing firms during the period 1982-2010.

Adding non-manufacturing firms substantially expands the sample and the statistical significance of our estimates, but it does not affect the overall goodness of fit. On the other hand, eliminating the first ten years of data from our baseline sample slightly improves overall performance. The main results are also confirmed on the smaller balanced sample,

²³Several different subsamples were also examined without noticeable changes in the findings. All results are available upon request.

which shows that our findings are not driven by the attrition in database due to firms' entry and exit.

IV. Extensions

In this section we extend the basic methodology to address the impact on investment of (i) unobserved aggregate state variables entering the investment equation in a non-additive fashion; (ii) firm-specific shocks to the price of variable inputs of production such as wages; (iii) financial market frictions; and (iv) alternative adjustment cost specifications such as those proposed in Eberly, Rebelo and Vincent (2011).

A. Aggregate Shocks and Time-Specific Coefficients

A complete state-variable representation of investment in (11) would also include the *aggregate* state variables, Ω , as part of the exogenous state Z . The set of *aggregate* state variables affecting investment may include, among others, *aggregate* shocks to productivity, wages, capital adjustment costs, relative price of investment goods, and representative household preferences. While the measurement of our firm level state variables, like sales and size, already captures part of the variation in these underlying aggregate state variables, there may still be substantial investment variation attributable to omitted variation in these aggregate state variables. For instance, part of the variation in aggregate productivity shocks would be captured by our measure of firm level productivity, however, aggregate productivity shocks may still affect firm investment indirectly through the stochastic discount factor, M , by affecting risk premia.

Given a large panel of firms, the complete knowledge of the aggregate state variables in Ω is not necessary for the purpose of estimating investment. In fact, we can capture the impact of all unobserved aggregate variation, above and beyond the variation already incorporated in the measured firm level state variables, by allowing not only for time fixed effects, but also for time-specific polynomial slope coefficients. While introducing time

fixed effects in the approximate investment equation captures unobserved aggregate variation that affects all firms equally, including time-varying polynomial slope coefficients also accounts for unobserved variation in the aggregate state variables that affects firms differently.

More precisely, allowing for time-specific polynomial coefficients in our baseline firm level state variables, k and y , is equivalent to a tensor product polynomial representation of investment which includes a complete set of time dummies, η , as state variables:

$$\frac{I_{jt+1}}{K_{jt}} \simeq \sum_{i_k=0}^{n_k} \sum_{i_y=0}^{n_y} \sum_{i_\eta=0}^{n_\eta} b_{i_k, i_y, i_\eta} \times k_{jt}^{i_k} \times y_{jt}^{i_y} \times \eta_t^{i_\eta} = \sum_{i_k=0}^{n_k} \sum_{i_y=0}^{n_y} d_{i_k, i_y, t} \times k_{jt}^{i_k} \times y_{jt}^{i_y} \quad (15)$$

where the equality follows from the fact that $\eta_t^{i_\eta} = \eta_t$ for any $i_\eta \geq 0$, and $d_{i_k, i_y, t} \equiv \eta_t \times \sum_{i_\eta=0}^{n_\eta} b_{i_k, i_y, i_\eta}$. Given the correct set of firm level state variables, allowing for time-specific polynomial coefficients fully captures all relevant unobserved variation in aggregate economic conditions.

We implement empirically the baseline polynomial approximation with time-specific coefficients including firm fixed effects. Table IX reports the estimates of the average partial effects for each firm-level state variable in the polynomial approximation. The first column provides estimates based on the benchmark unbalanced panel of manufacturing firms for the period 1972-2010. The average coefficients are in line with previous estimates for the same polynomial without time-specific slopes as reported in column (4) of Table V. The baseline firm level state variables, namely sales and size, are on average significant, except for the second order term in firm size. The introduction of time-specific slopes, while allowing for a more flexible investment specification, trades off efficiency for goodness-of-fit. Figure 1 plots the time-specific coefficients on each of the polynomial terms along with the corresponding 95 percent confidence intervals. Slope coefficients exhibit substantial variation over time, with estimates preserving their sign over the sample period, except for few, mainly statistically insignificant, exceptions. The efficiency loss in the estimation of the time-specific polynomial is compensated, however, only with a marginal improvement in the overall fit for investment. Even though allowing

for time-specific coefficients increases only marginally the total explained sum of squares (R^2 moves from 42 percent to 43 percent only), the fraction attributable to the state variable polynomial representation increases from 25 percent up to 37 percent as shown by the variance decomposition in Table XIII (column 2).

The different size of each cross section may affect the efficiency and the overall goodness-of-fit for our previous estimates, which are based on the unbalanced panel. For comparison, we provide the time-specific estimates based on the balanced panel of manufacturing firms for the period 1982-2010 in the second column of Table IX and Figure 2. The average coefficients in Table IX are consistent both in terms of magnitude and significance with previous estimates for the same polynomial without time-specific slopes as reported in column (3) of Table VIII. As shown in Figure 2, the slope coefficients exhibit substantial time variation also in the balanced sample. The point estimates tend to consistently preserve their sign over time, with very few exceptions. Differently from the unbalanced panel, the added flexibility of the time-varying polynomial produces now an overall better fit for investment.

B. Labor Market Shocks and Cash Flow Data

The use of employment data in equation (13) raises two issues. The first is practical. Data on employment is relatively sparse in Compustat and data on hours is simply not available. The second concern is theoretical. If firm-specific wage shocks are very important, then $Z \neq A$ and equation (13) can actually be misspecified.²⁴

Although we think this is unlikely for most firms, the profit maximizing equation (1) does offer one possible way to address this concern. Whenever the labor share is constant, we can use the definition of operating profits in (1) and construct the unobserved state variable z instead from

$$z = \pi - \theta_K k \tag{16}$$

²⁴Recall that time and industry fixed effects will capture any other variation in wages and indeed in the price of any other variable input of production.

where $\pi = \log \Pi$, and z now captures both the productivity/demand, A , and wage shock, W , and θ_K is the share of capital in operating profit. In this case, the empirical investment equation can be rewritten as:

$$\frac{I}{K} \simeq \sum_{i_k=0}^{n_k} \sum_{i_\pi=0}^{n_\pi} a_{i_k, i_\pi} k^{i_k} \pi^{i_\pi} \quad (17)$$

which now can be implemented using data on operating profits and stock of capital. Imposing a constant labor share, however, restricts the production function to be Cobb-Douglas, which is generally not at odds with the empirical evidence.²⁵

Table X implements our polynomial approximation using several alternative measures of cash flows. The first column reports the results of estimating equation (17) using the classic measure of cash flow (earnings before extraordinary items plus depreciation). The next two columns are estimated using measures constructed from the uses and sources of funds. In the second column, we adjust operating income for investments in working capital and, more significantly, for non-recurring charges as in Lewellen and Lewellen (2011). Estimates in the third column use a measure that further adds interest and related expense and subtracts nonoperating income (expense). The main downside about using these measures of cash flow is the reduced sample size, given that data on sources and uses of funds are not always readily available. However, at least in our specifications, this does not have any noticeable impact on the results. The final column in the table returns to the more widely available income statement data, which defines cash flow as simply operating income plus depreciation expenses.

Although there are some differences across the various measures of cash flows, point estimates, levels of significance and goodness of fit are all substantively comparable. Overall, we find that these specifications perform slightly less well than the baseline specification which includes firm sales instead of cash flow. However, the variance decomposition in Table XIII (column 3) shows that, even though using cash flow instead

²⁵Let $F(K, N) = K^{\alpha_K} N^{\alpha_N}$. Profit maximization implies that, $\Pi(K, Z) = ZK^{\theta_K}$, where $\theta_K = \alpha_K / (1 - \alpha_N)$, and $Z = (1 - \alpha_N) [A (\alpha_N / W)^{\alpha_N}]^{\frac{1}{1 - \alpha_N}}$. This includes shocks to productivity, output demand, as well as variations in factor prices.

of sales reduces the total explained sum of squares (R^2 moves from 42 percent to 40 percent), about 35 percent of it can be attributed to the covariation with the polynomial in cash flow and size alone.

C. Capital Market Imperfections and Leverage

For clarity of exposition, our structural model in Section I. purposely avoids any discussion of financial market imperfections and assumes that the Modigliani-Miller theorem holds. An important benefit of this simplified environment is that it demonstrates very clearly how we can generate cash flow and size effects purely as result of adopting more realistic technologies (as in Gomes, 2001; and Abel and Eberly, 2010, for example).

Since our approach does not rely on the estimation of marginal q , it can naturally accommodate modern models where marginal q is no longer a sufficient statistics for investment, such as models with financial frictions, by allowing for the presence of additional state variables in the optimal investment policy, $G(\cdot)$.

While an exhaustive analysis of the impact of financial market imperfections on investment is beyond the scope of this paper, it is well known that most modifications of the firm problem in (4), that allow for such plausible frictions as tax benefits of debt, collateral requirements and costly external financing, often imply that debt, B , becomes an additional state variable for the optimal investment policy, so that:²⁶

$$I/K = G(K, B, Z). \tag{18}$$

It follows that we can generalize our procedure to derive empirical investment equations that now include leverage terms as additional state variables.²⁷

²⁶Examples of such models with capital market imperfections where net financial liabilities represents an additional state variable for the optimal investment policy include Whited (1992), Bond and Meghir (1994), Gilchrist and Himmelberg (1999), Hennessy, Levy, and Whited (2007), Bustamante (2011), and Bolton, Chen, and Wang (2012), among others.

²⁷Without loss of generality, we normalize debt by the stock of capital and use B/K in the empirical analysis.

Table XI shows the results of introducing leverage to our baseline state variable approximation of investment. Like our measure of cash flow, there are several measures of the (book) leverage ratio. In Table XI, we only report results for the two most widely used measures of leverage.²⁸ The first column measures debt as the sum of short-term plus long-term debt, while the second column uses a measure of net leverage, by subtracting cash and short-term investments from debt.

Our estimates show that all leverage terms are generally statistically significant confirming that there is indeed some degree of interaction between financing and investment decisions of firms. The negative point estimates are also generally consistent with theoretical restrictions imposed by most models of financing frictions.

The variance decomposition in Table XIII, however, raises some concerns about the quantitative importance of leverage for the investment decisions of this broad cross-section of firms. A Type III variance decomposition for the state variables approximation including net leverage (column 4) shows that only 3 percent of the explained sum of squares can be captured by the covariation with firm leverage.

This marginal contribution indicates that leverage per se does not contain much valuable information beyond the information identifiable through firm size and sales. However since leverage is itself an endogenous variable, it can still affect investment indirectly through size and sales. As such, we do not interpret this result as evidence against capital market imperfections. It is also possible that leverage may be of greater importance for some specific subsets of firms and/or different formulations of the structural model may lead to more precise restrictions on the form of investment equations.²⁹

²⁸Using several alternative measures of leverage does not alter the main findings. These results are available upon request.

²⁹Gala and Gomes (2012) explore the implications of several alternative financing constraints on the empirical investment equation and estimate it on various sub-panels.

D. Alternative Adjustment Costs and Lagged Investment

Our investigation of the role of leverage can be further expanded to discuss the issue of econometric specification more broadly. Time-to-build, non-smooth adjustment costs, and even labor market distortions, can all lead to more complex forms of investment equation (10).

Notably, Eberly, Rebelo and Vincent (2011) provide some evidence that lagged investment is empirically the most significant determinant of contemporaneous investment, and attribute this to more elaborate adjustment cost functions, $\Phi(\cdot)$, such as those proposed in Christiano, Eichenbaum, and Evans (2005).³⁰ Alternatively, building lags offer also an explanation to the documented serial correlation in investment expenditures and a possible micro foundation for more complex adjustment costs.³¹

Our approach can naturally accommodate such frictions as time-to-build and more complex adjustment cost specifications like in Eberly, Rebelo and Vincent (2011), by including lagged investment as additional state variable in the optimal investment policy, $G(\cdot)$.

We investigate the role of lagged investment in Tables XII and XIII. To address endogeneity issues in dynamic panel data with a lagged dependent variable, we instrument lagged investment with prior two lags of its first-difference. Table XII provides the results of several investment specifications including lagged investment. We report for comparison the investment specification in Eberly, Rebelo and Vincent (2011), which includes lagged investment and cash flow to a conventional Q-type regression. Consistent

³⁰Eberly, Rebelo and Vincent (2011) specify capital adjustment costs in the capital accumulation equation, and assume a formulation that depends on the growth rate of investment. Specifically, they use the following linear-quadratic adjustment cost function:

$$\Phi(I_t, I_{t-1}) = \left[1 - \xi \left(\frac{I_t}{I_{t-1}} - \gamma \right)^2 \right] I_t$$

where ξ controls the size of the adjustment cost and γ denotes the firm steady-state growth rate. This adjustment cost specification makes lagged investment, I_{t-1} , a relevant state variable for the firm optimal investment policy.

³¹Of course the lagged investment effect can also reflect the presence of more complicated, i.e. non-Markov, exogenous shocks.

with the evidence in Eberly, Rebelo and Vincent (2011), lagged investment enters with a positive and significant coefficient, and increases the overall fit to investment. Table XII also reports the estimates of our baseline state variable approximation augmented with lagged investment. While lagged investment enters significantly, its inclusion affects neither the point estimates nor the significance of our baseline state variable approximation, and only increases the total adjusted R^2 from 42 percent to 46 percent. Table XIII provides its variance decomposition (column 5). Even though lagged investment remains an important determinant, its share of the total explained variation in investment is less than one third of that accounted by our core state variables, which still capture a large fraction of the explained variation in investment.³²

V. Conclusion

This paper questions the widespread use of Q-ratios in empirical work. Instead, we propose an asset price-free alternative relying on the insight that the optimal investment policy is a function of much more easily measurable state variables. Under very general assumptions about the nature of technology and markets, our approach ties investment rates directly to firm size and either sales or cash flows, even in the absence of financial market frictions. Our methodology can also account for capital markets imperfections by augmenting the set of measurable state variables with observable leverage. Similarly, it can accommodate such frictions as time-to-build and more complex adjustment cost specifications like in Eberly, Rebelo and Vincent (2011), by including lagged investment as additional state variable for the optimal investment policy. Furthermore, our approach can easily account for the impact of aggregate variation, above and beyond the variation already incorporated in the measured firm level state variables, by augmenting the set of state variables in the polynomial approximation with time dummies. We show that the empirical performance of our methodology is noticeably superior to that of standard Q-based investment equations.

³²These results are virtually unchanged when leverage is included to the state variable approximation of investment.

Are measures of Tobin's Q at all informative? Measures of Tobin's Q may still contain valuable information beyond the information identifiable through measurable state variables. As such, whenever measurement error in market values does not represent a major concern, we can always use Tobin's Q as a useful catch-all variable to account for omitted state variables rather than as primary determinant of investment. Indeed, using Tobin's Q as "residual" variable in investment regressions may capture relevant variation in unobserved firm-specific time-varying state variables including, for instance, firm-specific persistent shocks to input adjustment costs. However, the empirical evidence suggests otherwise.

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Table I: Summary Statistics

This table reports summary statistics for the primary variables of interest from Compustat over the period 1972-2010. Investment rate, I/K , is defined as capital expenditures in property, plant and equipment scaled by the beginning-of-year capital stock. The capital stock, K , is defined as net property, plant and equipment. Firm size, $\ln(K)$, is the natural logarithm of the beginning-of-year capital stock. The sales-to-capital ratio, $\ln(Y/K)$, is computed as the natural logarithm of end-of-year sales scaled by the beginning-of-year capital stock. The employment-to-capital ratio, $\ln(N/K)$, is defined as the natural logarithm of the number of employees scaled by the capital stock. The cash flow rate, CF/K , is calculated as the sum of end-of-year earnings and depreciation scaled by the beginning-of-year capital stock. Tobin's Q is defined as the market value of assets scaled by the book value of assets.

	Obs	Mean	Std. Dev.	25th	50th	75th
I/K	32,890	0.26	0.24	0.12	0.20	0.32
$\log K$	32,890	3.56	2.27	1.88	3.44	5.20
$\log \frac{Y}{K}$	32,890	1.72	0.77	1.23	1.69	2.18
$\log \frac{N}{K}$	32,890	-3.16	1.00	-3.81	-3.15	-2.46
CF/K	32,890	0.54	0.79	0.19	0.37	0.66
Q	32,890	1.56	1.00	0.95	1.24	1.80

Table II: Investment Rate by Single-Sort State-Variable Portfolios

This table reports mean investment rates across firm size (Panel A), sales-to-capital ratio (Panel B), and employment-to-capital ratio (Panel C) deciles. Portfolios are formed each year by allocating firms into size, sales-to-capital ratio, and employment-to-capital ratio deciles, respectively. We report both equal- and capital-weighted averages of firm investment rates. The sample period is 1972 to 2010.

Panel A: Firm Size (K)										
	Q1	2	3	4	5	6	7	8	9	Q10
EW Mean (I/K)	0.37	0.31	0.29	0.28	0.26	0.25	0.24	0.22	0.21	0.21
CW Mean (I/K)	0.35	0.30	0.29	0.28	0.26	0.25	0.24	0.22	0.21	0.20
Panel B: Sales-to-Capital Ratio (Y/K)										
	Q1	2	3	4	5	6	7	8	9	Q10
EW Mean (I/K)	0.17	0.19	0.21	0.22	0.24	0.24	0.26	0.30	0.35	0.47
CW Mean (I/K)	0.16	0.20	0.21	0.23	0.24	0.24	0.25	0.26	0.32	0.42
Panel C: Employment-to-Capital Ratio (N/K)										
	Q1	2	3	4	5	6	7	8	9	Q10
EW Mean (I/K)	0.20	0.22	0.23	0.23	0.25	0.25	0.27	0.30	0.32	0.37
CW Mean (I/K)	0.18	0.21	0.23	0.22	0.24	0.23	0.25	0.25	0.26	0.28

Table III: Investment Rate by Double-Sort State-Variable Portfolios

This table reports equal-weighted average investment rates for portfolios based on conditional sorts on firm size and sales-to-capital ratio (Panel A), and on firm size and employment-to-capital ratio (Panel B). The sample period is 1972 to 2010.

Panel A					
Sales-to-Capital Ratio (Y/K)					
Firm Size (K)	1	2	3	4	5
1	0.18	0.25	0.31	0.41	0.54
2	0.19	0.23	0.26	0.33	0.42
3	0.19	0.22	0.25	0.26	0.36
4	0.19	0.21	0.22	0.23	0.31
5	0.17	0.20	0.21	0.23	0.25

Panel B					
Employment-to-Capital Ratio (N/K)					
Firm Size (K)	1	2	3	4	5
1	0.24	0.30	0.34	0.38	0.43
2	0.24	0.27	0.29	0.33	0.31
3	0.24	0.26	0.25	0.26	0.28
4	0.21	0.22	0.22	0.24	0.25
5	0.18	0.20	0.21	0.22	0.24

Table IV: Investment Polynomials

This table reports estimates from the investment regression specification:

$$\frac{I_{jt+1}}{K_{jt}} = \beta X_{jt} + \delta_j + \eta_t + \varepsilon_{jt+1}$$

where the left-hand-side is end-of year capital expenditures scaled by beginning-of-year property, plant and equipment, δ_j is a firm fixed effect, η_t is a year fixed effect, and X denotes a set of explanatory variables including firm size, $\ln K$, sales-to-capital ratio, $\ln(Y/K)$, and employment-to-capital ratio, $\ln(N/K)$. Standard errors are clustered by firm and t-statistics are reported in parenthesis. R^2 denotes adjusted R^2 . The sample period is 1972 to 2010.

	(1)	(2)	(3)	(4)	(5)	(6)
$\ln \frac{Y}{K}$	0.20 (26.83) ^{***}	0.10 (10.47) ^{***}	0.10 (9.14) ^{***}	0.22 (31.15) ^{***}	0.11 (11.48) ^{***}	0.10 (3.70) ^{***}
$\ln K$	-0.01 (-4.76) ^{***}	-0.02 (-4.01) ^{***}	-0.02 (-3.35) ^{***}	-0.02 (-6.02) ^{***}	-0.03 (-5.30) ^{***}	-0.04 (-4.69) ^{***}
$\ln \frac{N}{K}$				-0.03 (-5.71) ^{***}	-0.06 (-3.43) ^{***}	-0.06 (-2.63) ^{***}
$(\ln \frac{Y}{K})^2$		0.03 (11.30) ^{***}	0.03 (9.77) ^{***}		0.03 (10.84) ^{***}	0.03 (7.51) ^{***}
$(\ln K)^2$		0.00 (2.87) ^{***}	0.00 (2.86) ^{***}		0.00 (3.71) ^{***}	0.00 (0.78)
$(\ln \frac{N}{K})^2$					0.00 (-1.84) [*]	-0.01 (-3.20) ^{***}
$\ln \frac{Y}{K} \times \ln K$			0.00 (0.26)			0.00 (0.40)
$\ln \frac{Y}{K} \times \ln \frac{N}{K}$						0.00 (-0.47)
$\ln K \times \ln \frac{N}{K}$						-0.01 (-3.29) ^{***}
$R^2(\text{within})$	0.22	0.24	0.24	0.22	0.24	0.24
R^2	0.41	0.42	0.42	0.41	0.42	0.42
Obs	32,890	32,890	32,890	32,890	32,890	32,890

Table V: Investment Regressions (Fixed-Effect OLS)

This table reports estimates from the investment regression specification:

$$\frac{I_{jt+1}}{K_{jt}} = \beta X_{jt} + \delta_j + \eta_t + \varepsilon_{jt+1}$$

where the left-hand-side is end-of year capital expenditures scaled by beginning-of-year property, plant and equipment, δ_j is a firm fixed effect, η_t is a year fixed effect, and X denotes a set of explanatory variables including Tobin's Q, and the state-variable polynomial in firm size, $\ln K$ and sales-to-capital ratio, $\ln(Y/K)$. Standard errors are clustered by firm and t-statistics are reported in parenthesis. R^2 denotes adjusted R^2 . The sample period is 1972 to 2010.

	(1)	(2)	(3)	(4)	(5)
Q	0.08 (21.18)***	0.16 (20.04)***		0.06 (19.17)***	0.12 (16.22)***
Q^2		-0.01 (-13.55)***			-0.01 (-10.50)***
$\ln K$			-0.02 (-4.01)***	-0.02 (-4.42)***	-0.02 (-4.62)***
$\ln \frac{Y}{K}$			0.10 (10.47)***	0.08 (8.88)***	0.07 (8.29)***
$(\ln K)^2$			0.00 (2.87)***	0.00 (2.07)**	0.00 (2.01)**
$(\ln \frac{Y}{K})^2$			0.03 (11.30)***	0.03 (12.08)***	0.03 (12.15)***
$R^2(\text{within})$	0.12	0.13	0.24	0.27	0.28
R^2	0.33	0.34	0.42	0.45	0.45
Obs	32,890	32,890	32,890	32,890	32,890

Table VI: Investment Regressions (First-Difference OLS)

This table reports estimates from the investment regression specification:

$$\frac{I_{jt+1}}{K_{jt}} = \beta X_{jt} + \delta_j + \eta_t + \varepsilon_{jt+1}$$

where the left-hand-side is end-of year capital expenditures scaled by beginning-of-year property, plant and equipment, δ_j is a firm fixed effect, η_t is a year fixed effect, and X denotes a set of explanatory variables including Tobin's Q, and the state-variable polynomial in firm size, $\ln(K)$, and sales-to-capital ratio, $\ln(Y/K)$. Standard errors are clustered by firm and t-statistics are reported in parenthesis. R^2 denotes adjusted R^2 . The sample period is 1972 to 2010.

	(1)	(2)	(3)	(4)	(5)
Q	0.06 (12.52) ^{***}	0.13 (12.77) ^{***}		0.03 (9.11) ^{***}	0.07 (8.05) ^{***}
Q^2		-0.01 (-8.36) ^{***}			-0.01 (-4.65) ^{***}
$\ln K$			-0.28 (-10.42) ^{***}	-0.29 (-10.93) ^{***}	-0.29 (-11.00) ^{***}
$\ln \frac{Y}{K}$			0.11 (5.60) ^{***}	0.10 (5.13) ^{***}	0.10 (4.96) ^{***}
$(\ln K)^2$			0.01 (4.62) ^{***}	0.01 (5.07) ^{***}	0.01 (5.18) ^{***}
$(\ln \frac{Y}{K})^2$			0.05 (7.37) ^{***}	0.05 (7.61) ^{***}	0.05 (7.60) ^{***}
R^2	0.04	0.04	0.30	0.31	0.31
Obs	29,295	29,295	29,295	29,295	29,295

Table VII: Variance Decomposition

This table presents a variance decomposition for several investment specifications. Panel A reports estimates based on fixed effect estimators, while Panel B reports estimates based on first difference estimators. We compute the Type III partial sum of squares for each effect in the model and then normalize each estimate by the sum across the effects, forcing each column to sum to one. For example, in specification (2) of Panel A, 8% of the explained sum of squares captured by the included covariates can be attributed to Tobin’s Q . Similarly, in specification (2) of Panel B, 69% of the explained investment changes captured by the included covariates can be attributed to changes in Tobin’s Q . Firm FE are firm fixed effects. Year FE are calendar year fixed effects. Q denotes Tobin’s Q , and “Sales and Size” denotes the state-variable polynomial in firm size, $\ln(K)$, and sales-to-capital ratio, $\ln(Y/K)$: $[\ln(K), \ln(Y/K), \ln(K)^2, \ln(Y/K)^2]$. R^2 denotes adjusted R^2 . The sample period is 1972 to 2010.

	(1)	(2)	(3)	(4)
Panel A: Fixed Effect Estimators				
Firm FE	0.84	0.77	0.67	0.56
Year FE	0.16	0.15	0.08	0.05
Q		0.08		0.07
Sales and Size			0.25	0.32
R^2	0.28	0.33	0.42	0.45
Panel B: First Difference Estimators				
Year FE	1.00	0.31	0.02	0.02
Q		0.69		0.02
Sales and Size			0.98	0.96
R^2	0.01	0.04	0.30	0.31

Table VIII: Alternative Panels

This table reports estimates from the investment regression specification:

$$\frac{I_{jt+1}}{K_{jt}} = \beta X_{jt} + \delta_j + \eta_t + \varepsilon_{jt+1}$$

where the left-hand-side is end-of year capital expenditures scaled by beginning-of-year property, plant and equipment, δ_j is a firm fixed effect, η_t is a year fixed effect, and X denotes a set of explanatory variables including firm size, $\ln K$, and sales-to-capital ratio, $\ln(Y/K)$. Standard errors are clustered by firm and t-statistics are reported in parenthesis. R^2 denotes adjusted R^2 . Specification (1) report results for the sample including all firms except financial, regulated utilities and public services, for the period 1972-2010. Specification (2) uses manufacturing firms for the period 1982-2010. Specification (3) uses manufacturing firms on a balanced panel for the period 1982-2010.

	(1)	(2)	(3)
$\ln \frac{Y}{K}$	0.13 (19.71)***	0.10 (9.65)***	0.10 (5.01)***
$(\ln \frac{Y}{K})^2$	0.02 (8.38)***	0.03 (9.61)***	0.01 (2.13)**
$\ln K$	-0.05 (-8.12)***	-0.02 (-2.42)**	-0.03 (-2.44)**
$(\ln K)^2$	0.00 (5.61)***	0.00 (1.06)	0.00 (1.75)*
R^2 (within)	0.22	0.25	0.21
R^2	0.42	0.45	0.34
Obs	58,663	25,653	4,788

Table IX: Time-Specific Polynomial Coefficients

This table reports the average partial effects from the following investment regression specification with time-specific coefficients:

$$\frac{I_{jt+1}}{K_{jt}} = d_{1,0,t} \ln K_{jt} + d_{0,1,t} \ln (Y_{jt}/K_{jt}) + d_{2,0,t} [\ln K_{jt}]^2 + d_{0,2,t} [\ln (Y_{jt}/K_{jt})]^2 + \delta_j + \eta_t + \varepsilon_{jt+1}$$

where the left-hand-side is end-of year capital expenditures scaled by beginning-of-year property, plant and equipment, δ_j is a firm fixed effect, η_t is a year fixed effect, and the set of explanatory variables includes the state-variable polynomial in firm size and sales-to-capital ratio: $[\ln(K), \ln(Y/K), \ln(K)^2, \ln(Y/K)^2]$. Robust t-statistics are reported in parenthesis. R^2 denotes adjusted R^2 . Specification (1) report results for the benchmark unbalanced sample including manufacturing firms for the period 1972-2010. Specification (2) uses manufacturing firms on a balanced panel for the period 1982-2010.

	(1)	(2)
$\ln K$	-0.02 (-3.75)***	-0.05 (-7.98)***
$\ln \frac{Y}{K}$	0.08 (10.35)***	0.10 (7.40)***
$(\ln K)^2$	0.00 (1.14)	0.00 (6.17)***
$(\ln \frac{Y}{K})^2$	0.03 (15.75)***	0.01 (2.32)**
R^2 (within)	0.25	0.25
R^2	0.43	0.37
Obs	32,890	4,788

Table X: Cash Flow Data

This table reports estimates from the investment regression specification:

$$\frac{I_{jt+1}}{K_{jt}} = \beta X_{jt} + \delta_j + \eta_t + \varepsilon_{jt+1}$$

where the left-hand-side is end-of year capital expenditures scaled by beginning-of-year property, plant and equipment, δ_j is a firm fixed effect, η_t is a year fixed effect, and X denotes a set of explanatory variables including firm size, $\ln K$, and alternative measures of the cash flow-to-capital ratio, CF/K . We use the following measures of cash flow: (1) earnings before extraordinary items plus depreciation; (2) funds from operations as in Lewellen and Lewellen (2011); (3) funds from operations plus interest and related expense minus nonoperating income (expense); and (4) operating income before depreciation. All end-of-year measures of cash flow are scaled by the beginning-of-year capital stock. Standard errors are clustered by firm and t-statistics are reported in parenthesis. R^2 denotes adjusted R^2 . The sample period is 1972 to 2010.

	(1)	(2)	(3)	(4)
CF/K	0.14 (18.06)***	0.19 (23.33)***	0.18 (16.48)***	0.12 (25.34)***
$(CF/K)^2$	-0.01 (-9.81)***	-0.01 (-4.71)***	-0.01 (-4.73)***	-0.01 (-12.40)***
$\ln K$	-0.08 (-18.35)***	-0.09 (-6.96)***	-0.10 (-7.03)***	-0.08 (-15.43)***
$(\ln K)^2$	0.00 (7.41)***	0.00 (1.38)	0.00 (1.26)	0.00 (5.79)***
R^2 (within)	0.21	0.19	0.18	0.22
R^2	0.40	0.45	0.44	0.41
Obs	32,890	10,997	10,992	32,731

Table XI: Leverage

This table reports estimates from the investment regression specification:

$$\frac{I_{jt+1}}{K_{jt}} = \beta X_{jt} + \delta_j + \eta_t + \varepsilon_{jt+1}$$

where the left-hand-side is end-of year capital expenditures scaled by beginning-of-year property, plant and equipment, δ_j is a firm fixed effect, η_t is a year fixed effect, and X denotes a set of explanatory variables including firm size, $\ln K$, sales-to-capital ratio, $\ln(Y/K)$, and alternative measures of leverage, B/K . Specification (1) uses book leverage defined as the sum of short-term plus long-term debt scaled by total assets; and specification (2) uses net book leverage constructed as book leverage minus cash and short-term investments scaled by total assets. Standard errors are clustered by firm and t-statistics are reported in parenthesis. R^2 denotes adjusted R^2 . The sample period is 1972 to 2010.

	(1)	(2)
$\ln \frac{Y}{K}$	0.09 (9.74)***	0.09 (9.52)***
$(\ln \frac{Y}{K})^2$	0.03 (12.02)***	0.03 (12.32)***
$\ln K$	-0.02 (-3.41)***	-0.01 (-2.34)**
$(\ln K)^2$	0.00 (3.66)***	0.00 (3.19)***
$\frac{B}{K}$	-0.33 (-8.55)***	-0.20 (-14.92)***
$(\frac{B}{K})^2$	0.19 (3.50)***	0.02 (1.06)
R^2 (within)	0.25	0.26
R^2	0.43	0.44
Obs	32,837	32,834

Table XII: Lagged Investment

This table reports instrumental variable estimates from the investment regression specification:

$$\frac{I_{jt+1}}{K_{jt}} = \beta X_{jt} + \delta_j + \eta_t + \varepsilon_{jt+1}$$

where the left-hand-side is end-of year capital expenditures scaled by beginning-of-year property, plant and equipment, δ_j is a firm fixed effect, η_t is a year fixed effect, and X denotes a set of explanatory variables including lagged investment, Tobin's Q , cash flow rate, CF/K , firm size, $\ln K$, and sales-to-capital ratio, $\ln(Y/K)$. Lagged investment is instrumented using prior two lags of its first-difference. Standard errors are clustered by firm and t-statistics are reported in parenthesis. R^2 denotes adjusted R^2 . The sample period is 1972 to 2010.

	(1)	(2)
Lagged I/K	0.19 (14.02) ^{***}	0.18 (13.29) ^{***}
$\ln \frac{Y}{K}$		0.10 (9.12) ^{***}
$(\ln \frac{Y}{K})^2$		0.03 (8.24) ^{***}
$\ln K$		-0.02 (-3.27) ^{***}
$(\ln K)^2$		0.00 (2.03) ^{**}
Q	0.03 (12.02) ^{***}	
CF/K	0.11 (16.61) ^{***}	
R^2 (within)	0.21	0.27
R^2	0.42	0.46
Obs	23,802	23,802

Table XIII: Extended Variance Decomposition

This table presents a variance decomposition for several additional investment specifications. We compute the Type III partial sum of squares for each effect in the model and then normalize each estimate by the sum across the effects, forcing each column to sum to one. For example, in specification (4), 3% of the explained sum of squares captured by the included covariates can be attributed to firm net leverage. Firm FE are firm fixed effects. Year FE are calendar year fixed effects. “Sales and Size” denotes the second order polynomial in firm size, $\ln(K)$, and sales-to-capital ratio, $\ln(Y/K)$. “Time-Specific Sales and Size” denotes the second order polynomial in firm size, $\ln(K)$, and sales-to-capital ratio, $\ln(Y/K)$, with time-specific coefficients. “Cash Flow and Size” denotes the second order polynomial in firm size, $\ln(K)$, and cash flow-to-capital ratio, CF/K . R^2 denotes adjusted R^2 . The sample period is 1972 to 2010.

	(1)	(2)	(3)	(4)	(5)
Firm FE	0.67	0.62	0.60	0.66	0.48
Year FE	0.08	0.01	0.05	0.09	0.04
Sales and Size	0.25			0.23	0.36
Time-Specific Sales and Size		0.37			
Cash Flow and Size			0.35		
Net Leverage				0.03	
Lagged Investment					0.12
R^2	0.42	0.43	0.40	0.44	0.46

Figure 1: Time-Specific Coefficients (Unbalanced Panel)

This figure plots the time-specific coefficient estimates from the investment regression specification:

$$\frac{I_{jt+1}}{K_{jt}} = d_{1,0,t} \ln K_{jt} + d_{0,1,t} \ln \left(\frac{Y_{jt}}{K_{jt}} \right) + d_{2,0,t} [\ln K_{jt}]^2 + d_{0,2,t} \left[\ln \left(\frac{Y_{jt}}{K_{jt}} \right) \right]^2 + \delta_j + \eta_t + \varepsilon_{jt+1}$$

where the left-hand-side is end-of year capital expenditures scaled by beginning-of-year property, plant and equipment, δ_j is a firm fixed effect, η_t is a year fixed effect, and the set of explanatory variables includes the state-variable polynomial in firm size and sales-to-capital ratio: $[\ln(K), \ln(Y/K), \ln(K)^2, \ln(Y/K)^2]$. The solid line displays the time-specific estimates, and the dashed lines illustrate the upper and lower 95% confidence bounds. The sample consists of an unbalanced panel of manufacturing firms for the period is 1972-2010.

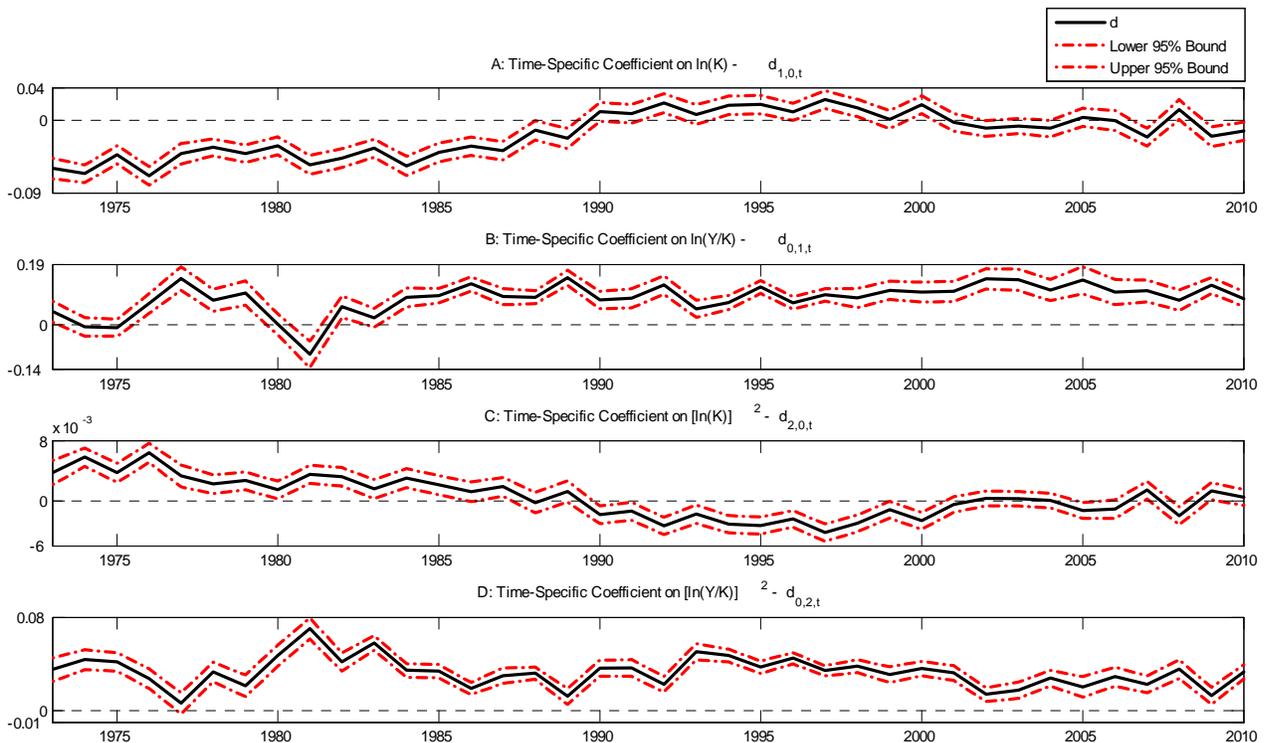


Figure 2: Time-Specific Coefficients (Balanced Panel)

This figure plots the time-specific coefficient estimates from the investment regression specification:

$$\frac{I_{jt+1}}{K_{jt}} = d_{1,0,t} \ln K_{jt} + d_{0,1,t} \ln \left(\frac{Y_{jt}}{K_{jt}} \right) + d_{2,0,t} [\ln K_{jt}]^2 + d_{0,2,t} \left[\ln \left(\frac{Y_{jt}}{K_{jt}} \right) \right]^2 + \delta_j + \eta_t + \varepsilon_{jt+1}$$

where the left-hand-side is end-of year capital expenditures scaled by beginning-of-year property, plant and equipment, δ_j is a firm fixed effect, η_t is a year fixed effect, and the set of explanatory variables includes the state-variable polynomial in firm size and sales-to-capital ratio: $[\ln(K), \ln(Y/K), \ln(K)^2, \ln(Y/K)^2]$. The solid line displays the time-specific estimates, and the dashed lines illustrate the upper and lower 95% confidence bounds. The sample consists of a balanced panel of manufacturing firms for the period is 1982-2010.

