Project Design with Limited Commitment and Teams*

George Georgiadis[†], Steven A. Lippman[‡], and Christopher S. Tang[§]

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Abstract

We study the interaction between a group of agents who exert costly effort over time to complete a project, and a manager who chooses the objectives that must be met in order for her to sign off on it. The manager has limited commitment power so that she can commit to the requirements only when the project is sufficiently close to completion. This is common in projects that involve *design* or *quality* objectives, which are difficult to define far in advance. The main result is that the manager has incentives to extend the project as it progresses: she is time-inconsistent. This result has two implications. First, the manager will choose a larger project if she has less commitment power. Second, if the agents receive a fraction of the project's worth upon its completion, then the manager should delegate the decision rights over the project size to the agents unless she has sufficient commitment power. In this case, the agents will choose a smaller project that is optimal for the manager, but their preferences are time-consistent.

1 Introduction

A key component of a project, such as the development of a new product, is choosing the features that must be included before the decision maker deems the product ready to market. Naturally, which features should be included must be communicated to the relevant stakeholders. When choosing these features, the decision maker must balance the added value

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[†]California Institute of Technology and Boston University. E-mail: georgiadis@hss.caltech.edu.

[‡]UCLA Anderson School of Management. E-mail: steven.lippman@anderson.ucla.edu.

[§]UCLA Anderson School of Management. E-mail: chris.tang@anderson.ucla.edu.

derived from a bigger or a more complex project (*i.e.*, one that contains more features) against the additional cost associated with designing and implementing the additional features. Such costs include not only engineering inputs but also the implicit cost associated with delayed cash flow.

An intrinsic challenge involved in choosing the requirements of a project is that the manager may not be able to commit to them in advance. This can be due to the fact that the requirements are difficult to describe; for example, if the project involves significant novelty in quality or design. What we have in mind about the incontractibility of the project requirements was eloquently posed by Tirole (1999):

> *In practice, the parties are unlikely to be able to describe precisely the specifics of an innovation in an ex ante contract, given that the research process is precisely concerned with finding out these specifics, although they are able to describe it ex post.*

In addition, committing to specific requirements may be difficult due to an asymmetry in the bargaining power of the parties involved. For example, if a project is undertaken in-house where the manager can significantly influence the team members' career paths and contracts are typically implicit, the manager will tend to be less able to commit relative to the case in which the project is outsourced and contracts are explicit.

Changing the requirements of a project (often referred to as *moving the goal post*), is common in project management (Brenner (2001)). One prominent example is Boeing's 787 Dreamliner project. The original set of project goals was to develop a lightweight, fuel-efficient aircraft that would meet the customer's needs for lower operating costs. On top of these original project goals, Boeing's senior management then decided to outsource parts of the design, engineering, and manufacturing processes to some fifty external "strategic partners" to reduce development costs and time. From the perspective of Boeing's engineers, a new goal was appended to the original set of project goals: restructure the design and manufacturing processes by overseeing and coordinating the work performed by internal engineers and those external strategic partners (Tang and Zimmerman (2009) and Brenner (2013)).

Anecdotal evidence from the development of Apple's first generation iPod indicates that Steve Jobs kept changing the requirements of the iPod as the project progressed (Wired Magazine (2004)). This suggests that committing to a set of features and requirements early on was infeasible in the development of an innovative new product such as the iPod back in 2001.

Similarly, consider the process of designing a new car. If it were possible to describe in advance what the design must look like for management to give its approval, then there would be far fewer delays as the new car makes its way to production and design would be relatively easy. However, as the final product takes shape, the decision maker can better guide the design team to fulfill her objectives.

We propose a tractable model to analyze a dynamic contribution game in which a group of agents collaborate to complete a project. The project progresses at a rate that depends on the agents' costly effort, and it generates a payoff upon completion. Formally, the state of the project q_t is equal to 0 at $t = 0$, and it progresses according to $dq_t = \sum_{i=1}^{n} a_{i,t}\, dt$, where $a_{i,t}$ denotes the effort level of agent i at time t. The project generates a payoff at the first stopping time τ such that $q_\tau = Q$, where Q is a one-dimensional parameter that captures the project requirements, or equivalently, the project size. The manager is the residual claimant of the project, and she possesses the decision rights over its requirements (*i.e.*, its size).

In Section 3, we analyze the agents' problem and we compute the manager's discounted profit for a fixed project size. We characterize the (essentially) unique Markov Perfect equilibrium, wherein at every moment, each agent's strategy depends solely on the current state of the project. In addition, we characterize a continuum of (non-Markovian) Public Perfect equilibria, in which agents choose their effort by maximizing a convex combination of their individual and the team's discounted payoff along the equilibrium path. Motivated by the concepts of *insiders* and *outsiders* (who act in the best interest of the team and in their own best interest, respectively) introduced by Akerlof and Kranton (2000), the weight that agents place on maximizing the team's payoff can be interpreted as a measure of the team's *cooperativeness*. A key result is that the agents exert greater effort the closer the project is to completion. Intuitively, this is because they discount time and they are compensated upon completion, so that their incentives become stronger as the project progresses.

In Section 4, we examine how the manager will choose the project size to maximize her discounted profit. The fundamental trade-off that she faces is that a larger project generates a bigger payoff upon completion but requires more effort to complete. To model the manager's limited ability to commit, we assume that given the current state of the project q_t, she can commit to any $Q \in [q_t, q_t + y]$, where $y \geq 0$ captures the commitment power. Therefore, the manager can commit to a project size $Q > y$ only after the agents have made sufficient progress such that $q_t \geq Q - y$. For example, y will tend to be larger in a construction project, where the requirements are typically standardized and easy to define, than in a project that involves a significant innovation or quality component, where the requirements of the final output cannot be contracted on until the project is at a sufficiently advanced stage.

The main result is that the manager's incentives propel her to extend the project as it progresses; for example, by introducing additional requirements. The manager chooses the project size by trading off the marginal benefit of a larger project against the marginal cost associated with a longer wait until the larger project is completed. However, because the agents increase their effort, this marginal cost decreases as the project progresses, while the respective marginal benefit is independent of the progress made. As the project size will be chosen such that the two marginal values are equal, it follows that the manager's optimal project size increases as the project progresses.

An implication of this result is that the manager's optimal project size decreases in her commitment power. If the manager has sufficiently large commitment power, then she will commit to her optimal project size at time 0. Otherwise, she can commit to a smaller than ideal project at time 0, or else she must wait until the project is at a sufficiently advanced stage so that she can commit to her optimal project size then. We show that the manager always finds the latter option preferable. However, once such an advanced stage has been reached, her optimal project size is larger than it was originally, and the manager faces the same dilemma as at time 0. As a consequence, the manager will choose a bigger project the smaller her commitment power.

Anticipating that the manager will choose a larger project if she has less commitment power, the agents decrease their effort, which renders the manager worse off. To mitigate her commitment problem, assuming that the agents receive a share of the project's worth upon completion (*i.e.*, an equity contract), the manager might delegate the decision rights over the project size to them. In this case, the agents will choose a smaller project than is optimal for the manager, but their preferences are time-consistent. Intuitively, because (unlike the manager) they also trade off the cost of effort when choosing the project size, their marginal cost associated with a larger project does not decrease as the project progresses. As a result, the manager's discounted profit is independent of the commitment power under delegation, while it increases in her commitment power when she retains the decision rights over the project size. We show that there exists an interior threshold such that delegation is optimal unless the manager has sufficient commitment power.

Motivated by the equilibrium selection concepts proposed by Kreps (1990), in Section 5, we consider the case in which the manager can influence the weight that each agent places on maximizing the team's discounted payoff, for example, by selecting the team members or encouraging interaction among them. Given a fixed project size, a fully cooperative environment (*i.e.*, placing all the weight on maximizing the team's payoff) renders all parties better off. However, when the project size is endogenous, then a fully cooperative environment is profit-maximizing only if the manager has sufficient commitment power. Otherwise, the degree of cooperativeness that maximizes her discounted profit is interior, and it increases in her commitment power. Intuitively, by cultivating a lower degree of cooperativeness, the manager can mitigate her ex-post incentives to extend the project, which are more severe the smaller her commitment power.

Related Literature

First and foremost, this paper is related to the literature on dynamic contribution games. The general theme of these games is that a group of agents interact repeatedly, and in every period (or moment), each agent chooses his contribution (or effort) to a joint project at a private cost. Contributions accumulate until they reach a certain threshold, at which point each agent receives a lump-sum payment that is independent of his individual contributions, and the game ends. Admati and Perry (1991) and Marx and Matthews (2000) show that contributing little by little over multiple periods, each conditional on the previous contributions of the other agents, helps mitigate the free-rider problem.[1] More recently, Yildirim (2006) and Kessing (2007) show that in contrast to the case in which the project generates flow payments while it is in progress as studied by Fershtman and Nitzan (1991), efforts are strategic complements when the agents receive a payoff only upon completion. Georgiadis (2013) examines how the incentives to contribute to a public good project depend on the team composition, and he focuses on how a manager should choose the team composition and the agents' compensation scheme. A feature common to most papers in this literature is that the size of the project is given exogenously. However, in applications (*e.g.*, new product development), the choice of the objectives of any given project is typically a central part of the problem.[2] Our contribution to this literature is to propose a tractable model to study this family of dynamic contribution games, to endogenize the size of the project, and to

[1] As some of our equilibrium characterization results parallel those of Marx and Matthews (2000), we discuss the similarities and differences between the two papers immediately after Proposition 2.

[2] One exception is Yildirim (2004), who studies the optimal piecewise procurement of a large-scale project. By using a repeated procurement auction to allocate each subproject to the agent with the lowest cost, he examines the effect of the project size to the agents' bidding behavior. In contrast, information is symmetric in our model, and we focus on the moral hazard problem and the manager's commitment problem.

examine how the optimal project size depends on who has the decision rights and on the magnitude of the decision maker's commitment power.

A second strand of related literature is that on incomplete contracting. In particular, our paper is closely related to the papers that study how ex-ante contracting limitations generate incentives to renegotiate the initial contract ex-post (Grossman and Hart (1986), Hart and Moore (1990), Aghion and Tirole (1994), Tirole (1999), and Al-Najjar, Anderlini and Felli (2006)). A subset of this literature focuses on situations wherein the involved parties have asymmetric information. Here, ratchet effects have been shown to arise in principal-agent models in which the principal learns about the agent's ability over time, and the agent reduces his effort to manipulate the principal's beliefs about his ability (Freixas, Guesnerie and Tirole (1985) and Laffont and Tirole (1988)). Another thread of this strand includes papers that consider the case in which the agent is better informed than the principal, or he has better access to valuable information. A common result is that delegating the decision rights to the agent is beneficial as long as the he is sufficiently better informed and the incentive conflict is not too large (Aghion and Tirole (1997) and Dessein (2002)). In our model however, all parties have full and symmetric information, so that ratchet effects and the incentives to delegate the decision rights to the agents arise purely out of moral hazard.

This paper is organized as follows. We introduce the model in Section 2, and in Section 3 we analyze the agents' as well as the manager's problem for a fixed project size. In Section 4, we study the manager's optimal choice of the project size, and we examine her option to delegate the decision rights over the project size to the agents. Section 5 examines the issue of equilibrium selection, and Section 6 concludes. In Appendix A, we extend our model to test the robustness of our results. All proofs are provided in Appendix B.

2 The Model

A group of n identical agents contracts with a manager to undertake a project. The agents exert (costly) effort over time to complete the project, they receive a lump-sum compensation upon completing the project, and they are protected by limited liability.[3] The manager is the residual claimant of the project, and she possesses the decision rights over its size. A project of size $Q \geq 0$ generates a payoff equal to Q upon completion. This payoff is split

[3]We assume that the agents are compensated only upon completing the project for tractability. In Appendix A.4, we consider the case in which, in addition to a lump-sum payment upon completion, they also receive a per unit of time compensation while the project is ongoing. All main results continue to hold.

between the parties as follows: each agent receives $\frac{\beta Q}{n}$, and the manager receives $(1-\beta)\,Q$.[4] Time $t \in [0, \infty)$ is continuous; all parties are risk neutral and they discount time at rate $r > 0$. The project starts at state $q_0 = 0$. At every moment t, each agent observes the state q_t of the project, and exerts costly effort to influence the process

$$dq_t = \left(\sum_{i=1}^{n} a_{i,t}\right) dt\,,$$

where $a_{i,t}$ denotes the (unverifiable) effort level of agent i at time t.[5] Each agent's flow cost of exerting effort level a is $\frac{a^2}{2}$, while his outside option is equal to 0. The project is completed at the first stopping time τ such that $q_\tau = Q$.

3 Results

In Section 3.1, we study the agents' problem, and we characterize the unique project-completing Markov Perfect equilibrium (hereafter MPE) wherein each agent conditions his strategy at t only on the current state of the project q_t, as well as a continuum of Public Perfect equilibria (hereafter PPE) wherein each agent's strategy at t is conditioned on the entire evolution path of the project $\{q_s\}_{s \leq t}$. Then in Section 3.2 we determine the manager's discounted profit. Throughout this Section we take the project size Q as given, and we endogenize it in Section 4.

3.1 Agents' Problem

Given a project of size Q and the current state q_t of the project, agent i's expected discounted payoff function satisfies

$$\Pi_{i,t}\left(q\,;Q\right) = \max_{\{a_{i,s}\}_{s \geq t}} \left[e^{-r(\tau-t)} \frac{\beta Q}{n} - \int_t^\tau e^{-r(s-t)} \frac{a_{i,s}^2}{2} ds \mid \{a_{-i,s}\}_{s \geq t} \right], \tag{1}$$

[4] This is essentially an equity contract. In Appendix A.2, we consider the case in which each agent receives a flat payment upon completion of the project that is independent of the project size Q. The main results continue to hold, but such contract is shown to aggravate the manager's commitment problem.

We assume that β is independent of Q; otherwise, the assumption that the manager has limited ability to commit to a project size would be violated. However, we defer a detailed justification until after we have formalized the notion of limited commitment in Section 3.1.2.

[5] Efforts are perfect substitutes. The case in which efforts are complementary is examined in Appendix A.1, and we show that all results continue to hold.

The assumption that the project progresses deterministically is made for tractability. In Appendix A.5, we consider the case in which the project progresses stochastically and we illustrate that all main results continue to hold.

where τ denotes the completion time of the project and it depends on the agents' strategies. Note that the first term captures the agent's net discounted payoff upon completion of the project, while the second term captures his discounted cost of effort for the remaining duration of the project. Because payoffs depend solely on the state of the project (*i.e.*, q) and not on the time t, this problem is stationary, and hence the subscript t can be dropped. Using standard arguments (Dixit (1999)), one can derive the Hamilton-Jacobi-Bellman equation for the expected discounted payoff function for agent i

$$r\Pi_i\left(q\,;Q\right) = \max_{a_i}\left\{-\frac{a_i^2}{2} + \left(\sum_{j=1}^{n} a_j\right)\Pi_i'\left(q\,;Q\right)\right\} \tag{2}$$

subject to the boundary conditions

$$\Pi_i\left(q\,;Q\right) \geq 0 \text{ for all } q \quad \text{and} \quad \Pi_i\left(Q\,;Q\right) = \frac{\beta Q}{n}\,. \tag{3}$$

The first boundary condition captures the fact that each agent's discounted payoff must be non-negative since he can guarantee himself a payoff of 0 by exerting no effort and hence incurring no effort cost. The second boundary condition states that upon completing the project, each agent receives his reward and exerts no further effort.

3.1.1 Markov Perfect Equilibrium (MPE)

In a MPE, at every moment t, each agent i observes the state of the project q, and chooses his effort a_i to maximize his expected discounted payoff while accounting for the effort strategies of the other team members. It follows from (2) that the first order condition for agent i's problem yields that $a_i\left(q\,;Q\right) = \Pi_i'\left(q\,;Q\right)$: at every moment, he chooses his effort to equate the marginal cost of effort to the marginal benefit of bringing the project closer to completion. By noting that the second order condition is satisfied and that the first order condition is necessary and sufficient, it follows that in any differentiable, project-completing MPE, the discounted payoff for agent i satisfies

$$r\Pi_i\left(q\,;Q\right) = -\frac{1}{2}\left[\Pi_i'\left(q\,;Q\right)\right]^2 + \left[\sum_{j=1}^{n}\Pi_j'\left(q\,;Q\right)\right]\Pi_i'\left(q\,;Q\right) \tag{4}$$

subject to the boundary conditions (3). The following Proposition characterizes the MPE, and establishes conditions under which it is unique.

Proposition 1. *For any given project size Q, there exists a Markov Perfect equilibrium (MPE) for the game defined by (1). This equilibrium is symmetric, each agent's effort* *strategy satisfies*

$$a\left(q\,;Q\right)=\frac{r}{2n-1}\left[q-C\left(Q\right)\right]^{+}\ \text{, where } C\left(Q\right)=Q-\sqrt{\frac{2\beta Q}{r}\frac{2n-1}{n}}\,,$$

and the project is completed at $\tau=\frac{2n-1}{rn}\ln\left[1-\frac{Q}{C(Q)}\right]$.[6] *In equilibrium, each agent's discounted payoff is given by*

$$\Pi\left(q\,;Q\right)=\frac{r}{2}\frac{\left(\left[q-C\left(Q\right)\right]^{+}\right)^{2}}{2n-1}\,.$$

If $Q<\frac{2\beta}{r}$*, then this equilibrium is unique, and the project is completed in finite time. Otherwise, there also exists an equilibrium in which no agent ever exerts any effort and the project is never completed.*

First note that if the project is too far from completion ($i.e.$, $q<C\left(Q\right)$), then the discounted cost to complete it exceeds its discounted net payoff, and hence the agents are better off not exerting any effort, in which the project is never completed. Because the project starts at $q_0=0$, this implies that the project is never completed if $C\left(Q\right)\geq 0$, or equivalently if $Q\geq\frac{2\beta}{r}\frac{2n-1}{n}$. On the other hand, if $Q<\frac{2\beta}{r}\frac{2n-1}{n}$, then each agent's effort level increases in the state of the project q. This is due to the facts that agents are impatient and they incur the cost of effort at the time it is exerted, while they are compensated only when the project is completed. As a result, their incentives are stronger, the closer the project is to completion.

Second, it is worth emphasizing that because the agents' effort costs are convex, the MPE is always symmetric. Finally, while the MPE need not be unique, it turns out that when the project size Q is endogenous, the manager will always choose it such that the equilibrium is unique (see Remark 1 in Section 4). As such, we shall restrict attention to the project-completing equilibrium characterized in Proposition 1.

3.1.2 Public Perfect Equilibria (PPE)

While the restriction to MPE is reasonable when teams are large and members cannot monitor each other, there typically exist other PPE with history-dependent strategies; *i.e.*, strategies that at time t depend on the entire evolution path of the project $\left\{q_s\right\}_{s\leq t}$. In this Section, we characterize a continuum of such equilibria in which at every moment, each agent chooses his effort to maximize a convex combination of his individual and the entire team's discounted payoff along the equilibrium path.

[6] To simplify notation, because the equilibrium is symmetric and unique, the subscript i is dropped throughout the remainder of this paper. Moreover, $\left[\cdot\right]^{+}=\max\left\{\cdot,0\right\}$.

Building upon the concepts introduced in the seminal paper on social identity by Tajfel and Turner (1979), Akerlof and Kranton (2000) argue that depending on the work environment, employees may behave as *insiders* who act in the best interest of the organization or as *outsiders* who act in their individual best interest. Therefore, the weight that an agent places on maximizing the team's discounted payoff can be interpreted as the degree to which he feels an insider, and we shall refer to an equilibrium as more cooperative the more weight each agent places on maximizing the team's discounted payoff.

We model this by assuming that given the current state of the project q, each agent chooses his effort to maximize the expected discounted payoff of $k \in [1, n]$ agents; *i.e.*, he solves

$$a\,(q\,;Q,\,k) \in \arg\max_a \left\{ a\,k\,\Pi'\,(q\,;Q,\,k) - \frac{a^2}{2} \right\} . \tag{5}$$

Note that $k = 1$ ($k = n$) corresponds to the case in which each agent places all the weight on maximizing his individual (the team's) discounted payoff, while $k \in (1, n)$ corresponds to intermediate cooperation levels in which each agent maximizes a convex combination of his individual and the team's discounted payoff. The following Proposition establishes that for all $k \in [1, n]$ there exists a PPE in which at every moment along the equilibrium path, each agent chooses his effort by solving (5).[7]

Proposition 2. *For any given $k \in [1, n]$ and project size Q, there exists a Public Perfect equilibrium (PPE) in which each agent's effort strategy satisfies*

$$a\,(q\,;Q,\,k) = \frac{r}{2n-k}\,[q - C\,(Q\,;k)]^+ \tag{6}$$

along the equilibrium path, where $C\,(Q\,;k) = Q - \sqrt{\frac{2\beta Q}{r}\frac{(2n-k)k}{n}}$, and the project is completed at $\tau = \frac{2n-k}{rn} \ln \left[1 - \frac{Q}{C(Q\,;k)}\right]$. After any deviation from the equilibrium path, all agents revert to the Markov Perfect equilibrium (i.e., $k = 1$) for the remaining duration of the project. In equilibrium, each agent's discounted payoff is given by

$$\Pi\,(q\,;Q,\,k) = \frac{r}{2k}\frac{\left([q - C\,(Q\,;k)]^+\right)^2}{2n-k} ,$$

[7] There also exist PPE where each agent's cooperation level varies as the project progresses, and the cooperation level differs across team members. However, we restrict attention to the case in which the cooperation level is constant throughout the duration of the project and identical across all agents (i) for tractability, and (ii) because we interpret k as part of the organization's corporate culture that is persistent over time.

and it increases in k for all q and Q.

The intuition behind the existence of cooperative PPE is as follows. First, if all agents choose their effort by solving (5) for some $k > 1$, then each agent is strictly better off relative to the case in which $k = 1$. Second, $k = 1$ corresponds to the MPE, so that the threat of punishment is credible.[8] Third, by examining the progress made until time t, each agent can infer whether all agents followed the equilibrium strategy; *i.e.*, if q_t corresponds to the progress that should occur if all agents follow (6). Because a deviation from the equilibrium path is detectable (and punishable) arbitrarily quickly, the gain from a deviation is infinitesimally small. As a result, no agent has an incentive to deviate from the strategy dictated by (6), so that it constitutes a PPE.[9]

Because our model is related to that of Marx and Matthews (2000) (hereafter MM), it is instructive to relate our results (*i.e.*, Propositions 1 and 2) to theirs. The main modeling differences are that time is continuous and effort costs are convex in our model, whereas time is discrete and effort costs are linear in MM. The two papers share the existence of completing and non-completing equilibria, and similar to MM, a completing equilibrium exists in our model if the agents are sufficiently patient; *i.e.*, if $C\left(Q\,;k\right) < 0$ or equivalently if $r < \frac{2\beta}{Q}\frac{(2n-k)k}{n}$. However, in MM, if the project generates a payoff only upon completion, then the game reduces to a static one (see their footnote 7): because effort costs are linear, agents cannot benefit by spreading their effort over time (unlike the case in which they are convex as in our model).

The following result establishes some comparative statics about how each agent's effort level depends on the parameters of the problem.

Result 1. Other things equal, each agent's effort level $a\left(q\,;Q,\,k\right)$:
(i) increases in k (and β) ;
(ii) there exists a threshold Θ_r such that it increases in r if and only if $q \geq \Theta_r$; and
(iii) there exists a threshold Θ_n such that it increases in n if and only if $q \geq \Theta_n$.

To see the intuition behind statement (i), note that the free-rider problem is mitigated as the agents' cooperation level k increases. As a result, the agents' incentives become stronger in

[8] Note that punishment is never inflicted along the equilibrium path. Therefore, the results will not change even if the agents use a more severe punishment after detecting a deviation.

[9] There is a well known problem associated with defining a trigger strategy in continuous-time games. To see why, suppose that a deviation occurs at some t', and agents revert to the MPE at t''. However, because there is no first time after t', there always exists some $t \in (t', t'')$ such that the agents are better off reverting to the MPE at that t; *i.e.*, subgame perfection fails. To resolve this problem, we use the concept of inertia strategies proposed by Bergin and MacLeod (1993).

k (and it is eliminated when $k = n$). The intuition behind the second part of statement (i) is straightforward: if the agents receive a larger reward upon completion, then their incentives are stronger. The threshold results in statements (ii) and (iii) are similar to Georgiadis (2013) who studies a stochastic version of this model with a fixed project size.[10]

Before we proceed to analyze the manager's problem, it is instructive to characterize the first best outcome of this game.

Result 2. Consider a social planner whose objective it is to maximize the total surplus of the team. For any given project size Q, the discounted payoff and the optimal strategy for each agent is given by

$$\Pi^{FB}\left(q\,;Q\right) = \frac{r}{2n}\left(\left[q - Q + \sqrt{\frac{2Qn}{r}}\right]^{+}\right)^{2} \text{ and } a^{FB}\left(q\,;Q\right) = \frac{r}{n}\left[q - Q + \sqrt{\frac{2Qn}{r}}\right]^{+},$$

respectively, and the project is completed at $\tau^{FB} = \frac{1}{r}\ln\left(\frac{\sqrt{2\beta n}}{\sqrt{2\beta n}-\sqrt{rQ}}\right)$.

To see why this is the first best discounted payoff and effort path, respectively, recall that at every moment, the social planner chooses the agents' effort levels to maximize the total discounted payoff of the team. Therefore, the socially efficient effort function satisfies (6) after substituting $\beta = 1$ and $k = n$. It follows that $\Pi^{FB}\left(q\,;Q\right) = \Pi\left(q\,;Q,\,n\right)|_{\beta=1}$.

3.2 Manager's Problem

Given a project of size Q, the agents' belief $\tilde{Q}$ about the manager's choice of the project size, and the agents' cooperation level k, the manager's discounted profit can be written as $W\left(q\,;Q,\tilde{Q},\,k\right) = \left[e^{-r\tau}\left(1-\beta\right)Q \,|\, Q,\tilde{Q}\right]$, where the project's completion time τ depends on the current state q and the agents' strategies, which in turn depend on $\tilde{Q}$ and k.[11] Of course, in equilibrium beliefs must be correct; *i.e.*, $Q = \tilde{Q}$. Using standard arguments, one can derive the HJB equation for the manager's discounted profit

$$rW\left(q\,;Q,\tilde{Q},\,k\right) = n\,a\left(q\,;\tilde{Q},\,k\right)\,W'\left(q\,;Q,\tilde{Q},\,k\right)$$

[10]Note that when examining how each agent's effort level depends on the team size, one must first consider how the agents' cooperation level k depends on n. Statement (iii) holds for any fixed k as well as for $k = n$.

[11]The manager's discounted payoff depends on the agents' beliefs about her choice of Q because this belief influences their effort strategy. In addition, note that because the equilibrium is symmetric for any given Q, all agents will have the same belief $\tilde{Q}$.

subject to the boundary conditions

$$W\left(q\,;Q,\tilde{Q},\,k\right)\geq 0 \text{ for all } q \quad \text{and} \quad W\left(Q\,;Q,\tilde{Q},\,k\right)=(1-\beta)\,Q\,.$$

To interpret these conditions, note that manager's discounted profit is non-negative at every state of the project, because she does not incur any cost or disburse any payments to the agents while the project is in progress. On the other hand, she receives her net profit $(1-\beta)\,Q$, and the game ends as soon as the state of the project hits Q for the first time. It is straightforward to show that this ODE has the following solution

$$W\left(q\,;Q,\tilde{Q},\,k\right)=(1-\beta)\,Q\left(\frac{\left[q-C\left(\tilde{Q}\,;k\right)\right]^{+}}{Q-C\left(\tilde{Q}\,;k\right)}\right)^{\frac{2n-k}{n}}. \tag{7}$$

Note that $(1-\beta)\,Q$ represents the manager's net profit upon completion of the project, while the next term can be interpreted as the present discounted value of the project, which depends on the current state q, the agents' beliefs about the project size and their cooperation level k, which in turn influence their strategies characterized in Proposition 1.

Before we proceed with the analysis of the optimal project size, one assumption that deserves discussion is that the agents' compensations are independent of the completion time of the project; *i.e.*, that the manager does not use deadlines to incentivize the agents.[12] This assumption is made primarily for tractability, as it enables us to characterize the agents' and the manager's payoff functions in closed form by solving two ordinary differential equations (hereafter ODE). In addition, one drawback of deadlines is that they are not renegotiation proof: if the agents fail to make the pre-specified amount of progress by the deadline (and assuming that neither party receives a reward unless the project is completed), the manager has incentives to renegotiate it. Because the agents will anticipate this behavior, deadlines may be useful only if the manager can credibly commit to not renegotiate them.

4 Project Design and the Commitment Problem

In this Section we endogenize the project size Q. The manager has the decision rights over the choice of the project size, but she may not be able to commit to a specific Q until the

[12]Because the completion time of the project is deterministic along any equilibrium path, it suffices to consider the case in which each agent receives $\frac{\beta Q}{n}$ as long as $\tau\leq T$ and his reward drops to 0 otherwise, where T is a pre-specified deadline.

project is sufficiently close to that Q. Formally, we assume that given the current state of the project q, the manager can only commit to a project size in the interval $[q,\, q+y]$, where $y \geq 0$ is common knowledge, and it can be thought of as capturing the describability (or verifiability) of the project requirements.

The extreme case $y = \infty$ represents the situation in which the requirements are perfectly describable. Therefore, the manager can (and will) commit to her optimal project size at time 0. On the other hand, if $y = 0$, then the requirements are completely indescribable, and the manager only knows that the project is complete *when she sees it*. In this case, at every moment the manager observes the current state of the project q, and decides whether it is *good enough* (in which case its size will be $Q = q$), or whether to let the agents continue to work and re-evaluate the completion decision an instant later. Therefore, y can be interpreted as the manager's commitment power, where a larger y indicates greater commitment power.

For example, y is likely to be large in a construction project where the requirements are relatively standardized and easy to define. On the other hand, in a project that involves a significant innovation or quality component, y is likely to be small, because the manager cannot contract on the requirements of the final output until the project is at a sufficiently advanced stage. Similarly, y is typically small in design-related projects such as automotive design, as the requirements are difficult to describe. Alternatively, if the project is outsourced and contracts are explicit, then y will tend to be larger than the case in which the project is undertaken in-house so that the manager can influence the team members' career paths and contracts are typically implicit.

4.1 Optimal Project Size

To examine the manager's optimal project size, we first consider the case in which she has full commitment power (*i.e.*, $y = \infty$), so that she can commit to any project size before the agents begin to work. Second, we consider the opposite extreme case in which she has no commitment power (*i.e.*, $y = 0$), so that at every moment she observes the current state of the project q and decides whether to complete it immediately, or to let the agents continue working and re-evaluate her option to complete it a moment later. Finally, we consider the case in which she has intermediate commitment power (*i.e.*, $0 < y < \infty$), and we examine how her optimal project size depends on y. Throughout this Section, we take the agents' cooperation level $k \in [1, n]$ as given. As such, we suppress k for notational convenience.

4.1.1 Full Commitment Power ($y = \infty$)

If the manager has full commitment power, then she can commit to a project size before the agents begin to work. Therefore, at $q_0 = 0$, the manager leads a Stackelberg game in which she chooses the project size that maximizes her discounted profit and the agents follow by adopting the equilibrium strategy characterized in Proposition 2. As a result, her optimal project size with full commitment (FC) satisfies $Q_{FC}^M \in \arg\max_Q W(0\,; Q, Q)$.[13] Noting from (7) that $W(0\,; Q, Q)$ is concave in Q, differentiating it with respect to Q yields

$$Q_{FC}^M = \frac{\beta}{r}\frac{k\,(2n-k)}{2n}\left(\frac{4n}{4n-k}\right)^2 .$$

Note that the concavity of her discounted profit function implies that she commits to Q_{FC}^M at $q = 0$ for any commitment power $y \geq Q_{FC}^M$.

4.1.2 No Commitment Power ($y = 0$)

On the other hand, if the manager has no commitment power, then at every moment she observes the current state of the project q, and she decides whether to stop work and collect the net profit $(1-\beta)\,q$ or to let the agents continue working and re-evaluate her decision to complete the project a moment later. In this case, the manager and the agents engage in a simultaneous-action game, where the manager chooses Q to maximize her discounted profit given the agents' belief $\tilde{Q}$ and the corresponding strategies, and the agents form their beliefs by anticipating the manager's choice Q. Therefore, her optimal project size with no commitment (NC) satisfies $Q_{NC}^M(k) \in \arg\max_Q \left\{W\left(q\,; Q, \tilde{Q}\right)\right\}$, where in equilibrium beliefs must be correct; *i.e.*, $Q = \tilde{Q}$. By solving $\left.\frac{\partial W\left(q\,; Q, \tilde{Q}\right)}{\partial Q}\right|_{q=\tilde{Q}=Q} = 0$, we have

$$Q_{NC}^M = \frac{\beta}{r}\frac{2kn}{2n-k} .$$

Observe that if $y = 0$, then the manager will choose a strictly larger project relative to the case in which she has full commitment power: $Q_{NC}^M > Q_{FC}^M$. We shall discuss the intuition behind this result in Section 4.1.3 after we determine the manager's optimal project size for intermediate levels of commitment power.

This case raises the question of what happens to the agents' beliefs off the equilibrium

[13] Because the manager leads the agents in a Stackelberg game, given any choice Q, the agents will choose their strategy based on that Q, and the agent's belief $\tilde{Q}$ will coincide with Q ex-ante.

path if the manager does not complete the project at Q^M_{NC}. Suppose that the manager did not complete the project at Q^M_{NC} so that $q > Q^M_{NC}$. Clearly, Q and $\tilde{Q} > Q^M_{NC}$, and it is straightforward to verify that $\frac{\partial W\left(q\,;Q,\tilde{Q}\right)}{\partial Q} < 0$ for all q, Q and $\tilde{Q} > Q^M_{NC}$, which implies that the manager would be better off had she completed the project at Q^M_{NC} irrespective of the agents' beliefs..

Conceptually, this commitment problem could be resolved by allowing β to be contingent on the project size. In particular, suppose that the manager can fix β, and let $\hat{\beta}\left(Q\right)$ equal β if $Q = Q^M_{FC}$, and 1 otherwise. Then, her optimal project size is equal to Q^M_{FC} regardless of her commitment power because any other project size will yield her a net profit of 0. However, this implicitly assumes that Q^M_{FC} is contractible at $q = 0$, which is clearly not true for any $y < Q^M_{FC}$. Therefore, we rule out this possibility by assuming that β is independent of Q.

4.1.3 Partial Commitment Power ($0 < y < \infty$)

Recall that for any given cooperation level k, the manager's optimal project size is equal to Q^M_{FC} for all $y \geq Q^M_{FC}$, and it is equal to Q^M_{NC} if $y = 0$. To determine her optimal project size when $y \in \left(0, Q^M_{FC}\right)$, we solve an auxiliary problem, and we show that there is a one-to-one correspondence between this auxiliary problem and the original problem.

Suppose that the manager can credibly commit to her optimal project size as soon as the project hits (some exogenously given) x. In this case, the manager leads a Stackelberg game, where she chooses Q^M_x to maximize her discounted profit at x, so that $Q^M_x \in \arg\max_{Q \geq x} \left\{W\left(x\,; Q, Q\right)\right\}$, and the agents follow by choosing their strategies based on Q^M_x. We then show that for all $y \in \left(0, Q^M_{FC}\right)$, there exists a unique $x\left(y\right) \in \left(0, Q^M_{NC}\right)$, such that the manager will commit to the project size $Q^M_{x(y)}$ as soon as the project hits $x\left(y\right)$.

Proposition 3. *Suppose that given the current state q, the manager can commit to any project size $Q \in [q,\, q + y]$. Then at $x\left(y\right)$ the manager will commit to $Q^M_{x(y)}$, where*

$$Q^M_{x(y)} = \left(\frac{2n}{4n-k}\right)^2 \left(\sqrt{\frac{\beta}{r}\frac{k\left(2n-k\right)}{2n}} + \sqrt{\frac{\beta}{r}\frac{k\left(2n-k\right)}{2n} + \frac{k\left(4n-k\right)}{4n^2}x\left(y\right)}\right)^2 , \qquad (8)$$

$x\left(y\right)$ is the unique solution to $\max\left\{Q^M_{x(y)} - y,\, 0\right\} = x\left(y\right)$, *and it decreases in y.*
Therefore, the manager's optimal project size decreases in her commitment power: $Q^M_{x(y)}$ decreases in y.

The first part of this Proposition asserts that the manager has incentives to extend the project as it progresses: Q_x^M increases in x. Intuitively, the manager trades off a larger project that yields a larger net profit upon completion against having to wait longer until that profit is realized, but she ignores the additional effort cost associated with a larger project. Moreover, recall that the agents raise their effort, and hence the manager's marginal cost associated with choosing a larger project decreases as the project progresses. On the other hand, her marginal benefit from choosing a larger project is independent of the progress made. Since the project size will be chosen such that the two marginal values are equal, it follows that the manager's optimal project size increases as the project progresses.[14]

After re-arranging terms in (8), it is possible to write the manager's optimal project size explicitly as a function of her commitment power y as

$$Q^M(y) = \frac{1}{2}\left(\sqrt{\frac{\beta}{r}\frac{kn}{2n-k}} + \sqrt{\frac{\beta}{r}\frac{kn}{2n-k} - \frac{k}{2n-k}\min\left\{y, Q_{FC}^M\right\}}\right)^2 .$$

Remark 1. *Recall that (i) the MPE is unique if* $Q < \frac{2\beta}{r}$*, (ii)* $Q_{NC}^M < \frac{2\beta}{r}$ *for all* $n \geq 2$*, , and (iii)* $Q^M(y) \leq Q_{NC}^M$ *for all* y*. Therefore, the game has a unique MPE for any level of commitment power when the project size is chosen by the manager.*

The implication of Proposition 3 is that if the manager has less commitment power, then she will commit at a later state and to a larger project; *i.e.*, $x(y)$ and $Q^M(y)$ decrease in y. By noting that the extreme cases in which the manager has full (no) commitment power correspond to $y = 0$ ($y = \infty$), this intuition also explains why $Q_{NC}^M > Q_{FC}^M$. Figure 1 illustrates an example. In addition, Proposition 3 together with the expression for the completion time of the project computed in Proposition 2 imply that the duration of the project will be larger if the manager has less commitment power.

It is important to emphasize that the agents internalize the manager's limited ability to commit, and they choose their effort strategy appropriately. In particular, each agent's effort increases in the manager's commitment power ($i.e.$, $a\left(q\,;Q^M(y)\right)$ increases in y) since $C(Q\,;k)$ increases in Q for all $Q > \frac{\beta}{r}\frac{(2n-k)k}{2n}$ and $Q^M(y) > \frac{\beta}{r}\frac{(2n-k)k}{2n}$ for all y and k. This implies that the manager's ability to commit induces a ratchet effect: anticipating that she

[14] To reinforce this intuition, suppose that each agent exerts constant effort $a > 0$ throughout the duration of the project. Given the current state q, the project will be completed in $\frac{Q-q}{na}$ units of time so that the manager's discounted profit is equal to $(1-\beta)\,Qe^{-\frac{r(Q-q)}{na}}$. It follows that the her optimal project size is $Q = \frac{na}{r}$ and it is independent of q, which implies that the manager's time-inconsistency arises due to the agents increasing their effort along the evolution path of the project.

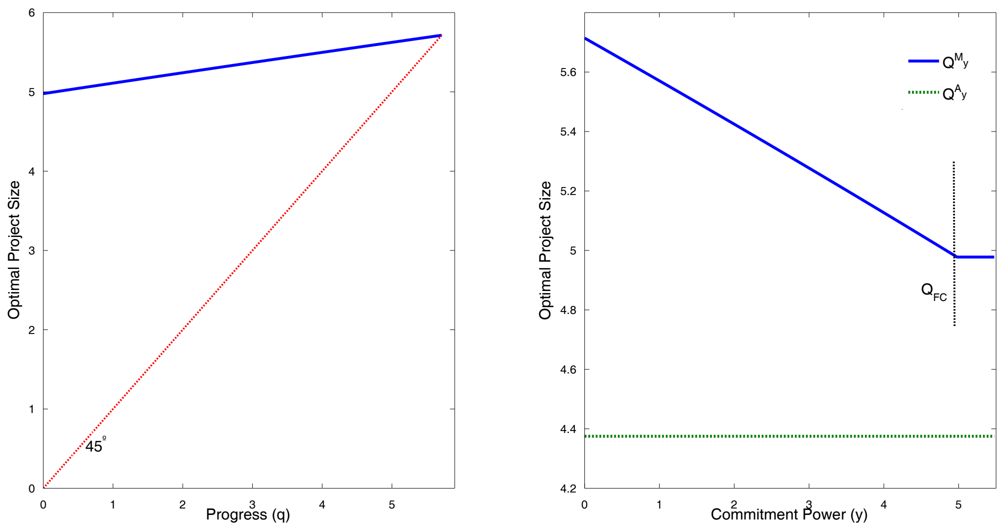

Figure 1: **Optimal project size** when $\beta = 0.5$, $r = 0.1$, $k = 1$, and $n = 4$. The left panel illustrates the manager's incentives the extend the project as it progresses: her optimal project size increases in the state of the project q, and there exists a state at which the manager is better off completing the project without further delay. The right panel illustrates that her optimal project size (solid line) decreases in her commitment power, while the agents' optimal project size (dashed line) is independent of their commitment power.

will choose a larger project, the agents respond by scaling down their effort. While ratchet effects have been shown to arise in settings with asymmetric information (e.g., Freixas, Guesnerie and Tirole (1985) and Laffont and Tirole (1988)), in our model they arise under moral hazard with full information.

The following result examines how the manager's optimal project size depends on the parameters of the problem.

Result 3. Other things equal, the manager's optimal project size $Q^M(y)$:
(i) increases in k (and β) for all y ;
(ii) decreases in r for all y ; and
(iii) if $k = 1$, then there exists a threshold Φ such that it increases in n if and only if $y \geq \Phi$. On the other hand, if $k = n$, then it increases in n.

Statements (i) and (ii) are not surprising. Because each agent's effort increases in k, the team can achieve more progress during any given time interval by playing a more cooperative equilibrium, and hence the manager has incentives to choose a larger project. If the agents receive a larger share of the project's value upon completion, then they will work harder along the equilibrium path, and as a result, the manager will choose a bigger project. For the intuition behind (ii), recall that the manager trades off the higher payoff of a larger project against the longer delay until that payoff is collected. As the parties become more patient (*i.e.*, as r decreases), the cost associated with the delay decreases, and hence the optimal project size increases.

To examine how the manager's optimal project size depends on the team size, one must first consider how the agents' cooperation level k depends on n. To obtain sharp results, we consider the cases $k = 1$ (which corresponds to the MPE) and $k = n$ (which corresponds to the efficient PPE). In the first case, observe from Proposition 1 that $\frac{\partial}{\partial q} a\left(q\,; Q\right) = \frac{r}{2n-1}$ decreases in n. Because the manager's incentive to extend the project is driven by the agents raising their effort as the project progresses, it follows that this incentive becomes weaker in n. As a result, the manager's optimal project size increases in n if and only if she has sufficient commitment power. On the other hand, if $k = n$, then the result follows from the fact that the aggregate effort of the team increases in n at every state of the project.

This analysis also raises the question about the manager's optimal team size. Considering the cases $k = 1$ and $k = n$ as above, by substituting (8) in (7) and differentiating with respect to n, it is straightforward to show that with no commitment power (*i.e.*, $y = 0$), the manager's optimal team size is $n^* = 2$ when $k = 1$, while the project is never completed and the manager's discounted profit equals 0 for any team size if $k = n$. On the other hand, with full commitment power (*i.e.*, $y = \infty$), it is $n^* = 1$ and $n^* = \infty$ when $k = 1$ and $k = 2$, respectively. With intermediate levels of commitment power, the expression for the manager's discounted profit is not sufficiently tractable to optimize with respect to n, but numerical analysis for the cases with $k = 1$ and $k = n$ indicates that the manager's optimal project size decreases in her commitment power.

4.2 Optimal Delegation

The manager's limited ability to commit, in addition to disincentivizing the agents from exerting effort, is detrimental to her ex-ante discounted profit; *i.e.*, $W\left(0\,; Q^M\left(y\right), Q^M\left(y\right)\right)$ increases in y.[15] Thus, unable to commit sufficiently early, the manager might consider delegating the decision rights over the project size to the agents.

[15] This follows from the facts that $W\left(0\,; Q, Q\right)$ is concave in Q, the manager's ex-ante discounted profit is maximized at Q^M_{FC}, $Q^M\left(y\right) \geq Q^M_{FC}$ for all y, and $Q^M\left(y\right)$ decreases in y.

We begin by examining how the agents would select the project size. Let $Q^A \in \arg\max_Q \{\Pi(x; Q)\}$ denote the agents' optimal project size given the current state x. Solving this maximization problem yields

$$Q^A = \frac{\beta}{r} \frac{k(2n-k)}{2n}.$$

[16] First, observe that the agents' optimal project size is independent of the current state x. This is also illustrated in the right panel of Figure 1. Intuitively, this is because they incur the cost of their effort, so that their effort cost increases together with their effort level as the project progresses. As a result, unlike the manager, their marginal cost associated with choosing a larger project does not decrease as the project evolves, and consequently they do not have incentives to extend the project as it progresses.

Second, observe that $Q^A < Q^M(y)$ for all y; *i.e.*, the agents always prefer a smaller project than the manager.[17,18] This is because they incur the cost of their effort, so that their marginal cost associated with a larger project is greater than that of the manager's.

Proposition 4. *Suppose that given the current state q, the manager can commit to any project size $Q \in [q, q+y]$. There exists an interior threshold θ such that $W\left(0; Q^A, Q^A\right) > W\left(0; Q^M(y), Q^M(y)\right)$ if and only if $y < \theta$; i.e., she should delegate the choice of the project size to the agents unless she has sufficient commitment power.*

Recall that the agents' optimal project size is time-consistent, which implies that if the manager delegates the decision rights to the agents, then her ex-ante discounted profit is independent of when the project size is chosen. The key part of this result is that if the manager has no commitment power (*i.e.*, $y = 0$), then she is always better off delegating the decision rights over the project size to the agents. By noting that the manager's optimal project size (and hence her ex-ante discounted profit) increases in her commitment power, the Proposition follows.

4.3 Socially Optimal Project Size

In this Section, we characterize the optimal project size of a social planner who seeks to maximize the team's total discounted payoff.

[16] The comparative statics established in Result 3 (i) and (ii) also hold for Q^A. However, Q^A increases in the team size n both when $k = 1$ and $k = n$.

[17] An implication of this observation, together with Remark 1, is that the equilibrium of the game is unique also when the project size is chosen by the agents.

[18] One might envision an intermediate decision rule, wherein all parties need to unanimously agree on a project size. Because the manager prefers a larger project than the agents regardless of her commitment power, under a unanimity requirement, effectively, they will agree to the manager's optimal project size.

Result 4. Consider a social planner who maximizes the sum of the agents' and the manager's discounted payoffs (but cannot control the agents' effort strategies).
(i) His optimal project size Q_y^{FB} satisfies $Q^A < Q_y^{FB} < Q^M(y)$ for all y.
(ii) With 1 agent, his optimal project size decreases in the commitment power y.[19]

The social planner seeks to maximize the sum of the manager's and the agents' discounted payoff. As such, his optimal project size will lie between the agents' and the manager's optimal project size for all y. In addition, because the agents' (manager's) optimal project size is independent of (decreases in) y, it is intuitive that the social planner's optimal project size decreases in the commitment power y. With $n \geq 2$ agents, this problem becomes intractable. However, numerical analysis indicates that the social planner's optimal project size continues to decrease in the commitment power y. This is illustrated in the left panel of Figure 2.

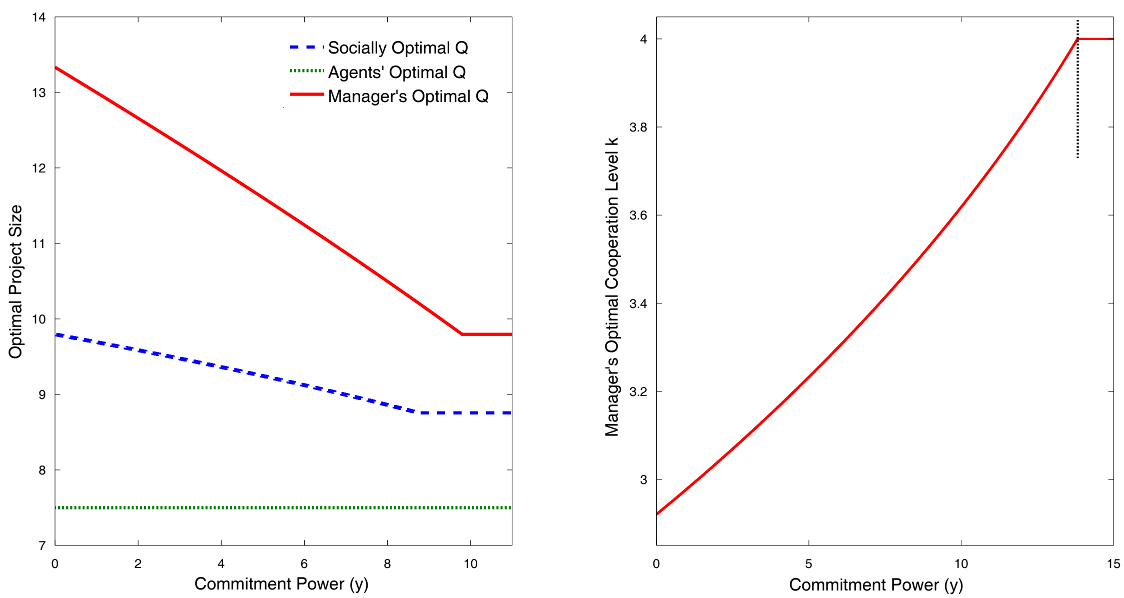

Figure 2: **The social planner's optimal project size** (left panel) **and the manager's optimal cooperation level** (right panel) when $\beta = 0.5$, $r = 0.1$, and $n = 4$. The left panel illustrates (with $k = 2$) that the social planner's optimal project size decreases in the commitment power y. The right panel illustrates that the manager's optimal cooperation level k increases in her commitment power and a fully cooperative equilibrium (*i.e.*, $k = n$) is optimal only if she has sufficient commitment power (*i.e.*, $y \geq \varphi$).

[19]Given y, the social planner commits to $Q_{x(y)}^{SP} = \arg\max_Q \{\Pi(q\,;Q) + W(q\,;Q,Q)\}$ at $q = x(y)$, where $x(y)$ satisfies $\max\left\{Q_{x(y)}^{SP} - y,\, 0\right\} = x(y)$. The result follows from the facts that this problem is strictly concave and $\frac{d}{dQ}[\Pi(q\,;Q) + W(q\,;Q,Q)]$ increases in q for all $Q \leq Q_{x(0)}^{SP}$, which in turn implies that $Q_{x(y)}^{SP}$ decreases in y.

If the social planner can also control the agents' strategies, then it is straightforward to verify from the social planner's discounted payoff characterized in Result 1 that his optimal project size equals $\frac{n}{2r}$ and it is independent of his commitment power.

5 Equilibrium Selection

Insofar we have taken the agents' cooperation level as given. However, experiments in social identity theory have demonstrated that it is surprisingly easy to affect subjects' behavior as insiders or outsiders within a group (Akerlof and Kranton (2005)). Using equilibrium selection concepts proposed by Kreps (1990), we consider the possibility that the manager can influence the agents' cooperation level (*i.e.*, the PPE that will be played) by cultivating a more (or less) cooperative environment within the team; for example, by organizing sponsored activities, encouraging interaction among the team members, and engaging the agents when making decisions, as well as with appropriate selection of those who join the team.

Taking into account her commitment power y at time 0, the manager chooses the agents' cooperation level to maximize her ex-ante discounted profit:

$$k_y \in \arg\max_{1 \leq k \leq n} \left\{ W\left(0 \,; Q^M\left(y\right), Q^M\left(y\right), \, k\right) \right\} .$$

To obtain tractable results, we restrict attention to the extreme cases $y = \infty$ and $y = 0$.

Proposition 5. *Suppose that the manager can choose the agents' cooperation level k at q_0.*
(i) If $y = \infty$, then a fully cooperative environment is optimal: $k_{FC} = n$.
(ii) If $y = 0$, then the optimal cooperation level $k_{NC} < n$.

Choosing a higher cooperation level has two opposite effects. For a fixed Q, the agents work harder, which increases the manager's discounted profit. However, because the agents work harder and they ramp up their effort faster as the project progresses, the manager has stronger incentives to extend the project (as evidenced by the fact that $\frac{\partial Q_x^M}{\partial x}$ increases in k), which harms her ex-ante discounted profit. If $y = \infty$, then the latter effect is absent and hence she is better off fostering a fully cooperative environment within the team; *i.e.*, $k = n$. On the other hand, if $y = 0$, then the latter effect dominates the former, so that a less than fully cooperative environment within the team renders her better off; *i.e.*, $k < n$. The manager's optimal cooperation level is illustrated in the right panel of Figure 2 as a function of her commitment power.

If the manager delegates the decision rights over the project size to the agents, then a fully cooperative environment (*i.e.*, $k = n$) is always optimal. This is intuitive, because the agents' preferences are time-consistent and a fully cooperative environment eliminates the free-rider problem.[20]

6 Concluding Remarks

We develop a tractable model to study the interaction between a group of agents who collaborate over time to complete a project and a manager who chooses its size. A central feature of the model is that the manager has limited commitment power, in that she can only commit to the project size when the project is sufficiently close to completion. This is common in projects that involve a significant innovation or quality or design component that is difficult to contract on in advance.

In a setting in which both the manager and the agents are rational and they do not obtain new information about the difficulty or the value of the project, we show that the manager has incentives to extend the project as it progresses. As a result, if the manager has lower commitment power, then she will eventually commit to a bigger project. To mitigate her commitment problem, the manager might consider delegating the decision rights over the project size to the agents, who will choose a smaller project than is optimal for the manager, but their preferences are time-consistent. We show that delegation is optimal unless the manager has sufficient commitment power.

In our model, agents are compensated upon completion of the project and their compensation is independent of the completion time of the project. While Georgiadis (2013) shows that backloading all rewards is optimal when the project size is given exogenously, it is unclear that this continues to be optimal when the project size is endogenous and the manager has limited commitment power. It is possible that a more complex scheme in which the manager provides agents with flow payments while the project is in progress (*e.g.*, Sannikov (2008)) can improve her discounted profit by mitigating her commitment problem. In addition, we know from Bonatti and Hörner (2011) and Campbell, Ederer and Spinnewijn (2013) that time-dependent contracts (*e.g.*, deadlines) can be employed to mitigate the free-rider problem. Relaxing these restrictions is an interesting avenue for future research.

[20]Similarly, if the manager acts as a social planner who maximizes the sum of the manager's and the agents' discounted payoffs (but cannot control the agents' effort strategies), then numerical analysis suggests that cultivating a full cooperative environment is optimal.

A Extensions

In this Appendix we consider five extensions to our model to test the robustness of the main results.

A.1 Production Synergies and Team Coordination Costs

First, we consider the case in which at every moment, the total effort of the team is greater (due to production synergies) or smaller (due to coordination costs among the team members) than the sum of the agents' individual efforts. We show that all three main results continue to hold for any degree of complementarity.

To obtain tractable results, we consider the production function proposed by Bonatti and Hörner (2011), so that the project evolves according to $dq = \left(\sum_{i=1}^{n} a_i^{1/\gamma}\right)^{\gamma} dt$, where $\gamma > 0$. Note that $\gamma \in (0,1)$ ($\gamma > 1$) captures the case in which the total effort of the team is smaller (greater) than the sum of the agents' individual efforts, and a larger γ indicates smaller coordination costs or a stronger degree of complementarity. By assuming symmetric strategies, it follows that given the current state of the project q, cooperation level k, and the completion state Q, each agent's discounted payoff and effort strategy are given by

$$\Pi\left(q\,;Q,k\right) = \frac{rn^{2-2\gamma}}{2k}\frac{\left(\left[q-C\left(Q\,;k\right)\right]^{+}\right)^{2}}{2n-k} \quad \text{and} \quad a\left(q\,;Q,\,k\right) = \frac{rn^{1-\gamma}}{2n-k}\left[q-C\left(Q\,;k\right)\right]^{+} \;,$$

respectively, where $C\left(Q\,;k\right) = Q - \sqrt{\frac{2\beta Q}{r}\frac{n^{2\gamma-2}(2n-k)k}{n}}$.[21] Because (with other things equal) $\Pi\left(q\,;Q,k\right)$ increases in k for all γ, it follows that for all $k \in [1,n]$ there exists a PPE such that each agent follows the strategy dictated by $a\left(q\,;Q,\,k\right)$, and after any deviation from the equilibrium path, all agents revert to the MPE; *i.e.*, $k=1$. Furthermore, each agent's discounted payoff, his equilibrium effort, as well as the aggregate effort of the entire team, increase in the degree of complementarity γ.

By using the agents' strategies, it follows that the manager's discounted profit satisfies

$$W\left(q\,;Q,\tilde{Q},\,k\right) = (1-\beta)\,Q\left(\frac{\left[q-C\left(\tilde{Q}\,;k\right)\right]^{+}}{Q-C\left(\tilde{Q}\,;k\right)}\right)^{\frac{2n-k}{n}} .$$

[21] As the algebra is straightforward and similar to that used to derive Propositions 1 and 2, it is omitted here in order to streamline the exposition.

To streamline the exposition, we focus on the extreme cases in which the manager has either full or no commitment power. It follows that

$$Q_{FC}^{M}=\frac{\beta}{r}\frac{k\,(2n-k)}{2n}\left(\frac{4n}{4n-k}\right)^{2}n^{2\gamma-2}\quad\text{and}\quad Q_{NC}^{M}=\frac{2\beta}{r}\frac{kn}{2n-k}n^{2\gamma-2}\,.$$

Observe that the manager's optimal project size increases in the degree of complementarity, and similar to the case analyzed in Section 4, $Q_{NC}^{M}>Q_{FC}^{M}$. Moreover, the counterpart of Proposition 2 continues to hold; *i.e.*, if the manager has less commitment power, then she will choose a bigger project.

We now examine the manager's option to delegate the choice of Q to the agents, as well as her optimal choice of the agents' cooperation level. To begin, note that the agents' optimal project size satisfies $Q^{A}=\frac{\beta}{2r}\frac{k(2n-k)}{n}n^{2\gamma-2}$. By following a similar approach as in Section 4.2, it follows that there exists a threshold θ such that the manager is better off delegating the choice of the project size to the agents if and only if her commitment power $y<\theta$.

A.2 Fixed Compensation

In the base model, we have assumed that the agents' net payoff upon completion of the project is proportional to its value. While a more valuable project will typically yield a larger net payoff to the agents - for example a bigger bonus, a salary increase, greater job security, or a larger outside option, this assumption can be thought of as an extreme case, since any incentive scheme will likely consist of a fixed component that is independent of the project size, and a performance-based component. In this Section, we consider the opposite extreme where each agent's net payoff is fixed and independent of the project size, while efforts are perfect substitutes; *i.e.*, $dq_t=\left(\sum_{i=1}^{n}a_{i,t}\right)dt$.

The main results continue to hold. In fact, the manager's commitment problem becomes so aggravated in this case, that the project may never be completed in equilibrium. Moreover, because the agents' net reward is independent of Q, their optimal project size is always 0. As such, the manager can no longer use delegation to mitigate her commitment problem.

To begin, suppose that each agent receives a lump-sum $\frac{V}{n}$ as soon as the project is completed regardless of its size. Then given the current state of the project q, the cooperation level k, and the completion state Q, each agent's equilibrium effort is given by

$$\bar{a}\left(q\,;Q,\,k\right)=\frac{r}{2n-k}\left[q-\bar{C}\left(Q\,;k\right)\right]^{+}\quad\text{where }\bar{C}\left(Q\,;k\right)=Q-\sqrt{\frac{2V}{r}\frac{\left(2n-k\right)k}{n}}\,,$$

while the manager's discounted profit satisfies

$$\bar{W}\left(q\,;Q,\tilde{Q},\,k\right)=\left(Q-V\right)\left(\frac{\left[q-\bar{C}\left(\tilde{Q}\,;k\right)\right]^{+}}{Q-\bar{C}\left(\tilde{Q}\,;k\right)}\right)^{\frac{2n-k}{n}}\,.$$

Using the same approach as in Section 3, one can show that for all $k\in(1,n]$ there exists a PPE such that each agent follows the strategy dictated by $\bar{a}\left(q\,;Q,\,k\right)$ contingent on all other agents following the same strategy, and reverts to the MPE (*i.e.*, $k=1$) after observing a deviation.

By examining the manager's optimal project size, it follows that with full and with no commitment power, we have

$$\bar{Q}_{FC}^{M}=\frac{2n-k}{3n-k}V+\frac{n}{3n-k}\sqrt{\frac{2V}{r}\frac{\left(2n-k\right)k}{n}}\quad\text{and}\quad\bar{Q}_{NC}^{M}=V+\sqrt{\frac{2V}{r}\frac{kn}{2n-k}}\,,$$

respectively. Observe that $\bar{Q}_{NC}^{M}>\bar{Q}_{FC}^{M}$, and by solving for $\bar{Q}_{x}^{M}\in\arg\max_{Q}\bar{W}\left(q\,;Q,Q,\,k\right)$, it follows that $\bar{Q}_{x}^{M}$ increases in x. Therefore, similar to the base model, the manager has incentives to extend the project as it progresses. In fact, these incentives can be so strong that the project is never completed in equilibrium. To see why, note that the project is completed only if $\bar{C}\left(Q\,;k\right)<0$, and this inequality is true at $Q=\bar{Q}_{NC}^{M}\left(k\right)$ if and only if $rV<\frac{2k(n-k)^{2}}{n(2n-k)}$. Moreover, if each agent's net payoff is independent of the project size, then delegating the choice of the project size to the agents is not beneficial, because they will choose a project of size 0.

To examine the manager's optimal choice of k, note that the last inequality is violated if $k=n$, which implies that if the agents play the fully cooperative PPE and the manager has no commitment power, then the project is never completed. Therefore, the manager can increase her discounted profit by choosing some $k<n$ such that the project is completed. On the other hand, by noting that $\sqrt{\frac{2(2n-k)k}{n}}>\left.\sqrt{\frac{2kn}{2n-k}}\right|_{k=1}$ for all $n\geq 2$, and observing that V is the only parameter that the manager can choose, it follows that there always exists some $V>0$ and $k\in[1,n]$ such that the project is completed in equilibrium even if the manager has no commitment power.

Therefore, the manager's commitment problem is severely aggravated if the agents' net payoffs are independent of the project size. Intuitively, this is because the manager obtains the entire marginal benefit from a larger project (as opposed to $1-\beta$ thereof), which provides her with stronger incentives to extend it as it progresses. As a result, anticipating this behavior, the agents prefer to exert no effort and abandon the project altogether.

A.3 Flow Payments while the Project is in Progress

Throughout the analysis we have maintained the assumption that the agents receive a lump-sum payment upon completing the project, but they do not receive any flow payments while the project is ongoing. Therefore, to extend the project, the manager must only incur the cost associated with having to wait longer until the project is completed. In this Section, we consider the case in which the manager compensates each agent with a flow payment $\frac{w}{n} > 0$ per-unit of time while the project is in progress, in addition to a lump-sum payment upon completing it.

Similar to the base case, the manager has incentives to extend the project as it progresses, and she is better off delegating the decision rights to the project size to the agents unless she has sufficient commitment power. Moreover, her optimal cooperation level k increases in her commitment power, and a fully cooperative equilibrium is optimal only if she has sufficient commitment power.

It is straightforward to show that for a given project size Q, each agent's discounted payoff, and the manager's discounted profit satisfy

$$\begin{aligned} r\bar{\Pi}\left(q\,;Q\right) &= \frac{w}{n}+\frac{k\left(2n-k\right)}{2}\left[\bar{\Pi}'\left(q\,;Q\right)\right]^{2} \quad \text{s.t. } \bar{\Pi}\left(Q\,;Q\right)=\beta Q \\ r\bar{W}\left(q\,;Q,\tilde{Q}\right) &= -w+\left[na\left(q\,;\tilde{Q},\,k\right)\right]\bar{W}'\left(q\,;Q,\tilde{Q}\right) \quad \text{s.t. } \bar{W}\left(Q\,;Q,\tilde{Q}\right)=\left(1-\beta\right)Q\,, \end{aligned}$$

respectively, where $\bar{a}\left(q\,;\tilde{Q},\,k\right) = k\bar{\Pi}'\left(q\,;Q\right)$. As this model is analytically not tractable, to examine how the main results extend to this case, we present a numeric illustration (see Figure 3). The takeaway from this analysis is that the main results continue to hold when the agents receive flow payments while the project is in progress.

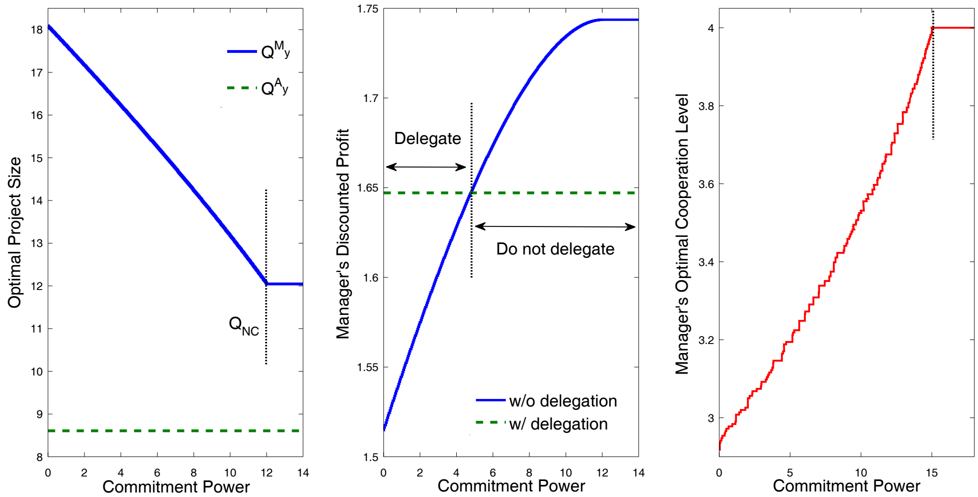

Figure 3: **An example in which the manager compensates the agents per unit of time while the project is in progress** when $\beta = 0.5$, $r = 0.1$, $n = 4$, and $w = 0.001$. The **left panel** illustrates that her optimal project size decreases in her commitment power, while the agents' optimal project size is independent of their commitment power. The **middle panel** illustrates that delegating the decision rights over the project size to the agents is beneficial if and only if the manager doesn't have sufficient commitment power. Finally, the **right panel** illustrates that the manager's optimal cooperation level increases in her commitment power.

A.4 Sequential Projects

Insofar, we have assumed that the manager interacts with the agents for the duration of a single project. However, because in practice, relationships between a manager and work teams are often persistent, it is important to verify that the main results of this paper are robust to repeated interactions. In this Section we consider the case in which as soon as a project is completed, the manager and the agents interact for the duration of another project with probability $\alpha < 1$, while the relationship is terminated with probability $1 - \alpha$ and each party receives its outside option which is normalized to 0.[22]

Indeed, we find that when the manager and the agents engage in sequential projects, all the main results continue to hold. Moreover, we observe that if the relationship is more persistent (*i.e.*, α is larger), then the manager has stronger incentives to delegate the choice of the project size to the agents, and her optimal cooperation level is larger.

[22] If $\alpha = 0$, then this case reduces to the base model. On the other hand, because the value of the project has been assumed to be linear in its size, and it generates a payoff only upon completion, as $\alpha = \rightarrow 1$, both the manager and the agents choose an arbitrarily small project, which is completed arbitrarily quickly. Therefore, we restrict attention to the cases in which $\alpha < 1$.

Since the problem is stationary, the manager will choose the same project size every time. Both the agents' and the manager's problem remain unchanged, except for the boundary conditions, which become $\bar{\Pi}(Q\,;Q) = \frac{\beta Q}{n} + \alpha\bar{\Pi}(0\,;Q)$ and $\bar{W}\left(Q\,;Q,\tilde{Q}\right) = (1-\beta)\,Q + \alpha\bar{W}\left(0\,;Q,\tilde{Q}\right)$, respectively. The interpretation of these conditions is that upon completion of each project, each party receives its net payoff from this project, plus the expected continuation value from future projects.

Unfortunately, it is not possible to derive the desired results analytically. As such, we use a numerical example to illustrate how the main results of the paper extend to this case. Figure 4 illustrates that the main results continue to hold. In particular, (i) the manager's optimal project size decreases in her commitment power, whereas the agents' optimal project size is independent of their commitment power, (ii) the manager should delegate the decision rights over Q unless she has sufficient commitment power, and (iii) the manager's optimal cooperation level increases in her commitment power.

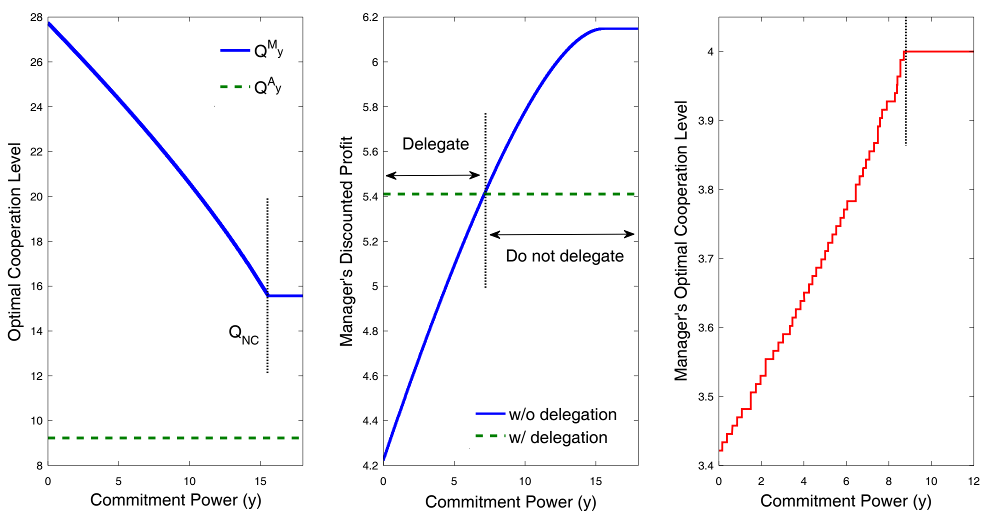

Figure 4: **An example in which the manager interacts with the agents repeatedly** when $\beta = 0.5$, $r = 0.1$, $n = 4$, and $\alpha = 0.25$. Similar to Figure 3, it illustrates that the main results continue to hold in this case.

A.5 Stochastic Evolution of the Project

A key assumption that provides tractability to our model is that the project progresses deterministically. To obtain some insights as to how the results in this paper depend on this assumption, consider the case in which the project progresses stochastically according to

$$dq_t = \left(\sum_{i=1}^{n} a_{i,t} \right) dt + \sigma dW_t \,,$$

where $\sigma > 0$ captures the degree of uncertainty associated with the evolution of the project, and W_t is a standard Brownian motion. It is straightforward to show that for a given project size Q, in any MPE, each agent's expected discounted payoff and the manager's expected discounted profit satisfy

$$\begin{aligned} r\bar{\Pi}\left(q\,;Q\right) &= \frac{(2n-1)}{2}\left[\bar{\Pi}'\left(q\,;Q\right)\right]^2 + \frac{\sigma^2}{2}\bar{\Pi}''\left(q\,;Q\right) \text{ and} \\ r\bar{W}\left(q\,;Q,\tilde{Q}\right) &= \left[n\bar{a}\left(q\,;\tilde{Q}\right)\right]\bar{W}'\left(q\,;Q,\tilde{Q}\right) + \frac{\sigma^2}{2}\bar{W}''\left(q\,;Q,\tilde{Q}\right) \end{aligned}$$

subject to $\bar{\Pi}\left(Q\,;Q\right) = \frac{\beta Q}{n}$, $\bar{W}\left(Q\,;Q,\tilde{Q}\right) = (1-\beta)\,Q$, $\lim_{q\to-\infty}\bar{\Pi}\left(q\,;Q\right) = 0$, and $\lim_{q\to-\infty}\bar{W}\left(q\,;Q\right) = 0$, respectively, where $\bar{a}\left(q\,;\tilde{Q}\right) = \bar{\Pi}'\left(q\,;Q\right)$. This problem is studied by Georgiadis (2013) for a fixed project size, who shows that similar to Proposition 1, a unique solution to this system of ODE exists, and $\bar{\Pi}'\left(q\,;Q\right) > 0$ and increasing in q, which in turn implies that the MPE is unique. It is worth noting that the non-Markovian strategies characterized in this paper no longer constitute a PPE if the project progresses according to the above stochastic process. Intuitively, this is because a deviation from the equilibrium path cannot be detected instantaneously with probability 1.

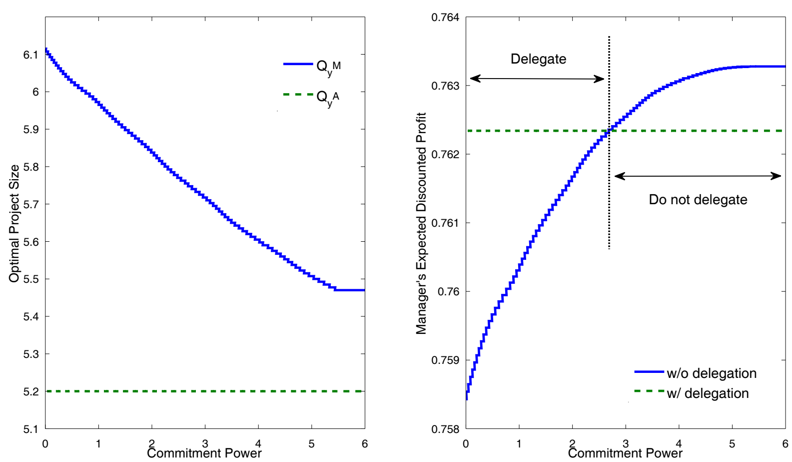

Figure 5: **An example in which the project progresses stochastically** when $\beta = 0.5$, $r = 0.1$, $n = 3$, and $\sigma = 1$. Similar to Figure 3, it illustrates that the main results continue to hold in this case

Georgiadis (2011) shows that the agents are time-consistent with respect to their optimal project size, which implies that similar to the deterministic case, their optimal project size is independent of the commitment power y. Unfortunately however, the analysis of the manager's choice of Q is not tractable. Therefore, in Figure 5, we use a numerical example to illustrate that the main results continue to hold when the project progresses stochastically: the manager's optimal project size decreases in her commitment power, while the agents' optimal project size is independent of their commitment power, and delegation is optimal unless the manager has sufficient commitment power.

B Proofs

Proof of Proposition 1. To show that a MPE with differentiable strategies exists for this game, it suffices to show that a solution to (4) exists. To show this, we derive a symmetric solution analytically. In particular, for symmetric strategies (*i.e.*, $\Pi_i(q\,;Q) = \Pi_j(q\,;Q)$ for all i and j), (4) can be re-written as

$$r\Pi(q\,;Q) = \frac{2n-1}{2}\left[\Pi'(q\,;Q)\right]^2 \,, \tag{9}$$

and the solution to this ODE satisfies

$$\Pi(q\,;Q) = \frac{r}{2}\frac{\left([q-C(Q)]^+\right)^2}{2n-1}\,,\text{ where } C(Q) = Q - \sqrt{\frac{2\beta Q}{r}\frac{2n-1}{n}}$$

is determined by the value matching condition. By using the first order condition, it follows that each agent's effort strategy is given by

$$a(q\,;Q) = \frac{r}{2n-1}\left[q - C(Q)\right]\mathbf{1}_{\{q\geq C(Q)\}}\,.$$

To show that there do not exist any asymmetric solutions to (4) we proceed by contradiction. Fix $Q > 0$, and suppose there exist at least two agents a and b whose discounted payoff functions $\Pi_a(q\,;Q)$ and $\Pi_b(q\,;Q)$ satisfy (4), but $\Pi_a(q\,;Q) \neq \Pi_b(q\,;Q)$ for at least some $q < Q$. Then let $D(q) = \Pi_a(q\,;Q) - \Pi_b(q\,;Q)$, and note that $D(Q) = 0$ and $D(\cdot)$ is differentiable. Then using (4) we can write $2rD(q) = \left[2\sum_i \Pi_i(q\,;Q) - \Pi_a(q\,;Q) - \Pi_b(q\,;Q)\right]D'(q)$. Moreover, because agents are impatient ($r > 0$) and the amount of effort that needs to be exerted until the project is completed diverges to infinity as $q \to -\infty$, it must be true that $\Pi_i(q\,;Q) \to 0$ as $q \to -\infty$. Therefore, $\lim_{q\to-\infty} D(q) = 0$, so if $D(q) \neq 0$ for at least some $q < Q$, then it must be the case that there exists some interior $z < Q$ such that $D(z) \neq 0$ and $D'(z) = 0$, which yields a contradiction. Hence we conclude that (4) cannot admit an asymmetric solution.

To show that (9) has a unique symmetric solution, we use a similar approach. Fix $Q > 0$, and suppose that there exist $\Pi_A(q\,;Q)$ and $\Pi_B(q\,;Q)$ that both satisfy (9). Then let $\Delta(q) = \Pi_A(q\,;Q) - \Pi_B(q\,;Q)$, and note that $\Delta(Q) = 0$ and $\Delta(\cdot)$ is differentiable. Therefore, (9) can be re-written as $2r\Delta(q) = (2n-1)\left[\Pi_A'(q\,;Q) + \Pi_B'(q\,;Q)\right]\Delta'(q)$. Moreover, $\lim_{q\to-\infty}\Delta(q) = 0$ by the same argument as above, so if $\Delta(q) \neq 0$ for at least some $q < Q$, then it must be the case that there exists some interior $z < Q$ such that $\Delta(z) \neq 0$ and $\Delta'(z) = 0$, which yields a contradiction. Therefore, there exists a unique symmetric solution to (4).

We have insofar shown that there exists a unique solution to (4), and that this solution is symmetric. Moreover, note that if $C(Q) \geq 0$ (or equivalently $Q \geq \frac{2\beta}{r}\frac{2n-1}{n}$), then the equilibrium strategy dictates that no agent ever exerts any effort, in which case the project is never completed. On the other hand, as long as $C(Q) < 0$, the strategy $a(q\,;Q)$ constitutes the unique *project-completing* MPE. Next, suppose that $C(Q) < 0 \leq C(Q)|_{n=1}$ (or equivalently $\frac{2\beta}{r} \leq Q < \frac{2\beta}{r}\frac{2n-1}{n}$), and fix all effort strategies except of that of agent i to 0. Then agent i's best response is to also exert 0 effort, since $C(Q)|_{n=1} \geq 0$; *i.e.*, he is not willing to undertake the entire project by himself. As a result, if $Q \geq \frac{2\beta}{r}$, then in addition to the *project-completing* MPE, there also exist an equilibrium in which no agent exerts any effort, and the project is never completed.

Finally, to compute the completion time of the project, we substitute the agents' effort function $a(q;\,Q)$ into $dq_t = na_t dt$, we solve the resulting ODE $q'(t) = \frac{r\,n}{2n-1}\left[q(t) - C(Q)\right]^+$ subject to $q(0) = 0$, and we obtain the completion time of the project τ by solving for $q(\tau) = Q$.

□

Proof of Proposition 2. This proof is organized as follows. First, we show that $\Pi(q\,;Q,\,k)$ is the solution to a game in which each agent chooses his effort according to (5), and that (6) is the corresponding effort strategy. Then we show that this strategy constitutes a PPE.

From (5), given the current state of the project q, the first order condition yields $a(q\,;Q,\,k) = k\Pi'(q\,;Q,\,k)$, and the second order condition is always satisfied. Substituting the first order condition into each agent's HJB equation yields

$$\Pi(q\,;Q\,k) = \frac{(2n-k)\,k}{2r}\left[\Pi'(q\,;Q,\,k)\right]^2$$

subject to $\Pi(Q\,;Q,\,k) \geq 0$ for all q and $\Pi(Q\,;Q,\,k) = \frac{\beta Q}{n}$.

It is straightforward to verify that $\Pi(q\,;Q,\,k) = \frac{r}{2k}\frac{\left([q-C(Q\,;k)]^+\right)^2}{2n-k}$ solves the above HJB equation, and by using the first order condition, it follows that each agent's effort strategy satisfies (6). If $k=1$, then (6) corresponds to the Markov equilibrium. To compute the completion time of the project, we substitute (6) into $dq_t = na_t dt$, we solve the resulting differential equation $q'(t) = \frac{rn}{2n-k}\left[q(t) - C(Q\,;k)\right]^+$ subject to $q(0)=0$, and we obtain the completion time of the project τ by solving for $q(\tau)=Q$.

We now show that the strategy defined above is indeed a PPE for any $k \in (1,n]$. First note that any deviation from the described strategy is detectable arbitrarily quickly. Since the agents can react *quickly*, such deviation can be punished with arbitrarily small delay, so that the gains from a deviation are arbitrarily small. Second, reverting to the Markov equilibrium after a deviation is sequentially rational since the MPE is (by definition) a PPE. Third, observe that $\Pi(q\,;Q,\,k) > \Pi(q\,;Q,\,1)$ for all $k>1$ and $q \geq C(Q\,;k)$, which implies that for any $k \in (1,n]$, as long as each agent chooses his effort to maximize the expected discounted payoff of k agents, no agent has an incentive to unilaterally deviate. Finally, by applying Theorem 4 of Bergin and MacLeod (1993) it follows that there exists a Public Perfect equilibrium in which each agent follows 6 along the equilibrium path.

□

Proof of Proposition 3. To begin, fix $k \in [1,n]$, and note that for any x, $W(x\,;Q,Q,k)$ is strictly concave in Q. Applying the first order condition yields (8). It is straightforward to verify that $\frac{\partial}{\partial x}Q_x^M > 0$ and $\frac{\partial^2}{\partial x^2}Q_x^M < 0$ for all $q>0$. Finally, solving the fixed point $Q_Q^M = Q$ yields $Q_Q^M = \frac{\beta}{r}\frac{2kn}{2n-k}$.

Next, let $g(x) = Q_x^M - x$, and observe that $g(0) = Q_0^M > 0$ and $g\left(Q_Q^M\right) = 0$. Moreover, it is easy to check that $g'(x) < 0$ on $\left[0,\, Q_Q^M\right]$, which implies that given any $y \leq Q_0^M$, there exists a unique $x(y)$ such that $g(x(y)) = y$.

Clearly, if $y \geq Q_0^M$, then the manager finds it optimal to commit to Q_0^M at $x=0$. Therefore, for all $y \geq 0$, there exists a unique $x(y)$ that solves $\max\left\{Q_{x(y)}^M - y,\, 0\right\} = x(y)$.

To proceed, suppose that $y < Q_0^M$, and note that $W(q\,;Q,Q)$ is strictly concave in Q for all $Q \geq q$. Given the current state of the project q, the manager can either commit to a completion state in the interval $[q,\, q+y]$, in which case her discounted payoff is equal to $\max_{q\leq Q\leq q+y} W(q\,,Q,Q)$, or she can delay committing, anticipating that she will be able to commit to some completion state $q' > q+y$ later, which will yield her a discounted payoff $W(q\,,q',q')$. Therefore, the manager will choose to commit to a completion state at q if and only if $\max_{q\leq Q\leq q+y} W(q\,,Q) \geq W(q\,,q',q')$ for all $q' > q+y$, or equivalently if and only if $\arg\max_{Q\geq q} W(q\,,Q,Q) \leq q+y$. By noting that $Q_q^M \in \arg\max_{Q\geq q}\{W(q\,;Q,Q)\}$, it follows that the manager finds it optimal to commit to project size $Q_{x(y)}^M$ at $q = x(y)$, where $x(y)$ is the unique solution to the equation $\max\left\{Q_{x(y)}^M - y,\, 0\right\} = x(y)$, and $Q_{x(y)}^M$ is given by (8).

$\square$

Proof of Proposition 4. To begin, fix $n \geq 1$ and $k \in [1, n]$. If the project size is chosen by the agents, then they will choose $Q^A = \frac{\beta}{r}\frac{k(2n-k)}{2n}$, and by substituting this into the manager's expected discounted profit yields $W\left(0\,; Q^A, Q^A\right) = \frac{(1-\beta)\beta}{r}\frac{k(2n-k)}{2n}\left(\frac{1}{2}\right)^{\frac{2n-k}{n}}$.

Next, consider the case in which the completion state is chosen by the manager, and she has no commitment power (*i.e.*, $y = 0$) so that she eventually completes the project at $Q^M_{NC} = \frac{\beta}{r}\frac{2kn}{2n-k}$. By substituting this the manager's expected discounted profit we have that $W\left(0\,; Q^M_{NC}, Q^M_{NC}\right) = \frac{(1-\beta)\beta}{r}\frac{2kn}{2n-k}\left(\frac{n-k}{2n-k}\right)^{\frac{2n-k}{n}}$.

Now consider the ratio $\frac{W\left(0\,;Q^M_{NC},Q^M_{NC}\right)}{W\left(0\,;Q^A,Q^A\right)} = \left(\frac{2n}{2n-k}\right)^2\left(\frac{2n-2k}{2n-k}\right)^{\frac{2n-k}{n}}$, and for the purpose of this proof, let $h(n,k) = \left(\frac{2n}{2n-k}\right)^2\left(\frac{2n-2k}{2n-k}\right)^{\frac{2n-k}{n}}$. Observe that $h(k,k) = 0$ and $\lim_{n\to\infty} h(n,k) = 1$. Differentiating with respect to n yields $\frac{d}{dn}h(n,k) = \frac{4\left[(2n-k)(n-k)\ln\left(\frac{2n-2k}{2n-k}\right)+nk\right]}{(2n-k)^3(n-k)}\left(\frac{2n-2k}{2n-k}\right)^{\frac{2n-1}{n}} > 0$ if and only if $(2n-k)(n-k)\ln\left(\frac{2n-2k}{2n-k}\right)+nk > 0$ or equivalently if $\ln\left(\frac{2n-2k}{2n-k}\right)+\frac{nk}{(2n-k)(n-k)} > 0$. Now observe that $\lim_{n\to\infty}\left[\ln\left(\frac{2n-2k}{2n-k}\right) + \frac{nk}{(2n-k)(n-k)}\right] = 0$, and $\frac{\partial}{\partial n}\left[\ln\left(\frac{2n-2k}{2n-k}\right) + \frac{nk}{(2n-k)(n-k)}\right] < 0$ for all $n \geq k$. This implies that $\ln\left(\frac{2n-2k}{2n-k}\right) + \frac{nk}{(2n-k)(n-k)} > 0$, and hence $\frac{d}{dn}h(n,k) > 0$. By noting that $h(k,k) = 0$ and $\lim_{n\to\infty} h(n,k) = 1$, it follows that $h(n,k) < 1$ for all $n \geq k$, which implies that $W\left(0\,; Q^A, Q^A\right) > W\left(0\,; Q^M_{NC}, Q^M_{NC}\right)$ for all $n \geq k$.

We have thus far established that $W\left(0\,; Q^A, Q^A\right) > W\left(0\,; Q^M_{NC}, Q^M_{NC}\right)$. Moreover, it is straightforward to verify that $W\left(0\,; Q^M_{FC}, Q^M_{FC}\right) > W\left(0\,; Q^A, Q^A\right)$; *i.e.*, the manager should not delegate the choice of Q to the agents if she has full commitment power. Because $Q^M(y)$ is strictly decreasing in y for all $y < Q^M_{FC}$, $W(0\,; Q, Q)$ is strictly concave in Q, and $Q^M_{FC} < Q^M_{NC}$, it follows that $W\left(0\,; Q^M(y), Q^M(y)\right)$ is strictly increasing in y on $\left[0, Q^M_{FC}\right)$. By noting that $W\left(0\,; Q^A, Q^A\right)$ is independent of y, it follows that there exists some threshold $\theta < Q^M_{FC}$ such that $W\left(0\,; Q^A, Q^A\right) > W\left(0\,; Q^M(y), Q^M(y)\right)$ if and only if $y < \theta$.

$\square$

Proof of Proposition 5. Suppose first that the manager has full commitment power. Then, her optimal project size is equal to $Q^M_{FC} = \frac{2\beta}{r}\frac{k(2n-k)}{n}\left(\frac{2n}{4n-k}\right)^2$, and it follows that

$$W\left(0\,; Q^M_{FC}, Q^M_{FC}, k\right) = \frac{2\beta(1-\beta)}{r}\frac{k(2n-k)}{n}\left(\frac{2n}{4n-k}\right)^2\left(\frac{2n-k}{4n-k}\right)^{\frac{2n-k}{n}}.$$

By differentiating this with respect to k we have that $\frac{\partial}{\partial k}W\left(0\,; Q^M_{FC}, Q^M_{FC}, k\right) > 0$ if and only if $2n(n-k) - k(2n-k)\ln\left(\frac{2n-k}{4n-k}\right) > 0$. This condition holds for all $k \in [1, n]$. Therefore, in this case the manager's optimal coordination level is $k_{FC} = n$.

Next, suppose that the manager has no commitment power, so that she eventually completes the project at $Q_{NC}^{M}=\frac{2\beta}{r}\frac{kn}{2n-k}$. Then it follows that

$$W\left(0\,;Q_{NC}^{M},Q_{NC}^{M},\,k\right)=\frac{2\beta\left(1-\beta\right)}{r}\frac{kn}{\left(2n-k\right)}\left(\frac{n-k}{2n-k}\right)^{\frac{2n-k}{n}},$$

and by differentiating this with respect to k we have that $\frac{\partial}{\partial k}W\left(0\,;Q_{NC}^{M},Q_{NC}^{M},\,k\right)>0$ if and only if $\frac{k(2n-k)}{n}\left(n-k\right)\ln\left(\frac{n-k}{2n-k}\right)-\left[k-\left(2+\sqrt{2}\right)n\right]\left[k-\left(2-\sqrt{2}\right)n\right]<0$. Because $\lim_{k\to n}\left(n-k\right)\ln\left(\frac{n-k}{2n-k}\right)=0$ and $\left(\left[k-\left(2+\sqrt{2}\right)n\right]\left[k-\left(2-\sqrt{2}\right)n\right]\right|_{k=n}<0$, the last inequality is violated as $k\to n$. Therefore, $\lim_{k\to n}\frac{\partial}{\partial k}W_{k}\left(0\,;Q_{NC}^{M},Q_{NC}^{M},\,k\right)<0$, so that $\arg\max_{k}W\left(0\,;Q_{NC}^{M},Q_{NC}^{M},\,k\right)<n$.

Therefore, we have show that with full commitment power, the manager's optimal cooperation level $k_{FC}=n$, while with no commitment power, her optimal cooperation level $k_{NC}<n$.

□

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