

A Theory of Patent Portfolios

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Abstract

This paper develops a theory of patent portfolios in which firms accumulate an enormous amount of related patents in diverse technology fields such that it becomes impractical to develop a new product that with certainty does not inadvertently infringe on other firms' patent portfolios. We analyze how litigation incentives for the holders of patent portfolios impact the incentives to introduce new products and draw welfare implications.

Keywords: patent portfolios, patent litigation, practicing and non-practicing entities, patent troll

JEL: D43, L13, O3

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1 Introduction

Recent years have seen a dramatic increase in the number of patent applications and patents granted as a result of firms amassing vast patent portfolios, leading to "patent portfolio races." This paper develops a theory of patent portfolios in which firms accumulate a large amount of related patents in diverse technology fields to mitigate potential "hold-up" problems and use them as bargaining chips in negotiations with other patent owners. We analyze how the relative position of patent portfolios vis-à-vis competitors influences incentives to litigate and how they in turn impact incentives to develop a new product.

We consider a situation in which the sheer number of patents held by other firms makes it impractical for firms to develop new products that avoid inadvertent infringement on other firms' patent portfolio with certainty. For instance, Cotropia and Lemley (2009) report that only a very small fraction of patent infringement cases involve defendants who have copied the patented technology, implying that most cases entail inadvertent infringement. Bessen and Meurer (2006) also provide empirical evidence suggesting that most defendants in patent litigation are inadvertent infringers rather than firms attempting to copy or invent around patents. This type of situation is particularly pertinent in many high-tech industries where technologies are rapidly advancing and draw upon existing stocks of knowledge. The convergence of digital media and the emergence of the Internet have also blurred the boundaries of the previously separate information and communication technology (ICT) industries. As a result, the development of new products in the ICT industry often requires access to and integration of numerous complementary technologies, as illustrated by smartphones that employ a variety of technologies in the areas of wireless communication, GPS, camera, digital technology, high speed broadband, and so on. The semiconductor industry provides another example of an industry that "requires access to a 'thicket' of intellectual property rights in order to advance the technology or to legally produce or sell" new products (Hall and Ziedonis, 2001).

Since 2000, for instance, Apple has filed 1,298 patents (as of September 2012) in the field of hand-held mobile radio telephone technologies, with the vast majority filed after the launch of the iPhone in 2007 (Thomson Reuters, 2012). According to Drummond (2001),

Senior Vice President and Chief Legal Officer of Google, a smartphone may contain as many as 250,000 patent claims, portraying the rapidly increasing technological complexity of mobile devices.

The importance of building patent portfolios is also demonstrated by recent episodes of patent portfolio acquisitions. The acquisition of Nortel Network's patent portfolio by the Rockstar consortium (whose members include Apple, Microsoft, Research in Motion, Ericsson and Sony) is a case in point. When Nortel went bankrupt and its patent portfolio of approximately 6,000 patents was auctioned off as part of the bankruptcy proceeding, the Rockstar consortium acquired it with a \$4.5 billion bid. Google, which lost its bid for Nortel patents, responded with its own acquisition of Motorola Mobility at the price of \$12.5 billion. The transaction involved Motorola Mobility's entire asset portfolio, including its handset businesses, but Google's primary interest was known to be Motorola's more than 17,000 patents in wireless technologies (Rusli and Miller, 2011).

As firms expand their patent portfolios, perhaps as a response to potential hold-up by other firms' patent portfolios, the amassment of patents inevitably leads to overlapping claims and litigations. In conjunction with the build-up of its patent portfolios, Apple was embroiled in more than 150 IP lawsuits in 2012 as a plaintiff, defendant, and counter-claimant, with the highest profile lawsuit being the global litigation with Samsung, which resulted in the jury awarding Apple with \$1.05 billion in damage in the US. The recent explosion of patent-related litigation and strategic patent portfolio acquisitions demand a new paradigm of patent analysis that shifts away from isolated patents and towards patent portfolios.

We develop a model to analyze how the accumulation of patent portfolios affects litigation incentives and how this feeds into incentives to develop new products. In particular, we analyze the effect of relative positions on litigation incentives and settlement terms, and compare litigation incentives of practicing entities (hereafter, PE) vis-à-vis non-practicing entities (NPE). The conventional wisdom is that NPEs have higher incentives to litigate because they do not have any product that would be subject to counter litigation. We show that this is true in most circumstances, but PEs may have higher incentives when the product market competition is intense. The intuition is that litigation provides a mechanism to change each firm's market position from a duopolist to a stochastic monopolist. The benefit of this change becomes more important as the profit in a duopolistic market

decreases with the intensity of competition.

Based on the analysis of litigation incentives, we further investigate the effects of patent portfolios on the incentives to develop a new product in the shadow of ex post patent litigation. We show that as one firm accumulates, it is necessary that at least one firm's investment in new product development decreases. A typical scenario would be the accumulating firm increases its investment while the rival firm decreases. However, it is possible that the accumulating firm decreases its development efforts and opts to operate as an NPE if the rival firm already has a strong patent portfolio position and is more likely to develop a new product. Another possibility is that both firms reduce investment in new products, but both firms investing more is not possible as one firm accumulates more patents.

In light of recent high profile patent portfolio sales, we also explore a patent portfolio acquisition game. We consider two scenarios. When the competition is between PEs, we show that the firm with the larger portfolio acquires the additional portfolio in equilibrium while consumers would be better off if the portfolio were acquired by the firm with the weaker portfolio. When the competition is between a PE and an NPE, the only incentive for the PE to acquire the patent portfolio is for defensive purposes while the incentive for an NPE is to extract licensing fees from the PE. In this case, the willingness to pay for the patent portfolio is the same for both firms. The equilibrium price will be at the point where both firms are indifferent between acquiring and not acquiring. Either way, the PE pays a price.

In our model, an NPE arises as a firm fails to develop a new product. We also explore NPE as a business model in which firms acquire patent portfolios without any intention to produce any products: their business model is to litigate (or threat to litigate) and extract licensing revenues from PEs.

Despite the importance of patent portfolios in the innovation market and much discussion in popular press, academic papers on this topic are sparse. Hall and Ziedonis (2001) conduct an empirical analysis of patenting behavior in the U.S. semiconductor industry between 1979 and 1995 to rationalize the so-called "patent paradox," a recent phenomenon of an unprecedented surge in patenting unaccounted for by increases in R&D spending alone even as the expected value of each patent decreases (Kortum and Lerner, 1998). They explore the link between the pro-patent policy shift via the creation of the Court of Appeals for the Federal Circuit (CAFC) in 1982 and intensified patenting behavior by analyzing the

patent data in the semiconductor industry complemented by interviews with industry representatives. They find that large-scale manufacturers have invested far more aggressively in patents with the pro-patent policy shift, engaging in "patent portfolio races" aimed at reducing concerns about being held up by external patent owners and at negotiating access to external technologies on more favorable terms. Ziedonis (2004) expands on Hall and Ziedonis (2001) and finds that firms patent more aggressively than otherwise expected when markets for technology are highly fragmented and ownership rights are widely dispersed. Thus, an aggressive patent portfolio acquisition strategy is an organizational response to mitigate hazards in markets for technology when *ex ante* solutions are infeasible due to fragmentation and heightened transactions costs.

Morton and Shapiro (2013) provide a related and complementary analysis to our paper. More specifically, they conduct an analysis of the tactics used by NPEs to monetize the patents they acquire. They analyze the effects of enhanced patent monetization on innovation and on consumers and how they change depending on the type of seller, the type of buyer and the patent portfolio involved. Our model deals with a broader set of issues including litigation incentives of both PEs and NPEs and an explicit analysis of patent acquisition games.

Bessen and Meurer (2006) develop a model of patent litigation similar to ours. They consider a game in which a patent owner invests in a level of patent protection that influences the probability of successfully suing a potential entrant and the strength of this probability is known once two firms invest in product developments. Their main purpose is to derive testable empirical predictions based on reduced form profit functions. Our framework provides a microfoundation by explicitly considering a litigation game to analyze the incentives to litigate and the terms of settlement. Our model also allows for an analysis of a patent acquisition game, welfare effects of strategic patent portfolios, and other related issues without resorting to any *ad hoc* assumptions.

Chiou (2013) touches upon similar issues addressed in this paper, but in a very different framework. He builds a model with a continuum of firms, all of whom can acquire a patent. In terms of manufacturing capability, there are two types of firms. One type of firm has no manufacturing capacity and only serves as a non-practicing entity. The other type of firm can invest in manufacturing facilities. As in our model, a patent can be used as a defensive mechanism to be used as a credible countersuit to threats or as a purely offensive

one. Depending on their patenting and investment costs, firms self-select into NPE, pure manufacturing firm (without a patent), or a vertically integrated firm (that has a patent and manufactures). He analyzes how the industry configuration depends on what he calls the "defensive premium." In such a framework, he shows that an (exogenous) increase in the defensive premium induces more investment by PEs but can have the side effect of increasing incentives for offensive patenting by NPEs. His model, however, is devoid of strategic interactions due to the continuum assumption and thus is incapable of analyzing the effects of industry competitiveness on strategic incentives to litigate and on investment incentives.¹

Law scholars have also waded in the debate. In an attempt to provide a resolution to the so-called "patent paradox," Parchomovsky & Wagner (2004) develop a patent portfolio theory that "the true value of patents lies not in their individual worth, but in their aggregation into a collection of related patents." They posit that the amassment of patent portfolios generates "scale" and "diversity" that would confer advantages over individual patents. Scale allows the freedom to innovate, avoiding costly litigation, improving bargaining position, and facilitating capital investments, whereas diversity allows firms to hedge against the uncertainties regarding a product, future market conditions, future competitors, and possible changes in patent law. In short, well-crafted patent portfolios act as a "super-patent" and as a result, "the whole is greater than the sum of its parts" as a patent acquisition strategy. However, they do not formalize how the mechanism by which such advantages arise. In addition, their analysis is focused on explaining the incentives to build patent portfolios while our analysis concerns how patent portfolios affect litigation incentives and new product developments. Chien explores implications of "patent-assertion entities," sometimes derisively called "patent trolls," in the patent ecosystem. The sole purpose of patent-assertion entities is to use patents primarily to obtain license fees rather than to support the development of technology, which creates a secondary market for patents that would otherwise sit on the shelf. She proposes a framework that includes both the "arms race," in which the goal is to provide entities with the freedom to operate, and the marketplace, through which entities leverage their freedom to litigate. She argues that the

¹See also Siebert and von Graevenitz (2010) and Denicolo and Zanchettin (2012) for models of patent portfolio acquisition. Once again, our model is very different and asks a different set of questions with a focus on litigation incentives.

value of a patent can be based on the "exclusion value" rather than the "intrinsic value" when it is held by patent-assertion entities. Our paper formalizes how the exclusion value is created by the credible threat to litigate and explores the implications on incentives to develop new products.

The remainder of the paper is organized in the following way. In Section 2, we set up a very simple model of patent portfolios and investigate litigation incentives. Section 3 analyzes how the relative strength of patent portfolios affects the incentives to introduce a new product. In Section 4, we analyze welfare implications for consumers of strategic patent portfolios. Section 5 considers a patent portfolio acquisition game in which a third party's patent portfolio is up for sale. Section 6 considers NPE as a business model. Section 7 extends the analysis and check the robustness of the main results. Section 8 closes the paper with concluding remarks. The proofs for lemmas and propositions are relegated to the Appendix.

2 Model

We consider two firms competing to introduce a new product into a market. Each firm i has a patent portfolio of size S_i , where $i = 1, 2$. When firm i develops a new product, there is a chance that its new product may infringe on some of the patents in the other firm's patent portfolio, which is an increasing function of the other firm's patent portfolio size S_j , $j \neq i$. Let us denote these infringing probabilities by α_j , which can be interpreted as the strength of firm j 's patent portfolio.² The new product contains many new features and functionalities, such as smartphones do. By this formulation, we envision a situation in which "the high cost of evaluating which patents in the rival firm's portfolio of thousands might apply" to each functionality makes it impractical to avoid infringement on other firm's patents with certainty.³ We assume that the α_j 's are common knowledge for both firms.

We assume that when a firm invests in I , the probability of successful introduction of a new product is given by $p(I)$, where $p'(I) > 0$, $p''(I) < 0$, and $0 < p(I) < 1$, for any positive I . More generally, we could assume that the probability of success depends on

²More generally, the probability of infringing firm j 's patent portfolio, α_j , will depend not only on firm j 's patent portfolio size, but also the patent quality.

³Chien (2010), p. 308.

the size and quality of each firm’s patent portfolio. By assuming that the probability of success does not depend on the existing patent portfolio, we essentially consider only patent portfolios of non-core technologies whose value derives from their exclusion value rather than intrinsic value and their impact on successful product design is of second order importance. Alternatively, we can interpret investment I as marketing efforts. In the introduction of feature-laden high-tech products, success is difficult to assess because how the key features of the new product will appeal to consumers is hard to predict in advance.

Depending on the outcomes of each firm’s product introduction, there are several sub-games to consider. If both firms fail to introduce a new product, the game ends and there is nothing further to analyze. There are two meaningful cases: one in which only one firm is successful and the other in which both firms are successful.

2.1 Litigation and Settlement with PE and NPE

Suppose that firm i is the only successful one in introducing a new product. Thus, firm i is the only practicing entity (PE) and the other firm $j (\neq i)$ is a non-practicing entity (NPE). The monopoly profit associated with the new product is denoted by π^m . In this case, firm j has an option to litigate, claiming that successful firm i ’s new product infringes on its patent portfolio. With probability α_j the litigating firm will prevail in court. In such a case the court grants an injunction and firms engage in Nash bargaining. With equal bargaining power, the innovating firm has to pay a licensing fee of $\pi^m/2$ to the NPE. Let L be the litigation costs for both firms. Thus, firm j will litigate if the following condition holds:

$$\alpha_j \frac{\pi^m}{2} \geq L \tag{1}$$

This implies that firm j as a non-practicing entity (NPE) will have a credible threat to litigate the innovating firm if $\alpha_j \geq \alpha^* = 2L/\pi^m$. However, in order to save on litigation costs, the two firms always find it profitable to settle out of court. In ex ante settlement negotiations with equal bargaining powers, the PE agrees to pay $\alpha_j \pi^m/2$ to the NPE, anticipating court outcomes and subsequent bargaining on ex post licensing fees. Let Π_i^{XY} denote firm i ’s expected payoffs when firm i is in state X and the rival firm $j (\neq i)$ is in state Y where states 1 and 0, respectively, represent a successful introduction of a new product and a failure. Each firm’s expected payoffs when only one firm is successful can be written

as:

$$\begin{aligned}\Pi_i^{10}(\alpha_j) &= \begin{cases} \pi^m - \alpha_j\pi^m/2 = (1 - \alpha_j)\pi^m + \alpha_j\pi^m/2 & \text{for } \alpha_j \geq \alpha^*, \\ \pi^m & \text{for } \alpha_j < \alpha^*, \end{cases} \\ \Pi_i^{01}(\alpha_i) &= \begin{cases} \alpha_i\pi^m/2 & \text{for } \alpha_i \geq \alpha^*, \\ 0 & \text{for } \alpha_i < \alpha^*. \end{cases}\end{aligned}$$

In other words, for a patent portfolio to have an impact, it needs to achieve a certain level of critical mass to make its litigation threat credible. Note that at the threshold value at which the litigation threat becomes credible (i.e., at $\alpha_j = \alpha^*$), both profit functions are discontinuous. The profits of the PE decrease by $L(= \alpha^*\pi^m/2)$ whereas the profits of the NPE increase by the same amount.

2.2 Litigation and Settlement with two PEs

Now consider a scenario in which both firms successfully launch new products. Let the duopoly profit be denoted by π^d , where $2\pi^d \leq \pi^m$.⁴ We consider firms' incentives to litigate or to settle. When firm i files a claim against firm j , we assume that firm j 's optimal strategy is to counter-litigate as is typically the case in the real world. This implies that firm i risks its own product being subject to injunction as a practicing entity (PE) when it initiates litigation. There are several potential outcomes in the presence of litigation. One possibility is that neither firm is found to infringe on the other's patent portfolio. This leads to a duopoly outcome and takes place with probability $(1 - \alpha_1)(1 - \alpha_2)$. Another outcome that leads to a status quo is when both firms are found to infringe on the other's patent portfolio. In such a case, we assume that they cross-license each other and maintain a duopoly outcome. The remaining possibility is that one firm, say firm i , is found not to infringe on firm j 's while firm j is found to infringe on firm i 's patent portfolio. With the assumption of $2\pi^d \leq \pi^m$, there is no possibility of settlement and firm i will be a monopolist in the market. Thus, firm 1 will litigate if it holds that

$$\alpha_1(1 - \alpha_2)\pi^m + (1 - \alpha_1)(1 - \alpha_2)\pi^d + \alpha_1\alpha_2\pi^d - L \geq \pi^d$$

⁴This is the efficiency effect (Gilbert and Newbery, 1982). In section 7.1, we consider the case where $2\pi^d > \pi^m$. This would be the case where the two firms are operating in different industries or competition has the effect of expanding the market.

or

$$\alpha_1(1 - \alpha_2)(\pi^m - \pi^d) - \alpha_2(1 - \alpha_1)\pi^d \geq L. \quad (2)$$

Litigation provides firm 1 with the opportunity to monopolize the market in the case where firm 2 is found infringing while firm 1 is not. However, this benefit has to be weighed against the cost of litigation and the potential loss of duopoly profits in the case of the reverse litigation outcome. Similarly, firm 2 has a unilateral incentive to litigate if

$$\alpha_2(1 - \alpha_1)(\pi^m - \pi^d) - \alpha_1(1 - \alpha_2)\pi^d \geq L. \quad (3)$$

Solving (2) and (3) for the respective firm's own portfolio strength yields that firm i has an incentive to litigate if $\alpha_i > \alpha_i^{**}(\alpha_j)$, where

$$\alpha_i^{**}(\alpha_j) = \frac{\alpha_j \pi^d + L}{(1 - \alpha_j)(\pi^m - \pi^d) + \alpha_j \pi^d}.$$

Given the other's patent portfolio strength, a firm needs to acquire a sufficient level of its own patent portfolio strength to make its litigation threat credible. In addition, it can be easily verified that $\alpha_i^{**}(\alpha_j)$ is an increasing function of α_j . This means that as the other firm's patent portfolio increases, the incentive to litigate decreases. This captures the idea that building a patent portfolio can be used as a defensive mechanism against potential litigation. Notice that this defensive mechanism works only against PEs, but not NPEs, because the incentive to litigate for NPEs depends only on its own patent portfolio strength, not the defendant's.

To further analyze the litigation incentives of PEs, let us define the litigation set for each firm as

$$\mathcal{L}_i(\alpha_j) = \{(\alpha_1, \alpha_2) | \alpha_i > \alpha_i^{**}(\alpha_j)\}$$

Then, a litigation threat by at least one firm is credible if $(\alpha_1, \alpha_2) \in \mathcal{L} = \mathcal{L}_1(\alpha_2) \cup \mathcal{L}_2(\alpha_1)$. Otherwise, there will be no litigation. However, litigation does not always take place when $(\alpha_1, \alpha_2) \in \mathcal{L}$. Firms can negotiate an out-of-court settlement to avoid the cost of litigation before bringing an infringement suit. A settlement occurs and litigation is avoided if the firms' joint profits from a duopoly outcome are higher than the joint expected profits from

litigation, that is, if the following condition holds:

$$[\alpha_1(1 - \alpha_2) + \alpha_2(1 - \alpha_1)]\pi^m + [1 - (\alpha_1(1 - \alpha_2) + \alpha_2(1 - \alpha_1))]2\pi^d - 2L < 2\pi^d$$

or

$$(\alpha_1 + \alpha_2 - 2\alpha_1\alpha_2)(\pi^m - 2\pi^d) < 2L.$$

Let S be the set of (α_1, α_2) for which the condition above holds. Litigation takes place if and only if $(\alpha_1, \alpha_2) \in \tilde{\mathcal{L}} = \mathcal{L} \setminus S$.

By comparing the condition that defines each set, it can be easily verified that when both firms have unilateral incentives to litigate, a settlement is not possible. To see this, note that litigation occurs if the sum of LHS of conditions (2) and (3) is greater or equal than the sum of the RHS of (2) and (3), that is, if

$$(\alpha_1 + \alpha_2 - 2\alpha_1\alpha_2)(\pi^m - 2\pi^d) \geq 2L. \quad (4)$$

This condition always hold when both (2) and (3) are satisfied and both firms have an incentive to litigate. However, when only one firm, say only firm i , has an incentive to litigate, i.e., $(\alpha_1, \alpha_2) \in (\mathcal{L} - \mathcal{L}_j(\alpha_i))$, a settlement is possible if $(\alpha_1, \alpha_2) \in S$. This leads us to the following lemma.

Lemma 1 $(\mathcal{L}_1(\alpha_2) \cap \mathcal{L}_2(\alpha_1)) \cap S = \phi$ and $(\mathcal{L} - \mathcal{L}_j(\alpha_i)) \cap S \neq \phi$, where $i = 1, 2$, and $j \neq i$.

The lemma says that we can always find a set of parameters (α_1, α_2) where settlement takes place when only one firm has the incentive to litigate. However, litigation occurs despite the possibility of out-of-court settlements if the expected gains in industry profit in the case of asymmetric litigation outcomes outweigh the cost of litigation. This holds if either both firms prefer unilateral litigation or if one firm's expected gains from litigation outweighs the rival's expected losses. Reflecting the possibility of out-of-court settlements, we can write each firm's expected profit when both firms are PEs as follows:

$$\Pi_i^{11}(\alpha_i, \alpha_j) = \begin{cases} \alpha_i(1 - \alpha_j)\pi^m + (1 - \alpha_1)(1 - \alpha_2)\pi^d + \alpha_1\alpha_2\pi^d - L & \text{for } (\alpha_1, \alpha_2) \in \tilde{\mathcal{L}} \\ \pi^d + (\alpha_i - \alpha_j)\pi^m/2 & \text{for } (\alpha_1, \alpha_2) \in \mathcal{L} \cap S \\ \pi^d & \text{for } (\alpha_1, \alpha_2) \notin \mathcal{L} \end{cases}$$

The following proposition further characterizes the equilibrium outcome of the litigation game with two PEs.

Proposition 1 (i) Suppose litigation costs are small in that $L \leq (\pi^m/2 - \pi^D)/2$. In this case, firms have a strong incentive to litigate and settle only if either both firms portfolios are sufficiently weak or both firms' portfolio are sufficiently strong. (ii) Suppose litigation costs are moderate in that $(\pi^m/2 - \pi^D)/2 < L \leq \pi^m/2 - \pi^D$. If both firms' patent portfolios are of similar size and either sufficiently small or sufficiently large, firms settle out of court. If one firm's portfolio is sufficiently large while the rival's portfolio is sufficiently small, litigation occurs. (iii) Otherwise, firms never litigate. Overall, the more intense the product market competition, the more litigation in the industry.

The litigation and settlement equilibrium is summarized in FIGURE 1 and 2 below.

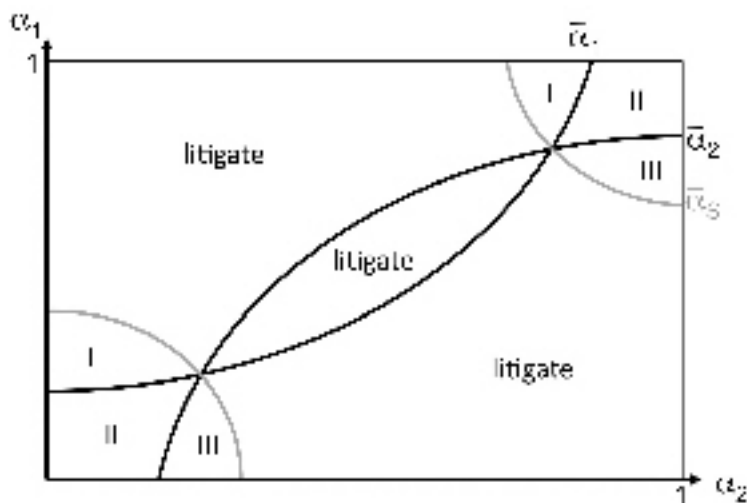


Figure 1: *Litigation and Settlement with two PEs for low Litigation Costs.*

If the litigation costs are small relative to the difference between monopoly and duopoly profits, firms have a strong incentive to litigate and settle only if either both firms portfolios are sufficiently weak or both firms' portfolio are sufficiently strong. This is illustrated in FIGURE 1. When firms settle, three possibilities arise. In region II, which corresponds to values $(\alpha_1, \alpha_2) \notin \mathcal{L}$, no firm has a credible threat of litigation and firms maintain their duopoly position. In regions I and III, where $(\alpha_1, \alpha_2) \in \mathcal{L} \cap S$, firms settle out-of-court. In region I, only firm 1 has a credible threat of litigation and leverages this threat to secure

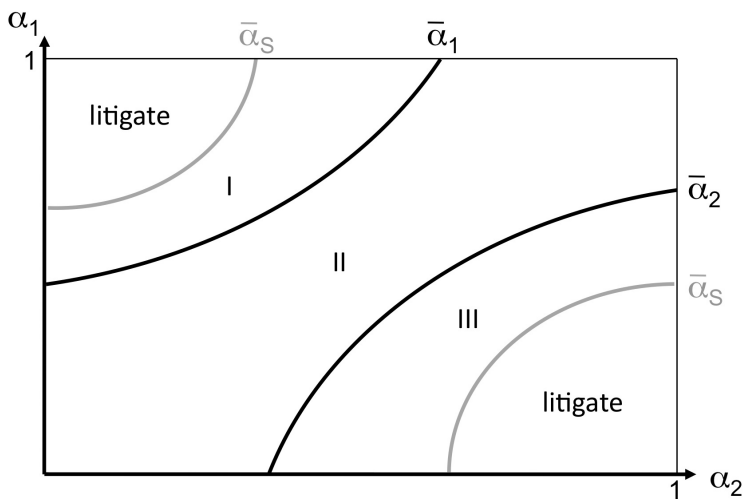


Figure 2: *Litigation and Settlement for intermediate Litigation Costs.*

itself a higher profit in the settlement negotiations. Vice versa, in region III, only firm 2 has a credible threat of litigation and receives a higher payoff after settlement.

When litigation costs are intermediate, settlement also occurs when patent portfolios are of similar intermediate size. In other words, litigation only occurs if one firm has a sufficiently large patent portfolio while the other one has a sufficiently small portfolio. This is illustrated in FIGURE 2.

We can also analyze the incentives to litigate as industry competitiveness changes. Fiona M. Scott Morton (2012) poses an empirical puzzle of why “we see global litigation among platform competitors, rather than ‘patent peace’ some observers thought would occur with heavily armed competitors” with sizeable patent portfolios. Our model identifies conditions under which such litigation takes place. In particular, as the competitiveness of the industry intensifies, the relative gains from excluding a rival through litigation are higher (see condition (4)). Hence, we would expect to see more litigation among PEs in industries where competition is more intense.

2.3 Comparison of Litigation Incentives between NPE and PE

In recent years, serious concerns have been expressed regarding the role of NPEs in patent litigation. Our analysis sheds some light on this. The above model partially confirms the conventional wisdom that NPEs have more incentives to litigate because they have

nothing to lose beyond the litigation costs whereas PEs risk their own products being subjected to injunction when they initiate litigation. However, we can also show that there is a countervailing mechanism that may induce PEs to have higher incentives to litigate compared to NPEs. When NPEs litigate, the reward for a successful litigation is to share the monopoly profit with the infringer. With Nash bargaining between the NPE and the infringer, the payoff from successful litigation is $\pi^m/2$. For PEs, the payoff from a successful litigation outcome is the value of excluding its rival from the marketplace, $(\pi^m - \pi^d)$. In the event of a negative litigation outcome, a PE is excluded from the marketplace and incurs a loss of π^d . Hence, the value of litigation is increasing in the intensity of product market competition. Thus, when π^d is small and competition is intense, we cannot rule out the case where PEs have higher incentives to litigate compared to NPEs.

To be more precise, consider firms with patent portfolios of equal size $\alpha_1=\alpha_2=\alpha$ and compare the litigation incentive constraints (1) and (4). FIGURE 3 depicts the two constraints in an $\alpha - \pi^d$ diagram. NPEs have an incentive to litigate when their portfolio is sufficiently strong and $\alpha\pi^m/2 \geq L$. The incentive to litigate for PEs is maximized when the probability that exactly one firm is found infringing, $2\alpha(1-a)$ is highest. Furthermore, as derived above, litigation among PEs does not occur when $\pi^d \geq \pi^m/2 - 2L$. At the other extreme, $\pi^d = 0$, NPEs have an incentive to litigate if $\alpha(1-a)\pi^m \geq L$. Thus, for $\alpha < 1/2$, there must exist parameter scenarios such that PEs would litigate whereas a NPE would not. Furthermore, since the expected payoff from litigation decreases in π^d , there is a unique threshold value $\bar{\pi}^d$ with

$$\bar{\pi}^d = \frac{\pi^m}{4} \frac{\pi^m - 4L}{\pi^m - 2L},$$

such that $\pi^d \leq \bar{\pi}^d$ is a necessary condition for PEs to have stronger litigation incentives. We thus obtain the following result.

Lemma 2 *If product market competition is intense and patent portfolios are neither too small nor too big, then PEs have stronger incentives to litigate compared to NPEs. Otherwise, NPEs have (weakly) more incentives to litigate.*

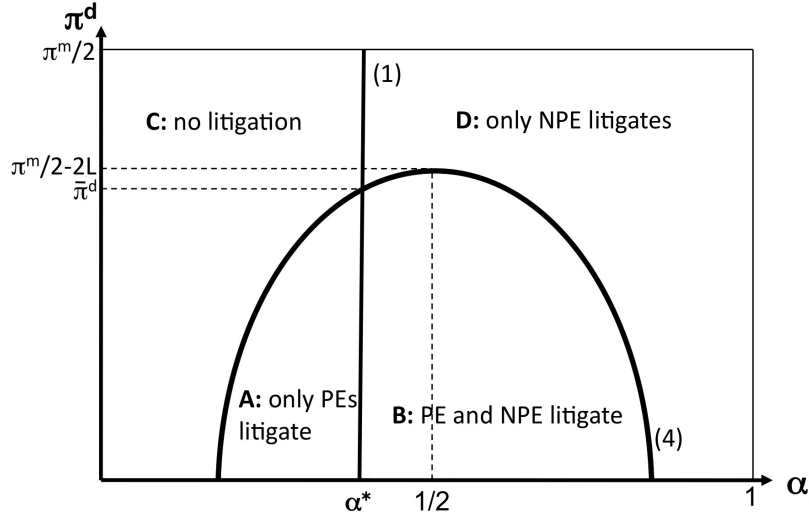


Figure 3: *Litigation Incentives PEs versus NPE*

3 Investment in New Product Development

One of the major concerns about the patent thicket and the accumulation of strategic patent portfolios is their impact on innovative activities. In this section we analyze the effects of patent portfolios on the incentives to invest in R&D. In the next section we conduct a welfare analysis of strategic patent portfolios.

For given patent portfolio sizes (α_1, α_2) , firm i 's expected payoff when it invests I_i and the rival firm invests I_j can be written as

$$\begin{aligned} \Psi_i(I_1, I_2; \alpha_1, \alpha_2) = & p(I_i)p(I_j)\Pi_i^{11}(\alpha_1, \alpha_2) + p(I_i)[1 - p(I_j)]\Pi_i^{10}(\alpha_j) \\ & + [1 - p(I_i)]p(I_j)\Pi_i^{01}(\alpha_i) - I_i \end{aligned}$$

Firm i 's optimal investment level on new product development, given I_j , can be derived by solving the following problem:

$$\text{Max } \Psi_i(I_1, I_2; \alpha_1, \alpha_2)$$

The first order condition for firm i 's optimal investment I_i , $\partial\Psi_i/\partial I_i = 0$, can be rewritten as

$$p(I_j)[\Pi_i^{11}(\alpha_i, \alpha_j) - \Pi_i^{01}(\alpha_i)] + [1 - p(I_j)]\Pi_i^{10}(\alpha_j) = \frac{1}{p'(I_i)}. \quad (5)$$

This equation implicitly defines firm i 's reaction function $I_i = R_i(I_j; \alpha_1, \alpha_2)$. The LHS is the expected benefit of investing in a higher R&D success rate. The rival is successful with probability $p(I_j)$. In this case, a higher success rate for firm i makes it more likely that both firms introduce new products and less likely that firm i is an NPE facing a successful rival. By contrast, when the rival is not successful, more investment leads to a higher probability that firm i is the only PE in the industry. Hence, higher profits as a PE increase the incentive to invest whereas higher profits as a NPE, $\Pi_i^{01}(\alpha_i)$, lower R&D incentives.

The Nash equilibrium investment levels $I_1^*(\alpha_1, \alpha_2)$ and $I_2^*(\alpha_1, \alpha_2)$ are at the intersection of the firms' reaction functions. We now conduct a comparative static analysis of how changes in (α_1, α_2) affect the equilibrium investment in product development (I_1^*, I_2^*) . Throughout this analysis we assume that the stability condition (see the appendix to the next proposition) is satisfied and we focus on situations where the unique Nash equilibrium is an interior solution. As a first step, compare the profit functions of PEs and NPEs.

Lemma 3 $\Pi_i^{10}(\alpha_j) \geq \max[\Pi_i^{11}(\alpha_1, \alpha_2), \Pi_i^{01}(\alpha_i)]$. *The relative magnitudes of $\Pi_i^{11}(\alpha_1, \alpha_2)$ and $\Pi_i^{01}(\alpha_i)$, however, are ambiguous and depend on the competitiveness of the duopoly outcome.*

The lemma states that for any configuration of patent portfolio positions, a firm strictly prefers to be the sole firm that succeeds in product development. However, when the other firm is successful in the development of a new product, it is not necessarily better to develop its own product and compete in the product market. It may be better to be a NPE, especially when competition is intense and the other firm has built a strong position in its patent portfolio that can be used against the firm in consideration. Moreover, Lemma 3 directly implies the following property.

Lemma 4 *Investments in new product development are strategic substitutes.*

We are now in a position to analyze the effect of a unilateral increase in one firm's patent portfolio position on investment.

Lemma 5 $\partial R_i / \partial \alpha_j < 0$, but the sign of $\partial R_i / \partial \alpha_i$ is ambiguous. In particular, (i) if firm i has incentives to litigate only when it is a PE, $\partial R_i / \partial \alpha_i > 0$, (ii) if firm i has incentives

to litigate only when it is an NPE, then $\partial R_i/\partial \alpha_i < 0$, and (iii) if firm i has incentives to litigate whenever firm j develops a new product and $\alpha_j < 1/2$, then $\partial R_i/\partial \alpha_i > 0$.

Lemma 5 states that when firm i 's patent portfolio size increases, the rival firm j 's reaction function in investment of new product development shifts inwards. However, the effect on its own product development is ambiguous. When firm i 's litigation threat is credible for firm i whenever firm j develops a new product, an increase in firm i 's patent portfolio induces its own reaction function to shift out only when $\alpha_j < 1/2$, that is, the rival firm's patent portfolio size is not substantial.

Proposition 2 *Let us assume that the Nash equilibrium investment levels $I_1^*(\alpha_1, \alpha_2)$ and $I_2^*(\alpha_1, \alpha_2)$ satisfy the stability condition. When one firm's patent portfolio size increases, it is never the case that both firms invest more in new product development. If $\partial R_i/\partial \alpha_i > 0$, an increase in firm i 's patent portfolio size induces firm i to invest more and firm j to invest less in new product development. If $\partial R_i/\partial \alpha_i < 0$, both firms may invest less with an increase in one firm's patent portfolios.*

When $\partial R_i/\partial \alpha_i > 0$, firm i 's reaction function shifts out as it accumulates more patents in its portfolio while the rival firm's reaction function shift in. As a result, firm i increases its investment in new product development whereas the rival firm responds by investing less. When $\partial R_i/\partial \alpha_i < 0$, both firms' reaction function shifts in. In this case, the most likely outcome is that both firms reduce investments as one firm builds a stronger patent portfolio. However, it is possible that one of them increases its investments if the other firm's reaction curve shifts relatively more. Yet, it is never possible that both firms increase their investment as a result of patent accumulation by one firm.

4 Welfare effects of strategic patent portfolios

In this section we analyze the welfare implications for consumers of strategic patent portfolios by considering two scenarios. The first scenario is the set-up from the previous section. Patent validity and scope are uncertain and firms hold incomplete information about the patent positions of their rivals. Patent portfolios increase the risk of inadvertent infringement and ex post litigation. We dub this the "patent uncertainty" scenario. In the second scenario patents are iron-clad and firms have ex ante complete information. That is, firms are

aware of all possible infringements and are able to invent around their rival's patents. This is the "patent certainty" scenario. In the following we compare ex ante consumer surplus in these two scenarios. First, we derive investment levels in the patent certainty scenario and compare with the previous section. Then, we investigate overall ex ante expected consumer surplus.

Consider investment incentives in the patent certainty scenario. Firms know the patent claims held by their rivals and are able to invent around. There is no inadvertent infringement and no litigation ex post. In this case, firm i 's optimal investment, for a given rival investment I_j , is given by

$$p(I_j)\pi^d + [1 - p(I_j)]\pi^m = \frac{1}{p'(I_i)}. \quad (6)$$

Compare this condition with the first-order condition (5) in the previous section. A sufficient condition for both firms to invest more with patent uncertainty is that each firm's respective LHS in (5) is smaller than the LHS of (6).⁵ The first term in each of these conditions is the marginal value of investing given the rival innovates. The value in (5), $\Pi_i^{11}(\alpha_i, \alpha_j) - \Pi_i^{01}(\alpha_i)$ can be smaller or larger than π^d as a function of whether firms litigate or not. However, the marginal value with patent uncertainty can be strictly larger for each firm only if both firms innovate and litigate, that is for $(\alpha_1, \alpha_2) \in \tilde{\mathcal{L}}$. The second term is the marginal investment value given the rival is not active. In this case the marginal value from investing is (at least weakly) larger in the complete information scenario.

From this follows that a necessary condition for firms investing more with patent uncertainty is that firms litigate in the event that both introduce new products. Two cases are possible. First, NPEs do not have an incentive to litigate while PEs litigate. From Lemma 2 follows that such situations arise when product market competition is intense and patent portfolios are neither too small nor too large. In those cases, we have $\Pi_i^{10}(\alpha_j) = \pi^m$, $\Pi_i^{01}(\alpha_i) = 0$ and, by (2), it holds that $\Pi_i^{11}(\alpha_i, \alpha_j) \geq \pi^d$. Hence, both firms invest strictly more with the possibility of inadvertent infringement compared to the scenario with patent certainty. The second case arises when both NPEs and PEs litigate after at least one innovation is introduced. Firms have a stronger incentive to invest with patent uncertainty

⁵This is a sufficient condition when the firms' patent portfolio positions are sufficiently similar in size.

if

$$p(I_j)[\Pi_i^{11}(\alpha_i, \alpha_j) - \pi^d + \pi^m - \Pi_i^{01}(\alpha_i) - \Pi_i^{10}(\alpha_j)] > \pi^m - \Pi_i^{10}(\alpha_j).$$

For simplicity, consider the symmetric case when both firms hold patent portfolios of the same size α . With litigation incentives in place, it holds that $\pi^m = \Pi_i^{01}(\alpha) + \Pi_i^{10}(\alpha)$ and we can rewrite the condition as

$$p(I_j) > \frac{\alpha\pi^m/2}{\alpha(1-\alpha)(\pi^m - 2\pi^d) - L}.$$

If the rival's investment is sufficiently high, the firm's reaction function is shifted outwards. Hence, if the equilibrium investment with complete information is less than this threshold value, equilibrium investment with patent uncertainty is higher.

Lemma 6 *Suppose $(\alpha_1, \alpha_2) \in \tilde{\mathcal{L}}$ and ex post litigation arises when both firms innovate. There always exist parameter values such that firms invest more under patent uncertainty.*

Litigation can increase industry profits as it raises the probability of monopolistic market outcomes. This implies that firms may invest more in R&D when they hold patent portfolios and there is the possibility of inadvertent infringement. This argument naturally raises the question of whether patent uncertainty might also be salutatory from the point of view of consumers. To analyze this issue, let S^d and S^m denote consumer surplus in a duopoly and monopoly outcome, respectively. Assume $0 \leq S^m \leq S^d$. For simplicity, let us focus on the case where patent portfolios unambiguously increase R&D investment. That is the case for patent portfolio positions such that two innovating PEs litigate whereas NPEs have no incentive to litigate. Furthermore, suppose that the firms have patent portfolios of the same size α and invest the same amount in equilibrium. The ex ante expected consumer surplus without patent portfolios and a common success rate of p is given by

$$CS^0(p) = p^2 S^d + 2p(1-p)S^m.$$

Note that ex ante consumer surplus increases in the success rate p . Similarly, the ex ante expected consumer surplus with patent portfolios, such that PEs litigate while NPE do not litigate, is

$$CS^p(p) = CS^0(p) - p^2 2\alpha(1-\alpha)(S^d - S^m).$$

At equal success rates, the consumer surplus is lower in the presence of patent portfolios due to the fact that when both firms innovate and litigation ensues, there is a probability that one firm is able to exclude its rival from the marketplace. Overall ex ante consumer surplus with patent portfolios is higher if

$$CS^0(p(I^*)) - CS^0(p(I^0)) \geq p(I^*)^2 2\alpha(1-\alpha)(S^d - S^m) \quad (7)$$

or the increase in consumer surplus due to more innovations outweighs the price effect of exclusionary litigation outcomes. This condition is satisfied, for instance, in the following example.

Example 1 Consider a market for a homogenous good with a linear demand function $D(p)=1-p$. When both firms develop a new product, they compete in quantities. Suppose firms hold patent portfolios of equal size α . Further, firms invest in R&D using $p(I) = \sqrt{I}$. In this set-up, the equilibrium hazard rates are

$$p(I^0) = \frac{\pi^m}{2 + \pi^m - \pi^d}, p(I^*) = \frac{\pi^m}{2 + \pi^m - \pi^d}.$$

Substituting these values and $\pi^m = 1/4, \pi^d = 1/9, S^d = 2/9, S^m = 1/4$ into (7) yields

$$\frac{2 + L}{[77 - \alpha(1 - a) + 36L]^2} \geq \frac{2}{77^2}$$

which holds since (4) requires $L \leq \alpha(1 - a)/36$.

We thus derive the following result.

Proposition 3 In the presence of patent uncertainty, patent portfolios have two effects on consumer surplus. There is a negative, static effect as litigation can reduce competition in the marketplace. There is also a dynamic effect as the prospect of litigation might increase or decrease investment incentives. The latter effect can dominate - and consumer surplus can be higher - for patent portfolio positions such that litigation arises when both firms introduce new products.

5 Patent Portfolio Acquisition

Suppose that a patent portfolio of strength $\Delta > 0$ has been put up for sale. In other words, the probability that any new product infringes on some patents in the portfolio for sale is given by Δ . Let us assume that the sale is via an ascending price auction. When this portfolio is acquired, the acquiring firm's patent portfolio size and its strength increases. Let α_i^+ be the ex post patent portfolio strength when firm i acquires the patent portfolio for sale. Let $\delta_i (= \alpha_i^+ - \alpha_i)$ denote the incremental patent portfolio strength due to the acquisition where α_i is the strength of firm i 's existing patent portfolio. Further, suppose that the infringing probabilities on each patent portfolio are independent. Then, we have $\delta_i = (1 - \alpha_i)\Delta$, that is,

$$\alpha_i^+ = \alpha_i + (1 - \alpha_i)\Delta$$

Note that δ_i is decreasing in α_i even though α_i^+ is increasing in α_i . In other words, the effect of acquiring the additional patent portfolio on the strength of the existing patent portfolio is decreasing in the original strength. For instance, if $\alpha_i = 1$, there would be no impact on the strength of the patent portfolio, that is, $\delta_i = 0$.

5.1 Patent Portfolio Acquisition between Two PEs

Consider two PEs with existing patent portfolios of size α_1 and $\alpha_2 (\geq \alpha_1)$, respectively, bidding for the available patent packet. Firm i 's willingness-to-pay is the difference in profits from securing the patent portfolio itself and having its rival acquire it, that is,

$$\Pi_i^{11}(\alpha_i + \delta_i, \alpha_j) - \Pi_i^{11}(\alpha_i, \alpha_j + \delta_j).$$

It is easy to verify that firm 2 has a higher willingness-to-pay for the patents if industry profits are higher when firm 2 buys relative to the case when firm 1 buys, or

$$\Pi^{11}(\alpha_1, \alpha_2 + \delta_2) \geq \Pi^{11}(\alpha_1 + \delta_1, \alpha_2), \quad (8)$$

where

$$\Pi^{11}(\alpha_1, \alpha_2) = \sum_{i=1}^2 \Pi_i^{11}(\alpha_1, \alpha_2).$$

The following lemma states that this condition always holds.

Lemma 7 *The firm with the larger patent portfolio has a (weakly) higher willingness-to-pay for additional patents.*

The firm with the larger patent portfolio has a stronger incentive to accumulate more patents. This is due to the fact that industry profits are higher the more asymmetric the patent portfolio distribution. Asymmetric patent portfolios increase the probability that exactly one firm can affirm its patents in litigation and exclude its rival from the marketplace. Lemma 7 thus implies that, in equilibrium, firm 2 bids slightly more than the willingness-to-pay of firm 1 and secures the patent packet. It follows that the difference in patent portfolio strength between the two firms increases.

An interesting question is how the patent packet sale affects consumer surplus. Let $CS^{11}(\alpha_1, \alpha_2)$ denote expected consumer surplus when both firms are PEs and hold patent portfolios of strength α_1 and α_2 , respectively. Consumers face a duopoly except for patent portfolio constellations, in which firms litigate and exactly one firm is excluded. Hence, we have

$$CS^{11}(\alpha_1, \alpha_2) = S^d - [1 - \alpha_1\alpha_2 - (1 - \alpha_1)(1 - \alpha_2)] \begin{cases} S^d - S^m & \text{if } (\alpha_1, \alpha_2) \in \tilde{\mathcal{L}}, \\ 0 & \text{otherwise.} \end{cases}$$

Now compare expected consumer surplus when firm 1 and firm 2 acquire the patent packet, respectively. Suppose litigation always occurs independent of which firm acquires the packet. Then we get

$$CS^{11}(\alpha_1 + \delta_1, \alpha_2) - CS^{11}(\alpha_1, \alpha_2 + \delta_2) = 2\Delta(\alpha_2 - \alpha_1)(S^d - S^m) > 0.$$

Furthermore, if there is litigation when firm 2 acquires but not after firm 1's acquisition, consumer surplus is always higher in the latter case. Thus, consumers are weakly better off when the firm with the smaller portfolio acquires the patents. This leads to a more even distribution of patents and a lower probability that one firm is excluded from the marketplace in case litigation arises.

Proposition 4 *In equilibrium the firm with the larger patent portfolio acquires the additional patent packet while consumers would be better off if the packet would be purchased by the firm with the weaker portfolio. The acquisition price (weakly) decreases in the degree of product market competition between firms.*

[CAN WE RELATE THIS BACK TO NORTEL CASE?]. The acquisition price reflects the profit difference for firm 1 between winning and losing the auction. More intense competition reduces the acquisition price because the probability of ending up in a duopoly after litigation has a positive cross derivative with respect to α_1 and α_2 . This implies that adding the packet for sale to the smaller portfolio of firm 1 increases the likelihood of a duopoly outcome more than adding it to the larger portfolio of firm 2. Hence, the higher duopoly profits the larger the difference between winning and losing for firm 1, and the larger the acquisition price.

5.2 Patent Portfolio Acquisition between PE and NPE

By contrast, consider a patent portfolio acquisition game between one PE and one NPE. Without any loss of generality, let firm 1 be PE and firm 2 be NPE. As is clear from the profit definitions in Section 2, the PE's portfolio strength does not figure into each firm's payoff functions. The only reason for the PE to acquire the available patent packet is to prevent an NPE from using it in settlement negotiations or in litigation against the PE. Since PE and NPE never litigate and always settle on license terms in equilibrium, it is clear that the willingness-to-pay for the patent portfolio is the same for both firms, that is,

$$\begin{aligned} \Pi_1^{11}(\alpha_1 + \delta_1, \alpha_2) - \Pi_1^{11}(\alpha_1, \alpha_2 + \delta_2) &= \Pi_2^{11}(\alpha_1 + \delta_1, \alpha_2) - \Pi_2^{11}(\alpha_1, \alpha_2 + \delta_2) \\ &= \begin{cases} 0 & \text{if } \alpha_2 < \alpha^* - \delta_2 \\ (\alpha_2 + (1 - \alpha_2)\Delta)\pi^m/2 & \text{if } \alpha^* - \delta_2 \leq \alpha_2 < \alpha^* \\ (1 - \alpha_2)\Delta\pi^m/2 & \text{if } \alpha_2 \geq \alpha^*. \end{cases} \end{aligned}$$

Notice that the equilibrium acquisition price exhibits a non-monotonicity in the NPE's ex ante patent strength α_2 . This is illustrated in FIGURE 4 below. When α_2 is small and the NPE does not have any credible threat to litigate even after acquiring the patent packet, no firm has an incentive to pay a positive price for the new patent portfolio.⁶ When the acquisition makes the litigation threat credible, the value of acquisition is highest. In this case, the acquisition price does not only reflect the incremental strength, but also the existing patent strength. When the existing patent portfolio is already strong enough

⁶However, this conclusion does not hold if there is a possibility of further patent portfolio acquisition in the future. EXPLAIN MORE

to make the litigation threat credible, the acquisition price decreases in α_2 because the incremental value is less. For instance, when $\alpha_2 = 1$, firm 1's new product already infringes NPE's patent portfolio for sure, so there is no need to acquire additional patents.

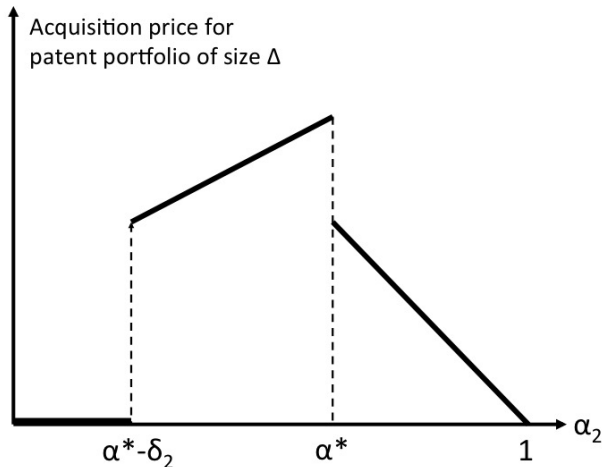


Figure 4: *Acquisition price for patent portfolio*

6 NPE as a Business Model

The analysis so far has assumed that all firms have the ability to manufacture and market new products. A firm becomes an NPE when its investment fails to produce a new product. However, in recent years the number of companies whose business model is purely based on converting intellectual property into licensing revenues (“patent trolls”) has sharply increased. In this section, we analyze NPEs as a business model to accommodate this possibility. Section 5 analyzes a patent portfolio acquisition game at the litigation stage after the outcomes of new product development. In this section, we analyze the incentive to acquire a patent portfolio for sale in anticipation of new product development. To simplify the analysis, we consider a case where firm 1 is potentially a PE, but firm 2 is an NPE without any manufacturing capacity whose main source of revenues is through licensing. When firm 2 is an NPE that does not engage in any new product development, the only thing that matters is the strength of firm 2’s patent portfolio because firm 1 cannot litigate against firm 2. We have to consider three cases.

Case 1: $\alpha_2 \geq \alpha^*$. In this case, firm 2 has incentives to litigate even if it does not acquire

a new patent portfolio for sale when firm 1 has a new product. Let us define

$$\varphi(\alpha_2) = \text{Max } p(I_1)\Pi_1^{10}(\alpha_2) - I_1.$$

Let $I_1^*(\alpha_2)$ denote the maximizer of this objective. Note that $I_1^*(\alpha_2)$ is decreasing in α_2 , and $\partial\varphi(\alpha_2)/\partial\alpha_2 = -p(I_1^*(\alpha_2))\pi^m/2 < 0$ by the envelope theorem. Firm 1's incentive to acquire the additional patent packet is purely for defensive purposes to prevent the NPE from acquiring it. Firm 1's maximum willingness to pay to acquire the patents for sale is given by

$$B_1 = \varphi(\alpha_2) - \varphi(\alpha_2 + \delta_2) = - \int_{\alpha_2}^{\alpha_2 + \delta_2} \frac{\partial\varphi(x)}{\partial x} dx = \frac{\pi^m}{2} \int_{\alpha_2}^{\alpha_2 + \delta_2} p(I_1^*(x)) dx$$

The incentives for firm 2 to acquire the additional patents come from the exclusionary value, and firm 2's maximum willingness to pay is given by

$$B_2 = [p(I_1^*(\alpha_2 + \delta_2))(\alpha_2 + \delta_2) - p(I_1^*(\alpha_2))\alpha_2] \frac{\pi^m}{2}.$$

Case 2: $\alpha_2 < \alpha^* < \alpha_2^+$. In this case, firm 2 does not have a credible threat to litigate against firm 1 without acquiring the patent portfolio for sale, but its threat becomes credible after acquisition. In other words,

$$\Pi_1^{10}(\alpha_2) = \pi^m, \quad \Pi_1^{10}(\alpha_2^+) = (1 - \frac{\alpha_2^+}{2})\pi^m$$

In this case, the PE's willingness to pay for the patent packet for sale is

$$\begin{aligned} B_1 &= \varphi(\alpha_2) - \varphi(\alpha_2 + \delta_2) \\ &= [p(I_1^*(\alpha_2))\pi^m - I_1^*(\alpha_2)] - [p(I_1^*(\alpha_2^+))(1 - \frac{\alpha_2^+}{2})\pi^m - p(I_1^*(\alpha_2^+))] \end{aligned}$$

whereas the NPE's maximum willingness to pay is given by

$$B_2 = p(I_1^*(\alpha_2^+)) \frac{\alpha_2^+}{2} \pi^m.$$

Case 3: $\alpha_2^+ < \alpha^*$. Here, the patent portfolio for sale has no value to the PE and NPE.

Comparing the willingness-to-pay in all cases, we obtain the following outcome of the

patent sale.

Proposition 5 *If the patent sale occurs before the development of the new product, then the PE has a (weakly) higher willingness-to-pay and acquires the patent portfolio.*

The intuition for this result is that the NPE can extract rents only when the PE develops a new product. The acquisition of additional portfolio is beneficial ex post, but adversely affects the PE’s investment incentives. This adverse impact on the PE’s investment incentives discourages the NPE’s patent portfolio acquisition. This effect is absent when the acquisition auction takes place after the development of the new product.

Our analysis also reveals the incentives for NPEs to acquire a patent portfolio in secret. This is in sharp contrast to PEs’ practice. Chien (2010) makes a distinction between contrasting strategies of “patent signal” and “patent secrecy”. When firms acquire a patent portfolio to deter litigation by other PEs, they publicize their patent portfolio to send a message to competitors: “If sued, I have the ability to retaliate.”⁷ However, the so-called patent trolls exploit secrecy. Intellectual Ventures, Acadia, and many others have assigned their patents to thousands of shell companies and subsidiaries, making them hard to track. For instance, Ewing and Feldman (2011) identified 1276 shell companies created by Intellectual Ventures. It is their explicit strategy to wait until PEs develop products that infringe on their patent portfolios to “surprise them with a suit.”

7 Extensions and Robustness

In this section, we extend our analysis into two directions and check the robustness of our main results.

7.1 The Market Expansion Effect

So far we have assumed that the two firms are competing in the same market. With the assumption of $2\pi^d \leq \pi^m$, this implies that there is no licensing between PEs when one firm is found to infringe upon the other’s patent portfolio, but the latter does not infringe upon the former’s. We now consider the possibility of market expansion with licensing between PEs. To formalize this, suppose that each firm’s new product covers a market size of 1.

⁷Chien (2010), p. 319.

However, there is an overlap between the two firms' customer base of size $(1 - s)$. In other words, for the market size of s , each firm is a monopolist, but for the remaining area of $(1 - s)$ they compete. Thus, the parameter s represents the market expansion effect when both PEs produce compared to only one PE producing.⁸ When $s = 1$, their markets do not overlap and the market expansion effect is the largest. Our previous analysis is the special case of $s = 0$.

When one firm is a PE and the other is NPE, market expansion is not possible and the previous analysis applies. Now let us consider the case of two PEs. If they do not engage in litigation, their individual payoffs are given by $s\pi^m + (1 - s)\pi^d$. When firm i litigates against firm j , again, it is firm j 's best interest to counter-litigate. With the probability of $(1 - \alpha_1)(1 - \alpha_2)$, neither firm is found to infringe on the other's patent portfolio. In this case, the pre-litigation situation persists with each firm earning $s\pi^m + (1 - s)\pi^d$. Another outcome that leads to a status quo is when both firms are found to infringe on the other's patent portfolio. The remaining possibility is that one firm, say firm i , is found not to infringe on firm j 's while firm j is found to infringe on firm i 's patent portfolio. In this case, firm i can license its patent portfolio and enable firm j to enter its markets.⁹ A license agreement is feasible if licensing increases industry profits relative to the situation where firm i supplies the competitive market segment as well as its exclusive segment as a monopolist. This holds if the gain from supplying firm j exclusive market segment via licensing outweighs the introduction of duopolistic competition in the contested market segment,

$$s\pi^m \geq (1 - s)(\pi^m - 2\pi^d) \quad \text{or} \quad s \geq \frac{\pi^m/2 - \pi^d}{\pi^m - \pi^d} \equiv s_L.$$

Licensing occurs when the market expansion effect is sufficiently large and product market competition in the contested segment is weak. What are the effects of licensing and the market expansion effect on litigation incentives? First, suppose s is sufficiently large such that licensing occurs if exactly one firm can assert its patent portfolio in litigation. Licensing then yields an industry profit of $2s\pi^m + (1 - s)\pi^d$, which is exactly the industry profit when firms refrain from litigation. Since litigation is costly, it will never occur if firms have ex

⁸To be more precise, the parameter s captures two effects, a market size expansion and a relative increase in the monopolistic versus the competitive market segment. Since both effects individually yield the same qualitative results in our framework, we have subsumed them into one parameter.

⁹Alternatively, firm i could license its patents for use in firm j 's monopolistic market segment only. This would affect the qualitative nature of our results.

post incentives to license. By contrast, if s is sufficiently small, licensing does not arise ex post. In this case litigation is optimal for firms if the industry profits from asymmetric litigation outcomes and monopolization of the contested market segment exceed the status quo industry profits for symmetric outcomes and the cost of litigation, that is,

$$[\alpha_1(1 - \alpha_2) + \alpha_2(1 - \alpha_1)]\pi^m - 2L \geq [1 - \alpha_1(1 - \alpha_2) - \alpha_2(1 - \alpha_1)] [2s\pi^m + (1 - s)\pi^d]$$

or

$$s \leq s_L - \frac{L}{[1 - \alpha_1(1 - \alpha_2) - \alpha_2(1 - \alpha_1)](\pi^m - \pi^d)}.$$

Thus, three parameter regions exist. For high values of s and low product market competition, there is no litigation and no licensing. For intermediate values of the market expansion effect, firms have no incentive to license ex post but litigation is too costly. Finally, if the market expansion effect is not too strong and product market competition in the contested segment is intense, firms litigate and refrain from licensing when exactly one firm asserts its property rights during litigation.

Proposition 6 *When product market competition is less intense, the market expansion effect may induce firms to license ex post rather than to litigate.*

7.2 Asymmetric Product Market Positions

In our above analysis we allow firms to hold patent portfolios of different sizes but assume that they have symmetric positions in product market competition. In this extension, we investigate the effect of product market position on litigation incentives for given symmetric patent portfolios of size $\alpha_1 = \alpha_2 = \alpha$. Which firm has a stronger incentive to litigate, the market leader or the firm with the smaller market share? What is the impact of firm asymmetry on litigation in equilibrium? To address these questions in a simple way, assume that firms differ in their marginal cost of production. In particular, let c_i denote the marginal cost of production of firm i such that $c_1 = c - \epsilon$ and consider $c_2 = c + \epsilon$ where $\epsilon \geq 0$ is a measure for product asymmetry. Further let $\pi^m(c)$ denote the monopoly profit with cost c and $\pi^d(c_i; \theta)$ the duopoly profit of firm i . The parameter $\theta \geq 0$ represents the intensity of competition in the market. For $\theta = 0$ the firms' products are independent and as θ goes to

infinity, products become perfect substitutes. We impose the following two assumptions:¹⁰

$$\begin{aligned} \text{A.1 } & \frac{\partial [\pi^d(c_1; \theta) - \pi^d(c_2; \theta)]}{\partial \theta} \geq 0 \\ \text{A.2 } & \frac{\partial [\pi^d(c_1; \theta) + \pi^d(c_2; \theta)]}{\partial \epsilon} \geq 0, \quad \frac{\partial^2 [\pi^d(c_1; \theta) + \pi^d(c_2; \theta)]}{\partial \epsilon \partial \theta} \geq 0. \end{aligned}$$

The first condition states that the profit advantage of the low-cost firm increases as products become closer substitutes. The second set of assumptions implies that cost asymmetry increases industry duopoly profits and that this effect is stronger when products are closer substitutes.

First, consider firms' unilateral incentives to litigate. Firm i has an incentive to litigate if

$$L \leq \alpha(1 - \alpha) [\pi^m(c_i) - 2\pi^d(c_i)].$$

Comparing the individual litigation constraints yields that the firm with the higher marginal cost (firm 2) has a stronger incentive to litigate if and only if

$$\pi^m(c_2) - 2\pi^d(c_2) \geq \pi^m(c_1) - 2\pi^d(c_1). \quad (9)$$

This condition always hold under assumption A.1 as the smaller firm stands to gain more from excluding its rival. Similarly, firms prefer litigation over settlement if

$$\alpha(1 - \alpha) [\pi^m(c - \epsilon) + \pi^m(c + \epsilon) - 2\pi^d(c - \epsilon) - 2\pi^d(c + \epsilon)] \geq 2L. \quad (10)$$

Monopoly profits are decreasing and convex in cost. Hence, the sum of monopoly profits in the squared bracket increases in the degree of cost asymmetry. However, under assumption A.2 cost asymmetries increase duopoly industry profits more and the LHS is decreasing in the parameter ϵ . It follows that in industries with asymmetric product market positions, firms litigate less in equilibrium.

Proposition 7 *The firm with the smaller market share has a stronger incentive to litigate. Asymmetric product market positions reduce overall litigation incentives in the industry.*

¹⁰These assumptions are satisfied for the most commonly used demand structures such as the ones in Singh and Vives (1984) or Shubik and Levitan (1980).

8 Concluding Remarks

The patent system is created as a mechanism to encourage discovery and development of new ideas and technologies. However, the current patent system has been criticized and described to be under siege due to an explosion of suspect quality, overlapping, and excessively broad patents. With the convergence of technologies in various high-tech fields, it is inevitable for new products to incorporate complementary technologies and inadvertently infringe on patented technologies developed elsewhere. This has led to patent portfolio races in which firms competitively build up an ever increasing size of patent portfolios by internal R&D and/or acquisition of patents held by other firms. In this paper, we have developed a model to analyze the implications of such patent portfolios on the incentives to develop new products in the shadow of patent litigation.

We showed that the incentives to litigate for practicing entities depend crucially on the competitiveness of the industry. The effects of an increase in one firm's patent portfolios unambiguously reduce the rival firm's incentives to develop a new product. However, an increase in its own patent portfolio does not necessarily induce more incentives to develop its own new product. In such a case, the patent build-up by one firm can unambiguously reduce the overall rate of new product developments.

Our analysis can be extended to address many other unexplored issues. For instance, we have assumed that the extent of patent portfolios held by each firm is common knowledge. However, there are many examples in which companies with new products and services have been held up by patent asserting entities unknownst to them. Our model can help identify circumstances under which firms with a large patent portfolio would have incentives to exploit secrecy to their advantage. One way to achieve secrecy is to create shell companies and subsidiaries, which makes it difficult to track the ownership of patents. For instance, Intellectual Ventures has created more than 1200 shell companies (Ewing and Feldman, 2012). The secrecy allows patent-assertion entities to use a surprise tactic by litigating (unknowingly) infringing firms at the most vulnerable time when they have sunk their resources in designing new products (Shapiro, 2010) while maintaining other firms' incentives to introduce new products by keeping them in the dark.

Finally, we have analyzed the effects of given sizes of patent portfolios on new product developments. We need to analyze the strategic incentives to build up patent portfolios

and the optimal composition of patent portfolios.

Appendix

Proof of Lemma 1 and Proposition 1 From (4) follows that firms litigate if and only if

$$\alpha_1(1 - 2\alpha_2)(\pi^m - 2\pi^d) \geq 2L - \alpha_2(\pi^m - 2\pi^d).$$

For $\alpha_2 < 1/2$, firms settle if

$$\alpha_1 \leq \frac{2L - \alpha_2(\pi^m - 2\pi^d)}{(1 - 2\alpha_2)(\pi^m - 2\pi^d)} \equiv \bar{\alpha}_S(\alpha_2).$$

When α_2 is sufficiently small, this threshold value is strictly positive and continuous in α_2 .

Thus, firms settle if both portfolios are sufficiently small. For $\alpha_2 > 1/2$, firms settle if

$$\alpha_1 \geq \bar{\alpha}_S(\alpha_2).$$

It holds that

$$\bar{\alpha}_S(\alpha_2 = 1) = \frac{\pi^m - 2\pi^d - 2L}{\pi^m - 2\pi^d} < 1.$$

As $\bar{\alpha}_S(\alpha_2)$ is continuous, there must exist values such that firms settle if both portfolios are sufficiently strong. This proves the first point in Proposition 1. Further note that

$$\bar{\alpha}_S(\alpha_2 = 0) = \frac{2L}{\pi^m - 2\pi^d} < 1$$

if and only if

$$\pi^m/2 - \pi^d \geq L. \tag{11}$$

Hence, there exist values (α_1, α_2) such that firms litigate if firm 1's portfolio is sufficiently strong while firm 2's portfolio is sufficiently weak. Similarly, it holds under assumption (11) that

$$\bar{\alpha}_S(\alpha_2 = 1) \geq 0.$$

Thus, firms litigate if firm 1's portfolio is sufficiently weak and firm 2's portfolio is sufficiently strong. This proves the second statement. In order to further characterize the settlement

and litigation behavior, check that

$$\frac{\partial \bar{\alpha}_S}{\partial \alpha_2} = -\frac{\frac{\pi^m}{2} - \pi^D - 2L}{(1 - 2\alpha_2)^2(\frac{\pi^m}{2} - \pi^D)}$$

which implies

$$\text{sign}\left(\frac{\partial \bar{\alpha}_S}{\partial \alpha_2}\right) = \begin{cases} -1 & \text{if } \frac{\pi^m}{2} - \pi^D > 2L, \\ 0 & \text{if } \frac{\pi^m}{2} - \pi^D = 2L, \\ 1 & \text{if } \frac{\pi^m}{2} - \pi^D < 2L. \end{cases}$$

Further let $\bar{\alpha}_1^1(\alpha_2)$ denote the value of α_1 such that (2) holds with equality and above which firm 1 has a strict incentive to litigate. It follows that

$$\bar{\alpha}_1^1(\alpha_2) = \frac{\alpha_2 \pi^d + L}{(1 - \alpha_2)(\pi^m - \pi^d) + \alpha_2 \pi^d}$$

and

$$\begin{aligned} \frac{\partial \alpha_1^{**}}{\partial \alpha_2} &= \frac{L(\pi^m - 2\pi^d) + \pi^d(\pi^m - \pi^d)}{[(1 - \alpha_2)(\pi^m - \pi^d) + \alpha_2 \pi^d]^2} > 0, \\ \frac{\partial^2 \alpha_1^{**}}{(\partial \alpha_2)^2} &= \frac{2(\pi^m - 2\pi^d) [L(\pi^m - 2\pi^d) + \pi^d(\pi^m - \pi^d)]}{[(1 - \alpha_2)(\pi^m - \pi^d) + \alpha_2 \pi^d]^3} > 0 \end{aligned}$$

Similarly, let $\bar{\alpha}_1^{**}(\alpha_2)$ denote the value of α_1 such that (3) holds with equality,

$$\bar{\alpha}_1^{**}(\alpha_2) = \frac{\alpha_2(\pi^m - \pi^d) - L}{\alpha_2(\pi^m - 2\pi^d) + \pi^d}$$

and

$$\begin{aligned} \frac{\partial \bar{\alpha}_1^{**}(\alpha_2)}{\partial \alpha_2} &= \frac{L(\pi^m - 2\pi^d) + \pi^d(\pi^m - \pi^d)}{[\alpha_2(\pi^m - 2\pi^d) + \pi^d]^2} > 0, \\ \frac{\partial^2 \bar{\alpha}_1^{**}(\alpha_2)}{(\partial \alpha_2)^2} &= -\frac{2(\pi^m - 2\pi^d) [L(\pi^m - 2\pi^d) + \pi^d(\pi^m - \pi^d)]}{[\alpha_2(\pi^m - 2\pi^d) + \pi^d]^3} < 0. \end{aligned}$$

Finally note that $\alpha_1^{**}(\alpha_2) \geq \bar{\alpha}_1^{**}(\alpha_2)$ if and only if

$$\alpha_2(1 - \alpha_2)\left(\frac{\pi^m}{2} - \pi^D\right) \leq \frac{L}{2}$$

This condition holds for any α_2 if

$$\frac{\pi^m}{2} - \pi^D < 2L.$$

The qualitative properties of the graphs in FIGURE 1 and 2 in the main text follow. ■

Proof of Lemma 3 $\Pi_i^{10}(\alpha_j)$ is a decreasing function of α_j with the minimum value of $\frac{\pi^m}{2}$ when $\alpha_j = 1$ whereas $\Pi_i^{01}(\alpha_i)$ is an increasing function of α_i with the maximum value of $\frac{\pi^m}{2}$ when $\alpha_1 = 1$. Thus, $\Pi_i^{10}(\alpha_j) \geq \Pi_i^{01}(\alpha_i)$ for all (α_1, α_2) with the equality holding only at $\alpha_1 = \alpha_2 = 1$. Note that $\Pi_i^{11}(\alpha_1, \alpha_2)$ depends on the competitiveness of duopoly outcome and increases in $\theta (= \frac{2\pi^d}{\pi^m})$. It achieves the highest value when $\theta = 1$, in which case its value is given by $\frac{1+\alpha_i-\alpha_j}{2}\pi^m$, which is less than $\frac{2-\alpha_j}{2}\pi^m (= \Pi_i^{10}(\alpha_j))$. Taken together, this implies that $\Pi_i^{10}(\alpha_j) \geq \max[\Pi_i^{11}(\alpha_1, \alpha_2), \Pi_i^{01}(\alpha_i)]$. However, the relative magnitudes of $\Pi_i^{11}(\alpha_1, \alpha_2)$ and $\Pi_i^{01}(\alpha_i)$ are ambiguous. For instance, $\Pi_i^{11}(\alpha_1, \alpha_2) > \Pi_i^{01}(\alpha_i)$ if $\theta = 1$ (i.e., $\pi^d = \frac{\pi^m}{2}$), whereas $\Pi_i^{01}(\alpha_i) > \Pi_i^{11}(\alpha_1, \alpha_2)$ if $\theta = 0$ and $\alpha_j > 1/2$. Since $\Pi_i^{11}(\alpha_1, \alpha_2)$ is an increasing in θ and decreasing in α_j while $\Pi_i^{01}(\alpha_i)$ is independent of θ and α_j , $\Pi_i^{11}(\alpha_1, \alpha_2)$ is more likely to be larger than $\Pi_i^{01}(\alpha_i)$ for a higher θ and a lower α_j . ■

Proof of Lemma 4 By totally differentiating the first order condition (5) with respect to I_1 and I_2 , we derive

$$R'_i(I_j; \alpha_1, \alpha_2) = \frac{dI_i}{dI_j} = -\frac{p'(I_i)p'(I_j)[\Pi_i^{11}(\alpha_i, \alpha_j) - \Pi_i^{10}(\alpha_j) - \Pi_i^{01}(\alpha_i)]}{[SOC_i]} < 0,$$

where $[SOC_i] = p''(I_i)[p(I_j)\Pi_i^{11}(\alpha_i, \alpha_j) + (1 - p(I_j))\Pi_i^{10}(\alpha_j) - p(I_j)\Pi_i^{01}(\alpha_i)] < 0$. The inequality follows from Lemma 3. ■

Proof of Lemma 5 By totally differentiating the first order condition (5) with respect to I_i and α_j , we have

$$\frac{\partial R_i}{\partial \alpha_j} = -\frac{p'(I_i)}{[SOC_i]} \left[p(I_j) \frac{\partial \Pi_i^{11}(\alpha_i, \alpha_j)}{\partial \alpha_j} + (1 - p(I_j)) \frac{\partial \Pi_i^{10}(\alpha_j)}{\partial \alpha_j} \right] \leq 0$$

since $\partial \Pi_i^{11}(\alpha_i, \alpha_j) / \partial \alpha_j \leq 0$ and $\partial \Pi_i^{10}(\alpha_j) / \partial \alpha_j \leq 0$. Similarly, a total differentiation of (5) with respect to I_i and α_i yields

$$\frac{\partial R_i}{\partial \alpha_i} = -\frac{p'(I_i)p(I_j)}{[SOC_i]} \left[\frac{\partial \Pi_i^{11}(\alpha_i, \alpha_j)}{\partial \alpha_i} - \frac{\Pi_i^{01}(\alpha_i)}{\partial \alpha_i} \right].$$

Thus, the sign of $\frac{\partial R_i}{\partial \alpha_i}$ equals the sign of $[\partial \Pi_i^{11}(\alpha_i, \alpha_j)/\partial \alpha_i - \partial \Pi_i^{01}(\alpha_i)/\partial \alpha_i]$, which is ambiguous since

$$\frac{\partial \Pi_i^{11}(\alpha_i, \alpha_j)}{\partial \alpha_i} = \begin{cases} (1 - \alpha_j)(\pi^m - \pi^d) + \alpha_j \pi^d & \text{for } (\alpha_1, \alpha_2) \in \tilde{\mathcal{L}} \\ \pi^m/2 & \text{for } (\alpha_1, \alpha_2) \in \mathcal{L} \cap S \\ 0 & \text{for } (\alpha_1, \alpha_2) \notin \mathcal{L} \end{cases}$$

and

$$\frac{\partial \Pi_i^{01}(\alpha_i)}{\partial \alpha_i} = \begin{cases} \pi^m/2 & \text{for } \alpha_i \geq \alpha^*, \\ 0 & \text{for } \alpha_i < \alpha^*. \end{cases}$$

In situations where PEs have an incentive to litigate whereas NPEs do not, the sign is definitely positive. Vice versa, when PEs have no incentive to litigate whereas NPEs would, the sign is negative. When PEs litigate and NPE have an incentive to litigate, we get

$$\frac{\partial \Pi_i^{11}(\alpha_i, \alpha_j)}{\partial \alpha_i} - \frac{\partial \Pi_i^{01}(\alpha_i)}{\partial \alpha_i} = \left(\frac{1}{2} - \alpha_j\right)(\pi^m - 2\pi^d)$$

which is positive if and only if $\alpha_j < 1/2$. ■

Proof of Proposition 2 Let us totally differentiate the first order conditions for I_1 and I_2 with respect to α_1 .

$$\begin{aligned} \frac{\partial^2 \Psi_1}{\partial I_1^2} dI_1 + \frac{\partial^2 \Psi_1}{\partial I_1 \partial I_2} dI_2 + \frac{\partial^2 \Psi_1}{\partial I_1 \partial \alpha_1} d\alpha_1 &= 0 \\ \frac{\partial^2 \Psi_2}{\partial I_1 \partial I_2} dI_1 + \frac{\partial^2 \Psi_2}{\partial I_2^2} dI_2 + \frac{\partial^2 \Psi_2}{\partial I_2 \partial \alpha_1} d\alpha_1 &= 0 \end{aligned}$$

To derive comparative statics result with respect to α_1 , we can write the expression above in the following matrix form.

$$\begin{bmatrix} \frac{\partial^2 \Psi_1}{\partial I_1^2} & \frac{\partial^2 \Psi_1}{\partial I_1 \partial I_2} \\ \frac{\partial^2 \Psi_2}{\partial I_1 \partial I_2} & \frac{\partial^2 \Psi_2}{\partial I_2^2} \end{bmatrix} \begin{bmatrix} \frac{dI_1}{d\alpha_1} \\ \frac{dI_2}{d\alpha_1} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 \Psi_1}{\partial I_1 \partial \alpha_1} \\ -\frac{\partial^2 \Psi_2}{\partial I_2 \partial \alpha_1} \end{bmatrix}$$

By applying Cramer's rule, we can derive

$$\begin{aligned} \frac{dI_1}{d\alpha_1} &= \frac{\begin{bmatrix} -\frac{\partial^2 \Psi_1}{\partial I_1 \partial \alpha_1} & \frac{\partial^2 \Psi_1}{\partial I_1 \partial I_2} \\ -\frac{\partial^2 \Psi_2}{\partial I_2 \partial \alpha_1} & \frac{\partial^2 \Psi_2}{\partial I_2^2} \end{bmatrix}}{|A|} = \frac{-\frac{\partial^2 \Psi_1}{\partial I_1 \partial \alpha_1} \frac{\partial^2 \Psi_2}{\partial I_2^2} + \frac{\partial^2 \Psi_2}{\partial I_2 \partial \alpha_1} \frac{\partial^2 \Psi_1}{\partial I_1 \partial I_2}}{|A|} \\ \frac{dI_2}{d\alpha_1} &= \frac{\begin{bmatrix} \frac{\partial^2 \Psi_1}{\partial I_1^2} & -\frac{\partial^2 \Psi_1}{\partial I_1 \partial \alpha_1} \\ \frac{\partial^2 \Psi_2}{\partial I_1 \partial I_2} & -\frac{\partial^2 \Psi_2}{\partial I_2 \partial \alpha_1} \end{bmatrix}}{|A|} = \frac{-\frac{\partial^2 \Psi_1}{\partial I_1^2} \frac{\partial^2 \Psi_2}{\partial I_2 \partial \alpha_1} + \frac{\partial^2 \Psi_2}{\partial I_1 \partial I_2} \frac{\partial^2 \Psi_1}{\partial I_1 \partial \alpha_1}}{|A|} \end{aligned}$$

where $|A| = \begin{vmatrix} \frac{\partial^2 \Psi_1}{\partial I_1^2} & \frac{\partial^2 \Psi_1}{\partial I_1 \partial I_2} \\ \frac{\partial^2 \Psi_2}{\partial I_1 \partial I_2} & \frac{\partial^2 \Psi_2}{\partial I_2^2} \end{vmatrix} > 0$ by the stability condition. Thus,

$$\begin{aligned} \text{sign}\left[\frac{dI_1}{d\alpha_1}\right] &= \text{sign}\left[-\frac{\partial^2 \Psi_1}{\partial I_1 \partial \alpha_1} \frac{\partial^2 \Psi_2}{\partial I_2^2} + \frac{\partial^2 \Psi_2}{\partial I_2 \partial \alpha_1} \frac{\partial^2 \Psi_1}{\partial I_1 \partial I_2}\right] \\ \text{sign}\left[\frac{dI_2}{d\alpha_1}\right] &= \text{sign}\left[-\frac{\partial^2 \Psi_1}{\partial I_1^2} \frac{\partial^2 \Psi_2}{\partial I_2 \partial \alpha_1} + \frac{\partial^2 \Psi_2}{\partial I_1 \partial I_2} \frac{\partial^2 \Psi_1}{\partial I_1 \partial \alpha_1}\right] \end{aligned}$$

We know that $\frac{\partial^2 \Psi_1}{\partial I_1^2} < 0$ and $\frac{\partial^2 \Psi_2}{\partial I_2^2} < 0$ by the second order condition. In addition, $\text{sign}\left[\frac{\partial^2 \Psi_2}{\partial I_2 \partial \alpha_1}\right] = \text{sign}\left[\frac{\partial R_2}{\partial \alpha_1}\right] < 0$ by lemma 5.

(i) When $\frac{\partial R_1}{\partial \alpha_1} > 0$, we have an unambiguous result that $\frac{dI_1}{d\alpha_1} > 0$ and $\frac{dI_2}{d\alpha_1} < 0$.

(ii) When $\frac{\partial R_1}{\partial \alpha_1} < 0$, the signs of $\frac{dI_1}{d\alpha_1}$ and $\frac{dI_2}{d\alpha_1}$ are ambiguous. However, both cannot be positive. We prove this by contradiction. Suppose that $\frac{dI_1}{d\alpha_1} > 0$ and $\frac{dI_2}{d\alpha_1} > 0$. For this to happen, it must be that

$$\left| \frac{\partial^2 \Psi_1}{\partial I_1 \partial I_2} \right| > \left| \frac{\frac{\partial^2 \Psi_1}{\partial I_1 \partial \alpha_1} \frac{\partial^2 \Psi_2}{\partial I_2^2}}{\frac{\partial^2 \Psi_2}{\partial I_2 \partial \alpha_1}} \right| \quad \text{and} \quad \left| \frac{\partial^2 \Psi_2}{\partial I_1 \partial I_2} \right| > \left| \frac{\frac{\partial^2 \Psi_1}{\partial I_1^2} \frac{\partial^2 \Psi_2}{\partial I_2 \partial \alpha_1}}{\frac{\partial^2 \Psi_1}{\partial I_1 \partial \alpha_1}} \right|$$

This implies that

$$\left| \frac{\partial^2 \Psi_1}{\partial I_1 \partial I_2} \frac{\partial^2 \Psi_2}{\partial I_1 \partial I_2} \right| > \left| \frac{\partial^2 \Psi_1}{\partial I_1 \partial \alpha_1} \frac{\partial^2 \Psi_1}{\partial I_1^2} \right|$$

However, the condition above contradicts our stability condition. ■

Proof of Lemma 7 First check the condition for the case when litigation occurs no matter

who buys the patent portfolio,

$$\begin{aligned} & \Pi^{11}(\alpha_1, \alpha_2 + \delta_2)|_{(\alpha_1, \alpha_2 + \delta_2) \in \tilde{\mathcal{L}}} - \Pi^{11}(\alpha_1 + \delta_1, \alpha_2)|_{(\alpha_1 + \delta_1, \alpha_2) \in \tilde{\mathcal{L}}} \\ & = \Delta(\alpha_2 - \alpha_1)(\pi^m - 2\pi^d) \geq 0 \end{aligned} \quad (12)$$

Next check the case where litigation arises if firm 2 obtains the patent portfolio but not when firm 1 acquires it. This holds when

$$\Pi^{11}(\alpha_1, \alpha_2 + \delta_2)|_{(\alpha_1, \alpha_2 + \delta_2) \in \tilde{\mathcal{L}}} \geq 2\pi^d > \Pi^{11}(\alpha_1 + \delta_1, \alpha_2 + \Delta)|_{(\alpha_1 + \delta_1, \alpha_2) \in \tilde{\mathcal{L}}}.$$

In this case, $\Pi^{11}(\alpha_1 + \delta_1, \alpha_2) = 2\pi^d$ and condition (8) always holds. From (12) follows that it is never possible that after firm 1's acquisition, there is litigation while there is no litigation after firm 2's acquisition. Finally, if there is no litigation after acquisition by any of the two firms, the industry profits equal $2\pi^d$ and condition (8) is satisfied with equality.

■

Proof of Proposition 5 We show $B_1 > B_2$. First, consider Case 1. Since $I_1^*(\alpha_2)$ is decreasing in α_2 , we have

$$B_1 = \frac{\pi^m}{2} \int_{\alpha_2}^{\alpha_2 + \delta_2} p(I_1^*(x)) dx > \frac{\pi^m}{2} p(I_1^*(\alpha_2 + \delta_2)) \delta_2$$

Note that

$$\begin{aligned} \delta_2 p(I_1^*(\alpha_2 + \delta_2)) & > p(I_1^*(\alpha_2 + \delta_2)) \delta_2 - [p(I_1^*(\alpha_2)) - p(I_1^*(\alpha_2 + \delta_2))] \alpha_2 \\ & = p(I_1^*(\alpha_2 + \delta_2))(\alpha_2 + \delta_2) - p(I_1^*(\alpha_2)) \alpha_2 \end{aligned}$$

Therefore, $B_1 > B_2$. For Case 2 we have

$$B_1 - B_2 = [p(I_1^*(\alpha_2))\pi^m - I_1^*(\alpha_2)] - [p(I_1^*(\alpha_2^+))\pi^m - I_1^*(\alpha_2^+)] > 0$$

by a revealed preference argument. ■

Proof of Proposition 6 First, check that condition (9) is always satisfied for $\theta=0$ since $\pi^d(c_i; \theta=0) = \pi^m(c_i)$. By assumption A.1, this condition becomes less restrictive as θ increases and the result follows. Second, take the derivative of the LHS of (10) with respect

to ϵ

$$\begin{aligned}
& \frac{\partial \pi^m(c + \epsilon)}{\partial \epsilon} - \frac{\partial \pi^m(c - \epsilon)}{\partial \epsilon} + 2 \frac{\partial \pi^d(c - \epsilon; \theta = 0)}{\partial \epsilon} - 2 \frac{\partial \pi^d(c + \epsilon; \theta = 0)}{\partial \epsilon} \\
&= \frac{\partial \pi^m(c + \epsilon)}{\partial \epsilon} - \frac{\partial \pi^m(c - \epsilon)}{\partial \epsilon} + 2 \frac{\partial \pi^m(c - \epsilon)}{\partial \epsilon} - 2 \frac{\partial \pi^m(c + \epsilon)}{\partial \epsilon} \\
&= \frac{\partial \pi^m(c - \epsilon)}{\partial \epsilon} - \frac{\partial \pi^m(c + \epsilon)}{\partial \epsilon} < 0.
\end{aligned}$$

Furthermore, by assumption A.2 the derivative of the LHS decreases further as θ increases and the result follows. ■

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