Bargaining with Endogenous Entry of New Traders *

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(Preliminary – please do not circulate without permission)

August 14, 2017

Abstract

This paper studies bargaining dynamics when the original counterparties face the endogenous entry of other agents. Time is continuous, there is a privately informed buyer and an uninformed seller who can make offers every instant, and they bargain while waiting for the possible entry of another privately informed buyer who observes their interactions. I show that allowing for endogenous entry fundamentally changes the equilibrium dynamics. In a typical case, there is a burst of trade at the first instant of bargaining, followed by a long impasse of slow, gradual agreement. In the burst of initial trade, the seller “drives an easy bargain,” making an offer that an atom of buyer types accept instantly. During the impasse, the seller “drives a hard bargain,” making offers that only be accepted by the (measure 0) highest buyer type who has not already been ruled out by prior play. By contrast, bargaining under any exogenous time-varying rate of entry is always atomless, with no bursts of trade. As a corollary, I show that making offers private (hiding them from the entrants) slows down trade for some initial buyer types.

Real world bargaining happens in the shadow of other parties who can insert themselves in the interaction, changing the bargaining power of the original counterparties. Consider, for instance, the recent acquisition of Straight

*Many thanks to my advisors Andy Skrzypacz, Matt Jackson, and Kyle Bagwell for guidance throughout the project. I would also like thank to Gabriel Carroll, Alex Bloedel, Shota Ichihashi, and Giorgio Martini for helpful comments. I gratefully acknowledge the support from the Kapnick Foundation Fellowship, granted through the Stanford Institute for Economic Policy Research.
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Path by Verizon. Straight Path was an unprofitable telecom company (it had 2016 revenues of $2.1 million), but it had the unique attraction that it owned the licenses to spectrum in the 28 and 39 gigahertz frequencies, which industry observers think will be crucial for 5G networks. Initially, AT&T approached Straight Path privately, and after negotiations, it struck a deal to acquire Straight Path for $1.25 Billion, a 160% premium on Straight Path’s stock price. In addition, the firms agreed that the transaction would close within 12 months (a relatively lax deadline that allowed for further due diligence and clearing of regulatory hurdles). Since Straight Path was publicly traded, the terms of the deal, including the long closing period, were released in press statements at the time.

Verizon is a main competitor of AT&T that is also in need of scarce spectrum. When the terms of the deal were first released, Verizon decided to approach Straight Path and counter AT&T’s offer. This triggered a bidding war between AT&T and Verizon. One month after the initial agreement was made public, Verizon approached Straight Path offered $2.3 Billion. A week later, Verizon returned with an even higher offer of $3.1 Billion, a 500% premium over Straight Path’s stock price before the initial AT&T agreement was announced. AT&T did not counter the final offer, and three months after its last offer (keep in mind the initial deal had a 12 month closing window), Straight Path shareholders approved the merger with Verizon.

In order to make predictions, models of bargaining impose strong assumptions on the bargaining protocol that may be hard to map to real-world practices. On that front this paper is no exception. However, the economic logic in the Straight Path case has two distinct features that are amenable to modeling. First, real world bargaining happens in the shadow of other parties who can insert themselves in the interaction, changing the bargaining power of the original counterparties. Second, delay in bargaining conveys information to the potential entrants about the possible gains from trade and the stiffness of competition to expect if they interrupt the bargaining process. This delay can take the form of rejected offers (as will be the case in this paper), or it can take the form of conditionality clauses, long closing periods (as in the Straight Path example), and “go-shop” provisions. Presumably, both

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the price AT&T announced and the length of the closing window conveyed information to Verizon about AT&T’s value for Straight Path, its assessment of the risks involved in the transaction, and the likelihood that AT&T would eventually back down in a bidding war.

The presence of potential entrants and the information content of delays in reaching agreement imply that, if entry is costly (say, because of due diligence costs), then it will be endogenous. Third parties will pay the cost of entry when they expect entry to be profitable, and whether it seems profitable or not depends on their beliefs. Since these beliefs are themselves determined the history of offers and counter-offers, the probability of entry will be an equilibrium object. For example, rejecting a low price is a strong signal to the market that the buyer’s value is low, so competing buyers who might face a bidding war with the original buyer would be induced to enter by that rejection (the reverse would apply to some extent if the buyer rejected very high price).

This paper therefore studies bargaining dynamics when the original counterparties are face the endogenous entry of other agents. In the model, time is continuous, there is a privately informed buyer and an uninformed seller who can make offers every instant, and they bargain while waiting for the possible entry of another privately informed buyer who observes their interactions. The main questions I ask are as follows. How are the trading dynamics affected by the endogeneity of entry? Is trade faster than exogenous entry (or no entry at all)? Is trade slower? Faster, then slower? Is it more efficient? Who is made better or worse off by the fact that entrants respond to the history of offers and rejections?

I show that allowing for endogenous entry fundamentally changes the equilibrium dynamics. This is so above and beyond the effect of adding entry, even time-varying exogenous entry. In a typical case, there is a burst of trade at the first instant of bargaining, followed by a long impasse of slow, gradual agreement. In the burst of initial trade, the seller “drives an easy bargain,” making an offer that a positive measure of buyer types accept instantly. During the impasse, the seller “drives a hard bargain,” making offers that would only be accepted by a measure 0 set of buyers types who have not already been ruled out by prior play. (In other words, the seller prices only for the “top of the market”.) Recall the classic durable-good monopolist analogy for repeated bargaining, where one thinks of the seller offering a divisible durable good to a demand curve $q = 1 - F(p)$ of buyers, instead of offering an indivisible good to a single buyer with private type distributed according to $F$. In terms of this analogy, one can describe the starkly different trading dynamics that co-exist in equilibrium as jumping down the demand curve and walking down the demand curve. When the
seller jumps down the demand curve (pictured in red in Figure 1), it drives an easy bargain in the sense that the jumps give away triangles of consumer surplus to the buyers. When the seller walks down the demand curve (moving along the blue arrows), it drives a hard bargain in the sense that only the top of the market would accept the going price at any given time.

This paper uses a canonical dynamic bargaining setup that follows Bellow (1982) and Stokey (1981): there is one-sided incomplete information, and the uninformed party (to fix ideas, let it be the seller) makes repeated offers in discrete time with exponential discounting. In particular, I build on the literature, started by Fudenberg et al. (1985) and Gul et al. (1986), that studies what happens as the seller loses all commitment power not to renegotiate price offers. Most often this is modeled by looking at the limit as the time between successive offers vanishes, whereas I model the bargaining directly in continuous time, i.e., the seller can propose a different price every instant. The literature overwhelmingly focuses on Markovian or stationary equilibria, where continuation play only conditions on a sufficient statistic for the seller’s beliefs. In continuous-type models, the seller’s beliefs after any history are typically truncations of the prior (I describe this “skimming property” below in more detail) so the sufficient statistic is the truncation cutoff. I follow that approach here and study only Markovian strategies with the cutoff as state variable. A special concern in this literature is whether a Coase Conjecture (Coase, 1972) holds. The conjecture was initially taken to mean that trade will be efficient and happen in the first instant, but as I discuss below, newer models have produced a different interpretation of the conjecture.

This paper is especially in line with recent models that enrich the canonical setup with different kinds of arrivals—arrivals either of news or of other players (Inderst, 2008; Fuchs and Skrzypacz, 2010; Daley and Green, 2017). The stage game here follows closely that of Fuchs and Skrzypacz (2010). They study a discrete time bargaining game where some event can arrive at a constant, exogenous rate. The arrival of the event produces some reduced-form payoffs for the buyer and the seller as a function of the buyer’s type, so that different functional forms can model the arrival of new traders or of new information. The authors look at “atomless” limits of discrete-time Markovian equilibria, where the probability of trade in each period shrinks as at the same rate as the delay between periods. Therefore, in the limit trade happens slowly, but a Coase Conjecture still holds in the sense that the seller’s payoffs are pinned down to its outside option of waiting for the exogenous event. In Inderst (2008), the seller can only interact with newly

\[2\] But see Ausubel and Deneckere (1989a) for notable exceptions.
arriving buyers sequentially, so the payoff from terminating the bargaining interaction with the current buyer is independent of that buyer’s type. Unlike [Fuchs and Skrzypacz (2010)], [Inderst (2008)] finds trading dynamics that converge to instantaneous efficient trade with the first buyer as the interval between offers shrinks.

The differences between [Fuchs and Skrzypacz (2010)], [Inderst (2008)], and this paper can be seen by looking again at Figure 1. A durable good monopolist would always want to move along the blue directed curve, so as not to give up grey triangles of consumer surplus. Absent other considerations, the seller prefers going down that curve as fast as possible, to avoid delay costs. In continuous time the seller always has the choice of going down the blue curve faster. In particular, if there is no entry, the seller will go down that curve at infinite speed; but this means that prices as a function of time become flat at 0 in the limit, resulting in the classical Coase Conjecture outcome. When, as in [Fuchs and Skrzypacz (2010)], exogenous entry gives the seller an outside option payoff that increases in the belief truncation, the seller will want to slow down as it traverses the blue curve: belief truncations are high when not much trade has happened before, so delaying trade increases the seller’s outside option. When, as in [Inderst (2008)], exogenous entry give the seller an outside option payoff that is independent of the belief truncation, then the seller’s incentives to speed down the blue curve are much as they were in the case with no entry: nothing motivates it to slow down, so it flashes down the curve, leading to instant trade. When, as in the present paper, endogenous entry gives the seller an outside option payoff that sometimes increases, sometimes decreases in the belief truncation, the seller will sometimes want to move slowly down the curve, sometimes move very fast down the curve. Therefore, equilibria will feature both smooth trade and jumps.

[Daley and Green (2017)] focus on the exogenous arrival of news above the informed type, whereas I focus on the endogenous arrival of other traders, but this paper shares their approach to formulating the canonical dynamic bargaining model directly in continuous time. They focus on the case where the informed party is either high or low, values are interdependent so there is a lemons problem, and both parties observe Brownian news that gradually reveals the informed party’s type. My model has neither a lemons problem nor a news process, but I rely closely on their definition of equilibria in order to avoid the non-existence and circularity issues that are often present in continuous time game-theoretic models (see, for instance, [Simon and Stinchcombe (1989)]). They focus on a class of Markovian equilibria where

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3 Fuchs and Skrzypacz (2010) call this “speeding up the clock.”
trade is smooth until a random time at which the uninformed party “buys
the market” and trades with both informed types. The authors define a
modified Coase Conjecture result: the perfect lack of commitment by the
uninformed party implies that its payoffs are exactly the same as if it were
unable to make offers at all and could only choose a random time at which
to buy the market. That is, without commitment power (as is the case when
offers happen in continuous time) the uninformed party does not benefit
from the ability to screen going forward. This form of the Coase Conjecture
is in line with [Fuchs and Skrzypacz (2010)], where a seller with no ability to
screen through price offers would obtain the outside option of waiting for the
buyer to arrive. As I show below, this form of the Coase Conjecture does not
always hold in my model. It will hold for initial states (beliefs truncations)
at which there is smooth trade in the equilibrium with screening. For states
which trade happens in atoms, however, the seller does strictly better than
the outside option at that state, which is exactly what it would receive if it
were unable to make screening offers.

To my knowledge, very few works have focused on the issue of endogenous
entry in dynamic models with incomplete information. [Zryumov (2015)]
is a notable exception. The author analyzes a dynamic lemons model where
the uninformed side of the market is competitive and makes all the offers
(so the offers are pinned down by a zero-profit condition). There are good
and bad entrepreneurs, and bad entrepreneurs can strategically choose when
to enter the market for funding. In particular, [Zryumov (2015)] shows that
trade flows can respond discontinuously to changes in market conditions.
This is reminiscent of the bursts of trade in my model, and it suggests that
discontinuous dynamics may be a more general feature of endogenous entry
models.

Finally, this work related to a recent discrete-time literature on trans-
parency in bargaining. In [Fuchs et al. (2016)], informed sellers face a sequence
of uninformed short-term buyers, one per period, with two periods. Buyers
make all the offers. The authors introduce interdependent values, so buyers
face a lemons problem. [Kaya and Liu (2015)] study the case with independent
values and many periods. In both these works, making the price offers pri-
ivate speeds up trade and can help efficiency. By contrast, in my model both
sides are long-lived, and I find that making offers private can delay trade for
a positive measure of types.

Section 1 presents the model and the equilibrium concept. Writing down
the strategic interaction directly in continuous time greatly simplifies the
analysis, but it requires me to introduce some technicalities and use an equi-
librium concept that, while not being fully Nash, captures the key features of
a discrete time Perfect Bayes analysis. Section 2 presents the main result. In
particular, Section \[2.2.1\] presents the exogenous entry benchmark, and it discusses the novel aspect of the dynamics relative to that benchmark. Section 3 describes the implications for trading dynamics of making prices private (hiding them from entrants). Section \[4\] concludes.

1 Model

1.1 Stage Game and Entry Dynamics

There is a long-lived seller \( S \) and a long-lived buyer \( B \) bargaining over an indivisible asset owned by \( S \). They face a sequence of possible short-lived second buyers (\( E \), for “entrant”) who observe the course of negotiations and can choose to enter and trigger a bidding war. \( B \)’s willingness to pay for the good is a privately known type \( v^B \sim F[0, 1] \). Each entrant is characterized by a private cost of entry \( c \sim G[0, \bar{c}] \), which when paid also reveals \( E \)’s value for the asset \( v^E \sim F[0, 1] \). The entrant privately observes \( v^E \). The \( c \)’s and \( v^E \)'s are independent across entrants, and they are independent of \( v^B \). \( F \) and \( G \) are absolutely continuous, with full support, continuous densities \( f \) and \( g \), respectively.

Time is continuous, with an infinite horizon. A heuristic description of the stage game happening within each \( dt \) is as follows. At each \( t \) before the arrival of an entrant, the seller offers a price \( P_t \) to \( B \), who can accept or reject. If \( B \) accepts, the game ends, but if \( B \) rejects, one of three things can happen. First, \( E \) can arrive. If no entrant arrives, bargaining between \( S \) and \( B \) continues, with \( S \) making another offer. If an entrant \( E \) arrives and
then chooses to pay its entry cost $c$, it learns its value $v^E$, a bidding war breaks out between $E$ and $B$ (formally, there is an ascending auction with no reserve price), and the game ends. If $E$ does not enter, it leaves the game and bargaining between $S$ and $B$ continues as though $E$ had never arrived.

If $S$ and $B$ agree at time $t$ on a price $p$ before entry occurs, they receive payoffs $e^{-rt}p$ and $e^{-rt}(v^B - p)$, while any entrants that may have arrived but not entered receive 0. Let $\Pi(v^B) = \int_{v^B}^{\infty} \min\{x, v^E\} dF(x)$ and $w(v^B) = E_{v^E \sim F}[(v^B - v^E)_+]$ be $S$ and $B$’s expected payoffs from the bidding war (i.e., the ascending auction) when $B$ has type $v^B$. If there is entry at time $t$ and a bidding war is triggered, $S$ receives $e^{-rt}\Pi(v^B)$ (in expectation over $v^E$), $B$ receives $e^{-rt}w(v^B)$ (also in expectation over $v^E$), and $E$ receives $(v^E - v^B)_+ - c$.

For future reference, denote by $\Pi(k) = E[\Pi(v^B)|v^B \leq k]$ the seller’s expected profit from the bidding war if it knows that $v^B \leq k$.

Entrants arrive to the market according to a Poisson process with constant rate $\lambda_0$, which is independent of $v^B$, and of all draws of $c$ and $v^E$. Let $\mathcal{A} = (\mathcal{A}_t)_{t \geq 0}$ be the filtration for this arrival process on some sufficiently rich probability space. Let $\tilde{c}(\mu^E) = \int_0^1 \int (v - \tilde{v})_+ d\mu^E(\tilde{v}) dF(v)$ be $E$’s expected surplus in the bidding war when its belief is about $v^B = \mu^E$. Then an entrant who upon arrival has belief $\mu^E$ about $B$’s type will enter iff $c \leq \tilde{c}(\mu^E)$. As discussed in more detail below, the game satisfies a skimming property (higher $v^B$’s accept first), so that after every history, conditional on no entry, $E$ and $S$ beliefs about $v^B$ are truncations of $F$. Let $K_t$ denote the point of truncation at $t$. Then, by standard results on sampling Poisson processes and abusing notation for $\tilde{c}$ slightly, the game ends at a stopping time $\sigma$ distributed according to a Poisson process with rate $\Lambda(K_t) = \lambda_0 G(\tilde{c}(K_t))$. Hence, if beliefs jump after a rejection at $t$, then the intensity of entry induced corresponds to the beliefs after the jump. For future reference, let $D(k) \equiv \frac{\Lambda(k)}{\Lambda(k) + r}$ denote the present value of a dollar at the time of entry, when the current state is $k$. Figure 2 illustrates this heuristic “stage game” being played within each infinitesimal period of time $dt$.

Finally, let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space satisfying the “usual conditions”\footnote{The space is complete and $\mathcal{F}$ is complete and right-continuous.} (Harrison 2013, p. 172), with $\mathcal{F}$ independent of all other random elements in the model ($v^B$, $c$’s and $v^E$’s, and the Poisson process governing arrivals). Here $\mathcal{F}_t$ is a public correlation device introduced purely for technical convenience, to be able to define continuous-time strategies consistently.
1.2 Equilibrium Concept

1.2.1 Discussion

Making formal the heuristic stage game I described above requires an ad hoc equilibrium concept that avoids several well-known pitfalls in continuous time game-theoretic modeling (Simon and Stinchcombe, 1989). My equilibrium notion follows Daley and Green (2017), adapted to the current setting. In essence, the equilibrium notion captures the following basic intuitions from the typical discrete-time formulation in the Coase-Conjecture literature:

- Buyers solve an optimal stopping problem: conditional on no entry, when should they accept the offer and stop the bargaining process?

- The equilibrium satisfies a “skimming property”: Higher types of $B$ accept earlier than lower types, so $S$ and $E$’s posterior beliefs about $v^B$ after any history are a truncation of the prior.\(^5\)

- Given the above, the current truncation (hereafter, the “cutoff type”) forms a natural state variable for the game, and the literature focuses on Markovian or stationary strategies, where the seller’s price offers at any point depend only on the current cutoff.

- Since any given price history induces a history of realized cutoff types, along the equilibrium path the seller can be thought of as choosing its own future beliefs as a function of its current beliefs.

\(^5\) Standard arguments, which I omit, show that when $v - W(v)$ is increasing (as is the case here) the buyer’s payoffs satisfy a single-crossing property between time until acceptance and type.
Aside from technical minutia, the main difference between a discrete time formulation (even one that considers the limit as the delay between periods goes to zero) and a continuous time formulation is that, in the latter, the seller solves an “impulse control problem” ([Harrison] 2013). In discrete time, in every period the seller is always choosing a mass of $B$ types to trade with. While these masses may differ in size across periods, there is no qualitative difference between them. Modeling the seller’s choice as an impulse control over beliefs means that the seller can now engage in fundamentally different kinds of trading: periods (instants) during which the probability mass of trade is infinitesimal, so that $S$ screens through types of $B$ slowly, and periods during there are atoms of trade, so that $S$ screens through a chunk of $v^B$’s infinitely fast.

1.2.2 Formalities

**Definition 1.** A stationary equilibrium consists of $\{K_t\}_{t \geq 0}, (\tau^v)_{v \in [0,1]}, P$ such that

1. $K_t$ is time-homogenous, Markov with respect to $\mathcal{F}_t$, and offers are given by a fixed function $P(k)$.

2. The time of entry $\sigma$ follows a Poisson process adapted to $\mathcal{F}_t \otimes \mathcal{A}_{\tau}$ with intensity $\Lambda(K_t) = \lambda_0 G(\tilde{c}(K_t))$.

3. $v^B$, taking $P$ and the law of motion for $K_t$ as given, solves

$$J_B^v(k) = \sup_{\tau \in \mathcal{T}} \mathbb{E}_k^v \left[ 1_{\tau > \tau^v} e^{-r\tau} (v^B - P(K_\tau)) + 1_{\tau \leq \tau^v} e^{-r\tau} w(v^B) \right]$$  

(1)

where $\mathbb{E}_k^v$ is the expectation operator with respect to the law of $K_t$ and $\sigma$ given in points 1. and 2., conditional on $K_t = k$ and $v^B = v$.

4. For $t < \sigma$, $K_t = \int 1_{\tau^v \geq t} dv$.

5. $S$ takes $P(k) = k - J_B^k(k)$ as given, and $K_t$ solves

$$J_S(k) = \sup_{Q \in \Gamma} \mathbb{E}_Q^k \left[ \int_0^\sigma e^{-rt} P(Q_t)dQ_t + e^{-r\sigma} \Pi(Q_\sigma) \right]$$  

(2)

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6 From Condition 1, at any time $t$ $v^B$’s payoffs going forward depend only on $\{K_{t+s}, s \geq 0\}$, and the law of motion for $\{K_{t+s}, s \geq 0\}$ depends only on the current state $K_t = k$. Therefore, $B$’s value is a function of depends on $\mathcal{F}_t$ only through $K_t = k$.

7 From the discussion in [6], conditional on no entry, $S$’s payoffs going forward depend on $\mathcal{F}_t$ only through $Q_t$, which it chooses taking $Q_t = k$ as given. Similarly, its payoffs in the case of entry depend only on $\mathcal{F}_t$ only through $\sigma$, but this stopping time itself depends on $\mathcal{F}_t$ only through $\{Q_t, t \geq 0\}$. Therefore, $S$’s value function depends on $\mathcal{F}_t$ only through the current state $K_t = k$. 

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where $\Gamma$ is the set of decreasing process on the support of $\nu^B$ adapted to $\mathcal{F}_t$, and $E^Q_k$ is the expectation with respect to the law of $\sigma$ induced by $K = Q = \{Q_t, t \geq 0\}$ according to point 2., conditional on initial state $K_0 = k$.

In words, prices depend only on $S$'s beliefs, these prices “skim” buyer types from the top down, $S$ solves an impulse control problem in its own belief, and this belief must be consistent with the buyer types who actually accept. I focus on a subset of stationary equilibria that where the seller alternatives between sufficiently smooth periods of hard bargains and few periods of easy bargains.

**Definition 2.** A stationary equilibrium $(\{K_t\}_{t \geq 0}, (\tau^v)_{v \in [0,1]}, P)$ is regular if $K_t = K_t^{ac} + K_t^J$, where $K_t^{ac}$ is absolutely continuous in $t$, and $K_t^J$ is a step function with finitely many jumps.

Since $K_t$ is monotone, it has a Lebesque decomposition. This already implies that $K_t = K_t^{ac} + K_t^J + K_t^s$, where $K_t^J$ is a piecewise constant jump function and $K_t^s$ is a singular continuous function (i.e., a continuous function whose points of increase are measure zero). Therefore, the additional content of this restriction on equilibria is twofold:

- the jumps in $K_t$ are rare (the seller drives an easy bargain infrequently).
- the continuous part of $K_t$ is sufficiently smooth: when the seller is driving a hard bargain, the buyer can actually see the price dropping gently over time, rather than $K_t$ only moving in “flashes.”

Regular equilibria do rule out potential dynamics, but in a model without Brownian noise, these dynamics are seem pathological and an artifice of the continuous time formulation.

**Remark 1.** In the no-gap case, the filtration $\mathcal{F}_t$ is a technical aid to avoid the usual circularity and non-existence issues that plague continuous time models of strategic interaction (Simon and Stinchcombe 1989). In the gap case (in progress), the additional randomness introduced by $\mathcal{F}_t$ serves as a public correlation device that the seller can use to implement quiet periods of random length.

This is in contrast to, say, Daley and Green (2017) where there is Brownian news and two types, and singular trading dynamics correspond to certain kinds of mixing by the informed party. The dynamics in the continuous limit of Deneckere and Liang (2006) are not regular according to this definition, and they are generated by strategies that are not Markovian in the limit.
2 Endogenous Entry and Trading Dynamics

In the following two sections, I show how the trading dynamics are largely determined by the hypothetical price path that would have obtained if the equilibrium consisted only of a single smooth trade region (what Fuchs and Skrzypacz [2010] call “atomless equilibria”).

2.1 Derivation of Smooth Trade Locus

By assumption, the equilibrium consists of at most finitely many jumps, so there exist regions with smooth trading where the probability of leaving that region in the next $dt$ is negligible. In any such region, the seller’s payoffs must evolve according to

$$r J_S(k) = \sup_{K \in (0, \infty)} \begin{cases} \text{flow payoff from offer accepted offers} & (P(k) - J_S(k)) \frac{f(k)}{F(k)} (\dot{K}(k)) + \Lambda(k)[\Pi(k) - J_S(k)] \\ \text{arrival of entrant, game ends} & + \frac{J'_S(k)(-\dot{K}(k))}{\Lambda(k) + r} \end{cases}$$

(3)

The seller can choose $\dot{K} \in \mathbb{R}_+ \cup \{\infty\}$ freely, and this variable enters (3) linearly. Therefore, if the coefficients on $\dot{K}$ did not add up to exactly 0, so that $S$ was not indifferent over all possible speeds, it would choose either $\dot{K} = \infty$ (when the coefficients add up to something positive) or $\dot{K} = 0$ (when the coefficients add up to something negative). Setting the coefficients on $\dot{K}$ to zero in (3), for any $k$ in the interior of in a smooth trading region of any candidate equilibrium,

$$J_S(k) = \frac{\Lambda(k)}{\Lambda(k) + r} \Pi(k)$$

and

$$P(k) = J_S(k) + F(k)J'_S(k)/f(k)$$

(4)

The following lemma re-expresses smooth-trade prices in a useful way that is helpful for further calculations and points to a key difference between my setup and one with exogenous entry. Note first that $D(k)\pi(k)$ is the value to the seller at state $k$ of waiting for entry indefinitely if $B$’s type is in fact $k$.

Lemma 1. On a smooth trade region of equilibrium, the seller experiences ex post regret. That is, the price $S$ obtains by trading with a given $\nu^B$ is lower
that the value of waiting for entry forever if both \(E\) and \(S\) knew \(B\)’s type:

\[
P(k) = D(k)\pi(k) + D'(k)\Pi(k)F(k)/f(k) < D(k)\pi(k).
\]

(5)

The brief proof, which I omit, plugs \(JS\) from (4) into the \(P\) expression in (4).

Lemma 1 says that the seller loses money (relative to the outside option) on every sale and after selling at a given speed would have rather slowed down trade. This is not a contradiction—\(S\) is willing to face this regret because by “losing money” on the marginal type \(k\) at state \(k\), it exposes all all inframarginal types below \(k\) to more entry, which increases the profits coming from those types. This contrasts to the result in Fuchs and Skrzypacz (2010), where entry is exogenous with a constant Poisson rate \(\lambda\), and in any atomless limit of their equilibria prices satisfy “no ex-post regret” property.

Smooth trade prices are completely determined by the seller’s indifference between speeds of trade. Correspondingly, the speed of trade (when trade is smooth) is completely determined by the buyer \(B\)’s indifference between accepting and rejecting the smooth-trade price. For a state \(k\) in the interior of a smooth trade region of equilibrium, the initial buyer’s payoffs satisfy the following HJB:

\[
r.J_B^{'k}(k) = \Lambda(k)(W(k) - J_B^k(k)) + J_B^{'k}(k)(-\dot{K}(k))
\]

(6)

where \(J_B^{'k}(k)\) is short-hand for \(\frac{\partial}{\partial k}J_B^k(k)\big|_{v=k}\). Unlike the seller’s HJB in (3), the HJB for buyer \(k\) at state \(k\) has no term corresponding to the flow payoffs in (6). This term would consist of the difference between the stopping payoff from accepting \(P(k)\) and the continuation payoff from rejecting it, but by the definition of equilibrium, the price \(P(k)\) makes \(k\) just indifferent between accepting and rejecting.

Differentiating both sides of the equilibrium condition \(J_B^k(k) = k - P(k)\) with respect to the state \(k\) (as opposed to the type \(v^B\) that happens to equal \(k\)) yields

\[
J_B^{'k}(k) = -P'(k)
\]

Plugging that into the buyer’s HJB in (6) and solving for \(\dot{K}\), one finds that

\[
\dot{K} = \frac{D(k)W(k) - k + P(k)}{-P'(k)}
\]

(7)

must be the speed of trade in the interior of a smooth trading region of any candidate equilibrium.
Definition 3. Let \( \hat{P}, \hat{K}, \) and \( \hat{J}_S \) denote functions obtained by naively extending (5) and (7) to the entire support of \( v^B \). Then \( (\hat{P}, \hat{K}, \hat{J}_S) \) is the smooth trade locus.

2.2 Characterization of Trading Dynamics

A truly minimal requirement for the speed of trade (7) to form part of an actual equilibrium is that it be non-negative. Plugging the expression for smooth-trade prices in (5) into the numerator of the smooth-trade speed of trade yields

\[
D(k)[W(k) + \pi(k)] - k + D'(k)F(k)/f(k) < 0,
\]

since \( W(k) + \pi(k) \) is at most \( k \). The expression in (7) will therefore be positive if and only if \( \hat{P} \) is strictly increasing at \( k \).

If \( \hat{P} \) were strictly increasing, then (4) and (7) could be used to construct an equilibrium where trade between \( B \) and \( S \) always happens smoothly along the path of play. However, even in highly regular cases, \( \hat{P} \) can have strictly decreasing regions, which are incompatible with smooth trade. Figure 3 below plots \( \hat{P} \) and \( \hat{J}_S \) for the case where \( F \) and \( G \) are both \( U[0,1] \). \( \lambda \) and \( r \) affect these loci only through \( \gamma = \frac{\lambda}{r} \), the relative intensity of arrival. Dotted lines trace \( \hat{J}_S \), and solid lines trace \( \hat{P} \). The figure shows that both \( \hat{P} \) and \( \hat{J}_S \) for this example are strictly single-peaked at an interior point. As the relative intensity of arrival increases, \( \hat{J}_S \) becomes more likely to be only increasing. And yet, the decreasing region of \( \hat{P} \) becomes more and more pronounced as \( \gamma \) increases.

Having more optimistic beliefs about \( B \)’s value (in the form of a higher \( k \)) makes entry more valuable to \( S \) when it finally happens. Since \( k \) is high when not much trade has happened between \( B \) and \( S \), delays in bargaining make entry more valuable to the seller; if in addition the likelihood of entry is unaffected by this delay (as when entry is exogenous), then the seller will have a strong incentive, relative to the case without entry, to slow down the rate of trade. In the present case, however, the seller’s beliefs are also the entrant’s beliefs, and the entrant prefers facing low \( v^B \); thus entry is less likely, happens later, and is more discounted by the seller, when \( k \) is high. Overall, since higher \( k \)’s make entry more valuable but less likely, the seller’s outside option could sometimes be increasing and sometimes decreasing in \( k \), and the possibility of entry will sometimes push \( S \) to trade slower and sometimes push \( S \) to trade faster than it would have in the absence of entry.

Therefore, since trade is instantaneous in the absence of entry, intuition would suggest that the seller will want to trade in bursts in regions where the outside option is decreasing. Consider, then, the seller’s payoffs from causing a downwards jump in \( K_t \). The price that induces a jump in \( K_t \) down
to \( k' \) must be precisely the price that leaves the buyer with type \( v^B = k' \) indifferent between accepting and rejecting, i.e., \( \hat{P}(k') = k' - J_S^{k'}(k') \). Hence, the seller’s payoff to jumping from a state \( k \) to a state \( k' < k \) at which smooth trade commences is

\[
U(k, k') = \left( 1 - \frac{F(k')}{F(k)} \right) \hat{P}(k') + \frac{F(k')}{F(k)} \hat{J}_S(k')
\]

(8)

\( U \)'s behavior significantly pares down the possible regular equilibria. On the one hand, if there is smooth trade at \( k \) along the equilibrium path, it must be that \( k \in \arg\max_{k \in [0,k]} U(k, \tilde{k}) \)—otherwise, the seller would want to jump down to some \( k' < k \) and trade smoothly from there instead. On the other hand, recall that, within the class of regular equilibria, all jumps in \( K \) happen from the smooth trade locus and to the smooth trade locus. Therefore, if there is a jump at \( k \) to \( k' < k \) along the equilibrium path, it must be that \( k' \in \arg\max_{k \in [0,k]} U(k, \tilde{k}) \) and \( k' \in \arg\max_{k' \in [0,k']} U(k', \tilde{k}') \)—otherwise, either the seller would not want to jump to \( k' \), or once there would want to jump again. These kinds of considerations lead to the main result:

**Theorem 1 (Trade Dynamics).** In any candidate equilibrium under endogenous entry,

1. Trade can happen smoothly in states \( k \) where \( \hat{P}(k) \) is strictly increasing.
2. Trade happens in bursts at any state \( k \) where \( \hat{P}(k) \) is strictly decreasing.

3. At any such \( k \), the state jumps to a local maximum of \( \hat{P} \) to the left of \( k \).

In smooth trading regions, trade happens at a positive but finite rate

\[
\dot{K} = \frac{D(k)W(k) - k + \hat{P}(k)}{-\hat{P}'(k)}
\]

The proof, which I present in the appendix, relies solely on local optimality arguments, the key for which is the following expression:

\[
U_2(k, k') = \left(1 - \frac{F(k')}{F(k)}\right) \hat{P}'(k').
\]

In words, \( U_2(k, k') \) is proportional to \( \hat{P}'(k') \) for any \( k \), which implies that the local maxima and local minima of \( k' \mapsto \hat{P}(k') \) and \( k' \mapsto U(k, k') \) will coincide for the sub-domain \([0, k)\).\(^9\)

Therefore, if \( \hat{P} \) is single-peaked, as in the uniform-uniform example from Figure 3, there is a burst of trade at the first instant of bargaining, followed by a long “impasse.” During the impasse, the seller drives a hard bargain, making offers that are only a measure 0 set of remaining buyers (i.e., the cutoff buyer) would accept.

2.2.1 Discussion and Comparison to Exogenous Entry

The novel aspect of Theorem 1 is that two very different kinds of trading dynamics co-exist in equilibrium: slow, gradual trading, interspersed with burst of trade of instantaneous trade. The latter bears some resemblance to the classical Coase-conjecture result without entry, where as the delay between periods shrinks, trade happens infinitely fast in the first instant at the seller’s reservation value (e.g., Fudenberg et al. (1985) and Gul et al. (1986)). Here, there are instants with atoms of trade (trade at infinite speed) at “bargain” prices, but the atoms have mass smaller than 1, so that there is some equilibrium delay. The prices are not equal to the seller’s cost, but they are still an “easy bargain” in the sense that a measure 1 of the buyer’s who accept them get strictly more than their continuation value. These bursts

\(^9\) For instance, if \( k' \) is a local minimum of \( \hat{P} \), then \( \hat{P}' \) goes from negative to positive in the neighborhood around \( k' \). This sign change is transferred to \( U_2(k, k') \), which means \( U(k, k') \) is also at a local minimum.
of trade also differ from the Coase Conjecture benchmark in that they can happen after some delay—for instance, if $\hat{P}$ has a trough in $(0, 1)$.

These jump dynamics come about entirely because of the endogeneity of entry. Neither entry itself, nor the fact that entry is time varying here, could generate this pattern of trade. Following the same steps as in Section 2.1 one can rule out jumps within the class of regular equilibria.

To see this, consider candidate regular equilibria where in the model where entry happens according to an exogenous inhomogenous Poisson process with strictly positive rate $\lambda_t$. Expanding the equilibrium notion to this case is straightforward, the main difference being that strategies, and by extension value functions, now depend on both $K_t$ and calendar time. I omit the formalities since they are very similar to the endogenous entry case. This modification yields the formal result:

**Theorem 2.** Fix any exogenous and strictly positive entry rate $(\lambda_t)_{t \geq 0}$. Then all regular equilibria have smooth trade at all states $(k, t)$.

The only reason $S$ induced jumps under endogenous entry was because of the effect this had on its outside option and the effects that the changed outside option had on its future payoffs. When entry was endogenous, sometimes the outside option payoffs by accelerating trade. Faster previous trade (resulting in a lower current $K_t$) made entry less valuable when it finally happened, but also made it happen faster; the second force sometimes dominated, making the outside option rise with faster previous trade. When entry is exogenous, the seller’s actions cannot affect the speed of entry, and can so accelerating trade today can only decrease the value of eventual entry. The only force leading $S$ to accelerate trade vanishes, so that $S$ never wants to create jumps in $K_t$.

**Proof of Theorem 2.** Suppose, by way of contradiction, that a regular equilibrium under exogenous entry features jumps. Since regular equilibria have only finitely many jumps, there must be a final one. Say this last jump happens at time $\bar{t}$, and goes from $\bar{k}$ to $k \in (0, k)$. (The endpoint $\bar{k}$ must be strictly greater than 0, since $v^B = 0$ would only accept a price of 0 and the seller can do better than that by trading smoothly.) Since the equilibrium is regular and there are no jumps after arriving at $(\bar{k}, \bar{t})$, there has to be smooth trade until the end of the game.

By identical arguments to the ones in section 2.1 the seller’s payoff must satisfy the HJB below in this final smooth-trading region:

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10Essentially, remove requirement 2. in Definition 1, add calendar time to the state space, and make strategies meetable with respect to $\mathcal{F}_t \times \mathcal{B}[0, t)$, where $\mathcal{B}[0, t)$ is the Borel $\sigma$-algebra on $[0, t)$.
where $J^l_l, l \in \{k, t\}$ is shorthand for $\frac{\partial J_S(k,t)}{\partial l}$. Continuing as in section 2.1, all terms with $\dot{K}$ must vanish from (10), so $J_S(k,t)$ must satisfy the differential equation

$$rJ_S(k,t) = \lambda_t [\Pi(k) - J_S(k,t)] + J^{(t)}_S(k,t)$$

(11)

(with boundary conditions to be determined).

In particular, $S$ is equally well off as if it chose $\dot{K} = 0$ for all $k \leq k^\bar{t}, t \geq \bar{t}$.

So instead of solving (11) directly, one can use the fact that, with $\dot{K} = 0$, $S$ is not trading, and its payoff must equal the outside option of waiting for an entrant:

$$J_S(k,t) = \hat{J}_S(k,t) \equiv \left[ \int_t^\infty \lambda_s e^{-rs + \int_t^s \nu d\nu} d\nu \right] \Pi(k), k \leq k, t \geq \bar{t}$$

(12)

This expression suffices to determine the smooth-trade locus and derive the contradiction. Let $D(t)$ equal the coefficient on $\Pi(k)$ in the above display. The same manipulations as in (5) imply a smooth trade locus satisfying (superscript “X” for “eXogenous“):

$$\hat{P}_X(k,t) = \hat{J}_S(k,t) + \hat{J}^{(k)}_S(k,t) F(k) = D(t) \pi(k).$$

(13)

and corresponding jump payoffs:

$$U_X(k,k', t) = \left( 1 - \frac{F(k')}{F(k)} \right) \hat{P}_X(k', t) + \frac{F(k')}{F(k)} \hat{J}_S(k', t)$$

Instead of jumping from from $\bar{k}$ to $k$, the seller could have jumped either closer ($k' > k$) or farther ($k' < k$) before resuming smooth trade. A necessary condition for this final jump to have been optimal is therefore

$$U_2^X(\bar{k}, k, t) = \left( 1 - \frac{F(k)}{F(k)} \right) \hat{P}_1^X(k, t) = 0,$$

The argument here generalizes Lemma 6 in , making use of the continuous time formulation to obtain formal optimality conditions that are easier to manipulate.
where the simplified expression for $U^X_2$ follows by identical arguments to those in the proof of Theorem 1. However, since $π(k)$ is strictly increasing, it is clear from (13) that $\hat{P}^X(k, t)$ must be strictly increasing in $k$, and $U_2^X(\bar{k}, k, t) > 0$. A final jump to $\bar{k}$ cannot be optimal for any $\bar{k} < \bar{k}$, and the result follows.

3 Effects of Transparency

I have assumed throughout that entrants observe the entire history of offers and rejections. However, if entry is indeed endogenous and $B$ and $S$ know that, they will have induced preferences over the transparency of their bargaining history. This transparency is presumably under their (joint) control. Will making the offers private speed up trade, or will it slow it down? Will trade still have the same stylized dynamics under private offers, with bursts alternating with long impasses? Who is made better or worse off by private offers, and to what extent is there a conflict of interest between $S$ and $B$ over the transparency of their bargaining? One might guess, for instance, that $S$ wants its rejected offer to be public if the offer were low (in which case its rejection is a sign of $B$’s weakness, and $E$ is likely to enter), but not if the offer were high, whereas $B$, who presumably wants to dissuade entry, would have the opposite reasoning.

First, I modify the notion of equilibrium to account for the fact that entrants do not observe price offers.

Definition 4. A (pure) stationary equilibrium with private offers consists of $((K_t)_{t\geq0}, (\tau^v)_{v\in[0,1]}, (\lambda_t)_{t\geq0}, P)$ such that

1. $K_t$ is time-homogenous, Markov with respect to $\mathcal{F}_t$, and offers are given by a fixed function $P(k)$.

2. The time of entry $\sigma$ follows a Poisson process adapted to $\mathcal{A}_t$ with intensity $\lambda_t$.

3. $v^B$, taking $P$ and the law of motion for $K_t$ as given, solves

$$J_B^v(k) = \sup_{\tau \in T} \mathbb{E}^B_k \left[ 1_{\sigma > \tau} e^{-rt} (v^B - P(K_\tau)) + 1_{\sigma \leq \tau} e^{-r\sigma} w(v^B) \right]$$

where $\mathbb{E}^v_k$ is the expectation with respect to the law of $K_t$ and $\sigma$ given in points 1. and 2., conditional on $K_0 = k$ and $v^B = v$.

4. For $t < \sigma$, $K_t = \int 1_{\tau^s \geq t} dv$. 

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5. $S$ takes $P(k) = k - J_B^k(k)$ as given, and $K_t$ solves

$$J_S(k) = \sup_{Q \in \Gamma} E_k^\lambda \left[ \int_0^\sigma e^{-rt} P(Q_t) dQ_t + e^{-r\sigma} \Pi(Q_\sigma) \right]$$

where $\Gamma$ is the set of decreasing process on the support of $v^B$ adapted to $\mathcal{F}_t$, and $E_k^\lambda$ is the expectation with respect to the law of $\sigma$ given in point 2., conditional on initial state $K_t = k$.

6. $\lambda_t$ satisfies

$$\lambda_t = \lambda_0 G(\tilde{c}(K_t)) \quad (14)$$

The key difference between this definition and Definition 1 with public offers is the law of motion for $\sigma$. In point 5., the seller now takes that law as given. This law depends on the entrants’ beliefs about the path of cutoffs $(K_t)_{t \geq 0}$, which the seller cannot affect. Suppose, for instance, the entrants believe prices leading to a particular path $(K_t)_{t \geq 0}$ are being played. Since they cannot see the particular offers that were made, if the seller deviates to some other $(\tilde{K}_t)_{t \geq 0}$ by making different offers, the entrants’ beliefs at $t^*$ when the game has not ended will still be given by $K_{t^*}$. Nonetheless, in equilibrium the entrants’ beliefs about $v^B$ must be consistent with $S$ and $B$’s choices. Given the focus on pure strategies for the seller, this means that their entry rate must be equal what it would have been if they knew $K_t$, which is exactly (14).

Theorem 2 already has strong implications for the possible dynamics under private offers.

**Corollary 1.** Regular equilibria under private offers have only smooth trade.

**Proof.** Consider $S$’s best response to a possible belief path by the entrants. Since entrants cannot observe the particular offers, their beliefs can only depend on the calendar time since the beginning of the game. Therefore, any belief path for $E$ will generate some time-varying rate of entry $(\tilde{\lambda}_t)_{t \geq 0}$. By the assumptions on $G$, $\tilde{\lambda}_t$ must be strictly positive at all times. Therefore, by Theorem 2, $S$’s best response to any beliefs of the entrants will not feature any jumps in $K_t$, and in consequence the fixed point in entrant beliefs corresponding to the private offers equilibria cannot have any jumps in $K_t$ either.

One implication of the above is that, in the typical case where $\hat{P}$ (the endogenous entry smooth trade locus) is single-peaked at an interior point, making offers private will slow down trade for all $v^B$'s to the right of $\hat{P}$’s peak.
Saying more about the public-private contrast in the full model would require characterizing the fixed point of entry rates, but this fixed point in continuous time is extremely intractable, since it is characterized by a non-linear functional equation. In particular, with exogenous entry, the smooth trade locus $\dot{K}^X_t$ depends on all of $(\lambda_s)_{s \geq 0}$ through (12).

4 Conclusion

Endogenous entry is an important feature of real-world bargaining, largely unexplored in the literature. I have presented a dynamic bargaining model with this feature, and I have shown that allowing entry to be endogenous, even in a simple way, dramatically alters the dynamics of trade. Modeling the strategic interaction directly in continuous time (as opposed to the limit of discrete time bargaining) greatly simplifies the study of equilibrium dynamics with frequent offers. Even in very standard settings, the equilibrium alternates between phases when the seller and initial buyer will sometimes reach agreement in bursts (atoms of of trade in an instant), and sometimes reach agreement only gradually, after a long impasse that screens the buyers type by type. This is unlike a world with exogenous entry; exogenous entry, even if it is time-varying, cannot generate bursts of agreement. A corollary of the above is that making offers private can delay trading times for a positive measure of buyers.

I have only identified necessary conditions for conditions for equilibrium, and even then only in a restricted class. A next step (in progress) is using these conditions to fully construct an equilibrium via a verification approach. If, for instance, one assumes that $\hat{P}$ is single-peaked, the argument above almost suffices for verifying buyer and seller optimality; more complicated shapes for $\hat{P}$ will require special arguments. As for the restrictions on strategies, there are two possible directions, one feasible and one infeasible. The (probably) infeasible one is to allow for non-stationarity, as this would create major technical complications in a continuous time setting. The more feasible direction is to allow for, perhaps with the goal of ruling them out, singular dynamics, where $K_t$ moves continuously but always at zero or infinite speed. In their model with Brownian news and a two-type buyer, Daley and Green (2017) suggest that the high-contact conditions that characterize optimal singular dynamics (e.g., as is the case with a reflecting boundary) can be used to generate contradictions that rule out these dynamics. Perhaps that approach can be made to work here, too.
5 Appendix

Proof of Theorem 1. First, I derive (9)—the remainder of the proof relies on local optimality arguments from that expression. Let \( A(k, k') = 1 - \frac{F(k')}{F(k)} \) be the probability of immediate acceptance by \( B \) in a jump from \( k \) to \( k' < k \). Then differentiating jump payoffs in (8) with respect to \( k' \),

\[
U_2(k, k') = A_2(k, k') \left[ \hat{P}(k') - \hat{J}_S(k') \right] + (1 - A(k, k')) \hat{J}'_S(k')
\]

\[+ A(k, k') \hat{P}'(k') \quad (15)\]

The result then follows from plugging the expression for \( \hat{P} \) in (4) in the above, and using the identity

\[
A_2(k, k') = -f(k') \frac{F(k')}{F(k') F(k)} = \frac{f(k')}{F(k')} (1 - A(k, k'))
\]

With (9) in hand, I verify points 1-3 of the theorem.

For 1 and 2, note that, since \( k \) is at a corner, there will be smooth trade only if the \( k \) satisfies the first and second order conditions for optimality, i.e., \( U_2(k, k^-) \leq 0, \) and for some \( \varepsilon > 0, \) \( k' \mapsto U_2(k, k') \) is decreasing for all \( k' \in [k - \varepsilon, k] \). The former is true for any \( k \): using (??) and the continuity of \( F, \) \( U_2(k, k^-) = 0. \) The latter also follows from (9). Since \( \hat{P} \) is strictly increasing at \( k \) by assumption, there exists some \( \varepsilon > 0 \) such that \( U_2(k, k') = \left(1 - \frac{F(k')}{F(k)}\right) \hat{P}'(k') > 0 \) for all \( k' \in (k - \varepsilon, k). \) In particular, keeping in mind that \( U_2(k, k^-) = 0, \) one can find a further \( \varepsilon' \) such that \( k' \mapsto U_2(k, k') \) is strictly decreasing in \( k' \) for all \( k' \in (k - \varepsilon', k]. \) A symmetric argument shows that \( k \) is a local minimum of \( k' \mapsto U(k, k') \) when \( \hat{P} \) is decreasing at \( k^- \), which means that there must be a jump at any such \( k. \)

Next I verify point 3. Since jumps must end on the smooth trade locus, and \( U(k, k') \) gives the payoff from jumping onto the smooth trade locus at \( k' \) from \( k, \) then \( k' \in \arg \max_{k \in [0, k)} U(k, k') \) is a necessary condition for an equilibrium jump from \( k \) to \( k'. \) As remarked in the main text, expression (9) implies that \( \hat{P}(k') \) and \( U(k, k') \) have the same local maxima in \( k' \in [0, k). \) This suggests local maxima of \( \hat{P} \) on \( [0, k) \) as candidate endpoints for the jump, and they will be valid candidates so long as smooth trade can be supported at those endpoints. Since \( \hat{P} \) is left-differentiable everywhere and \( k' \) is a local maximum of \( \hat{P}, \) it follows that, for some \( \varepsilon > 0, \) \( \hat{P}' > 0 \) for all \( (k' - \varepsilon, k'). \) Therefore, smooth trade can be supported at \( k', \) by the argument in previous paragraph.

\[\square\]
References


