Uncertainty-driven Cooperation*

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Abstract

We study the dynamics of team production with unknown true prospects. Team members receive interim feedback that is informative of their current effort levels and the project's prospects. We show that the presence of uncertainty alleviates inefficiencies arising from free-riding. Team members exaggerate their effort to influence the interim feedback signal, which in turn, affects their partners' beliefs about the prospects and consequently affects their future effort choices. The free-riding problem can vanish in the limit where feedback is sufficiently responsive. Our result implies that introducing uncertainty into team production can be welfare improving. Utilizing the tractability of our framework, we analyze various implications for optimal team design, such as the effects of team flexibility and asymmetric information among team members.

Keywords: Team Production, Free-riding, Dynamic Games, Uncertainty, Learning

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1 Introduction

The fast evolution of the modern economic environment creates considerable uncertainty about the prospects of many economic activities. In this light, many firms increasingly utilize small teams as more agile and adaptable means of production, moving away from rigid and highly structured means.[1] Understanding the dynamics of team incentives under uncertainty is therefore of central economic interest. These dynamics are complex: as uncertainty resolves over time, positive feedback boosts team members' confidence and subsequent effort, while setbacks undermine team morale. At the same time, individual team members may have incentives to keep their teammates motivated by exerting themselves in order to improve feedback.

In this paper, we present a framework of dynamic team production that enables us to analyze various features of team dynamics in the presence of uncertainty. The questions we address include: How does the dynamic resolution of uncertainty interact with the classic free-riding that naturally arises in teams? What is the effect of uncertainty on team welfare? Are there implications for optimal team design?

We consider a team of agents working on a joint project with unknown true prospects and finite horizon.At the end of the project, agents share the common output. This inherently creates free-riding incentives. Each agent's effort level is unobservable by the others. Over time, the agents receive interim public feedback about team performance. This feedback is noisy but informative about the agents' efforts and the project's prospects: both high effort and good prospects (statistically) improve feedback.

Such dynamic models, where the learning process interacts with unobservable actions, are typically not tractable. Specifically, the characterization of behavior off the equilibrium path is severely complicated as a deviation by an agent may cause her private belief about the prospects to diverge from the public belief. Moreover, in such environments, effort incentives

[1] To quote an article from the Economist: "...a network of teams is replacing the conventional hierarchy. The fashion for teams is driven by a sense that the old way of organising people is too rigid for both the modern marketplace and the expectations of employees. Technological innovation puts a premium on agility." ("Team spirit", 2016) Using survey data, Osterman (1994, 2000) estimates that among private, for-profit establishments that have at least 50 employees, approximately 40% have at least half of their employees organized in teams. Similarly, Lawler et al. (2001) reports that 47% of Fortune1000 companies make use of self-managed teams.

can be confounded by incentives to experiment. This paper contributes to the literature by proposing a model that overcomes these difficulties while isolating and highlighting the main economic forces at work. In particular, the marginal product of effort depends on uncertain prospects, while feedback is additively separable in effort and prospects with Gaussian noise. The former aspect guarantees that the dynamics of agents' incentives are linked with the evolution of their beliefs—thus remaining relevant to our central economic question—while the latter eliminates motives for experimentation, rendering our analysis tractable.

Two aspects of our equilibrium characterization deserve emphasis. First, our model admits a unique perfect Bayesian equilibrium (PBE). Second, the equilibrium strategies have a particularly simple and intuitive structure. Namely, the agents' effort choices after any history (both on and off the equilibrium path) are a linear function of the mean of their own private belief. Here, the coefficient multiplying the posterior mean—the belief sensitivity of effort—captures the impact of uncertainty on effort incentives. Utilizing the simple structure of the unique PBE, we describe the non-stationary dynamics of a team project whose true prospects are gradually revealed.

Our first main result is that *the presence of uncertainty alleviates the free-riding problem*. The presence of uncertainty boosts the effort incentives of the agents, because working harder today improves the interim feedback, rendering team members more optimistic about the project. Optimistic agents exert more effort in the subsequent phases because better prospects provide the agents with a higher marginal product. Essentially, the presence of uncertainty endogenously generates strategic complementarity between one agent's current effort and the other agents' future efforts.[2] This strategic complementarity leads to an equilibrium effort level that is higher than the myopically optimal level.[3]

The main result of this paper leads to an important corollary: *Introducing uncertainty into*

[2]By strategic complementarity we refer to the following: Fix all agents' expectations about the effort choice of a given agent. Then, *provided that the effort is unobservable*, an increase in the agent's effort level would lead to an increase in the expected marginal returns to the other agents' future efforts. Although this is not precisely the standard definition of strategic complementarity (Bulow et al., 1985), we use the term with some abuse, as we believe that it effectively captures the essence of the mechanism we identify.

[3]This mechanism can be interpreted as a novel application of "signal jamming," which has been identified in various contexts, including early work in industrial organization (Riordan, 1985; Fudenberg and Tirole, 1986) and the seminal paper of Holmström (1999) in the context of agency theory. We discuss our contribution relative to these models in the literature review below.

team production can be welfare improving. While uncertainty always entails a cost resulting from uninformed action choices, the benefit from mitigating the free-riding problem could outweigh that cost. In this case, adopting a project with uncertain prospects—even without a premium on returns —would lead to a Pareto improvement for the team members.[4]

After characterizing the unique equilibrium, we conduct several comparative statics exercises. We show that the effort-boosting impact of uncertainty is stronger when the project uncertainty is higher, the interim feedback is more precise, and the agents are more patient. Moreover, there exists a share structure under which the free-riding problem *vanishes* when interim feedback is sufficiently responsive to the agents' effort.

We also consider an infinite-horizon version of our model in which the project's prospects evolve stochastically over time. We construct a Markov perfect equilibrium whose structure is similar to that of the unique PBE of our main model. In analyzing the Markov perfect equilibrium, we show when the state is stochastic, the effort boosting impact of uncertainty exists permanently so that the equilibrium belief sensitivity of effort remains higher in the presence of uncertainty than in its absence.

The tractability of the framework we propose can help answer various economic questions. We take advantage of this feature in our discussion of *optimal team design*, focusing on the following four aspects:

- *Role of imperfect monitoring*: If individual effort choices are perfectly observable, then exerting more effort does not create an optimistic bias in others' beliefs. Therefore, the effort-boosting impact of uncertainty disappears. From the perspective of team design, this result implies that in environments in which monitoring is costly, it may be beneficial to choose a project with uncertain prospects instead of investing in the monitoring structure and establishing formal contracts.

- *Optimal level of project uncertainty*: Suppose that a team faces a choice of projects with

[4]Our result provides a novel explanation of risk-taking behavior in entrepreneurial organizations. Risk taking is considered one of the main elements of entrepreneurial behavior (Miller, 1983), and the economics literature has suggested various motivations for risk-taking behavior, such as the desire to receive a higher premium (Heaton and Lucas, 2000) or the desire to smooth out the entrepreneur's value as a function of wealth (Vereshchagina and Hopenhayn, 2009). In this paper, we identify an alternative motivation related to team incentives.

various levels of uncertainty regarding their prospects. We demonstrate that there exists an optimal level of project uncertainty that balances the trade-off between the benefit of alleviating free-riding problem and the cost of uninformed effort choices.

- *Effect of team flexibility*: We show that if an organization is more flexible—that is, if team members receive feedback more frequently and adjust their actions—the free-riding is further alleviated in equilibrium. This result provides an interesting contrast to the repeated partnership literature in which the scope of cooperation could be limited when actions are flexible (Abreu et al., 1991; Sannikov and Skrzypacz, 2007).

- *Effect of asymmetric information among team members*: We consider an asymmetric information model in which some team members are "experts" who are perfectly informed about the prospects. We show that the essential structure of the unique equilibrium of our model extends to these cases. Additionally, using the asymmetric information model, we discuss the interaction between the incentives of the informed experts and those of the uninformed agents. The main trade-off arises between the speed of learning and the strength of effort incentives. When a team member is replaced by an expert, learning takes place faster as the expert's informed actions lead to more informative feedback. However, team members' effort incentives become weaker since it is not possible to manipulate the expert's beliefs. Either side of the trade-off may dominate in equilibrium, depending on various aspects of the economic environment.

The remainder of the paper is organized as follows. Section 1.1 discusses the related literature. Section 2 formally describes the model. Section 3 characterizes the equilibrium and undertakes the comparative statics exercises. Section 4 extends the main model to an infinite-horizon version. Section 5 analyzes the implications for team design. Section 6 concludes. The Appendix contains all the omitted proofs. The supplementary material analyzes a two-period example, discusses the potential non-monotonicity of the belief sensitivity and analyzes a continuous-time version of the main model.

1.1 Literature Review

This paper contributes to the literature on free-riding in groups (Olson, 1965; Alchian and Demsetz, 1972; Holmström, 1982). The literature generally suggests that cooperation can be sustained by "punishments" based on past behavior in the form of either lower monetary transfers or future non-cooperation from other team members.[5] Our paper analyzes dynamic moral hazard in team production with uncertainty over a project's prospects and demonstrates that the presence of uncertainty could alleviate free-riding.

Our paper is related to the literature on experimentation in teams. The literature focuses on the effect of either a pure informational externality (Bolton and Harris, 1999; Keller et al., 2005; Rosenberg et al., 2007) or combinations of information and payoff externalities (Bonatti and Hörner, 2011; Guo and Roesler, 2016; Halac et al., 2017). In contrast, our model considers a *pure payoff externality*: In our model, the speed of learning is independent of the agents' actions, and thus, the agents do not have incentives for experimentation. Moreover, the welfare effect of uncertainty is typically negative in the literature—the equilibrium payoff is higher if the state is known—but we show that uncertainty mitigates the free-riding problem, possibly leading to a welfare improvement.

In this literature, the closest papers to ours are Bolton and Harris (1999) and Bonatti and Hörner (2011). Bolton and Harris (1999) consider a multi-agent experimentation problem in which agents' actions are observable and the agents share the information, but not the payoff, resulting from experimentation. Their symmetric Markov perfect equilibrium demonstrates that the possibility of eliciting future experimentation by others encourages current experimentation. While our unique equilibrium exhibits similar incentives, the underlying channels are distinct. Whereas in Bolton and Harris (1999), the agents are encouraged to demonstrably generate new information (*convincing*), in our model, the agents' incentives are generated by secretly manipulating the feedback (*cheating*). Indeed, for the "encouragement effect" to exist in our case, it is essential that the agents' effort choices are unobservable. Bonatti and Hörner (2011) consider dynamic moral hazard in teams with an uncertain state. In their paper, the

[5] In the literature on contracts with many agents, a group contract based on total output can mitigate moral hazard in teams (Holmström, 1982; Legros and Matthews, 1993); in repeated partnership games, the threat of future non-cooperation following a deviation sustains various equilibrium dynamics (Radner et al., 1986).

game ends when the common project has a "breakthrough," the arrival rate of which depends on the agents' current effort levels and the unknown quality of the project. This instantaneity of potential success implies that one agent's current effort and the others' future efforts are strategic substitutes, leading to inefficiencies in the form of procrastination. In contrast, in our model, uncertainty over the project's prospects generates a form of strategic complementarity between an agent's current effort and the future efforts of others, strengthening the incentives to exert effort and sometimes leading to an (approximately) socially efficient outcome.

Our paper is also related to the literature on dynamic contributions to public goods. Admati and Perry (1991) and Marx and Matthews (2000) show that a public project can be completed by agents who contribute small amounts from time to time. Yildirim (2006) and Georgiadis (2014) assume that the payoff is realized only when the project's state reaches a pre-specified threshold. In these papers, the threshold-payoff assumption implies that the effort choices at different points in time are strategic complements, which plays a key role in mitigating the free-riding problem.[6] Importantly, these papers do not feature uncertainty over the project type. In contrast, our repeated partnership game does not assume a completion threshold, and the complementarity between current and future effort arises endogenously because an agent's effort affects the inferences of others.

As noted above, the signal-jamming mechanism of our paper has been investigated in various contexts. Since Holmström (1999), the literature on career concerns has analyzed the "market-based" incentives of a manager who attempts to affect the market belief about his innate ability. Riordan (1985) (oligopoly) and Fudenberg and Tirole (1986) (entrant-incumbent game) consider cases in which a firm has a signal-jamming incentive to make the competing firm more pessimistic about future profitability. In this paper, we identify the role of such a mechanism in the context of team production and optimal team design. Cisternas (2017b) expands the career concerns model to allow general (non-linear) payoffs for the long-run player in a stationary environment. Using the first-order approach, he shows how "ratchet effect" shapes player's equilibrium incentives.[7] Compared to Cisternas (2017b), we describe the dy-

[6] See also Georgiadis (2016) for how deadlines and the frequency of the monitoring affect free-riding incentives.

[7] Cisternas (2017a) generalizes the career concerns model along another dimension by allowing investment in human capital.

namics of the ratchet behavior in a *non-stationary* model with a specific form of non-linear payoffs.[8]

Our infinite-horizon model in Section 4 is closely related to the literature on repeated games with frequent actions.[9] Since Abreu et al. (1991), the literature has shown how frequent actions can be detrimental to cooperation (Sannikov and Skrzypacz, 2007; Fudenberg and Levine, 2007, 2009). In contrast, we show that frequent actions increase the level of cooperation in our model.

2 Model

A team of N agents undertakes a common project. Time $t = 0, \ldots, T$ is discrete and finite. Each period has length $\Delta > 0$, and $\tau = T\Delta$ is the real-time length of the project. At the beginning of the game, nature draws a persistent state of the world θ from a Gaussian distribution $\mathcal{N}(\mu_0, 1/\nu_0)$, which defines the initial common prior about θ.[10] In each period, agent i chooses an effort level $a_{it} \in \mathbb{R}$. Each agent's effort level is not observable by others. We assume that agent i incurs a quadratic cost of effort $\Delta c_i a_{it}^2/2$, where $c_i > 0$. The agents have a common discount factor $\delta = e^{-r\Delta}$, where $r > 0$.

At the end of each period, the agents publicly observe feedback y_t. This can be the outcome of an internal review or feedback from an employer. We assume that the period-t feedback is

$$y_t = \Delta \left[\kappa_\theta \theta + \kappa_a \sum_{i=1}^{N} a_{it} + \varepsilon_t \right],$$

where $\varepsilon_t \sim \mathcal{N}(0, 1/\nu_\varepsilon)$ is a stochastic noise term with precision $\nu_\varepsilon = \Delta \eta_\varepsilon$, and $\kappa_\theta, \kappa_a > 0$ are positive constants. We interpret η_ε as the information disclosure rate. Note that the informativeness of feedback increases in Δ.[11] The parameters κ_a and κ_θ determine how sensitive feedback is to agents' actions and to the realization of the true state, respectively. We assume

[8] This result is in contrast to the outcome in Holmström (1999)'s model, where the optimal effort level of the long-run player may diverge as the horizon becomes longer.

[9] Recently, Iijima and Kasahara (2015) prove an equilibrium uniqueness result for a class of finite horizon frequent action games which however does not include our finite horizon model.

[10] In Section 4, we extend our result to the case of a stochastic state.

[11] As $\Delta \to 0$, a linear interpolation of the feedback process y_t converges in distribution to $dY_t = \left(\kappa_\theta \theta + \kappa_a \sum_{i=1}^{N} a_{i,t}\right) dt + \frac{1}{\sqrt{\eta_\varepsilon}} dW_t$, where W_t is a standard Brownian motion (Whitt, 1980).

that the ε_ts are independent and identically distributed over time.

Total production P is realized at the end of period T and is given by

$$P = e^{r\tau} \sum_{t=0}^{T} e^{-rt\Delta} P_t,$$

where $P_t = \Delta\theta \sum_{i=1}^{N} a_{it}$ is period-t production and $e^{r\tau}$ is a normalization term.[12] Note that P_t is linear in each agent's period-t effort, and the state θ is the marginal product. Further, note that in this specification, output is additively separable in effort across agents and over time.

The agents share total production according to a rule $(s_1, \cdots, s_N)$, where s_i represents agent i's share of the total output with $\sum_{i=1}^{N} s_i = 1$ and $s_i > 0$ for all i. The agents are risk-neutral expected utility maximizers, with agent i maximizing

$$\begin{aligned} U &= \mathbb{E}\left[s_i e^{r\tau} P - \sum_{t=0}^{T} e^{-rt\Delta} \Delta c_i \frac{a_{it}^2}{2} \right] \\ &= \sum_{t=0}^{T} \Delta e^{-rt\Delta} \mathbb{E}\left[s_i \theta \sum_{j=1}^{N} a_{jt} - c_i \frac{a_{it}^2}{2} \right]. \end{aligned}$$

Remark 1. The agent's payoff in our model is *not* additively separable in the agent's action (a_{it}) and the state (θ). Such complementarity between the action and the state is crucial for generating our main result. Without this complementarity, the agent's marginal benefit of effort—and, therefore, the optimal effort level—would be independent of the state; thus, the incentive to manipulate others' beliefs would disappear.

Remark 2. The agent's action and the state enter in an additively separable way into feedback y_t. As we demonstrate in a two-period example in the supplementary material, such additive separability is not necessary for our results, but it renders our dynamic model very tractable. In particular, as Section 3 clarifies, this assumption implies that the speed of learning is independent of agents' actions, and thus, the agents in our model do not have incentives

[12] Our assumption that output is realized at the end of the game is not essential for our main mechanism. This assumption allows the linear feedback to be the only source of belief updating throughout the game and, thus, significantly simplifies the analysis. Nevertheless, one can find various real-world examples in which the returns to effort are realized at a specific future date, such as the release of a new product or the issuance of an IPO.

for experimentation. This makes the underlying mechanism of the model different from those in the literature on experimentation in teams (Bonatti and Hörner, 2011; Keller et al., 2005).

A public history $h^t \in \mathscr{H}$ is a feedback sequence $\{y_k\}_{k=0}^{t-1}$. Agent i's private history $h_i^t \in \mathscr{H}^i$ is the combination of the public history and the sequence of his own past effort choices, that is, $h_i^t = \{(a_{ik}, y_k)\}_{k=0}^{t-1}$.[13] A pure strategy for agent i is a function $a_i : \mathscr{H}_i \to \mathbb{R}$, where $a_{it} = a_i(h_i^t)$ is agent i's effort level in period t. We focus on pure strategy profiles.

The solution concept is perfect Bayesian equilibrium (PBE).[14] A PBE is a strategy profile $a = (a_1, \ldots, a_N)$ and a belief system such that the beliefs on and off the equilibrium path are derived using Bayes' rule from the strategies whenever possible, and each player's strategy is optimal given his beliefs and the strategies of others.

Benchmark cases We conclude this section by considering two benchmark cases for future reference. The proofs are straightforward and thus omitted.

1. Static setting $(T = 0)$: Agent i's effort in the unique equilibrium of the static setting is $a^*_{i,static} = \mathbb{E}[\theta] = \frac{s_i}{c_i}\mu_0$. Note that the socially efficient level of effort (the one without free-riding) is $\frac{1}{c_i}\mu_0$.

2. Complete information case ($v_0 = \infty$): Suppose that the state of the world θ is perfectly known. Then, the unique equilibrium profile is $a^*_{it} = \frac{s_i}{c_i}\theta$ for any $t = 0, \ldots, T$, while the socially efficient level is $\frac{1}{c_i}\theta$.

3 Equilibrium

In this section, we derive the unique PBE of our model. We also discuss the resulting equilibrium dynamics and the mechanisms underlying these dynamics.

[13] As usual, we define $h^0 = h_i^0 = \emptyset$ for all i.

[14] For the formal definition of PBE, see Fudenberg and Tirole (1991) Definition 8.2.

3.1 Belief Updating

We first analyze the evolution of beliefs on and off the equilibrium path. Observe that the agent's deviation is never detected because of the full-support assumption for feedback. Thus, any public history h^t is on the equilibrium path, and hence, the posterior belief is pinned down by Bayes' rule. Then, the Gaussian information structure of the game implies that all posteriors are also Gaussian; thus, any posterior belief is characterized by its mean and precision.

Define the *public belief* as the common posterior belief under the expectation that the agents follow the equilibrium strategy profile. Formally, let $a^* = (a_1^*, \ldots, a_N^*)$ be an equilibrium strategy profile, and given a public history $h^t = \{y_k\}_{k=0}^{t-1}$, define $\bar{h}_i^t = \{(\bar{a}_{ik}, y_k)\}_{k=0}^{t-1}$ recursively as $\bar{a}_{i0} = a_i^*(\emptyset)$ and $\bar{a}_{it} = a_i^*(\bar{h}_i^t)$. Note that $\bar{h}_i^t$ is a private history of agent i in which he follows the equilibrium strategy *for all* $t' < t$. Then, given the expectation of "no-deviation," an element of feedback y_t that is purely informative about the state θ is

$$z_t \equiv y_t - \Delta\kappa_a \sum_i a_i^*(\bar{h}_i^t),$$

Note that if the agents follow the equilibrium strategy for all $t' < t$, the signal z_t follows a normal distribution with mean $\Delta\kappa_\theta\theta$ and precision η_ε/Δ.

In each period, the public belief is updated based on z_t. Let μ_t and ν_t be the mean and the precision, respectively, of the public belief in period t. Then, by standard Gaussian updating, μ_t and ν_t are recursively determined by

$$\mu_t = \frac{\nu_{t-1}\mu_{t-1} + \kappa_\theta\eta_\varepsilon z_{t-1}}{\nu_{t-1} + \Delta\kappa_\theta^2\eta_\varepsilon} \quad \text{and} \quad \nu_t = \nu_{t-1} + \Delta\kappa_\theta^2\eta_\varepsilon. \tag{1}$$

The *private belief* of agent i does not necessarily follow the public belief, as he privately knows his effort level. Specifically, agent i updates his private belief based on the signal

$$\hat{z}_{it} \equiv y_t - \Delta\kappa_a \left(a_{it} + \sum_{j \neq i} a_j^*(\bar{h}_j^t) \right),$$

where a_{it} is the actual effort choice of agent i, which can be different from $a_i^*(\bar{h}_i^t)$. Then, the mean $\hat{\mu}_{it}$ and precision $\hat{\nu}_{it}$ of the private belief in period t are recursively determined by

$$\hat{\mu}_{it} = \frac{\nu_{t-1}\hat{\mu}_{it-1} + \kappa_\theta \eta_\varepsilon \hat{z}_{it-1}}{\nu_{t-1} + \Delta\kappa_\theta^2 \eta_\varepsilon} \quad \text{and} \quad \hat{\nu}_{it} = \nu_t. \tag{2}$$

Note that $\hat{\mu}_{it} = \mu_t$ as long as agent i follows the equilibrium strategy. In contrast, once an agent deviates from his equilibrium effort choice, his private belief and the public belief diverge. For example, suppose that agent i deviates in period t and plays $a_{it} = a_i^*(\overline{h}_i^t) + \alpha$ for some $\alpha > 0$ and, thereafter, follows the strategy that the other agents anticipate (that is, he plays $a_i^*(\overline{h}_i^s)$ for any $s = t+1, \ldots, T$).[15] Then, for all future periods, the public belief is more optimistic than agent i's private belief. In particular, for any $s > t$,

$$\hat{\mu}_{is} = \mu_s - \rho_s \alpha,$$

where

$$\rho_s = \left(\prod_{\tau=t+2}^{s} \frac{\partial \mu_{\tau+1}}{\partial \mu_\tau} \right) \cdot \frac{\partial \mu_{t+1}}{\partial z_t} \cdot \frac{\partial z_t}{\partial a_{it}} = \frac{\Delta\kappa_a \kappa_\theta \eta_\varepsilon}{\nu_s}$$

is the rate at which the deviation in period $t < s$ affects the belief divergence.[16]

Note that agent i's deviation does not bias his own belief about the state, since he discounts feedback according to his actual effort level. However, agent i's deviation biases the public belief, which discounts the observed feedback through the equilibrium action. Specifically, by devoting greater effort, each agent can increase the mean of the public belief μ_s (for any realization of noise ε_t) at a rate of ρ_s. This is precisely the mechanism that leads each agent to have additional incentives to increase his effort.

Finally, note that the precision of the posterior belief is deterministic and independent of any history. Since the speed of learning is independent of the action, the agents in our model do not have incentives for experimentation in choosing their optimal effort levels. This property greatly simplifies our equilibrium analysis, as becomes clear in the next subsection.

[15] Clearly, playing $a_i^*(\overline{h}_i^s)$ need not be optimal for agent i following a deviation, as his belief off the equilibrium path may be distinct from the public belief. In Subsection 3.2, we provide a detailed discussion of the off-equilibrium behavior in the unique PBE.

[16] We define $\prod_{\tau=t+2}^{s} \frac{\partial \mu_{\tau+1}}{\partial \mu_\tau} = 1$ for $s = t+1$.

3.2 Equilibrium

We first state our main result.

Proposition 1. *There exists a unique perfect Bayesian equilibrium. In equilibrium, agent i's period-t action is*

$$a_i^*(h_i^t) = \xi_{it}\hat{\mu}_{it}, \tag{3}$$

where $\xi_{iT} = s_i/c_i$, *and*

$$\xi_{it} = \frac{s_i}{c_i}\left[1 + \sum_{k=t+1}^{T} e^{-r(k-t)\Delta} \sum_{j\neq i} \xi_{jk}\rho_k \prod_{l=t+1}^{k-1}(1-\xi_{il}\rho_l)\right], \tag{4}$$

for $t = 0,\ldots,T-1$.

The unique PBE of our model has a remarkably simple structure: *After any history*, the equilibrium action of each agent is linear in the mean of his private posterior belief.[17] We call the coefficient ξ_{it} agent i's *belief sensitivity of effort* in period t: It captures the rate at which the agent responds to changes in his belief ($\hat{\mu}_{it}$). Note that ξ_{it} is deterministic and varies only with the calendar time t. If the agents are homogeneous (that is, $c_i = c$ and $s_i = 1/N$ for all i), then ξ_{it}s are identical across agents and the unique PBE becomes symmetric. However, the agents may choose different actions off the equilibrium path, as their beliefs could diverge.

In the Appendix, we formally prove Proposition 1. Here, we provide an intuitive explanation. First, note that since we assume a quadratic cost function $c_i a^2/2$, agent i's optimal effort level equals his expected marginal benefit divided by c_i. Rewriting equations (3) and (4), we express the marginal benefit of effort as follows:

$$c_i a_{it}^* = \underbrace{s_i\hat{\mu}_{it}}_{\text{myopic benefit}} + \underbrace{\underbrace{e^{-r\Delta} s_i\hat{\mu}_{it} \sum_{j\neq i} \xi_{j,t+1}\rho_{t+1}}_{\text{effect on period } t+1} + \underbrace{e^{-2r\Delta} s_i\hat{\mu}_{it} \sum_{j\neq i} \xi_{j,t+2}\rho_{t+2}(1-\xi_{i,t+1}\rho_{t+1})}_{\text{effect on period } t+2} + \cdots}_{\text{signal-jamming}}. \tag{5}$$

[17] This result can be explained as follows: Due to the Gaussian signal structure, the marginal impact of increased effort on belief is independent of the level of others' beliefs. Moreover, the marginal payoff gained from the belief divergence is constant due to the quadratic cost function.

The first term in (5) captures the (direct) myopic benefit of effort, which is equal to the expected social benefit $(\hat{\mu}_{it})$ times agent i's share (s_i). The rest of the right-hand side captures the (indirect) benefit from manipulating others' future beliefs. Specifically, this term captures the extent to which (agent i's share of) expected output increases as a result of an increase in i's effort in period t followed by optimal effort choices based on his private belief.

To understand the benefit from the signal-jamming effect, consider an upward deviation by agent i in period t in which he chooses $a_{it} = a_{it}^* + \alpha$, with $\alpha > 0$. For any realization of ε_t, such a deviation increases the mean of the period-$(t+1)$ public belief (μ_{t+1}) by $\rho_{t+1}\alpha$. Then, in the next period, each agent $j \neq i$ would increase his effort by $\xi_{j,t+1}\rho_{t+1}\alpha$ (recall that ξ_{jt} is the response rate of agent j's effort to a change in the posterior mean he holds in period t). Therefore, agent i's expected benefit from this increase in others' effort is $e^{-r\Delta}s_i\hat{\mu}_{it} \cdot (\sum_{j\neq i}\xi_{j,t+1}\rho_{t+1}\alpha)$. Since the benefit from deviation is linear in α, the marginal benefit is constant and equal to the second term of (5).

For period $t+2$ (and thereafter), the effect of a deviation in period t becomes more complicated because the agents' beliefs diverge off the equilibrium path. If agent i plays $a_{it} = a_{it}^* + \alpha$ in period t, his private belief in period $t+1$ is more pessimistic than the public belief: $\hat{\mu}_{i,t+1} = \mu_{t+1} - \rho_{t+1}\alpha$. Then, in period $t+1$, the equilibrium effort level of agent i is smaller than what the others anticipate:

$$a_{i,t+1}^* = \xi_{i,t+1}\hat{\mu}_{i,t+1} = \xi_{i,t+1}(\mu_{t+1}\underbrace{-\rho_{t+1}\alpha}_{\text{divergence}}).$$

This divergence of agent i's effort makes the public belief in period $t+2$ downward biased. In particular, μ_{t+2} is smaller by $\rho_{t+2}\cdot\xi_{i,t+1}\rho_{t+1}\alpha$ than its level would have been had the agent taken the anticipated action. Consequently, belief divergence in period $t+1$ negatively affects agent i's incentive in period t. We refer to this negative incentive as the *ratchet effect,* as the agent's current incentive to work is affected by the other agents' expectations for the future.[18]

As a result, the agent's marginal benefit of effort consists of the (positive) direct signal-

[18] The ratchet effect—the effect of potentially causing high future expectations of the agent's current incentives—is extensively analyzed in the literature on dynamic agency models with asymmetric information (Weitzman, 1980; Freixas et al., 1985) and dynamic moral hazard with learning and symmetric uncertainty (Bhaskar, 2014; Prat and Jovanovic, 2014; Cisternas, 2017b; Bhaskar and Mailath, 2016).

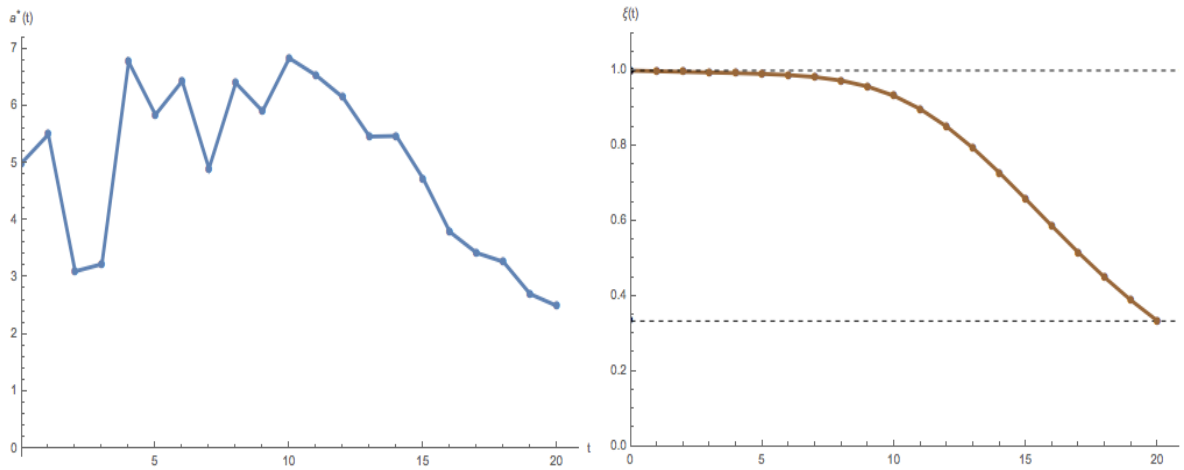
jamming effect and the (negative) ratchet effect. Note that similar to its effect on period $t+1$, agent i's deviation in period t directly increases μ_{t+2} by $\rho_{t+2}\alpha$. Therefore, agent i's deviation in period t, followed by the corresponding equilibrium strategy in period $t+1$, has a net effect of

$$\left[\underbrace{\xi_{j,t+2}\rho_{t+2}}_{\text{direct signal-jamming}} - \underbrace{\xi_{j,t+2}\rho_{t+2}\xi_{i,t+1}\rho_{t+1}}_{\text{ratcheting}}\right]\alpha = \xi_{j,t+2}\rho_{t+2}\left(1-\xi_{i,t+1}\rho_{t+1}\right)\alpha$$

on agent j's period-$(t+2)$ effort choice. Summing over all agents $j \neq i$, multiplying by $s_i\hat{\mu}_{it}$ and discounting yields the coefficient (of α) equal to the third term of (5). Iterating this reasoning yields the expression in Proposition 1.

Remark 3. The direct signal-jamming effect in period t is linear in each $\xi_{jk}\,(k > t)$. By devoting greater effort, each agent can directly change the future posterior to which the other agents respond in a linear way. However, the ratchet effect is (at least) quadratic in ξ_{ik} and ξ_{jk}: A deviation creates belief divergence in future periods, and the resulting divergence in the expected effort level in turn leads to belief distortion in periods further in the future. The implication of this difference becomes more evident when we consider the continuous-time limit of the equilibrium below.

We prove uniqueness by backward induction. Note that the above argument for the marginal benefit holds after any history, regardless of whether an agent has previously deviated. In the final period ($t = T$), after any history h_i^T, each agent has a unique best response $a_i^*(h_i^T) = (s_i/c_i)\hat{\mu}_{iT}$, which is linear in the mean of the private belief. Now, suppose that for some t, the equilibrium strategy after any history h_i^k is linear in $\hat{\mu}_{ik}$ for all $k = t+1, \ldots, T$. Then, each agent's best response in period t is unique since the cost of effort is convex while the benefit is linear. Furthermore, our linear-quadratic-Gaussian structure implies that the unique best response is also linear in $\hat{\mu}_{it}$. In the Appendix, we present a formal proof based on this argument.



(a) A realization of the equilibrium effort a_{it}^*

(b) Belief sensitivity ξ_t

Figure 1: Dynamics of the unique PBE with the homogeneous agents $(N = 3, T = 20)$

Equilibrium dynamics Figure 1 illustrates the dynamics of the unique PBE when the agents are homogeneous ($c_i = 1$ and $s_i = 1/N$ for all i). Note that with homogeneous agents, the unique PBE is symmetric; i.e., $\xi_{it} = \xi_t$ for all i. The left panel shows a realization of the equilibrium effort on the equilibrium path (where $\hat{\mu}_{it} = \mu_t$). The equilibrium effort level $a_{it}^* = \xi_t \mu_t$ is stochastic and typically non-monotonic over time. This is because the dynamics of the posterior mean μ_t depend on realized feedback. However, the coefficient of the equilibrium action (belief sensitivity of effort) is deterministic and has more consistent properties. In what follows, we analyze the equilibrium properties by mainly focusing on the dynamics of the belief sensitivity.

The dynamics of a symmetric ξ_t over time are depicted in the right panel of Figure 1. Recall that the myopically optimal level of belief sensitivity—that is, the level without the signal-jamming effect—is $s_i/c_i = 1/N$ (lower dashed line), while the socially efficient level is $1/c_i = 1$ (upper dashed line). In the graph, the equilibrium ξ_t decreases over time and lies between the two dashed lines.

The intuition for decreasing belief sensitivity is twofold. First, as t increases, there are fewer remaining periods during which coworkers make effort choices, and thus, the agents' return to influencing the others' beliefs declines. Second, as the agents learn θ more precisely over time, they place a smaller weight on new feedback in updating their beliefs, making it more difficult to affect this belief by changing the effort level.

Although the above intuition suggests that the equilibrium ξ_{it} should generally be monotonic, this is not always the case. Non-monotonicity may result when the ratchet effect dominates the direct signal-jamming effect. Such a phenomenon may occur when the belief sensitivity in the next period is large. For example, suppose that the (homogeneous) agents in period t expect a very large (symmetric) ξ_{t+1} (where $t+1<T$). Then, contributing more effort in period t creates a very large divergence in expectations of $a_{i,t+1}$ between agent i and the other agents, which in turn, downward biases the period-$(t+2)$ public belief by a large amount. This quadratic ratchet effect may dominate the direct signal-jamming effect, and thus, ξ_t may be lower than ξ_{t+1}. In the supplementary material, we discuss this issue in detail.

Such non-monotonicity disappears when the "real-time" length of a period becomes shorter. Note that as the period length becomes shorter, the agents receive more frequent feedback and can frequently adjust their effort levels. In this case, as we solve the equilibrium by backward induction, the equilibrium ξ_{it} cannot exhibit large "sudden jumps". Therefore, although the ratchet effect strengthens gradually (as t goes backward), it does not dominate the signal-jamming effect, leading to monotonic dynamics.

In the next subsection, we consider the continuous-time limit of the model where the period length becomes arbitrarily small. In this limiting environment, we establish the monotonicity of ξ_{it} and conduct comparative statics.

3.3 Continuous-Time Limit and Comparative Statics

In this subsection, we consider a continuous-time limit in which feedback (and the corresponding effort adjustment) is arbitrarily frequent. Specifically, we fix the real-time length $\tau = T\Delta$ of the game and consider the limit of equilibrium behavior as $\Delta \to 0$.

3.3.1 Equilibrium in the Continuous-Time Limit

To derive the continuous-time limit of the equilibrium, we first describe the equilibrium strategies in recursive form. Define $S_{ij,t}(t=0,\ldots,T)$ recursively as

$$S_{ij,t} = \xi_{j,t+1}\rho_{t+1} + e^{-r\Delta}(1-\xi_{i,t+1}\rho_{t+1})S_{ij,t+1}, \tag{6}$$

with $S_{ij,T} = 0$. The variable $S_{ij,t}$ captures the effect of agent i's signal-jamming in period t, resulting from changes in agent j's effort level. Then, (4) can be rewritten as

$$\xi_{it} = \frac{s_i}{c_i}\left[1 + e^{-r\Delta}\sum_{j\neq i} S_{ij,t}\right]. \tag{7}$$

Summing (6) over $j \neq i$ and substituting for $\sum_{j\neq i} S_{ij,t}$ and $\sum_{j\neq i} S_{ij,t+1}$ from (7) yields a recursive formulation for ξ_{it}:

$$\xi_{it} = \frac{s_i}{c_i} + e^{-r\Delta}\left[\frac{s_i}{c_i}\sum_{j\neq i}\xi_{j,t+1}\rho_{t+1} + (1-\xi_{i,t+1}\rho_{t+1})\left(\xi_{i,t+1} - \frac{s_i}{c_i}\right)\right]. \tag{8}$$

Then, writing the variables in terms of Δ, re-arranging, and taking $\Delta \to 0$, we obtain the following system of differential equations: For $i = 1, \ldots, N$,

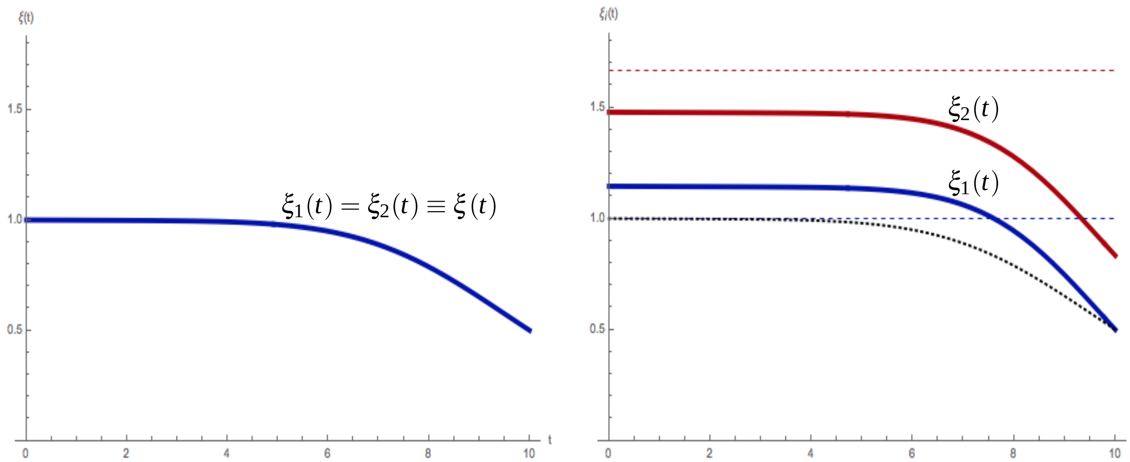
$$\dot{\xi}_i(t) = \underbrace{r\left(\xi_i(t) - \frac{s_i}{c_i}\right)}_{\text{discounting}} \underbrace{- \frac{\eta_\varepsilon \kappa_\theta \kappa_a}{v_0 + \eta_\varepsilon t \kappa_\theta^2}\left(\frac{s_i}{c_i}\sum_{j=1}^{N}\xi_j(t) - \xi_i(t)^2\right)}_{\text{signal-jamming}}. \tag{9}$$

Together with the terminal conditions $\xi_i(\tau) = s_i/c_i$, the above system of differential equations fully describes the equilibrium dynamics over time.[19,20] In the supplementary material, we formulate the continuous-time counterpart of the main model and show that (i) there exists an equilibrium of the continuous-time model that is described by (9) and (ii) the unique PBE of the discrete-time model (weakly) converges to that continuous-time equilibrium as $\Delta \to 0$.

The differential equation (9) has a simple and intuitive form. The first term captures the effect of discounting. Note that whenever $\xi_i(t)$ is above the myopically optimal level (s_i/c_i), the first term is positive, and thus, the discounting effect decreases the incentive in the earlier phases (bear in mind that we compute $\xi_i(t)$ backwards from $t = \tau$). The second term captures the effect of signal jamming, and its coefficient is a function of the information parameters (v_0, η, κ_θ and κ_a). It consists of a linear component and a quadratic component, which capture the direct signal-jamming effect and the ratchet effect, respectively.

[19] In this subsection, we slightly abuse notation and refer to t as real time in the game.

[20] This system is a backward Riccati equation. In the proof of Proposition 2, we show the existence and uniqueness of the solution. When s_i and c_i are the same for all i, the system has a closed-form solution expressed by confluent Hypergeometric and Laguerrel functions.



(a) Homogeneous agents ($c_1 = c_2 = 1$) (b) Heterogeneous agents ($c_1 = 1, c_2 = 0.6$)

Figure 2: Equilibrium $\xi_i(t)$ under the continuous-time limit $(N = 2, \tau = 10, s_i = 1/2)$

Figure 2 illustrates the evolution of belief sensitivity in the limit, where $\Delta \longrightarrow 0$, with homogeneous agents (left panel) and heterogeneous agents (right panel). It shows that the signal-jamming effect remains nontrivial in the continuous-time limit. In what follows, we use the simple form of (9) to further analyze the properties of the equilibrium.

3.3.2 Equilibrium Properties

We begin by establishing the monotonicity of equilibrium belief sensitivity $\xi_i(t)$.

Proposition 2. *For any* i, $\xi_i(t)$ *is monotonic and decreasing over time. Moreover, for any* $t \in [0, \tau]$, $\xi_i(t) \in [\underline{\xi}_i, \overline{\xi_i})$, *where*

$$\underline{\xi}_i = \frac{s_i}{c_i}, \qquad \overline{\xi_i} = \sqrt{\frac{s_i}{c_i}} \sum_{j=1}^{N} \sqrt{\frac{s_j}{c_j}}.$$

The intuition for a monotonically decreasing $\xi_i(t)$ is provided in the previous subsection. In addition, Proposition 2 establishes lower and upper bounds on the equilibrium $\xi_i(t)$. The lower bound $\underline{\xi}_i$ is the myopically optimal level that would be attained in the absence of signal-jamming incentives.

The existence of the upper bound $\overline{\xi_i}$ is due to the ratchet effect, which appears in the quadratic term of (9). Suppose that we solve (9) backwards from the terminal point $t = \tau$. At

$t = \tau$, as $\xi_i(\tau) = s_i/c_i$, the linear term $(\frac{s_i}{c_i}\sum_{j=1}^{N}\xi_j(t))$ is greater then the quadratic term $(\xi_i(t)^2)$, and thus, the signal-jamming incentive becomes greater as t goes backward. However, as $\xi_i(t)$ becomes larger, the quadratic term catches up to the linear term, which prevents the belief sensitivity from being greater than $\overline{\xi}_i$.[21]

The next proposition states the equilibrium properties with respect to the cost parameter and the share structure. Its proof is straightforward from equation (9), and Proposition 2 and thus is omitted.

Proposition 3. *Consider the continuous-time limit of the unique PBE.*

1. *For any $t \in [0,T]$, $\xi_i(t)$ decreases in c_i.*
2. *For any $t \in [0,T)$, $\xi_i(t)$ decreases in $c_j (j \neq i)$.*
3. *Suppose that the share structure $s^* = (s_1^*, \ldots, s_N^*)$ is set by $s_i^* \equiv \frac{\frac{1}{c_i}}{\sum_{j=1}^{N}\frac{1}{c_j}}$. Then, $\overline{\xi}_i = 1/c_i$ for all $i = 1, \ldots, N$.*

The intuition for part 1 is straightforward: The agent contributes less effort when his marginal cost increases. Perhaps more interestingly, part 2 states that agent i's effort level decreases in the marginal cost of other agents. This is because the agent's marginal benefit of effort is increasing in the other agents' belief sensitivity, which is decreasing in their own cost. This effect is illustrated in Figure 2. The left panel illustrates the symmetric $\xi(t)$ of a homogeneous two-person team, and the right panel shows $\xi_1(t)$ and $\xi_2(t)$ when agent 2's marginal cost has decreased. Note that *both* $\xi_1(t)$ and $\xi_2(t)$ lie above the symmetric $\xi(t)$ (black dotted line in the right panel): If one agent's cost is reduced, then both agents choose higher effort.

Part 3 shows that there exists a sharing rule that makes the upper bound on agent i's belief sensitivity ($\overline{\xi}_i$) coincide with the socially efficient level ($1/c_i$). Figure 2 (right panel) shows that, generally, $\overline{\xi}_i$ does not coincide with $1/c_i$ (depicted as dashed lines of the respective color): $\xi_1(t)$ initially lies above the socially efficient level, while agent 2 always underinvests in his

[21] In contrast, in Holmström (1999)'s career concerns model, the ratchet effect does not preclude equilibrium action from diverging under limits (e.g., $\kappa_a \to \infty$). This is because in our model, the return to jamming the feedback signal is endogenous and based on others' belief sensitivity, which is also subject to the ratchet effect. The reduction in others' belief sensitivity due to the ratchet effect compounds the negative impact on each agent's effort choice, eventually bounding it.

effort. If an agent's cost is higher than that of the other agents, his signal-jamming incentives may be inefficiently strong, as the other agents are more responsive to belief changes.

The next proposition establishes the comparative statics results for the discount rate and the information parameters.

Proposition 4. *In the unique PBE of the model, for any $t \in [0, \tau)$,*

1. *$\xi_i(t)$ decreases in r.*
2. *$\xi_i(t)$ decreases in v_0 and increases in η_ε.*
3. *$\xi_i(t)$ increases in κ_a but is non-monotone in κ_θ.*

That $\xi_i(t)$ decreases in r is intuitive: A larger r makes the future less important and thus decreases the signal-jamming incentive. The intuition for parts 2 and 3 can be explained by the coefficient $\left(\frac{\eta_\varepsilon \kappa_\theta \kappa_a}{v_0 + \eta_\varepsilon t \kappa_\theta^2}\right)$ of the second term of (9): This coefficient becomes larger when future beliefs become more sensitive to variations in current effort, and consequently, the marginal benefit of current effort increases. This, in turn, happens when the impact of effort on feedback (κ_a) increases or when future beliefs become more sensitive to feedback either due to a decrease in initial precision (v_0) or an increase in the signal precision (η). The effect of κ_θ on the signal-jamming incentives is non-monotonic. Specifically, signal-jamming incentives disappear when κ_θ is too low (feedback contains almost no information about θ) or too high (feedback is extremely precise). This implies that there is an interior value of κ_θ that maximizes the belief sensitivity of effort.

The next proposition establishes the effect of team size, and it states that as the individual effort level decreases in N, the total effort level increases. The result follows immediately by inspecting (9) and is therefore stated without proof.

Proposition 5. *If the agents are homogeneous ($c_i = c$ and $s_i = 1/N$ for all i), then for any $t \in [0, T]$, $\xi(t)$ decreases in N, and for any $t \in [0, T)$, $N\xi(t)$ increases in N.*

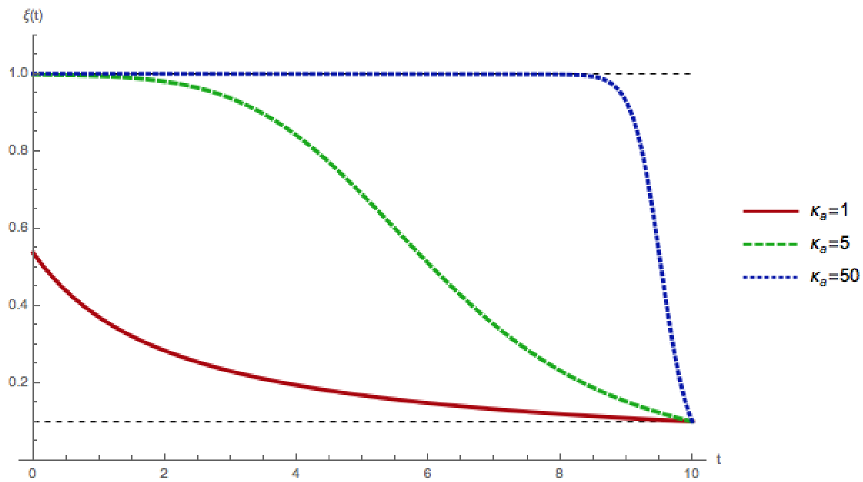


Figure 3: Equilibrium $\xi(t)$ in the homogeneous agents case, with different values of κ_a

3.3.3 Vanishing Free-riding Problem

Finally, we identify a limit condition under which the free-riding problem vanishes. The following proposition shows that with the share structure established in Proposition 3, the equilibrium $\xi_i(t)$ can be arbitrarily close to the socially efficient level in certain limiting cases.

Proposition 6. *Suppose that the share structure $s^* = (s_1^*, \ldots, s_N^*)$ is set as*

$$s_i^* \equiv \frac{\frac{1}{c_i}}{\sum_{j=1}^{N} \frac{1}{c_j}}.$$

Then, as $\kappa_a \to \infty$, the agents' equilibrium belief sensitivity $\xi_i(t)$ for any $t \in [0, \tau)$ converges (pointwise) to $1/c_i$.

Figure 3 shows the equilibrium $\xi(t)$ in the homogeneous agents case with different values of κ_a ($\kappa_a = 1, 5, 50$). Note that if the agents have the same c_i, then $s_i^* = 1/N$; thus, the equal share structure induces the socially efficient effort level when κ_a is large. Note that as κ_a increases, the equilibrium $\xi(t)$ becomes arbitrarily close to the socially efficient level ($1/c = 1$) for almost the full length of the horizon.

4 Stationary Model

In this section, we construct an infinite-horizon version of our model in which the state θ evolves stochastically over time. We show that our main result and underlying strategic incentives continue to hold in the infinite-horizon environment. Moreover, the stationary model generates an equilibrium with a simpler structure, providing a framework for a broader set of applications.[22]

Assume that the time horizon is infinite ($t = 0, 1, \ldots$) and that each period has length $\Delta > 0$.[23] Let θ_t be the state of the world in period t. We assume that θ_t follows a random walk

$$\theta_{t+1} = \theta_t + \sigma_t,$$

where σ_t is independently and identically drawn from the distribution $\mathcal{N}(0, \Delta/\eta_\sigma)$, and $\eta_\sigma > 0$ is the persistence of the state. Similar to the main model, the period-t feedback is given by $y_t = \Delta\left[\kappa_\theta \theta_t + \kappa_a \sum_{i=1}^N a_{it} + \varepsilon_t\right]$, where $\kappa_\theta, \kappa_a > 0$ are constants and $\varepsilon_t \sim \mathcal{N}(0, 1/\Delta\eta_\varepsilon)$. Assume that σ_t and ε_t are independent of one another. For simplicity, we consider a case with homogeneous agents $(c_i = 1)$ and equal share structure $(s_i = 1/N)$.

As in our main model, the posterior belief about the state after any history follows a normal distribution. Let μ_t and ν_t be the mean and precision of the public belief about θ_t at the beginning of period t, and let μ_t' and ν_t' describe the public belief *after* feedback y_t is realized. Then, we have $\mu_t' = \frac{\nu_t \mu_t + \kappa_\theta \eta_\varepsilon z_t}{\nu_t + \Delta\kappa_\theta^2 \eta_\varepsilon}$ and $\nu_t' = \nu_t + \Delta\kappa_\theta^2 \eta_\varepsilon$, where $z_t = y_t - \Delta\kappa_a \sum_i a_i^*(\overline{h}_i^t)$ is the signal for updating the public belief given the equilibrium strategy a_i^* and the "no-deviation" history $\overline{h}_i^t$ (defined in Subsection 3.1). Taking into account the effect of σ_t, the period-$(t+1)$ public

[22] In Subsections 5.3 and 5.4, we conduct the analysis using the stationary model.

[23] In the infinite-horizon model, we reinterpret the discount factor $e^{-r\Delta}$ as the probability of project survival: In each period, the project ends with probability $1 - e^{-r\Delta}$, and the team members share the total production.

belief is characterized by[24]

$$\mu_{t+1} = \mu_t' = \frac{\nu_t \mu_t + \kappa_\theta \eta_\varepsilon z_t}{\nu_t + \Delta \kappa_\theta^2 \eta_\varepsilon}, \tag{10}$$

$$\nu_{t+1} = \left(\frac{1}{\nu_t'} + \frac{\Delta}{\eta_\sigma}\right)^{-1} = \frac{\left(\nu_t + \Delta \kappa_\theta^2 \eta_\varepsilon\right)\eta_\sigma}{\Delta\left(\nu_t + \Delta \kappa_\theta^2 \eta_\varepsilon\right) + \eta_\sigma}. \tag{11}$$

It is easy to show that, for any value of the initial precision ν_0, as t goes to infinity, ν_t converges to a stationary level ν^*, which is given by[25]

$$\nu^* = \frac{\eta_\varepsilon \kappa_\theta^2}{2}\left(-\Delta + \sqrt{\Delta^2 + \frac{4\eta_\sigma}{\eta_\varepsilon \kappa_\theta^2}}\right). \tag{12}$$

We are interested in constructing a Markov perfect equilibrium that has a structure similar to the unique PBE of our main model: In equilibrium, the agent's action is linear in the mean of his private belief, that is, $a_i^*(h_i^t) = \xi_t \hat{\mu}_{it}$.

Even though we construct such Markov perfect equilibrium in general environments, for heuristic purposes, let us first consider the environment with stationary precision, that is, $\nu_t = \nu^*$ for all t. In this case, there exists a Markov perfect equilibrium with a stationary sensitivity level, that is, $\xi_t = \xi^*$ for all t. To compute ξ^*, consider the effect of a deviation in period t on the future beliefs in this equilibrium. Suppose that agent i deviates to $a = \xi^* \hat{\mu}_{it} + \alpha$. Then, the period-$(t+1)$ public posterior mean is given by $\mu_{t+1} = \hat{\mu}_{i,t+1} + \Lambda_a \alpha$, where $\Lambda_a \equiv \frac{\partial \mu_{t+1}}{\partial a_t} = \frac{\Delta \eta_\varepsilon \kappa_\theta \kappa_a}{\nu^* + \Delta \eta_\varepsilon \kappa_\theta^2}$. From period $(t+2)$ onward, the public belief and agent i's private belief diverge, creating a ratchet effect. A similar calculation as one in the main model yields $\mu_{t+k} = \hat{\mu}_{i,t+k} + \Lambda_a(\Lambda_\mu - \xi^* \Lambda_a)^{k-1}\alpha$ for any $k \geq 2$, where $\Lambda_\mu \equiv \frac{\partial \mu_{t+1}}{\partial \mu_t} = \frac{\nu^*}{\nu^* + \Delta \eta_\varepsilon \kappa_\theta^2}$. Therefore, the optimal effort level (which equals the marginal benefit of effort) is given by

$$a_{it}^* = \xi^* \hat{\mu}_{it} = \frac{\hat{\mu}_{it}}{N}\left(1 + (N-1)\xi^* \sum_{k=1}^{\infty} e^{-r\Delta k} \Lambda_a (\Lambda_\mu - \xi^* \Lambda_a)^{k-1}\right).$$

[24]Similar to Section 3, the private belief of agent i is updated using the same Gaussian updating process as (10) and (11) but with the signal $\hat{z}_{it} = y_t - \Delta \kappa_a (a_{it} + \sum_{j \neq i} a_j^*(\overline{h}_j^t))$ instead of z_t.

[25]The stationary level of ν^* is derived by setting $h_t = \nu_{t+1} = \nu^*$ in equation (11):

$$\nu^*(\Delta\left(\nu^* + \Delta \eta_\varepsilon \kappa_\theta^2\right) + \eta_\sigma) = (\nu^* + \Delta \eta_\varepsilon \kappa_\theta^2)\eta_\sigma.$$

This quadratic equation has a unique positive solution, as shown above.

Simplifying, we have a quadratic equation for ξ^*:

$$N(\xi^*)^2 - N(1-\Gamma)\xi^* - \Gamma = 0, \tag{13}$$

where $\Gamma = \frac{1-e^{-r\Delta}\Lambda_\mu}{e^{-r\Delta}\Lambda_a}$. There exists a unique positive solution for ξ^*, which is given by[26]

$$\xi^* = \frac{1-\Gamma+\sqrt{(1-\Gamma)^2+4\Gamma/N}}{2}. \tag{14}$$

The next proposition states the general result that for *any* initial precision v_0 under a sufficiently small Δ, there exists a linear Markov perfect equilibrium in which the belief sensitivity of effort converges to the stationary level.

Proposition 7. *Fix $v_0 > 0$. There exists $\bar{\Delta} > 0$ such that for any $\Delta < \bar{\Delta}$, there exists a sequence $\{\xi_t\}_{t=0}^{\infty}$ such that the agent's Markovian strategy $a^*_{it}(\hat{\mu}_{it}) = \xi_t \hat{\mu}_{it}$ is a Markov perfect equilibrium. Moreover, $\xi_t \to \xi^*$ as $t \to \infty$, where ξ^* is given by* (14).

In the Appendix, we provide the detailed construction of the linear Markov perfect equilibrium. First, we construct an agent's dynamic programming problem with μ_t and $\hat{\mu}_{it}$ as the state variables. Making use of our linear-quadratic-Gaussian framework, we guess a quadratic value function and a linear policy of the agent. Then, solving the problem yields recursive equations for the coefficients. Using a phase diagram, we show that for any v_0, there exists a unique value of ξ_0 such that the corresponding sequence $\{\xi_t\}$ converges to ξ^*. We show that this sequence satisfies the transversality condition and therefore constitutes an equilibrium strategy profile.

From (14), it is easy to derive various properties of the stationary belief sensitivity ξ^*, which we state in the next proposition.

Proposition 8. *In the stationary Markov perfect equilibrium,*

1. *for any $N \geq 2$, $\xi^* \in (1/N, 1)$;*

[26] Note that as $\Delta \to 0$, Γ converges to $\left(r\sqrt{\frac{\eta_\sigma}{\eta_\varepsilon}} + \kappa_\theta\right)/\kappa_a$. Therefore, the equilibrium in the continuous-time limit satisfies the "square-root law": The effect of doubling the discount rate on equilibrium behavior is equivalent to the effect of multiplying the volatility of the stochastic process by $\sqrt{2}$. A similar property is observed in various continuous-time models (Faingold and Sannikov, 2011; Daley and Green, 2012; Frei and Bernard, 2015).

2. $\frac{\partial \xi^*}{\partial N} < 0$;

3. $\frac{\partial \xi^*}{\partial r} < 0$;

4. $\frac{\partial \xi^*}{\partial \kappa_\theta} < 0$;

5. $\frac{\partial \xi^*}{\partial \kappa_a} > 0$*; and* $\lim_{\kappa_a \to \infty} \xi^* = 1$.

Part 1 implies that the main result in Section 3—that the presence of uncertainty alleviates the free-riding problem—-does not depend on the existence of a deadline. Moreover, it shows that the signal-jamming incentives are *permanent* with stochastically evolving states.

The comparative statics results of Parts 2-5 are analogous to the results in Section 3. Only Part 4 differs from the results of the main model, where the impact of κ_θ on ξ_t is non-monotone (Proposition 4). In the main model, the signal-jamming incentive disappears when κ_θ is too low, as feedback becomes uninformative *relative to* the prior belief about the persistent state θ. With stochastic states, however, the stationary precision v^* also becomes smaller as κ_θ decreases. In this case, feedback remains informative relative to the existing information, leading to a monotonic relationship between κ_θ and ξ^*.

5 Discussion: Team Design

Our analysis implies that the dynamics of the agents' incentives depend on the design of team structures, especially ones that affect the agents' beliefs and learning processes. In this section, we take advantage of the tractability of our framework to address questions concerning *optimal team design*. We discuss the following four aspects of teams: (1) imperfect monitoring of effort; (2) uncertainty of a project; (3) flexibility of a team; and (4) asymmetric information among team members. For the last two items, we conduct the analyses using the stationary model introduced in Section 4.

To keep the analysis simple and transparent, we assume throughout the section that the agents are homogeneous $(c_i = 1)$ and that they share the output equally $(s_i = 1/N)$ unless otherwise stated.

5.1 Role of Imperfect Monitoring

Our results highlight a mechanism whereby the presence of uncertainty indirectly induces signal-jamming incentives that alleviate free-riding. This mechanism contrasts with direct reward/punishment schemes that are extensively studied in the classic literature on teams (Alchian and Demsetz, 1972; Radner et al., 1986). It is interesting to note that while accurate performance measures enhance the effectiveness of direct reward schemes, such measures are detrimental in our model. In fact, our assumption that the agent's effort level is unobservable is crucial for the existence of the signal-jamming incentive. The following proposition shows that if a_{it} is perfectly observable to others, then each agent chooses the myopically optimal effort level. Its proof is straightforward and is thus omitted.

Proposition 9. *In the perfect monitoring case, there exists a unique PBE where for any* $t = 0, \ldots, T$,

$$a_{it}^* = \frac{s_i}{c_i}\mu_t.$$

From the perspective of team design, Proposition 9 highlights the role of signal-jamming incentives as an alternative to standard reward/punishment schemes. In the classic literature on teams, the inability to monitor individual effort (or its high cost) is typically considered a major obstacle to inducing cooperation.[27] Proposition 9 suggests that if the monitoring of individual effort is indeed costly, it may be advantageous to choose a project with uncertain prospects instead of investing in the monitoring structure and establishing formal incentive schemes.[28]

5.2 Optimal Level of Project Uncertainty

Risk taking is considered one of the main elements of entrepreneurial behavior. The literature suggests various explanations for risk-taking behavior, such as the higher premiums or

[27] For example, Alchian and Demsetz (1972) write, "...In team production, marginal products of cooperative team members are not so directly and separably (i.e., cheaply) observable...The costs of metering or ascertaining the marginal products of the team's members is what calls forth new organizations and procedures."

[28] Bonatti and Hörner (2011) also show that perfect monitoring may lead to more inefficient outcomes. Nevertheless, the mechanism underlying their result differs from that in our paper. In Bonatti and Hörner (2011), when it is *observed* that an agent worked hard, the agents are incentivized against contributing effort in the future. In contrast, in our paper, the agent's effort shows strategic complementarity over time *when effort is unobservable*, which provides positive incentives under imperfect monitoring.

risk-loving preferences of entrepreneurs. In this paper, we identify an alternative motivation: *undertaking an uncertain project can benefit organizations by mitigating the free-rider problem*. Our result indicates a natural trade-off between such a benefit and the standard cost of uncertainty due to the potential mismatch of the effort level and the state. In this subsection, we show that there exists an optimal level of uncertainty that balances the trade-off.

Suppose that the manager of a team faces a choice of projects with varying uncertainty. The team manager tries to maximize the ex ante total payoff of the team. To clarify our analysis of the trade-off, we consider the case in which all projects have *the same ex ante value under complete information*. Recall that if the project state θ is perfectly observed at the beginning, then the equilibrium action is $a_i^*(t) = \theta/N$ for all $t \in [0,T]$. Since the state θ is normally distributed with mean μ_0 and precision v_0, the agent's ex ante expected payoff *before* the realization of θ is

$$\begin{aligned}\mathbb{E}_0\left[\int_0^T \left(\theta \cdot a_i^*(t) - \frac{(a_i^*(t))^2}{2}\right) dt\right] &= \frac{T}{N}\left(1-\frac{1}{2N}\right)\mathbb{E}_0\left[\theta^2\right] \\ &= \frac{T}{N}\left(1-\frac{1}{2N}\right)\left(\mu_0^2+\frac{1}{v_0}\right).\end{aligned}$$

Note that the payoff structure of our model implies that the value of project is convex in θ. Therefore, choosing a risky project (one with a small v_0) is always beneficial under complete information.

Now, consider the original model where θ is unknown. We consider the optimal choice of uncertainty v_0 *subject to a constraint* $\mu_0^2 + \frac{1}{v_0} = k$ for some $k > 0$. This constraint requires that the mean of the project decreases as its level of uncertainty increases, offsetting the inherent benefit of risk taking described above.

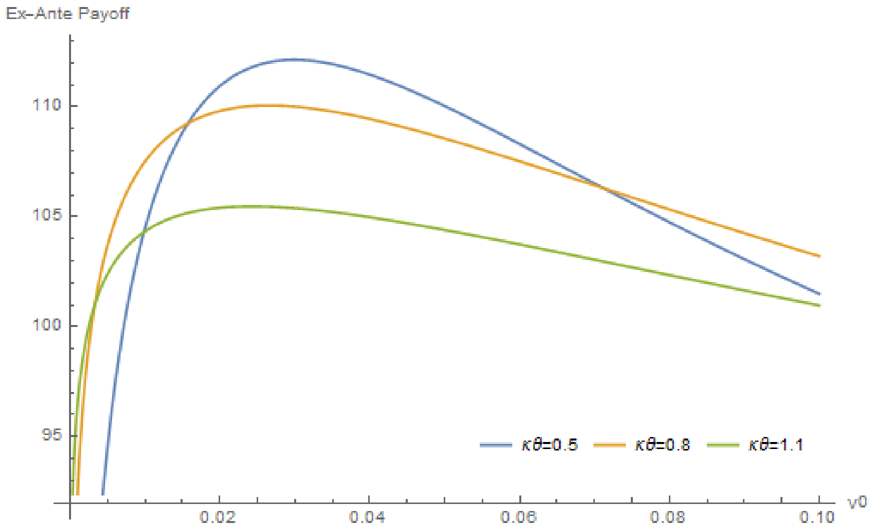


Figure 4: Ex ante payoff as a function of v_0, with different levels of κ_θ

Given the constraint, the ex ante expected equilibrium payoff is given by[29]

$$\begin{aligned}\mathbb{E}_0\left[\int_0^T\left(\theta\cdot a_i(t)-\frac{(a_i(t))^2}{2}\right)dt\right]&=\int_0^T\xi(t)\left(1-\frac{\xi(t)}{2}\right)\mathbb{E}_0\left[\mu(t)^2\right]dt.\\&=\int_0^T\xi(t)\left(1-\frac{\xi(t)}{2}\right)\left(k-\frac{1}{v(t)}\right)dt.\end{aligned}$$

Note that as the project uncertainty becomes larger, the cost of uncertainty (captured by the term $1/v(t)$) increases, while the free-riding problem is alleviated since $\xi(t)$ uniformly increases in v_0 for all $t\in(0,T)$.

Figure 4 illustrates that the trade-off between the two effects leads to an interior optimal level of uncertainty. The optimal level of uncertainty naturally depends on other parameters of the model. For example, Figure 4 shows that the larger is κ_θ, the larger is the optimal level of uncertainty. Intuitively, a larger κ_θ means that learning takes place faster. Therefore, the cost of uncertainty (via a mismatch between effort levels and the state) quickly disappears, leading to a higher optimal level of uncertainty.

[29]The second equality is derived by the distribution of equilibrium posterior mean μ_t: Since $z_t=\theta+\varepsilon_t$ on the equilibrium path, and the ε_ts are independent across time, we have $\sum_{s=0}^{t-1}z_s=t\theta+\sum_{s=0}^{t-1}\varepsilon_s\sim\mathcal{N}\left(t\mu_0,\frac{t^2}{v_0}+\frac{t}{v_\varepsilon}\right)$. Since $\mu_t=\frac{v_0}{v_t}\mu_0+\frac{v_\varepsilon}{v_t}\sum_{s=0}^{t-1}z_s$, $\mu_t\sim\mathcal{N}\left(\mu_0,\left(\frac{v_\varepsilon}{v_t}\right)^2\left(\frac{t^2}{v_0}+\frac{t}{v_\varepsilon}\right)\right)=\mathcal{N}\left(\mu_0,\frac{1}{v_0}-\frac{1}{v_t}\right)$. This result naturally extends to the continuous-time limit.

5.3 Effect of Flexibility

Another interesting question regarding team design is how the ability to frequently receive feedback and adjust actions accordingly affects the dynamics of team incentives. In our model of dynamic team production, flexibility is captured by (the inverse of) the period length Δ. In the literature on repeated games with imperfect public monitoring, it is well known that in the limit of flexible actions $(\Delta \to 0)$, cooperation may weaken (Abreu et al., 1991) or completely break down (Sannikov and Skrzypacz, 2007). In sharp contrast to these results, we show in this subsection that the degree of free-riding diminishes as the team becomes more flexible. This result highlights an interesting contrast between the signal-jamming mechanism in our paper and the classic punishment mechanism in the repeated games literature.

In our formal analysis of team flexibility, we use the stationary model developed in Section 4. Recall that in the stationary model, there exists (ξ^*, v^*) such that if $v_0 = v^*$ there exists a stationary Markov perfect equilibrium in which $a_{it}^* = \xi^* \hat{\mu}_{it}$ and $v_t = v^*$ for all t. Next, the proposition states that as the agents' actions become more flexible, the signal-jamming incentives in the stationary equilibrium become stronger.

Proposition 10. *In the stationary Markov perfect equilibrium,* $\frac{\partial \xi^*}{\partial \Delta} < 0$.

The intuition is straightforward. Suppose that Δ is arbitrarily large so that the agents are not able to frequently adjust their effort levels. Then, the effect of information on the future effort level would occur far in the future, and thus, the signal-jamming incentive vanishes. However, as Δ becomes smaller, the agents have more frequent opportunities to manipulate others' beliefs, leading to a higher benefit of effort.

Proposition 10 contrasts with the results in the classic literature on repeated games. Sannikov and Skrzypacz (2007) consider a repeated partnership game with imperfect public monitoring (but no uncertainty regarding the underlying state). They show that as as $\Delta \to 0$, it is impossible to achieve cooperation using the punishment scheme. Under flexible actions, any strategy profile must punish the agents based on the noisy information, which increases the cost of type I error, which eventually outweighs the benefit from future cooperation. In contrast, the equilibrium strategy profile of our paper does not involve any direct punishment, and the incentives become stronger as a team becomes more flexible. Our result suggests that the

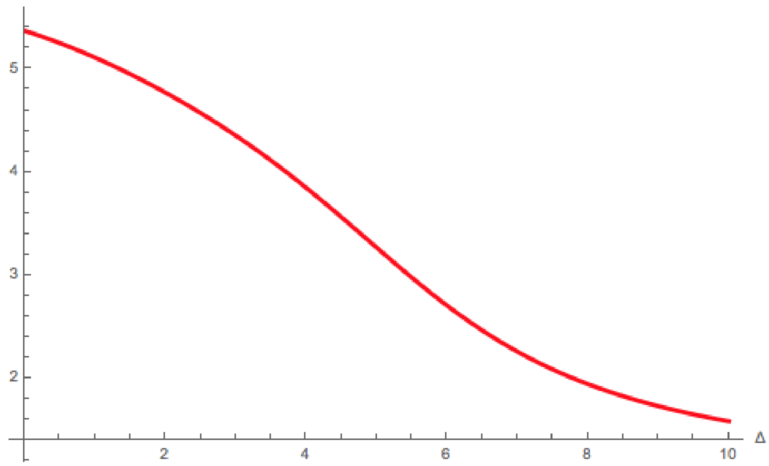


Figure 5: Ex ante payoff as a function of Δ

signal-jamming incentive in our model may work better than the standard punishment scheme under certain conditions of organizations.

To analyze welfare, note that in the stationary equilibrium, given any realization of θ_t, the mean of the public belief μ_t follows a normal distribution with mean θ_t and variance $1/\nu^*$. Therefore, conditional on the true state being θ_t, each agent's expected stage payoff is[30]

$$\xi^* \left(1 - \frac{\xi^*}{2}\right) \theta_t^2 - \frac{\xi^{*2}}{2\nu^*}. \tag{15}$$

Observe that the variable Δ can impact the payoff through two channels: (i) its impact on effort incentives (ξ^*); and (ii) its impact on the stationary level of uncertainty ($1/\nu^*$). As discussed above, more flexibility improves effort incentives. Moreover, inspecting (12) reveals that more flexibility increases the stationary precision ν^*. As a result, increased flexibility (i.e., smaller Δ) is generally welfare improving, as depicted in Figure 5.[31]

[30] Note that we analyze the welfare outcomes by computing an expected stage payoff given a fixed state θ_t. The ex ante payoff of the stationary model is not well defined, since θ_t follows a random walk so that the long-run expectation of θ_t^2 diverges. Yet, we claim that our exercise is valuable. First, if θ_t follows a mean-reverting process, the expectation of θ_t^2 would not diverge. In this case, we conjecture that the welfare result is consistent with our analysis. Second, for any initial finite number of periods, the expectation of θ_t^2 is finite, and in this case, the above qualitative discussion remains valid.

[31] This discussion ignores a second-order effect that appears in welfare calculations: as ξ^* increases, the cost of mismatch increases (as captured by the second term of (15)). If θ_t^2 is sufficiently large, such a negative effect becomes relatively weaker than the benefit from high ξ^*.

5.4 Asymmetric Information: Role of Experts

Since Hermalin (1998), the economics literature on leadership has analyzed the effect of information transmission and incentive provision by an informed member (leader) of a team.[32] Our framework enables us to analyze such effects of leadership in dynamic environments. For this, we extend the stationary model presented in the previous subsection to the asymmetric information case in which some team members perfectly know the true state. We refer to such team members as *experts*. The other team members, or *novices*, are uninformed, as in the previous model.

Consider a team with N^e experts and N^n novices $(N^e + N^n = N)$. We assume that the state θ_t follows a random walk, as in the previous subsection. The experts are perfectly informed of θ_t in each period, while the novices update their beliefs based on feedback. The period-t feedback is given by $y_t = \Delta[\kappa_\theta \theta_t + \kappa_a(N^e a_t^e + N^n a_t^n) + \varepsilon_t]$, where a_t^e and a_t^n are the period-t effort levels of experts and novices, respectively. We assume that the experts do not have a direct method of communication.

We construct an equilibrium in Markovian strategies in which the expert's effort level is linear in the current state and the novice's effort level is linear in the mean of his private belief, that is,

$$a_t^e = \gamma_t \theta_t, \qquad a_t^n = \xi_t \hat{\mu}_{it}.$$

Then, a sequence $\{(\gamma_t, \xi_t)\}_{t=0}^{\infty}$, combined with a sequence of belief precision levels $\{\nu_t\}_{t=0}^{\infty}$, completely describes the equilibrium strategy profile.

Note that under asymmetric information, the experts' actions can affect the precision of novices' belief ν_t. Given the above linear Markovian strategy profile (and the belief that the novices have never deviated in the past), the novices understand that feedback y_t on the equilibrium path is given by

$$y_t = \Delta[m_t \theta_t + \kappa_a N^n \xi_t \mu_t + \varepsilon_t],$$

[32] In the leadership literature, Hermalin (1998) and Komai et al. (2007) show two different channels through which an expert can improve the overall welfare of a team. In the former paper, expert effort (perfectly monitored) provides extra information that reduces the payoff loss resulting from uncertainty. In the latter paper, by working hard, the expert can *encourage* other members to work hard. In our model, both channels are present.

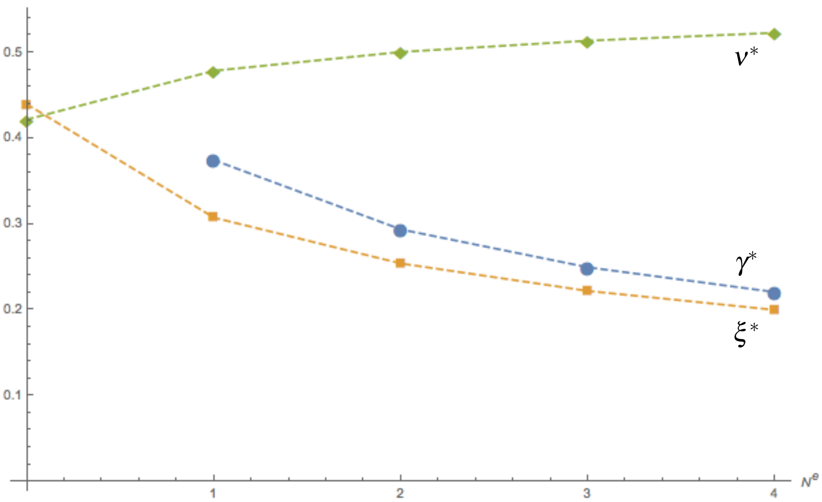


Figure 6: Value of (γ^*, ξ^*, ν^*) when $N^e = 0, \ldots, 4$ $(N = 5)$

where $m_t = \kappa_\theta + \kappa_a N^e \gamma_t$. Therefore, as the experts' response rate (γ_t) increases, feedback y_t becomes more informative about θ_t, leading to more precise novice beliefs.

We show that there exists a unique stationary Markov perfect equilibrium of the asymmetric information model, as stated in the next proposition.

Proposition 11. *There exists a unique triplet* (γ^*, ξ^*, ν^*) *such that if* $\nu_0 = \nu^*$*, the linear Markovian strategies*

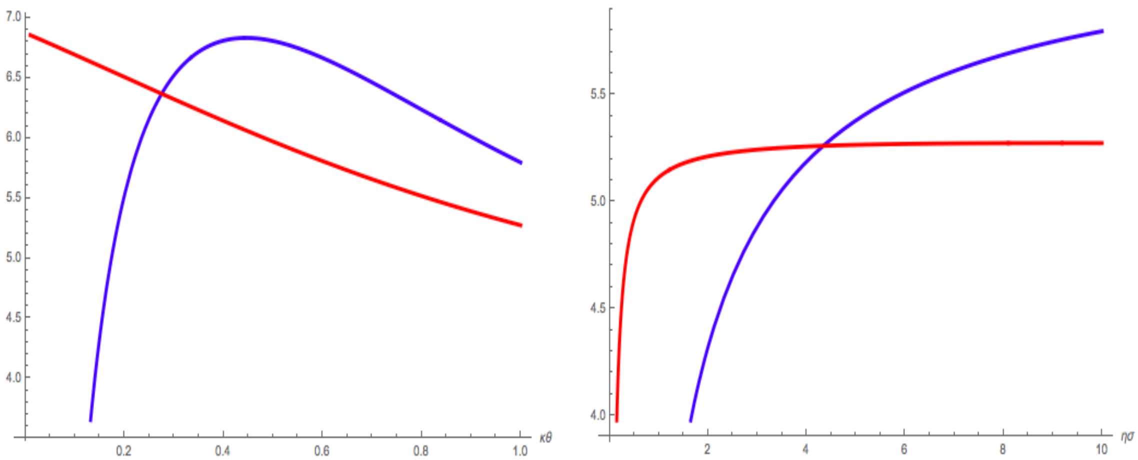
$$a_{it}^{e*}(\theta_t) = \gamma^* \theta_t, \qquad a_{it}^{n*}(\hat{\mu}_{it}) = \xi^* \hat{\mu}_{it},$$

constitute a Markov perfect equilibrium. In this equilibrium, $\nu_t = \nu^*$ *after any history.*[33]

Figure 6 plots the equilibrium values of ξ^* and γ^* and the stationary precision ν^* against the number of experts (N^e) on a 5-person team. First, note that as N^e increases, the team members' incentives (captured by ξ^* and γ^*) decrease. The intuition is straightforward: As there are fewer novices who are influenced by feedback, the incentives for signaling (for experts) and signal-jamming (for novices) decrease. However, the stationary precision ν^* increases in the number of experts, as the experts' efforts make feedback more informative about the state.

To measure the welfare effect of team composition as in the previous subsection, we calculate the expected stage payoff of the experts and the novices in the stationary equilibrium.

[33] For a general value of ν_0, finding a sequence (γ_t, ξ_t, ν_t) that converges to the stationary value is analytically intractable. However, the numerical simulations suggest that, similar to the analysis in Section 4, the continuity of the vector field may lead to the existence of such a sequence.



(a) aAs a function of κ_θ (b) As a function of η_σ

Figure 7: Welfare of a team with $(N^e, N^n) = (0,5)$ (blue lines) and a team with $(N^e, N^n) = (1,4)$ (red lines)

Given the period-t state θ_t, the sum of expected payoffs of all agents is given by

$$N^e \mathbb{E}\left[\frac{\theta_t}{N}(N^e\gamma^*\theta_t + N^n\xi^*\mu_t) - \frac{\gamma^{*2}\theta_t^2}{2}\right] + N^n \mathbb{E}\left[\frac{\theta_t}{N}(N^e\gamma^*\theta_t + N^n\xi^*\mu_t) - \frac{\xi^{*2}\mu_t^2}{2}\right]$$
$$= N^e\left(\gamma^* - \frac{\gamma^{*2}}{2}\right)\theta_t^2 + N^n\left[\left(\xi^* - \frac{\xi^{*2}}{2}\right)\theta_t^2 - \frac{\xi^{*2}}{2\nu^*}\right].$$

The above argument implies that having an expert helps a team when the benefit of providing more information outweighs the cost of an increasing free-riding effect. Figure 7 illustrates the impact of feedback responsiveness κ_θ (left panel) and state persistence η_σ (right panel) on the expected payoff of two teams: one with five novices (blue lines) and another with one expert and four novices (red lines). When feedback is uninformative of the state (small κ_θ) or the state is less persistent (small η_σ), then the informational contribution of an expert is important, so the team with an expert does better than the all-novice team. This advantage of having an expert disappears as κ_θ becomes large (so that the team members receive precise information even in the absence of an expert) or when η_σ becomes large (so that there is not much uncertainty).

Finally, we consider the impact of the share structure between experts and novices on team welfare. Figure 8 plots the total welfare of a team with $(N^e, N^n) = (1,4)$ as a function of the

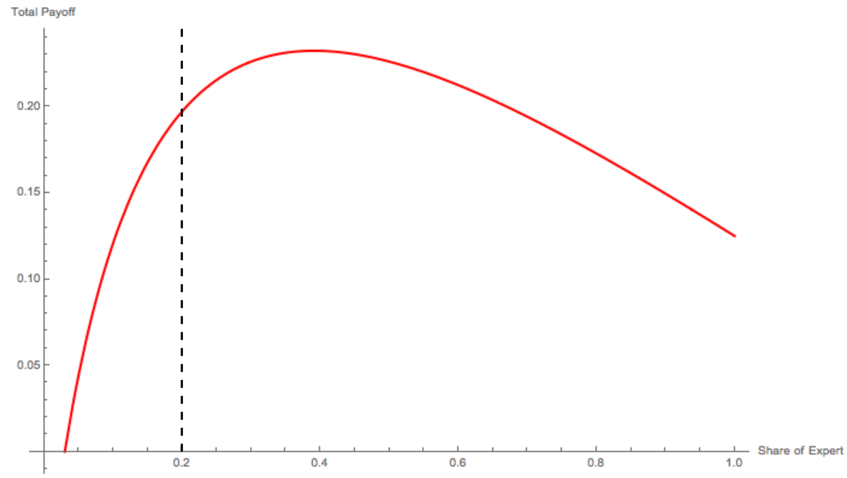


Figure 8: Team welfare as a function of the expert's share

expert's share, assuming that the remaining output is shared equally among the novices. Note that the optimal share structure is asymmetric, with the expert's share exceeding $1/N = 0.2$ (dashed line). Intuitively, the impact of increasing the share of experts is similar to that of replacing a novice with an expert. Thus, when the informational contribution of an expert becomes important (such as with small κ_θ and η_θ), the expert receives a larger share in the optimal share structure. Our numerical analysis is preliminary, and analyzing the optimal share structure (which may be history dependent) is left for future research.

6 Concluding Remarks

The main insights of our paper, along with the tractability of the linear-quadratic-Gaussian framework, present new opportunities for future research. Here, we discuss three potential research directions.

First, one could set up a model in which the payoff and informational externalities can be disentangled, and thus, the agents' efforts over time exhibit either strategic substitutability or complementarity conditional on the parameter values. This would provide a unified analysis of the effect of uncertainty on dynamic team production.

Second, it may be interesting to consider the case with private feedback. Suppose that the agents receive private feedback in addition to public feedback. Then, in contrast to our main model, other agents' beliefs also become relevant to making the effort choice, as they convey additional information about the true state. Then, in addition to signal-jamming, there will be a signaling incentive for each agent. Extending our model to this case would allow us to analyze additional issues, such as the optimal design of feedback structures in organizations.

Finally, comparison between the signal-jamming mechanism in our paper and the standard punishment mechanism deserves further exploration. In the infinite-horizon model, there exist many other non-Markovian equilibria that rely on a trigger mechanism to induce cooperation. Following the insight of Sannikov and Skrzypacz (2007), we conjecture that such trigger equilibria would not survive under flexible production, whereas the Markovian equilibrium of our paper would survive and continue to exhibit cooperation. Comparing other aspects of the two mechanisms and analyzing possible combinations of the two are left for future research.

Appendix

Proof of Proposition 1

First, recall that given any pure strategy profile, the Gaussian belief updating process (equations 1 and 2) implies that the agents' beliefs after every period-t history can be summarized by $(\mu_t, (\hat{\mu}_{1t}, \ldots, \hat{\mu}_{Nt}))$, where μ_t and $\hat{\mu}_{it}$ are the mean of the public belief and agent i's private belief in period t, respectively. Furthermore, since every public history is on the equilibrium path, all agents believe that the others have not deviated after any history, and thus, agent i believes that $\hat{\mu}_{jt} = \mu_t$ for all $j \neq i$.

We employ backward induction to prove the proposition. In the last period, each agent maximizes his stage payoff:

$$a_i^*(h_i^T) = \arg\max_a \mathbb{E}_T \left[\Delta s_i \theta \left(a + \sum_{j \neq i} a_{jT}^* \right) \right] - \Delta c_i \frac{a^2}{2} = \arg\max_a s_i \hat{\mu}_{iT} \left(a + \sum_{j \neq i} a_{jT}^* \right) - c_i \frac{a^2}{2}.$$

The first-order condition yields agent i's unique equilibrium effort $a_{iT}^* = (s_i/c_i)\hat{\mu}_{iT} = \xi_{iT}\hat{\mu}_{iT}$.

Now, suppose that the claim of the proposition holds for period $t+1$ onward—that is, in any equilibrium of the game, agent i plays $a_i^*(h_i^k) = \xi_{ik}\hat{\mu}_{ik}$ for $k = t+1, \ldots, T$, where ξ_{ik} is defined in (4). To solve agent i's optimization problem in period t, we first compute the impact of his effort choice on future public beliefs. Recall (from Section 3.1) that $\bar{a}_{it} = a^*(\bar{h}_i^t)$ is agent i's equilibrium effort at public history h^t if he has not deviated in the past—that is, the other agents expect agent i to play $\bar{a}_{it}$ at h^t. Suppose that agent i plays a in period t. Then, using (1) and (2), we have

$$\mu_{t+1} - \hat{\mu}_{i,t+1} = \frac{\nu_t}{\nu_{t+1}}(\mu_t - \hat{\mu}_{it}) + \frac{\Delta\kappa_a\kappa_\theta\eta_\varepsilon}{\nu_{t+1}}(a - \bar{a}_{it}) = \frac{\rho_{t+1}}{\rho_t}(\mu_t - \hat{\mu}_{it}) + \rho_{t+1}(a - \bar{a}_{it}). \tag{16}$$

For periods $t+2$ onward, we use the induction hypothesis that in period $l = t+1, \ldots, T$ agent i plays $a_{il} = \xi_{il}\hat{\mu}_{il}$, while the others expect him to play $\bar{a}_{il} = \xi_{il}\mu_l$. Therefore, we have

$$\mu_k - \hat{\mu}_{ik} = (\mu_{t+1} - \hat{\mu}_{i,t+1}) \prod_{l=t+1}^{k-1} \left(\frac{\nu_t}{\nu_{t+1}} - \xi_{il}\rho_{l+1} \right) = (\mu_{t+1} - \hat{\mu}_{i,t+1}) \frac{\rho_k}{\rho_{t+1}} \prod_{l=t+1}^{k-1} (1 - \xi_{il}\rho_l), \tag{17}$$

for $k = t+2, \ldots, T$. Substituting (16) into (17) and re-arranging, we obtain

$$\mu_k = \hat{\mu}_{ik} + \left(\frac{1}{\rho_t}(\mu_t - \hat{\mu}_{it}) + (a - \bar{a}_{it}) \right) \rho_k \prod_{l=t+1}^{k-1} (1 - \xi_{il}\rho_l). \tag{18}$$

In period t, agent i's optimization problem is given by

$$\begin{aligned} a_i^*(h_i^t) &= \arg\max_a s_i \hat{\mu}_{it} \left(a + \sum_{j \neq i} a_j^*(\bar{h}_{jt}) \right) - c_i \frac{a^2}{2} \\ &+ \mathbb{E}_t \left[\sum_{k=t+1}^{T} e^{-r\Delta(k-t)} \left(\left(s_i\xi_{ik} - c_i \frac{\xi_{ik}^2}{2} \right) \hat{\mu}_{ik}^2 + s_i \sum_{j \neq i} \xi_{jk}\mu_k\hat{\mu}_{ik} \right) \mid a \right]. \end{aligned}$$

Note that $\hat{\mu}_{ik}$ for $k = t+1, \ldots, T$ is independent of a and has expectation $\hat{\mu}_{it}$. Substituting μ_k with (18), eliminating additive terms that are independent of a, and replacing $\hat{\mu}_{ik}$ with its expectation whenever appropriate, agent i's problem is rewritten as

$$a_i^*(h_i^t) = \arg\max_a \; s_i\hat{\mu}_{it} \left[1 + \sum_{k=t+1}^{T} e^{-r\Delta(k-t)} \sum_{j \neq i} \xi_{jk}\rho_k \prod_{l=t+1}^{k-1} (1 - \xi_{il}\rho_l) \right] a - c_i \frac{a^2}{2}.$$

It is clear that the problem is concave in a and has a unique solution. The first-order condition immediately yields the desired result.

Proof of Proposition 2

First, we show that there exists a unique solution to (9), which defines an autonomous first-order nonlinear system of differential equations. Define $\mathbf{u}(t) = (\xi_1(t), \dots, \xi_N(t), v(t))$; then $d\mathbf{u}(t) = F(\mathbf{u}(t))$, where F is a Lipschitz continuous function given any $(\kappa_a, \kappa_\theta, r, \eta_\varepsilon, \eta_\theta)$. Then, by the Picard-Lindelöf theorem (Teschl (2012), Theorem 2.2), there exists a unique solution to this system in the domain $[0,T]$ with the boundary values $\xi_i(\tau) = s_i/c_i$ for all i.

To establish monotonicity, we first observe that $\dot{\xi}_i(\tau) < 0$. Suppose, for a contradiction, that there exist i and $\tilde{t} \in [0,\tau)$ such that $\dot{\xi}_i(\tilde{t}) > 0$. By the continuity of $\dot{\xi}_i(t)$, there exists $\hat{\Delta}_i > 0$ such that $\dot{\xi}_i(t) < 0$ for $t \in (\tau - \hat{\Delta}_i, \tau]$ and $\dot{\xi}_i(\tau - \hat{\Delta}_i) = 0$. Without loss of generality, assume that $i = 1$ attains $\min\{\hat{\Delta}_i | i = 1, \cdots, N\}$, with the convention that $\hat{\Delta}_j = \infty$ if $\dot{\xi}_j(t) < 0$ for all t. This in particular implies that $\dot{\xi}_i(\tau - \hat{\Delta}_1) \le 0$ for $i \neq 1$.

Next, we claim that $\ddot{\xi}_1(\tau - \hat{\Delta}_1) > 0$. By taking derivatives of both sides of (9) and using $\dot{\xi}_1(\tau - \hat{\Delta}_1) = 0$, we obtain

$$\ddot{\xi}_1(\tau - \hat{\Delta}_1) = \frac{\eta_\varepsilon^2 \kappa_\theta^3 \kappa_a}{\left(v_0 + \eta_\varepsilon t \kappa_\theta^2\right)^2} \left(\frac{s_1}{c_1} \sum_{j=1}^{N} \xi_j(\tau - \hat{\Delta}_1) - \xi_1(\tau - \hat{\Delta}_1)^2 \right) - \frac{\eta_\varepsilon \kappa_\theta \kappa_a}{v_0 + \eta_\varepsilon t \kappa_\theta^2} \frac{s_1}{c_1} \sum_{j=1}^{N} \dot{\xi}_j(\tau - \hat{\Delta}_1).$$

Since over $(\tau - \hat{\Delta}_1)$, ξ_1 is strictly decreasing, $\xi_1(\tau - \hat{\Delta}_1) > s_1/c_1$. Therefore, the first additive term in (9) is positive. Then, $\xi_1(\tau - \hat{\Delta}_1) = 0$ implies by (9) that

$$\frac{s_1}{c_1} \sum_{j=1}^{N} \xi_j(t) - \xi_1(t)^2 > 0.$$

This, together with $\dot{\xi}_j(\tau - \hat{\Delta}_1) \le 0$, establishes our claim that $\ddot{\xi}_1(\tau - \hat{\Delta}_1) > 0$. Now, since $\dot{\xi}_1$ is continuous, $\dot{\xi}_1(\tau - \hat{\Delta}_1) = 0$ and $\ddot{\xi}_1(\tau - \hat{\Delta}_1) > 0$, there exists $\varepsilon > 0$ such that $\dot{\xi}_1(t) > 0$ whenever $t \in (\tau - \hat{\Delta}_1 - \varepsilon, \tau - \hat{\Delta}_1)$, a contradiction establishing that for all t, $\dot{\xi}_t \le 0$.

Monotonicity immediately implies the lower bound on $\xi_i(t)$. Again, $\dot{\xi}_i(t) \le 0$ implies, by

(9), that the term in parentheses on the right-hand side must be positive. That is,

$$\frac{s_i}{c_i}\sum_{j=1}^{N}\xi_j(t) > \xi_i(t)^2.$$

Taking the square roots of both sides, summing over i and rearranging establishes the upper bound. The above argument implies that a solution does not blow up in finite time; therefore, a solution exists $\forall\tau\in[0,\infty)$.

Proof of Proposition 6

By (9), whenever $\xi_i(t)$ is less than the upper bound, $\dot{\xi}_i(t)\to-\infty$ as $\kappa_a\to\infty$. Together with monotonicity shown in Proposition 2, this implies that $\xi_i(t)$ converges pointwise to its upper bound, which is $1/c_i$ under the specified sharing rule.

Proof of Proposition 7

Dynamic Programming

Recall from (10) and (11) that the mean and the precision of the period-t public belief is recursively defined as

$$\mu_{t+1}=\frac{v_t\mu_t+\kappa_\theta\eta_\varepsilon z_t}{v_t+\Delta\kappa_\theta^2\eta_\varepsilon}, \qquad v_{t+1}=\frac{\left(v_t+\Delta\kappa_\theta^2\eta_\varepsilon\right)\eta_\sigma}{\Delta\left(v_t+\Delta\kappa_\theta^2\eta_\varepsilon\right)+\eta_\sigma},$$

where $z_t=y_t-\Delta\kappa_a\sum_i a_i^*(\overline{h}_i^t)$ and $a_i^*(\overline{h}_i^t)$ is the equilibrium strategy given that the agents have never deviated in the past. The private belief of agent i—characterized by the mean $\hat{\mu}_{it}$ and precision $\hat{v}_{it}$—is updated using the same Gaussian updating process but with the signal $\hat{z}_{it}=y_t-\Delta\kappa_a(a_{it}+\sum_{j\neq i}a_j^*(\overline{h}_j^t))$ instead of z_t. Note that the public and private beliefs have the same precision ($v_t=\hat{v}_{it}$ for any t), and v_t is independent after any public history.

Now, consider the agent's optimization problem. Given the linear Markovian strategy $a_{it}^*=\xi_t\hat{\mu}_{it}$, the agent's expected continuation payoff is a function of $t,\hat{\mu}_{it}$ and μ_t. Denote the agent's period-t continuation payoff as $V_t(\hat{\mu}_{it},\mu_t)$. Then, agent i's the optimization problem is written as

$$V_t(\hat{\mu}_{it},\mu_t) = \max_a \frac{\hat{\mu}_{it}}{N}(a+(N-1)\xi_t\mu_t) - \frac{a^2}{2} + e^{-r\Delta}\mathbb{E}_t\left[V_{t+1}(\hat{\mu}_{i,t+1},\mu_{t+1})\right].$$

Suppose that there exist constants $\omega_{t+1}, \alpha_{t+1}$ and β_{t+1} such that $V_{t+1}(\hat{\mu}_{i,t+1},\mu_{t+1}) = \omega_{t+1} + \alpha_{t+1}\hat{\mu}_{i,t+1}^2 + \beta_{t+1}\hat{\mu}_{i,t+1}\mu_{t+1}$. Then, we have

$$\begin{aligned} V_t(\hat{\mu}_{it},\mu_t) &= \max_a \frac{\hat{\mu}_{it}}{N}(a+(N-1)\xi_t\mu_t) - \frac{a^2}{2} \\ &\quad + e^{-r\Delta}\left(\omega_{t+1} + \alpha_{t+1}\mathbb{E}_t[\hat{\mu}_{i,t+1}^2] + \beta_{t+1}\mathbb{E}_t[\hat{\mu}_{i,t+1}\mu_{t+1}]\right). \end{aligned}$$

Since

$$\begin{aligned} \mathbb{E}_t[y_t] &= \Delta[\kappa_\theta\hat{\mu}_{it} + \kappa_a(a+(N-1)\xi_t\mu_t)] \\ \mathbb{E}_t[z_t] &= \mathbb{E}_t[y_t - \Delta\kappa_a N\xi_t\mu_t] = \Delta[\kappa_\theta\hat{\mu}_{it} + \kappa_a(a-\xi_t\mu_t)] \\ \mathbb{E}_t[\hat{z}_{it}] &= \mathbb{E}_t[y_t - \Delta\kappa_a(a+(N-1)\xi_t\mu_t)] = \Delta\kappa_\theta\hat{\mu}_{it} \\ \mathbb{E}_t[\hat{z}_{it}z_t] &= \mathbb{E}_t[y_t^2] - \mathbb{E}_t[y_t]\Delta\kappa_a(N\xi_t\mu_t + (a+(N-1)\xi_t\mu_t)) + \Delta^2\kappa_a^2(a+(N-1)\xi_t\mu_t)N\xi_t\mu_t \\ &= (\mathbb{E}_t[y_t] - \Delta\kappa_a(a+(N-1)\xi_t\mu_t))(\mathbb{E}_t[y_t] - \Delta\kappa_a N\xi_t\mu_t) + Var_t(y_t) \\ &= \mathbb{E}_t[\hat{z}_{it}]\mathbb{E}_t[z_t] + Var_t(y_t) \\ \mathbb{E}_t[\hat{z}_{it}^2] &= (\mathbb{E}_t[\hat{z}_{it}])^2 + Var_t(y_t), \end{aligned}$$

where $Var_t(y_t) = \Delta^2[\kappa_\theta^2 Var_t(\theta_t) + Var_t(\varepsilon_t)] = \frac{\Delta^2\kappa_\theta^2}{\nu_t} + \frac{\Delta}{\eta_\varepsilon}$, we have

$$\begin{aligned} \mathbb{E}_t[\hat{\mu}_{i,t+1}^2] &= \mathbb{E}_t\left[\left(\frac{\nu_t\hat{\mu}_{it} + \kappa_\theta\eta_\varepsilon\hat{z}_{it}}{\nu_t + \Delta\kappa_\theta^2\eta_\varepsilon}\right)^2\right] = \hat{\mu}_{it}^2 + \left(\frac{\kappa_\theta\eta_\varepsilon}{\nu_t + \Delta\kappa_\theta^2\eta_\varepsilon}\right)^2 Var_t(y_t), \\ \mathbb{E}_t[\hat{\mu}_{i,t+1}\mu_{t+1}] &= \mathbb{E}_t\left[\left(\frac{\nu_t\hat{\mu}_{it} + \kappa_\theta\eta_\varepsilon\hat{z}_{it}}{\nu_t + \Delta\kappa_\theta^2\eta_\varepsilon}\right)\left(\frac{\nu_t\mu_t + \kappa_\theta\eta_\varepsilon z_t}{\nu_t + \Delta\kappa_\theta^2\eta_\varepsilon}\right)\right] \\ &= \hat{\mu}_{it}\frac{\nu_t\mu_t + \Delta\kappa_\theta\eta_\varepsilon(\kappa_\theta\hat{\mu}_{it} + \kappa_a(a-\xi_t\mu_t))}{\nu_t + \Delta\kappa_\theta^2\eta_\varepsilon} + \left(\frac{\kappa_\theta\eta_\varepsilon}{\nu_t + \Delta\kappa_\theta^2\eta_\varepsilon}\right)^2 Var_t(y_t). \end{aligned}$$

Therefore, the first-order condition for a yields

$$a_{it}^{*}=\left(\frac{1}{N}+e^{-r\Delta}\beta_{t+1}\frac{\Delta\kappa_{\theta}\kappa_{a}\eta_{\varepsilon}}{\nu_{t}+\Delta\kappa_{\theta}^{2}\eta_{\varepsilon}}\right)\hat{\mu}_{it},$$

which is linear in $\hat{\mu}_{it}$, with

$$\xi_{t}=\frac{1}{N}+e^{-r\Delta}\beta_{t+1}\frac{\Delta\kappa_{\theta}\kappa_{a}\eta_{\varepsilon}}{\nu_{t}+\Delta\kappa_{\theta}^{2}\eta_{\varepsilon}}. \tag{19}$$

Plugging in $a_{it}^{*}=\xi_{t}\hat{\mu}_{it}$ into the payoff function, we have

$$\begin{aligned}V_{t}(\hat{\mu}_{it},\mu_{t}) &= \frac{\hat{\mu}_{it}}{N}(\xi_{t}\hat{\mu}_{it}+(N-1)\xi_{t}\mu_{t})-\frac{\xi^{2}}{2}\hat{\mu}_{it}^{2}\\ &+e^{-r\Delta}\left[\omega_{t+1}+\alpha_{t+1}\left(\hat{\mu}_{it}^{2}+\left(\frac{\kappa_{\theta}\eta_{\varepsilon}}{\nu_{t}+\Delta\kappa_{\theta}^{2}\eta_{\varepsilon}}\right)^{2}Var_{t}(y_{t})\right)\right)\\ &+\beta_{t+1}\left(\left(\hat{\mu}_{it}\frac{\nu_{t}\mu_{t}+\Delta\kappa_{\theta}\eta_{\varepsilon}(\kappa_{\theta}\hat{\mu}_{it}+\kappa_{a}\xi_{t}(\hat{\mu}_{it}-\mu_{t}))}{\nu_{t}+\Delta\kappa_{\theta}^{2}\eta_{\varepsilon}}+\left(\frac{\kappa_{\theta}\eta_{\varepsilon}}{\nu_{t}+\Delta\kappa_{\theta}^{2}\eta_{\varepsilon}}\right)^{2}Var_{t}(y_{t})\right)\right].\end{aligned}$$

Simplifying, we have the following recursive equations for $(\omega_{t},\alpha_{t},\beta_{t})$:

$$\begin{aligned}\omega_{t} &= e^{-r\Delta}\left(\omega_{t+1}+(\alpha_{t+1}+\beta_{t+1})\left(\frac{\kappa_{\theta}\eta_{\varepsilon}}{\nu_{t}+\Delta\kappa_{\theta}^{2}\eta_{\varepsilon}}\right)^{2}Var_{t}(y_{t})\right)\\ \alpha_{t} &= \frac{\xi_{t}}{N}-\frac{\xi_{t}^{2}}{2}+e^{-r\Delta}\left(\alpha_{t+1}+\beta_{t+1}\frac{\Delta\kappa_{\theta}\eta_{\varepsilon}(\kappa_{\theta}+\kappa_{a}\xi_{t})}{\nu_{t}+\Delta\kappa_{\theta}^{2}\eta_{\varepsilon}}\right)\\ \beta_{t} &= \frac{N-1}{N}\xi_{t}+e^{-r\Delta}\beta_{t+1}\frac{\nu_{t}-\Delta\kappa_{\theta}\kappa_{a}\eta_{\varepsilon}\xi_{t}}{\nu_{t}+\Delta\kappa_{\theta}^{2}\eta_{\varepsilon}}.\end{aligned} \tag{20}$$

Combining (19) and (20) yields the formula of β_{t} in terms of ξ_{t}, which is

$$\beta_{t}=\xi_{t}+\frac{\nu_{t}}{\Delta\kappa_{\theta}\kappa_{a}\eta_{\varepsilon}}\left(\xi_{t}-\frac{1}{N}\right)-\xi_{t}^{2}. \tag{21}$$

Then, by plugging (21) back into (20), we derive the recursive formulation for ξ_{t}:

$$\xi_{t}=\frac{1}{N}+e^{-r\Delta}\frac{1}{\nu_{t}+\Delta\kappa_{\theta}^{2}\eta_{\varepsilon}}\left(\Delta\kappa_{\theta}\kappa_{a}\eta_{\varepsilon}(\xi_{t+1}-\xi_{t+1}^{2})+\nu_{t+1}\left(\xi_{t+1}-\frac{1}{N}\right)\right). \tag{22}$$

Equilibrium Profile

Given the recursive formation, we conduct the phase diagram analysis to construct linear Markov perfect equilibrium. Equation (22), combined with the recursive equation for v_t

$$v_{t+1} = \frac{\left(v_t + \Delta\kappa_\theta^2 \eta_\varepsilon\right)\eta_\sigma}{\Delta\left(v_t + \Delta\kappa_\theta^2 \eta_\varepsilon\right) + \eta_\sigma}, \tag{23}$$

determines the evolution of (v_t, ξ_t) in equilibrium. Equation (23) implies that given an initial precision $v_0 > 0$, the sequence $\{v_t\}_{t=0}^{\infty}$ is deterministic and is independent of the strategy profile. Since $v_{t+1} - v_t = -\Delta\frac{v_t^2 + \Delta\kappa_\theta^2\eta_\varepsilon v_t - \kappa_\theta^2\eta_\varepsilon\eta_\sigma}{\Delta\left(v_t+\Delta\kappa_\theta^2\eta_\varepsilon\right)+\eta_\sigma}$, there exists a unique positive fixed point $v^* = \frac{\eta_\varepsilon\kappa_\theta^2}{2}\left(-\Delta + \sqrt{\Delta^2 + \frac{4\eta_\sigma}{\eta_\varepsilon\kappa_\theta^2}}\right)$. Furthermore, since $\frac{\partial v_{t+1}}{\partial v_t} = \left(\frac{\eta_\sigma}{\Delta\left(v_t+\Delta\kappa_\theta^2\eta_\varepsilon\right)+\eta_\sigma}\right)^2 \in (0,1)$ for any $v_t > 0$, the sequence $\{v_t\}$ converges to v^* as $t \to \infty$ for any value of $v_0 > 0$.

To analyze the dynamics of ξ_t, we rewrite (22) as

$$\xi_t - \frac{1}{N} = e^{-r\Delta}\Lambda_{\mu t}\left(\xi_{t+1} - \frac{1}{N}\right) + e^{-r\Delta}\Lambda_{at}\left(\xi_{t+1} - \xi_{t+1}^2\right), \tag{24}$$

where $\Lambda_{\mu t} = \frac{v_{t+1}}{v_t + \Delta\kappa_\theta^2\eta_\varepsilon}$ and $\Lambda_{at} = \frac{\Delta\kappa_\theta\kappa_a\eta_\varepsilon}{v_t+\Delta\kappa_\theta^2\eta_\varepsilon}$. Solving for ξ_{t+1} yields

$$\xi_{t+1}(\xi_t) = \begin{cases} \xi_{t+1}^{(+)}(\xi_t) \equiv \dfrac{(\Lambda_{\mu t}+\Lambda_{at}) + \sqrt{(\Lambda_{\mu t}+\Lambda_{at})^2 - \frac{4\Lambda_{at}}{e^{-r\Delta}}\left(\xi_t - \frac{1-e^{-r\Delta}\Lambda_{\mu t}}{N}\right)}}{2\Lambda_{at}}, \\ \xi_{t+1}^{(-)}(\xi_t) \equiv \dfrac{(\Lambda_{\mu t}+\Lambda_{at}) - \sqrt{(\Lambda_{\mu t}+\Lambda_{at})^2 - \frac{4\Lambda_{at}}{e^{-r\Delta}}\left(\xi_t - \frac{1-e^{-r\Delta}\Lambda_{\mu t}}{N}\right)}}{2\Lambda_{at}}. \end{cases} \tag{25}$$

Note that both $\xi_{t+1}^{(+)}(\xi_t)$ and $\xi_{t+1}^{(-)}(\xi_t)$ are well defined if and only if $\xi_t \le \tilde{\xi}$, where

$$(\Lambda_{\mu t}+\Lambda_{at})^2 - \frac{4\Lambda_{at}}{e^{-r\Delta}}\left(\tilde{\xi} - \frac{1-e^{-r\Delta}\Lambda_{\mu t}}{N}\right) = 0 \implies \tilde{\xi} = \frac{1-e^{-r\Delta}\Lambda_{\mu t}}{N} + \frac{e^{-r\Delta}(\Lambda_{\mu t}+\Lambda_{at})^2}{4\Lambda_{at}},$$

and $\xi_{t+1}^{(+)}(\tilde{\xi}) = \xi_{t+1}^{(-)}(\tilde{\xi}) = \frac{\Lambda_{\mu t}+\Lambda_{at}}{2\Lambda_{at}}$.

Fix $\hat{v} > 0$. Then, it is easy to check that for any $\varepsilon > 0$, there exists $\bar{\Delta}$ such that if $\Delta < \bar{\Delta}$, then $\Lambda_{\mu t} > 1-\varepsilon$ and $\Lambda_{at} < \varepsilon$ for any $v_t > \hat{v}$. Moreover, $\lim_{\Delta\to 0}\frac{\xi_{t+1}^{(+)}(\tilde{\xi})}{\tilde{\xi}} = \frac{2}{e^{-r\Delta}} > 1$. Thus, for sufficiently small Δ, $\xi_{t+1}^{(+)}(\tilde{\xi}) > \tilde{\xi} > 1$ for any $v_t > \hat{v}$. Since $\xi_{t+1}^{(+)}(\xi_t) > \xi_{t+1}^{(+)}(\tilde{\xi})$ for all $\xi_t < \tilde{\xi}$, if we set $\xi_{t+1} = \xi_{t+1}^{(+)}(\xi_t)$, then $\xi_{t+1} > \tilde{\xi}$, so ξ_{t+2} is not well defined. Therefore, in order for the sequence $\{\xi_t\}$ to be well defined, it must be that $\xi_{t+1} = \xi_{t+1}^{(-)}(\xi_t)$.

On the other hand, from (24) we have

$$\xi_{t+1} - \xi_t = e^{-r\Delta}\Lambda_{at}(\xi_{t+1} - \hat{\xi}(v_t))(\xi_{t+1} - \underline{\xi}(v_t)), \tag{26}$$

where

$$\hat{\xi}(v_t) = \frac{1-\Gamma_t + \sqrt{(1-\Gamma_t)^2 + 4\Gamma_t/N}}{2},$$

$\underline{\xi}(v_t) = \frac{1-\Gamma_t - \sqrt{(1-\Gamma_t)^2+4\Gamma_t/N}}{2}$, and $\Gamma_t = \frac{1-e^{-r\Delta}\Lambda_{\mu t}}{e^{-r\Delta}\Lambda_{at}} = \frac{(v_t+\Delta\kappa_\theta^2\eta_\varepsilon)(v_t+\Delta\kappa_\theta^2\eta_\varepsilon+(1-e^{-r\Delta})v_\sigma)}{(v_t+\Delta\kappa_\theta^2\eta_\varepsilon+v_\sigma)e^{-r\Delta}\Delta\kappa_\theta\kappa_a\eta_\varepsilon} > 0$. It is easy to check that $\hat{\xi}(v_t) \in (1/N, 1)$ and $\underline{\xi}(v_t) < 0$.

The above argument implies that $(v_{t+1}, \xi_{t+1}) = G(v_t, \xi_t)$, where $G_1(v_t, \xi_t) = \frac{(v_t+\Delta\kappa_\theta^2\eta_\varepsilon)\eta_\sigma}{\Delta(v_t+\Delta\kappa_\theta^2\eta_\varepsilon)+\eta_\sigma}$, and $G_2(v_t, \xi_t) = \xi_{t+1}^{(-)}(\xi_t)$. Figure 9 describes the phase diagram of (v_t, ξ_t) induced by G. Note that the space is divided into four regions by the two lines $v_t = v^*$ and $\xi_t = \hat{\xi}(v_t)$. The horizontal line $v_t = v^*$ illustrates the dynamics of v_t: If $v_t < v^*(v_t > v^*)$, then v_{t+1} is greater than (smaller than) v_t but v_{t+1} never crosses v^*. Therefore, regardless of the initial precision v_0, the sequence $\{v_t\}$ is monotone and converges to v^* as $t \to \infty$. On the other hand, the dynamics of ξ_t are depicted by the line $\xi_t = \hat{\xi}(v_t)$. Note that $\hat{\xi}(v_t)$ is downward sloping and intersects with the $v_t = v^*$ line at (v^*, ξ^*).[34]

Then, by the continuity of the vector field G, there must exist a unique curve $\mathscr{P}$ that passes through (v^*, ξ^*) such that if $(v_t, \xi_t) \in \mathscr{P}$, then $G(v_t, \xi_t) \in \mathscr{P}$. Note that $\mathscr{P}$ must lie on the upper-right and lower-left regions. In Figure 9, $\mathscr{P}$ is depicted as a blue line. Then, $\mathscr{P}$

[34] To prove that $\hat{\xi}(v_t)$ is downward sloping, note that

$$\begin{aligned} \frac{\partial\hat{\xi}}{\partial\Gamma_t} &= \frac{1}{2}\left(-1 + \frac{-(1-\Gamma_t)^2 + \frac{2}{N}}{\sqrt{(1-\Gamma_t)^2 + \frac{4}{N}\Gamma_t}}\right) < 0, \\ \frac{\partial\Gamma_t}{\partial v_t} &= \frac{(v_t+\Delta\kappa_\theta^2)\eta_\varepsilon{}^2 + 2v_\sigma(v_t+\Delta\kappa_\theta^2)\eta_\varepsilon) + (1-e^{-r\Delta})v_\sigma^2}{(v_t+\Delta\kappa_\theta^2)\eta_\varepsilon + v_\sigma)^2 e^{-r\Delta}\Delta\kappa_\theta\kappa_a\eta_\varepsilon} > 0, \end{aligned}$$

and thus,

$$\frac{\partial\hat{\xi}}{\partial v_t} = \frac{\partial\hat{\xi}}{\partial\Gamma_t}\frac{\partial\Gamma_t}{\partial v_t} < 0.$$

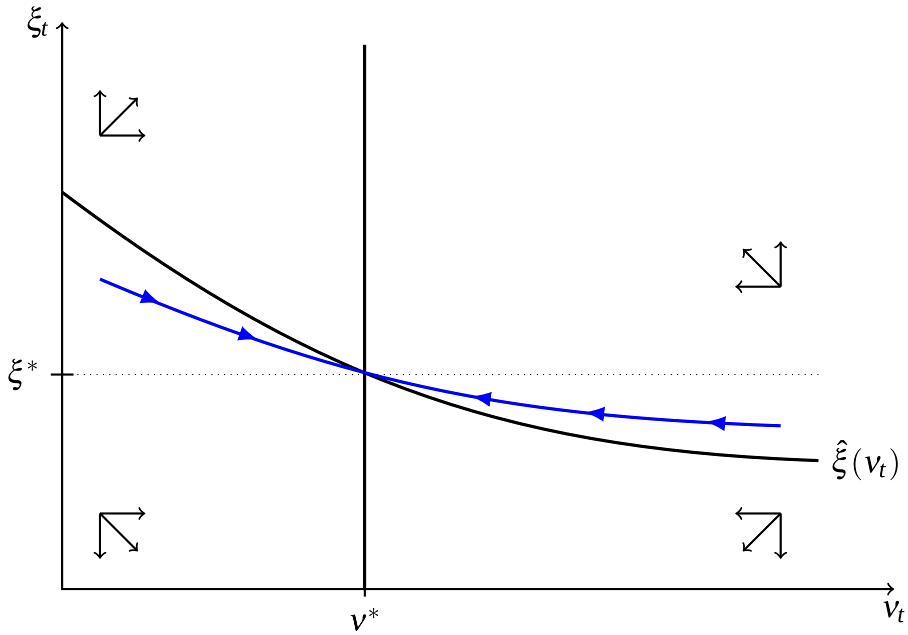


Figure 9: Phase diagram of (v_t, ξ_t)

defines a function $\Xi : \mathbb{R}_+ \to \mathbb{R}_+$ such that for any $v_0 > 0$, $(v_0, \Xi(v_0)) \in \mathscr{P}$. Therefore, for any initial precision v_0, there exists a unique value $\Xi(v_0)$ such that the corresponding sequence $\{v_t, \xi_t\}_{t=0}^{\infty}$ converges to (v^*, ξ^*). Since the sequence satisfies the transversality condition, it constitutes the Markov perfect equilibrium.

Proof of Proposition 8

(Parts 1-2) Since Γ is independent of N, it is straightforward from (14) that ξ^* decreases in N. Define $f(x) = Nx^2 - N(1-\Gamma)x - \Gamma$, then for any $N \geq 2$, $f(1/N) = -1 + 1/N < 0$ and $f(1) = (N-1)\Gamma > 0$. Thus, the solution to (13) must lie between $1/N$ and 1.

(Parts 3-5) Notice that

$$\begin{aligned} \frac{\partial \xi^*}{\partial \Gamma} &= \frac{1}{2}\left(-1 + \frac{-(1-\Gamma)^2 + \frac{2}{N}}{\sqrt{(1-\Gamma)^2 + \frac{4}{N}\Gamma}}\right) \\ &\leq \frac{1}{2}\left(-1 + \sqrt{\frac{(1-\Gamma)^2 + \frac{4}{N}(\Gamma - 1 + \frac{1}{N})}{(1-\Gamma)^2 + \frac{4}{N}\Gamma}}\right) < 0. \end{aligned}$$

Then, from

$$\Gamma = \frac{1-e^{-r\Delta}\Lambda_\mu}{e^{-r\Delta}\Lambda_a} = \frac{(e^{r\Delta}+1)+(e^{r\Delta}-1)\sqrt{1+\frac{4\eta_\sigma}{\Delta^2\eta_\varepsilon\kappa_\theta^2}}}{2}\frac{\kappa_\theta}{\kappa_a},$$

it is easy to show that $\frac{\partial\Gamma}{\partial r}>0$, $\frac{\partial\Gamma}{\partial\kappa_\theta}>0$, and $\frac{\partial\Gamma}{\partial\kappa_a}<0$. The limit result for κ_a is obtained from the fact that $\lim_{\Gamma\to 0}\xi^*=1$ and that $\Gamma\to 0$ as $\kappa_a\to\infty$.

Proof of Proposition 10

Rewrite Γ as

$$\Gamma(\Delta) = C_1\left(e^{r\Delta}+1+(e^{r\Delta}-1)\sqrt{1+\frac{C_2}{\Delta^2}}\right),$$

where $C_1=\frac{\kappa_\theta}{2\kappa_a}$ and $C_2=\frac{4\eta_\sigma}{\eta_\varepsilon\kappa_\theta^2}$. Then, we have,

$$\begin{aligned}\Gamma'(\Delta) &= C_1\left(re^{r\Delta}+re^{r\Delta}\sqrt{1+\frac{C_2}{\Delta^2}}+(e^{r\Delta}-1)\frac{-C_2/\Delta^3}{\sqrt{1+\frac{C_2}{\Delta^2}}}\right)\\ &= C_1e^{r\Delta}\left(r+\sqrt{1+\frac{C_2}{\Delta^2}}\left(r-\frac{1-e^{-r\Delta}}{\Delta}\frac{C_2}{\Delta^2+C_2}\right)\right)>0,\end{aligned}$$

since $r>1-e^{-r\Delta}/\Delta$ for any $\Delta>0$. Since $\frac{\partial\xi^*}{\partial\Gamma}<0$, (see the proof of Proposition 8) we have the desired result.

Proof of Proposition 11

Given a linear Markovian strategy profile $a_t^e=\gamma^*\theta_t$ and $a_t^n=\xi^*\hat{\mu}_{it}$, and given the belief that the novices have never deviated in the past (which implies $\hat{\mu}_{it}=\mu_t$ for all $i=1,\dots,N$), the novices understand that feedback y_t is of the following form:

$$\begin{aligned}y_t &= \Delta[\kappa_\theta\theta_t+\kappa_a(N^ea_t^e+N^na_t^n)+\varepsilon_t],\\ &= \Delta[m^*\theta_t+\kappa_aN^n\xi^*\mu_t+\varepsilon_t],\end{aligned}$$

where $m^*=\kappa_\theta+\kappa_aN^e\gamma^*$. Then, after observing y_t, the novices use the signal $z_t=y_t-\Delta\kappa_aN^n\xi^*\mu_t$ to update the public belief about the state. Let (μ_t,ν_t) and (μ_t',ν_t') be the mean and precision of the period-t public belief before and after observing y_t. Then, by standard Gaussian updating,

$$\mu_t' = \frac{v_t\mu_t + m_t\eta_\varepsilon z_t}{v_t + \Delta m_t^2\eta_\varepsilon}, \qquad v_t' = v_t + \Delta m_t^2\eta_\varepsilon.$$

Then, the public belief in period $t+1$ is

$$\begin{aligned} \mu_{t+1} &= \mu_{t+1}' = \frac{v_t\mu_t + m^*\eta_\varepsilon z_t}{v_t + \Delta m^{*2}\eta_\varepsilon}, \\ v_{t+1} &= \left(\frac{1}{v_t'} + \frac{\Delta}{\eta_\sigma}\right)^{-1} = \frac{(v_t + \Delta m^{*2}\eta_\varepsilon)\eta_\sigma}{\Delta(v_t + \Delta m^{*2}\eta_\varepsilon) + \eta_\sigma}. \end{aligned} \tag{27}$$

Solving (27) with $v_t = v_{t+1} \equiv v^*$, we obtain the stationary precision

$$v^* = \frac{m^{*2}\eta_\varepsilon}{2}\left(-\Delta + \sqrt{\Delta^2 + \frac{4\eta_\sigma}{m^{*2}\eta_\varepsilon}}\right). \tag{28}$$

Similar to the proof of Proposition 7, we solve the dynamic programming problems of the expert and the novice to obtain the values of γ^* and ξ^*. Denote the stationary continuation payoffs of the expert and the novice under the stationary Markovian profile as $W^*(\theta_t, \mu_t)$ and $V^*(\hat{\mu}_{it}, \mu_t)$, respectively. Then, we have

$$\begin{aligned} W^*(\theta_t, \mu_t) &= \max_a \frac{\theta_t}{N}(a + (N^e - 1)\gamma^*\theta_t + N^n\xi^*\mu_t) - \frac{a^2}{2} + e^{-r\Delta}\mathbb{E}_t^e\left[W^*(\theta_{t+1}, \mu_{t+1})\right]. \\ V^*(\hat{\mu}_{it}, \mu_t) &= \max_a \frac{\hat{\mu}_{it}}{N}(a + N^e\gamma^*\hat{\mu}_{it} + (N^n - 1)\xi^*\mu_t) - \frac{a^2}{2} + e^{-r\Delta}\mathbb{E}_t^n\left[V^*(\hat{\mu}_{i,t+1}, \mu_{t+1})\right]. \end{aligned}$$

Applying a guess and verify method—similar to one used in the proof of Proposition 7—to each dynamic programming problem, we obtain the equations for $m^* = \kappa_\theta + \kappa_a N^e\gamma^*$ and ξ^*:[35]

$$\frac{(1 - e^{-r\Delta})v^* + \Delta\eta_\varepsilon m^{*2}}{m^*}(m^* - \underline{m}) = e^{-r\Delta}\Delta\eta_\varepsilon\kappa_a^2\frac{N^eN^n}{N}\xi^*, \tag{29}$$

$$\frac{(1 - e^{-r\Delta})v^* + \Delta\eta_\varepsilon m^{*2}}{m^*}(\xi^* - \frac{1}{N}) = e^{-r\Delta}\Delta\eta_\varepsilon\kappa_a\left(\frac{N^n}{N}\xi^* - \xi^{*2}\right), \tag{30}$$

[35] The detailed calculations are available upon request.

where $\underline{m} = \kappa_\theta + \kappa_a \frac{N^e}{N}$. By dividing (30) by (29) and simplifying, we have

$$\xi^* = \frac{N^n}{N} \frac{m^* - \kappa_\theta}{m^* - \kappa_\theta + \frac{N^e}{N}(N^n - 1)\kappa_a}. \tag{31}$$

Finally, plugging (28) and (31) into (29), we obtain the equation for m^* given by

$$\frac{(e^{r\Delta}+1)+(e^{r\Delta}-1)\sqrt{1+4\frac{\nu_\sigma}{\Delta^2 m^{*2}\eta_\varepsilon}}}{2\kappa_a^2} m^* \left(m^* - \underline{m} + \frac{N^e N^n}{N}\kappa_a\right)(m^* - \underline{m}) = \frac{N^e (N^n)^2}{N^2}(m^* - \kappa_\theta), \tag{32}$$

which has a unique solution that satisfies $m^* > \underline{m}$.

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