Firm Size and Capital Structure*

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Abstract

Firm size has been empirically found to be strongly positively related to capital structure. A number of intuitive explanations can be put forward to account for this stylized fact, but none have been considered theoretically. This paper starts bridging this gap by investigating whether a dynamic capital structure model can explain the cross-sectional size-leverage relationship. The driving force that we consider is the presence of fixed costs of external financing that lead to infrequent restructuring and create a wedge between small and large firms. We find four firm-size effects on leverage. Small firms choose higher leverage at the moment of refinancing to compensate for less frequent rebalancings. Their longer waiting times between refinancings lead to lower levels of leverage at the end of restructuring periods. Within one refinancing cycle the intertemporal relationship between leverage and firm size is negative. Finally, there is a mass of firms opting for no leverage. The analysis of dynamic economy demonstrates that in cross-section the relationship between leverage and size is positive and thus fixed costs of financing contribute to the explanation of the stylized size-leverage relationship. However, the relationship changes sign when we control for the presence of unlevered firms.

Keywords: Capital structure, leverage, firm size, transaction costs, default, dynamic programming, dynamic economy, refinancing point, zero leverage

JEL Classification Numbers: G12, G32
Firm size has become such a routine control variable in empirical corporate finance studies that it receives little or no discussion in most research papers, even though it is not uncommonly among the most significant variables. This paper’s goal is to provide a rationale for one of the size relationships, that is between firm size and capital structure. Cross-sectionally, it has been consistently found that large firms in the US tend to have higher leverage ratios than small firms. International evidence suggests that in most, though not all countries leverage is also cross-sectionally related positively to size.\footnote{Titman and Wessels (1988), Rajan and Zingales (1995), and Fama and French (2002) are among many others documenting cross-sectional evidence for the US. International evidence is documented in Rajan and Zingales (1995) for developed countries and Booth et al. (2001) for developing countries.} Intuitively, firm size matters for a number of reasons. In the presence of non-trivial fixed costs of raising external funds large firms have cheaper access to outside financing per dollar borrowed. Similarly, larger firms are more likely to diversify their financing sources. Alternatively, size may be a proxy for the probability of default, for it is sometimes contended that larger firms are more difficult to fail and liquidate, or, once the firm finds itself in distress, as a proxy for recovery rate.\footnote{This is demonstrated by e.g. the cases of Chrysler in the US and Fiat in Italy. Shumway (2001) finds that the size of outstanding equity is an important predictor of bankruptcy probability.} Size may also proxy for the volatility of firm assets, for small firms are more likely to be growing firms in rapidly developing and thus intrinsically volatile industries.\footnote{To this effect, Fama and French (2002), among others, justify the conditioning of cross-sectional leverage regressions on size; this and default probability explanations are related since, conditional on the financial structure, low volatility firms are less likely to default.} Yet another explanation is the extent of the wedge in the degree of information asymmetry between insiders and the capital markets, which may be lower for larger firms, for example because they face more scrutiny by ever-suspicious investors.

All these explanations are intuitively appealing and it is very likely that all of them are, to a smaller or larger extent, at work. All these explanations, however, lie solely within the realm of intuition: while economic theories have been preoccupied with the determinants of firm size and its optimality at least since Coase (1937), existing theories are silent on the firm-size effect on the extent of external financing and, in particular, on quantitative implications that would provide a rationale for the observed size-leverage relationship. Empirical research has also not attempted so far to investigate and decompose the size effect, while some of the stylized facts that have been observed appear to be inconsistent with the proposed explanations. This state of affairs has led researchers to conclude that “…we do not really understand why size is correlated with leverage” (Rajan and Zingales, 1995, p. 1457). In this paper we make the very first step toward bridging this important gap in our knowledge. We choose, for clarity and relative simplicity, only one source of the size effect, namely the “fixed transaction cost” explanation, and attempt to see the extent to which this explanation alone can lead to an economically reasonable relationship between size and leverage. An important economic rationale for modelling fixed costs comes from indirect evidence...
provided by studies documenting infrequent restructuring and the mean reversion of leverage (see e.g. Leary and Roberts (2004)). The presence of external financing issuance costs suggests the dynamic nature of the problem. Thus, the exact question that we ask in this paper, is: Can a theory of dynamic capital structure produce an economically reasonable size effect resulting from fixed transaction costs of financing?

This paper’s contribution to the field is two-fold. First, we develop a theoretical framework and the solution method that can be applied to a wide class of dynamic financing problems. Second, by applying this framework to the problem at hand, we advance an intuition behind the dynamic relationship between firm size and leverage both at the level of an individual firm and in cross-section, where we investigate the cross-sectional relationship quantitatively. While we are ambivalent to the choice between a number of worthy theoretical ideas explaining leverage, we choose the trade-off model, which balances tax benefits of debt with various distress costs, as our workhorse. One of the reasons for our choice is that it is the only theory at present to produce viable dynamic models delivering quantitative predictions. However, the theoretical framework is invariant to a particular driving force of capital structure decisions, as also, to some extent, are a number of results.

To understand the economic intuition and empirical predictions behind our first set of findings, consider an all-equity firm contemplating debt issuance for the first time. The firm will choose an optimal leverage ratio that will balance the trade-off between expected tax benefits of debt and distress costs. In the absence of fixed costs, the firm will find it optimal to lever up immediately and will subsequently increase its debt continuously, as its fortunes improve, to restore the optimal balance. With fixed costs, however, it is suboptimal to refinance too often. The timing decision of the next refinancing will now balance fixed debt-issuance costs with the benefits of having more debt. The infrequency of refinancings will lower expected tax benefits and, to compensate for that, at each refinancing the firm will take on more leverage. (Of course, very small firms will find it optimal to postpone their first debt issuance until their fortunes improve substantially relative to the costs of issuance, and so the model also delivers the result that some firms are optimally zero-levered). The higher expected costs of future financing also imply that firms default sooner, at a higher level of asset value. As firm size increases, fixed costs become relatively less important and thus expected waiting times between refinancings are shorter and leverage at refinancings is closer to the no-fixed-cost case. In the limit, the firm’s optimal decision coincides with the decision in the absence of fixed costs.

It follows from the above discussion that, if we consider the comparative statics of a firm’s

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4Moreover, Hennessy and Whited (2006) find that external financing costs are substantially larger for small firms that for large firms. Altinkilic and Hansen (2000) and Kim, Palia, and Saunders (2003) find costs of public debt issuance in the order of 1% of debt principal. In addition, Altinkilic and Hansen estimate fixed costs to constitute about 10% of issuance costs on average (they are, of course, relatively more important for small firms). Section III.2 provides further detailed discussion.
optimal decision at the time of refinancing, the relationship between size and leverage will be negative, for smaller firms will have, conditional on issuing, an additional benefit of sustaining a higher level of debt. This comparative statics result, which we call the \textit{beginning-of-cycle effect}, is inconsistent with the observed empirical cross-sectional relationship that larger firms have higher leverage. However, firms are rarely at their refinancing points; they are more likely to be in the midst of a refinancing cycle, between two restructurings. This leads to two more effects on an individual firm level. The first one, which we call \textit{the within-cycle effect}, implies that between any two consecutive restructurings the market value of equity increases as the firm grows, decreasing the quasi-market leverage ratio and thus inducing a negative intertemporal correlation between firm size and leverage. The second effect, which we call \textit{the end-of-cycle effect}, states that while smaller firms rely on issuing more debt, they issue less often and so their longer waiting times lead to lower levels of leverage at the end of the refinancing cycle. With non-trivial waiting times, the presence of distress costs creates an asymmetry between the costs of being a low levered and a high levered firm. This leads firms, in pursuit of decreasing expected costs of distress, to increase leverage at refinancing only slightly, but to wait longer. For an individual firm, the “total” comparative statics within one refinancing cycle is thus contaminated with additional two effects and the resulting relationship between firm size and capital structure is more complicated.

Some of the above relationships (e.g. the within-cycle effect) are purely mechanical and by itself not surprising. What makes them important, both theoretically and empirically, is the aggregate outcome of complex interplay between all these effects especially in the cross-section. Indeed, in any cross-section, firms are likely to be at different stages of their refinancing and it is impossible to predict the implications of the above three effects on cross-sectional behavior by considering only the dynamic comparative statics of an individual firm. To complicate the matter, cross-sectional results are likely to be contaminated by the presence of zero-leverage firms in the economy which also happen to be the smallest in size. Since they constitute a mass point, they may induce a positive relationship between size and firm. We call this purely cross-sectional effect \textit{the zero-leverage effect}. To see the joint outcome of the four effects discussed above, we investigate the dynamic cross-sectional properties of the model to which our second set of results relates. In particular, we are interested in whether transaction costs can be the driving factor in the cross-sectional relationship between size and leverage and whether they can also be responsible for the mean reversion of leverage. For the benchmark case, replicating the standard empirical approach on artificially generated data, we find a strongly positive relationship between firm size and leverage, consistent with empirical findings. Thus, we find that transaction costs can, in theory, explain the sign of the leverage-size relationship. Quantitatively, the slope coefficient of size is very similar to the one found in the COMPUSTAT data set.
Interestingly, we also find that the positive relationship is an artefact of the presence of small unlevered firms in the economy. When we control for unlevered firms, the relationship between firm size and leverage becomes slightly negative, but still statistically significant. Thus, one of the important empirical messages of this paper is that in running cross-sectional capital structure regressions it is instrumental to control for the presence of unlevered firms. Whether controlling for the zero-leverage effect overturns the empirical result, demonstrating that the beginning-of-cycle and within-cycle effects dominate the end-of-cycle effect, is still largely an open issue. For example, in a recent study on the impact of access to the public debt market on leverage, Faulkender and Petersen (2006) find that excluding zero-debt firms changes the slope of the size coefficient, consistent with our results. This can also be related to why size has a negative impact on leverage in Germany (reported by Rajan and Zingales (1995)), for the German capital markets are less developed and only relatively large firms are publicly traded.

We also develop a number of empirical implications that can be derived from the cross-sectional dynamics of our model and which easily lend themselves to testing using standard corporate finance data sets. Small firms should restructure less frequently, add more debt at each restructuring, and have higher likelihood of default. The slope coefficient of firm size in standard leverage-level regressions is predicted to be negative for small levered firms, increasing with firm size, and be insignificant for large firms.

It may seem surprising that the question we are investigating here has not been addressed before. A traditional dynamic capital structure framework, rooted in the trade-off explanation, has been developed and successfully applied to many problems in works by Fisher, Heinkel and Zechner (1989), Leland (1998), Goldstein, Ju, and Leland (2001), Ju, Parrino, Poteshman, and Weisbach (2003), Christensen, Flor, Lando, and Miltersen (2002), and Strebulaev (2006). However, this framework can not be applied for our purposes. The major modelling trick the above models utilize is the so-called “scaling feature” (or first-order homogeneity property) which implies that at rebalancing points firms are replicas of themselves, just proportionally larger. All dollar-denominated variables (such as the value of the firm) are scaled and all ratios (such as debt to equity ratio) are invariant to scaling. The necessity of such an assumption comes from difficulties associated with the dimensionality of the optimization problem. Essentially, the scaling assumption allows researchers to map a dynamic problem into a static problem, for the optimal firm behavior at one refinancing point is identical to its behavior at all other refinancing points. Since all ratios are invariant to size, there are really no fixed costs and therefore the true size of the firm never enters the equation. To avoid instantaneous readjustments of debt, which will happen if costs are proportional to a marginal increase in debt, some of these models make an unrealistic but necessary
assumption that refinancing costs are proportional to the total debt outstanding.\footnote{Mauer and Triantis (1994) develop a framework, with the solution based on the finite difference method, of dynamic financing and investment decisions that allows for fixed costs of refinancing. In their model, equityholders always maximize the value of the firm, i.e. there is no ex-post friction between equityholders and debtholders where equity does not internalize the value of debt outstanding in making default or issuance decisions.}

To be able to characterize the solution for our question, we reconsider the existing framework and develop a new solution method, which is the second contribution of this paper. The modified framework enables us to solve for the optimal dynamic capital structure for truly fixed costs of external financing in a time-consistent rational way. Time consistency means that the optimal solution can be formulated as a certain rule at the initial date and that this rule will be optimal at all subsequent dates. Importantly, our framework introduces both firm size and realistic marginal proportional costs, which are now proportional only to new net debt issued. The solution method is both computationally feasible and intuitive, and can potentially be applied to a wide array of corporate finance problems featuring fixed costs, for example, the role of firm size in determining other corporate finance variables.

The intuitive idea behind the solution method is as follows. Instead of the case when the firm pays fixed costs for every restructuring, assume that no fixed costs are to be paid after a (sufficiently large) number of restructurings. Right after the last “fixed-cost” restructuring, the model will then represent a standard dynamic capital structure model (as modelled e.g. by Goldstein, Ju, and Leland (2001)). At the penultimate “fixed-cost” restructuring, the optimal solution will depend on the value of assets at this and next restructurings, both of which are unknown at the initial date. We proceed by guessing the firm size (measured by asset value) at the last “fixed-cost” restructuring. The firm size summarizes all past history of the firm. Having guessed the asset value, we can find a conditional optimal solution for the previous restructuring. Using a recursive procedure, we find optimal firm sizes at all previous restructurings conditional on our initial guess. Finally we solve for the optimal size at the initial date. Since the initial firm value is known, we compare these two values. If our initial guess did not lead to indistinguishable values, we readjust the initial guessed value and repeat the recursive procedure until our guess and the initial firm value coincide. We show that this procedure converges to the optimal solution in a computationally friendly way. Since this framework features a more realistic description of the transaction costs that have been so prominent in dynamic financing models, one can of course make these models more lifelike by incorporating modified transaction costs into them, and solving using this method.

To investigate the dynamic properties of the model, we use the method developed for related capital structure issues in Strebulaev (2006) based on the simulation procedure of Berk, Green, and Naik (1999). In particular, we simulate a number of dynamic economies and, after calibrating for the initial distribution of firm size to match the distribution of firms in COMPUSTAT, we replicate

\footnote{Another approach is to introduce exogenously fixed intervals between rebalancings (e.g. Ju et al. (2003)).}
the empirical analysis conducted by cross-sectional capital structure studies.

The remainder of the paper is organized as follows. Section I develops the theoretical framework and the solution algorithm. Section II applies the general framework to develop a dynamic trade-off model with fixed costs and analyzes the optimal solution and firm size effects in comparative statics. Section III contains the main dynamic analysis of the paper. Section IV discusses empirical implications of the model. Section V considers a number of extensions and Section VI concludes. Appendices A and B contain all proofs and Appendix C provides details of the simulation procedure.

I Theoretical framework

In this section, we first develop a general model of the dynamics of firms’ financing decisions, then provide the solution framework and, finally, apply the framework to the problem at hand by considering the dynamic trade-off model.

I.1 A general model of dynamic financing

Our model of the dynamics of firm financing is based on the following economic assumptions, a number of which can be relaxed at the expense of sacrificing the simplicity of the exposition: (1) External financing is costly and includes a fixed component; (2) At every point in time, owners of the firm choose a financing policy that will maximize their wealth; (3) Markets are perfectly rational and foresee all future actions by the owners of the firm; (4) Investment policy is independent of financial policy. The crucial assumption to what follows is the first one. The second assumption creates the necessary agency costs between the owners of the firm and other claimants to firm assets and cash flows, for the owners do not internalize the ex-post value of other claimant’s securities, but particulars can be relaxed. For example, by considering “owners” we assume away equityholder-manager conflicts, which could easily be introduced. The third assumption, while standard in most dynamic financing models, can be relaxed by allowing, for instance, behavioral biases and restrictions on no-arbitrage conditions. The fourth is the standard Modigliani-Miller assumption.

The first assumption prevents the firm from changing the book composition of its financial structure “too frequently”. In the continuous framework we consider, the set of time points when the firm takes no active financial decisions has a full measure: most firms prefer to be passive most of the time. The times when firms decide to change their financial structure we call interchangeably “refinancings”, “restructurings”, or “recapitalizations” followed terminology used by Fischer, Heinkel, and Zechner (1989), Goldstein, Ju, and Leland (2001) and Strebulaev (2006).

Other ingredients of the model are as follows. The owners of the firm choose between two types of external financing: common equity and debt. For simplicity, we do not consider more
complex financial structures. The decision to refinance depends on changes in firm fortunes and so we introduce two types of refinancings: default, when the future is bleak, and “upper” refinancing, when fortunes are excellent.\(^7\)

The firm owns a profit-generating project with the present value of cash flows at any date \(t\) denoted by \(V_t\). The random behavior of \(V\) stands for what we call “firm fortunes” and thus many variables measured relative to \(V\) are called \(V\)-adjusted. At any date \(t\), the firm has gone through \(k\) upper restructurings, \(k = 0, 1, ..., \) and we will describe the firm as being in the \((k+1)st\) refinancing cycle (period) if it is between the \(k\)th and \((k+1)st\) restructurings. The firm starts at date \(t_0\) as an all-equity firm and, if it decides to restructure immediately, restructuring zero takes place at date \(t_0\). The presence of fixed costs, however, may lead the firm to postpone introducing debt. Restructuring zero is thus defined as the first restructuring when the firm changes its status from an all-equity firm to a partially debt-financed firm.

At any restructuring, owners’ actions can be described in terms of changing the level of debt payments and future default and refinancing boundaries. In particular, at the beginning of the \(k\)th refinancing cycle, the firm adjusts its debt level by promising a constant \(V\)-adjusted (in units of \(V_{k-1}\)) coupon payment \(c_k\) to debtholders as long as the firm remains solvent. The owners also determine the new \(V\)-adjusted threshold \(\psi_kV_{k-1}, 0 < \psi_k < 1\), at which they choose to default, and the timing of the next restructuring by finding \(\gamma_k = V_{k}/V_{k-1}, \gamma_k > 1\). Given the initial value of assets, \(V_{t_0}\), the asset value at which the \(k\)th restructuring takes place can be written as \(\Gamma_kV_{t_0}\), where

\[
\Gamma_k = \prod_{m=0}^{k-1} \gamma_m.
\] (1)

The firm equityholders’ claim to intertemporal cash flows during refinancing cycle \(k\) in units of \(V_{k-1}\) is denoted by \(e(x_k), x_k = (\gamma_k, \psi_k, c_k)\), and the similar claim of debtholders is defined as \(d(x_k)\). Both \(e(x_k)\) and \(d(x_k)\) include the present value of default payouts. The combined value of debt and equity within the \(k\)th refinancing cycle, \(F(x_k)\), is the sum of \(e(x_k)\) and \(d(x_k)\) subtracting any proportional transaction costs associated with period-\(k\) debt or equity issuance. The sum of equity and debt intertemporal cash flows does not necessarily equal the total payout of the project because of issuance costs and the presence of other claims (e.g., government taxes). Fixed costs of refinancing paid at the moment of refinancing are constant at \(q\).

Let \(p(x_k)\) be the value of a claim at refinancing date \(k-1\) that pays $1 contingent on the asset value reaching the next refinancing before the bankruptcy threshold (in other words, \(V\) reaches

\(^7\)Other refinancing types can be introduced easily. For example, Fischer, Heinkel, and Zechner (1989) consider “lower” restructuring and Strebulaev (2006) considers liquidity-driven refinancing, both of which are intermediate states between “upper” restructuring and default.
\(\gamma_k V_{k-1}\) before \(\psi_k V_{k-1}\). The value of this claim at date \(t_0\) can then be written as

\[
P_k = \prod_{m=0}^{k} p(x_m).
\]  

(2)

The present value at date 0 of the firm owners’ claim to cash flows in refinancing cycle \(k + 1\) is

\[
P_k [V_{t_0} \Gamma_k F(x_{k+1}) - q],
\]  

(3)

and the present value of their total claim is thus

\[
W = V_{t_0} F(x_0) + \sum_{k=0}^{\infty} P_k [V_{t_0} \Gamma_k F(x_{k+1}) - q].
\]  

(4)

The first term above is the value of the claim before restructuring zero. Note that in equation (4) transaction costs, \(q\), do not change with the changes in firm size, which makes them truly fixed costs.

The owners’ objective is to find all \(x_k = (\gamma_k, \psi_k, c_k), k = 0, 1, \ldots\), that maximize the present value of their claim, \(W\), subject to limited liability:

\[
\begin{align*}
W & \to \max_{x_0, x_1, \ldots} \\
\text{s.t.} & \quad \frac{\partial E_k (v; x)}{\partial v} \bigg|_{v = \psi_k} = 0, \quad k \geq 1, \quad \gamma_0 \geq 1, \quad \psi_0 = 0, \quad c_0 = 0.
\end{align*}
\]  

(5)

The last three constraints specify that an initially all-equity firm can issue debt only starting at date \(t_0\). The first set of constraints are smooth-pasting conditions (see Leland (1994) and Morellec (2001)) determining when the firm will default on its debt in corresponding refinancing cycles, and \(E_k (v; x)\) is the total equity claim for arbitrary asset value \(v = V/V_{k-1}\) during period \(k\), measured in units of \(V_{k-1}\):

\[
E_k (v; x) = e (v; x_k) - p (v; x_k) D^0 (x_k) + p (v; x_k) \sum_{m=k}^{\infty} \frac{P_m}{P_k} \left( \frac{\Gamma_m}{\Gamma_{k-1}} F(x_{m+1}) - q \right),
\]  

(6)

where \(e (v; x_k)\) and \(p (v; x_k)\) are claims defined similarly to \(e (x_k)\) and \(p (x_k)\) but calculated within the \(k^{th}\) refinancing cycle at the moment when the asset value, measured in units of \(V_{k-1}\), reaches \(v\). Finally, if debt is callable, \(D^0 (x_k)\) is the \(V\)-adjusted called value of the \(k^{th}\)-period debt.
I.2 Solution method: Heuristic Approach

In this section we present an heuristic description of the method developed to solve our problem. Appendix A provides a rigorous treatment of the method.

Extant dynamic capital structure models with no fixed costs (e.g. Goldstein, Ju, and Leland (2001)) incorporate infrequent restructuring by assuming that costs of issuing debt are proportional to the total debt issued (rather than to its incremental amount). The absence of fixed costs simplifies the problem substantially since proportionality leads the firm at every refinancing be a scaled replica of itself. However, in addition to modelling proportional costs rather unrealistically, it does not allow to address any issues related to firm size. Technically, this method is equivalent to the dynamic programming approach in discrete time where the nominal value of the value function in the next period is the same as the total maximized value, which, in turn, allows the solution to be found as a root of a stationary Bellman equation. If issuance costs have a component that is independent of firm size, the scaling property is lost and the above method does not work. Specifically, the dynamic Bellman equation’s value function changes from period to period. Moreover, standard discrete-time dynamic programming methods developed for example by Abel and Eberly (1994) for investment problems (see also Dixit and Pindyck (1994) for general treatment) cannot be applied for at least two reasons. First, in our case, the discount factor is a function of control variables and it cannot be bounded from above by a constant that is strictly less than one. And second, the smooth-pasting condition (of equityholders to default) depends on the value function.

Our method for solving the problem can be intuitively described as follows. Imagine that, rather than fixed costs being paid at every restructuring, as would be the case in a truly dynamic model of financing that we envision, the firm has to incur fixed costs for only a finite number of restructurings $K$. We assume that $K$ is perfectly known by all market participants. After the $K^{th}$ restructuring, the firm is back into the no-fixed-cost problem, which can be easily solved using existing methods (since the scaling property holds). We also assume for now that is the firm will immediately lever up (i.e. that “restructuring zero ” coincides with initial date $t_0$).

The presence of fixed costs leads the firm’s optimal decision to depend on the absolute level of asset value. For the sake of exposition we will refer in this section to asset value $V$ as firm size, even though strictly speaking it is not true since $V$ includes also such claims as government taxes.

The equityholders’ decision depends on the accumulation of information about past firm financing. The most important point to observe is that the information content of the firm size in the $k^{th}$ refinancing cycle, represented by $\Gamma_{k-1}V_{t_0}$ (see equation (1)), is sufficient information in decision making. In other words, in restructuring space, firm size follows a Markov process. Intuitively, this holds because the owners care not about the “dollar value” of fixed restructuring costs, but about what may be called “firm-size adjusted” costs, which depend only on the current level of
asset value: the larger the firm, the smaller the relative costs of refinancing. Intuitively, \( \Gamma_{k-1} \) can be thought of as a sufficient statistic in the \( k^{th} \) refinancing cycle.

With that intuition, we start with the last “fixed-cost” restructuring \( K \) and guess firm size at this restructuring, \( \Gamma_K V_{t_0} \). Knowing \( \Gamma_K \) and the solution right after the last “fixed-cost” restructuring we can solve for the optimal decision at the \( (K-1)^{st} \) restructuring, which includes coupon and default levels in the \( K^{th} \) cycle, as well as restructuring level, \( \gamma_K \). But then, we also know, conditional on \( \Gamma_K \), the firm size at the previous restructuring. Recursively, we find optimal firm sizes at all restructurings \( K-2, \ldots, 2 \). For the first restructuring there are two ways to find \( \gamma_1 \). First, knowing all other \( \gamma \)s and the initial firm value \( V_{t_0} \), we can find \( \gamma_1 \) directly; we denote this value by \( \gamma_1^* \). But also, given \( V_{t_0} \), we can solve for optimal \( \gamma_1 \), which we denote \( \gamma_1^{**} \), in the same way as for previous restructurings. If our guess for the value of \( \Gamma_K \) was a correct one, these two methods of finding \( \gamma_1 \) will give the same solution. If \( \gamma_1^* > \gamma_1^{**} \), then optimal \( \Gamma_K \) is smaller than our guess, and vice versa. If the two values differ, we refine our guess of \( \Gamma_K \) and repeat the procedure.

There are two additional economic features that have so far been left unsatisfactorily. The first, a finite number of restructurings with fixed-cost payments, is addressed by showing that the solution to the above problem converges to the original one as the number of refinancings with fixed costs, \( K \), increases. Second, restructuring zero does not necessarily takes place at date \( t_0 \). For relatively small firms, in the presence of fixed costs, it is optimal to wait until the firm fortunes improve (and so firm-size adjusted costs decrease) rather than to issue debt immediately. It is sufficient to find the threshold level of asset value at which the firm will lever up and thus the condition for restructuring zero, \( \gamma_0 \).

II Firm size and leverage: Refinancing point analysis

II.1 A dynamic trade-off model

The dynamic trade-off framework lends itself easily to the introduction of firm size. For benchmark comparisons and for simplicity, we focus here on the modification of the benchmark Goldstein, Ju, and Leland (2001) model, though other frameworks also can be easily adapted.

The state variable in the model is the total time \( t \) net payout to claimholders, \( \delta_t \), where “claimholders” include both insiders (equity and debt) and outsiders (government and various costs). The evolution of \( \delta_t \) is governed by the following process under pricing measure \( Q^8 \)

\[
\frac{d\delta_t}{\delta_t} = \mu dt + \sigma dZ_t, \quad \delta_0 > 0, \tag{7}
\]

Since we consider an infinite time horizon, some additional technical conditions on Girsanov measure transformation (e.g. uniform integrability) are assumed here. In addition, the existence of traded securities that span the existing set of claims is assumed. Thus, the pricing measure is unique.
where $\mu$ and $\sigma$ are constant parameters and $Z_t$ is a Brownian motion defined on a filtered probability space $(\Omega, \mathcal{F}, \mathbb{Q}, (\mathcal{F}_t)_{t \geq 0})$. Here, $\mu$ is the risk-neutral drift and $\sigma$ is the instantaneous volatility of the project’s net cash flow. The default-free term structure is assumed flat with an instantaneous after-tax riskless rate $r$ at which investors may lend and borrow freely.

Therefore, the value of the claim to the total payout flow is given by

$$V_t = \mathbb{E}_t^Q \int_t^\infty e^{-r(u-t)} \delta_u du = \frac{\delta_t}{r - \mu}.$$  \hspace{1cm} (8)

The marginal corporate tax rate is $\tau_c$. The marginal personal tax rates, $\tau_d$ on dividends and $\tau_i$ on income, are assumed to be identical for all investors. Finally, all parameters in the model are assumed to be common knowledge.

All corporate debt is in the form of a perpetuity entitling debtholders to a stream of continuous coupon payments at the rate of $c$ per annum and, in line with previous trade-off models, allowing equityholders to call the debt at the face value at any time. The main features of the debt contract are standard in the literature. If the firm fails to honor a coupon payment in full, it enters restructuring. Restructuring, either a work-out or a formal bankruptcy, is modelled in reduced form. The absolute priority rule is enforced and all residual rights to the project are transferred to debtholders. However, distressed restructuring is costly and, in the model, restructuring costs are assumed to be a fraction $\alpha'$ of the value of assets on entering restructuring. In addition, debt contracts are assumed to be non-renegotiable and restrict the rights of equityholders to sell the firm’s assets.

The fundamental driving force of the model is the inherent conflict of interest between the different claimholders since ex-ante (prior to the issuance of debt) and ex-post (after debt has been issued) incentives of equityholders are not aligned. Equityholders maximize the value of equity (including debt still to be issued), and thus do not internalize a debtholders’ claim in a default decision. Debtholders foresee the future actions by equityholders and value debt accordingly.

The only rationale for issuing debt in the model is the existence of tax benefits of debt. In the absence of debt, cash flow to equityholders is $\delta_t(1 - \tau_c)(1 - \tau_d)$. The interest expense of $c$ changes cash flow to equity and debtholders to $(\delta_t - c)(1 - \tau_c)(1 - \tau_d) + c(1 - \tau_i)$. The maximum tax advantage to debt for one dollar of interest expense paid is thus $(1 - \tau_i) - (1 - \tau_c)(1 - \tau_d)$.

Up to this point our model is similar to the Goldstein, Ju, and Leland (2001) model. In our model, debt issuance costs consist of two components, a proportional and a fixed. The proportional cost, $q'$, is proportional to the marginal amount of debt issued and thus represents what most would think of as truly proportional costs. Fixed costs, $q$, are given in dollars measured as the fraction of all future net payouts at date $t_0$, $V_{t_0}$. Importantly, this way of modeling leads to the convex costs.

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9 The callability assumption is for technical convenience as long as only one seniority level is allowed.
of refinancing per dollar of debt issued, consistent with empirical results of, for example, Leary and Roberts (2005).

At every date \( t \), equityholders decide on their actions. Firms whose net payout reaches an upper threshold choose to retire their outstanding debt at par and sell a new, larger issue to take advantage of the tax benefits to debt. Refinancing thus takes the form of a debt-for-equity swap. An additional feature of realism in which we follow Goldstein, Ju, and Leland (2001) is that the firm’s financial decisions affect its net payout ratio. Empirically, higher reliance on debt leads to a larger net payout. Here, for simplicity, we assume that the net payout ratio depends linearly on the after-tax coupon rate:

\[
\frac{\delta}{V} = a + (1 - \tau_c)c. \tag{9}
\]

Given the general description of our model in Section I.1, the only features remaining to be elaborated are the values of \( p(x), e(x), d(x), F(x), \) and \( D^0(x) \).

Consider an arbitrary refinancing cycle that starts, say, at date \( 0 \) and let \( v_t = V_t/V_0 \) be the \( V_0 \)-measured value of firm assets. Then, at any date \( t \), the \( V_0 \)-measured values of equity and debt cash flows in one refinancing cycle that finishes either at \( TR = \inf \{ u \geq 0 : v_u = \gamma \}, \gamma > 1 \), in case of upper restructuring, or at \( TB = \inf \{ u \geq 0 : v_u = \psi \}, \psi < 1 \), in case of bankruptcy, are

\[
e(v_t; x) = \mathbb{E}_{v_t}^Q \left[ \int_T^{T_R \wedge T_B} e^{-r(u-t)} (1 - \tau_c) (1 - \tau_d) \left( \frac{\delta u}{V_0} - c \right) du \right], \tag{10}
\]

\[
d(v_t; x) = \mathbb{E}_{v_t}^Q \left[ \int_T^{T_R \wedge T_B} e^{-r(u-t)} (1 - \tau_i) cdv \right]
+ \mathbb{E}_{v_t}^Q \left[ e^{-r(T_B - t)} (1 - \alpha') (1 - \tau_c) (1 - \tau_d) \delta T_B \frac{T_B - T_R}{V_0} \right]. \tag{11}
\]

The second term in equation (11) is the present value of the recovery payback that debtholders expect at default.

Then, the date-\( t \) value of a \( V_0 \)-measured debt claim issued at date \( 0 \) is

\[
D(v_t; x) = d(v_t; x) + \mathbb{E}_{v_t}^Q \left[ e^{-r(T_R - t)} D(x) \mid T_R < T_B \right] = d(v_t; x) + p(v_t; x) D(x), \tag{12}
\]

where \( D(x) \) is the par value of the debt claim and

\[
p(v_t; x) = \mathbb{E}_{v_t}^Q \left[ e^{-r(T_R - t)} \mid T_R < T_B \right] \tag{13}
\]

is the date-\( t \) value of a claim that pays one dollar contingent on the asset value reaching the refinancing boundary \( \gamma \) before reaching the bankruptcy threshold \( \psi \).
Finally, the date-0 claims corresponding to (10)–(13) are given by

\[ e(x) \equiv e(1;x), \quad d(x) \equiv d(1;x), \quad p(x) \equiv p(1;x), \quad \text{(14)} \]

and

\[ D(x) \equiv D(1;x) = \frac{d(x)}{1 - p(x)}. \quad \text{(15)} \]

The combined value of debt and equity within the refinancing cycle, \( F(x) \), is the sum of \( e(x) \) and \( d(x) \) minus the present value of this and the next period’s transaction costs that are relevant for this period debt value \( D(x) \). The restructuring costs paid at the beginning of this period are \( q'[D(x) - D(x_{\text{prev}})] \), and the present value of those to be paid at the beginning of the next period is \( q'p(x)[D(x_{\text{next}}) - D(x)] \), where \( D(x_{\text{prev}}) \) and \( D(x_{\text{next}}) \) are the values of the previous and next period debt claims, respectively. We can now finally write the value of \( F(x) \) as

\[ F(x) = e(x) + d(x) - q'D(x) + q'p(x)D(x) = e(x) + (1 - q')d(x). \quad \text{(16)} \]

In the case of marginal restructuring costs, \( D^0(x) \), as defined through equation (6) in Section I.1, now represents both the called value of this period debt, \( D(x) \), and the portion of the next period restructuring costs in \( F(x) \) (the fourth term in (16)):

\[ D^0(x) = D(x) - q'D(x) = \frac{(1 - q')d(x)}{1 - p(x)}. \quad \text{(17)} \]

Note that all the above expressions can also be applied to restructuring zero by substitution \( x_0 = (\gamma_0, 0, 0) \) for \( x = (\gamma, \psi, c) \).

II.2 Comparative statics analysis at the refinancing point

To understand the economic intuition behind our first set of findings on how fixed costs of financing affect the leverage decision of firms, it is worth starting by considering the workings of the dynamic capital structure models such as those of Goldstein, Ju, and Leland (2001) and Strebulaev (2006). To allow for infrequent refinancing in the absence of truly fixed costs, they model the costs that are proportional to the total debt outstanding rather than to marginal debt issuance. To this end we introduce a state-space representation of these models, shown in Figure 1. We can think of \( V \) (defined in equation (8)), which stands for the firm’s asset value and is depicted on the horizontal

\[ F(x) = e(x) + d(x) - q' \frac{d(x)}{1 - p(x)}, \quad D^0(x) = D(x) = \frac{d(x)}{1 - p(x)}. \]
axis, as a proxy for firm size, and $C$, which denotes the level of coupon payment (measured in dollars in contrast to $V$-adjusted coupon rate $c$), as a proxy for the extent of debt financing. At the time of refinancing, the firm will choose the optimal level of leverage corresponding to its size on the middle line (for example, $C_1$ for $V_0$). Thus, the middle straight line (which we call here the beginning-of-cycle line) is the relationship between optimal leverage and firm size at refinancing points. The firm then moves along the horizontal within-cycle line (the dashed horizontal line at $C_1$ in Figure 1). If its fortunes substantially improve and it reaches the lower straight line (which is often called the upper restructuring line and which we call the end-of-cycle line) the firm refinances again (from $C_1$ to $C_2$, at $V_1$). If the firm’s fortunes deteriorate materially enough, the firm will default when it reaches the upper-left default line. In dynamics, firms can be at any point in the shadow area. Notice that since all three lines are straight, firm size does not matter for any financing decisions and, in particular, optimal leverage is constant. Moreover, all firms are optimally levered and large firms refinance as frequently as do small firms.

Figure 1: **Firm size and leverage: costs are proportional to total debt outstanding.** The figure shows the relationship between the level of firm’s asset value ($V$) and the level of debt payments ($C$) for the model with costs proportional to total debt outstanding.

What happens in our model in the presence of fixed and marginal proportional costs? If there were no fixed costs, the firm would find it optimal to restructure continuously as its asset value increases. Graphically, the beginning- and end-of-cycle lines in Figure 1 would coincide. However, adding fixed costs prevents infinitesimal increases from being the optimal strategy. To start with,
very small unlevered firms, for which the present value of tax benefits is less than the cost of refinancing, will abstain from issuing any debt. Such firms postpone debt issuance until their fortunes improve sufficiently. Figure 2 shows these firms on the horizontal zero-leverage segment between 0 and $V = V^*_0$ with the optimal coupon of 0. Once the threshold $V^*_0$ is reached, the firm will issue debt up to the level of $C_1$. Fixed costs lead to the discontinuity in the firm’s initial decision to use external debt financing.

Figure 2: Firm size and leverage: model with fixed and marginal proportional costs. The figure shows the relationship between the level of firm’s asset value ($V$) and the level of debt payments ($C$) for the model with fixed and marginal proportional costs.

The larger the fixed costs, the less often the firm restructures and, to compensate for less frequent external debt issuance in order to maximize the present value of tax benefits, at each restructuring the firm takes on more leverage. Thus, coupon $C_1$ is optimal for a smaller firm than in the absence of fixed costs (geometrically, $(V^*_0, C_1)$ is above the no-fixed-cost beginning-of-cycle threshold). Analogously, firms now defer the restructuring decision for longer so that the new end-of-cycle line is below the no-fixed-cost solid line. The evolution of the firm is within the shadow area and has the following pattern: it either moves horizontally to the right along the within-cycle line (if the present value of future cash flows becomes larger) until the new upper threshold is reached (in this case, $V_1$) and the firm refinances and immediately moves vertically to the new optimal coupon level $C_2$. Or, the firm moves horizontally to the left (if its future prospects become less favorable) until it reaches the default threshold. The new default boundary is reached sooner than
in the no-fixed-cost case, for the present value of the shareholders’ claim of ownership continuation is decreased by the present value of expected future fixed costs.

As firm size increases, fixed costs become of less importance, which is evident in Figure 2 where all three optimal dashed curves approach the optimal-decision solid lines of the no-fixed-cost case. The timing between subsequent refinancings decreases and attenuates to zero. In particular, the ratio of fixed to proportional costs in total issuance costs is larger for small firms and attenuates as firm size increases. There is another noteworthy pattern that emerges from Figure 2. If, upon default, the value of the firm is less than the initial threshold $V_0^*$, the firm emerges from bankruptcy restructuring optimally unlevered.

Figure 3: Optimal leverage and next refinancing decision. The left panel of the figure shows the relationship between the fixed costs of debt issuance ($q/V_0$) and optimal leverage at refinancing ($L_k$), where $k$ is the number of the refinancing cycle. The right panel of the figure demonstrates the relationship between the fixed costs of debt issuance and the scale factor ($\gamma_k$), which shows by how much firm size should increase before the firm refinances.

We now turn to investigation of the dependence of optimal leverage decisions on the fixed costs of issuance. The upper solid curve in the left panel of Figure 3 shows how the leverage ratio, $L_1$, changes as fixed costs measured in units of a firm’s initial asset value increase at the very first restructuring. (We address the numerical properties of the solution in the next section.) The plot shows that, conditional on issuance, higher fixed costs lead to higher leverage, for firms insure themselves against lengthened waiting times between refinancings. It also shows that, for fixed costs larger than a certain threshold level, the leverage decision at the first restructuring is independent of fixed costs. Facing exorbitant fixed costs of issuance, firms defer issuance decisions until some threshold level of firm size is reached and at that point, as Figure 2 shows (point $V_0^*$), the leverage decision is invariant to initial conditions.

The three dashed curves in the left panel of Figure 3 show the relationship between optimal leverage and fixed costs at the second, fifth, and tenth refinancings. Not surprisingly, given our previous discussion, at each subsequent refinancing, firm size-adjusted costs are smaller and so the
leverage ratio decreases. The leverage curve attenuates to the horizontal line with the value of the optimal leverage ratio for the no-fixed costs solution on the vertical axis.

The right panel of Figure 3 demonstrates by how much firm size should increase (the scale factor as represented by $\gamma$ on the vertical axis) before the firm refines again as a function of fixed costs. A similar pattern to optimal leverage at a refinancing point emerges. The upper curve shows the scale factor at the first refinancing, $\gamma_1$. As firm size-adjusted costs decrease, the firm restructures more often as evidenced by attenuation of the $\gamma$-curves in the figure. In the no-fixed costs case $\gamma$ is equal to one.

III Firm size and leverage: Dynamic analysis

The objective of this section is to investigate the cross-sectional relationship between firm size and leverage. The cross-sectional relationship in dynamics is impossible to investigate by studying only the comparative statics of optimal leverage decisions at refinancing points; the failure of this comparative statics approach and, intrinsically, static analysis to explain the cross-sectional dependencies was investigated in the context of capital structure by Strebulaev (2006). To see the intuition of why it is impossible to proceed in this way with our problem, consider Figure 4, which shows on the vertical axis the firm’s leverage ratio, $L$, as opposed to the coupon payment $C$ shown by Figure 2.

In dynamics, at any point in time firms can be either on the zero-leverage segment line (between 0 and $V_0^*$ on the horizontal axis) or in the shadow area. When the firm’s leverage ratio reaches the lower boundary, the firm optimally restructures (and the leverage ratio jumps to the middle beginning-of-cycle curve). Further evolution of the firm within the shadow area can be described similarly to that in Figure 2. It is only worth noting that the bankruptcy curve is now a horizontal line at $L = 1$, since equity is worthless at the time of default and the firm’s capital consists only of debt.

The figure shows the complex relationship between firm size and leverage. First, zero-leverage firms form a cluster that would tend to make the relationship positive. We call this the zero-leverage effect. Second, the end-of-cycle boundary, as an increasing function of size, also enhances the positive relationship (we call this the end-of-cycle effect). Third, the optimal leverage ratio at the refinancing point is a decreasing function of firm size (we call this the beginning-of-cycle effect). Finally, a firm’s path is, for any given positive debt level, a decreasing function of size (as demonstrated for example by the left vertical within-cycle boundary) and so it induces a negative relationship between firm size and leverage. We call this the within-cycle effect. The last three effects can be seen at the individual firm level while the zero-leverage effect is a purely cross-sectional one.
Figure 4: **Firm size and leverage: implications for dynamic analysis.** The figure shows the relationship between the level of firm’s asset value ($V$) and the leverage ratio ($L$) for the model with fixed and marginal proportional costs. $L_\infty$ is starts the no-fixed-cost optimal level of leverage.

The dynamic relationship will depend on the distribution of firms in the zero leverage segment and the shadow area. To relate the model to empirical studies, it is necessary to produce within the model a cross-section of leverage ratios structurally similar to those that would have been studied by an empiricist. Thus, we proceed to generate artificial dynamic economies from the model and then use the generated data to relate the leverage ratio to firm size. We use the method developed for corporate finance applications by Strebulaev (2006) based on the simulation method developed in the asset-pricing context by Berk, Green, and Naik (1999). Our first aim is to replicate a number of cross-sectional regressions used in empirical studies that produced stylized facts on the relation between leverage and firm size. The two questions which we wish to illuminate are whether fixed costs of issuance can produce results that are qualitatively similar to those found in empirical research, and, if so, whether the empirical estimates could have been generated by the model with reasonable probability under a feasible set of parameters.

**III.1 Data generation procedure**

This section describes the simulation procedure. Technical details are given in Appendix C.
To start with, observe that while only the total risk of the firm matters for pricing and capital structure decisions (since each firm decides on its debt levels independently of others), economy-wide shocks lead to dependencies in the evolution of the cash flow of different firms. To model such dependencies, shocks to their earnings are drawn from a distribution that has a common systematic component. Thus, cross-sectional characteristics of leverage are attributable both to firm-specific characteristics and to dependencies in the evolution of their assets. Following Strebulaev (2006), we model the behavior of the cash flow process as

$$\frac{d\delta_t}{\delta_t} = \mu dt + \sigma_I dZ^I + \beta \sigma_S dZ^S, \quad \delta_0 > 0.$$  

(18)

Here, $\sigma_I$ and $\sigma_S$ are constant parameters and $Z^I_t$ and $Z^S_t$ are Brownian motions defined on a filtered probability space $(\Omega, \mathcal{F}, \mathbb{Q}, (\mathcal{F}_t)_{t \geq 0})$. The shock to each project’s cash flow is decomposed into two components: an idiosyncratic shock that is independent of other projects ($\sigma_I dZ^I$) and a systematic (market-wide or industry) shock that affects all firms in the economy ($\sigma_S dZ^S$). The parameter $\beta$ is the systematic risk of the firm’s assets, which we will refer to as the firm’s “beta”. Systematic shocks are assumed independent from idiosyncratic shocks. The Brownian motion $dZ$ in equation (7) is thus represented as an affine function of two independent Brownian motions, $dZ = dZ^I + \beta dZ^S$, and the total instantaneous volatility of the cash flow process, $\sigma$, is

$$\sigma \equiv (\sigma_I^2 + \beta^2 \sigma_S^2)^{\frac{1}{2}}.$$  

(19)

At date 0 all firms in the economy are “born” and choose their optimal capital structure. Our benchmark scenario will be the case when all firms are identical at date 0 but for their asset value. This will allow us to concentrate on the relationship between firm size and leverage since the only difference between firms will be firm-size-adjusted fixed costs of external financing. For the benchmark estimation we simulate 300 quarters of data for 3000 firms. To minimize the impact of the initial conditions, we drop the first 148 observations leaving a sample period of 152 quarters (38 years). We refer to the resulting data set as one “simulated economy”. Using this resulting panel data set we perform cross-sectional tests similar to those in the literature. The presence of a systematic shock makes cross-sectional relations dependent on the particular realization of the market-wide systematic component. Therefore we repeat the simulation and the accompanying analysis a large number of times. This allows us to study the sampling distribution for statistics of interest produced by the model in dynamics.

In any period each firm observes its asset value dynamics over the last quarter. If the value does

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11 Thus, we assume that events ex ante are uncorrelated among firms and, say, default of firm $i$ neither increases nor decreases firm $j$'s chances of survival.
not cross any boundary, the firm optimally takes no action. If its value crosses an upper refinancing boundary (including the very first levering up), it conducts a debt-for-equity swap, re-setting the leverage ratio to the optimal level at a refinancing point, and so starting a new refinancing cycle. If the firm defaults, bondholders take over the firm and it emerges in the same period as a new (scaled down) firm with the new optimal leverage ratio.

III.2 Parameter calibration

This section describes how firms’ technology parameters and the economy-wide variables are calibrated to satisfy certain criteria and match a number of sample characteristics of the COMPUSTAT and CRSP data. An important caveat is that for most parameters of interest there is not much empirical evidence that permits precise estimation of their sampling distribution or even their range. Overall, the parameters used in our simulations must be regarded as ad hoc and approximate. To simplify the comparison, whenever possible we employ parameters used elsewhere in the literature.

In the model the rate of return on firm value is perfectly correlated with changes in earnings. In calibrating the standard deviation of net payout we therefore use data on securities’ returns. Firms differ in their systematic risk, represented by $\beta$. A distribution of $\beta$ is obtained by running a simple one-factor market model regression for monthly equity returns for all firms in the CRSP database having at least three years of data between 1965 and 2000, with the value-weighted CRSP index as proxy for the market portfolio. The resulting $\beta$ distribution is censored at 1% left and right tails and used as an estimate of the asset beta.

The volatility of firm assets, $\sigma$, is chosen to be 0.25, to coincide with a number of previous studies. It is also very close to the mean volatility of assets found by Schaefer and Strebulaev (2005) in the cross-section of firms that issued public debt. The standard deviation of the systematic shock, $\sigma_S$, is estimated to be 0.11. The volatility of idiosyncratic shocks, $\sigma_I$, is then chosen to be consistent with the value of total risk.

The proportional cost of restructuring in default, $\alpha'$, is equal to 0.05, consistent with a number of empirical studies. As was previously found, the model results are but slightly affected by the variation in distress costs, since the chances of defaulting are quite small.

All corporate taxes have the same value as in Goldstein, Ju, and Leland (2001) for ease of comparison. These values are also largely supported by empirical evidence. The corporate tax rate is equal to the highest existing marginal tax rate, $\tau_c = 0.35$. The marginal personal tax rate on interest income, $\tau_i$, is estimated by Graham (1999) to be equal to 0.35 over the period of 1980–1994. The marginal personal tax rate on dividend payments, $\tau_d$, is 0.2. Thus, the maximum tax benefit to debt, net of personal taxes, is $(1 - \tau_i) - (1 - \tau_c)(1 - \tau_d) = 13$ cents per one dollar of debt. The after-tax risk-free interest-rate is assumed to be 0.045 and the risk premium on the rate of return...
on firm assets is equal to 0.05. The net payout ratio increases with interest payments and the parameter $a$ (equation (9)) depends, ultimately, on firms’ price-earnings ratios and dividend policy. Its value is taken to be 0.035 – the same as used by Goldstein, Ju, and Leland (2001). When the net payout flow is very small, firms start losing part of their tax shelter. Since the remaining tax shelter depends on carry-forward and carry-back benefit provisions it is likely that firms lose a substantial part of the tax shield when current income is not sufficient to cover interest payments. In modelling the partial loss offset boundary, we follow Goldstein, Ju, and Leland (2001) and assume that firms start losing 50% of their debt offset capacity if the ratio of earnings to debt is relatively small.

Proportional costs of marginal debt issuance, $q'$, are assumed to be equal to 0.007 (or 0.7%). Fixed costs of restructuring, $q$, are calibrated in such a way that the total costs in a dynamic economy are on average about 1.2% of the amount of debt issuance. Datta, Iskandar-Datta, and Patel (1997) report total expenses of new debt issuance over 1976–1992 of 2.96%; Mikkelson and Partch (1986) find underwriting costs of 1.3% for seasoned offers and Kim, Palia, and Saunders (2003), in a study of underwriting spreads over the 30-year period, find them to be 1.15%. Altinkilic and Hansen (2000) also find that costs are in the order of 1% and that fixed costs on average constitute approximately 10% of total issuance costs. To obtain the ratio of total transaction costs to debt issuance to be about 1.2% per dollar of debt issued in the simulated economy in the last 35 of 75 years of simulations, we calibrate the initial distribution of $V$. The benchmark scenario’s initial $V$ distribution is censored lognormal with mean of $6\frac{1}{4}$, standard deviation of $2\frac{1}{4}$ and the minimum threshold value of $3\frac{1}{4}$. We introduce censoring, for COMPUSTAT contains records of relatively large firms in the economy. The ratio of the mean to the median firm size, one measure of skeweness of the distribution, is about 15 for an average annual COMPUSTAT sample year. Without censoring, the ratio in the simulations is about 75, showing that the sample is dominated by small firms. To avoid this artificial dependence on small firms and thus on fixed costs, we introduce the minimum threshold value. Other parameters of the distribution are chosen for the distribution of $V$ to resemble the observed distribution of firms in COMPUSTAT. Appendix C provides further details on calibration.

### III.3 Preliminary empirical analysis

We now bring together the calibrated model in dynamics with the results of comparative statics at the refinancing point and some empirical results from the literature. We use two definitions of leverage, both based on the market value of equity. The first, the market leverage ratio, can be defined as

$$ML_t = \frac{D(v_t; x)}{E(v_t; x) + D(v_t; x)},$$

(20)
where $v_t$ is the firm’s assets adjusted by their value at last refinancing and $E(v_t; x)$ and $D(v_t; x)$ are, respectively, the market values of equity and debt outstanding in a current refinancing cycle as defined in (6) and (12).

Typically, however, market values of debt are not available and book values are used. We therefore introduce a second definition, the quasi-market leverage ratio, defined as the ratio of the par value of outstanding debt to the sum of this par value and the market value of equity:

$$QML_t = \frac{D(x)}{E(v_t; x) + D(x)},$$

where $D(x)$ is the book value of debt as defined in (15). Typically, the difference between $ML$ and $QML$ is very small. For financially distressed firms, however, it can be more substantial. Intuitively, these ratios reflect how the firm has financed itself in the past since both the par and market values of debt reflect decisions taken early in a refinancing cycle.

Table I summarizes the cross-sectional distribution of these various measures in a dynamic economy and at the initial refinancing point. The average leverage ratio at the initial refinancing point is 0.33, compared with 0.37 in a model by Goldstein, Ju, and Leland (2001). To gauge the reasons for such a difference, consider the distribution of optimal leverage at the refinancing point. Notice that firms in the first percentile have zero leverage. In fact, for reasons of discontinuity in leverage decisions, about 9.3% of firms at the initial refinancing point are unlevered. If we exclude firms that do not have leverage, the average leverage ratio goes up from 0.33 to 0.36. This observation suggests that the low leverage puzzle (referring to the stylized fact that average leverage in the actual economy is lower that most trade-off models would predict) can be driven to a large extent by unlevered firms.\footnote{This fact is established empirically by Strebulaev and Yang (2005) who show that taking out all of the almost zero-leverage firms increases average leverage on a sample of COMPUSTAT firms between 1987 and 2003 from 25% to 35%.

The distribution of firm size in dynamics is similar to the distribution of firm size in COMPUSTAT. Of more importance, however, are the descriptive statistics for dynamics. Means for dynamic statistics are estimated in a two-step procedure. First, for each simulated economy statistics are calculated for each year in the last 35 years of data. Second, statistics are averaged across years for each simulated economy and then over economies. To get a flavor of the impact of systematic shocks, we also present minimum and maximum estimates over all economies. We begin by com-

\[\text{LogSize}_t = \log \left( E(v_t; x) + D(x) \right).\]
paring the leverage statistics in the dynamic economy with those at the initial refinancing point where the impact of the dynamic evolution of firm’s assets is ignored. Table I shows, in line with the results obtained by Strebulaev (2006), that leverage ratios in the dynamic cross-section are larger than at refinancing points. An intuition for this observation is quite general: unsuccessful firms tend to linger longer than successful firms who restructure fairly soon, especially so because firms who opt for higher leverage at refinancing points also choose a lower refinancing boundary, as demonstrated by Figure 4. In addition, firms that are in distress or close to bankruptcy typically have leverage exceeding 70%, and these firms have a strong impact on the mean. One of the major differences between Strebulaev (2006) and our results is that the dynamic cross-section in our model has a substantial number of firms that are unlevered. Table II shows, in particular that, on average, 6% of firms are unlevered at any point in time. Thus, our model is able to deliver low leverage for a large fraction of firms in cross-section and explain partially the low leverage puzzle in dynamics.\(^\text{13}\)

What Table I also shows is that the distribution of all parameters of interest (leverage, firm size and credit spreads) is much wider and closer to the empirically observed distribution than at the point of refinancing. In summary, because firms at different stages in their refinancing cycle react differently to economic shocks of the same magnitude, the cross-sectional distribution of leverage, as well as the other variables in Table I, is drastically different in dynamics as compared with the initial refinancing point.

Panel (a) of Table II shows that the annual default frequency is around 120 basis points. Every year about 10% of firms experience liquidity-type financial distress (their interest expense is larger than their pre-tax profit) and they have to resort to equity issuance to cover the deficit (recall that asset sales are not allowed in the model). Around 16% of firms restructure every year. This statistic, of course, hides a substantial cross-sectional variation between large and small firms and also between years when firms in the economy were relatively small and years when firms were relatively large. To gauge the effect of size, panels (a) and (b) give the same statistics for the smallest and largest 25% of firms (where these subsamples of firms are updated every year). Small firms exhibit a higher preponderance to default. While about 5% of the smallest 25% of firms (see panel (b)) are unlevered, the remainder have higher leverage ratios and lower default boundaries than the largest firms. Conditional on issuing debt, the likelihood of default within one year for these firms is about 1.8% compared with 0.6% for the 25% largest firms. Not surprisingly, small firms are also more likely to find themselves in financial distress (demonstrated by the higher fraction of small firms issuing equity). While our model allows only for one type of debt, it is interesting to note that along a number of dimensions our results are consistent with those of

\(^{13}\)Between 1987 and 2003 about 11% of COMPUSTAT firm-year observations have zero leverage as measured by book interest-bearing debt (the sum of data items 9 and 34) as reported by Strebulaev and Yang (2005).
Hackbarth, Hennessy, and Leland (2005) who find that young and small firms are more likely to have bank debt, while older and larger firms tend to have either a mix of bank and public debt or public debt only. If the fixed costs of obtaining bank debt is lower, then small firms will tend to have bank debt in our economy.

Table II also shows that the largest firms restructure on average almost every second year, while small firms may wait for decades without refinancing. Finally, panel (a) also provides some insight into the importance of systematic shock by sketching the distribution of frequency of events across generated economies. A systematic shock of realistic magnitude can lead to substantial variation in quantitative results. For example, in the “best-performing” economy out of a thousand simulated, on average less than 1% of firms were unlevered at any point in time, and in the “worst-performing” economy 36% of firms were unlevered. The results of all empirical studies are naturally based on only one realized path of a systematic shock.

Table III demonstrates the relative importance of fixed and proportional costs in simulated dynamic economies. The ratio of total costs to debt issuance, of 1.20%, was calibrated with the choice of initial distribution of asset value \( V \). The ratio of fixed to total costs is on average about 25%, somewhat higher than the 10% figure reported by Altinkilic and Hansen (2000). (The sample of Altinkilic and Hansen is likely to contain, on average, larger firms than in COMPUSTAT.) Panels (b) and (c) again report the same results for the subsamples of smallest and largest firms. Not surprisingly, smallest firms pay dearly to restructure with fixed costs being by far the largest component, consistent with recent empirical findings by Hennessy and Whited (2006).

### III.4 Cross-sectional regression analysis

This section examines the cross-sectional dynamic relationship between leverage and firm size. Recall that there are four general effects that may have opposite effects in cross-section. Firstly, some firms are unlevered (the zero-leverage effect). Secondly, smaller firms tend to take on more debt at refinancing to compensate for longer waiting times (the beginning-of-cycle effect). Thirdly, an increase in firm size increases the value of equity and decreases leverage (the within-cycle effect). Fourthly, smaller firms wait longer before restructuring and the leverage ratio deviates more from the leverage at refinancing point (the end-of-cycle effect). Our first task is to investigate the joint outcome of these four effects in the cross-section by replicating standard empirical tests.

Recall that each simulated data set (“economy”) consists of 3000 firms for 300 quarters. As described in section III.1, we simulate a large number of economies, dropping the first half of the observations in each economy. For each economy we then conduct the standard cross-sectional regression tests. We choose the procedure used by Fama and French (2002) where they first estimate year-by-year cross-sectional regressions and then use the Fama-MacBeth methodology to estimate
time-series standard errors that are not clouded by the problems encountered in both single cross-section and panel studies.\textsuperscript{14} First we run the following regression for each year of the last 35 years of each simulated economy:

\[ QML_{it} = \beta_{0,t} + \beta_{1,t} \log \text{Size}_{it-1} + \epsilon_{it}. \] (23)

We then average the resulting coefficients across economies. In addition, to control for substantial autocorrelation (since leverage is measured in levels), we report the \( t \)-statistics implied by Rogers robust standard errors clustered by firm.\textsuperscript{15} Note that we do not need to control for heterogeneity in the data resulting from the presence of omitted variables, which is a substantial problem in empirical research, for purposely the only source of the heterogeneity in the simulated cross-sectional data is the distribution of asset value.

Table IV reports the results of this experiment. Panel (a) shows the results of a standard regression of leverage on firm size and a constant. The relationship for the whole sample is positive, consistent with the existing empirical evidence. Thus, a dynamic trade-off model of capital structure is able to produce qualitatively the relationship between firm size and leverage as observed in empirical studies. To gauge whether empirically observed coefficients are consistent with our data, we also present the 10th and 90th percentiles of the distribution of size coefficients across economies. Empirically, Rajan and Zingales (1995) report a coefficient of 0.03 and Fama and French (2002) of 0.02-0.04, which is in the same range as the distribution of coefficients on \( \log \text{Size} \) in the simulated data.\textsuperscript{16} The coefficient of 0.03 roughly means that a 1\% increase in the value of assets increases leverage by 3 basis points.

This result demonstrates that the joint outcome of the end-of-cycle and zero-leverage effects dominates that of the beginning-of-cycle and within-cycle effects. However, interestingly, the coefficient on size is in fact negative in about every seventh simulated economy, implying that there is a relation between the evolution of systematic shock and the size-leverage relation. To investigate this and the relative importance of the effects further, we proceed to study the extent to which our neglect of firms with no leverage may affect our results both in simulated and actual economies. The reasons for our concentration on the zero-leverage effect are two-fold. First, it is the only

\textsuperscript{14}Strebulaev (2006) demonstrates that other cross-sectional methods (e.g. those of Bradley, Jarrell, and Kim (1984) and Rajan and Zingales (1995)) produce the same results when applied to the generated data.

\textsuperscript{15}Petersen (2005) shows that both the Newey-West method corrected for the presence of panel data and unadjusted Fama-MacBeth procedure lead to understated standard errors.

\textsuperscript{16}An important caveat is in order. Our definition of size is, while obviously highly correlated with theirs, not a one-to-one mapping. Rajan and Zingales (1995) use the logarithm of annual sales, and Fama and French (2002) use the logarithm of total book assets as their proxy for size, while our proxy is the logarithm of quasi-market value of firm assets. To study the consequences of that, we investigated the empirical relationship between firm size (as measured by the sum of book debt and market value) and proxies that empirical researchers use in the COMPUSTAT data. Not reported, they are significantly positively correlated as one would expect.
cross-sectional effect and thus relatively easy to delineate. Second, it is well-known that when a cross-section has mass in the extreme point, the results can be biased, and the geometric intuition behind Figure 4 suggests that in our case the bias can be substantial. Therefore, we proceed by estimating the same regression as (23) but controlling for the presence of unlevered firms by including a dummy that equals 1 if a firm is unlevered in a given year and 0 otherwise:

\[ QML_{it} = \beta_{0,t} + \beta_{1,t} \text{LogSize}_{it-1} + \beta_{2,t} \text{UnleveredDummy}_{it} + \epsilon_{it}. \]  

(24)

Panel (b) of Table IV reports the results of this exercise. Unsurprisingly, the coefficient on the dummy is large and negative. Perhaps surprisingly, incorporating the dummy changes the sign of the size effect. The coefficient of -0.009 implies that a 1% increase in the value of assets decreases leverage by about 0.9 basis points. Thus, controlling for the presence of unlevered firms, the higher leverage ratio at refinancing (the beginning-of-cycle effect) and the negative intertemporal cross-sectional relation (the within-cycle effect) dominate the effect of longer waiting times. Interestingly, this finding can provide, for example, an explanation of why Rajan and Zingales (1995) find a positive relation between firm size and leverage for most countries but Germany, for Germany has a less developed capital market with only relatively larger firms publicly traded (and thus present in their data set). Empirically, Faulkender and Petersen (2006) find that excluding zero-debt firms may change the slope of the size coefficient, consistent with our results. Additionally, it also provides an economic intuition of why in some economies the regression without a dummy produces a negative coefficient on size. In very successful economies the number of zero-levered firms shrinks (in a number of years it can be zero) and the only effects in play are those working at the individual firm level. This leads the coefficient of LogSize to be an increasing function of the value of the common shock.

IV Empirical implications

In this section we summarize a number of empirical implications that can be derived from our model and that easily lend themselves to testing using standard corporate finance data sets. A word of caution is in place: our usage of adjectives “small” and “large” with respect to firms may appear somewhat superficial in this section. The size of the firm is a relative term, in this context specifically to the value of fixed costs which can vary, for example, among industries, and this has to be taken into account in empirical research.

The first set of predictions is directly related to the presence of zero-leverage firms. Obviously, the exclusion of zero leverage firms will tend to increase average leverage in the economy. In the benchmark model, the average leverage ratio in dynamics increases from 45% to 50%. More
interestingly, the model predicts that a zero-leverage policy is more likely to be followed by small firms. Strebulaev and Yang (2005) find that unlevered firms are more likely to be the smallest firms in their industries, consistent with this result. Another prediction is that, conditional on issuance, zero-leverage firms will “jump” by issuing a lot of debt. Certainly, in reality there are some number of large firms that follow either a low-leverage or a zero-leverage policies not explained by the model. Two interrelated reasons are that our model does not consider optimal cash policy of the firm or internal agency conflicts. For example, large zero-leverage firms tend to have higher cash balances. Likewise, a number of small firms are levered. In addition to short-term bank debt that presumably is associated with lower costs, the financing decisions of these firms can be driven by their investment policy, which we do explicitly model here.

Two testable empirical predictions are at the core of the difference between small and large firms. Small firms restructure less often. At the same time, conditional on restructuring, small firms issue relatively more debt (relative to the amount of debt outstanding). This leads to another prediction that the propensity to default is a negative function of firm size and so small firms are more likely to fail. In addition, the observation that large firms will allow smaller deviations from optimal leverage ratios at restructuring implies that there is a negative relation between firm size and firm leverage volatility.

The model also predicts that an economy with relatively higher fixed costs will display a higher fraction of zero-leverage firms, lower frequency of restructurings and relatively larger amount of debt issuance conditional on restructurings taking place. This could possible be examined empirically by comparing the data from the 1960–1970s with the data from the 1980–1990s, for it is often argued that external financing issuing costs had decreased substantially over recent decades (see e.g. Kim, Palia, and Saunders (2003)).

How would firms facing high fixed costs of refinancing proceed to issuing debt in reality? One choice open to them is shelf registration that allows relatively large firms to register all securities that they expect to issue over the following couple of years. The model predicts that, first, firms with higher costs are more likely to file for shelf registration and, second, once self registration goes through, firms will issue debt more often.

For the results of cross-sectional regressions, our simulation predicts that if we condition on firms that have no leverage, the slope of the \( \log \text{Size} \) coefficient will become less significant and may become negative, as Faulkender and Petersen (2006) find on a COMPUSTAT sample. To the extent that we do not expect to observe empirically a perfect separation between small and large firms, as produced by the model, controlling for unlevered firms may result in a lesser effect. If empirical investigation finds that the coefficient is substantially less positive, it will also suggest that a number of other established results and stylized facts can be affected. We also conjecture
that if regressions are run separately on the subsamples of small and large levered firms, the slope coefficient will be more negative in the subsample of small firms.

Finally, to the extent that systematic shocks can be thought of as a proxy for a business cycle variable, we hypothesize that the fraction of unlevered firms increases in downturns and decreases in times of growth, with the frequency of restructurings revealing the opposite tendency.\textsuperscript{17} This has a bearing on the regression coefficient on firm size, for the decrease in the number of unlevered firms leads to the attenuation of the zero-levered cross-sectional effect. Also, an increase in aggregate leverage due to systematic shocks results in higher stock return volatility (see Schwert (1989) for empirical evidence).

V Fixed costs of bankruptcy

Economic intuition corroborated by empirical evidence suggests that, like refinancing costs, bankruptcy costs are also likely to consist of two components, proportional and fixed.\textsuperscript{18} To investigate whether adding the fixed component of bankruptcy changes our conclusions we modify the benchmark model by postulating that if the firm defaults in refinancing cycle $k$, then the fixed costs of size $\alpha$ have to be paid in addition to proportional costs, bringing the total bankruptcy costs to $\min[\alpha + \alpha'V, V]$. This lowers the recovery value debtholders receive at default so that the value of debt cash flows measured in units of $\delta_{k-1}$ in the $k^{th}$ refinancing cycle defined earlier in (11) is now reduced by the dollar-measured quantity $\alpha p_B(x_k)$, where

$$p_B(x_k) = \mathbb{E}_Q\left[e^{-rT_B} \mid T_B < T_R \right]$$

is the value of a claim that pays $1$ at the moment of bankruptcy contingent on the firm defaulting in cycle $k$. Appendix B formulates the problem specified by equation (5) with fixed bankruptcy costs and provides its solution.

The evolution of the economy with fixed bankruptcy costs is shown on Figure 5. Introducing fixed bankruptcy costs raises the threshold value $V_0^*$ at which very small firms restructure for the first time. For the case of the fixed component of bankruptcy costs being five times larger than fixed debt issuance costs, $\alpha = 5q$, the fraction of unlevered firms in the dynamic economy increases from 6% to 11%. Moreover, to reduce the likelihood of now costlier default, firms issue less debt than in the benchmark case. As a result, the beginning-of-cycle curve is no longer a monotonically decreasing function of firm size. It is always below its benchmark-case counterpart and first rises

\textsuperscript{17}See also Hackbarth, Miao, and Morellec (2006) for the model of the business cycle impact on capital structure.

\textsuperscript{18}For example, Hennessy and Whited (2006) find that bankruptcy costs, relative to capital, are much higher for small than for large firms.
Figure 5: **Firm size and leverage: fixed costs of bankruptcy.** The figure shows the relationship between the level of firm’s asset value ($V$) and the leverage ratio ($L$) for the model with fixed and marginal proportional restructuring costs and fixed bankruptcy costs.

with firm size and then, at some point, starts its decline attenuating to the no-fixed-cost optimal level of leverage, $L_\infty$. To understand the rationale for this behavior, note that both fixed costs of issuance and fixed costs of bankruptcy become less important as firm size increases. Two effects counterbalance each other. First, fixed costs of issuance become less crucial and this dampens optimal leverage at refinancing. Second, the importance of fixed costs of bankruptcy also decreases, but this leads to an increase in optimal leverage. Since in the absence of fixed bankruptcy costs small firms prefer larger optimal leverage, their probability of bankruptcy is higher, and therefore for these firms the latter effect dominates.

Having more unlevered firms in our simulated economies results in a higher (and more significant) firm size coefficient in our standard regression (the coefficient rises from 0.02 to 0.03). On the other hand, the non-monotonicity of the beginning-of-cycle curve is the main reason for an increase in the size coefficient when we control for the presence of unlevered firms (from $-0.004$ in the benchmark case to $-0.002$ once fixed bankruptcy costs are introduced).
VI Concluding remarks

This paper investigates, in a rigorous and economically intuitive model, the cross-sectional relationship between firm size and capital structure. We construct and find a solution for a general dynamic financing model with infrequent adjustment in the presence of fixed costs of external financing. Using an application of this model to the dynamic trade-off model of capital structure, we show that there are four effects on the relationship. Smaller firms issue more debt conditional on refinancing but they wait longer before restructuring again. Between refinancings the relation between firm size and leverage is negative at the individual firm level. Finally, in cross-section the presence of fixed costs may give rise to zero leverage being an optimal policy. The presence of unlevered firms has a positive effect on the size-leverage relation. We generate data that structurally resembles data used in empirical studies. In this way, the method allows us to compare the predictions of a capital structure model in “true dynamics” both to the findings of the empirical literature and to the comparative statics predictions of the same model. We find that the model results are consistent with the empirically observed positive relationship between firm size and leverage. However, conditional on the presence of zero-leveraged firms, the relation turns negative. Whether the model can give rise to data that are consistent with this relationship quantitatively is so far an open question. But these findings provide a clear signal of the need for further research in this area.

An important line of future studies is to put to empirical investigation a number of cross-sectional and time-series predictions developed here. A clear and principal direction that future theoretical work should take is to investigate other factors affecting the relationship between firm size and leverage. The developed framework could also usefully be extended to allow for other corporate finance effects on capital structure.

Appendix A. Solution method

Assuming that at the moment of refinancing \( k \) the firm pays fixed restructuring costs \( q_k \), and substituting (6) into (5), we obtain another way of writing the main problem:

\[
\begin{align*}
W &= V_{t_0} F(x_0) + \sum_{k=0}^{\infty} P_k [V_{t_0} \Gamma_k F(x_{k+1}) - q_k] \rightarrow \max_{x_0, x_1, \ldots} \\
\text{s.t. } &\bar{p}(x_k) \left[ \sum_{m=k}^{\infty} \frac{P_m}{P_k} [V_{t_0} \Gamma_m F(x_{m+1}) - q_m] - V_{t_0} \Gamma_{k-1} D^0(x_k) \right] + V_{t_0} \Gamma_{k-1} \bar{e}(x_k) = 0, \quad k \geq 1, \\
&\gamma_0 \geq 1, \quad \psi_0 = 0, \quad c_0 = 0,
\end{align*}
\]

(A.1)

where

\[
\bar{p}(x_k) = \frac{\partial p(v; x_k)}{\partial v} \bigg|_{v=\psi_k}, \quad \bar{e}(x_k) = \frac{\partial e(v; x_k)}{\partial v} \bigg|_{v=\psi_k}.
\]

(A.2)

The original model we want to solve is problem (A.1) with fixed costs \( q_k = q \). We now introduce an
auxiliary model, in which fixed costs \( q_k \) after the \( K^{th} \) restructuring are assumed to be zero:

\[
q_k = \begin{cases} 
1, & k \leq K, \\
0, & k > K.
\end{cases}
\]  

(A.3)

Intuitively, since the maximized function in (A.1) is restricted from above by \( V_{t_0} \), the present-value sum of fixed-cost payments for all restructurings is a convergent series and therefore, for sufficiently large \( K \), its residual goes to zero. On the other hand, this residual, the present value of the sum of claims to the fixed costs paid for restructurings \( K + 1, K + 2, \ldots \), is exactly the difference between the solutions to the firm’s problem (A.1) with fixed costs \( q_k = q \) and the problem with fixed costs given by (A.3), so that for sufficiently large \( K \) the auxiliary problem approximates the original one.

The reason why we wish to solve the auxiliary problem with fixed costs (A.3) is that after the \( K^{th} \) restructuring we are back to the standard no-fixed-cost problem, for which the scaling property holds. The solution to this problem, following, for example, Goldstein, Ju, and Leland (2001), can be represented (in units of \( V_K \)) as:

\[
\begin{cases}
W_\infty = \max_x \left[ \frac{F(x)}{1 - p(x)\gamma} \right] \\
s.t. \quad \bar{p}(x) \left[ \gamma \frac{F(x)}{1 - p(x)\gamma} - D^0(x) \right] + \bar{c}(x) = 0.
\end{cases}
\]  

(A.4)

Define \( x = (x_1, x_2, x_3) = (\gamma, \psi, c) \) and the state variable \( s_k \) as the “size of the firm” in the beginning of the \( k^{th} \) refinancing cycle

\[
s_k = V_{k-1} = \Gamma_{k-1} \times V_{t_0},
\]  

(A.5)

and the period-\( k \) transition function as

\[
s_{k+1} = f(s_k, x_k) = s_k x_{1,k} = s_k \gamma_k.
\]  

(A.6)

Observe that in problem (4) the shareholders’ decision about the \( k^{th} \) restructuring, \( x_k \), depends on all past states and actions \( \{s_0, x_0, \ldots, x_{k-1}, s_k\} \) only through the value of the period-\( k \) state \( s_k \). This allows us to reformulate problem (4) as a Markovian infinite-dimensional dynamic programming problem.

More formally, define the state space \( S = (0, +\infty) \) and the control space \( X = [1, +\infty) \times [0, 1] \times [0, 1] \). The value function \( W(s_k) \) is going to be the \( s_k \)-measured shareholder’s wealth at date \( k \).

Imposing an additional condition \( x_1 = \infty \) on a one-cycle model leads to the solution \( W_0 \) of a static financing problem as described e.g. in Leland (1994). The fact that \( \lim_{x_1 \to \infty} x_1 p(x) = \lim_{x_1 \to \infty} x_1 \bar{p}(x) = 0 \) implies that the static solution \( W_0 \) is not affected by future restructurings and is thus independent of firm size:  

\[
\begin{cases}
W_0 = \max_{x_2, x_1} \lim_{x_1 \to \infty} F(x) \\
s.t. \quad \lim_{x_1 \to \infty} \bar{e}(x) = 0.
\end{cases}
\]  

(A.7)

It also follows immediately that \( W(s) \geq W_0 \).

In the proof of the theorem below we also show a quite intuitive fact that the value function \( W(s) \) is bounded from above by the no-fixed-cost solution \( W_\infty \), as given by (A.4). This allows us to consider the value function set consisting of all functions \( W: S \to [W_0, W_\infty] \).

Define the mapping \( T \) by

\[
\begin{cases}
T(W)(s) = \max_{x \in X} \left[ F(x) - \frac{q^0(x)}{s} + p(x) x_1 W(s x_1) \right] \\
s.t. \quad \bar{p}(x) \left[ x_1 W(s x_1) - \frac{q^0}{s} - D^0(x) \right] + \bar{c}(x) = 0,
\end{cases}
\]  

(A.8)

\[19\text{Since the static problem can be seen as a problem with infinitely large future-restructuring fixed costs, } q = \infty, \text{ one can think of } W_0 \text{ as a solution for an infinitely small firm (if its size is measured in units of fixed costs), } W(0) = W_0. \]

\[20\text{The no-fixed-cost condition implies that the firm is infinitely large and } W(\infty) = W_\infty. \]
and let $W^*(s)$ be the solution to problem (4) (measured in units of $s = V_{t_0}$) assuming that the firm has optimally chosen a certain debt level.

The following theorem shows that $W^*(s)$ is the fixed point of the functional mapping $T$ and that for sufficiently large $K$, problem (4) with fixed costs given by (A.3) approximates the benchmark problem (4) with $q_k = 1$.

**Theorem 1** Assume that the functions $p(x)$, $F(x)$ and

$$B(x) = D^0(x) - \frac{\bar{c}(x)}{p(x)}$$

(A.9)

are continuously differentiable on $X$ and that there exists a constant $M > 0$ such that either for $\iota = 2$ or for $\iota = 3$, for every $x \in X$:

$$\{B(x)/x_1 | x_i \in [0, 1]\} \supset [W_0, W_\infty],$$

(A.10)

and

$$\frac{\partial B(x)}{\partial x_i} \neq 0,$$

(A.11)

and

$$0 \leq p(x) + \left[ \frac{\partial F(x)}{\partial x_i} + \frac{\partial p(x)}{\partial x_i} B(x) \right] \left( \frac{\partial B(x)}{\partial x_i} \right)^{-1} \leq \frac{M}{x_1}.$$  

(A.12)

Then $W^*(s)$ satisfies the Bellman equation given by

$$W^*(s) = T(W^*) (s).$$

(A.13)

Furthermore,

$$W^*(s) = \lim_{K \to \infty} T^K(W_\infty) (s),$$

(A.14)

and there exists a stationary optimal policy.

**Proof.** Denote

$$x' = \begin{cases} 
(x_1, x_3), & \text{if } \iota = 2, \\
(x_1, x_2), & \text{if } \iota = 3, 
\end{cases}$$

(A.15)

and $X' = [1, +\infty) \times [0, 1]$.

From (A.10) and (A.11), for every $x' \in X'$ there exists a function $\varphi_{x'} : [W_0, W_\infty] \to [0, 1]$ such that

$$\bar{p}[x', \varphi_{x'}(w)] \left(x_1 w - D^0[x', \varphi_{x'}(w)]\right) + \bar{c}[x', \varphi_{x'}(w)] = 0$$

(A.16)

and

$$\frac{\partial \varphi_{x'}}{\partial w} = x_1 \left( \frac{\partial B}{\partial x_i} \right)^{-1}.$$  

(A.17)

Define

$$R(x', w) = F[x', \varphi_{x'}(w)] + p[x', \varphi_{x'}(w)] x_1 w,$$

(A.18)

$$\Pi(s, x', W) = R[x', W(s x_1) - q/(s x_1)].$$

(A.19)

For any function $\nu : S \to X'$, let the mapping $T_\nu$ be

$$T_\nu(W)(s) = \Pi(s, \nu(s), W).$$

(A.20)

---

21While for the general case of the dynamic trade-off model described in Section II we were able to verify conditions (A.10)-(A.12) only numerically, relaxing the less important assumption that the net payout ratio is a function of the coupon rate (equation(9)) allows to show analytically that conditions (A.10)-(A.12) hold true (for $x_1 = x_3 = c$).
Then

$$W^*(s) = \sup_{v_0, v_1, \ldots, v_K} \lim_{K \to \infty} (T_{v_0} \ldots T_{v_K}) (W_{\infty}) (s).$$
(A.21)

Assumption (A.12) implies that

$$0 \leq \frac{\partial R}{\partial w} \leq M,$$
(A.22)

and thus the monotonicity property for the return function $\Pi$ holds: for every $s \in S$, $x' \in X'$ and $W_1, W_2 \in [W_0, W_{\infty}]$

$$\Pi (s, x', W_1) \leq \Pi (s, x', W_2), \quad \text{if} \quad W_1 \leq W_2.$$
(A.23)

Moreover, since $W_{\infty}$ solves

$$W_{\infty} = \sup_{x'} R (x', W_{\infty}),$$
(A.24)

it is immediate that for every $s \in S$ and $x' \in X'$

$$\Pi (s, x, W_{\infty}) \leq W_{\infty}.$$  
(A.25)

Therefore, Assumptions I, I.1 and I.2 in Bertsekas and Shreve (1996) (see Chapter 5, pp.70–71) hold (the latter holds because of the upper bound in (A.22)). Thus, by Proposition 5.2 (p.73) $W^*(s)$ satisfies the Bellman equation (A.13).

Since we are not interested in the values of $W^*(s)$ for very small firm size $s$ (because in the true model (4) the firm is initially unlevered) and since $f(s, x) \geq s$, we can restrict the values of the state variable $s$ to be greater than a certain small yet strictly positive constant $\gamma$. This, in turn, bounds from above the values of the control variable $x_1 = \gamma$. Therefore, the assumptions of Proposition 5.10 (Bertsekas and Shreve, p. 86) are satisfied so that the dynamic programming equation (A.14) holds and there exists a stationary optimal policy. This completes the proof of the theorem. ■

We propose the following algorithm to solve the firm’s dynamic financing problem, when fixed costs are given by (A.3) and the initial size of the firm is $s_0 = V_{t_0}$ (and $K$ is sufficiently large). We guess the size of the firm just before restructuring $K$, $\hat{s}_K$, and define $G_K = W_{\infty}$, and $H_K = q$.

Then, we solve the model backward. In the beginning of the $k^{th}$ refinancing cycle, $k = K - 1, K - 2, \ldots, 1$, we maximize the present $V$-adjusted value of equityholders’ claims:22

$$\begin{cases}
F (x_k) + p(x_k) \left[ \gamma_k G_{k+1} - \frac{H_{k+1}}{s_k} \right] \to \max_{x_k} \\
\text{s.t.} \quad \tilde{p}(x_k) \left[ \gamma_k G_{k+1} - \frac{H_{k+1}}{s_k} - D^0 (x_k) \right] + \tilde{e} (x_k) = 0,
\end{cases}$$
(A.26)

obtain optimal $x_k^* = (\gamma_k^*, \psi_k^*, c_k^*)$ and define recursively

$$\begin{align*}
G_k &= F (x_k^*) + \gamma_k^* p (x_k^*) G_{k+1}, \\
H_k &= q + p(x_k^*) H_{k+1}, \\
\hat{s}_k &= \frac{\hat{s}_{k+1}}{\gamma_k^*}. 
\end{align*}$$
(A.27) (A.28) (A.29)

Finally, we find when it is optimal to issue debt for the first time:

$$F (\gamma_0, 0, 0) + p (\gamma_0, 0, 0) \left[ \gamma_0 G_1 - \frac{H_1}{s_0} \right] \to \max_{\gamma_0 \geq 1}$$
(A.30)

22Note that according to our recursive procedure, in (A.26), $H_{k+1}$ and $G_{k+1}$ are given, but not $s_k$ (at date $k$, we know only $\hat{s}_{k+1}$). In order to be able to solve the maximization problem (A.26) correctly, we have to derive its first order conditions taking $s_k$ as given, then replace the unknown $s_k$ by $\hat{s}_{k+1}/\gamma_k$, and solve the system with respect to $\gamma_k$, $c_k$ and $\psi_k$.  

33
Then, we check if \( \hat{s}_{1}/s_{0} \) and the solution to (A.30), \( \gamma_{0}^{*} \), are sufficiently close numbers. If they are, the model is solved. Otherwise, we refine our guess about the size of the firm before the very last restructuring

\[
\hat{s}_{K}^{new} = \omega s_{0} \prod_{m=0}^{K-1} \gamma_{m}^{*} + (1 - \omega) \hat{s}_{K}, \quad \omega \in (0, 1),
\]  

(A.31)

and repeat the procedure.

**Appendix B. The case of fixed bankruptcy costs**

If fixed costs of size \( \alpha \) have to be paid in addition to proportional costs, the total bankruptcy costs are \( \min [\alpha + \alpha'V, V] \). This lowers the recovery value debtholders receive at default so that the \( V_{k-1} \)-measured value of debt cash flows in the \( k^{th} \) refinancing cycle defined in (11) is now reduced by the dollar-measured quantity \( \alpha p_{B}(x_{k}) \), where \( p_{B}(x_{k}) \) is given by (25). Using (16) and (17), the shareholders’ problem (5) is modified to

\[
\begin{align*}
\left\{ 
W = V_{t_{0}}F(x_{0}) + \sum_{k=0}^{\infty} P_{k} [V_{t_{0}}\Gamma_{k}F(x_{k+1}) - q - (1 - q')\alpha p_{B}(x_{k+1})] & \rightarrow \max_{x_{0},x_{1},...} \\
\right.
\end{align*}
\]

s.t. \( \bar{p}(x_{k}) \left[ \sum_{m=k}^{\infty} \frac{P_{m}}{P_{k}} [V_{t_{0}}\Gamma_{m}F(x_{m+1}) - q - (1-q')\alpha p_{B}(x_{m+1})] - V_{t_{0}}\Gamma_{k-1}D^{0}(x_{k}) \right] + V_{t_{0}}\Gamma_{k-1}\bar{e}(x_{k}) = 0, \quad k \geq 1,
\]

\( \gamma_{0} \geq 1, \quad \psi_{0} = 0, \quad c_{0} = 0. \)

(B.1)

Similar to the benchmark case, for sufficiently large \( K \) we approximate the above problem with the one for which both fixed restructuring and bankruptcy costs are zero after the \( K^{th} \) restructuring. Noticing that firm size (A.5) still summarizes all information about past restructuring that is necessary for the shareholders to make optimal decisions about next refinancings, we adopt a similar backward-induction approach to solve this approximating problem. The only difference with our solution procedure (A.26)–(A.30) is that at step \( k \) we now solve:

\[
\begin{align*}
\left\{ 
F(x_{k}) - (1-q')\alpha \frac{p_{B}(x_{k})}{x_{k}} + p(x_{k}) \left[ \gamma_{k}G_{k+1} - \frac{H_{k+1}}{s_{k}} \right] & \rightarrow \max_{x_{k}} \\
\right.
\end{align*}
\]

s.t. \( \bar{p}(x_{k}) \left[ \gamma_{k}G_{k+1} - \frac{H_{k+1}}{s_{k}} - (1-q')\alpha \frac{p_{B}(x_{k})}{x_{k}} - D^{0}(x_{k}) \right] + \bar{e}(x_{k}) = 0, \)

(B.2)

and for optimal \( x_{k}^{*} \) define the cycle-\( k \) value of all future fixed-cost payments as:

\[
H_{k} = q + (1-q')\alpha p_{B}(x_{k}^{*}) + p(x_{k}^{*}) H_{k+1}.
\]

(B.3)

**Appendix C. Details of simulation analysis**

The details are similar to those reported in Strebulaev (2006). The process for \( \delta \) is discretized using the following approximation:

\[
\delta_{t} = \delta_{t-\Delta t} e^{\left(\mu_{A} - \frac{\sigma^{2}}{2}\right)\Delta t + \sigma \sqrt{\Delta t} z_{t}},
\]

(C.1)

where \( \Delta t \) is one quarter, \( z_{t} \) is a standard normal variable and \( \mu_{A} \) is the growth rate of the net payout ratio under the physical measure. The benchmark simulation is for 300 quarters and 3000 firms. Note that while we discretize the model for the purpose of simulation, firms still operate in a continuous environment. In particular, it must happen now that firms will sometimes “overshoot” over boundaries and make their financial decisions not exactly at the prescribed optimal times. Unreported robustness checks show that increasing the frequency of observations does not change the results materially.

To choose the number of observations that will be dropped to minimize the impact of initial conditions the following ad hoc procedure has been implemented. We simulate the panel dataset for 3000 firms with the benchmark set of parameters in the absence of systematic shock 250 times. For each economy the average
leverage ratio is calculated. The rolling sum of the first differences in average leverage ratios (quarter by quarter) over the last 10 quarters is estimated. The stopping rule is for this variable to be less than 0.5% in absolute magnitude (for comparison, the average value of this variable in the first 10 quarters is 5%). The economy is then defined as converged to its steady state. The resulting distribution of steady state stopping times across all economies has the mean of 30 quarters, 95th percentile value of 50 quarters and the maximum of 76 quarters. For a conservative estimate we double the maximum. Since this procedure is largely ad hoc, we check the result by simulating 20 economies for 1000 quarters and confirm that there is no difference in the average leverage ratio behavior for the last 900 quarters by investigating rolling sums over the entire period.

References

DIXIT, AVINASH K., AND ROBERT S. Pindyck, 1994, Investment under Uncertainty, Princeton University Press.
Table I
Descriptive statistics

The table reports descriptive statistics for the following variables: market leverage ($ML$), quasi-market leverage ($QML$), $LogSize$ and credit spreads ($CS$). ‘Initial’ refers to the case when all firms are at their refinancing points. All other statistics are given for dynamics. 1000 data sets are generated, each containing 75 years of quarterly data for 3000 firms. For each dataset the statistics are first calculated for each year in the last 35 years of data and then are averaged across years. Finally, they are averaged over data sets. Min and Max give the minimum and maximum over the 1000 data sets of the annual averages. For credit spreads, only observations with positive debt are considered.

<table>
<thead>
<tr>
<th>Percentiles</th>
<th>Mean</th>
<th>1%</th>
<th>50%</th>
<th>90%</th>
<th>95%</th>
<th>99%</th>
<th>St. Dev.</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Market leverage, $ML$</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial</td>
<td>0.33</td>
<td>0</td>
<td>0.36</td>
<td>0.40</td>
<td>0.41</td>
<td>0.42</td>
<td>0.11</td>
<td>3000</td>
</tr>
<tr>
<td>Average</td>
<td>0.46</td>
<td>0.01</td>
<td>0.44</td>
<td>0.73</td>
<td>0.82</td>
<td>0.95</td>
<td>0.20</td>
<td>3000</td>
</tr>
<tr>
<td>Min</td>
<td>0.33</td>
<td>0</td>
<td>0.36</td>
<td>0.58</td>
<td>0.67</td>
<td>0.86</td>
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<tr>
<td>Max</td>
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<td>0.26</td>
<td>0.50</td>
<td>0.84</td>
<td>0.92</td>
<td>0.99</td>
<td>0.30</td>
<td>3000</td>
</tr>
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<td><strong>Quasi-market leverage, $QML$</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial</td>
<td>0.33</td>
<td>0</td>
<td>0.36</td>
<td>0.40</td>
<td>0.41</td>
<td>0.42</td>
<td>0.11</td>
<td>3000</td>
</tr>
<tr>
<td>Average</td>
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<td>0.01</td>
<td>0.45</td>
<td>0.77</td>
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<td>0.97</td>
<td>0.21</td>
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<tr>
<td>Min</td>
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<td>0.36</td>
<td>0.61</td>
<td>0.71</td>
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<td>3000</td>
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<td>Max</td>
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<td>0.25</td>
<td>0.51</td>
<td>0.88</td>
<td>0.94</td>
<td>0.99</td>
<td>0.31</td>
<td>3000</td>
</tr>
<tr>
<td><strong>LogSize</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial</td>
<td>6.05</td>
<td>3.02</td>
<td>5.94</td>
<td>8.51</td>
<td>9.22</td>
<td>10.34</td>
<td>1.78</td>
<td>3000</td>
</tr>
<tr>
<td>Average</td>
<td>7.14</td>
<td>1.92</td>
<td>7.05</td>
<td>10.42</td>
<td>11.35</td>
<td>13.03</td>
<td>2.47</td>
<td>3000</td>
</tr>
<tr>
<td>Min</td>
<td>4.38</td>
<td>-0.91</td>
<td>4.18</td>
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<td>8.90</td>
<td>10.93</td>
<td>2.37</td>
<td>3000</td>
</tr>
<tr>
<td>Max</td>
<td>9.54</td>
<td>3.82</td>
<td>9.51</td>
<td>13.02</td>
<td>13.91</td>
<td>15.86</td>
<td>2.65</td>
<td>3000</td>
</tr>
<tr>
<td><strong>Credit spreads, $CS$</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial</td>
<td>1.99</td>
<td>1.99</td>
<td>1.99</td>
<td>2.01</td>
<td>2.02</td>
<td>2.02</td>
<td>0.01</td>
<td>2717</td>
</tr>
<tr>
<td>Average</td>
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<td>0.19</td>
<td>2.27</td>
<td>4.16</td>
<td>5.16</td>
<td>7.41</td>
<td>1.26</td>
<td>2722</td>
</tr>
<tr>
<td>Min</td>
<td>1.98</td>
<td>0</td>
<td>2.03</td>
<td>2.92</td>
<td>3.56</td>
<td>5.53</td>
<td>0.71</td>
<td>2677</td>
</tr>
<tr>
<td>Max</td>
<td>3.01</td>
<td>1.85</td>
<td>2.52</td>
<td>5.30</td>
<td>6.48</td>
<td>8.60</td>
<td>1.88</td>
<td>2787</td>
</tr>
</tbody>
</table>
Table II  
Frequency of events

The table reports the frequency of various events in the generated data sets. ‘Default’ is the fraction of defaulting firms. ‘Restructure’ is the fraction of firms restructuring at the upper boundary. ‘Equity Issues’ is the fraction of firms for which interest expense is larger than current cash flow. ‘Unlevered’ is the fraction of firms that have zero leverage. 1000 data sets are generated, each containing 75 years of quarterly data for 3000 firms. For each dataset frequencies are computed across the last 35 years of data and then averaged over data sets. Min, 25%, 75% and Max give, correspondingly, the minimum, 25th percentile, 75th percentile and maximum annual averages over all data sets. All frequencies are annualized and given in percentages.

<table>
<thead>
<tr>
<th></th>
<th>Default</th>
<th>Restructure</th>
<th>Equity Issues</th>
<th>Unlevered</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel (a): All firms</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>1.21</td>
<td>16.06</td>
<td>10</td>
<td>5.85</td>
</tr>
<tr>
<td>Median</td>
<td>1.12</td>
<td>15.17</td>
<td>9.80</td>
<td>4.45</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.50</td>
<td>6.04</td>
<td>2.59</td>
<td>4.42</td>
</tr>
<tr>
<td>Min</td>
<td>0.22</td>
<td>4.56</td>
<td>3.03</td>
<td>0.50</td>
</tr>
<tr>
<td>25%</td>
<td>0.83</td>
<td>11.37</td>
<td>8.12</td>
<td>2.74</td>
</tr>
<tr>
<td>75%</td>
<td>1.51</td>
<td>19.86</td>
<td>11.84</td>
<td>7.83</td>
</tr>
<tr>
<td>Max</td>
<td>3.48</td>
<td>52.03</td>
<td>17.68</td>
<td>35.38</td>
</tr>
</tbody>
</table>

| **Panel (b): Smallest 25% firms** |          |             |               |           |
| Mean               | 1.84     | 2.31        | 12.22         |           |
| Median             | 1.76     | 2.01        | 12.05         |           |
| Std. Dev.          | 0.60     | 1.22        | 6.20          |           |

| **Panel (c): Largest 25% firms** |          |             |               |           |
| Mean               | 0.62     | 38.36       | 5.27          |           |
| Median             | 0.54     | 37.25       | 4.14          |           |
| Std. Dev.          | 0.36     | 11.72       | 4.31          |           |
### Table III

**Fixed and proportional costs**

The table reports the relative size of fixed and proportional costs of debt issuance in the generated data sets. ‘Total Costs’ is the sum of fixed and proportional costs. ‘Firm Value’ is the sum of book debt and market equity. 1000 data sets are generated, each containing 75 years of quarterly data for 3000 firms. For each dataset frequencies are computed across the last 35 years of data and then averaged over data sets. Min, 25%, 75% and Max give, correspondingly, the minimum, 25th percentile, 75th percentile and maximum annual averages over all data sets. All ratios are given in percentages.

<table>
<thead>
<tr>
<th></th>
<th>Fixed Total Costs</th>
<th>Total Costs Debt Issuance</th>
<th>Total Costs Firm Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel (a): All firms</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>24.70</td>
<td>1.20</td>
<td>0.14</td>
</tr>
<tr>
<td>Median</td>
<td>24.39</td>
<td>1.18</td>
<td>0.13</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>4.80</td>
<td>0.15</td>
<td>0.05</td>
</tr>
<tr>
<td>Min</td>
<td>12.13</td>
<td>0.90</td>
<td>0.05</td>
</tr>
<tr>
<td>25%</td>
<td>21.36</td>
<td>1.09</td>
<td>0.10</td>
</tr>
<tr>
<td>75%</td>
<td>28.05</td>
<td>1.29</td>
<td>0.17</td>
</tr>
<tr>
<td>Max</td>
<td>41.19</td>
<td>1.85</td>
<td>0.38</td>
</tr>
<tr>
<td><strong>Panel (b): Smallest 25% firms</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>81.03</td>
<td>4.63</td>
<td>1.64</td>
</tr>
<tr>
<td>Median</td>
<td>82.44</td>
<td>4.66</td>
<td>1.61</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>6.14</td>
<td>1.11</td>
<td>0.66</td>
</tr>
<tr>
<td><strong>Panel (c): Largest 25% firms</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>11.36</td>
<td>0.86</td>
<td>0.04</td>
</tr>
<tr>
<td>Median</td>
<td>10.69</td>
<td>0.85</td>
<td>0.03</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>4.44</td>
<td>0.05</td>
<td>0.01</td>
</tr>
</tbody>
</table>
Table IV
Cross-sectional regressions

The table reports the results of cross-sectional regressions on the level of the quasi-market leverage ratio, QML. The independent variable is firm size measured as the log of the sum of book debt and market equity. 1000 data sets are generated, each containing 75 years of quarterly data for 3000 firms. Coefficients and t-statistics are means over 1000 simulations. Fama-MacBeth (1973) method is used, with the regressions run over the last 35 years of each data set and then averaged. The last three columns report additional information on the regression: the standard deviation of coefficients and t-statistics, and the 10th and 90th percentile values of these coefficients across simulations. t-statistics are from Rogers robust standard errors clustered by firm.

<table>
<thead>
<tr>
<th></th>
<th>10%</th>
<th>90%</th>
<th>Std.dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel (a): Standard regressions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.397</td>
<td>0.062</td>
<td>0.489</td>
</tr>
<tr>
<td></td>
<td>(33.792)</td>
<td>(4.724)</td>
<td>(54.330)</td>
</tr>
<tr>
<td>LogSize</td>
<td>0.013</td>
<td>-0.007</td>
<td>0.030</td>
</tr>
<tr>
<td></td>
<td>(8.217)</td>
<td>(-27.470)</td>
<td>(18.770)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.034</td>
<td>0</td>
<td>0.091</td>
</tr>
<tr>
<td></td>
<td>(35)</td>
<td>(35)</td>
<td>(35)</td>
</tr>
<tr>
<td>$N$</td>
<td>3000</td>
<td>3000</td>
<td>3000</td>
</tr>
</tbody>
</table>

|                      | 10%     | 90%     | Std.dev. |
| Panel (b): Regressions with unlevered dummy |
| Constant             | 0.568   | 0.532   | 0.603    | 0.027    |
|                      | (41.558)| (22.915)| (63.354) | (17.401) |
| LogSize              | -0.009  | -0.010  | -0.007   | 0.001    |
|                      | (-15.931)|(-26.082)|(-7.672)  | (7.995)  |
| Unlevered firms      | -0.529  | -0.569  | -0.490   | 0.030    |
|                      | (-35.280)|(-52.871)|(-20.246)| (14.085) |
| $R^2$                | 0.270   | 0.131   | 0.442    | 0.119    |
|                      | (35)    | (35)    | (35)     |          |
| $N$                  | 3000    | 3000    | 3000     |          |