Debt restructuring and voting rules

Preliminary and incomplete

Comments welcome *

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Abstract

The voting arrangements used by creditors during debt restructuring are prespecified, often by statute. For example, U.S. law stipulates the use of a unanimous vote outside bankruptcy, but a supermajority vote in Chapter 11 bankruptcy. We analyze the effect of voting rules on the welfare of debtors and their creditors in restructuring negotiations. When creditors are close to rational and markets are liquid, the “toughness” engendered by a requirement of unanimous agreement benefits creditors by more than the rise in the probability of disagreement hurts them. Conversely, if markets are illiquid or creditors are far from rational, a unanimity requirement makes successful restructuring almost impossible, hurting creditors and the debtor alike. We apply our results to U.S. regulations governing debt restructuring, and to the current debate on the desirability of a sovereign debt restructuring mechanism. On a more technical level, our analysis extends the existing strategic voting literature (see especially Feddersen and Pesendorfer 1997) to the case in which the issue being voted over is endogenous to the voting rule used.
1 Introduction

An important literature within financial economics has sought to understand how creditors can protect themselves against a debtor who seeks to renegotiate the amount owed. For example, consider a debtor who owes $100 and values staying in business at $100. Suppose moreover that creditors would obtain only $50 from liquidation. The obvious problem for the creditors is that if the debtor defaults on the $100 owed, and then suggests restructuring the debt so that only $50 is owed, it is in the creditors’ best interests to accept. The inability of the creditors to credibly commit to punish the debtor in the event of default both directly reduces the amount that can be borrowed, and worsens any moral hazard problems that may exist.

Previous research has highlighted the role played by the size\(^1\) and numerosity\(^2\) of the creditors, along with the structure of the claims held, in strengthening the \textit{ex post} bargaining power of creditors vis-à-vis the debtor. In this paper, we consider a related but largely neglected question: how does the institutional structure of debt renegotiation impact creditors’ ability to protect themselves?

In practice, debt renegotiation often takes place in a highly structured way. For example, in U.S. style Chapter 11 proceedings, a debtor proposes a reorganization plan, which is then voted upon. The plan is accepted if and only if a supermajority of creditors accept. The details of this procedure are fixed by law — and so, in particular, cannot themselves be renegotiated \textit{ex post}.

Debt renegotiation outside bankruptcy is likewise highly structured. In the U.S., the Trust Indenture Act (1939) stipulates that any changes in the debt contract must be voted upon by creditors, with unanimous agreement required for the proposed change to be accepted. (The alternative of exchange offers are afflicted by the well-known hold-up problem. As a consequence, most exchange offers require acceptance by a very large fraction of credi-

\(^1\)See Dewatripont and Maskin (1995).
\(^2\)See Bolton and Scharfstein (1990), Berglöf and von Thadden (1994), von Thadden, Berglöf and Roland (2003), and Diamond (2004).
tors — often over 95% — turning them into de facto votes, with close to unanimity required for acceptance. In other legal jurisdictions,\(^3\) bond contracts may contain a majority agreement clause, whereby a proposal to change debt terms must be approved only by some (pre-specified) fraction of bondholders.

In all the cases just described, the basic structure of the renegotiation game is the same: the debtor makes a proposal, and the creditors then vote to accept or reject this proposal. In this paper we ask the following question: is the optimal voting rule some form of majority rule, or a unanimous agreement rule?

At a basic level, the trade-off between adopting a majority rule and a unanimity rule is straightforward to describe. On the one hand, requiring unanimity among creditors makes agreement harder to obtain. This hurts creditors when liquidation is worse for them than the proposed restructuring. But on the other hand, the fact that agreement is less likely should induce the debtor to propose an offer which delivers more to the creditors.

By explicitly modelling the voting game, we are able to establish the net impact of these two countervailing effects. When creditors are sufficiently rational, and financial markets are sufficiently liquid, creditors receive more under a unanimity agreement rule: the increased probability of failing to reach an agreement is more than compensated for by the improvement in the offer engendered by the tougher bargaining stance. But if either condition is violated — that is, if either creditors are far from rational, or if markets are very illiquid — then creditors are better off \textit{ex post} under a majority voting rule. In this second case, agreement is essentially impossible to reach under a unanimous agreement rule.

Although a unanimous agreement rule sometimes increases the payoff of creditors \textit{ex post} relative to a majority rule, it always reduces social efficiency \textit{ex post} — both the probabilities of inefficient liquidation and inefficient restructuring rise. Thus even when a unanimity voting rule bolsters the bargaining power of creditors, it does so at a cost. As we formally establish, this cost is worth bearing when the debtor is sufficiently credit constrained at the financing stage.

\(^3\)For example, English law and Luxembourg law.
We apply these results to the U.S. restructuring law, and to sovereign debt. As already noted, U.S. law embodies two diametrically opposing voting rules: outside bankruptcy, unanimity is required, while inside Chapter 11 bankruptcy a majority rule is in effect. Our results suggest that this combination may be quite effective — provided that filing for bankruptcy is costly to the debtor, creditors are protected outside bankruptcy, but at the same time there is an escape hatch provided for those instances in which unanimity cripples any possibility of agreement.

In recent years a large number of observers have called for the creation of a some form of sovereign debt restructuring mechanism (SDRM). A key component of most proposals is to allow a debtor to put a restructuring agreement to a binding majority vote. Proponents of an SDRM argue that without such a possibility, it is too hard for debtors and creditors to reach agreement. On the other side, critics caution that creditors are already poorly protected in sovereign markets, and that facilitating \textit{ex post} renegotiation will weaken their position still further.\footnote{See, e.g., Dooley (2000), or Shleifer (2003).} Our paper provides a theoretical framework in which to assess these competing effects.

\section*{Related literature}

Existing models of bargaining in Chapter 11 bankruptcy treat the creditors as a unified actor and hence are not suited for studying the effect of the voting rules on the creditors’ welfare (see Baird and Picker 1991, Bebchuk and Chang 1992, Eraslan 2003). One partial exception is Kordana and Posner (1999) who discuss many issues, among which is the choice of voting rule. Haldane \textit{et al} (2003) consider voting in the specific context of sovereign debt restructuring. They restrict themselves to an analysis of majority voting among creditors with private values preferences (see below).

On a technical level our analysis builds heavily on the strategic voting literature — see in particular Feddersen and Pesendorfer (1996, 1997, 1998). However, in that literature the issue being voted over is always exogenously fixed — whereas in the case of debt re-
organization the choice of proposed plan is clearly endogenous. To handle this, the heart of the paper consists of extending the existing analysis of strategic voting to the case of endogenously proposed alternatives. As such, our analysis has implications beyond debt reorganization for issues such as union wage bargaining, and perhaps even jury voting when a prosecutor has different preferences from jury members.

Within the finance literature, Maug and Yılmaz (2002) have analyzed a strategic voting game in which creditors with divergent interests are divided into two classes. They show that under some conditions information revelation is enhanced. However, in common with other strategic voting papers, they take the issue being voted over as exogenous. Detragiache and Garella (1996) apply the techniques of mechanism design derived in public goods provision settings to analyze exchange offers. Like us, they observe that when creditors have common value preferences agreement is often easier to obtain. They do not consider voting arrangements. In another non-voting context, Bagnoli and Lipman (1988) study takeover offers. As in the more recent voting literature, they pay careful attention to the probability that an individual shareholder's decision is pivotal.

Paper outline

The paper proceeds as follows. Section 2 outlines the basic model. Section 3 gives an informal overview of our results. Section 4 analyzes the voting game; Section 5 then analyzes the debtor's choice of offer in response to voting behavior. Section 6 characterizes the equilibrium payoffs. Section 7 briefly considers the robustness of our results. Section 8 embeds our analysis of debt restructuring into a simple model of the original financing decision. Finally, section 9 explores several applications of our results.

2 Model

We consider a negotiation between a financially distressed debtor and its $n$ creditors. We will denote a typical creditor by $i \in \{1, \ldots, n\}$. In order to focus on the effect of different
voting rules, we abstract from interclass conflicts: each creditor holds an identical claim against the debtor.

The reorganization and liquidation value of the firm need not be the same. The total liquidation value of the firm is normalized to be 1. There are two types of firms identified by their reorganization value $R \in \{L, H\}$ where $L < 1 < H$. Firms with $R = H$ are not economically distressed, so it is optimal to reorganize them, whereas firms with $R = L$ are economically distressed, so it is optimal to liquidate them. The fraction of the firms with $R = H$ is $p$.

**Restructuring offers and voting**

For the most part we abstract from issues associated with signalling and assume that the debtor is uninformed about the reorganization value. (We briefly consider the opposite extreme of a fully informed debtor in Section 7.) In reorganization, the debtor makes a proposal that allocates a fraction $x$ of the reorganized firm to the creditors (so each receives $x/n$), with the debtor holding on to the remaining fraction $(1 - x)$. Creditors then vote simultaneously on whether to accept or reject the proposal according to the voting rule in place. We allow for any voting rule of the type: the offer is accepted if at least a fraction $\alpha \in (0, 1]$ of the $n$ creditors vote in favor of the offer. If the number of creditors voting for the proposal is less than $\alpha n$, then the firm is liquidated and the game ends. Note that when $\alpha = 1/2 + 1/n$ the agreement rule is the simple majority rule, while when $\alpha = 1$ the agreement rule is the unanimity rule.

We assume that the firm is insolvent in the event of liquidation, so that the payoffs are $1/n$ for each creditor and 0 for the debtor if an agreement cannot be reached. The debtor’s expected payoff is consequently $(1 - x) E[R|\text{offer } x \text{ is accepted}]$.

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5The equilibrium we characterize is also part of a pooling equilibrium of the signalling game in which the debtor is fully informed about the reorganization value. See Section 7.
Creditor preferences: common values or private values?

A key ingredient in our analysis is the extent to which each individual creditor acts as though their payoff from accepting an offer $x$ will be same as the payoff of other creditors. This is exactly the distinction between common values and private values that is made in auction theory, and we will use this same terminology here:

*Common values*

In the benchmark case of common values, each creditor does indeed act as though their payoff will be same as that of other creditors. In many ways, this is the most natural assumption: although each creditor does not know whether the reorganization value is high or low, they know that conditional on the offer being accepted there exists a well-defined probability distribution over these two states — and that this distribution is common to all creditors. That is, each creditor values the offer as $\frac{x}{n} E [R | offer accepted]$.

*Private values*

In practice, there are at least two reasons why the creditors’ behavior may not be correctly described by a common values framework.

First, if capital markets are imperfectly liquid, as will often be the case for shares in a distressed company, then creditors holding the same beliefs about the reorganization value may nonetheless value their shares differently. For example, creditors may have different opportunity costs of funds; or they may face different tax rates. If the shares in the reorganized firm cannot be easily traded, they will not end up in the hands of those who value them the most.

Second, creditors may be imperfectly rational. For example, suppose each creditor $i$ believes that the probability of the reorganization value being high is $\hat{p}_i \in [0, 1]$. Then it may be tempting for that creditor to vote to accept an offer of $x$ if and only if

$$\frac{x}{n} (\hat{p}_i H + (1 - \hat{p}_i) L) > \frac{1}{n}.$$
Behavior of this type is termed *naive* in the voting literature, because in acting in this way creditor $i$ is failing to condition on the information revealed by other creditors’ voting behavior. One reason creditors may vote in this manner is if they fail to follow Bayes’ rule and update correctly based on other creditors’ votes. Alternatively, each creditor may believe that their own estimate of the probability that the reorganization value is $H$ is *much more precise* than the estimates of all other creditors.

**A general framework**

To capture these various possibilities, we proceed as follows. Prior to voting, each individual creditor $i$ receives a “signal” $\sigma_i \in [\sigma, \bar{\sigma}]$. Depending on the context, we interpret $\sigma_i$ in different ways.

If creditors are fully rational and markets liquid, then $\sigma_i$ is simply an informative signal about the liquidation value. Creditor $i$ values the offer as

$$\frac{x}{n} E [R | \sigma_i \text{ and the other creditors’ signals } \sigma_j \text{ are such that offer accepted}].$$

Alternatively, if markets are completely illiquid, $\sigma_i$ is creditor $i$’s valuation of a share in the reorganized firm. Creditor $i$ values the offer as simply $\frac{x}{n} \sigma_i$. Likewise, if creditors are either completely non-Bayesian, or very overconfident of their own expertise relative to other creditors, then $\sigma_i$ is the creditor’s estimate of the reorganization value — and again, creditor $i$ values the offer at $\frac{x}{n} \sigma_i$.

To capture possibilities inbetween, we define a preference parameter $\lambda \in [0, 1]$ and a utility function

$$u(\sigma_i, R, x) \equiv x(\lambda \sigma_i + (1 - \lambda) R).$$

Each creditor values an offer $x$ as

$$\frac{1}{n} E [u(\sigma_i, R, x) | \sigma_i \text{ and the other creditors’ signals } \sigma_j \text{ are such that offer accepted}].$$

*The difficulties of updating one’s own belief to incorporate the information revealed by other votes are akin to those faced by a bidder in an auction. See, for example, the “Winner’s curse” chapter in Thaler (1991) for evidence that bidders are not always up to this challenge.*
Clearly by setting the preference parameter \( \lambda = 0 \), we recover the extreme of fully rational creditors in a liquid market, i.e., *common values*; while by setting \( \lambda = 1 \), we are modelling creditors inhabiting one of the *private values* settings discussed.

**Distributional assumptions**

Independent of the interpretation we are placing on \( \sigma_i \), we make the following distributional assumptions. (a) After conditioning on the true reorganization value, \((\sigma_i)_{i \in 1, \ldots, n}\) are independent and identically distributed random variables. Conditional on the reorganization value \( R \), let \( F(\sigma|R) \) and \( f(\sigma|R) \) denote the distribution and density functions of \( \sigma_i \). (b) The density function \( f(\sigma|R) \) is a continuous function of \( \sigma \) for both \( R \in \{L, H\} \). (c) After either realization of the reorganization value \( R \in \{L, H\} \), \( \sigma_i \) has full support over \([\underline{\sigma}, \bar{\sigma}]\). (d) The limits of the support satisfy \( \underline{\sigma} < 1 < \bar{\sigma} \). That is, even when preferences are at the private values extreme, a creditor sometimes prefers liquidation to reorganization, and sometimes prefers reorganization to liquidation. (e) The realizations of \( \sigma_i \) are informative about the true reorganization value, in the sense that the monotone likelihood ratio property (MLRP) holds strictly: \( f(\sigma|H)/f(\sigma|L) \) is strictly increasing in \( \sigma \in [\underline{\sigma}, \bar{\sigma}] \). (f) Not too much information is revealed by a signal \( \sigma \in [\underline{\sigma}, \bar{\sigma}] \): \( f(\sigma|H)/f(\sigma|L) \) is bounded away from both 0 and \( \infty \).

**3 Overview of results**

Our main result is:

**Proposition 1** (*Unanimity better for creditors*)

Consider any majority voting rule. Then whenever creditors have preferences sufficiently close to *common values* and the number of creditors is sufficiently large the creditor payoff is higher under a requirement of unanimity for acceptance.

The proof of this result is quite long and involved. In this section, we offer an informal overview of the logic underlying this result.
First, consider the case in which the coalition uses a majority voting rule. It is a now standard result in strategic voting models that majority voting aggregates the available information very well. Consequently, if the debtor makes an offer such that \( xH \) lies just above the liquidation value \( 1 \), it will be accepted whenever the true reorganization value is \( R = H \), and rejected whenever the true reorganization value is \( R = L \). Figure 1 graphically displays these observations. Together, they imply that the debtor will make an offer close to \( 1/H \): if he makes a lower offer, it is rejected for sure, while the only way to further raise the acceptance probability would be to offer close to \( 1/L \), which would amount to giving away more than 100% of the firm.

Next, consider the case in which unanimous agreement is required in order for the coalition of creditors to accept an offer, displayed in Figure 2. Such a rule does not efficiently aggregate information. So when the true reorganization value is \( R = H \), the coalition will sometimes wrongly reject offers that satisfy \( xH > 1 \), while when the reorganization value is \( R = L \), the coalition will sometimes wrongly accept an offer. When the offer \( x \) is close to \( 1/H \), however, it can be shown that the first of these effects is much stronger. Consequently, offers in this range are rejected with high probability. It follows that in order to secure a reasonable acceptance probability, the debtor is forced to make an offer such that \( xH \) is significantly above the liquidation value \( 1 \).

4 Voting over an offer

To characterize of the outcome of the debt reorganization, we work backwards: we first find the probability that any given offer \( x \) is accepted (this section), and then solve for the debtor’s preferred offer (section 5).

Consider a creditor \( i \) who has received a signal \( \sigma_i \). Let \( piv \) denote the event that his vote is pivotal, and \( \overline{piv} \) denote the event that his vote is not pivotal. Define \( K(\sigma_i) = E[\text{payoff} | \overline{piv}] \Pr(\overline{piv} | \sigma_i) \), the expected payoff of creditor \( i \) in states in which he is not
Debtor’s offer

Acceptance probability

Lemma 4: accept iff $R = H$

Lemma 4: accept always

Lemma 4: reject always

Figure 1: The acceptance probability as a function of the debtor’s offer $x$ under majority voting
Lemma 5: reject offers close to $1/H$ with very high probability

Lemma 6: accept offers away from $1/H$ with positive probability

Figure 2: The acceptance probability as a function of the debtor’s offer $x$ under a unanimous acceptance rule
pivotal. His payoff from voting against the proposal is
\[
\text{Pr} (R = H, \text{piv} | \sigma_i) + \text{Pr} (R = L, \text{piv} | \sigma_i) + K(\sigma_i)
\]
while his payoff from voting for the proposal is
\[
u(\sigma_i, H, x) \text{Pr} (R = H, \text{piv} | \sigma_i) + \nu(\sigma_i, L, x) \text{Pr} (R = L, \text{piv} | \sigma_i) + K(\sigma_i)
\]
Thus creditor \(i\) votes to accept proposal \(x\) if and only if
\[
u(\sigma_i, H, x) \text{Pr} (R = H, \text{piv} | \sigma_i) + \nu(\sigma_i, L, x) \text{Pr} (R = L, \text{piv} | \sigma_i) \geq \text{Pr} (R = H, \text{piv} | \sigma_i) + \text{Pr} (R = L, \text{piv} | \sigma_i)
\]
Note that
\[
\text{Pr} (R, \text{piv} | \sigma_i) = \frac{\text{Pr} (R) \text{Pr} (\text{piv} | R) f (\sigma_i | R)}{p f (\sigma_i | H) + (1 - p) f (\sigma_i | L)}
\]
Lemma 1 (Each agent follows a cutoff strategy)
In any equilibrium, for each creditor \(i\) there must exist a cutoff signal \(\sigma_i^*(x, \alpha) \in [\bar{\sigma}, \bar{\sigma}]\) such that creditor \(i\) vote to accept proposal \(x\) if his signal is more positive than \(\sigma_i^*(x, \alpha)\), i.e., \(\sigma_i > \sigma_i^*(x, \alpha)\); and votes to reject the proposal if his signal is more negative, i.e., \(\sigma_i < \sigma_i^*(x, \alpha)\).

The highest offer the debtor ever contemplates making is clearly \(x = 1\). We assume throughout that the information content of signals is such that if an individual creditor receives the most negative signal (in terms of the reorganization value), \(\sigma_i = \bar{\sigma}\), then he will decline the offer \(x = 1\) — as well as any less generous offer. That is:

Assumption 1 (Reject the best offer given information \(\sigma_i = \bar{\sigma}\))
For all \(\lambda \in (0, 1)\) and \(x \leq 1\),
\[
1 > \frac{u(\sigma, H, x) p f (\sigma | H) + u(\sigma, L, x) (1 - p) f (\sigma | L)}{p f (\sigma | H) + (1 - p) f (\sigma | L)}.
\]

Equilibrium existence

Throughout, we focus on symmetric equilibria in which all creditors follow the same voting strategy. In light of Lemma 1, let \( \sigma^*(x, \alpha) \in [\underline{\sigma}, \bar{\sigma}] \) denote the common cutoff signal. For clarity of exposition, we will suppress the arguments \( x \) and \( \alpha \) unless needed. Evaluating explicitly, the probability that an agent is pivotal is given by

\[
\Pr(\text{piv}|R) = \left( \frac{n}{n\alpha - 1} \right) (1 - F(\sigma^*|R))^{n\alpha - 1} F(\sigma^*|R)^{n - n\alpha}.
\]

Substituting into inequality (1), creditor \( i \) votes to accept proposal \( x \) after observing signal \( \sigma_i \) if and only if

\[
(u(\sigma_i, H, x) - 1) pf(\sigma^*|H) (1 - F(\sigma^*|H))^{n\alpha - 1} F(\sigma^*|H)^{n(1-\alpha)} \\
\geq (1 - u(\sigma_i, L, x)) (1 - p) f(\sigma^*|L) (1 - F(\sigma^*|L))^{n\alpha - 1} F(\sigma^*|L)^{n(1-\alpha)}
\]

If there exists a \( \sigma^* \in (\underline{\sigma}, \bar{\sigma}) \) such that creditor \( i \) is indifferent between accepting and rejecting the offer \( x \) exactly when he observes the signal \( \sigma_i = \sigma^* \), then the equilibrium can be said to be a responsive equilibrium: there is a positive probability that each creditor votes to accept, and a positive probability that each creditor votes to reject. That is, a responsive equilibrium exists whenever the equation

\[
\frac{u(\sigma^*, H, x) - 1}{1 - u(\sigma^*, L, x)} \frac{pf(\sigma^*|H)}{(1 - p) f(\sigma^*|L)} \frac{1 - F(\sigma^*|L)}{1 - F(\sigma^*|H)} = \left( \frac{(1 - F(\sigma^*|L))^{\alpha} F(\sigma^*|L)^{1-\alpha}}{(1 - F(\sigma^*|H))^{\alpha} F(\sigma^*|H)^{1-\alpha}} \right)^n
\]

has a solution \( \sigma^* \in (\underline{\sigma}, \bar{\sigma}) \).

**Lemma 2 (Existence of a responsive equilibrium)**

Fix a voting rule \( \alpha \), and some \( \epsilon > 0 \). Then there exists an \( N \) such that a responsive equilibrium exists whenever \( n \geq N \) and \( x \in \left[ \frac{1}{\lambda \sigma + (1-\lambda)H} + \epsilon, 1 \right] \).

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7Throughout, we ignore the issue of whether or not \( \alpha n \) were an integer. This issue could easily be handled formally by replacing \( \alpha n \) with \( \lfloor \alpha n \rfloor \) everywhere, where \( \lfloor \alpha n \rfloor \) denotes the smallest integer weakly greater than \( \alpha n \). Since this formality has no impact on our results, we prefer to avoid the extra notation and instead proceed as if \( \alpha n \) is an integer.
For any offer such that \( x (\lambda \bar{\sigma} + (1 - \lambda) H) \leq 1 \), voting to accept is clearly a dominated strategy, since liquidation gives at least as much — and maybe more. For such offers, voting to reject with probability 1 is the only equilibrium to survive elimination of dominated strategies.

What about offers such that \( x (\lambda \bar{\sigma} + (1 - \lambda) H) \) is above 1 but the number of creditors is not large enough for Lemma 2 to hold? In general, for any offer \( x \) there is an equilibrium in which all creditors vote to reject; and except for the case of unanimity voting, there is also an equilibrium in which all creditors vote to accept. However:

**Lemma 3 (What if no responsive equilibrium exists?)**

Suppose a voting rule \( \alpha \geq \frac{1}{2} + \frac{1}{2n} \) is in effect. Then if no responsive equilibrium exists for an offer \( x \in (\frac{1}{\lambda \sigma + (1 - \lambda) H} + \varepsilon, 1] \), the only symmetric perfect equilibrium is that in which all creditors reject the offer with probability 1.

In light of Lemmas 2 and 3, we focus on the responsive equilibrium whenever one exists, and otherwise use the rejection equilibrium. Notationally, we represent responsive equilibria by their cutoff value \( \sigma^* \in (\underline{\sigma}, \bar{\sigma}) \), and the rejection equilibrium by \( \sigma^* = \bar{\sigma} \).

**Notation and conventions**

Given a voting rule \( \alpha \) and reorganization value \( R \in \{L, H\} \), let \( \sigma_R \) be the signal realization such that there is a probability of \( \alpha \) that a creditor receives a signal at least as positive:

\[
\Pr (\sigma > \sigma_R | R) = 1 - F (\sigma_R | R) = \alpha.
\]

Given \( \sigma_R \), we define \( x_R(\lambda) = 1/(\lambda \sigma_R + (1 - \lambda) R) \). As we will see, the pair of offers \( x_H \) and \( x_L \) play an important role in our analysis.

For use below, note that since \( \alpha > 0 \), \( \sigma_R < \bar{\sigma} \). Moreover, observe that \( \sigma_L \leq \sigma_H \), with the inequality strict if strict MLRP holds. To see this, suppose to the contrary that \( \sigma_H < \sigma_L \). Then \( 1 - \alpha = F (\sigma_H | H) < F (\sigma_L | H) \leq F (\sigma_L | L) = 1 - \alpha \) which is not possible.\(^8\) Since \( \sigma_L \leq \sigma_H \), the definition of \( x_R \) immediately implies that \( x_H(\lambda) \leq x_L(\lambda) \) for all \( \lambda \).

\(^8\)The second inequality here is a consequence of MLRP — see Lemma 13 in the appendix.
Note that as $\lambda \to 0$, $x_R \to 1/R$ and as $\lambda \to 1$, $x_R(\lambda) \to 1/\sigma_R$. Unless needed, we will suppress the dependence of $x_R$ on $\lambda$.

In the following subsections, we characterize the probability of an offer being accepted. Let $A$ denote the event that $n\alpha$ out of $n$ voters vote for the proposal. Then

$$ \Pr(A|R) = \sum_{j=n\alpha}^{n} \binom{n}{j} (1 - F(\sigma^*|R))^j F(\sigma^*|R)^{n-j} $$

and

$$ \Pr(A) = p \Pr(A|H) + (1 - p) \Pr(A|L) $$

Let $\sigma^*_n(\lambda)$ denote the equilibrium cutoff signal and let $x_n(\lambda)$ denote the equilibrium offer when there are $n$ creditors. Again, we will suppress the dependence of these variables on $\lambda$ unless needed. In what follows, we are interested primarily in the asymptotic properties of conditional and unconditional probabilities of agreement as $n$ grows large.

This is a convenient point to remark on a technical detail. Recall that a key component of our analysis is that the debtor’s offer $x_n$ is endogenously selected. As such, we need to be able to characterize the acceptance probabilities for arbitrary offers $x_n$. Consequently, we cannot assume that the corresponding equilibrium cutoff signals $\sigma_n(\lambda)$, nor the associated agreement probabilities, converge to well-defined limits as the number of creditors $n$ grows large. To deal with this difficulty, we will state almost all of our results in terms of the limit infimum (lim inf) and limit supremum (lim sup). Recall that for any sequence of real numbers $z_n$, these mathematical objects are defined by

$$ \liminf z_n = \lim_{N \to \infty} \inf \{z_n : n \geq N\} \quad \text{and} \quad \limsup z_n = \lim_{N \to \infty} \sup \{z_n : n \geq N\}. $$

For example, if $\limsup \sigma^*_n < \sigma_R$ then the cutoff signal $\sigma^*_n$ stays bounded above by $\sigma_R - \varepsilon$, for some $\varepsilon > 0$. Since by definition the probability that a creditor receives a more positive signal than $\sigma_R$ is $\alpha$, by the weak law of large numbers it follows that $\Pr(A|R) \to 1$. Similarly, if $\liminf \sigma^*_n > \sigma_R$ then $\sigma^*_n$ is bounded below by $\sigma_R + \varepsilon$ for some $\varepsilon > 0$, and $\Pr(A|R) \to 0$. 

15
Majority rule

We start by characterizing the acceptance probabilities for different offers \( x \) under majority voting (see Figure 1). Proceeding informally, it is useful to consider the two extremes of private values and common values.

Private values

Under private values (\( \lambda = 1 \)), the analysis is straightforward. Each creditor accepts an offer \( x \) only if his or her own private valuation of this offer, \( x\sigma \), exceeds the liquidation value 1. Consider an offer \( x \) by the debtor such that \( x\sigma R < 1 \) when the true reorganization value is \( R \). Since the probability that a creditor values the reorganization at more than this is less than \( \alpha \), the probability that \( \alpha \) or more vote to accept the offer is essentially zero when there are many creditors. Likewise, an offer \( x \) such that \( x\sigma R > 1 \) when the true reorganization value is \( R \) is almost certain to be accepted when the number of creditors is large.

Common values

Analysis of the common values case (\( \lambda = 0 \)) is somewhat more complicated. The existing literature on strategic voting has established that for any majority voting rule the private information of voters is effectively aggregated when the number of voters is large. In the current context, when the number of creditors is large then their individual estimates \( \sigma \) of the reorganization value jointly reveal the true reorganization value \( R \). Efficient aggregation of information then means that offers such that \( xR > 1 \) will be accepted almost for sure, while offers such that \( xR < 1 \) will be rejected almost for sure.

A rough intuition for this result is as follows. For concreteness, consider the case of a supermajority rule requiring the consent of two-thirds of the creditors (\( \alpha = 2/3 \)). Clearly there exist values for the cutoff signal \( \sigma^* \) such that when the true reorganization value is \( L \), each creditor votes to accept with a probability a little less than 2/3, while when the true
liquidation value is $H$ each creditor votes to accept with a probability a little more than $2/3$. In the former case, an individual creditor is pivotal when an unusually large number of creditors receive a high signal; while in the latter case, an individual is pivotal when an unusually small number of creditors receive a high signal.

By choosing the cutoff signal $\sigma^*$ appropriately, it is possible to make the probabilities that an individual creditor is pivotal arbitrarily close for the two underlying reorganization values $R = H, L$. This is essentially the nature of the voting equilibrium under a majority rule: if all but one creditors follow the cutoff rule $\sigma^*$, the remaining creditor views the probability of high and low reorganization values as roughly equal after conditioning on the event that he turns out to be pivotal. As such, he will then vote to accept when his own signal is high, and to reject when his own signal is low. Information aggregation occurs in this equilibrium since all creditors vote to accept when their private signals are high, and to reject when their private signals are low.

**Intermediate preferences**

Our formal result generalizes these extreme cases. It can be interpreted in a straightforward way. Just as we have parameterized the payoffs as a convex combination of private and common values, the voting equilibrium is essentially a convex combination of the equilibria for these two cases: in the limit, an offer is accepted if and only if it is above $x_R = 1/((\lambda \sigma_R + (1 - \lambda) R)$. The first component of the denominator corresponds to behavior under private values, while the second component corresponds to the information revelation that occurs under common values.

**Lemma 4 (Acceptance probabilities under majority voting)**

If $\lim \inf x_n > x_R$ then $\Pr(A | R) \rightarrow 1$ and if $\lim \sup x_n < x_R$ then $\Pr(A | R) \rightarrow 0$. 

We now turn to an analysis of voting behavior when unanimity in required for agreement. The existing strategic voting literature has established that voting in the common values case does not aggregate information. As such, the equilibrium acceptance probabilities differ starkly from those arising under any majority voting arrangement, even in the limit. Although the limiting acceptance probabilities do not have a convenient analytical form, as they do in Lemma 4, we are nonetheless able to characterize the key qualitative properties. In doing so, we extend the existing theory to link the extent to which information aggregation fails with the issue being put to the vote.

It is convenient to start by stating the results of this subsection in a somewhat informal manner. Recall from Lemma 4 that under majority voting, as the number of creditors grows large the debtor is able to have an offer arbitrarily close to $x_H$ accepted with probability arbitrarily close to 1 conditional on the true reorganization value being $H$. This is not the case under a requirement of unanimity: if the debtor tries to drive the offer down towards $x_H$ as the number of creditors grows large, that offer will be accepted with a vanishingly small probability.

**Lemma 5 (Informal version)**

If $x_n \to x_H$, the acceptance probability converges to 0.

Does Lemma 5 imply that the debtor is doomed to have his offer rejected when he faces a large coalition of creditors bound by a unanimous agreement rule? The answer is no. Provided he makes offers that stay bounded away from $x_H$, those offers will be accepted at least sometimes.

**Lemma 6 (Informal version)**

If $x_n$ is bounded away from above, the acceptance probability is likewise bounded away from 0.
The sharp difference between Lemmas 4 and 5 is traceable to the fact that when voters have common values, information is aggregated poorly under a unanimity voting rule. The reason is that as the number of creditors grows large, for any given cutoff rule $\sigma^*$ a creditor believes he is vastly more likely to be pivotal when the true reorganization value is $H$. Consequently, if all but one creditors employs a cutoff rule $\sigma^*$, the remaining creditor finds it best to ignore his own signal entirely — reasoning that conditional on actually holding the pivotal vote, the true reorganization value must in fact be $H$. Given this, the only possibilities for an equilibrium are those in which $\sigma^*$ converges to a limit of the signal support as the number of creditors grows large. But in these cases, creditors either always vote to accept, or always to reject, independent of their own signal. As a result, information aggregation fails.

Because of the failure of voting to aggregate information, the voting outcome is inefficient \textit{ex post}. That is, offers such that $xH > 1$ are rejected even when the true reorganization value is $H$. Lemma 5 shows that offers such that $xH$ is close to the liquidation value 1 are almost always mistakenly rejected in this manner. However, in Lemma 6 we show that the incidence of mistakes declines as the debtor improves his offer $x$.

Looking ahead to the debtor’s decision of what offer to make, Lemma 4 suggests that when facing a large coalition of close-to-Bayesian creditors employing a majority voting rule, the debtor will make an offer such that $xH$ is very close to the liquidation value 1. On the other hand, Lemmas 5 and 6 imply that if the same coalition is bound by a requirement of unanimous agreement, the debtor will make an offer such that $xH$ is strictly more than 1. Together, these observations clearly suggest that when the creditors have close to common values, they will obtain more in debt renegotiations by employing a unanimous voting rule.

Formalizing this argument requires some care, however. First, we must check whether higher offers actually improve the welfare of creditors when they use a unanimous voting rule — after all, since they are making mistakes, a higher offer from the debtor might cause them to mistakenly accept reorganizations when they should be liquidating, and so actually reduce creditor welfare. We return to this point in Lemma 10 below.
Second, our formal result involves a “double limit” — the coalition of creditors must be both sufficiently large, and their preferences sufficiently close to common values. Reflecting this need to handle the double limit, and also the fact that there is no reason to suppose that the equilibrium cutoff values converge to anything at all, the formal statements of Lemmas 5 and 6 are as follows:

**Lemma 5 (Low offers rejected under unanimity)**

For any $\lambda_0 \geq 0$, if

$$\sup_{\lambda \geq \lambda_0, N} \inf_{\lambda_0 \leq \lambda \leq \lambda_0 > 0, n \geq N} x_n(\lambda) - x_H(\lambda) \leq 0$$

then

$$\sup_{\lambda \geq \lambda_0, N} \inf_{\lambda_0 \leq \lambda \leq \lambda_0 > 0, n \geq N} \Pr(A|\lambda, n, x_n(\lambda)) = 0$$

**Lemma 6 (Intermediate offers accepted under unanimity)**

For any $\lambda_0 \geq 0$, if

$$\sup_{\lambda \geq \lambda_0, N} \inf_{\lambda_0 \leq \lambda \leq \lambda_0 > 0, n \geq N} x_n(\lambda) - x_H(\lambda) > 0$$

then

$$\sup_{\lambda \geq \lambda_0, N} \inf_{\lambda_0 \leq \lambda \leq \lambda_0 > 0, n \geq N} \Pr(A|\lambda, n) > 0$$

Remark: To interpret inequalities (6) - (9), it may help to observe that if $\lambda_0 > 0$ then

$$\sup_{\lambda \geq \lambda_0, N} \inf_{\lambda_0 \leq \lambda \leq \lambda_0 > 0, n \geq N} x_n(\lambda) - x_H(\lambda) = \lim_{N \to \infty} \inf_{n \geq N} x_n(\lambda_0) - x_H(\lambda_0) = \lim \inf_{n \geq N} x_n(\lambda_0) - x_H(\lambda_0)$$

while if $\lambda_0 = 0$ then

$$\sup_{\lambda \geq \lambda_0, N} \inf_{\lambda_0 \leq \lambda \leq \lambda_0 > 0, n \geq N} x_n(\lambda) - x_H(\lambda) = \sup_{\lambda_0 \leq \lambda \leq \lambda_0 > 0, n \geq N} x_n(\lambda) - x_H(\lambda).$$

5 The debtor’s choice of offer

Recall that the liquidation payoff of the debtor is assumed to be zero. Consequently the debtor chooses the offer $x$ to maximize

$$(1 - x)(pH \Pr(A|x, H) + (1 - p)L \Pr(A|x, L)).$$
Since the equilibrium cutoff signal $\sigma^*(x, \alpha)$ is continuous in $x$, and the probability of acceptance is continuous in $\sigma^*$, a solution exists.

As discussed above, when the debtor faces a coalition of creditors employing a majority voting rule, as the number of creditors grows large an offer that is arbitrarily close to $x_H$ is accepted with a probability arbitrarily close to 1 when the true reorganization value is $H$. In this case, the only way for the debtor to significantly increase the probability that his offer is accepted is to offer something close to $x_L$. However, in many cases — and in particular creditors’ preferences are sufficiently close to the extreme of common values — $x_L$ exceeds 100%. In such circumstances the debtor’s best offer converges to $x_H$:

**Lemma 7 (The debtor’s offer under a majority voting rule)**

Suppose a majority voting rule $\alpha < 1$ is in effect, and that for a given preference parameter $\lambda$, $x_H < 1$ and $(1 - x_H) p_H > (1 - x_L) E[R]$. Let $x_n$ be a sequence of offers that maximize the debtor’s payoff. Then $x_n \to x_H$. Moreover, the acceptance probability conditional on the true reorganization value being $R = H$ converges to 1.

In contrast, when the creditors employ a unanimity voting rule, offers close to $x_H$ are rejected with very high probability — even when the number of creditors is large. As such, the debtor is forced to offer an amount that stays bounded strictly away from $x_H$.

**Lemma 8 (The debtor’s offer under a unanimity voting rule)**

Suppose a unanimity voting rule is in effect ($\alpha = 1$). For each preference parameter $\lambda$, let $x_n(\lambda)$ be a sequence of offers that maximize the debtor’s payoff. Then

$$\sup_{\Lambda, N} \inf_{\lambda < \Lambda, n \geq N} x_n(\lambda) - x_H(\lambda) > 0.$$ 

6 Equilibrium payoffs

In the previous two sections we have characterized the creditors’ voting responses to a debtor’s offer, and then the debtor’s optimal choice of offer given those voting responses. We turn now to an evaluation of the equilibrium payoffs.
Let $\Pi_{\alpha,n}(x;\lambda)$ denote the creditors’ aggregate expected payoff under voting rule $\alpha$ given coalition size $n$ and an offer from the debtor of $x$. Decomposing,

$$
\Pi_{\alpha,n}(x_n(\lambda);\lambda) = pE[u(\sigma_i, H, x_n(\lambda)) | A, H, x_n(\lambda)] \Pr(A|H, x_n(\lambda)) + (1-p)E[u(\sigma_i, L, x_n(\lambda)) | A, L, x_n(\lambda)] \Pr(A|L, x_n(\lambda)) + (1 - \Pr(A|x_n(\lambda)))
$$

(10)

Close-to-common values creditors

When creditors have close to common values, and employ a majority voting rule, the debtor makes an offer such that $xH$ is just more than the liquidation value 1. With very high probability the creditors vote to accept this offer when the true reorganization value is $H$, and vote to reject it when the true reorganization value is $L$. As such, in either state the creditors are left with a payoff that is close to their liquidation payoff of 1:

**Lemma 9 (Creditor payoff under majority voting rule)**

Suppose a majority voting rule $\alpha < 1$ is in effect. For each preference parameter $\lambda$, let $x_n(\lambda)$ be a sequence of offers that maximize the debtor’s payoff. Then the creditors’ payoff satisfies

$$
sup_{A,N} \inf_{\lambda \leq A, n \geq N} \Pi_{\alpha,n}(x_n(\lambda);\lambda) \leq 1.
$$

In contrast to the situation arising under majority voting, when close-to-common values creditors instead require unanimous agreement to accept the debtor’s offer, the debtor responds by offering an amount such that $xH$ stays bounded strictly away from the liquidation value 1 (Lemma 8). At first sight it might seem that this trivially implies that the creditors’ payoff is higher under the unanimity requirement.

However, matters are not quite that simple. Recall that creditors are receiving an offer of more than $1/H$ precisely because information is aggregated poorly under unanimity voting, and so they sometimes mistakenly reject offers above $1/H$. But by the same token, creditors sometimes wrongly accept the debtor’s offer when the reorganization value is $L$. A priori it is by no means clear that the net effect of receiving a higher offer, but mistakenly passing up liquidation opportunities, is positive.
To deal with this complication, observe first that by Lemma 5 an offer of $1/H$ is always rejected by close-to-common values creditors using a unanimity voting rule. Since liquidation always ensues, the creditors expected payoff in this case coincides with the expected liquidation value of 1.

Suppose now that we exogenously increase the offer made to the creditors. How does this affect their payoff? On the one hand, if the creditors accept the offer in exactly the same states an increase in the offer clearly raises welfare. The potential caveat is that the set of states in which the offer is accepted changes. Since acceptance is suboptimal for the creditors when the true reorganization value is $L$, in principle this change in the acceptance states may lead to a decrease in their welfare. However, our next result establishes that the welfare of close-to-common values creditors is increased by an amount at least as great as if the acceptance states remained unchanged.

**Lemma 10 (Effect of an exogenous increase in the offer $x$)**

Suppose that a unanimity voting rule is in effect ($\alpha = 1$). Then if an offer $x$ is such that a responsive voting equilibrium exists, the derivative of the creditors’ payoff with respect to the offer $x$ is at least equal to the expected utility of creditors over states in which the offer is accepted.

Combined with the above observations, Lemma 10 is enough to establish that the welfare of close-to-common values creditors is indeed higher under a unanimity voting rule than under a majority voting rule. That is, if the debtor’s offer is exogenously fixed at $1/H$ in the case of unanimity voting, the creditors’ welfare coincides with their welfare under a majority voting rule and the debtor’s optimal offer. From Lemma 8 we know the creditors in fact receive an offer above $1/H$ when they use a unanimous agreement rule, and from Lemma 10 we know this increase in the offer does in fact benefit them. Stated formally,

**Lemma 11 (Creditor payoff under a unanimous rule)**

Suppose a unanimity voting rule is in effect ($\alpha = 1$). Then the creditors’ payoff satisfies

$$\sup_{\Lambda,N} \inf_{\lambda \leq \lambda, n \geq N} \Pi_{1,n}(x_n(\lambda); \lambda) > 1.$$
Lemmas 9 and 11 together give our main result:

**Proposition 1 (Unanimity better for common values creditors)**

Consider any majority voting rule $\alpha < 1$. Then whenever creditors have preferences sufficiently close to common values and the number of creditors is sufficiently large, the creditors’ payoff is higher under a requirement of unanimity for acceptance.

**Close-to-private values creditors**

Conventional wisdom identifies two opposing effects of adopting a unanimous voting rule. On the one hand, unanimity makes agreement harder to obtain. On the other hand, this “toughness” may be useful in negotiation. Proposition 1 identifies a fairly general circumstance in which the latter effect dominates: whenever creditors are close enough to common values, the increase in the debtor’s offer relative to that obtained under majority voting more than compensates for the increased probability of mistakenly rejecting the offer.

One way to think about this result is that when creditors vote strategically, the requirement of unanimity is not as inimical to agreement as it might at first seem. Recall that each creditor conditions his or her vote only on the circumstances under which it is actually pivotal. Given a unanimity agreement rule, this means that a creditor considers the impact of voting to accept an offer conditional on all other creditors accepting — in other words, conditional on all other voters viewing the offer as attractive. Such a creditor will vote to accept unless his own signal is very pro-liquidation.

In contrast, as we move to a situation in which creditors are further away from the common values benchmark, we reach a situation in which agreement is indeed extremely difficult to obtain under unanimity. This is most easily seen at the extreme of fully private values preferences: each creditor will vote to accept only if his own valuation of the offer on the table beats the liquidation value. When there are many creditors, it is almost certain that at least one of them will regard liquidation as more attractive. As such, agreement becomes extremely rare.
Lemma 12 (Private values creditor payoff under unanimity)

Suppose a unanimity voting rule is in effect \((\alpha = 1)\), and that creditors have preferences sufficiently close to private values such that \(\lambda \sigma + (1 - \lambda) H < 1\). Then under any feasible set of offers \(x_n \leq 1\), the acceptance probability converges to 0 and the creditor payoff converges to 1.

Under a majority voting arrangement, agreement is also harder to obtain when creditors do not have common values preferences. Again, this is most easily seen from a comparison of the extreme cases. As we have established, when creditors’ preferences embody close-to-common values, if the true reorganization value is \(R = H\) they will accept an offer \(x\) such that \(xH > 1\) with a probability very close to 1. But if creditors’ preferences embody close-to-private values, the debtor must make and offer \(x\) such that \(x\sigma H > 1\) to be confident that it will be accepted when the true reorganization value is high. Since we can always choose \(\alpha\) such that \(\sigma H < H\), the offer made in the latter case is higher, at least for some majority rule.

Loosely speaking then, private values creditors drive a tougher bargain than common values ones — and are impossible to satisfy if they adopt a unanimous agreement rule, in which case liquidation ensues for sure. Since the creditors are themselves worse off under liquidation, we then obtain the following converse to Proposition 1:

Proposition 2 (Unanimity worse for private values creditors)

Suppose the signal quality is high enough such that \(E[\sigma|H] > 1\). Then there exists a \(\Lambda < 1\) such that whenever \(\lambda \geq \Lambda\), i.e., creditors’ preferences are sufficiently close to private values, then there exists a majority voting rule \(\alpha < 1\) such that the creditor payoff is higher under that rule than under a unanimity voting rule.

7 Robustness

We pause briefly to discuss the robustness of our results to allowing for the case that the debtor knows the true reorganization value, \(R \in \{L, H\}\). We restrict attention to the
benchmark case of close-to-common values preferences.

First, observe that since majority voting rules aggregate information, asymptotically whether or not the debtor knows the true reorganization value makes no difference. That is, any offer from a low reorganization value debtor will be rejected, while an offer just above $1/H$ from a high reorganization debtor will be accepted almost for sure. The equilibrium matches that when the debtor is completely uninformed.

We now turn to the case of a unanimous agreement rule:

1. There is no non-trivial pure strategy separating equilibrium: Clearly there is no separating equilibrium in which the $L$-debtor makes an offer that is accepted. But nor can there be an equilibrium in which the $L$-debtor makes an offer that is rejected, while the $H$-debtor makes an offer that is accepted with positive probability. So the only possible separating equilibrium is one in which both debtor types make (distinct) offers that are rejected with probability 1.

2. In contrast, the equilibrium behavior of the game in which the debtor does not know his type forms part of a pooling equilibrium in the game in which the debtor is fully informed. This is easily seen: if creditors interpret any deviation from the pooling offer as being made by a $L$-debtor, they will reject such an offer.

3. In fact, it is a pooling equilibrium for both debtor types to pool at any offer $x$. But by construction, pooling at the offer associated with an uninformed debtor is the equilibrium that maximizes debtor welfare.

4. There may also exist a semi-separating equilibrium of the following type: the $L$-debtor offers $x^P$, while the $H$-debtor mixes between $x^P$ and $x^H > x^P$. Creditors accept the offer $x^P$ with a probability strictly between 0 and 1, and the offer $x^H$ with probability 1.

In such an equilibrium, the conclusion of Proposition 1 continues to obtain: creditors are better off under the unanimity voting rule. This is easily seen. The logic of Proposition 1 directly implies that over the pooling portion of the equilibrium,
creditors receive more than they would under a majority voting rule — where they receive the liquidation value 1. Moreover, in the separating part of equilibrium, creditors must receive at least their liquidation value.

8 Trading off *ex post* inefficiency and creditor recovery

Proposition 1 tells us that a unanimity voting rule improves creditor welfare when creditor preferences are sufficiently close to common values. However, it does so at a cost: creditors accept some reorganization offers even when the reorganization value is less than the liquidation value of the firm, and may also reject some offers even when the reorganization value is greater than the liquidation value. In contrast, under majority voting the probability of both types of mistakes approaches 0 as the number of creditors grows large (see Lemma 7).

Clearly, if creditors are able to decide on a voting rule after the debt has been issued, they will opt for a unanimity voting rule whenever they are sufficiently numerous and sufficiently close to common values. In practice, however, voting rules are likely to be fixed in advance: either by law, or by the terms of the debt contract. Given the cost it imposes in terms of inefficient liquidation *ex post*, is a unanimous voting rule ever socially efficient? Relatedly, would a prospective borrower ever want to issue debt specifying that unanimous agreement by creditors is required in order to change the terms of the debt agreement?

One instance in which a requirement of unanimous agreement among creditors is likely to be socially valuable is that in which the debtor is credit constrained. Under such a circumstance, unanimity permits a debtor to promise a higher repayment to creditors, thus loosening the credit constraint. Even though the unanimous agreement rule imposes a social cost *ex post*, this cost will be worthwhile provided that relaxing the credit constraint

\[ \frac{u(\sigma^*, H, x) - 1}{1 - u(\sigma^*, L, x)} \left( 1 - F(\sigma^*|L) \right) \frac{1 - F(\sigma^*|H)}{p f(\sigma^*|H)} = \frac{\Pr(A|L, x)}{\Pr(A|H, x)}. \]  

(11)

For the debtor’s optimal offers, the lefthand side is bounded away from 0 — and so the righthand side is also.

\[ \text{To see this formally, the equilibrium condition (3) can be rewritten as} \]

27
is sufficiently valuable.

To illustrate this point more formally, consider the following simple model of a firm’s ex ante financing decision. At date 0, a firm (with no resources of its own) seeks to raise funds for a constant returns to scale investment opportunity with maximum size $\bar{I}$. If the firm invests $I \leq \bar{I}$, then with probability $1 - Q$ the firm receives $\rho I$ at date 1, where $\rho > 1$ is the return. For simplicity, we assume that in this case the continuation value of the firm is simply 0. Moreover, because of the possibility of diversion, the firm can credibly pledge at most $K$ to its investors. It is this limited pledgeability that raises the possibility that the firm is credit-constrained.

With probability $Q$, the firm has no liquid resources at date 1. It is forced to attempt to restructure its debt. Let $\mu I$ denote the liquidation value of the firm. If the firm is allowed to continue, it will produce either $H \mu I$ or $L \mu I$ at date 2, with probabilities $p$ and $1 - p$ respectively. For simplicity we assume that these date 2 incomes are fully pledgeable, and that there are no further cash flows subsequent to date 2.

Restructuring takes place according to the voting game described in earlier sections. The creditors’ expected payoff from restructuring is $\Pi^C \mu I$. Let $\Pi^D \mu I$ denote the firm’s expected payoff.

The firm issues one-period debt at date 0. Let $k$ denote the face value of the debt. Normalizing the market interest rate to 1, the face value must satisfy $(1 - Q) k + Q \Pi^C \mu I = I$, i.e.,

$$k = \frac{1 - Q \Pi^C \mu}{1 - Q} I.$$ 

So the firm chooses its project size $I \leq \bar{I}$ to maximize

$$V(I) \equiv (1 - Q) \rho I - (1 - Q) k + Q \Pi^D \mu I$$

$$= (1 - Q) \rho I - I + Q (\Pi^C + \Pi^D) \mu I$$

subject to the credit constraint

$$I \leq \frac{1 - Q}{1 - Q \Pi^C \mu} K.$$  \hspace{1cm} (12)

One immediate implication is:
Proposition 3 (Non-binding credit constraint)
When the credit constraint (12) does not bind, the optimal voting arrangement is one that maximizes the combined ex post welfare of the creditors and debtor, \((\Pi^C + \Pi^D) \mu I\) — i.e., a majority rule. A firm is less likely to be credit constrained when \(K\) is high, \(\mu\) is high, and (provided \(\Pi^C \mu < 1\)) when \(Q\) is low.

What happens if the credit constraint (12) is binding? Then moving from a majority voting rule \(\alpha\) to a unanimity voting rule increases the creditors’ payoff in restructuring from \(\Pi^C_\alpha \mu I\) to \(\Pi^C_1 \mu I\), while reducing the combined ex post payoff from \((\Pi^C_\alpha + \Pi^D_\alpha) \mu I\) to \((\Pi^C_1 + \Pi^D_1) \mu I\). Define
\[
\phi = \frac{(\Pi^C_1 + \Pi^D_1) - (\Pi^C_\alpha + \Pi^D_\alpha)}{\Pi^C_1 - \Pi^C_\alpha},
\]
i.e., \(\phi\) is the “cost” in social efficiency of increasing the creditors’ payoff.

Proposition 4 (Ex ante optimality of unanimity rule)
Suppose that a firm is credit constrained under both majority voting rule \(\alpha\) and a unanimity voting rule. Then the unanimity voting rule maximizes the firm’s welfare if and only if
\[
\frac{(1 - Q) \rho - 1 + Q (\Pi^C_\alpha + \Pi^D_\alpha) \mu}{1 - Q \Pi^C_\alpha \mu} > \phi
\]
In particular, the unanimity agreement rule is preferred whenever the investment opportunity is sufficiently productive (\(\rho\) large enough).

9 Applications
To reiterate, our main findings are that when creditors are close to the common values benchmark a unanimity voting rule increases their payoff in ex post restructuring negotiations. In contrast, if creditors are close to the private values benchmark, they are better off selecting a (suitably chosen) majority voting rule. We have argued that the former case is likely to arise when creditors are close-to-rational and markets are liquid, while the latter case obtains if either (or both) of these conditions is violated. We turn now to several applications of these results.
United States reorganization law

At first sight, U.S. law embodies two diametrically opposing approaches to debt reorganization. On the one hand, the Trust Indenture Act (1939) stipulates that the unanimous agreement of bondholders is required in order to change the terms of a debt agreement. On the other hand, if a firm files for bankruptcy under Chapter 11, debt agreements can be restructured if a supermajority of debtholders agree.

Our results help shed some light on these two very different approaches. On the one hand, as Proposition 1 indicates, when markets are liquid and creditors behave close-to-rationally, the Trust Indenture Act’s requirement of unanimous agreement for a change in the debt contract serves to protect creditors ex post.

On the other hand, if a firm is severely distressed, its securities may no longer be very liquid. Listed securities are usually delisted once the firm’s financial condition no longer allows it to meet the listing requirements. In addition, transaction costs (commissions and bid-ask spreads) tend to be higher in percentage terms on lower priced issues. Under such circumstances, Proposition 2 may apply: requiring unanimous agreement makes restructuring almost impossible, and hurts both the debtor and the creditors. Chapter 11 then provides a venue in which the firm can improve its own welfare — along with that of its creditors — by putting a reorganization plan to a majority vote.

Importantly, filing for Chapter 11 is itself costly. For large firms the direct costs of bankruptcy are estimated to be 3%-5% of the prebankruptcy value of the firm (Altman 1984, Betker 1997, Warner 1977). For smaller firms, the direct costs of bankruptcy are much larger (in the order of 20%, see Lawless, et. al. 1994). In addition there are indirect costs that are hard to estimate precisely but are believed to be large (Altman 1984, Andrade and Kaplan 1998, Cutler and Summers 1988).

Without such a cost, the Trust Indenture Act would have no bite, and all firms would file for bankruptcy whenever they needed to restructure. (For both common and private values, the debtor’s payoff is higher under a majority vote.) The cost of filing acts to ensure that only firms for whom a unanimity vote would work poorly will take this route — in
other words, exactly those firms whose securities are illiquid, or whose bondholders are believed to be far from rational.

SOVEREIGN DEBT

In recent years, a large number of observers and policymakers have advocated the creation of a Sovereign Debt Restructuring Mechanism (SDRM).\textsuperscript{10} A key element of most proposals is that (as in U.S. Chapter 11 reorganizations) a supermajority of creditors would be able to vote to accept new contract terms.\textsuperscript{11} At present, the vast majority of sovereign debt issued under U.S. law requires the unanimous consent of bondholders for such a change.\textsuperscript{12} In contrast, international debt issued under English law typically \textit{does} allow for a binding supermajority vote to reschedule payments.\textsuperscript{13}

Proponents of an SDRM argue that without the possibility of a binding majority vote, it is too hard for debtors and creditors to reach agreement. On the other side, critics caution that creditors are already poorly protected in sovereign markets, and that facilitating \textit{ex post} renegotiation will weaken their position still further.\textsuperscript{14} Our paper provides a theoretical framework in which to assess these competing effects, and suggests there is some truth in both positions. The impossibility of holding a majority vote does make agreement harder. But under some circumstances (liquid markets and reasonably rational investors) a majority agreement rule would reduce the creditors' \textit{ex post} recovery. A sovereign borrower who is highly credit constrained and in need of funds would then be hurt by a move to facilitate restructuring (Proposition 4).

As Eichengreen and Mody (2004) observe, the fact that different jurisdictions have adopted different stances on the admissibility of majority agreement rules allows for these

\textsuperscript{10}See in particular Krueger (2002).
\textsuperscript{11}See, e.g., Eichengreen (2003).
\textsuperscript{12}See, e.g., White (2002).
\textsuperscript{13}Given that the Trust Indenture Act does not apply to sovereign debt (see, e.g., Bucheit \textit{et al} 2002), the reasons for this sharp contrast across jurisdictions are unclear.
\textsuperscript{14}See, e.g., Dooley (2000), or Shleifer (2003).
issues to be addressed empirically. Our theoretical results\textsuperscript{15} are consistent with two of their main empirical findings: lower quality borrowers are less likely to issue bonds under English law — but when they do so, they raise the interest rate they must pay on their debt, even after controlling for borrower quality.\textsuperscript{16} In terms of our analysis, lower quality borrowers are more likely to be credit constrained and to potentially prefer restructuring under a unanimity vote — see Propositions 3 and 4.\textsuperscript{17} (Our analysis cannot easily account for Eichengreen and Mody’s third main finding: high quality borrowers issuing under English law actually pay lower interest rates. However, sovereign bonds issued under English law differ from in ways other than the inclusion of a majority restructuring clause. For example, the ability of an individual creditor to litigate is constrained.)

APPLICATIONS TO AREAS OTHER THAN DEBT RESTRUCTURING

Although we have couched our analysis in terms of creditors voting over a restructuring offer made by a debtor, our results have implications for other settings. For example, the case of an employer bargaining with a union is similar in many ways: the employer proposes a wage offer, and union members then vote to accept or reject the offer. Union members potentially differ in their alternative employment opportunities (private values), and in their assessment of future economic conditions (common values). The same tradeoffs between

\textsuperscript{15}Although our analysis is couched in terms of a firm seeking to reorganize, its main results apply to the restructuring of sovereign debt. Consider the following stylized account. A sovereign has defaulted on its debt. If the debt is not renegotiated, the creditors expect to recover \( z \) cents on the dollar. The sovereign offers to reschedule the debt, for an NPV (at the world interest rate) of \( x \) cents on the dollar. There are two possible states of the world — one in which the rescheduled debt is repaid with probability \( L \), and one in which it is repaid with probability \( H > L \). Provided that the indebted sovereign prefers rescheduling with an NPV of \( z/H \) to defaulting, and defaulting to rescheduling with an NPV of \( z/L \), we are in an analogous situation to the model studied in this paper.

\textsuperscript{16}Becker \textit{et al} (2003) also empirically assess the effect of including a majority agreement rule. They find no statistically significant impact.

\textsuperscript{17}Eichengreen and Mody also find that the very lowest quality borrowers are more likely to issue under U.S. law. Proposition 4 can account for this: when \( \mu \) is low, borrowers with \textit{higher} default probabilities \( Q \) are \textit{less} likely to satisfy condition (13).
holding a unanimous vote and a majority vote then apply as in the case of debtor-creditor negotiations. To the extent to which union members are closer to being characterized by private values, our results suggest that they will opt to use a majority vote — exactly as is observed.

The observation that a unanimous agreement rule leads to poor aggregation of information was originally made in the context of jury voting — see Feddersen and Pesendorfer (1998). The counterintuitive implication of this result is that requiring unanimity for conviction actually provides less protection for innocent defendants than a majority vote would. However, one potentially important factor missing from this analysis is that the jury is voting over whether or not to convict a defendant of a charge proposed by a prosecutor. If the prosecutor is more pro-punishment than the jurors — either because of career concerns, or from selection bias of those who choose to become prosecutors — then the analysis of our paper suggests that a requirement of unanimity may lead the prosecutor to modulate the severity of punishment he seeks.18 Put differently, a unanimity rule may protect defendants not by reducing the probability of wrongful conviction, but instead by reining in an overaggressive prosecutor.

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18While the interests of a debtor are exactly opposite to those of creditors — the debtor prefers smaller offers $x$, creditors prefer larger offers — the same is unlikely to be the case for a prosecutor and jurors. But provided the interests of the two side are sufficiently distinct, we conjecture that our main results will hold.
References


A Appendix

We repeatedly use the following minor observation:

**Lemma 13** \( F(\sigma|H)/F(\sigma|L) \) is increasing in \( \sigma \), and is bounded above by 1. As a consequence, \( F(\sigma|H) \leq F(\sigma|L) \), and is strict if strict MLRP holds. Finally, \( (1 - F(\sigma|L))/(1 - F(\sigma|H)) \) is decreasing in \( \sigma \).

**Proof:** Rewriting, we must show that

\[
\frac{\int_{\sigma} f(\bar{\sigma}|L) \frac{f(\bar{\sigma}|H)}{f(\bar{\sigma}|L)} d \bar{\sigma}}{\int_{\sigma} f(\bar{\sigma}|L) d \bar{\sigma}}
\]

is increasing in \( \sigma \). Differentiating, we must show

\[
f(\sigma|L) \frac{f(\bar{\sigma}|H)}{f(\sigma|L)} \int_{\sigma} f(\bar{\sigma}|L) d \bar{\sigma} > f(\sigma|L) \int_{\sigma} f(\bar{\sigma}|L) \frac{f(\bar{\sigma}|H)}{f(\bar{\sigma}|L)} d \bar{\sigma},
\]

which is immediate from MLRP.

The proof that \( (1 - F(\sigma|L))/(1 - F(\sigma|H)) \) is decreasing is exactly parallel.

**Proof of Lemma 1**

Substituting equation (2) in equation (1), the claim immediately follows from MLRP.

**Proof of Lemma 2**

Consider the function defined by

\[
Z(\sigma) \equiv (u(\sigma, H, x) - 1) pf(\sigma|H) + (u(\sigma, L, x) - 1) (1 - p) f(\sigma|L) \left( \frac{1 - F(\sigma|L)}{1 - F(\sigma|H)} \right)^{n-1} \left( \frac{F(\sigma|L)}{F(\sigma|H)} \right)^{n(1-\alpha)} \tag{14}
\]

A responsive equilibrium exists if and only if \( Z(\sigma) = 0 \) has a solution in the interval \((\underline{\sigma}, \bar{\sigma})\).

As \( \sigma^* \to \underline{\sigma} \),

\[
Z(\sigma) \to (u(\underline{\sigma}, H, x) - 1) pf(\underline{\sigma}|H) + (u(\underline{\sigma}, L, x) - 1) (1 - p) f(\underline{\sigma}|L) \left( \frac{f(\underline{\sigma}|L)}{f(\underline{\sigma}|H)} \right)^{(1-\alpha)n}.
\]

37
By MLRP, \( \frac{f(\sigma|L)}{f(\sigma|H)} \geq 1 \). Assumption 1 implies \( u(\sigma, L) < 1 \), and so

\[
\lim_{\sigma \to \bar{\sigma}} Z(\sigma) \leq (u(\sigma, H, x) - 1) pf(\sigma|H) + (u(\sigma, L, x) - 1) (1 - p) f(\sigma|L),
\]

which again by Assumption 1 is less than 0.

On the other hand, as \( \sigma^* \to \bar{\sigma} \),

\[
Z(\sigma) \to (u(\bar{\sigma}, H, x) - 1) pf(\bar{\sigma}|H) + (u(\bar{\sigma}, L, x) - 1) (1 - p) f(\bar{\sigma}|L) \left( \frac{f(\bar{\sigma}|L)}{f(\bar{\sigma}|H)} \right)^{n\alpha - 1}.
\]

Certainly \( u(\bar{\sigma}, L, x) - 1 \geq -1 \). By strict MLRP \( \frac{f(\bar{\sigma}|L)}{f(\bar{\sigma}|H)} < 1 \). So if \( x \geq \lambda \bar{\sigma} + (1 - \lambda) H + \varepsilon \), then certainly \( u(\bar{\sigma}, H, x) - 1 \geq \varepsilon \min \{ H, \bar{\sigma} \} \), and so there exists an \( N \) (independent of \( \lambda \)) such that \( Z(\sigma) > 0 \) whenever \( n \geq N \). Continuity of \( Z \) in \( \sigma \) then implies that for all \( n \) large enough, there is a value of \( \sigma \) in the range \( (\sigma, \bar{\sigma}) \) such that \( Z(\sigma) = 0 \).

**Proof of Lemma 3**

Since there is no responsive equilibrium, from the proof of Lemma 2

\[
(u(\bar{\sigma}, H, x) - 1) pf(\bar{\sigma}|H) + (u(\bar{\sigma}, L, x) - 1) (1 - p) f(\bar{\sigma}|L) \left( \frac{f(\bar{\sigma}|L)}{f(\bar{\sigma}|H)} \right)^{(1-\alpha)n} < 0 \tag{15}
\]

Inequality (15) can only hold if \( u(\bar{\sigma}, L, x) < 1 \). By MLRP, It follows that for all \( \sigma_i \in [\sigma, \bar{\sigma}] \) and any \( m \leq n\alpha - 1 \),

\[
(u(\sigma_i, H, x) - 1) pf(\sigma_i|H) + (u(\sigma_i, L, x) - 1) (1 - p) f(\sigma_i|L) \left( \frac{f(\sigma_i|L)}{f(\sigma_i|H)} \right)^{m} < 0. \tag{16}
\]

This last inequality is equivalent to

\[
(u(\sigma, H, x) - 1) \Pr(H|a signal \sigma_i and m signals of \bar{\sigma}) + (u(\sigma, L, x) - 1) \Pr(L|a signal \sigma_i and m signals of \bar{\sigma}) < 0.
\]

In words, inequality (17) says that creditor \( i \), having observed his own signal \( \sigma_i \), will reject the offer \( x \) even if he conditions on the event that \( m \leq n\alpha - 1 \) other creditors observe the most pro-acceptance signal \( \bar{\sigma} \). This has two implications.
First, the equilibrium in which all creditors reject always is a perfect equilibrium: for if all creditors tremble and accept with probability \( \varepsilon \) independent of their own signal, it remains a best response to reject the offer (this is just inequality (17) with \( m = 0 \)).

Second, we claim that the equilibrium in which all creditors accept is not perfect. Recall that a creditor \( i \)'s vote only matters if \( n\alpha - 1 \) other creditors vote to accept. This event only arises if \( n - n\alpha \) of the \( n - 1 \) other creditors tremble. By assumption \( n - n\alpha \leq n\alpha - 1 \). Consequently, inequality (17) implies that no matter what information creditor \( i \) infers from conditioning on the event that \( n - n\alpha \) of the other creditors tremble, it is a best response to reject the offer. As such, the equilibrium in which all creditors accept always cannot be perfect.

**Proof of Lemma 4**

We prove the lemma in four steps.

**Claim 1** If \( \lim \sup x_n < x_H \) then \( \lim \inf \sigma_n^* > \sigma_H \).

**Proof:** By hypothesis, there exists an \( \varepsilon > 0 \) such that \( x_n \leq x_H - \varepsilon \) for all \( n \) large. Suppose that contrary to the claim \( \lim \inf \sigma_n^* \leq \sigma_H \). So for any \( \delta \), there exists a subsequence such that \( \sigma_n^* \leq \sigma_H + \delta \). Since \( \sigma_H < \hat{\sigma} \), for \( \delta \) sufficiently small we have a responsive equilibrium. But

\[
\begin{align*}
    u(\sigma_n^*, H, x_n) - 1 &= x_n(\lambda \sigma_n^* + (1 - \lambda) H) - 1 \\
    &\leq x_n(\lambda \sigma_H + (1 - \lambda) H) + \lambda \delta x_n - 1 \\
    &= \frac{x_n}{x_H} + \lambda \delta x_n - 1 \\
    &\leq \frac{x_H - \varepsilon}{x_H} + \lambda \delta (x_H - \varepsilon) - 1.
\end{align*}
\]

The last expression is strictly negative for all sufficiently small choices of \( \delta \). Moreover, it implies \( u(\sigma_n^*, L, x_n) - 1 < 0 \). As such, on receiving the signal \( \sigma_n^* \) a creditor strictly prefers liquidation to the offer \( x_n \) in both states \( R = L, H \), a contradiction to the equilibrium condition.
**Claim 2** If $\lim \sup x_n < x_L$ then $\lim \inf \sigma^*_n > \sigma_L$.

**Proof:** By hypothesis, there exists $\varepsilon$ such that $x_n \leq x_L - \varepsilon$ for all $n$ large enough. Suppose now to the contrary that $\lim \inf \sigma^*_n \leq \sigma_L$. So for any $\delta > 0$, there exists a subsequence of $\sigma^*_n$ such that $\sigma^*_n \leq \sigma_L + \delta$. Again, we are in a responsive equilibrium for $\delta$ chosen small enough. As in the proof of Claim 1, for all $\delta$ sufficiently small there exists a $\hat{\varepsilon} > 0$ such that $u(\sigma^*_n, L, x_n) - 1 \leq -\hat{\varepsilon}$ for all $n$ sufficiently large.

Next, define
\[
\phi = \min_{\sigma \in [\sigma_L, \sigma_L + \delta]} \left( \frac{(1 - F(\sigma|L))^{\alpha} F(\sigma|L)^{1-\alpha}}{(1 - F(\sigma|H))^{\alpha} F(\sigma|H)^{1-\alpha}} \right)^n
\]
Note that the function $(1 - q)^{\alpha} q^{1-\alpha}$ is increasing for $q \in (0, 1 - \alpha)$ and decreasing for $q \in (1 - \alpha, 1)$. Recall that by definition $F(\sigma|L) = 1 - \alpha$, and by Lemma 13 $F(\sigma|H) < F(\sigma|L)$ for all $\sigma \in (\underline{\sigma}, \bar{\sigma})$. It follows that $\phi > 1$ for $\delta$ chosen small enough, and so
\[
\left( \frac{(1 - F(\sigma^*|L))^{\alpha} F(\sigma^*|L)^{1-\alpha}}{(1 - F(\sigma^*|H))^{\alpha} F(\sigma^*|H)^{1-\alpha}} \right)^n \geq \phi^n \to \infty
\]
This contradicts the equilibrium condition (3). \(\blacksquare\)

**Claim 3** If $\lim \inf x_n > x_L$ then $\lim \sup \sigma^*_n < \sigma_L$.

**Proof:** By hypothesis, there exists $\varepsilon$ such that $x_n \geq x_L + \varepsilon$ for all $n$ large enough. Moreover, since $x_L > \frac{1}{\lambda \sigma + (1-\lambda)H}$ then by Lemma 2 a responsive equilibrium exists for all $n$ large. Suppose that contrary to the claim $\lim \sup \sigma^*_n \geq \sigma_L$. So for any $\delta$, there exists a subsequence such that $\sigma^*_n \geq \sigma_L - \delta$. Then
\[
u(\sigma^*_n, L, x_n) - 1 = x_n (\lambda \sigma^*_n + (1 - \lambda) L) - 1
\]
\[
\geq x_n (\lambda \sigma_L + (1 - \lambda) L) - \lambda \delta x_n - 1
\]
\[
= \frac{x_n}{x_L} - \lambda \delta x_n - 1
\]
\[
> \frac{x_L + \varepsilon}{x_L} - \lambda \delta x_n - 1.
\]
The last expression is strictly positive for all sufficiently small choices of $\delta$. Moreover, it implies $u(\sigma^*_n, H, x_n) - 1 > 0$. As such, on receiving the signal $\sigma^*_n$ a creditor strictly prefers
the offer \( x_n \) to liquidation in both states \( R = L, H \), a contradiction to the equilibrium condition.

**Claim 4** If \( \lim \inf x_n > x_H \) then \( \lim \sup \sigma^*_n < \sigma_H \).

**Proof:** By hypothesis, there exists \( \varepsilon \) such that \( x_n \geq x_H + \varepsilon \) for all \( n \) large enough. Moreover, since \( x_H > \frac{1}{\lambda \sigma + (1-\lambda)H} \) then by Lemma 2 a responsive equilibrium exists for all \( n \) large. Suppose to the contrary that \( \lim \sup \sigma^*_n \geq \sigma_H \). So for any \( \delta > 0 \), there is a subsequence such that \( \sigma^*_n \geq \sigma_H - \delta \). As in the proof of Claim 3, there exists an \( \hat{\varepsilon} > 0 \) such that \( u(\sigma_n^*, H, x_n) - 1 \geq \hat{\varepsilon} \) for sufficiently small \( \delta \) and sufficiently large \( n \).

Next, define

\[
\phi = \max_{\sigma \in [\sigma_H - \delta, \sigma]} \frac{(1 - F(\sigma|L))^\alpha F(\sigma|L)^{1-\alpha}}{(1 - F(\sigma|H))^\alpha F(\sigma|H)^{1-\alpha}}
\]

Recall that by definition \( F(\sigma|H) = 1 - \alpha \), and by Lemma 13 \( F(\sigma|H) < F(\sigma|L) \) for all \( \sigma \in (\sigma, \bar{\sigma}) \). Parallel to the argument in Claim 2, \( \phi < 1 \) for \( \delta \) chosen small enough, and so

\[
\left( \frac{(1 - F(\sigma^*|L))^\alpha F(\sigma^*|L)^{1-\alpha}}{(1 - F(\sigma^*|H))^\alpha F(\sigma^*|H)^{1-\alpha}} \right)^n \leq \phi^n \rightarrow 0
\]

This contradicts the equilibrium condition (3).

Together claims 1, 2, 3 and 4 imply Lemma 4.

**Proof of Lemmas 5 and 6**

It is fractionally more convenient to prove Lemmas 5 and 6 in the reverse order to that in which they are stated in the main text:

**Proof of Lemma 6**

Fix \( \lambda_0 \). From condition (8), for all \( \varepsilon > 0 \) sufficiently small there exists a \( \Lambda \geq \lambda_0 \) and \( N_0 \) such that \( x_n(\lambda) - x_H(\lambda) \geq \varepsilon \) whenever \( \lambda_0 \leq \lambda \leq \Lambda, \lambda > 0 \) and \( n \geq N_0 \). Since

\[
x_H(\lambda) = \frac{1}{\lambda \sigma + (1-\lambda)H} > \frac{1}{\lambda \bar{\sigma} + (1-\lambda)H},
\]

by Lemma 2 it follows that there exists an \( N \geq N_0 \).
such that a responsive equilibrium exists whenever \( \lambda_0 \leq \lambda \leq \Lambda, \lambda > 0 \) and \( n \geq N \). Since \( \sigma^*_n (\lambda) \geq \bar{\sigma} = \sigma_H \), then \( u (\sigma^*_n (\lambda), H, x_n (\lambda)) - 1 \geq \varepsilon / x_H (\lambda) \) under these same conditions. So from the equilibrium condition (3), there exists a \( \delta > 0 \) such that

\[
\left( \frac{1 - F (\sigma^*_n (\lambda) | L)}{1 - F (\sigma^*_n (\lambda) | H)} \right)^n \geq \delta \text{ for } \lambda_0 \leq \lambda \leq \Lambda, \lambda > 0 \text{ and } n \geq N. \tag{18}\]

Note that (18) implies that

\[
\inf_{\lambda_0 \leq \lambda \leq \Lambda, \lambda > 0} \sigma^*_n (\lambda) \to \bar{\sigma} \text{ as } n \to \infty. \tag{19}\]

(It is immediate from Lemma 13 that \( \sigma^*_n (\lambda) \) cannot remain bounded away from both \( \bar{\sigma} \) and \( \bar{\sigma} \). A straightforward application of l’Hôpital’s rule, together with fact that \( f (\bar{\sigma} | L) / f (\bar{\sigma} | H) < 1 \), is then enough to establish (19).)

Given (19), we assume without loss that \( N \) was chosen large enough so that if \( n \geq N \) then

\[
\inf_{\lambda_0 \leq \lambda \leq \Lambda, \lambda > 0} \sigma^*_n (\lambda) \leq (\bar{\sigma} + \bar{\sigma}) / 2. \tag{20}\]

To establish our result, it is sufficient to show that

\[
\inf_{\lambda_0 \leq \lambda \leq \Lambda, \lambda > 0, n \geq N} \Pr (A | \lambda, n) > 0.
\]

Suppose to the contrary that

\[
\inf_{\lambda_0 \leq \lambda \leq \Lambda, \lambda > 0, n \geq N} \Pr (A | \lambda, n) = 0, \tag{21}\]

which in turn implies there exists some sequence \((\lambda_n, m(n))\) for which \(\lambda_0 \leq \lambda_n \leq \Lambda, \lambda_n > 0\) and \(m(n) \geq N\) and

\[
\left( \frac{1 - F (\sigma^*_{m(n)} (\lambda_n) | L)}{1 - F (\sigma^*_{m(n)} (\lambda_n) | H)} \right)^{m(n)} \to 0 \text{ as } n \to \infty. \tag{22}\]

Note that \(m(n) \to \infty\): if this were not the case, (22) would imply \(\sigma^*_{m(n)} (\lambda_n) \to \bar{\sigma}\), contradicting (20). As such, to satisfy (18) it must be the case that \(\sigma^*_{m(n)} (\lambda_n) \to \bar{\sigma}\).

Now, the sequence

\[
\left( \frac{1 - F (\sigma^*_{m(n)} (\lambda_n) | L)}{1 - F (\sigma^*_{m(n)} (\lambda_n) | H)} \right)^{m(n)} \tag{23}\]
is bounded, and so by Bolzano-Weierstrass has a convergent subsequence. Without loss, assume that the sequence \((\lambda_n, m(n))\) was chosen so that it has this property directly. By (18), the limit of expression (23) is above \(\delta\).

By the discrete version of l'Hôpital’s rule,

\[
\lim \ln \left( 1 - F \left( \sigma_{m(n)}^* (\lambda_n) | H \right) \right)^{m(n)} = \lim \frac{m(n + 1) - m(n)}{\ln \frac{1 - F \left( \sigma_{m(n+1)}^* (\lambda_{n+1}) | H \right)}{1 - F \left( \sigma_{m(n)}^* (\lambda_n) | H \right)}},
\]

provided the right hands side exists.\(^{19}\) Expanding, the right hand side is equal to

\[
\lim (m(n + 1) - m(n)) \frac{\ln \left( 1 - F \left( \sigma_{m(n+1)}^* (\lambda_{n+1}) | H \right) \right) \ln \left( 1 - F \left( \sigma_{m(n)}^* (\lambda_n) | H \right) \right)}{\ln \left( 1 - F \left( \sigma_{m(n+1)}^* (\lambda_{n+1}) | H \right) \right) - \ln \left( 1 - F \left( \sigma_{m(n)}^* (\lambda_n) | H \right) \right)}
\]

\[
= \lim (m(n + 1) - m(n)) \frac{\frac{1 - F \left( \sigma_{m(n+1)}^* (\lambda_{n+1}) | L \right)}{1 - F \left( \sigma_{m(n+1)}^* (\lambda_{n+1}) | H \right)} \ln \frac{1 - F \left( \sigma_{m(n)}^* (\lambda_n) | L \right)}{1 - F \left( \sigma_{m(n)}^* (\lambda_n) | H \right)} - \frac{1 - F \left( \sigma_{m(n+1)}^* (\lambda_{n+1}) | L \right)}{1 - F \left( \sigma_{m(n+1)}^* (\lambda_{n+1}) | H \right)} \ln \frac{1 - F \left( \sigma_{m(n)}^* (\lambda_n) | L \right)}{1 - F \left( \sigma_{m(n)}^* (\lambda_n) | H \right)}}{\ln \left( 1 - F \left( \sigma_{m(n+1)}^* (\lambda_{n+1}) | H \right) \right) - \ln \left( 1 - F \left( \sigma_{m(n)}^* (\lambda_n) | H \right) \right)}
\]

\[
\times \frac{\frac{1 - F \left( \sigma_{m(n+1)}^* (\lambda_{n+1}) | L \right)}{1 - F \left( \sigma_{m(n+1)}^* (\lambda_{n+1}) | H \right)} \ln \frac{1 - F \left( \sigma_{m(n+1)}^* (\lambda_{n+1}) | H \right)}{1 - F \left( \sigma_{m(n+1)}^* (\lambda_{n+1}) | L \right)} - \frac{1 - F \left( \sigma_{m(n)}^* (\lambda_n) | L \right)}{1 - F \left( \sigma_{m(n)}^* (\lambda_n) | H \right)} \ln \frac{1 - F \left( \sigma_{m(n+1)}^* (\lambda_{n+1}) | H \right)}{1 - F \left( \sigma_{m(n+1)}^* (\lambda_{n+1}) | L \right)}}{\ln \left( 1 - F \left( \sigma_{m(n+1)}^* (\lambda_{n+1}) | H \right) \right) - \ln \left( 1 - F \left( \sigma_{m(n)}^* (\lambda_n) | H \right) \right)}
\]

\[
\times \frac{\frac{1 - F \left( \sigma_{m(n+1)}^* (\lambda_{n+1}) | L \right)}{1 - F \left( \sigma_{m(n+1)}^* (\lambda_{n+1}) | H \right)} \ln \frac{1 - F \left( \sigma_{m(n)}^* (\lambda_n) | L \right)}{1 - F \left( \sigma_{m(n)}^* (\lambda_n) | H \right)} - \frac{1 - F \left( \sigma_{m(n+1)}^* (\lambda_{n+1}) | L \right)}{1 - F \left( \sigma_{m(n+1)}^* (\lambda_{n+1}) | H \right)} \ln \frac{1 - F \left( \sigma_{m(n)}^* (\lambda_n) | L \right)}{1 - F \left( \sigma_{m(n)}^* (\lambda_n) | H \right)}}{\ln \left( 1 - F \left( \sigma_{m(n+1)}^* (\lambda_{n+1}) | H \right) \right) - \ln \left( 1 - F \left( \sigma_{m(n)}^* (\lambda_n) | H \right) \right)}
\]

\(^{19}\)The discrete version of l'Hôpital’s rule holds provided that \(\lim_{n \to \infty} \frac{1}{\ln \frac{1 - F \left( \sigma_{m(n)}^* (\lambda_n) | H \right)}{1 - F \left( \sigma_{m(n)}^* (\lambda_n) | H \right)}} \to \infty\), which is satisfied since \(\sigma_{m(n)}^* (\lambda_n) \to \sigma\).
Since $\sigma^*_{m(n)}(\lambda_n) \to \sigma$,  

$$
\frac{\ln \frac{1 - F(\sigma^*_{m(n)}(\lambda_n))}{1 - F(\sigma^*_{m(n+1)}(\lambda_{n+1}))}}{\ln \left(1 - F\left(\sigma^*_{m(n)}(\lambda_n) \mid H\right)\right)} - \ln \left(1 - F\left(\sigma^*_{m(n+1)}(\lambda_{n+1}) \mid H\right)\right) 
\to \frac{f(\sigma|L)}{f(\sigma|H)} - 1
$$

By strict MLRP, $f(\sigma|L)/f(\sigma|H) > 1$. Thus provided

$$
\ln \frac{1 - F(\sigma^*_{m(n)}(\lambda_{n+1}))}{1 - F(\sigma^*_{m(n+1)}(\lambda_{n+1}))} - \ln \frac{1 - F(\sigma^*_{m(n)}(\lambda_n))}{1 - F(\sigma^*_{m(n+1)}(\lambda_{n+1}))} 
\to \left(\frac{f(\sigma|L)}{f(\sigma|H)} - 1\right)^{-1}
$$

exists and is finite, the right hand side limit exists and is finite. But again by the discrete version of l’Hôpital’s rule,

$$
\lim \ln \left(1 - F\left(\sigma^*_{m(n)}(\lambda_n) \mid L\right)\right) = \lim (m(n+1) - m(n))
$$

which is finite by assumption. Thus $\lim \ln \left(1 - F\left(\sigma^*_{m(n)}(\lambda_n) \mid H\right)\right)$ exists and is finite, contradicting (21) and thus completing the proof.

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20This follows since

$$
\ln (1 - F(\sigma|R)) = -F(\sigma|R) + o(F(\sigma|R)^2)
$$

$$
F(\sigma|R) = (\sigma - \bar{\sigma}) f(\sigma|R) + o((\sigma - \bar{\sigma})^2)
$$

and so

$$
\ln (1 - F(\sigma|R)) = -(\sigma - \bar{\sigma}) f(\sigma|R) + o((\sigma - \bar{\sigma})^2).
$$
Fix $\lambda_0$. Suppose to the contrary that (7) does not hold. So there exists a $\Lambda$ and $N$ such that $\inf_{\lambda_0 \leq \lambda \leq \Lambda, \lambda > 0, n \geq N} \Pr(A | \lambda, n) > 0$. From (6), we can construct a sequence $(\lambda_n, m(n))$ for which $\lambda_0 \leq \lambda_n \leq \Lambda, \lambda_n > 0, n \geq N$ and $m(n) \to \infty$ and

$$\inf_n x_{m(n)}(\lambda_n) - x_H(\lambda_n) \leq 0.$$  

By Bolzano-Weierstrass, $x_{m(n)}(\lambda_n) - x_H(\lambda_n)$ must have a convergent subsequence. Likewise, $\Pr(A | \lambda_n, m(n), x_{m(n)}(\lambda_n))$ must have a convergent subsequence. Thus without loss, assume that $(\lambda_n, m(n))$ is chosen so that

$$\lim x_{m(n)}(\lambda_n) - x_H(\lambda_n) \leq 0 \quad (24)$$

$$\lim \Pr(A | \lambda_n, m(n), x_{m(n)}(\lambda_n)) > 0 \quad (25)$$

Similar to in the proof of Lemma 6, from (25), $\sigma_{\lambda_n}(\lambda_n) \to 2$ since $m(n) \to \infty$. Given this, equation (24) and the equilibrium condition (3) together imply

$$\left( \frac{1 - F(\sigma_{\lambda_n}(\lambda_n) \mid L)}{1 - F(\sigma_{\lambda_n}(\lambda_n) \mid H)} \right)^{m(n)} \to 0 \text{ as } n \to \infty \quad (26)$$

By the discrete version of l'Hôpital’s rule,

$$\lim \ln \left( \frac{1 - F(\sigma_{\lambda_n}(\lambda_n) \mid L)}{1 - F(\sigma_{\lambda_n}(\lambda_n) \mid H)} \right) = \lim \frac{m(n + 1) - m(n)}{\ln \frac{1 - F(\sigma_{\lambda_{n+1}}(\lambda_{n+1}) \mid L)}{1 - F(\sigma_{\lambda_{n+1}}(\lambda_{n+1}) \mid H)}}$$
provided the right hand side exists. Expanding in the same manner as in Lemma 6, the right hand side is equal to

\[
\lim (m(n + 1) - m(n)) \times \frac{\ln \left(1 - F \left(\sigma^*_m(n+1) (\lambda_{n+1}) | H \right)\right)}{\ln \left(1 - F \left(\sigma^*_m(n) (\lambda_n) | H \right)\right)} \times \frac{\ln \left(1 - F \left(\sigma^*_m(n) (\lambda_n) | H \right)\right)}{\ln \left(1 - F \left(\sigma^*_m(n+1) (\lambda_{n+1}) | H \right)\right)}
\]

Again, as in Lemma 6, this limit exists and is finite provided \(\lim \left(1 - F \left(\sigma^*_m(n) (\lambda_n) | H \right)\right) > 0\), which is the case by (25). But this contradicts (26), completing the proof.

**Proof of Lemma 7**

**Part 1:** \(x_n \to x_H\)

We show that \(\lim \inf x_n \geq x_H \geq \lim \sup x_n\).

First, we claim that \(\lim \inf x_n \geq x_H\). If this were not the case, then for some \(\varepsilon > 0\) there must exist a subsequence \(x_n\) such that \(x_n < x_H - \varepsilon\), and thus \(\lim \sup x_n \leq x_H - \varepsilon\). By Lemma 4, \(\Pr(A|H, x_n) \to 0\). In contrast, the sequence of offers \(x_n \equiv x_H + \kappa\) is accepted with a probability converging to \(p\) for any \(\kappa \in (0, x_L - x_H)\). So for any \(\kappa < 1 - x_H\), the debtor’s payoff is higher under this alternative offer for all \(n\) large enough. But this contradicts the optimality of \(x_n\) from the debtor’s perspective.

Second, we claim that \(\lim \sup x_n < x_L\). If this were not the case, then for any \(\varepsilon > 0\) there would exist a subsequence \(x_n\) such that \(x_n \geq x_L - \varepsilon\). But then the debtor’s payoff is bounded above by \((1 - x_L + \varepsilon)E[R]\), while under the sequence of offers \(x_n\) defined above, the debtor’s payoff converges to \((1 - x_H - \kappa) pH\). Recall that by assumption \((1 - x_H) pH > \)
\[(1 - x_L)E[R].\] So for \(\varepsilon\) and \(\kappa\) chosen small enough, the debtor’s payoff from the offer \(\hat{x}_n\) is higher, again a contradiction.

Third, we claim that \(\lim \sup x_n \leq x_H\). If this were not the case, then for some \(\varepsilon > 0\), there must exist a subsequence \(\tilde{x}_n\) such that \(\tilde{x}_n \geq x_H + \varepsilon\), and thus \(\lim \inf \tilde{x}_n \geq x_H + \varepsilon\). By Lemma 4, \(\Pr (A|H, \tilde{x}_n) \to 1\). However, consider a second sequence of offers \(\hat{x}_n \equiv x_H + \varepsilon / 2\). Again by Lemma 4, \(\Pr (A|H, \hat{x}_n) \to 1\). The difference in the debtor’s payoff under the two sequences of offers is

\[
(1 - \hat{x}_n) (pH \Pr (A|H, \hat{x}_n) + (1 - p) L \Pr (A|L, \hat{x}_n))
- (1 - \tilde{x}_n) (pH \Pr (A|H, \tilde{x}_n) + (1 - p) L \Pr (A|L, \tilde{x}_n))
= (\tilde{x}_n - \hat{x}_n) (pH \Pr (A|H, \hat{x}_n) + (1 - p) L \Pr (A|L, \hat{x}_n))
- (1 - \tilde{x}_n) [pH (\Pr (A|H, \tilde{x}_n) - \Pr (A|H, \hat{x}_n))]
+ (1 - p) L (\Pr (A|L, \tilde{x}_n) - \Pr (A|L, \hat{x}_n))]
\] (27)

From our second claim, we know that \(\lim \sup x_n < x_L\), and so by Lemma 4 both \(\Pr (A|L, \tilde{x}_n)\) and \(\Pr (A|L, \hat{x}_n)\) converge to 0. Clearly \(\tilde{x}_n - \hat{x}_n \geq \varepsilon / 2\). Consequently, expression (27) is bounded below by \(pH \varepsilon / 2\) for \(n\) large enough. In other words, the offer \(\hat{x}_n\) is preferred to the offer \(\tilde{x}_n\), contradicting the optimality of \(\tilde{x}_n\).

**Part 2:** \(\Pr (A|H, x_n) \to 1\)

Suppose to the contrary that \(\Pr (A|H, x_n) \not\to 1\). So there is some \(\varepsilon > 0\) and some subsequence \(\tilde{x}_n\) for which \(\Pr (A|H, \tilde{x}_n) \leq 1 - \varepsilon\). Consider the alternative offer sequence \(\hat{x}_n \equiv x_H + \delta\). By Lemma 4, \(\Pr (A|H, \hat{x}_n) \to 1\), and so the debtor’s payoff converges to \((1 - x_H - \delta) pH\). In contrast, under the subsequence of original offers

\[
\lim \sup \text{(debtor’s payoff)} \leq (1 - \varepsilon) (1 - x_H) pH
\]

So for \(\delta\) chosen small enough, the optimality of the original offers is contradicted. \(\blacksquare\)
**Proof of Lemma 8**

Suppose otherwise, i.e.,

\[
\sup_{\lambda,\tilde{\lambda}, \lambda < \tilde{\lambda}, n \geq N} \inf x_n (\lambda) - x_H (\lambda) = 0. \tag{28}
\]

The proof consists of first selecting a pair of preference parameters \(\lambda, \tilde{\lambda}\), and then showing that when the true preference parameter is \(\tilde{\lambda}\), the debtor prefers to make the offer \(x_n (\tilde{\lambda})\). This contradicts the supposed optimality of \(x_n (\lambda)\).

**Step 1: Selection of \(\lambda\) and \(\tilde{\lambda}\)**

Choose \(\delta\) such that \(1 - 4\delta\) \(H > 1\). Observe that \(H x_H (\lambda) \to 1\) as \(\lambda \to 0\). Given this and our choice of \(\delta\), we can choose \(\bar{\lambda}\) such that \((H x_H (\bar{\lambda}))^2 < (1 - 4\delta) H\) and \(x_H (\tilde{\lambda}) < 1\).

By supposition, there exists \(\lambda < \bar{\lambda}\) such that \(\lim \inf x_n (\lambda) - x_H (\lambda) \leq \delta\). Define

\[
\hat{\delta} \equiv \lim \inf E \left[ R | A, \lambda, n, x_n (\lambda) \right] \Pr (A | \lambda, n, x_n (\lambda))
\]

Certainly \(\hat{\delta} > 0\). To see this, note that if instead \(\hat{\delta} = 0\) the debtor’s payoff must approach 0 for the preference parameter \(\lambda\). In contrast, for any constant offer \(\xi > x_H (\lambda)\) Lemma 6 implies that the acceptance probability stays bounded away from 0, and so the debtor’s payoff does also. But then the offers \(x_n (\lambda)\) would be suboptimal for all \(n\) large enough.

For use below, note that since \(\lim \inf \Pr (A | \lambda, n, x_n (\lambda)) > 0\), the equilibrium \(\sigma^*_n (\lambda)\) is responsive; and moreover, \(\sigma^*_n (\lambda) \to \sigma\), and so for all \(n\) large enough \(\sigma^*_n (\lambda) x_n (\lambda) < 1\) (since \(\sigma < 1\)).

Given \(\delta, \lambda,\) and \(\hat{\delta}\), choose an \(\varepsilon \in (0, \hat{\delta})\) such that

\[
\varepsilon < \hat{\delta} \frac{1 - H x_H (\lambda)^2 - 4\delta}{1 - x_H (\lambda) + \lambda (H - \sigma) x_H (\lambda)^2}. \tag{29}
\]

By supposition (28) and Lemma 5, there exists \(\tilde{\lambda} < \lambda\) such that

\[
\lim \inf E \left[ R | A, \tilde{\lambda}, n, x_n (\tilde{\lambda}) \right] \Pr (A | \tilde{\lambda}, n, x_n (\tilde{\lambda})) \leq \varepsilon. \tag{30}
\]

For the preference parameter \(\tilde{\lambda}\), we must have

\[
\lim \inf x_n (\tilde{\lambda}) - x_H (\tilde{\lambda}) > 0. \tag{31}
\]
— for if instead \( \liminf x_n(\tilde{\lambda}) - x_H(\tilde{\lambda}) \leq 0 \), then Lemma 5 (with \( \lambda_0 = \tilde{\lambda} \)) implies that \( \liminf \Pr(A|\tilde{\lambda}, n, x_n(\tilde{\lambda})) = 0 \), and the same argument we used above implies that this is incompatible with the optimality of the offers \( x_n(\tilde{\lambda}) \).

**Step 2: Showing that the debtor prefers \( x_n(\lambda) \) to \( x_n(\tilde{\lambda}) \)**

We now show that if the debtor switches to the offer \( x_n(\lambda) \) when the preferences are described by \( \tilde{\lambda} \), then his welfare is strictly increased, and hence we obtain a contradiction to the optimality of \( x_n(\tilde{\lambda}) \). Define \( \tilde{\sigma}_n^* \) as the equilibrium cutoff signal for the offer \( x_n(\lambda) \) when the preferences are described by \( \tilde{\lambda} \).

By construction, \( \tilde{\lambda} < \lambda \): the preference parameter \( \tilde{\lambda} \) is closer to the common values case than \( \lambda \). We claim that for all \( n \) large enough this implies \( \tilde{\sigma}_n^* \leq \sigma_n^*(\lambda) \) — in other words, the offer \( x_n(\lambda) \) is accepted with a higher probability under \( \tilde{\lambda} \) than \( \lambda \).

The proof of this subclaim is as follows. Recall that \( \sigma_n^*(\lambda) \) is a responsive equilibrium. Rewriting equation (3), the cutoff value \( \sigma_n^*(\lambda) \) is a solution of \( h(\sigma, \lambda) = g(\sigma) \) where

\[
h(\sigma, \lambda) = \frac{u(\sigma, H, x) - 1}{1 - u(\sigma, L, x)}
\]

is strictly increasing in \( \sigma \) and

\[
g(\sigma) = \frac{(1-p)f(\sigma|L)[1 - F(\sigma|L)]^{n-1}}{pf(\sigma|H)[1 - F(\sigma|H)]^{n-1}}
\]

is strictly decreasing in \( \sigma \) by MLRP and Lemma 13. Since

\[
\frac{\partial h(\sigma, \lambda)}{\partial \lambda} = \frac{x(H-L)(\sigma x - 1)}{(1 - u(\sigma, L, x))^2}.
\]

then \( h(\sigma, \tilde{\lambda}) > h(\sigma, \lambda) \) for \( \sigma x < 1 \). Recall that for all \( n \) large enough \( \sigma_n^*(\lambda) x_n(\lambda) < 1 \).

From equation (3), a responsive equilibrium exists for the offer \( x_n(\lambda) \) under preference parameter \( \tilde{\lambda} \) provided \( h(\sigma, \tilde{\lambda}) \) and \( g(\sigma) \) intersect at some point in the interval \((\sigma, \bar{\sigma})\). This condition is certainly satisfied: both functions are continuous in \( \sigma \), \( h(\sigma_n^*(\lambda), \tilde{\lambda}) > h(\sigma_n^*(\lambda), \lambda) \geq g(\sigma_n^*(\lambda)) \), and by Assumption 1, \( h(\sigma) < g(\sigma) \).
Given that it is responsive, the equilibrium \( \tilde{\sigma}_n^* \) is a solution of \( h(\sigma, \tilde{\lambda}) = g(\sigma) \). Suppose that contrary to our claim, \( \tilde{\sigma}_n^* > \sigma_n^*(\lambda) \). Then
\[
g(\tilde{\sigma}_n^*) = h(\tilde{\sigma}_n^*, \tilde{\lambda}) \geq h(\sigma_n^*(\lambda), \tilde{\lambda}) > h(\sigma_n^*(\lambda), \lambda) = g(\sigma_n^*(\lambda))
\]
which contradicts that \( g \) is decreasing. This completes the proof of the subclaim.

Fix the preference parameter equal to \( \tilde{\lambda} \). By inequality (30), for \( n \) large enough, the debtor’s payoff under the offer \( x_n(\tilde{\lambda}) \) is bounded above by \( \frac{\hat{\delta} + \varepsilon}{2} (1 - x_n(\tilde{\lambda})) \). Observe that since the equilibrium when the offer \( x_n(\lambda) \) is made to creditors with preferences \( \tilde{\lambda}, \tilde{\sigma}_n^* \), satisfies \( \tilde{\sigma}_n^* \leq \sigma_n^*(\lambda) \), then
\[
E \left[ R|A, \tilde{\lambda}, n, x_n(\lambda) \right] Pr \left( A|\tilde{\lambda}, n, x_n(\lambda) \right) \geq E \left[ R|A, \lambda, n, x_n(\lambda) \right] Pr \left( A|\lambda, n, x_n(\lambda) \right) = \hat{\delta}.
\]
Thus the debtor obtains a payoff of at least \( \hat{\delta} (1 - x_n(\lambda)) \) by offering \( x_n(\lambda) \). Let \( \Delta_n \) be the utility gain from offering \( x_n(\lambda) \) in place of \( x_n(\tilde{\lambda}) \). Evaluating,
\[
\Delta_n \geq \hat{\delta} (1 - x_n(\lambda)) - \frac{\varepsilon + \hat{\delta}}{2} (1 - x_n(\tilde{\lambda}))
\]
\[
= \frac{\hat{\delta} - \varepsilon}{2} (1 - x_n(\lambda)) + \frac{\varepsilon + \hat{\delta}}{2} (x_n(\tilde{\lambda}) - x_n(\lambda))
\]
Now, for \( n \) large enough
\[
x_n(\tilde{\lambda}) - x_n(\lambda) \geq x_H(\tilde{\lambda}) - x_H(\lambda) - 2\delta
\]
\[
\geq -(\lambda - \tilde{\lambda}) (H - \sigma) x_H(\lambda)^2 - 2\delta
\]
\[
\geq -\lambda (H - \sigma) x_H(\lambda)^2 - 2\delta
\]
and
\[
1 - x_n(\lambda) \geq 1 - x_H(\lambda) - 2\delta.
\]
Thus
\[
\Delta_n \geq \frac{\hat{\delta} - \varepsilon}{2} (1 - x_H(\lambda) - 2\delta) + \frac{\varepsilon + \hat{\delta}}{2} (-\lambda (H - \sigma) x_H(\lambda)^2 - 2\delta)
\]
\[
= \frac{\hat{\delta}}{2} \left( 1 - x_H(\lambda) - \lambda (H - \sigma) x_H(\lambda)^2 - 4\delta \right) - \frac{\varepsilon}{2} \left( 1 - x_H(\lambda) + \lambda (H - \sigma) x_H(\lambda)^2 \right)
\]
\[
= \frac{\hat{\delta}}{2} \left( 1 - H x_H(\lambda)^2 - 4\delta \right) - \frac{\varepsilon}{2} \left( 1 - x_H(\lambda) + \lambda (H - \sigma) x_H(\lambda)^2 \right)
\]
which is greater than 0 by (29).

\[ \square \]

**Proof of Lemma 9**

Recall that as \( \lambda \to 0 \), \( x_R(\lambda) \to 1/R \). For \( \lambda \) close enough to 0, \( x_L(\lambda) > 1 \) and \( x_H(\lambda) < 1 \).
For all \( \lambda \) small enough the conditions of Lemma 7 are thus satisfied, and \( x_n(\lambda) \to x_H(\lambda) \). So for any \( \varepsilon > 0 \), there exists an \( \Lambda \) and \( N \) such that if \( \lambda \leq \Lambda \) and \( n \geq N \) then \( x_n(\lambda) \leq 1/H + \varepsilon \).
By Lemma 4, the associated probability of acceptance when \( R = L \) approaches 0. The result then follows straightforwardly from expansion (10) of the creditors' payoff.

\[ \square \]

**Proof of Lemma 10**

The creditors' payoff is

\[
p x \left( \lambda E \left[ \frac{1}{n} \sum \sigma_i | A, H, x \right] + (1 - \lambda) H \right) Pr(A|H, x) + (1 - p) x \left( \lambda E \left[ \frac{1}{n} \sum \sigma_i | A, L, x \right] + (1 - \lambda) L \right) Pr(A|L, x) + (1 - Pr(A|x)).
\]

Now,

\[
\frac{\partial}{\partial x} E \left[ \frac{1}{n} \sum \sigma_i | R, A \right] Pr(A|R) = \frac{\partial}{\partial x} \left( \int_{\sigma^*} \ldots \int_{\sigma^n} \frac{1}{n} \sum \sigma_i dF(\sigma_i | R) \ldots dF(\sigma_n | R) \right) = \frac{\partial}{\partial x} \left( (1 - F(\sigma^* | R))^n \int_{\sigma^*} \sigma dF(\sigma | R) \right) = -\frac{\partial \sigma^*}{\partial x} f(\sigma^* | R) \left( (n - 1) (1 - F(\sigma^* | R))^{n-2} \int_{\sigma^*} \sigma dF(\sigma | R) + (1 - F(\sigma^* | R))^{n-1} \sigma^* \right).
\]

(For the particular case of a unanimity voting rule, it is straightforward to show that when a responsive equilibrium exists it is unique. The derivative \( \frac{\partial \sigma^*}{\partial x} \) is then easily shown to be well-defined.)
Since
\[ \frac{\partial}{\partial x} \Pr(A|R) = \frac{\partial}{\partial x} (1 - F(\sigma^*|R))^n = -n \frac{\partial \sigma^*}{\partial x} f(\sigma^*|R) (1 - F(\sigma^*|R))^{n-1}, \]
then
\[ \frac{\partial}{\partial x} \left[ \frac{1}{n} \sum \sigma_i | R, A \right] \Pr(A|R) = \left( \frac{n-1}{n} \int_{\sigma^*}^{\sigma} \sigma dF(\sigma|R) + \frac{1}{n} \sigma^* \right) \frac{\partial}{\partial x} \Pr(A|R) \]

For \( R = H, L, \) define \( I^R \) as
\[ I^R = \int_{\sigma^*}^{\sigma} \sigma' dF(\sigma'|R) \]

Note that \( I^R > \sigma^* \) for \( R = L, H \). Define
\[ C = E \left[ \lambda \sum \sigma_i + (1 - \lambda)R|A \right] \Pr(A|x), \]
the total expected payoff to creditors obtained in states in which the offer \( x \) is accepted.

The derivative of the creditor’s payoff with respect to \( x \) is then
\[
C + \frac{\partial}{\partial x} \Pr(A|H) p \left( \lambda \frac{n-1}{n} I^H x + \frac{\lambda}{n} \sigma^* x + (1 - \lambda) H x - 1 \right) \\
+ \frac{\partial}{\partial x} \Pr(A|L) (1 - p) \left( \lambda \frac{n-1}{n} I^L x + \frac{\lambda}{n} \sigma^* x + (1 - \lambda) L x - 1 \right)
\]
\[
= C + \frac{\partial}{\partial x} \Pr(A|H) p \left( u(\sigma^*, H, x) - 1 + \lambda \frac{n-1}{n} (I^H - \sigma^*) x \right) \\
+ \frac{\partial}{\partial x} \Pr(A|L) (1 - p) \left( u(\sigma^*, L, x) - 1 + \lambda \frac{n-1}{n} (I^L - \sigma^*) x \right)
\]

Now, from the equilibrium condition (3):
\[
\frac{\partial \Pr(A|H)}{\partial x} = \frac{f(\sigma^*|H) (1 - F(\sigma^*|H))^{n-1}}{f(\sigma^*|L) (1 - F(\sigma^*|L))^{n-1}} = \frac{(1 - p) (1 - u(\sigma^*, L, x))}{p (u(\sigma^*, H, x) - 1)}.
\]

Thus the derivative of the creditors’ payoff with respect to \( x \) is
\[
C + \frac{\partial}{\partial x} \Pr(A|H) p \lambda \frac{n-1}{n} (I^H - \sigma^*) x + \frac{\partial}{\partial x} \Pr(A|L) (1 - p) \lambda \frac{n-1}{n} (I^L - \sigma^*) x \geq C.
\]
**Proof of Lemma 11**

From Lemmas 8 and 6, we know that the debtor’s optimal offers and associated acceptance probabilities satisfy

\[
\sup_{\Lambda, N} \inf_{\lambda \leq \Lambda, n \geq N} x_n(\lambda) - x_H(\lambda) > 0
\]

\[
\sup_{\Lambda, N} \inf_{\lambda < \Lambda, n \geq N} \Pr(A|\lambda, n, x_n(\lambda)) > 0.
\]

Define a new set of offers by

\[
\hat{x}_n(\lambda) = \frac{1}{2} \left( x_n(\lambda) + x_H(\lambda) \right).
\]

Thus

\[
\sup_{\Lambda, N} \inf_{\lambda \leq \Lambda, n \geq N} \hat{x}_n(\lambda) - x_H(\lambda) > 0
\]

and so by Lemma 6 (with \(\lambda_0 = 0\)), it follows that

\[
\sup_{\Lambda, N} \inf_{\lambda < \Lambda, n \geq N} \Pr(A|\lambda, n, \hat{x}_n(\lambda)) > 0.
\]

Since the acceptance probabilities are positive, we are in a responsive voting equilibrium, and Lemma 10 implies that there exists an \(\varepsilon > 0\) such that for all \(n\) large enough and \(\lambda\) small enough,

\[
\frac{\partial}{\partial x} \Pi_{1,n}(x_n(\lambda); \lambda) \geq \varepsilon.
\]

It follows that

\[
\Pi_{1,n}(x_n(\lambda); \lambda) \geq \Pi_{1,n}(x_H(\lambda); \lambda) + (x_n(\lambda) - \hat{x}_n(\lambda)) \Pr(A|\lambda, n, \hat{x}_n(\lambda))
\]

\[
= \Pi_{1,n}(x_H(\lambda); \lambda) + \frac{\varepsilon}{2} (x_n(\lambda) - x_H(\lambda)).
\]

By Lemma 5, for any \(\lambda\) the offer \(x_H(\lambda)\) is rejected with probability approaching 1 as the number of creditors grows large. So

\[
\sup_{\Lambda, N} \inf_{\lambda \leq \Lambda, n \geq N} \Pi_{1,n}(x_H(\lambda); \lambda) = 1.
\]

So from expression (11),

\[
\sup_{\Lambda, N} \inf_{\lambda \leq \Lambda, n \geq N} \Pi_{1,n}(x_n(\lambda); \lambda) > 1,
\]

completing the proof. \(\square\)
Proof of Lemma 12

It is sufficient to show that the equilibrium cutoff signal $\sigma_n^*$ is bounded away from $\sigma$ for $n$ large. If the equilibrium is responsive, the equilibrium cutoff value is a solution to $Z(\sigma) = 0$, where $Z$ is as defined by expression (14) in the proof of Lemma 2. But by the assumption $\lambda \sigma + (1 - \lambda) H < 1$, it is immediate that $Z$ is strictly negative in the neighborhood of $\sigma = \sigma$. Thus if the equilibrium is responsive, $\sigma_n^*$ is bounded away from $\sigma$, while if it is unresponsive $\sigma_n^* = \bar{\sigma}$.

Proof of Proposition 2

Suppose for now that a majority voting rule $\alpha$ is such that $x_H(\alpha) < 1$ and $(1 - x_H(\alpha))pH > (1 - x_L(\alpha))E[R]$. Then from Lemma 7, $x_n(\alpha) \rightarrow x_H(\alpha)$, $Pr(A|x_n, H) \rightarrow 1$ and $Pr(A|x_n, L) \rightarrow 0$. So the creditors’ payoff $\Pi_{\alpha,n}(x_n)$ converges to

$$p \left( \frac{\lambda E[\sigma|H]}{\lambda \sigma_H(\alpha) + (1 - \lambda)H} \right) + (1 - p)$$

From Lemma 12, the creditors’ payoff converges to 1 under unanimity for $\lambda$ close to 1. So by the assumption that $E[\sigma|H] > 1$, the creditors’ payoff is strictly greater under the majority rule $\alpha$ provided $\sigma_H(\alpha) < 1$.

To complete the proof, it thus suffices to choose a value of $\alpha$ so that

$$x_H(\alpha) < 1$$

$$\sigma_H(\alpha) < 1$$

$$(1 - x_H(\alpha))pH > (1 - x_L(\alpha))E[R].$$

For $\lambda$ close to 1, $x_R \approx 1/\sigma_R$. Given strict MLRP, $\sigma_H(\alpha) < \sigma_L(\alpha)$ for all $\alpha$. So choosing an $\alpha$ such that $\sigma_H(\alpha) < 1 < \sigma_L(\alpha)$ is feasible, and satisfied the three inequalities above for $\lambda$ close enough to 1. ❑
Proof of Proposition 4

Since the firm is credit constrained, \( I \) is determined by the credit constraint (12) at equality. The derivative of the firm’s investment with respect to a change in the creditors’ payoff \( \Pi^C \) is

\[
\frac{\partial I}{\partial \Pi^C} = \frac{\partial}{\partial \Pi^C} \frac{1 - Q}{1 - Q \Pi^C \mu} K
\]

\[
= \frac{1 - Q}{(1 - Q \Pi^C \mu)^2} K Q \mu
\]

\[
= \frac{Q \mu I}{1 - Q \Pi^C \mu}
\]

First, consider a small increase \( \varepsilon \) in the creditors’ reorganization payoff \( \Pi^C \), accompanied by a reduction of \( (\phi + 1) \varepsilon \) in the firm’s payoff \( \Pi^D \). The effect on the firm’s payoff is

\[
\varepsilon \left( (1 - Q) \rho - 1 + Q \left( \Pi^C + \Pi^D \right) \mu \frac{\partial I}{\partial \Pi^C} - \phi Q \mu I \right)
\]

\[
= \varepsilon Q \mu I \left( (1 - Q) \rho - 1 + Q \left( \Pi^C + \Pi^D \right) \mu \frac{1}{1 - Q \Pi^C \mu} - \phi \right)
\]

(33)

Second, we claim that if (33) is positive for some \( \Pi^C \) and \( \Pi^D \), the same is true for \( \Pi^C + \varepsilon \) and \( \Pi^D - (\phi + 1) \varepsilon \) for any \( \varepsilon > 0 \). To see this, observe that a further increase of \( \varepsilon \) in \( \Pi^C \), accompanied by a decrease of \( (\phi + 1) \varepsilon \) in \( \Pi^D \), increases the term in parentheses by

\[
\varepsilon \left( -\frac{\phi Q \mu}{1 - Q \Pi^C \mu} + \frac{Q \mu}{(1 - Q \Pi^C \mu)^2} \left( (1 - Q) \rho - 1 + Q \left( \Pi^C + \Pi^D \right) \mu \right) \right)
\]

\[
\geq \varepsilon \left( -\frac{\phi Q \mu}{1 - Q \Pi^C \mu} + \frac{Q \mu}{(1 - Q \Pi^C \mu)^2} \phi \left( 1 - Q \Pi^C \mu \right) \right)
\]

\[
= 0.
\]

Third, an exactly parallel argument establishes that if (33) is negative for some \( \Pi^C \) and \( \Pi^D \), the same is true for \( \Pi^C + \varepsilon \) and \( \Pi^D - (\phi + 1) \varepsilon \) for any \( \varepsilon > 0 \).