Using Nash Bargaining to Design Project Management Contracts
Under Cost Uncertainty

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Abstract

In the design of procurement contracts, cost sharing, wherein the contractor receives a fixed fee plus a fraction of his cost, is common when the cost for completing the project is uncertain. We determine the best cost-sharing contract between a risk-neutral project manager and a risk-averse contractor when negotiation proceeds in accord with Nash bargaining. We examine the characteristics of the contract when the contractor can invest to reduce the mean and/or the uncertainty of the project cost. We show that cost-plus contracts dominate fixed-price contracts as well as all other cost-sharing contracts. In order to extend our analysis to the case when the value of the project itself is uncertain, we employ the recent method of embedded Nash bargaining.

Keywords: Project management, Nash bargaining, Fixed-price contracts, Cost-plus contracts.
1 Introduction

The inherent uncertainty in the cost of completing a project in industries such as construction, defense, management consultancy, and hardware and software development compels the project manager to design a contract that provides incentives to the contractor to work efficiently so as to minimize the cost of completing the project, with some regard for risk. In fact, survey studies found in Simister (1994) and in Akintoye and MacLeod (1997) indicate that contract design is the second most important mechanism for managing projects with uncertain cost.\(^1\) Some articles in the project management literature, notably Turner and Simister (2001), suggest the use of specific contract terms, but there is a notable lack of quantitative analysis to justify these suggestions.\(^2\)

The goal of this paper is to investigate cost-sharing contracts between a risk-neutral manager and a risk-averse contractor when the outcome of their negotiation is in accord with Nash bargaining.\(^3\)

Economists have studied contract design for at least 40 years (see McCall (1970)). In considering a multitude of issues, they have produced an enormous literature with some emphasis on moral hazard, adverse selection, signaling, asymmetric information, contracting in a dynamic setting, and contracting in competitive markets. (See the near encyclopedic book by Bolton and Dwatripont (2005) for background and more than 500 references.)

Of particular relevance to our work, Bajari and Tadelis (2001) argue that it is not practical to use a menu (i.e., a list) of contracts to entice the contractor to reveal his hidden information by selecting a contract from the menu (e.g., Laffont and Tirole (1993)). Instead, they argue that it is more practical to use either a fixed-price or a cost-plus contract, especially when managing projects in the construction industry where use of such contracts is a commonplace (see Bajari and Tadelis (2001), Bajari, McMillan, and Tadelis (2009), Bartholomew (1998),

\(^1\)The survey studies found in these two articles reveal that, after insurance, project contract design is the most common mechanism for managing projects with uncertain cost. Insurance is essentially a post-event compensatory mechanism. In contrast, contract design is a proactive approach to managing uncertain projects.

\(^2\)As discussed in Turner and Simister (2001), it is not clear which types of project contracts are effective in the face of uncertainty. By applying some basic concepts of organizational theory such as conflict, cooperation, and transaction cost analysis, they come to suggest two types of cost-based contracts for managing risky projects. The first is the cost-plus contract: the project manager pays the contractor the actual cost incurred plus a margin (either a fixed amount or a percentage of the actual cost). The second is the alliance contract: the project manager works closely with the contractor to improve productivity and reduce cost.

\(^3\)Because Nash bargaining is crucial for this paper and many readers may not be familiar with it, we provide a brief introduction in Appendix 1.
and Ibbs et al. (1986)). Under a fixed-price contract, the manager makes a fixed payment (an amount agreed upon before the project commences) to the contractor upon project completion; under a cost-plus contract, the manager pays a fixed fee plus all relevant costs incurred by the contractor. Clearly, use of a cost-plus contract finds both parties carefully monitoring and auditing the costs.

In this paper, we analyze the full class of cost-sharing contracts under which the payment made to the contractor equals the sum of a fixed fee $\alpha$ plus a fraction $\beta$ of the contractor’s total cost. In addition to fixed-price (when $\beta = 0$) and cost-plus contracts (when $\beta = 1$), this class of contracts includes contracts under which both parties share the uncertain cost (i.e., when $\beta \in (0, 1)$). While cost-sharing contracts have been examined in the economics literature, our exploration of the optimal contract design has three fundamental differences.

First, instead of using the traditional Stackelberg game in which the manager acts as the leader and specifies the contract (that is, he alone selects the pair $(\alpha, \beta)$), and the contractor acts as the follower by responding to the contract offered by the manager (e.g., Kwon et al. (2010a)), we model the process of contract selection as the outcome of Nash bargaining. What is particularly appealing is that the details of the negotiation process in our model – a game in which the players act together when each player’s purpose is to maximize his own utility or gain – do not need to be specified. The possibilities in bargaining are complex beyond description. By comparison, in a non-cooperative game, the rules of the negotiating process are specified in inctuous detail. We view the Nash bargaining method of contract design as more realistic than the “take-it-or-leave-it” contract that results from a Stackelberg game; this realism especially holds in the construction industry. For instance, the empirical study conducted by Bajari et al. (2009) found that nearly half of the private sector building contracts awarded in Northern California during the years 1995-2000 were negotiated.

Second, instead of the manager using cost sharing to entice the contractor to exert more

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4 This class of contracts is known as “incentive” contracts in the contract theory literature (e.g., Hiller and Tollison (1978) and McCall (1970)); in the project management literature, this class is known as the “cost-plus incentive fee” contract (e.g., Al-Subhi (1998)). When $\beta < 1$, the contractor has an incentive to control costs. Hiller and Tollison (1978) investigated the benefit of such cost sharing in defense contracts while Al-Subhi (1998) effected a numerical examination of the impact of $\beta$ on expected utility of the firm and the contractor.

5 For example, one possible set of rules is that the players make alternating take-it-or-leave-it offers, a game made exceedingly popular in Rubinstein (1982).

6 Besides negotiated contracts, Bajari et al. (2009) reported that 18% of the jobs were let on an open bidding process while the rest were awarded on invited bids from pre-selected contractors.
effort to reduce the project cost, in our model the manager and the contractor jointly determine how much to invest and how to share the cost of an investment that reduces the mean and/or the variance of the project cost. Such investments include, for example, process re-engineering, enhanced structural design, and the use of better construction equipment for a construction project.

Third, in addition to project-cost uncertainty, we extend our analysis to the case when both the value of project and the cost of the project are uncertain.

Given a value \( v \) for the project and a contract specified by a pair \((\alpha, \beta)\), the project manager and the contractor each receive an expected utility if the contract is accepted. They each receive a different, and by assumption smaller, utility if the contract is not accepted: this is known as the disagreement payoff. The outcome of Nash bargaining maximizes what we call the *Nash product*: the product of the two net gains from accepting the contract, each measured in the party’s utility. The contract \((\alpha^*, \beta^*)\) that maximizes the Nash product produces the Nash bargaining solution to this problem of contract design. Apart from the work of Gurnani and Shi (2006) wherein Nash bargaining is used to examine supply contracts with uncertain delivery dates, we are unaware of any other paper that employs Nash bargaining to analyze contract design.

Employing the Nash bargaining solution, we determine the fixed fee \( \alpha \) and investment level \( k \) for any cost-sharing contract with a given \( \beta \), and we provide appropriate comparative statics. Then we specifically consider both fixed-price and cost-plus contracts. Not only does the cost-plus contract selected via Nash bargaining dominate all fixed-price contracts, it dominates all cost-sharing contracts. In fact, the cost-plus contract is robust in the sense that it maximizes the total channel profit.

We extend our analysis to examine the case when the value of the project is also uncertain. Applying the embedded Nash bargaining approach developed in Lippman and McCardle (2004, 2011), we determine the unique cost-plus contract that results from embedded Nash bargaining. We show that both parties are made better off by reaching agreement before the value of the project is realized: the project manager pays a lower (expected) amount, and the contractor receives a greater expected utility from avoiding the risk.

This paper is organized as follows. Section 2 presents our model along with the problem formulation. In Section 3, we analyze the cost-sharing contract negotiated via Nash bargaining for any given \( \beta \) in the interval \([0, 1]\). We establish a key result in Section 4: the

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7In Kwon et al. (2010a), additional effort decreases the time until project completion.

8The reader is referred to Cachon (2003) and Tang (2007) for two comprehensive reviews of the supply contract literature.
cost-plus contract produced by Nash bargaining dominates all other cost-sharing contracts. In Section 5 we employ embedded Nash bargaining to extend our model to the case when the value of the project is uncertain. This paper concludes in Section 6.

2 The Model

Consider a project manager (manufacturer) for whom the value of a specific project when complete is $V$. We begin by assuming that the value of the project is fixed, $V = v$, though in Section 5 we allow for $V$ to be random. The project will be carried out by a contractor (supplier). The cost $X$ of completing the project is uncertain. We assume $X \sim \text{Normal}(\mu, \sigma^2)$, the distribution of $X$ is common knowledge, and its realized value is verifiable by both parties. In our static analysis with perfect information, we assume that monitoring is sufficient to eliminate moral hazard. Furthermore, we assume that $\mu$ is sufficiently large relative to $\sigma$ so that $\text{Prob}\{X < 0\}$ is negligible.\(^9\) To capture the differential financial positions of the two parties, we assume the manufacturer is risk-neutral and the supplier is risk-averse.\(^{10}\) In particular, the manufacturer’s utility function $U_m$ is given by

$$U_m(z) = z,$$

and the supplier’s utility function $U_s$ is exponential with coefficient $\lambda$:

$$U_s(z) = 1 - e^{-\lambda z}.$$

Exponential utility is the most commonly used model of risk aversion in decision analysis applications (c.f., Corner and Corner 1995).

In this paper, the manufacturer and the contractor select a contract from the class of cost-sharing contracts. The payment $p$ made to the contractor equals the sum of a fixed fee $\alpha$ and a fraction $\beta$ of the contractor’s cost: $p = \alpha + \beta X$, where $\alpha \geq 0$, and $\beta \in [0, 1]$. This class of cost-sharing contracts includes the two most common forms: fixed-price contracts ($\beta = 0$) and cost-plus contracts ($\beta = 1$).\(^{11}\) For ease of exposition we ignore time discounting, though

\(^9\)Similar results can be obtained when $X$ is exponentially distributed. However, we present the analysis only for the normal distribution.

\(^{10}\)This relative risk posture holds when the manufacturer is a much larger company than the supplier. For example, in the Boeing 787 Dreamliner development project (Nolan and Kotha, 2005), Boeing is much larger than its suppliers (e.g., Honeywell).

\(^{11}\)When $\beta \in (0, 1)$, this class of cost-sharing contracts is known in the economics literature as incentive contracts because the contractor has incentive to reduce the project cost. See Al-Subhi (1998) and McAfee and McMillan (1986) for details.
it is an easy matter to include the time value of money in our model. The manufacturer and contractor employ Nash bargaining as the negotiation process to determine the terms \((\alpha, \beta)\) of the contract.

Given \(\alpha \geq 0\) and \(\beta \in [0, 1]\), the expected utility of the manufacturer \(U_m(\alpha, \beta)\) and the expected utility of the supplier \(U_s(\alpha, \beta)\) are

\[
U_m(\alpha, \beta) = E(v - p) = v - \alpha - \beta E(X), \quad \text{and} \\
U_s(\alpha, \beta) = E(1 - e^{-\lambda p}) = E(1 - e^{-\lambda(\alpha - (1 - \beta)X)}).
\]

Before the manufacturer and the supplier negotiate the payment terms, both parties realize that by making an investment of size \(k\) the project cost can be changed from \(X \sim Normal(\mu, \sigma^2)\) to \(Y(k) \sim Normal(e^{-\eta k} \mu, [e^{-\gamma k} \sigma]^2)\) where \(\eta \geq 0\) and \(\gamma \geq 0\). Investment reduces the expected project cost when \(\eta > 0\) and reduces the project cost uncertainty when \(\gamma > 0\). This kind of improvement opportunity occurs in many projects. For example, when asking a consulting firm (supplier) such as Accenture or Deloitte to implement an Enterprise Resource Planning (ERP) system such as SAP or Oracle, many manufacturers have experienced a significant reduction in implementation cost if they invest in process re-engineering projects first (see Al-Mashari and Zairi (2000)). Also, in the product development literature, it is well known that project design can affect 80% of the total product development cost; investment in product design can substantially reduce the total product development cost (see Clark and Fujimoto (1991) and Ullman (1992)).

Including the investment \(k\), the total project cost changes from \(X\) to \(Y(k) + k\). Consequently, the manufacturer’s expected utility \(U_m(\alpha, \beta, k)\) and the supplier’s expected utility \(U_s(\alpha, \beta, k)\) satisfy

\[
U_m(\alpha, \beta, k) = v - \alpha - \beta E(Y(k) + k) = v - \alpha - \beta e^{-\eta k} \mu - \beta k, \quad \text{and} \\
U_s(\alpha, \beta, k) = E(1 - e^{-\lambda(\alpha - (1 - \beta)(Y(k) + k))}) = 1 - e^{-\lambda \alpha \cdot w(\beta, k)}, \quad \text{where} \\
w(\beta, k) = \exp\{\lambda(1 - \beta) \cdot [e^{-\eta k} \mu + k + \frac{\lambda(1 - \beta)}{2} e^{-2\gamma k} \sigma^2]\}. \quad (2.3)
\]

We refer to \(w(\beta, k)\) as the supplier’s expected risk exposure. As \(\mu > 0\), \(w(\beta, k) \geq 1\). Under the cost-plus contract (\(\beta = 1\)), \(w(1, k) = 1\). Note the different attitudes toward the investment \(k\) as evidenced by the utility functions. From (2.1), the manufacturer is only interested in investments that effectively lower the total expected cost inclusive of \(k\). Inspecting (2.2) makes clear that the contractor is interested in investing to reduce both the expected total cost as well as the variance of costs.

If the manufacturer and supplier fail to come to an agreement, the project is not begun, in which case the manufacturer receives utility 0 and the supplier receives utility \(U_s(0) = 6\).
\[ 1 - e^{-\lambda_0} = 0. \] The utility pair \((d_m, d_s) = (0, 0)\) is referred to as the *disagreement point*.\(^1\)

Any contract \((\alpha, \beta)\) with \(\alpha \geq 0\) and \(\beta \in [0, 1]\) is feasible. The goal of Nash bargaining is to find a mutually agreeable, feasible outcome.

As described in Appendix 1, the *Nash bargaining solution* is the unique feasible outcome which satisfies four axioms. It results in the outcome that maximizes the product of the respective distances in utilities to the disagreement point. We proceed by fixing the cost-sharing proportion \(\beta\) and solving for the associated fixed fee \(\alpha\) and investment \(k\). We then solve for \(\beta\).

Given \(\beta \in [0, 1]\), the manufacturer and the supplier solve the following problem that maximizes the Nash product \(N(\alpha, \beta, k)\):

\[
N^*(\beta) = \max_{\alpha, k \geq 0} N(\alpha, \beta, k) = \max_{\alpha, k \geq 0} U_m(\alpha, \beta, k) \cdot U_s(\alpha, \beta, k),
\]

(2.4)

where \(U_m(\alpha, \beta, k)\) and \(U_s(\alpha, \beta, k)\) are given in (2.1) and (2.2).

### 2.1 Condition for Participation

Fix \(\beta \in [0, 1]\). Before we analyze the Nash bargaining contract \(p^*_\beta = \alpha^*_\beta + \beta (Y(k^*_\beta) + k^*_\beta)\), where \((\alpha^*_\beta, k^*_\beta)\) is the optimal solution to the problem defined in (2.4), we establish a simple sufficient condition that ensures both parties elect to participate in the project.

**Proposition 1** Fix \(\beta \in [0, 1]\). If the project value \(v > \mu + \frac{1}{2}(1 - \beta)^2 \sigma^2\), then there exists a non-negative solution \((\alpha^*_\beta, k^*_\beta)\) that has \(U_m(\alpha^*_\beta, \beta, k^*_\beta) > 0\) and \(U_s(\alpha^*_\beta, \beta, k^*_\beta) > 0\).

Obtaining the condition for participation as stated in Proposition 1 is straightforward. Suppose no investment is made so that \(k = 0\). It follows from (2.2) that the supplier’s certainty equivalent of the project cost is \((1 - \beta)\mu + \frac{1}{2}(1 - \beta)^2 \sigma^2\). The supplier having positive expected utility requires \(\alpha > (1 - \beta)\mu + \frac{1}{2}(1 - \beta)^2 \sigma^2\). The manufacturer has positive expected utility if and only if \(v > \alpha + \beta \mu\). Putting the two conditions together yields the condition in Proposition 1.

### 3 Nash Bargaining Contracts

Given \(\beta \in [0, 1]\), we now determine the Nash bargaining contract by determining the payment and investment pair \((\alpha^*_\beta, k^*_\beta)\) that solves the problem defined in (2.4). We first establish

\(^{12}\)If either of the parties had an outside alternative in lieu of undertaking this project – for instance if the manufacturer could perform the project in-house for some specified cost – the utility of that outside alternative would be the disagreement point.
several properties of the supplier’s expected risk exposure $w(\beta, k)$ via a pair of lemmas.

**Lemma 1** Fix $\beta \in [0, 1]$. The supplier’s expected risk exposure $w(\beta, k)$ satisfies

$$\frac{d}{dk}(w(\beta, k)) = w(\beta, k) \cdot \lambda(1 - \beta)\left[-\eta \mu e^{-\eta k} + 1 - \lambda(1 - \beta)\gamma \sigma^2 e^{-2\gamma k}\right]. \quad (3.1)$$

Also, $w(\beta, k)$ is convex in the investment $k$, decreasing and convex in $\beta$, decreasing in $\gamma$ and $\eta$, and increasing in $\lambda$ and $\sigma$.

(All proofs are in Appendix 2.)

**Lemma 2** When $\beta = 0$, the investment $k_u$ that minimizes the supplier’s expected risk exposure solves

$$-\eta \mu e^{-\eta k} + 1 - \lambda \gamma \sigma^2 e^{-2\gamma k} = 0. \quad (3.2)$$

When $\sigma = 0$, the investment $k_l$ that minimizes the supplier’s expected risk exposure solves

$$-\eta \mu e^{-\eta k} + 1 = 0, \text{ so that } k_l = \frac{1}{\eta} \log(\eta \mu). \quad (3.3)$$

Moreover, $k_u \geq k_l$.

Of course, we restrict $k_l \geq 0$, so when $\eta$ is small ($\eta < 1/\mu$), $k_l = 0$. We hasten to note that when $\beta = 1$, $k_u = k_l$.

In the next sub-sections, we first determine the optimal fixed fee $\alpha^*_\beta(k)$ given the investment level $k$. We then determine the optimal investment $k^*_\beta$ and retrieve the corresponding fixed fee $\alpha^*_\beta(k^*_\beta)$.

### 3.1 Optimal Fixed Fee $\alpha^*_\beta(k)$

We now determine the optimal $\alpha^*_\beta(k)$ for any given investment $k$ for any $\beta \in [0, 1]$. To begin, differentiate the Nash product $N(\alpha, \beta, k) = U_m(\alpha, \beta, k) \cdot U_s(\alpha, \beta, k)$ given in (2.4) with respect to $k$. Inspecting the terms in (2.1), (2.2), and (2.3) and applying (3.1), we obtain the first-order condition.

**Lemma 3** Given $k$, the fixed fee $\alpha^*_\beta(k)$ that maximizes the Nash product $N(\alpha, \beta, k)$ satisfies

$$e^{\lambda \alpha} + w(\beta, k)[1 + \lambda(v - \alpha - \beta(\mu e^{-\eta k} + k))] = 0. \quad (3.4)$$

The fixed fee $\alpha^*_\beta(k)$ is decreasing in $\beta$ and $\gamma$; it is increasing in $\sigma$ for $\beta < 1$. More importantly, $\alpha^*_\beta(k)$ is decreasing in $k$ over the region $[k_l, k_u]$. 

The comparative static results stated in Lemma 3 are intuitive. As the manufacturer pays a higher proportion of the project cost (i.e., when $\beta$ becomes larger), the manufacturer should reduce the fixed-fee portion of the payment: $\alpha^*_\beta(k)$ is decreasing in $\beta$. Also, as either the investment $k$ increases or the investment becomes more effective (i.e., when $\gamma$ becomes larger), the effective project cost is reduced, leading to a lower fixed fee $\alpha^*_\beta(k)$. Finally, as the cost uncertainty $\sigma$ increases, the Nash bargaining solution in the presence of the risk-averse supplier will entail a higher fixed fee. Note, however, that the change in $\alpha^*_\beta(k)$ is ambiguous in $\lambda$ and $\eta$.

### 3.2 Optimal Investment $k^*_\beta$

As per (2.1) and (2.2), the manufacturer pays the portion $\beta$ of the investment and the supplier pays the remaining portion $1 - \beta$. Nash bargaining implies that the size $k$ of the investment is a joint decision. Using the corresponding fixed fee $\alpha^*_\beta(k)$ that satisfies (3.4), the manufacturer’s expected utility given in (2.1) can be rewritten as

$$U_m(\alpha^*_\beta(k), \beta, k) = v - \alpha^*_\beta(k) - \beta e^{-\eta k} \mu - \beta k,$$

(3.5)

Similarly, the supplier’s expected utility given in (2.2) can be rewritten as

$$U_s(\alpha^*_\beta(k), \beta, k) = 1 - \frac{1}{1 + \lambda(v - \alpha^*_\beta(k) - \beta e^{-\eta k} \mu - \beta k)}.$$

(3.6)

Consequently, the problem of maximizing the Nash product $N(\alpha, \beta, k)$ becomes

$$(P) : \max_{k \geq 0} \left\{(v - \alpha^*_\beta(k) - \beta e^{-\eta k} \mu - \beta k) \cdot \left[1 - \frac{1}{1 + \lambda(v - \alpha^*_\beta(k) - \beta e^{-\eta k} \mu - \beta k)}\right]\right\}. \tag{3.7}$$

Because $x \cdot \left[1 - \frac{1}{1 + ax}\right]$ is increasing in $x$, the solution to problem $(P)$ is the same as the solution to problem $(P')$, where

$$(P') : \min_{k \geq 0} [\alpha^*_\beta(k) + \beta e^{-\eta k} \mu + \beta k]. \tag{3.8}$$

By applying Lemma 3, we can establish the following results.

**Proposition 2** The investment $k^*_\beta$ resulting from Nash bargaining:

1. solves

$$ (1 - \beta) \cdot [-\eta \mu e^{-\eta k} + 1 - \lambda(1 - \beta)\gamma \sigma^2 e^{-2\gamma k}] + \beta \cdot [-\eta \mu e^{-\eta k} + 1] = 0; \tag{3.9} $$
2. is contained in the interval \([k_l, k_u]\), where \(k_u\) and \(k_l\) are given in (3.2) and (3.3), respectively. Furthermore \(k^*_0 = k_u\) (when \(\beta = 0\)) and \(k^*_1 = k_l\) (when \(\beta = 1\));

3. is strictly increasing in \(\sigma\) if \(\gamma > 0\) and \(\beta < 1\), and \(k^*_\beta\) is constant in \(\sigma\) when \(\gamma = 0\) or \(\beta = 1\);

4. is decreasing in \(\beta\) and \(\gamma\) and increasing in \(\lambda\); and

5. is decreasing in \(\eta\) if \(\eta > e/\mu\).

Proposition 2 shows that for any \(\beta \in [0, 1]\), the optimal investment \(k^*_\beta\) is the unique root of equation (3.9) that lies in the region \([k_l, k_u] = [k^*_1, k^*_0]\). The comparative statics of Proposition 2 have the following implications. First, when the cost uncertainty \(\sigma\) increases, \(k^*_\beta\) increases when investment reduces the variance of the cost (i.e., when \(\gamma > 0\)). Second, when the manufacturer covers a higher proportion of the project cost (i.e., as \(\beta\) increases), the adverse impact of the cost uncertainty upon the risk-averse supplier is reduced (while the risk-neutral manufacturer continues to encounter no impact from cost uncertainty); consequently, \(k^*_\beta\) is decreasing in \(\beta\). This result comports with the general intuition that increasing \(\beta\) reduces the contractor’s incentive to control costs.

Next, consider the case when the supplier becomes more risk-averse (as \(\lambda\) increases) or when the cost uncertainty \(\sigma\) increases, statements 4 and 3 reveal that the two parties invest more so as to contain project cost. Finally, observe from statements 4 and 5 that when the investment increases in effectiveness (i.e., when \(\gamma\) increases or \(\eta\) increases for \(\eta > e/\mu\)), it behooves the parties to invest less: \(k^*_\beta\) is decreasing in \(\eta\) and \(\gamma\).

By using Proposition 2 and Lemma 3, we can apply (3.9) and (3.4) to retrieve the associated Nash bargaining fixed fee \(\alpha^*_\beta = \alpha^*_\beta(k^*_\beta)\). Hence, given \(\beta \in [0, 1]\), we can determine the contract payment \(p^*_\beta = \alpha^*_\beta + \beta \cdot [e^{-nk^*_\beta} \mu + k^*_\beta]\).

### 4 Fixed-price vs. Cost-plus Contracts

We now consider the two most common contracts. The fixed-price contract entails \(\beta = 0\): the manufacturer pays a fixed amount to the contractor without regard to the cost of completing the project. The cost-plus contract entails \(\beta = 1\): the manufacturer covers the entire project cost, and the fixed-fee component \(\alpha\) of the payment is the supplier’s profit. In this section, we analyze each of the two contracts based on the corresponding Nash bargaining solutions. Then we compare the Nash product under these two different contracts and determine the conditions under which one type of contract outperforms the other.
4.1 Nash Bargaining Fixed-Price Contract

Under the fixed-price contract the manufacturer pays only the fixed fee $\alpha_0^*$ to the contractor. The contractor covers both the investment $k_0^*$ and the resulting (random) project cost $Y(k_0^*)$. Applying Proposition 2 and Lemma 3, we obtain the following corollary.

**Corollary 1** Under the fixed-price contract, the Nash bargaining solution $(\alpha_0^*, k_0^*)$ has the following properties. First, the fixed fee $\alpha_0^*$ satisfies

$$-e^{\lambda \alpha} + w(0, k_0^*)[1 + \lambda(v - \alpha)] = 0,$$

(4.1)

where $w(0, k_0^*) = \exp\{\lambda \cdot [e^{-nk_0^* \mu} + k_0^* + \frac{1}{2} \sigma^2 e^{-2\gamma k_0^*}]\}$, and the investment $k_0^* = k_u$ and satisfies (3.2). Second, the fixed fee $\alpha_0^*$ is increasing in $\sigma$. Third, the investment $k_0^*$ is increasing in $\sigma$ and $\lambda$, decreasing in $\gamma$, and decreasing in $\eta$ for $\eta > e/\mu$.

Even though the manufacturer is risk-neutral, an increase in $\sigma$ is deleterious because the contractor is risk averse. An increase in $\sigma$ makes both parties worse off as each party absorbs a portion of the loss imposed by the increase in risk. The harm imposed upon the manufacturer is clear: $\alpha_0^*$ increases in $\sigma$. Because Nash bargaining results in an even split of the (utility denominated) surplus, it follows that the harm to the contractor also increases in $\sigma$.

4.2 Nash Bargaining Cost-Plus Contract

Under the cost-plus contract produced by Nash bargaining, in addition to the fixed fee, the manufacturer covers the entirety of the investment and project cost: $p_1^* = \alpha_1^* + (Y(k_1^*) + k_1^*)$. Because the manufacturer covers both the project cost and the investment, the fixed fee $\alpha_1^*$ represents the contractor’s (riskless) net profit. Corollary 2 is obtained by applying Proposition 2 and Lemma 3.

**Corollary 2** Under the cost-plus contract, the Nash bargaining solution $(\alpha_1^*, k_1^*)$ entails $k_1^* = k_l$ as per (3.3) and $\alpha_1^*$ satisfies

$$-e^{\lambda \alpha} + [1 + \lambda(v - \alpha - (e^{-nk_1^* \mu} + k_1^*))] = 0.$$

(4.2)

Both $k_1^*$ and $\alpha_1^*$ are independent of $\sigma$ and $\gamma$. While $k_1^*$ is independent of $\lambda$, $\alpha_1^*$ is decreasing in $\lambda$. Finally, $k_1^*$ is decreasing in $\eta$ when $\eta > e/\mu$. 

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When \( \beta = 1 \), neither party is concerned with risk per se, whence neither the investment \( k_1^* \) nor the fixed fee \( \alpha_1^* \) is affected by \( \sigma \) or \( \gamma \). Because of the confounding of risk aversion with decreasing marginal utility for income, an increase in the contractor’s risk aversion \( \lambda \) has a deleterious effect on the contractor, even though the manufacturer absorbs all of the risk. That is, when \( \beta = 1 \), an increase in \( \lambda \) should be thought of as a decrease in the marginal utility of the contractor, not as an increase in the contractor’s risk aversion. Because of the decrease in the contractor’s marginal utility, the outcome of Nash bargaining finds the contractor’s net profit \( \alpha_1^* \) reduced. (See Dyer and Sarin 1982 for a full discussion.)

### 4.3 Comparisons

Coupling Lemma 2 with Corollaries 1 and 2 reveals that \( k_0^* = k_u \geq k_l = k_1^* \): the risk-averse contractor makes a larger investment \( k_0^* \) under the fixed-price contract even though he pays for it himself. This prompts us to inquire whether the manufacturer offers a larger payment under the fixed-price contract. Recall that the manufacturer’s payment is \( \alpha_0^* \) under the fixed-price contract while its expected value is \( \alpha_1^* + k_1^* + e^{-\eta k_1^*} \mu \) under the cost-plus contract.

**Corollary 3** When \( \sigma > 0 \), the manufacturer’s expected payment under the cost-plus contract is smaller than the payment under the fixed-price contract: \( \alpha_1^* + k_1^* + e^{-\eta k_1^*} \mu < \alpha_0^* \). When \( \sigma = 0 \), the payment is the same under both contracts.

Corollary 3 reveals that by having the risk-neutral manufacturer bear all costs, the manufacturer ends up paying less than when he bears none of the costs: \( \alpha_1^* + k_1^* + e^{-\eta k_1^*} \mu < \alpha_0^* \). This fact coupled with (3.6) makes clear that the contractor is also better off under the cost-plus contract.

**Corollary 4** When \( \sigma > 0 \), both the manufacturer and the contractor experience greater expected utility under the cost-plus contract than under the fixed-price contract.

We complete the comparison between the two contracts by considering the channel profit. When \( \beta = 0 \), the channel profit \( \Pi_0 \) under the fixed-price contract satisfies

\[
\Pi_0 = E(v - k_0^* - Y(k_0^*)) = v - k_0^* - e^{-\eta k_0^*} \mu.
\]

Similarly, the channel profit \( \Pi_1 \) under the cost-plus contract satisfies

\[
\Pi_1 = v - k_1^* - e^{-\eta k_1^*} \mu.
\]

Observe from Lemma 2 that \( k_1^* \) minimizes the function \( k + e^{-\eta k} \mu \).
Corollary 5 Under Nash bargaining, the channel profit $\Pi_1$ under the cost-plus contract is greater than the channel profit $\Pi_0$ under the fixed-price contract.

Together Corollaries 3, 4 and 5 demonstrate that the cost-plus contract dominates, in a robust manner, the fixed-price contract when there is cost uncertainty (i.e., when $\sigma > 0$).

4.4 Optimal Proportion $\beta^*$

Corollary 4 reveals that both parties attain a greater expected utility under the cost-plus contract than under the fixed-cost contract. This result raises a new question: can both parties attain an even greater utility if they share the relevant costs? That is, is there a cost-sharing contract for some $\beta \in (0, 1)$ that outperforms the cost-plus contract? The answer is no.

Proposition 3 The cost-plus contract dominates all cost-sharing contracts: $\beta^* = 1$.

Hence, under Nash bargaining, it is optimal for the manufacturer to bear all costs. This result is consistent with the one obtained by McCall (1970) using a different solution concept in a setting with information asymmetry. He argued that cost-sharing contracts are ineffective because they cannot be used to identify high- and low-cost contractors.

5 Extension: Uncertain Project Value

In addition to uncertain project cost, we now extend our model to the case when the value $V$ of the project to the manufacturer is also uncertain. To ease our exposition, we drop the investment option $k$ (i.e., we set $k \equiv 0$), and we consider the simplest case of uncertainty: $V = v_h$ with probability $q$ and $V = v_\ell$ with probability $(1 - q)$, where $v_h > v_\ell$. To ensure both parties will participate, we assume the condition stated in Proposition 1 holds for the lower value of $V$ so that $v_\ell > \mu + \frac{1}{2}(1 - \beta)^2\sigma^2$.

When $V$ is uncertain, we envision a two-period Nash bargaining model. In period 1, $V$ is unknown, and the manufacturer and supplier attempt to negotiate a contract. If an agreement is reached in period 1, the supplier will begin the project and receive the agreed-upon payment.\footnote{Naturally, the agreement is not contingent upon which of the states obtains because if it were, the problem would reduce to a one-period problem with $c = 0$.} If they fail to agree in period 1, the manufacturer expends some amount of money $c \geq 0$ to determine the value of $V$, where $c$ is given exogenously. In period 2,
$V$ is revealed to both parties, either $V = v_h$ or $V = v_\ell$, and then the two parties again attempt to negotiate a contract. Lippman and McCardle (2004, 2011) suggest embedded Nash bargaining as an approach to solve this two-period Nash bargaining model. In each period, the parties employ Nash bargaining, and the model is solved in a backward dynamic-programming fashion, much like subgame perfection.

We begin our analysis by starting at the end, in period 2, with the parties having failed to reach an agreement in period 1. The manufacturer has already paid $c$ to reveal the value of $V$. Once the value of $V$ is revealed in period 2, results from the previous sections prove that a cost-plus contract is optimal. If the parties enter the Nash bargaining game after $V$ is realized, the supplier will receive the payment $\alpha^*_h + X$ when the realized project value is $v_h$ and $\alpha^*_\ell + X$ when the realized project value is $v_\ell$. From Corollary 2, $\alpha^*_h$ and $\alpha^*_\ell$ satisfy

$$-e^{\lambda \alpha^*_h} + (1 + \lambda(v_h - \alpha^*_h - \mu)) = 0. \quad (5.1)$$

$$-e^{\lambda \alpha^*_\ell} + (1 + \lambda(v_\ell - \alpha^*_\ell - \mu)) = 0. \quad (5.2)$$

The supplier’s (ex-ante) expected period-2 utility equals

$$U^*_s = q(1 - e^{-\lambda \alpha^*_h}) + (1 - q)(1 - e^{-\lambda \alpha^*_\ell}). \quad (5.3)$$

Define $\bar{\alpha}^*$, the expected fixed-fee portion of the payment in period 2, by

$$\bar{\alpha}^* = q\alpha^*_h + (1 - q)\alpha^*_\ell.$$

Let $\alpha_{CE}$ denote the supplier’s certainty equivalent of the period-2 payment so that $1 - e^{-\lambda \alpha_{CE}} = U^*_s$. Hence, $\alpha_{CE} = -\ln(1 - U^*_s)/\lambda$. From Jensen’s inequality, we have

$$1 - e^{-\lambda \alpha_{CE}} = U^*_s = q(1 - e^{-\lambda \alpha^*_h}) + (1 - q)(1 - e^{-\lambda \alpha^*_\ell}) < 1 - e^{-\lambda \bar{\alpha}^*}.$$

Hence, $\alpha_{CE} < \bar{\alpha}^*$.

Similarly, once $V$ is realized in period 2, the manufacturer earns either $v_h - \alpha^*_h - X$ or $v_\ell - \alpha^*_\ell - X$. The manufacturer’s (ex-ante) expected period-2 utility equals

$$U^*_m = q(v_h - \alpha^*_h - \mu) + (1 - q)(v_\ell - \alpha^*_\ell - \mu). \quad (5.4)$$

By considering $U^*_m$ and $U^*_s$ given in (5.4) and (5.3), the disagreement point $(d_m, d_s)$ if bargaining fails in period 1 is

$$(d_m, d_s) = (U^*_m - c, U^*_s). \quad (5.5)$$
Moving now to period 1, we restrict attention to cost-plus contracts and determine the Nash bargaining solution. The fixed-fee portion $\alpha^*$ of the cost-plus contract in period 1 maximizes the Nash product and solves
\[
\max_{\alpha}(\bar{v} - \alpha - \mu - d_m)(1 - e^{-\lambda \alpha} - d_s),
\]
where $\bar{v} \equiv q v_h + (1 - q)v_{\ell}$.

**Proposition 4** When the project value $V$ is uncertain, the two-stage embedded Nash bargaining game in cost-plus contracts results in agreement at the first stage. If $c = 0$, the fixed-fee portion $\alpha^*$ of the cost-plus contract in period 1 lies in the (non-degenerate) interval $[\alpha_{CE}, \bar{\alpha}^*]$. Furthermore, $\alpha^*$ is increasing in $c$ decreasing in $\lambda$.

Proposition 4 asserts that under two-stage embedded Nash bargaining, reaching agreement in period 1 prior to the realization of the project value yields a larger positive Nash product. Observe that the fixed fee $\alpha^*$ in period 1 is smaller than the (ex-ante) expected fixed fee $\bar{\alpha}^*$ in period 2. Moreover, because the investment $k = 0$, the realized project cost is invariant as to when agreement is reached. Therefore, the manufacturer obtains a greater utility by reaching agreement in period 1. Similarly, the supplier obtains a greater utility by reaching agreement in period 1. That $\alpha^*$ is increasing in $c$ makes clear that the contractor employs his ability to disagree in period 1 (and thereby impose an increased cost upon the manager) to extract a larger payment. Conversely, the manager’s ability to disagree in period 1 enables him to extract more favorable terms as the contractor’s risk aversion $\lambda$ increases.

### 6 Conclusions

In this paper a risk-neutral project manager and a risk-averse contractor use Nash bargaining to negotiate a contract when the cost of completing the project is uncertain. We limit consideration to cost-sharing contracts, the class of contracts that is commonly observed in practice. We allow for the possibility of an investment that reduces the mean and/or the variance of the project cost. We found that the cost-plus contract dominates all other cost-sharing contracts: the cost-plus contract maximizes the joint-product expected utility function and it maximizes the channel profit. While cost-plus contracts are effective in our setting, in practice monitoring and auditing costs must be taken into consideration when adopting a specific contract.

The model presented in this paper serves as a starting point for examining project contracts under uncertainty using Nash bargaining. One of the axioms on which the Nash
A bargaining solution is based is symmetry: the ordering of the players does not matter. If symmetry were dropped, then weights \((\delta_1, \delta_2)\) could be introduced that model the relative “bargaining power” of the players (Harsanyi and Selten 1972). The resulting solution maximizes the product of the utility differences raised to the relative power weights. One difficulty from a modeling perspective is that there is no established procedure to measure the relative power of two parties. Instead, we model the manufacturer as risk neutral and the contractor as risk averse - in essence giving to the manufacturer more weight in the bargaining. Methods to measure (at least approximately) a decision maker’s risk attitudes do exist (Clemen and Reilly 2001).

Additional issues that deserve attention include: (1) information asymmetry (e.g., the cost reduction parameters \(\eta\) and \(\gamma\) are unobservable by the manufacturer); (2) joint investment opportunities for increasing the project value; and (3) other types of contracts such as revenue-sharing and profit-sharing contracts.

References


Appendix 1: Nash Bargaining

Because the project management contracts negotiated in this paper are the result of Nash bargaining, we briefly introduce the two-person Nash bargaining solution. (It is not to be confused with the two-person Nash equilibrium of a non-cooperative game.) In a bargaining game, the two players attempt to reach an agreement on how to split an asset of value $V$ (where $V$ can be random). The game is specified by a disagreement point $d = (d_1, d_2)$, measured in utilities, and a closed, convex, and bounded set $F$ of feasible utility allocations which contains $d$. If the two players fail to come to an agreement, player $i$ receives $d_i$, $i = 1, 2$. However, if they agree on a point $(x, y)$ in $F$, then player 1 receives $x$ and player 2 receives $y$. For simplicity, we assume that there is an allocation $(x, y)$ in $F$ for which agreement is strictly better for one of the players than is disagreement and the other player is not worse off. The players are assumed to be rational.

Nash bargaining is an axiomatic approach that employs a set of four axioms. The Nash bargaining machinery posits that a solution $\phi$ is any allocation in $F$ that satisfies these four axioms. Roughly, these axioms require the following. First, $\phi$ is invariant to positive linear transformations of the players’ utilities. Second, $\phi$ is Pareto efficient: there is no point $f$ in $F$ with $f > \phi$. The third axiom, independence of irrelevant alternatives, works as follows. Consider two Nash bargaining games in which the disagreement point is $d$ for both games, the sets of feasible allocations for the two games are $G$ and $F$ where $G$ is a subset of $F$. If $\theta$ is the solution to $(G, d)$ and $\phi$ is the solution to $(F, d)$ and, moreover, $\phi$ is in the set $G$, then $\theta = \phi$. This axiom says that eliminating feasible allocations (from $F$ to form $G$) that would not have been chosen when $F$ is the set of feasible allocations makes no difference in the solution to $G$. The fourth axiom states that if the game is symmetric (i.e., $d_1 = d_2$ and $(x, y)$ is in $F$ if and only if $(y, x)$ is in $F$), then the solution $\phi$ is symmetric: $\phi_1 = \phi_2$.

Nash (1950) proved that there is a unique solution $\phi$ that satisfies these four axioms; moreover, $\phi$ is that allocation in $F$ that maximizes the Nash product over pairs $(x, y)$ in $F$, $(x - d_1)(y - d_2)$. Because axiom 2 states that the solution $\phi$ is efficient, the players always avoid disagreement; instead, they agree to an allocation on the efficient frontier of the feasible set $F$. When the players are both risk-neutral, this results in each of the players garnering exactly one-half of the maximum surplus. That is, if the players are risk neutral and if the value of the asset that the players seek to divide is $v$, then $\phi_1 - d_1 = \phi_2 - d_2$ and $\phi_1 + \phi_2 = v$.

In short, when the negotiation process results in the Nash bargaining solution, as we assume throughout this paper, the outcome is that allocation of $v$ which maximizes the
Nash product. In addition to the one proposed by Nash (1950), there are other solutions to the bargaining problem that rely on different axioms. See Thomson (1994) for discussion and references.

8 Appendix 2: Proofs

Proof of Lemma 1: The proof follows immediately from differentiating $w(\beta, k)$ with respect to $k$, $\beta$, $\gamma$, $\eta$, $\lambda$ and $\sigma$. We omit the details.

Proof of Lemma 2: By Lemma 1 part (2), the supplier’s expected risk exposure $w(\beta, k)$ is convex in $k$ for any $\beta \in [0, 1]$; hence, the first-order condition (3.1) generates a maximum. Setting equation (3.1) equal to 0 reduces to (3.2) and (3.3) when $\beta = 0$ and $\sigma = 0$, respectively. It follows from (3.2) and (3.3) that $k_u$ and $k_l$ must satisfy: $\eta \mu (e^{-\eta k_l} - e^{-\eta k_u}) = \lambda (1 - \beta) \gamma \sigma^2 e^{-2\gamma k_u}$. Because the right-hand side is non-negative, we can conclude $k_u \geq k_l$.

Proof of Lemma 3: The first-order condition (3.4) can be obtained by differentiating $N(\alpha, \beta, k)$ with respect to $\alpha$ and rearranging terms. We omit the details. Differentiating (3.4) with respect to $\beta$ and rearranging terms, one can show that:

$$\frac{d}{d\beta}(\alpha^*(\beta)) = \frac{-w(\beta, k)\lambda [e^{-\eta k} + k] + \frac{d}{d\sigma}(w(\beta, k)) \cdot [1 + \lambda (v - \alpha - \beta (\mu e^{-\eta k} + k))]}{\lambda (e^{\lambda \alpha} + w(\beta, k))}.$$  

For any $\alpha^*_\beta(k) \geq 0$ that satisfies (3.4), the term $[1 + \lambda (v - \alpha - \beta (\mu e^{-\eta k} + k))] \geq 0$. Combining this observation with the result from Lemma 2 that $\frac{d}{d\beta}(w(\beta, k)) < 0$, we conclude that $\frac{d}{d\beta}(\alpha^*_\beta(k)) < 0$. The remainder of the statement can be obtained by using the same approach.

Next, let us differentiate (3.4) with respect to $k$ and rearranging the terms, getting:

$$\frac{d}{dk}(\alpha^*_\beta(k)) = \frac{e^{\lambda \alpha} \lambda (1 - \beta) [-\eta \mu e^{-\eta k} + 1 - \lambda (1 - \beta) \gamma \sigma^2 e^{-2\gamma k}] - \lambda \beta w(\beta, k)[-\eta \mu e^{-\eta k} + 1]}{\lambda (e^{\lambda \alpha} + w(\beta, k))} \tag{8.1}$$

For any $k \in [k_l, k_u]$, we can use the fact that $w(\beta, k)$ is convex in $k$ to show that $[-\eta \mu e^{-\eta k} + 1 - \lambda (1 - \beta) \gamma \sigma^2 e^{-2\gamma k}] \leq 0$ for $k \leq k_u$ and that $[-\eta \mu e^{-\eta k} + 1] \geq 0$ for $k \geq k_l$. Combining these observations along with (8.1), we have shown that $\alpha^*_\beta(k)$ is decreasing in $k$ over the region $[k_l, k_u]$.

Proof of Proposition 2: Differentiate (3.8), the objective function of problem $(P')$, apply (8.1), and simplify terms: we obtain (3.9). To show that $\frac{d}{d\sigma}(k^*_\beta) > 0$ when $\gamma > 0$, differentiate (3.9) with respect to $\sigma$ and apply the implicit function theorem. We can show $\frac{d}{d\sigma}(k^*_\beta) = 0$ when $\gamma = 0$. We omit the details.

Next, by observing that condition (3.9) reduces to condition (3.2) and (3.3) when $\beta = 0$ and $\beta = 1$, respectively, we confirm that $k^*_\beta = k_u$ when $\beta = 0$ and $k^*_\beta = k_l$ when $\beta = 1$. In
this case, we can prove the remainder of the proposition by showing that \( k^*_\beta \) is decreasing in \( \beta \). To do so, we use the fact that \( w(\beta, k) \) is strictly convex in \( k \) to show: (1) both 
\[ [-\eta \mu e^{-\eta k} + 1 - \lambda (1 - \beta) \gamma (1 - \beta) e^{2\gamma k}] < 0 \] 
and \( [-\eta \mu e^{-\eta k} + 1] < 0 \) when \( k < k_l \); and (2) both 
\[ [-\eta \mu e^{-\eta k} + 1 - \lambda (1 - \beta) \gamma (1 - \beta) e^{2\gamma k}] > 0 \] 
and \( [-\eta \mu e^{-\eta k} + 1] > 0 \) when \( k > k_u \). Therefore, the optimal \( k^*_\beta \) that satisfies (3.9) must lie in the region \([k_l, k_u]\) so that \( [-\eta \mu e^{-\eta k} + 1 - \lambda (1 - \beta) \gamma (1 - \beta) e^{2\gamma k}] \leq 0 \) for \( k \leq k_u \) and that \( [-\eta \mu e^{-\eta k} + 1] \geq 0 \) for \( k \geq k_l \). Using this observation, we can differentiate (3.9) with respect to \( \beta \) and apply the implicit function theorem to show that \( \frac{d}{d\beta}(k^*_\beta) < 0 \) for \( k^*_\beta \in [k_l, k_u] \) for any \( \beta \in [0, 1] \). Finally, by differentiating (3.9) with respect to \( \gamma, \sigma, \) and \( \lambda \) and applying the implicit function theorem, we obtain the desired results.

It remains to investigate the comparative static of \( k^*_\beta \) with respect to \( \eta \). To do so, differentiate (3.9) with respect to \( \eta \) and rearrange terms, yielding:

\[
\frac{d}{d\eta}(k^*_\beta) = \frac{(1 - \eta k^*_\beta)\mu e^{-\eta k^*_\beta}}{\eta^2 \mu e^{-\eta k^*_\beta} + 2\lambda (1 - \beta) \gamma \sigma^2 e^{-2\gamma k^*_\beta}}.
\]

Observe from the above equation that \( \frac{d}{d\eta}(k^*_\beta) > 0 \) if and only if \( (1/\eta) > k^*_\beta \). Also, observe from statement (3) and Lemma 3, \( k^*_\beta \geq k_l = \log(\eta \mu)/\eta \). By combining these two observations, we can conclude that \( \frac{d}{d\eta}(k^*_\beta) > 0 \) only if \( \eta < e/\mu \). This completes the proof.

Proof of Corollary 1: When \( \beta = 0 \), Lemma 2 proves that the optimal investment \( k^*_0 \) satisfies (3.2). The supplier covers that investment. To establish the comparative statics, consider statements (4) and (5) in Proposition 2.

Next, when \( \beta = 0 \) and when \( k = k^*_0 \), the first-order condition (3.4) as stated in Lemma 2 reduces to (4.1). To complete our proof, we first differentiate \( w(0, k^*_0) \) with respect to \( \sigma \). By using the chain rule and the fact that \( \frac{d}{d\sigma}(k^*_\beta) > 0 \), we can apply the implicit function theorem to show that \( \frac{d}{d\sigma}[w(0, k^*_0)] > 0 \). Knowing \( \frac{d}{d\sigma}[w(0, k^*_0)] > 0 \), we can differentiate (4.1) with respect to \( \sigma \) and apply the implicit function theorem to show that \( \frac{d}{d\sigma}(\alpha^*_0) > 0 \).

Proof of Corollary 2: When \( \beta = 1 \), the first-order condition (3.9) as stated in Proposition 2 reduces to (3.3). Hence, we can apply Lemma 2 to determine the optimal investment \( k^*_1 = \log(\eta \mu)/\eta \). Next, when \( \beta = 1 \) and when \( k = k^*_1 \), the first-order condition (3.4) in Lemma 2 reduces to (4.2). By noting that \( k^*_1 = \log(\eta \mu)/\eta \) is independent of \( \sigma, \gamma, \) and \( \lambda \), it is easy to check from (4.2) that \( \alpha^*_1 \) is also independent of \( \sigma \) and \( \gamma \). To see that \( \alpha^*_1 \) is decreasing in \( \lambda \), differentiate (4.2) with respect to \( \lambda \), substitute from (4.2), and rearrange terms to get

\[
\frac{\partial \alpha}{\partial \lambda} \lambda^2 (1 + e^\lambda) = e^\lambda (1 - \lambda \alpha) - 1.
\]

The function \( e^x(1 - x) - 1 \) is equal to 0 at \( x = 0 \) and is decreasing, so the right-hand side of the above equation is negative, hence, \( \alpha^*_1 \) is decreasing in \( \lambda \).
We obtain the remainder of the proof by differentiating \( k_1^* \).

**Proof of Corollary 3:** When \( \sigma = 0 \), it is easy to check \( k_1^* = k_0^* \) and \( w(0, k_0^*) = e^{\lambda [e^{-\eta k_0^*}]} \).

Combining this observation with (4.1) and (4.2), it follows that \( \alpha_0^* = \alpha_1^* + k_1^* + e^{-\eta k_1^*} \mu \) when \( \sigma = 0 \). We complete the proof by applying the result from Corollary 1 that \( \alpha_1^* \) is strictly increasing in \( \sigma \) and by applying Corollary 2 that \( \alpha_1^* \) and \( k_1^* \) are independent of \( \sigma \).

**Proof of Corollary 4:** From Corollary 3, when \( \sigma > 0 \), \( \alpha_0^* > \alpha_1^* + k_1^* + e^{-\eta k_1^*} \mu \). Then

\[
U_m(\alpha_0^*, 0, k_0^*) = v - \alpha_0^* < v - (\alpha_1^* + k_1^* + e^{-\eta k_1^*} \mu) = U_m(\alpha_1^*, 1, k_1^*),
\]

and

\[
U_s(\alpha_0^*, 0, k_0^*) = 1 - \frac{1}{1 + \lambda(v - \alpha_0^*)} < [1 - \frac{1}{1 + \lambda(v - \alpha_1^* - e^{-\eta k_1^*} \mu - k_1^*)}] = U_s(\alpha_1^*, 1, k_1^*).
\]

**Proof of Proposition 3:** Observe that for a cost-sharing contract based on any given \( \beta \in [0, 1] \), the corresponding effective payment is:

\[
g(\beta) = \alpha_0^* + \beta [e^{-\eta k_0^*} \mu + k_0^*], \tag{8.2}
\]

Then, by using the argument as presented in Section 5.4., we can complete our proof by showing that \( g(\beta) \) is strictly decreasing in \( \beta \) so that the cost-plus contract yields the lowest effective payment, hence the greatest utility to both the manufacturer and the supplier.

We now show that \( g(\beta) \) is strictly decreasing in \( \beta \). In preparation, let us determine \( \frac{d}{d\beta}(w(\beta, k_0^*)) \) and \( (\frac{d}{d\beta}(\alpha_0^*)) \). First, for any \( \beta \in [0, 1] \), the optimal investment \( k_\beta^* \) satisfies (3.9). By differentiating \( w(\beta, k_\beta^*) \) given in (2.3) with respect to \( \beta \), we can apply (3.9) to show that (after some algebra):

\[
\frac{d}{d\beta} w(\beta, k_\beta^*) = -\lambda w(\beta, k_\beta^*) \cdot [e^{-\eta k_\beta^*} \mu + k_\beta^* + \lambda(1 - \beta)\sigma^2 e^{-2\gamma k_\beta^*} + \beta(-ne^{-\eta k_\beta^*} \mu + 1) \frac{d}{d\beta}(k_\beta^*)]. \tag{8.3}
\]

Second, by considering the optimal investment \( k_\beta^* \) satisfies (3.9), we can differentiate (3.4) with respect to \( \beta \) and then apply the implicit function theorem and (3.9) to show that (after some algebra):

\[
\frac{d}{d\beta} \alpha_0^* = -(e^{-\eta k_\beta^*} \mu + k_\beta^*) - \beta(-ne^{-\eta k_\beta^*} \mu + 1) \frac{d}{d\beta}(k_\beta^*) - \frac{\lambda e^{\lambda \alpha_0^*}(1 - \beta)\sigma^2 e^{-2\gamma k_\beta^*}}{w(\beta, k_\beta^*) + e^{\lambda \alpha_0^*}}. \tag{8.4}
\]

By applying (8.3) and (8.4), we can show that:

\[
\frac{d}{d\beta}(g(\beta)) = \frac{d}{d\beta}(\alpha_0^*) + (e^{-\eta k_\beta^*} \mu + k_\beta^*) + \beta(-ne^{-\eta k_\beta^*} \mu + 1) \frac{d}{d\beta}(k_\beta^*)
\]

\[
= -\frac{\lambda e^{\lambda \alpha_0^*}(1 - \beta)\sigma^2 e^{-2\gamma k_\beta^*}}{w(\beta, k_\beta^*) + e^{\lambda \alpha_0^*}} < 0.
\]
This completes our proof.

**Proof of Proposition 4:** Observe that when \( c = 0 \),

\[
0 = (\tilde{v} - \tilde{\alpha}^* - \mu - d_m) < (\tilde{v} - \alpha_{CE} - \mu - d_m),
\]

and

\[
0 = (1 - e^{-\lambda\alpha_{CE}} - d_s) < (1 - e^{-\lambda\tilde{\alpha}^*} - d_s).
\]

That is, when \( c = 0 \), the Nash product given in (5.6) is positive if and only if \( \alpha \in (\alpha_{CE}, \tilde{\alpha}^*) \).

The Nash bargaining solution is monotone in the disagreement payoff, so \( \alpha^* \) is increasing in \( c \). To check that \( \alpha^* \) is decreasing in \( \lambda \), take the partial derivative with respect to \( \lambda \) of the first-order condition of (5.6) and collect terms to get

\[
\frac{\partial \alpha}{\partial \lambda}[-2\lambda e^{-\lambda\alpha} - \lambda^2 e^{-\lambda\alpha}(\tilde{v} - \alpha - \mu - d_m)] = \frac{\partial d_m}{\partial \lambda}(\lambda e^{-\lambda\alpha})
\]

\[
+ (\tilde{v} - \alpha - \mu - d_m)e^{-\lambda\alpha}(\lambda\alpha - 1) + \alpha e^{-\lambda\alpha} - \frac{\partial d_s}{\partial \lambda}.
\]

The term in brackets on the left is negative. It remains to show that the right-hand side is positive.

Corollary 2 establishes that both \( \alpha^*_h \) and \( \alpha^*_l \) are decreasing in \( \lambda \), from which it follows that both the first term \( (\partial d_m/\partial \lambda)(\lambda e^{-\lambda\alpha}) \) and the last term \( -(\partial d_s/\partial \lambda) \) on the right-hand side are positive. Substitution from the first-order condition reveals that the sum of the middle two terms is positive.