Risk, Risk Aversion and The Capital Asset Pricing Model

FINANCE 352

INVESTMENTS

Professor Alon Brav

Fuqua School of Business
Duke University
Motivation

- Asset prices should be calculated as the present value of expected cashflows to the asset-holder.
- The key inputs are the expected cashflows, the timing of the expected cashflows, and the discount rate(s).
  - With an asset pricing model we can calculate the required discount rate.
  - In this lecture we focus on the derivation of the basic asset pricing model (CAPM) and then, later on, other multi-factor extensions.
A Quick Review…

Recall from Finance 350 that…

- We formed the mean variance frontier using risky investments (the two-asset case).
- We then extended the analysis by introducing a riskless asset.
- We then introduced indifference curves and showed how a risk averse investor and a more risk tolerant individual would choose portfolios from the mean-variance frontier.
- Last, we talked about equilibrium and risk-expected return tradeoffs. That lead to the CAPM’s Security Market Line.
### Assumptions Used to Formally Derive the CAPM

- There is only one period.
- No transaction costs and no taxes.
- All assets are traded in perfectly liquid fractional amounts.
- Perfect competition (i.e., no investor can affect prices).
- Only the mean and variance matters which is true if (i) returns are normally distributed and/or (ii) investors have quadratic utility.
- Short sales are allowed.
- Borrowing and lending at a risk-free rate is possible.
- Everybody agrees on the inputs to MV-analysis (homogeneous expectations).

Should we care about the realism of the model’s assumptions?
Equilibrium

- For every asset $i$ we obtain:
  \[ E(R_i - R_f) = \beta_i E(R_p - R_f), \]
  where
  \[ \beta_i = \frac{\text{Cov}(R_i, R_p)}{\text{Var}(R_p)} \]

- All assets have a linear beta relation to the efficient portfolio
  - The above is true for any mean-variance efficient portfolio!
  - However, it doesn’t tell us anything we can use unless we identify an efficient held portfolio.

- **Major result**: The market portfolio is the tangency portfolio.
Intuition: The Market Portfolio and Equilibrium

- All investors choose their portfolios as in the MV-analysis.
- All investors should then hold a combination of the risk-free asset and the tangency portfolio.
- But if all investors hold the same portfolio of risky assets, then they act like a representative investor.
- As a result, the tangency portfolio must be the market portfolio if the supply and demand of assets for the representative investor are equal.
- The CAPM implies that it is optimal to hold a combination of the risk-free asset and the market portfolio, as characterized by the Capital Market Line (CML)
  \[ E(R_i) = R_f + \sigma_i [E(R_M) - R_f] / \sigma_M \]
- But we also have that the unique risk is diversifiable, and that the market risk (as measured by the beta) is all that matters, giving us the Security Market Line (SML)
  \[ E(R_i) = R_f + \beta_i [E(R_M) - R_f] \]
Figure 1: Expected Return – Standard Deviation Frontier and CML
Figure 2: Expected Return – Beta and the Security Market Line (“SML”)
Extensions

- In the absence of a risk-less assets one can derive the zero-beta CAPM of Black (1972)
  \[ E(R_i) = R_z + \beta_i[E(R_M) - R_z] \]
  where \( R_z \) is the return on a zero-beta portfolio (i.e., where \( \text{Cov}(R_z,R_M) = 0 \))
- Other extensions take into account (i) short-sales restrictions, (ii) different lending and borrowing rates, (iii) taxes, (iv) non-marketable assets, (v) heterogeneous expectations, (vi) non price-taking behavior, and (vii) multiple periods.
Extensions (Continued)

- CAPM with a Liquidity Premium (BKM 9.4)

\[ E(r_i) - r_f = \beta_i \left[ E(r_i) - r_f \right] + f(c_i) \]

\( f(c_i) \) = liquidity premium for security i

\( f(c_i) \) increases at a decreasing rate

- Over the period 1961-1980 average realized returns increase from 0.35% (lowest bid-ask spread) to 1.024% (highest bid-ask spread).
Inputs to the CAPM

- In the CAPM, the expected return on an investment is
\[ E(R_i) = R_f + \beta_i[E(R_M) - R_f], \]
where
- \( E(R_i) \) is the expected return on investment \( i \)
- \( R_f \) is the risk-free rate
- \( \beta_i \) measures the systematic risk (the covariance with the market divided by the variance of the market)
- \( E(R_M) - R_f \) is the market risk premium
Inputs: Risk-free Rate

- Short-term Treasury rates are often used as risk-free rates, but they are not risk-free in *long-term* investments.
- For an investment to be risk-free, it has to have
  - No default risk.
  - No reinvestment risk.
- Risk-free rates in valuation will then depend on when the cash flow is expected to occur, and they will vary across time (a simple approach is instead to match the duration of the analysis to the duration of a default-free bond).
Inputs: Beta

The standard procedure for estimating equity betas is to regress individual stock returns $R_{it}$ against market returns $R_{Mt}$, that is,

$$R_{it} = \alpha_i + \beta_i R_{Mt} + \epsilon_{it},$$

or alternatively, to run the characteristic line regression

$$R_{it} - R_{ft} = \alpha_i + \beta_i (R_{Mt} - R_{ft}) + \epsilon_{it},$$

where the $\alpha_i$s are the intercepts, and the $\beta_i$s are the slopes of the regressions which correspond to the betas of the stock, and the $\epsilon_{it}$ are error terms.
Inputs: Beta (Continued)

- One needs to choose a market index, time period, and return interval.
- It is common to choose monthly data spanning 5 years (i.e., in total 60 observations).
  - Why not use more data? 10 or 20 years?
  - Why not use higher frequency? Weekly or daily data?
  - Shrinkage techniques common.
- Further, one may need to augment the beta as it reflects the firm’s business mix over the period of the regression (not the current mix), and as it reflects the firm’s average financial leverage over the period (rather than the current leverage).
- Merrill Lynch (Shrinkage)
- Forecasting beta as a function of firm size, growth, leverage etc.
Example: Regression with the GM Data From BKM

- A regression in Excel results in the following
  \[(R_{GMt} - R_n) = -2.592 + 1.135(R_{Mt} - R_n),\]
  \[
  \begin{bmatrix}
  -2.592 \\ 1.135
  \end{bmatrix}
  \begin{bmatrix}
  [1.159] \\ [0.309]
  \end{bmatrix}
  \]

  where standard errors are given in parentheses below the point estimates, and the R-square is 57.5%
- Consider the graph on the following slide
  - What are each of these points?
  - How can we read off the estimates of \(\alpha_i\), \(\beta_i\), and \(\varepsilon_{it}\)?
Figure 3: Characteristic Line for GM
Inputs: Risk Premium

- This is what we are after \( \rightarrow [E(RM) - R_f] \)
- It is common to use the historical premium (the premium that stocks have historically earned over risk-free assets).
- Practitioners never seem to agree on the premium:
  - How far back should you go in history.
  - Whether you use T-Bill rates or T-Bond rates.
  - Whether you use geometric or arithmetic averages.

<table>
<thead>
<tr>
<th>Period</th>
<th>Stocks – T-Bills</th>
<th>Stocks – T-Bonds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Arithmetic</td>
<td>Geometric</td>
</tr>
<tr>
<td>1926-2001</td>
<td>8.1</td>
<td>6.2</td>
</tr>
<tr>
<td>1980-2001</td>
<td>8.9</td>
<td>8.0</td>
</tr>
</tbody>
</table>
Inputs: Risk Premium (Continued)

- Historical premia: A risk premium comes with a standard error, and given the annual standard deviation in stock prices of about 20%, the standard error in a historical premium estimated over 25 years is roughly: \( 20\%/\sqrt{25} = 4\% \)

- Risk premia in other markets: Typically from shorter histories (a possible solution is to rely on a World CAPM where the World risk premium is backed out from the U.S. premium).

- Implied risk premia: Risk premia that are “backed out” from discounted cash flow models give lower estimates (about 4% per year compared to 8% per year for the historical risk premium).
Figure 4: Equity Premium Estimates

[Graph showing equity premium estimates over time with two lines: one labeled 'historical 36-year moving average' and the other labeled 'implied from SDF-model.']
This figure provides the time-series of the value-weighted market expected return using individual firm annualized expected returns obtained from Value Line. We calculate for each end of quarter: beginning in 1975 through 2001 (March, June, September, and December) the value-weighted annual return using all expected returns issued within a given quarter. The resulting time-series of quarterly market expectations are given below.

Source: Brav, Lehavy and Michaely (2002)
This figure provides the time-series (in green) of the value-weighted market expected return using individual firm annualized expected returns data obtained from Value Line. We calculate for each end of quarter beginning in 1975 through 2001 (March, June, September, and December) the value-weighted annual return using all expected returns issued within a given quarter. The resulting time-series of quarterly market expectations are given below. The shorter time-series (in blue) is the value-weighted market expected return using individual firm annual expected return data obtained from First Call.

Source: Brav, Lehavy and Michaely (2002)
Equity Premium and Survival

- International equity investors in 1900 had no idea which countries to invest in
  - The U.S. looked promising, but so did Argentina.
  - The U.K. and Germany had far richer histories.
  - Japan was a rising power.
- Investors in 1900 might have chosen any of these countries
- What happened?
  - U.S. (did well), Germany (WW I), Russia (communism), U.K. (did well), Japan (WW II), etc.
- This teaches us an important lesson! If the return on the portfolio of countries is what investors required and expected, then the return in the U.S. was much higher than investors expected.
  - Thus it is unwise to use the experience of the U.S. in the 20th century to calculate the equity premium.
  - Data drawn from the 20th-century U.S. experience suffer from survival bias, see Goetzmann and Jorion (1999).
Figure 7: Real Returns on Global Stock Markets
The Equity Premium Puzzle

- Recall that we defined the equity premium as the expected excess return on stocks in excess of a risk-free investment, \( E(R_M - R_f) \).
- In the U.S., the historical average premium has been about 8% per year, with a standard deviation of 20% per year. The implied premia from discounted cash-flow models give somewhat lower estimates, about 4% per year.
- Standard economic models cannot explain the size of the risk premium.
- This is referred to as the equity premium puzzle.
The CAPM can potentially help us identify mispriced assets.
Let the alpha of an asset be the difference between the expected return minus the expected return as predicted by an asset pricing model.
In the CAPM case, we have
\[ \alpha_i = E(R_i) - R_f - \beta_i[E(R_M) - R_f] \]
• If $\alpha_i > 0$ ($< 0$), the expected return on an asset is too high (low) according to CAPM and the asset is hence underpriced (overpriced).
• The alpha is also referred to as an abnormal or risk-adjusted return.
• However, if $\alpha_i$ is different from zero, then the CAPM is either misspecified or the market is not efficient (a joint hypothesis problem).
  • When we calculate alphas, we cannot tell if it is because of market inefficiency, or simply because CAPM is a bad model! We will come back to this point in our market efficiency lecture.
A Single-Index Model

- Estimating a Mean-Variance frontier can be daunting.
- Suppose, we have N assets to invest in.
  - In order to calculate MV-frontier, we will have to estimate:
    - N variance terms
    - N(N - 1)/2 covariance terms
    - N expected returns
  - For N = 20 (100), we have 230 (5,150) parameters.
- However, we can analyze individual assets in terms of a single-factor model.
A Single-Index Model (Continued)

- Consider the market return as a single driving force:
  \[ R_i - R_f = \alpha_i + \beta_i (R_M - R_f) + \epsilon_i \]

- Given (i) the expected return on market portfolio, (ii) the variance of return on market portfolio, and (iii) the alphas, betas, and standard deviations of residuals, we have inputs for the frontier
  \[ E(R_i - R_f) = \alpha_i + \beta_i E(R_M - R_f) \]
  \[ \sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma_{\epsilon_i}^2 \]
  \[ \sigma_{ij} = \beta_i \beta_j \sigma_M^2 \]
A Single-Index Model (Continued)

- The expected return has two components:
  - The unique part, $\alpha_i$
  - The market related part $\beta_i E(R_M - R_f)$.

- The variance has also two components:
  - The unique risk, $\sigma^2_{\epsilon_i}$
  - The market related risk, $\beta_i^2 \sigma^2_M$

- We have a reduction in the number of parameters to estimate.
  - In fact we get ‘only’ $3N+2$ parameters.
  - For $N = 20$ (100), we now have 62 (302) parameters.
A Single-Index Model (Continued)

- This makes it easier to compute inputs for portfolios. The expected return and variance of the portfolio are:

\[
E(R_p) = \sum_{i=1}^{N} w_i E(R_i)
\]

\[
= R_f + \sum_{i=1}^{N} w_i \alpha_i + \sum_{i=1}^{N} w_i \beta_i E(R_M - R_f)
\]

\[
= R_f + \alpha_p + \beta_p E(R_M - R_f)
\]

\[
\sigma^2_p = \beta^2 \sigma^2_M + \sum_{i=1}^{N} w_i^2 \sigma^2_i
\]
The CAPM Debate: Agenda

- We start from earlier studies which find support of CAPM
- We then review the evidence in Fama and French (1992)
  - There is no relation between expected returns and beta.
  - Size and book-to-market describe the cross-section of realized rates of return.
- Finally, we consider critiques of Fama and French, and resurrections of the CAPM.
  - In our next lecture we will explore multi-factor extensions of the simple CAPM.
Empirical Tests and the CAPM Debate

- Recall the major conclusions of the CAPM:
  - The expected return on an asset can be written as
    \[ E(R_i) = R_f + \beta_i[E(R_M) - R_f] \]
    (a linear relation between the expected return and beta)
  - The market portfolio is mean-variance efficient.
  - Only systematic, or non-diversifiable, risk is compensated for (in terms of higher expected returns).
Time-Series versus Cross-Section

- We can evaluate whether $\alpha_i = 0$ in a first-pass regression:
  \[ R_{it} - R_{ft} = \alpha_i + \beta_i(R_{Mt} - R_{ft}) + \epsilon_{it} \]

- We can evaluate whether $\lambda_0 = 0$ and $\lambda_1 = E(R_M - R_f)$ in the second-pass regression:
  \[ \overline{R_i - R_f} = \lambda_0 + \lambda_1 \hat{\beta}_i \]

where $\overline{R_i - R_f}$ is the average excess return, $\hat{\beta}_i$ is the estimated beta, $\lambda_0$ is the intercept, and $\lambda_1$ is the slope coefficient.
1. Evaluations of the CAPM have been performed with various proxies (do they include gold, real estate, human capital, etc.?), and since we do not observe the true return on the aggregated portfolio of invested wealth, we can never really test the CAPM.

2. The linear beta relation holds for any mean-variance efficient portfolio (and, furthermore, the portfolio is efficient only if the beta relation holds), so all the implications of the CAPM boil down to the assertion that the market portfolio is mean-variance efficient, and the CAPM may therefore not be testable.
Empirical Tests

- The two versions of the CAPM:
  - Risk-free borrowing and lending as in Sharpe (1964)–Lintner (1965)
    \[ E(R_i) = R_f + \beta_i [E(R_M) - R_f] \]
  - Restricted borrowing and lending as in Black (1972)
    \[ E(R_i) = R_z + \beta_i [E(R_M) - R_z] \]
Empirical Tests (Continued)

- If expected returns and betas were known, we could just plot expected returns versus betas.
- However, we have to form estimates of them:

\[ R_i - R_f = \lambda_0 + \lambda_1 \hat{\beta}_i \]

- The Sharpe-Lintner version predicts \( \lambda_0 = 0 \) and \( \lambda_1 = \text{E}(R_M - R_f) \).
- We assume that sample versions correspond to population versions (plus some random noise which is typically large for individual securities, but less for portfolios of securities).
- The objective in portfolio construction is to (i) get as much dispersion in true betas as possible, but (ii) measure dispersion precisely.
Some Evidence from Black, Jensen, and Scholes (1972)

- The data are consistent with the prediction of a linear relation between average returns and betas.
- The estimated intercept and slope are about 4% and 13%.
- They are, however, significantly different from 0% and 17% (the predictions given the data in the studied period).
- This indicates a flatter relation than predicted (that is, low beta stocks do better than the model predicts, and high-beta stocks do worse than the model predicts).
Figure 8: Average Excess Returns versus Betas, BJS (1972)

Mean Excess Returns vs. Betas, 1931–65
Block, Jensen, and Scholes (1972)

- Fitted Line, 1931–65
- Variants ±5
- Data 1931–65

Source: Block, Jensen, and Scholes (1972)
Additional Terms

In the empirical test of CAPM, add a new term:

\[ R_i - R_f = \lambda_0 + \lambda_1 \hat{\beta}_i + \lambda_2 X_i + \varepsilon_i \]

where \( \lambda_2 \) is the coefficient on the new variable \( X_i \)

- Evaluate whether \( \lambda_2 = 0 \).
- A number of papers have found that other variables (such as market capitalization) can help “explain” the cross-sectional variation.
- This is counter to the predictions from the theory which says that only beta matters.
Fama and French (1992) argue that the relation between expected returns and betas is completely flat!

- They find that $\lambda_1$ (related to beta) is negative and not significant, whereas $\lambda_2$ (related to size) is significantly different from zero with inverse relation between firm size and realized return.
- They also include the ratio of book value of common equity to its market value (B/M) as an explanatory variable (B/M accounts for a substantial portion of the cross-sectional variation).

This was announced as “Beta is Dead”.
### Average Returns (in % per year) for Portfolios Sorted on Size and Betas

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<tr>
<th></th>
<th>All Firms</th>
<th>Low Beta</th>
<th>High Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Firms</td>
<td>15.0%</td>
<td>16.1%</td>
<td>13.7%</td>
</tr>
<tr>
<td>Small</td>
<td>18.2%</td>
<td>20.5%</td>
<td>17.0%</td>
</tr>
<tr>
<td>Large</td>
<td>10.7%</td>
<td>12.1%</td>
<td>6.7%</td>
</tr>
</tbody>
</table>

Source: Fama and French (1992)
### Average Returns (in % per year) for Portfolios Sorted on Size and B/M Ratios

<table>
<thead>
<tr>
<th></th>
<th>All Firms</th>
<th>Low B/M</th>
<th>High B/M</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Firms</td>
<td>14.8%</td>
<td>7.7%</td>
<td>19.6%</td>
</tr>
<tr>
<td>Small</td>
<td>17.6%</td>
<td>8.4%</td>
<td>23.0%</td>
</tr>
<tr>
<td>Large</td>
<td>10.7%</td>
<td>11.2%</td>
<td>14.2%</td>
</tr>
</tbody>
</table>

Source: Fama and French (1992)
Weaknesses

- Interpretation of the statistical tests: The standard errors in Fama and French (1992) are high, indicating that a wide range of plausible risk premia cannot be rejected.
  - Recall from stats class the meaning of the regression coefficient’s standard error.
- Sample period (“if beta died, it died recently”)
- Survivorship bias in data base for B/M: Firms that had high B/M early in the sample are less likely to survive (and are not included in the sample), whereas firms that do survive show high returns
  - This argument has been examined and shown to be implausible.
Concluding Comments

- The CAPM is a simple model, but very often used in practice!
  - Strong assumptions regarding expected utility, risk aversion, and formation of beliefs! We will see how these are relaxed within Behavioral Finance.
  - Finding inputs is more like an art than science! → This is important, I think, since it tells you a lot about financial economics as a “science.”
- Alphas (over- and under-performance measures) are often used relative to general market exposure.
- It seems that variables such as size and B/M are related to the expected returns (which leads us to a discussion of multi-factor models of asset returns).