Outline

- Background
- Multifactor models and applications
- Arbitrage pricing theory
- Value versus growth
- Losers versus winners
- Concluding comments
Background

- We observe several CAPM “anomalies” (characteristics other than market beta are important in determining expected returns)
- Example, firm size and book-to-market (B/M)
  - Fama and French (1992) find that size and B/M capture the cross-sectional variation in stock returns.
  - Later they have argued that these variables proxy for risk (high B/M firms are in distress and small firms are more sensitive to changes in business conditions).
  - If this is the case, we have three factors that may explain the returns on size and B/M sorted portfolios
Background (Continued)

- Form portfolios of value stocks (stocks with low market value relative to some measure of fundamentals), that is, high
  - Book-to-market (B/M) ratios
  - Cashflow-to-price (C/P) ratios
  - Earnings-to-price (E/P) ratios
- Form also a portfolio of growth stocks, that is, with low B/M, C/P, or E/P ratios.
- Value stocks dramatically outperform growth stocks.
Figure 1: Value versus Growth Strategies
What is the basic idea behind multifactor models?

- The CAPM’s intuition is that the expected return depends on an asset’s beta because it measures the effect of adding a little bit more of an asset on the portfolio variance.
- We have that

\[ E(R_i) = R_f + \beta_i \lambda_M (R_M - R_f) \]

\[ E(R_i) = R_f + \beta_i \lambda_M \lambda_M, \]

where \( \lambda_M \) is the market risk premium (or, the compensation for taking general market risk), and \( \beta_i \) is the market beta of asset \( i \).
What if investors also care about the portfolio performance in recessions?

- Adding a little bit more of an asset may affect how the portfolio does in recessions.
- Investors try to buy stocks that do well in recessions.
- This drives down expected returns

\[ E(R_i) = R_f + \beta_i \lambda \cdot E(R_M - R_f) + \beta_i \lambda_{\text{Recession}} \]

as captured by the exposure to recession risk, \( \beta_i \lambda_{\text{Recession}} \), and the compensation for taking this risk, \( \lambda_{\text{Recession}} \).

- BKM Chapter 10 (page 312): Investors might want to diversify labor income risk. If these hedging demands do not cancel out we should reject the CAPM in favor of a multi-factor extension.
There are two broad classes of models with multiple sources of risk:

- Arbitrage pricing theory (APT) based on no arbitrage.
- Multi-beta models which are based on investor optimization and equilibrium (like the CAPM, and one version is the Intertemporal CAPM, or “ICAPM”).

The two approaches lead to similar expressions for expected returns, and the biggest difference between APT and ICAPM in empirical work is the “inspiration” for the additional factors.
Applications of Multifactor Models

- Reducing the dimensionality of the investment task
- Estimation of cost of equity capital for valuation
- Risk management and hedging
- Performance evaluation
- Style analysis and performance attribution
Arbitrage Pricing Theory (APT): BKM, Chapter 11

- The APT is a multifactor model which recognizes that prices are affected by factors other than the market return
  - It relies on the concept of no arbitrage
  - It is not an equilibrium asset pricing model like the CAPM
- Only a few assumptions are necessary to get the APT
  - For example, APT does not require that everyone is optimizing and solving exactly the same Mean-Variance problem. No reliance on the observability of the market portfolio since we now need to identify whatever systematic factor is of interest to investors.
  - However, like in the CAPM, diversification is crucial and investors are only rewarded for non-diversifiable risk.
Key Ideas Behind the APT

- No arbitrage
  - Repackaging of assets neither creates nor destroys value
  - Securities with the same cash flows have identical prices
  - In a well-functioning financial market, arbitrage opportunities cannot exist (not for very long, at least).
  - Unlike equilibrium models such as CAPM, no arbitrage requires only that there is one intelligent investor in the economy with unlimited supply of capital.
    - Very powerful idea. How reasonable is it?

- All returns have a factor structure (i.e., a few common sources of risk and an asset specific risk)
  - Asset specific risk can be diversified which means that the expected rate of return is entirely determined by the sensitivity to the common factor risk
  - Asset specific risk is uncorrelated with factor risk
A Linear K-Factor Structure

- Assume that asset gross returns can be described by the following statistical model

\[ R_i = \alpha_i + \sum_{k=1}^{K} \beta_{ik} F_k + \epsilon_i \quad i = 1, 2, \ldots, N \]

\[ E(\epsilon_i) = 0, \quad E(\epsilon_i, F_k) = 0, \quad E(\epsilon_i, \epsilon_j) = 0, \]

Where
- \( F_k \) is the common factor \( k \)
- \( \beta_{ik} \) is the sensitivity of the return on asset \( i \) to factor \( k \)
- \( \epsilon_i \) is the asset specific risk
A Linear K-Factor Structure: The key Assumptions

- $E(\varepsilon_i) = 0$
- $E(\varepsilon_i F_k) = 0$, which implies that the factor risk and specific risk are uncorrelated.
- $E(\varepsilon_i \varepsilon_j) = 0$, which implies that the non-factor component is asset specific.
- Other terms for
  - Sensitivity coefficients are “factor loadings” or “characteristics”
  - Asset specific risk are “non-factor” component or “idiosyncratic” return.
It is common to decompose the return into an expected part and an unexpected part:

\[ R_i = E(R_i) + \sum_{j=1}^{k} \beta_{ij} (F_j - E(F_j)) + \varepsilon_i \]

\[ R = E(R) + \sum_{i=1}^{K} \beta_i F_i + \varepsilon \]

With \( E(\varepsilon_i) = 0 \), \( E(F_j) = 0 \), \( E(\varepsilon_i F_j) = 0 \), \( E(\varepsilon_i \varepsilon_j) = 0 \).

The return for a portfolio \( P \) can be written as

\[ R_p = E(R_p) + \sum_{j=1}^{K} \beta_{pj} \tilde{F}_j + \varepsilon_p \]

with \( \beta_{pi} = \sum_{i=1}^{N} w_i \beta_{pi} \), \( E(\tilde{F}_j) = 0 \), \( \varepsilon_p = \sum_{i=1}^{N} w_i \varepsilon_i \)

Where \( w_i \) is the proportion of asset \( i \) in the portfolio.
Variances and Covariances of Portfolios

- If the factors are uncorrelated (orthogonal)
  \[ \text{Cov} (\tilde{F}_i, \tilde{F}_j) = 0, \]

- The variance of asset i and its covariance with assets j are
  \[
  \text{Var} (R_i) = \beta_{i1}^2 \text{Var} (\tilde{F}_1) + \ldots + \beta_{ik}^2 \text{Var} (\tilde{F}_k) + \text{Var} (\varepsilon_i),
  \]
  \[
  \text{Cov} (R_i, R_j) = \beta_{i1}\beta_{j1} \text{Var} (\tilde{F}_1) + \ldots + \beta_{ik}\beta_{jk} \text{Var} (\tilde{F}_k)
  \]

- The factor model helps to identify a relatively small number of key factors which decompose risk into factor-related risk and idiosyncratic risk.
Variances and Covariances of Portfolios

- The idiosyncratic risk of a portfolio is determined by (i) the portfolio weights, and (ii) the idiosyncratic risk of the assets,
  \[ \text{Var} \left( \varepsilon_p \right) = \sum_{i=1}^{N} w_i^2 \text{Var} \left( \varepsilon_i \right) \]
- The effects of diversification is then that the idiosyncratic risk is reduced, for instance, in an equally-weighted portfolio:
  \[ \lim_{N \to \infty} \text{Var}(\varepsilon_p) = \lim_{N \to \infty} \frac{1}{N} \text{Var}(\varepsilon) = 0, \]
- The effects of diversification is then that the factor-related risk is averaged, the idiosyncratic risk is reduced virtually to zero, and the factor model at least holds as an approximation.
- This idea provides the basis for the APT.
Multi-beta representation

- By the principle of no-arbitrage it can then shown that the expected rate of return satisfies a multi-beta representation that is analogous to the CAPM result:

\[ E(R_i) = \lambda_0 + \sum_{k=1}^{K} \beta_{ik} \lambda_k \]

where \( \lambda_0 = R_f \), and \( \lambda_k \)'s are factor risk premia.

- The equality holds for well-diversified portfolios and for most individual assets (see BKM, Chapter 11.3).

- Without loss of generality, we can assume that the factors are traded zero-cost portfolios (for instance, the return on portfolio of small firms minus a portfolio of large firms, \( R_{SMB} = R_S - R_B \)), and we have that \( \lambda_k = \mathbb{E}(F_k) \).

- Key: Expected returns must depend on the covariance (beta) with factors that are common and affect every asset.
A One-Factor Example

- Based on diversification arguments, it should be the case that we cannot construct a well diversified portfolio with zero factor-risk and an excess return different from zero.

- A one-factor model, given by
  \[ R_i = E(R_i) + \beta_{i1}[F_1 - E(F_1)] + \epsilon_i, \]

- leads to the pricing equation
  \[ E(R_i) \approx \lambda_0 + \beta_{i1}\lambda_1 \]
Example (Continued)

- Suppose we have two portfolios A and B

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>E(R_i)</th>
<th>b_i1</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>14.0</td>
<td>1.0</td>
</tr>
<tr>
<td>B</td>
<td>8.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

- Then we observe a third portfolio C, which has a loading of 0.5 and an expected return of 10%
- However, we can form a portfolio P (50% in A and 50% in B) with a loading of 0.5 and an expected return of 11%
- Arbitrage opportunity: Short C and buy P to get $\beta_{i1} = 0$, but a higher expected return than $E(R_{P}) = 8\%$
Figure 2: Pricing Equation of One-Factor Model
Is this really arbitrage?

- It is a slight misnomer as the strategy involves some risk
  - The actual realizations differ from their expectations
  - There is some uncertainty here, but it is assumed that in well-diversified portfolios this is not priced
- A more proper term is maybe “arbitrage in expectations,” because an investor locks in a positive expected payoff, not a positive guaranteed payoff
- Expected returns will be determined such that expected returns in the economy plot on, or close to, the pricing equation
- Notice that we need two assets to create an arbitrage portfolio when we have one risk-factor (i.e., we have only one risk dimension)
- More generally, expected returns will be determined such that expected returns of assets plot on or close to a pricing equation with as many dimensions as there are factors.
A Two-Factor Example

- Consider the structure of a two-factor model
  \[ R_i = E(R_i) + \beta_{i1}[F_1 - E(F_1)] + \beta_{i2}[F_2 - E(F_2)] + \varepsilon_i \]

- If an investor holds a well-diversified portfolio, she needs to be concerned with three attributes: \( E(R_p) \), \( \beta_{p1} \), and \( \beta_{p2} \)

- The equation of a ‘plane’ in the \( E(R_i) \), \( \beta_{i1} \), \( \beta_{i2} \)-space:
  \[ E(R_i) = \lambda_0 + \beta_{i1}\lambda_1 + \beta_{i2}\lambda_2 \]
Example (Continued)

- Suppose that the following three portfolios are observed
  Portfolio A: $15.0 = \lambda_0 + 1.0\lambda_1 + 1.5\lambda_2$
  Portfolio B: $16.4 = \lambda_0 + 2.0\lambda_1 + 0.2\lambda_2$
  Portfolio C: $11.0 = \lambda_0 + 0.5\lambda_1 + 0.5\lambda_2$

- and we can solve for the risk premia (three equations and three unknowns!)
  \[ E(R_i) = 8 + 4\beta_{i1} + 2\beta_{i2} \]
Example (Continued)

- The expected return and risk measures of any portfolio of these three portfolios are given by:
  \[ E(R_P) = w_A E(R_A) + w_B E(R_B) + w_C E(R_C) \]
  \[ \beta_{P1} = w_A \beta_{A1} + w_B \beta_{B1} + w_C \beta_{C1} \]
  \[ \beta_{P2} = w_A \beta_{A2} + w_B \beta_{B2} + w_C \beta_{C2} \]
  \[ 1 = w_A + w_B + w_C \]

- A weighted combination of points on a plane (where the weights sum to one) also lies on the plane.
Figure 3: Pricing Equation of Two-Factor Model
Example (Continued)

- Suppose that a portfolio D with the following properties exists:

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>$E(R_p)$</th>
<th>$\beta_{i_1}$</th>
<th>$\beta_{i_2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>14.0</td>
<td>1.2</td>
<td>1.2</td>
</tr>
</tbody>
</table>

- Form a portfolio which has the same risk as portfolio D

\[ \beta_{P_1} = 1.0w_A + 2.0w_B + 0.5(1 - w_A - w_B) = 1.2 \]
\[ \beta_{P_2} = 1.5w_A + 0.2w_B + 0.5(1 - w_A - w_B) = 1.2 \]

- This results in weights

$w_A = 0.76$, $w_B = 0.21$, and $w_C = 0.03$, and an expected return:

\[
E(R_p) = w_A E(R_A) + w_B E(R_B) + w_C E(R_C)
\]
\[
= 0.76 \times 14.0 + 0.21 \times 15.4 + 0.03 \times 11.0
\]
\[
= 15.2
\]
Example (Continued)

- An arbitrage portfolio can be constructed!

<table>
<thead>
<tr>
<th></th>
<th>Initial cash-flow</th>
<th>E(R_i)</th>
<th>( \beta_{11} )</th>
<th>( \beta_{12} )</th>
<th>End-of-period cash-flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short sell D</td>
<td>+100</td>
<td>-14.0</td>
<td>-1.2</td>
<td>-1.2</td>
<td>-114.0</td>
</tr>
<tr>
<td>Buy P</td>
<td>-100</td>
<td>+15.2</td>
<td>+1.2</td>
<td>+1.2</td>
<td>+115.2</td>
</tr>
<tr>
<td>Results in</td>
<td>0</td>
<td>+1.2</td>
<td>0.0</td>
<td>0.0</td>
<td>+1.2</td>
</tr>
</tbody>
</table>

- All investments and portfolios must be on a plane in \( E(R_p), \beta_{P1}, \beta_{P2} \) - space.
The APT and the CAPM

- Consider a two-factor APT representation
  \[ R_i = E(R_i) + \beta_{i1}[F_1 - E(F_1)] + \beta_{i2}[F_2 - E(F_2)] + \epsilon_i \]
  \[ E(R_i) = R_f + \beta_{i1}\lambda_1 + \beta_{i2}\lambda_2 \]

- Suppose that both factors are priced according to CAPM
  \[ \lambda_1 = \beta_1[E(R_M) - R_f] \]
  \[ \lambda_2 = \beta_2[E(R_M) - R_f] \]

Where
- \( \beta_1 \) and \( \beta_2 \) are based on the factors’ covariation with the market.
The APT and the CAPM (Continued)

- Values can be substituted into the pricing equation for APT:
  \[ E(R_i) = R_f + b_{i1} \beta_1 [E(R_M) - R_f] + b_{i2} \beta_2 [E(R_M) - R_f] \]
  \[ = R_f + (\beta_{i1} \beta_1 + \beta_{i2} \beta_2) [E(R_M) - R_f] \]
  \[ = R_f + \beta_i [E(R_M) - R_f] \]

- Given the assumptions above, the CAPM and the APT are essentially just different approaches to the same problem
  - They are not necessarily contradictory as the CAPM betas are just: \( \beta_i = (\beta_{i1} \beta_1 + \beta_{i2} \beta_2) \).

- APT applies to well diversified portfolios and not necessarily to individual stocks

- With APT it is possible for some individual stocks to be mispriced - not lie on the SML

- APT is more general in that it gets to an expected return and beta relationship without the assumption of the market portfolio

- APT can be extended to multifactor models.
Implementing Factor Models

- To use a factor model, we need estimates of the risk premia (the $\lambda_k$s) and the sensitivities (the $\beta_{ik}$s) which are typically estimated in two steps.
- **Step 1**: If we know the factors, we just run a first-step time-series regression to find the betas of each asset:
  
  \[
  R_t - R_f = \alpha_i + \sum_{k=1}^{K} \beta_{ik} F_t + \epsilon_t \quad t = 1,2,\ldots,T,
  \]

  that is, we obtain estimates of $\alpha_i$ and the $\beta_{ik}$s.
- **Step 2**: Find the price of risk for each factor via a cross-sectional regression of the average return on the estimated betas:
  
  \[
  \overline{R_i} - R_f = \lambda_0 + \sum_{k=1}^{K} \lambda_k \beta_{ik} + \epsilon_i \quad i = 1,2,\ldots,N
  \]

- These estimates can then be used to implement the model.
Implementing Factor Models (Continued)

- An additional task is to *determine* the common factors.
- There are at least three approaches
  1. Factor analysis (or principal component analysis)
  2. Use of macroeconomic variables
  3. Use of firm specific variables
Factor Analysis

- Factor analysis is a purely statistical device to extract common factors from any arbitrary set of variables: Estimate the covariance matrix and construct a set of factors that best explain the covariances.
- The advantage of factor analysis is that you do not need to make a lot of assumptions on the common factors.
- The disadvantage is that the common factors generated shed no (or only little) light on the economic variables that are linked to the factors.
Macroeconomic Variables

- Pre-select some macroeconomic variables which have power in capturing the pattern of returns in the cross-section
- What can be a factor?
  - Macroeconomic variables like consumption, consumption growth, GDP growth, change in oil price, inflation, employment/unemployment.
  - Shifts in investment opportunity set; good or bad news for long-run investment, dividends, term premium.
  - Return on the market portfolio.
Which factors are used in practice?

- Business cycle risk: Unanticipated growth in industrial production.
- Default risk: The default spread, Baa-Aaa, which is a proxy for unanticipated changes in risk premia.
- Time horizon risk: The term premium, return on long bond minus short bond, which is a proxy for unanticipated changes in slope of yield curve.
- Inflation risk: Inflation minus expected inflation, or change in expected inflation.
- Market timing risk: Returns on equally- or value-weighted market portfolios.

- Construction of “factor mimicking” portfolios in this case: Form a well-diversified portfolio that has a beta of 1 on the chosen macroeconomic factor and a beta of zero on all the other factors. Then use the premium associated with the constructed portfolio.
Firm-Specific Sorted Portfolios

- Form factor mimicking portfolios which capture the factors, for instance, the following zero-cost portfolios:
  - Market, \( R_M - R_f \)
  - High-minus-low (HML), \( R_{HML} = R_H - R_L \)
  - Small-minus-big (SMB), \( R_{SMB} = R_S - R_B \)
  - Fama and French (1993)
- The advantage is that it is intuitive.
- The disadvantage is that there is a data mining concern (maybe these factors only pick up mispricing which may be unrelated to the correlation among assets).
The Fama-French (1993) three-factor model and extensions

- The first factor is the excess return on the value weighted market portfolio (RMRF).
- The second factor, SMB, is the return on a zero investment portfolio formed by subtracting the return on a large firm portfolio from the return on a small firm portfolio. The breakpoints for small and large are determined by NYSE firms alone, but the portfolios contain all firms traded on NYSE, AMEX, and NASDAQ exchanges.
- The third factor is the return of another mimicking portfolio, HML, defined as the return on a portfolio of high book-to-market stocks less the return on a portfolio of low book-to-market stocks. The high book-to-market portfolio represents the top 30% of all firms on COMPUSTAT while the low book-to-market portfolio contains firms in the lowest 30% of the COMPUSTAT universe of firms.

\[ r_{p,t} - r_{f,t} = \alpha + \beta \cdot RMRF_t + s \cdot SMB_t + h \cdot HML_t + \epsilon_t \]
The Fama-French (1993) three-factor model and extensions

- Carhart (1997), extends the “Fama and French model” by adding a Momentum “factor”. His factor, PR12, is formed by taking the return on high momentum stocks minus the return on low momentum stocks

\[ r_{pt} - r_{f} = \alpha + \beta \cdot \text{RMRF} + s \cdot \text{SMB} + h \cdot \text{HML} + p \cdot \text{PR}12 + \epsilon \]
Evaluating the Fama and French Model

- With traded assets, we can evaluate the model’s performance in the following time-series regression:

\[ R_{it} - R_f = \alpha_i + \beta_{iM}(R_{Mt} - R_f) + \beta_{iHML}R_{HMLt} + \beta_{iSMB}R_{SMBt} + \epsilon_{it}, \]

and if assets are correctly priced, the $\alpha_i$ should be zero.

- Alternatively, we can assess how much is explained in the cross-section.
Value versus Growth

- Form portfolios of value stocks (stocks with low market value relative to some measure of fundamentals), that is, high
  - Book-to-market (B/M) ratios
  - Cashflow-to-price (C/P) ratios
  - Earnings-to-price (E/P) ratios
- Form also a portfolio of growth stocks (low ratios)
- Value stocks dramatically outperform growth stocks. See Figure 1 on slide 5
  - Two explanations (i) risk and rational, or (ii) irrationally priced securities.
A Risk-Based Explanation

- Fama and French (1993) argue that
  - The above ratios represent a distress factor
  - The factors are present in firms’ earnings and returns
  - Value stocks are more prone to this source of risk than growth stocks
  - Value stocks tend to be “fallen angels”

- They conclude that the high return to small and high B/M stocks is an extra reward for taking on some type of risk.
Figure 4: Average Excess Returns versus Market Betas

Average Excess Returns vs. Market Betas
U.S. Data 1963–91

Observations
Figure 5: CAPM

Average Excess Returns vs. Predicted Excess Returns
U.S. Data 1963–91

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Finance 352, Multi-Factor Models
Figure 6: Market, HML and SMB Model

Average Excess Returns vs. Predicted Excess Returns
U.S. Data 1963-91

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Finance 352, Multi-Factor Models
Alternative (Behavioral) Explanations

- Lakonishok, Shleifer and Vishny (1994) argue that investors overreact to past earnings growth/declines, and that the value premium is consistently positive and unrelated to the business cycle
  - High B/M firms generally have experienced a decline in their earnings over the previous 3 to 5 years
  - The market overreacts to these firms’ poor performance and as a result the price of these firms gets pushed too low
  - The price recovers as the firms do not do as badly as expected, and the firms experience high average returns.
- LSV Asset Management (http://www.lsvasset.com/jsps/)
  - LSV Asset Management specializes in value equity management for institutional investors around the world. LSV currently manages approximately $18 billion in value equity portfolios for approximately 200 clients. LSV was established in 1994 and is based in Chicago, Illinois with a regional office in Norwalk, Connecticut.
Lakonishok, Shleifer and Vishny (1992) suggest that, because of the structure of the money management industry, fund managers are reluctant to buy the bad (value) stocks.

- Value stocks give higher returns on average (but they do also go bankrupt more often)
- Pension funds consistently underperform because funds buy “good” growth stocks which are “glamorous”
- Why do smart money managers not take advantage of this mispricing?
- They claim that the structure of the money management industry makes it costly for them to buy these bad firms as “You don’t get fired for buying IBM”.

Alternative Behavioral Explanations (Continued)
Long Term Losers versus Winners

- A contrarian strategy (DeBondt and Thaler, 1985)
  - Rank stocks on the basis of their performance over the last three to five years
  - Form a winner portfolio of the best 10% performing stocks
  - Form a loser portfolio of the worst 10% performing stocks
  - Look at their returns over the next three to five years
  - The loser portfolio outperforms the winner portfolio
- Again, there are two explanations
  1. Risk: The losers are riskier firms, and their higher returns are just compensation for risk
  2. Overreaction: The winners are firms that investors have become too excited about, and subsequently they realize that they were too optimistic which drives prices down (giving low returns)
Concluding Comments

- We find strategies which seem profitable, maybe just because they are risky and require fair compensation.
  - Hence, it may not be an "anomaly," but rather the case that we have ignored one or several risk factors. The “joint hypothesis” problem which we will talk about in our Market Efficiency class.
- A particular trading strategy may be spurious and a result of data mining (and have no bearing for the future).
  - There are frictions which have to be considered when exploiting an anomaly (transaction costs, liquidity, and taxes). We shall explore these frictions in our Limits on Arbitrage class.