Option Valuation in Corporate Finance

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Options in Corporate Finance
1. Payoffs and Valuation

Call Option Payoffs

Call option gives the holder the right, but not the obligation to purchase at a particular price.

Example: You have a call option on IBM stock gives you the right to purchase at $70 in 3 months. If stock is trading at $80 in 3 months, the payoff is $10.
Options in Corporate Finance
I. Payoffs and Valuation

Call Option Payoffs

At expiration, the call is worth

\[ C = \text{Max}(S-X,0) \]

\( C = \) call payoff
\( S = \) stock price at expiration
\( X = \) exercise price

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Options in Corporate Finance
I. Payoffs and Valuation

- Call Option \( X = 70 \)

Option worth nothing if stock price at expiration \$70 or less

"In-the-money", if stock price at expiration greater than \$70."
Put Option Payoffs

Put option gives the holder the right, but not the obligation to sell at a particular price.

Example: You have a put option on IBM stock gives you the right to sell at $70 in 3 months. If stock is trading at $60 in 3 months, the payoff is $10.

Put Option Payoffs

At expiration, the put is worth

\[ P = \text{Max}(X - S, 0) \]

\( P = \) put payoff
\( S = \) stock price at expiration
\( X = \) exercise price
Options in Corporate Finance
I. Payoffs and Valuation

- Put Option $X=70$

Option worth nothing if stock price at expiration $\geq 70$ or more.

"In-the-money", if stock price at expiration less than $70$.

Combined Position Payoffs

Consider the strategy of buying a call and selling a put option

\[ \text{Strategy 1} = C - P \]
Selling Put  $X=70$

We already know the call payoff.

Selling put is exactly the opposite of buying put.

Buy Call & Sell Put  $X=70$

Combine two payoffs.
Buy Call & Sell Put
\[ X = 70 \]

Net payoffs

Now consider an alternative strategy

Buy one share of the stock and borrow the present value of the exercise price

\[ \text{Strategy } 2 = S - \frac{X}{(1 + r)^T} \]
Borrowing the present value of the exercise price, means that you owe X at the expiration of the option.

Example, if r=10%, X=70, T=1 year, we would borrow $63.64 today.

- Combined Payoff

One share of stock will be worth S.

We owe X in debt.
The payoffs of Strategy 1 are exactly equal to the payoffs of Strategy 2 (no matter what happens to the stock price).

Hence, the today's value of the call minus the put (Strategy 1) must equal the value of the stock minus the present value of the exercise price.
This relation must hold to prevent arbitrage.

\[ c - p = s - \frac{X}{(1 + r)^T} \]

Known as "Put-Call Parity." This relation holds for today's value of the options and stock.
Options in Corporate Finance
II. Corporate Liabilities as Options

- Equity holder has a call option

Equity valuable only if firm is able to repay debt.

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II. Corporate Liabilities as Options

- Payoff to Debt

In one year,

- if firm's value \( (V) \) less than $100 \( (B) \), bondholders get liquidation value \( (V) \).

- if firm's value greater than $100, bondholders get paid B.
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II. Corporate Liabilities as Options

- Bond holder has value of firm and has written a call

Bond holder has given the upside away to the equity holder.

So value of debt today is $V-c$.

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II. Corporate Liabilities as Options

- The Firm as Options

By put-call parity, we can write:

$$V = c + \frac{X}{(1+r)^T} - p$$

But $E=c$, hence
The Firm as Options

So bond holders can also be thought of as:

\[ D = \frac{X}{(1+r)^T} - p \]

Riskfree debt minus a put on firm value

The intuition is simple. Risky debt plus a loan guarantee has the same value as riskfree debt.

Loan guarantee (G) is like insurance - it pays off when firm value less than B.
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II. Corporate Liabilities as Options

- Option definitions

European options cannot be exercised until the time to maturity.

American options can be exercised at any time.

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II. Corporate Liabilities as Options

- Coupon bearing debt

Equity is now analogous to a European call option on a dividend paying stock (the coupon is the dividend).

Note: the call option is on the firm value, $V$. 
Callable debt

Suppose the coupon bond is callable under a schedule of prices $Z$.

Equity now analogous to an American call option on a dividend paying stock where the exercise price changes according to $Z$.

Callable debt

The value of call protection against redemption for the first $T_1 < T$

time periods can be viewed as the difference in value of two American call options.
II. Corporate Liabilities as Options

- Callable debt

The first call can be exercised at any time.

The second call can only be exercised in the last $T - T_1$ periods.

- Loan guarantees

As shown already, simple combinations of options contracts.
Options in Corporate Finance
III. Option Valuation: An Overview

- Black and Scholes

Show it is possible to construct a portfolio involving stock and a risk free asset where the return exactly replicates the option return.

Show how this portfolio's composition changes through time and in response to changing stock prices.

- Black Scholes Option Model

Value of the replicating portfolio (stock and riskfree asset) must equal the value of the option.

If unequal, then arbitrage possible.
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III. Option Valuation: An Overview

- Two state example

\[ S^+ \quad S^- \]

\[ c^+ = \max(0, S^+ - X) \]

\[ c^- = \max(0, S^- - X) \]

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III. Option Valuation: An Overview

- Two state example

What appropriately levered position will perfectly replicate the payoff to the call option at the end of the period.

We will buy \( N \) shares and borrow \( b \) dollars at interest rate \( r \).
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III. Option Valuation: An Overview

- Possible outcomes

\[
\begin{align*}
NS^+ - b(1+r) \\
NS^- - b(1+r)
\end{align*}
\]

NS-b

To perfectly replicate the option, N and b must be chosen so that no matter what the final stock price, the levered position exactly matches the payoff of the call option.

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III. Option Valuation: An Overview

- For N and b it must be true that

\[
\begin{align*}
NS^+ - b(1+r) &= c^+ \\
NS^- - b(1+r) &= c^-
\end{align*}
\]

\[
N = \frac{c^+ - c^-}{S^+ - S^-} \quad b = \frac{S^+c^+ - S^-c^-}{(S^+ - S^-)(1+r)}
\]

This tells us how to solve for N and b.
Numerical example, $X=60$, $r=20\%$

- $S^+=120$
- $S^-=40$
- $c^+=\max(0, 120-60)=60$
- $c^- = \max(0, 40-60)=0$

Substitute into our expressions for $N$ and $b$ to get the levered position to replicate option payoff.

\[
N = \frac{c^+ - c^-}{S^+ - S^-} = \frac{60 - 0}{120 - 40} = 0.75
\]

Hence, we need 0.75 shares.
The amount of borrowing is determined by

\[ b = \frac{S^-c^+ - S^+c^-}{(S^+ - S^-)(1 + r)} = \frac{40(60) - 120(0)}{(120 - 40)(1.2)} \]

\[ b = \$25, \text{ hence } \$25 \text{ is borrowed to finance the purchase.} \]

Value of the call today must be equal to the value of the levered position in the stock -- to prevent arbitrage.

\[ c = NS - b \]

\[ = 0.75(80) - 25 \]

\[ = \$35 \]
Usual approach to liability valuation: different pricing approaches used for

- equity
- long-term debt
- warrants
- convertible securities

In contrast, option-based approach, or "contingent claims analysis", begins with the firm's total capital structure and uses a single evaluation technique to price all of the liabilities.

- CCA methodology takes interactions all liabilities into account.
Consider the two state example again.

\[ V^+ \]
\[ V \]
\[ V^- \]

Assume firm capitalized with equity and zero coupon debt.

Equity:
\[ E^+ = \max(V^+ - B, 0) \]
\[ E^- = \max(V^- - B, 0) \]

Debt:
\[ D^+ = \min(V^+, B) \]
\[ D^- = \min(V^-, B) \]
For any corporate security, we can replicate it with a portfolio of the entire firm's V and borrowings.

Example:

What levered position in the firm will exactly replicate the payoff to risky debt?

- Levered position is $N$ amount of the firm and borrowing $b$ at rate $r$.

\[
\begin{align*}
NV^+ - b(1 + r) \\
NV^- - b(1 + r)
\end{align*}
\]

In order to perfectly replicate the risky debt, $N$ and $b$ must be chosen so that no matter what the final firm value, the levered position exactly matches the payoff of the risky debt.
For this to work, $N$ and $b$ must be chosen such that

$$NS^+ - b(1 + r) = D^+$$
$$NS^- - b(1 + r) = D^-$$

$$N = \frac{D^+ - D^-}{V^+ - V^-} \quad b = \frac{V^- D^+ - V^+ D^-}{(V^+ - V^-)(1 + r)}$$

This tells us how to solve for $N$ and $b$.

To avoid arbitrage,

$$D = NV - b.$$  

The portfolio of $NV - b$ exactly replicates the payoff to the risky debt.
### Options in Corporate Finance
#### IV. Valuing Corporate Liabilities

- Numerical example, debt principal = $100, \( r = 10\% \), \( V = $100 \).

\[
\begin{align*}
V^+ &= $120 \\
V^- &= $80 \\
D^+ &= \min(120, 100) = 100 \\
D^- &= \min(80, 100) = 80 \\
V &= $100
\end{align*}
\]

Substitute into our expressions for \( N \) and \( b \) to get the levered position to replicate risky debt payoff.

\[
N = \frac{D^+ - D^-}{V^+ - V^-} = \frac{100 - 80}{120 - 80} = 0.5
\]

Hence, we need 0.5 shares.
The amount of borrowing is determined by

$$b = \frac{V^- D^+ - V^+ D^-}{(V^+ - V^-)(1 + r)} = \frac{80(100) - 120(80)}{(120 - 80)(1.1)}$$

$$b = -36.36$$, hence the portfolio is one-half of the firm and lending $36.36. This will replicate the payoff to risky debt.

→ Value of the risky debt today must be equal to the value of the portfolio -- to prevent arbitrage.

$$D = NV \cdot b$$

$$= 0.5(100) + 36.36$$

$$= 86.36$$
→ Value of equity is:

\[ E = V - D \]

\[ = 100 - 86.36 \]

\[ = 13.64 \]

Simple example of levered firm instructive but practical value of CCA approach is in complex securities encountered in practice:

- callable coupon debt
- callable convertible debt
- effects of mergers, acquisitions, scale expansions, spinoffs on risky debt and equity
- evaluation of specific bond indentures such as safety covenants, subordination agreements and restrictions on the financing of payouts
CCA model captures:

1. Business risk-(volatility of the rate of return to the firm)—the value and riskiness of the firm's liabilities is partially determined by the riskiness of its assets.

2. Financial risk—(knowledge of firm value \( V \) and the amount and timing of mandatory payout).

CCA model captures:

3. Level of interest rates—The level of interest rates plays a direct role through the specification of \( r \), the riskless rate of interest.

4. Covenants—Callability, convertibility are explicitly recognized.
Present Value Approach to Capital Budgeting often ignores the strategic value of project.

While PV method is sound, it is often implemented incorrectly. Important sources of value are sometimes ignored.

Options are well suited to quantify the "ignored sources of value".

Flexibility

Definition: Options available to management as part of the project.

Sometimes known as operating options.
Example:

Electric utility has choice of building a power plant that:

- (1) Burns oil
- (2) Burns either oil or coal

Plant (1) is cheaper to construct.

Example:

Naive implementation of present value might suggest that plant (1) be constructed.

While (2) costs more, it also provides greater flexibility.

- Management has the ability to select which fuel to use and can switch back and forth depending on energy conditions.
Example:

Proper implementation of present value analysis (or discounted cash flow, DCF) must take the value of this operating option into account.

The following is a catalog of how options could enter the capital budgeting decision.

Input Mix Options:

Electric utility example.

Many operating facilities (such as oil refineries and chemical plants) can use different mixes of inputs to produce same output.
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V. Options and Capital Budgeting

Output Mix Options:

Some facilities can use the same inputs to produce different arrays of outputs.

Stop Options:

Traditional capital budgeting assumes that a project will operate in each year of its lifetime.

However, for many projects with production facilities, it may not be optimal to operate a plant in a given year -- because revenue will not cover variable cost.
Stop Options (con't)

Explicit recognition of this type of flexibility is critical when choosing among alternative production technologies with different ratios of variable-to-fixed costs.

Intensity Options:

Closely related to the Stop Options. Intensity Options is the flexibility to expand or contract the scale of the project.

Examples--

- Change output rate per unit of time.
- Change total length of production run time.
Intensity Options (con't)

- Build production capacity in excess of expected level of output (so it can produce at higher rate if needed)

- Build plant whose physical life exceeds the expected duration of use (thereby providing the firm with the option of producing more by extending life of project).

Intensity Options (con't)

- Choose plant with high maintenance costs relative to construction costs. Management gains the flexibility to reduce the life of the plant and contract the scale of project by reducing expenditures on maintenance.
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V. Options and Capital Budgeting

Termination Options:

In contrast to the temporary Stop Option, the Termination Option is final.

- Particularly important for large capital intensive projects (such as nuclear plants).

- Also important for projects involving new products where their acceptance in the market is uncertain.

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V. Options and Capital Budgeting

Initiation Options:

Just as Termination Option is valuable, so is the option to initiate the project.

Example:

Purchaser of off-shore lease can choose when, if at all, to develop property. This option has significant value.
Example (con't):

If U.S. government required immediate development of leases:

- Prices paid for leases would decline
- Some leases would not be purchased at all.

Example (con't):

Also, true for exploration in general:

If natural resource companies were committed to produce all resources discovered, then they would never explore in areas where the estimated extraction cost exceeded the expected future price at which the resource could be sold.
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V. Options and Capital Budgeting

Sequencing Options:

Important strategic issue is the sequencing of projects.

Example:

Successful marketing of consumer products often requires "brand name."

Suppose a firm is evaluating projects to produce a number of consumer products.

Example (con't):

May be advantageous to implement projects sequentially rather than in parallel.

Pursuing the development of a single product, the firm can resolve some of the uncertainty surrounding its ability to establish "brand name."
Example (con't):

Once resolved, management has the option to proceed or not with the development of the other projects.

If taken in parallel, management would have already spent the resources and the value of the option not to spend them is lost.

Intra vs. Interproject Options:

The sequencing option was an interproject option. That is:

*The sequencing of projects creates options on one or more projects as the direct result of undertaking another project.*

Old-style capital budgeting will miss this option because projects evaluated on stand-alone basis.
Intra vs. Interproject Options (con't):

Ignoring interproject options, could lead to significant undervaluation of projects.

Extreme case example is R&D: The source of value is the options created to undertake other projects.

Interproject options are created whenever management makes an investment that places the firm in a position to use new technology to enter a different industry.

Present Value of Growth Opportunities:

Value of the firm can exceed the market value of the projects currently in place because the firm may have the opportunity to undertake positive NPV projects in the future.

Standard method is to establish the present value of these projects based on anticipated implementation dates.
PVGO (con't):

But this implicitly assumes that the firm is committed to go ahead with the projects!

However, management need not make such a commitment.

Standard valuation methods ignore the option not to go forward.

Under and Overvaluation:

Ignoring options usually causes undervaluation of projects.

Some projects may have little or no option component. However, standard valuation techniques may overvalue these projects by failing to recognize losses in flexibility to the firm that result from implementation.
Financial Flexibility:

Choice of capital structure can affect value of project.

Like operating flexibility, financial flexibility can be measured by the value of the financial options made available to the firm by its choice of capital structure.

Financial Flexibility (con't):

Interaction between financial and operating options can be strong -- especially for long-term investment projects with a lot of uncertainty.

*Option valuation framework is particularly useful to the corporate strategist because it provides an integrative analysis of both operating and financial options associated with the combined investment and financing decisions.*
References:

Mason, Merton, Perold and Tufano, Cases in Financial Engineering.